

# Delta Fluctuations and Option Returns

Yuan Lu  
*The Chinese University of Hong Kong*  
[yuan.lu@link.cuhk.edu.hk](mailto:yuan.lu@link.cuhk.edu.hk)

This version: July 14, 2023

## Abstract

The paper documents a significant, robust positive relationship between delta fluctuations and option returns. As absolute delta fluctuations introduce equal risks to both option buyers and sellers, the return predictability of delta fluctuations cannot be attributed to a rational risk-based explanation. Instead, it stems from the asymmetrical risk perceptions of option buyers and sellers. Our findings suggest that option buyers play a dominant role in pricing the delta fluctuations, while the option sellers are inclined to be more “risk-seeking”. The sellers’ relatively lower awareness of risk contributes to the mispricing of delta fluctuations in options. Furthermore, we explore the time-series and cross-sectional variations of this option mispricing and find that, when limits to arbitrage are higher, the return predictability of delta fluctuations becomes more prominent.

*Keywords:* option delta fluctuation; option mispricing; option demand and supply; risk perception; limits to arbitrage; return predictability.

*JEL Classifications:* G10, G12, G13.

# Delta Fluctuations and Option Returns

This version: July 14, 2023

## Abstract

The paper documents a significant, robust positive relationship between delta fluctuations and option returns. As absolute delta fluctuations introduce equal risks to both option buyers and sellers, the return predictability of delta fluctuations cannot be attributed to a rational risk-based explanation. Instead, it stems from the asymmetrical risk perceptions of option buyers and sellers. Our findings suggest that option buyers play a dominant role in pricing the delta fluctuations, while the option sellers are inclined to be more “risk-seeking”. The sellers’ relatively lower awareness of risk contributes to the mispricing of delta fluctuations in options. Furthermore, we explore the time-series and cross-sectional variations of this option mispricing and find that, when limits to arbitrage are higher, the return predictability of delta fluctuations becomes more prominent.

*Keywords:* option delta fluctuation; option mispricing; option demand and supply; risk perception; limits to arbitrage; return predictability.

*JEL Classifications:* G10, G12, G13.

## 1. Introduction

Option delta is a crucial measure that gauges the sensitivity of option prices to changes in underlying asset prices. It serves as a significant indicator for option investors in designing trading strategies and managing risks. However, option delta is not a constant measure and is influenced by various factors such as underlying asset price, volatility, time to maturity, and risk-free interest rate. Notably, the delta is prone to extreme volatility as the at-the-money option nears maturity. As delta values fluctuate over time, this study aims to investigate how these fluctuations in option delta impact both option pricing and option trading.

Traditional option theory (Black and Scholes, 1973) prices options through dynamic replication with continuous perfect delta hedging. In an ideal frictionless and complete market, such fluctuations in delta have no effect on option pricing. The realities of imperfect markets make it crucial to empirically investigate the effect of delta fluctuations on option pricing.

We measure the monthly delta fluctuations by the average daily absolute delta changes. The measure captures the realized delta volatility, encompassing various aspects of delta sensitivity such as stock price (gamma), stock volatility (vanna), and time (charm). Based on this measure, we investigate how option prices react to these delta fluctuations.

On the one hand, delta neutral trading is a common trading strategy that can remove the impact of stock price movement, in which daily rebalancing delta-hedge positions can eliminate 90% of directional risks (Hull, 2003; Tian and Wu, 2021). Hence, volatile deltas pose risks for both option buyers and sellers as they can make it challenging for both parties to maintain a delta-neutral position. On the other hand, as delta can be understood as an approximate estimate of the probability of profit for option buyers, delta fluctuations introduce uncertainty and risk to these buyers. In a zero-sum game, on the sellers' side, the probability of profit corresponds to one minus delta. For at-the-money options, when deltas fluctuate symmetrically around 50%, the delta fluctuations bring about the same uncertainty to both option buyers and option sellers. If both option buyers and sellers are rational and share a symmetrical perception of risks, the equilibrium prices of options would not be affected by varying levels of uncertainty caused by delta fluctuations. Therefore, the observed relationship between option returns and delta fluctuations provides us with insights into the asymmetry of risk perception between option buyers and sellers, rather than the asymmetry of risk itself.

In our empirical analysis, we first conduct portfolio sorting tests to examine the relationship between delta fluctuations and delta-hedged option returns. Our findings reveal a monotonically increasing relationship between option returns and delta fluctuations, along with the significant profitability of a long-short strategy. We also construct variance risk premiums as our alternative measures to proxy option expensiveness and, consistently, we find option expensiveness monotonically decreases in the delta fluctuations. Additionally, to address the concern that the impacts of delta fluctuations are driven by other characteristics that have been well examined in the previous literature, we control for a bunch of stock and option characteristic in the Fama-Macbeth regressions. We find the relationships of delta fluctuations on option returns are significantly positive in all specifications. This set of consistent results suggests that option buyers take the lead in pricing the delta fluctuations risk, as they tend to lower the purchase price and require a high return premium for their risk taking.

Then, we test why option buyers could affect the option prices. Inspired by the demand-based option pricing model (Garleanu, et al., 2008), as the net buying demand affects option price, we test the relationship between option demands and delta fluctuations. Our findings consistently demonstrate that option demands decrease as delta fluctuations increase, aligning with our earlier results. That is, as option buyers bear the risks associated with delta fluctuations, their lower demand leads to a decrease in option prices and subsequently higher returns.

We have so far examined how option buyers respond to delta fluctuations. At the same time, option sellers face identical risks and might, therefore, be expected to increase the option price accordingly. However, we don't see this pattern and the explanation is what we've mentioned before – their perception of risk differs from that of the buyers. Specifically, the option sellers are less aware of the risk than option buyers.

This prompts a subsequent question – why are option sellers less wary of risks than option buyers? Two possible reasons may underpin this phenomenon: (1) option sellers could be more risk-seeking in nature, and (2) they may face fewer financial restrictions and higher risk tolerance than option buyers. However, the latter one seems implausible, as option sellers are subject to margin requirements. If sellers are handcuffed by financial restrictions, a logical result is that they would demand higher compensation than buyers.

The most convincing explanation is that the option sellers tend to exhibit a relatively higher degree of “risk-seeking” behavior compared to option buyers. Given the flexibility to either buy

or sell options, the “risk-seeking” people may lean towards selling options, as selling an option involves assuming higher risks in comparison to buying an option. The inherent nature of options implies that selling an option carries the potential for unlimited losses that can far exceed the initial premiums received. On the other hand, buying an option entails no additional risks beyond the upfront option premiums. Consequently, option sellers, rather than buyers, are more likely to demonstrate “risk-seeking” behavior. In other words, option buyers tend to be more risk-averse, leading them to exhibit a greater awareness of the risks associated with delta fluctuations.

Therefore, we attribute the positive relationship between delta fluctuations and option returns to the mispricing caused by asymmetrical risk perceptions between option buyers and sellers. Next, we examine the variations in option mispricing over time and across different underlying stocks. According to [Shleifer and Vishny \(1997\)](#), limits to arbitrage hinder arbitrageurs from correcting the mispricing, causing it to become more prominent. To investigate this further, we consider two aspects. Firstly, we analyze the time-series variations in limits to arbitrage by the market sentiment and market uncertainty time series. We find that during periods of high market sentiment and uncertainty, the positive relationship between delta fluctuations and option returns becomes stronger. Secondly, we explore the cross-sectional variations in limits to arbitrage using underlying stock characteristics. Our findings indicate that for stocks with high arbitrage risk, information uncertainty, and transaction costs, the positive relationship between delta fluctuations and option returns is more pronounced.

In summary, our study reveals that asymmetrical risk perceptions between option buyers and sellers contribute to option mispricing, and the presence of limits to arbitrage hampers the correction of such mispricing. These factors ultimately enhance the predictability of option returns based on delta fluctuations.

Our paper is the first empirical study to explore delta fluctuations and their relationship with option returns. One of the clever aspects of using those around at-the-money delta fluctuations for option pricing research is that it introduces risks for both option buyers and sellers. This symmetrical risk nature allows us to quickly verify if any return predictability is driven by option mispricing rather than a fully rational, risk-based explanation. We contribute to the existing literature in several ways. Firstly, we identify a robust and significant positive relationship between delta fluctuations and option returns across various specifications, thereby expanding the literature of option return predictability. Secondly, we explicitly distinguish whether the return predictability

arises from risk or mispricing, shedding light on the underlying mechanisms of option mispricing. Thirdly, we compare the risk perceptions between option buyers and sellers, which is relatively understudied in the previous literature.

The remainder of this paper proceeds as follows. Section 2 reviews the literature. Section 3 demonstrate the motivations and hypotheses developments. Section 4 describes the data and key variables. For delta fluctuations, we introduce how to construct the measure and explain why it bring about equivalent risk for both option buyers and sellers. For delta-hedged option returns, we introduce how to construct the delta-neutral option portfolio to calculate the option return. Section 5 presents the empirical results and possible explanations. Section 6 concludes.

## **2. Literature Review**

### *2.1. Option Demand and Supply*

There is a large body of literature investigating the demand and supply of options. First, most of the existing literature focuses on the risks for the supply side. They suggest writing an option might bear boundless losses that can greatly exceed the initial premiums received whereas buying an option bears no additional risk aside from the upfront option premiums. In the Black-Scholes economy, the risk of selling an option can be perfectly hedged. However, the infeasibility of perfect hedging would result in “model risk” for option writers (Green and Figlewski, 1999). The “model risk” can be attributed to the volatility risk, jump risk from the underly asset (Bakshi, et al., 1997; Bakshi and Kapadia, 2003). Second, there are a strand of literature assessing the relation between option investors and option market makers. Bollen and Whaley (2004) find that changes in implied volatility are directly related to net buying pressure from public order flow. Garleanu, et al. (2008) propose the demand-based option pricing model and explore how net demands from end-users price options with competitive risk-averse dealers who cannot hedge perfectly. Muravyev (2016) decomposes the price impact of trades into inventory risk and asymmetric information components and finds that inventory risk plays a dominant role in option price formation. Most of the demand-based option pricing models are still based on the argument that the option supply side cannot fully hedge their inventories risk and therefore the net demand will affect the option pricing. On the other hand, Christoffersen, et al. (2018) demonstrate that market makers, as liquidity providers in the equity option market, are compensated for the risks and costs of market making, leading to a positive relationship between option return and option

illiquidity. Third, some literature examines why people buy or sell the options. [Leland \(1980\)](#) focuses on the insurance nature of options and characterize those investors who will benefit from purchasing insurance. [Franke, et al. \(1998\)](#) show that investors with low or no background risk tend to sell options on the market portfolio, whereas investors with high background risk buy these options. [Buraschi and Jiltsov \(2006\)](#) takes information heterogeneity into account to explain the dynamics of option volume and the smile.

## *2.2. Behavior Bias, Mispricing and Limits to Arbitrage*

Previous studies has identified a number of so-called anomalies, in which their returns cannot be justified by their systematic risk. [Shleifer and Vishny \(1997\)](#) provides us an insight on how to understand anomalies. They suggest that the first step is to understand the source of noise trading that might generate the mispricing in the first place and examine whether such noise trading is driven by behavioral bias or market friction. The second step is to evaluate the costs of arbitrage in the market as arbitrage will mitigate the mispricing. Stock return anomalies has been widely explored by previous studies, which can be associated with sentiment ([Baker and Wurgler, 2006](#); [Stambaugh, et al., 2012](#)), investor misperceptions and investors misevaluation ([Hirshleifer and Jiang, 2010](#); [Hirshleifer, et al., 2012](#)), financial distress([Avramov, et al., 2013](#)), institutional investors ([Edelen, et al., 2016](#)), biased expectations([Engelberg, et al., 2018](#)), prospect theory ([Barberis, et al., 2021](#))and etc. There are also some risk to arbitrageurs when they arbitrage mispriced stocks, which is so called limits to arbitrage([De Long, et al., 1990](#); [Shleifer and Vishny, 1997](#); [Liu and Longstaff, 2004](#)).

For option market, there are also a growing number of papers examining the option mispricing. [Poteshman \(2001\)](#) investigates the misreaction to information in the options market. [Han \(2008\)](#) examines the option mispricing during high sentiment, with limits to arbitrage. [Jones and Shemesh \(2018\)](#) find that stock option returns are more negative during nontrading periods. They suggest that these nontrading returns cannot be attributed to risk, but instead stem from widespread and persistent option mispricing caused by the incorrect handling of stock return variance during market closure periods. [Muravyev and Ni \(2020\)](#) attribute the day–night return asymmetry to that option prices’ failing to account for the well-known fact that stock volatility is substantially higher intraday than overnight. [Eisdorfer, et al. \(2022\)](#) demonstrate that short-term options exhibit significantly lower returns during months with 4 versus 5 weeks between expiration

dates. This finding implies that the mispricing can be attributed to investor inattention towards the precise expiration date, rather than underlying risk exposures or transaction costs.

### *2.3. Option Return Predictability*

My paper contributes to the expanding literature on option return predictability. [Bakshi and Kapadia \(2003\)](#) investigate the delta-hedged gains and suggest there is a negative market volatility risk premium. [Goyal and Saretto \(2009\)](#) demonstrate a positive relationship between option delta-hedged returns and the misestimation of volatility, measured as the difference between Historical Volatility and Implied Volatility (HV-IV). [Cao and Han \(2013\)](#) find that an increase in idiosyncratic volatility (IVOL) of the underlying stock leads to a decrease in delta-hedged option returns. [Bali and Murray \(2013\)](#) create skewness assets by removing exposure to delta and vega, revealing a negative relationship between skewness asset returns and risk-neutral skewness. [Boyer and Vorkink \(2014\)](#) discover that the ex-ante skewness of an option significantly impacts delta-hedged option returns due to premium compensation for intermediaries bearing investor demand for lottery-like options. [Byun and Kim \(2016\)](#) demonstrate that options on lottery stocks are overvalued, driven by optimism-induced gambling preference. [Huang, et al. \(2019\)](#) find a significant negative relationship between volatility, volatility of volatility, and future delta-hedged option payoffs. [Ramachandran and Tayal \(2021\)](#) investigate the impact of short-sell constraints on the delta-hedged returns of put options on overpriced stocks. [Choy and Wei \(2022\)](#) discover that option buying pressures are higher and future option returns are lower when underlying stocks attract more attention. [Zhan, et al. \(2022\)](#) examine the relationship between option returns and various firm fundamentals and stock characteristics. [Bali, et al. \(2023\)](#) demonstrate that their nonlinear machine learning models generate substantial profits in long-short portfolios of equity options, even after accounting for transaction costs.

### **3. Motivations and Hypotheses Developments**

Option delta measures the sensitivity of option prices to the underlying asset prices, which is an importance indicator for option investors to design and implement their trading strategies, as well as risk management. The delta of an option, however, stands not as a constant measure, but rather as a variable influenced by the underlying asset price, volatility, time to maturity, and risk-free interest rate. Particularly, when the at-the-money option approaches its maturity, the delta tends to



be extremely volatile. Thus, delta values fluctuate over time and this paper seeks to examine how these fluctuations in option delta influence both option pricing and option trading.

Classic option theory (Black and Scholes, 1973) prices option by dynamic replication with continuously delta hedging. In a frictionless and complete market, such delta fluctuations have no impact on option pricing. Nevertheless, markets in the real world are seldom perfect. Hence, the impact of delta fluctuations on option pricing makes for an intriguing empirical question.

On one hand, delta-neutral trading is a widely used strategy that aims to mitigate the impact of stock price movements. Through the daily rebalancing of delta-hedged positions, approximately 90% of directional risks can be eliminated (Hull, 2003; Tian and Wu, 2021). Consequently, the volatility of deltas presents risks for both option buyers and sellers, making it challenging for both parties to effectively maintain a delta-neutral position.

On the other hand, as option deltas are the approximate probability estimations of the likelihood that the options will be exercised by expiration<sup>1</sup>, the higher delta fluctuations bring about the uncertainty of being profitable for both option buyers and option sellers. Standing on the buyer side, for both the call and put options, buyers hope the options can be exercised, as they pay the option premiums to buy the rights to exercise the options. Standing on the seller side, no matter for the call or put options, sellers hope for non-exercise of the options as they have already collected the premiums and if the options get exercised, they have the obligations to deliver the underlying assets or buy them at unfavorable prices. Approximately, the probability of exercising an option corresponds to the absolute value of its delta, while the probability of not exercising it is equivalent to one minus the absolute delta. When the absolute delta increases, the situation becomes favorable (unfavorable) for option buyers (sellers), and vice versa. For the at-the-money options, delta fluctuates symmetrically around 50%. If the deltas are volatile, exhibiting fluctuations in either direction, the uncertainties introduce the same risks for both option buyers and sellers. And these are also discussed in later Section 4.2.

Bearing the increased uncertainty, if investors as fully rational, both sides of risk-averse investors would tend to require higher risk premium. As options are in zero net supply, the trade-

---

<sup>1</sup> Taking call option as an example, in Black-Scholes-Merton option pricing model,  $N(d_2)$  represents the risk neutral probability of being profitable(exercising) in option. Option delta is  $N(d_1)$  and represent conditional probability of exercising the option, which is under different probability measure with  $N(d_2)$ .  $N(d_1)$  is slightly higher than  $N(d_2)$  and their differences are minor especially for short-term options. So, in practice, investors always interpret option delta as the probability of profit. For put option, the absolute delta represents the probability of profit as the put delta is a negative value.

off between option buyers and sellers would be an interesting question and we can examine that by exploring the relationship between delta fluctuation uncertainties and the delta-hedged option returns. If option buyers have a higher (lower) bargaining power than option sellers, we will observe a positive (negative) relationship between the delta fluctuations and option returns for buyers. As mentioned in the previous literature, option investors are net sellers of equity options (Garleanu, et al., 2008; Muravyev, 2016) and we infer that equity options are in the buyer's market (Christoffersen, et al., 2018; Choy and Wei, 2020) and option buyers have a higher bargaining power. Therefore, we propose our first testable hypothesis as:

*Hypothesis 1: For both call and put options, the delta-hedged option returns are increasing in the delta fluctuation uncertainties.*

If Hypothesis 1 exists, the options with volatile deltas are cheaper than those with stable deltas. Inspired by the demand-based option pricing, we propose our second testable hypothesis as:

*Hypothesis 2: For both call and put options, the option demands are decreasing in the delta fluctuation uncertainties.*

Hypothesis 2 suggests that option investors are less likely to buy options with volatile deltas than those with stable deltas. Therefore, the lower demands from option buyers decrease the option prices and therefore induce higher subsequent option returns.

As option investors have the choice to either buy or sell options<sup>2</sup>, one might question why, in situations where both buying and selling options entail increased uncertainty and risks, option investors exhibit a greater reluctance to buy rather than sell. Option sellers may have a lesser awareness of delta fluctuations compared to option buyers. The reasoning behind this is straightforward: as selling an option carries higher risks than buying an option, “risk-seeking” investors are more likely to engage in selling options. In other words, option buyers tend to be more risk-averse than option sellers and, as a result, charge higher return compensations.

Thus, while both option buyers and sellers face similar risks arising from delta fluctuations, option sellers exhibit more “risk-seeking” behavior and are less conscious of delta fluctuations. Conversely, risk-averse option buyers demand a higher risk premium to account for delta fluctuations. Consequently, option buyers play a crucial role in determining the pricing of delta

---

<sup>2</sup> Option investors can sell options if they meet the margin requirement. The margin rules can refer to the CBOE margin manual: [https://cdn.cboe.com/resources/membership/Margin\\_Manual.pdf](https://cdn.cboe.com/resources/membership/Margin_Manual.pdf).

fluctuations, while the mispricing resulting from “risk-seeking” option sellers contributes to cross-sectional predictability in option returns.

To further validate our previous conjectures, we then examine time-serial variations and cross-sectional variations in return predictability from delta fluctuations, are more pronounced. As part of our analysis, we present our third set of hypotheses:

*Hypothesis 3.1: For both call and put options, the positive relationships between delta fluctuations and option returns are more pronounced during periods of higher market sentiment and market uncertainty.*

*Hypothesis 3.2: For both call and put options, the positive relationships between delta fluctuations and option returns are more pronounced for the underlying stocks with greater limits to arbitrage.*

During periods of higher market sentiment and uncertainties, the option mispricing by asymmetrical risk perceptions becomes more prominent, as the presence of greater limits to arbitrage makes it increasingly challenging for arbitrageurs to exploit these opportunities for profit and rectify the mispricing of options. Therefore, the mispricing induced by “risk-seeking” option sellers, along with the limits to arbitrage, makes the return predictability more pronounced. Our findings are consistent with on how to understand anomalies.

## **4. Data and Key Variables**

### *4.1. Sample Coverages*

Our analysis focuses on the options of common stocks and the sample period is from January 1996 to October 2021. The equity option related data are obtained from the Ivy DB OptionMetrics. The stock related data are obtained from the Center for Research on Security Prices (CRSP), Compustat and I/B/E/S. Each month, we select a pair of options (one call and one put) that is closest to being at-the-money, with a remaining maturity period of approximately fifty days for each optionable stock.

Following previous studies (Cao and Han, 2013; Zhan, et al., 2022), several filters are applied in our sample selection. First, we exclude stocks with a closing price below five dollars to avoid extremely illiquid stocks. Second, we exclude options with dividend payouts during the remaining life of the option. Third, to mitigate microstructure biases, we only consider options with positive bid quotes and bid-ask spreads, where the mid-points of the bid and ask quotes are

at least \$1/8. Furthermore, we remove any option observations that violate obvious no-arbitrage conditions. Specifically, we exclude call options outside the range  $[\max(S - Ke^{-rT}, 0), S]$ , and put options outside the range  $[\max(Ke^{-rT} - S, 0), Ke^{-rT}]$ . Additionally, options with moneyness lower than 0.8 or higher than 1.2 are excluded. Finally, we retain only those stocks that have both call and put options available after applying these filters.

Our sample consists of 236,226 option-month observations for both call and put options on individual stocks. The selected options have an average moneyness of 1, with a small standard deviation of 0.04, indicating their proximity to being at-the-money. The options have a maturity period ranging from 43 to 53 calendar days, with an average of 50 days. We examine a total of 6,844 underlying stocks, with an average of 762 optionable stocks available per month.

#### 4.2. Delta Fluctuations Risks for Both Option Buyers and Sellers

The key variable in the paper is the monthly measure of option delta fluctuations. Delta of an option indicates the change in option price for a \$1 change in underlying price. According to the Black-Scholes option pricing model (Black and Scholes, 1973), the call option and put option prices are calculated as:

$$c = S_0 N(d_1) - e^{-rT} KN(d_2)$$

$$p = e^{-rT} KN(-d_2) - S_0 N(-d_1)$$

where  $d_1 = \frac{\log(\frac{S_0}{K}) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$ ;  $d_2 = d_1 - \sigma\sqrt{T}$ ;  $N(x)$  is the standard normal cumulative distribution function;  $S_0$  is the underlying price;  $K$  is the strike price;  $\sigma$  is the underlying volatility;  $r$  is risk-free interest rate;  $T$  is time to expiration.

Delta is the first derivative of option price with respect to underlying price. The delta for call option is  $N(d_1)$  and for put option is  $N(d_1) - 1$ . Hence, the delta is not stable, which is sensitive to stock price, volatility, time to maturity, and risk-free interest rate. In an ideal and complete market, options can be perfectly delta-hedged, and the price of the options would not be affected by delta fluctuations. However, the real market can never be perfect, so it is worthwhile to explore the impact of delta fluctuations.

We measure the monthly delta fluctuations by average the daily absolute delta changes:

$$Avg|\Delta delta|_{o,s,t} = \frac{\sum_{t_1}^{t_n} |delta_{o,s,t_i} - delta_{o,s,t_{i-1}}|}{n}$$

where the  $Avg|\Delta\delta|_{o,s,t}$  is our key variable, which measures the monthly delta fluctuations for option  $o$  with the underlying stock  $s$  in month  $t$ . Month  $t$  has  $n$  deltas, in which  $t_1$  is the first day of month  $t$  and  $t_n$  is the last day of month  $t$ . Shown in Table 1, the mean  $Avg|\Delta\delta|$  is around 4% with standard deviation of 1.5%.

People may easily confuse our measure of delta fluctuations with the concept of option gamma. Option gamma is the sensitivity of option delta to the change of underlying price, while  $Avg|\Delta\delta|$  is the realized delta volatility which capture all the sensitivities of option delta to different parameters including stock price (option gamma), volatility (option vanna), time (option charm) and etc.  $Avg|\Delta\delta|$  directly measures the uncertainty brought by the delta fluctuations. Both option buyers and option sellers are affected by the delta fluctuations as the volatile deltas makes them harder to implement their strategies.

In practice, the absolute value of option delta are also interpreted as the probability of being profitable in options. To be more precise, taking call options for example, the probability of exercising the option by expiration is represented by  $N(d_2)$ , while option delta is expressed as  $N(d_1)$ , which is slightly higher than  $N(d_2)$ . Nonetheless, their differences are relatively small, particularly for short-term options. Furthermore,  $N(d_1)$  can also signify the probability of profit under different probability measures. Hence, it is reasonable to consider option delta as an indicator of the likelihood of profitability. As options are in zero net supply, the sum of the probability of profit (proxied by absolute delta) for option buyers and sellers should amount to one:

$$Prob(Profit)_{buyer} = 1 - Prob(Profit)_{seller}$$

For the At-The-Money options:

$$\begin{aligned} Var[Prob(Profit)_{buyer}] &= Var[Prob(Profit)_{seller}] \\ E[Prob(Profit)_{buyer}] &= E[Prob(Profit)_{seller}] = 50\% \end{aligned}$$

The simplified equations above help us understand that delta fluctuations introduce an risk<sup>3</sup> for both option buyers and sellers. When option buyers and sellers perceive and react to this risk in the same way, we are not able to observe any price variations at different levels of delta volatility. Thus, examining the relationship between delta fluctuations and option returns allows us to gain

---

<sup>3</sup> For the higher moments of ATM delta uncertainty, they are also symmetrical for buyers and sellers, which can be found in the appendix.

valuable insights into the differing reactions of option buyers and sellers when faced with identical levels of risk.

### 4.3. Delta-Hedged Option Returns

To measure option returns, I calculate the delta-hedged option return both with and without daily rebalancing. Following the methodology of previous studies (Bakshi and Kapadia, 2003; Goyal and Saretto, 2009; Cao and Han, 2013; Bali, et al., 2023), I construct a self-financing portfolio for calculating the delta-hedged option return. This involves longing one contract of the option and shorting delta shares of the underlying stock. This delta-neutral strategy ensures that changes in the stock price do not impact the portfolio return.

For the daily-rebalanced delta-hedged option return, I make adjustments to the portfolio to maintain delta-neutrality by buying or selling the appropriate amount of stock each trading day. The option position remains constant at one contract until the end of the month. For instance, in the case of an option portfolio with discrete hedging  $N$  times over month  $t+1$ , the calculation of the daily-rebalanced delta-hedged option return is as follows:

*Delta hedged option return*

$$= \frac{V_{t+1} - V_t - \sum_{n=0}^{N-1} \text{delta}_{V,t_n}(S_{t_{n+1}} - S_{t_n}) - \sum_{n=0}^{N-1} \frac{a_n r_{t_n}}{365} (V_{t_n} - \text{delta}_{V,t_n} S_{t_n})}{|V_t - \text{delta}_{V,t} S_t|}$$

where  $t_n$  ( $n=0,1,\dots,N-1$ ) is the hedge rebalancing date;  $t_0$  is the end of month  $t$  when we form the delta-hedged option portfolio;  $t_N=t+1$  is the end of month  $t+1$  when we close the option portfolio;  $V_t$  is the option price on date  $t$ ;  $S_t$  is the underlying stock price on date  $t$ ;  $\text{delta}_{V,t}$  is the option delta on date  $t$ ;  $r_m$  is the annualized risk free rate on date  $t_n$ ;  $a_n$  is the number of calendar days between  $t_n$  and  $t_{n+1}$ .

For the delta-hedged option return without daily rebalancing, to avoid the transaction cost brought by daily rebalancing, I hold the portfolio (long one contract of option and short delta shares of stock) for one month without adjusting the delta-hedge position. The buy-and-hold return is calculated as:

$$\text{Delta hedged option return w/o daily rebalancing} = \frac{V_{t+1} - V_t - \text{delta}_{V,t}(S_{t+1} - S_t)}{|V_t - \text{delta}_{V,t} S_t|}$$

The table 1 presents, with daily rebalancing, the average delta-hedged call option return is -1% and put option return is -1.3%. The buy-and-hold delta-hedged call option return is -3.1% and put option return is -2.8%. The negative delta-hedged returns mean that option prices are more expensive than Black-Scholes price. Because option sellers undertake the volatility risk and jump risk which has not been priced in the Black-Scholes model, they tend to charge higher option prices to compensate their risk-taking(Bakshi and Kapadia, 2003).

[Insert Table 1 about here]

## 5. Empirical Results

### 5.1. Option Mispricing and Return Predictability

As discussed in Section 3 and 4.2, the delta fluctuations bring about the same uncertainty regarding the probability of profit for both option sellers and option buyers. If option buyers and sellers perceive and react to the same level of risk in a similar manner, different levels of delta fluctuations would affect both parties equally, leading to no observable pattern in option returns. Therefore, examining the relationship between delta fluctuations and option returns provides valuable insights into how option buyers and sellers respond to equivalent risks. Biased perspectives may lead to option mispricing, resulting in cross-sectional return predictability.

#### 5.1.1. Delta-Hedged Option Returns

The Hypothesis 1 suggests a positive relationship between delta fluctuations and option delta-hedged returns. In the section, we verify our Hypothesis 1 by portfolio sorting tests. For each option, we use the average daily absolute delta changes ( $avg|\Delta delta|$ ) as our proxy for monthly delta fluctuations. At the end of each month, we select the nearest at-the-money short-term option pair (one call and one put) for each stock and then sort all call (put) options into quintiles based on their monthly delta fluctuations. After portfolio formation, for each group, we calculate the next-month average delta-hedged call(put) returns, FFC4 alpha adjusted by Fama and French (1993)-Carhart (1997) four factors, q5 alpha adjusted by Hou, et al. (2015) factors. Additionally, for each month, we do the long-short strategy by longing the most volatile delta (High) group and shorting the most stable delta (Low) group and compare the High-Low spreads.

In Table 2, the Panel A reports the delta-hedged option returns with daily rebalancing and Panel B reports the delta-hedged option returns without daily rebalancing. Taking Panel A as an

example, Group1 is the portfolio group with the lowest delta fluctuations and group 5 is the portfolio group with the highest delta fluctuations. For call options, the average delta-hedged option returns for the “High” group is -0.44%, whereas the returns for the “Low” group is -0.76%. The Spread of Long “High” and Short “Low” is 1.32%. The Spreads cannot be absorbed by common risk factors and are significant profitable, with a high annualized Sharpe Ratio of about 16. Similar for put options, the average delta-hedged option returns for the “High” group is -1.01%, whereas the returns for the “Low” group is -2.01%. The Spread of Long “High” and Short “Low” is 1.00%, which cannot be explained by common risk factors and with high Sharpe Ratios.

[Insert Table 2 about here]

Overall, in all the specifications, the delta-hedged option returns monotonically increase in response to delta fluctuations, confirming our Hypothesis 1. These findings suggest that when faced with equivalent risk driven by delta fluctuations, the option buyers have a higher bargaining power compared to option sellers. As discussed in Section 3, the possible explanation is that option sellers tend to exhibit more “risk-seeking” and have a lower awareness of the associated risks. The resulting mispricing caused by the different risk perceptions between option buyers and sellers contribute to the predictability pattern observed in option returns.

### 5.1.2. *Alternative Measures for Option Expensiveness*

In addition to delta-hedged option returns, I also calculate the variance risk premium as an alternative proxy for option expensiveness. Our Hypothesis 1 suggests that the options with volatile deltas are cheaper than those with stable deltas. Following the previous studies (Garleanu, et al., 2008; Bollerslev, et al., 2009; Goyal and Saretto, 2009), we construct two types of variance risk premium. The first one is based on the option implied volatility (Black and Scholes, 1973) minus the historical realized volatility and the second one is based on the Model-Free implied volatility (Britten-Jones and Neuberger, 2000) minus the ex-post realized volatility, both of which can reflect the option expensiveness:

$$VRP1 = BS\_Impl\_Vol_{o,s,t_N} - Historical\ Vol_{s,[t_1,t_N]}$$

$$VRP2 = Avg_{t_1}(MF\_Impl\_Vol_{s,30,t_1} - Realized\ Vol_{s,[t_1,t_1+30]})$$

where  $BS\_Impl\_Vol_{o,s,t_N}$  is the Black-Scholes implied volatility for the option o with the underlying stock s at the end of month t;  $Historical\ Vol_{s,[t_1,t_N]}$  is the annualized historical



volatility of month  $t$ ; Every monthend, we calculate the difference between  $BS\_Impl\_Vol_{o,s,t_N}$  and  $Historical\ Vol_{s,[t_1,t_N]}$  as our VRP1.  $MF\_Impl\_Vol_{s,30,t_{1i}}$  is the model-free implied volatility for options with 30-day to maturity for stock  $s$  on date  $t_{1i}$ ;  $Realized\ Vol_{s,[t,t+30]}$  is the annualized ex-post realized volatility of stock  $s$  during the period  $[t_{1i}, t_{1i} + 30]$ ; For every day  $t_{1i}$  in month  $t+1$ , we calculate  $MF\_Impl\_Vol_{s,30,t_{1i}} - Realized\ Vol_{s,[t_{1i},t_{1i}+30]}$  and then take the average of them as our VRP2.

Based on the similar portfolio sorting method in the Table 2, the results in Table 3 also suggest that options with stable deltas are more expensive than volatile deltas. We can observe for both call and put options, for both VRP1 and VRP2, the option expensiveness monotonically decreases in delta fluctuations. For call options, the VPR1 and VRP2 are 8.31% and 13.67% respectively in the most stable delta (Low) group, while they are 3.07% and 7.72% respectively in the most volatile delta (High) group. Their differences between the “High” group and “Low” group are -5.24% and -5.95%, which are highly significantly negative. For put options, the VPR1 and VRP2 are 8.14% and 15.50% respectively in the most stable delta (Low) group, while they are 3.43% and 7.98% respectively in the most volatile delta (High) group. Their differences between the “High” group and “Low” group are -4.7% and -7.53%, which are also highly significantly negative. With different proxies for option expensiveness, the results in Table 3 are still consistent with those in Table 2.

[Insert Table 3 about here]

### 5.1.3. Controlling for Stock and Option Characteristics

There is a concern that the effect of delta fluctuations on option returns are not driven by the delta fluctuations but driven by some related stock and option characteristics. Here we eliminate our concern by controlling a bunch of stock and option characteristics in the Fama-Macbeth regressions.

In Table 4, for both call options and put options, we have four specifications: regressing next-month delta-hedged option returns on delta fluctuations without any control variables in column (1); with both stock and option characteristics control variables in column (2); with only option characteristics control variables in column (3); with only stock characteristics control variables in column (4).

[Insert Table 4 about here]

The option characteristics control variables include: the idiosyncratic volatility ( $IVOL$ ), the difference between implied volatility and historical volatility ( $HV - IV$ ), the volatility of volatility ( $avg|\Delta IV|$ ), the model-free implied skewness and kurtosis ( $MFIS, MFIK$ ), the option percentage bid-ask spread ( $Option\ spread\ \%$ ), the average daily open interest scaled by stock shares outstanding ( $avg\frac{option\ open\ interest}{stock\ shrou}$ ), the average daily delta and gamma ( $avgdelta, avggamma$ ); the option vega scaled by stock price ( $\frac{vega}{stock\ price}$ ), the option theta( $theta$ ).

The stock characteristics control variables include: the idiosyncratic volatility ( $IVOL$ ), the difference between implied volatility and historical volatility ( $HV - IV$ ), the logarithm of stock price, size, book-to-market ratio and Amihud illiquid measure ( $\log(stockprc)$ ,  $\log(size)$ ,  $\log(BM)$  and  $\log(Illiquid)$ ), the past 12-1 return momentum ( $Momentum$ ), the gross profits scaled by total assets ( $Profit$ ), the net stock issuance in the past year ( $ISSUE\_1Y$ ), the lottery-like characteristics ( $Max5$ ), the analyst forecast dispersions ( $Opinion\ Dispersion$ ).

The results in all the specifications suggest that the delta fluctuation – option return relationship can survive after considering various related characteristics. For call options, when we do not include all the control variables in column (1), the coefficient estimate is 0.271 with t-statistics of 13.07, which means one standard deviation increase with  $avg|\Delta delta|$  is associated with about 0.4% increase in option returns. When we include both the stock and option characteristics in column (2), the coefficient estimate is 0.109 with t-statistics of 6.25, which means one standard deviation increase with  $avg|\Delta delta|$  is associated with about 0.2% increase in option returns. Though the magnitude of the coefficient is halved, it is still highly significant positive. Therefore, the positive relationship between delta fluctuations and option returns cannot be fully explained by the other characteristics that have been examined in the previous literature.

## 5.2. Option Demands

The demand-based option pricing model(Garleanu, et al., 2008) suggests that option net buying demand from option buyers will elevate option price because of the sellers' increased unhedgeable risk (Muravyev, 2016). On the other hand, Christoffersen, et al. (2018) suggests that equity option market makers are net buyers and charge higher return premiums from the large selling pressure

from option end-users. Inspired by them, we hypothesize that with higher(lower) delta fluctuations, the net buying demands for those options will be decreasing(increasing).

Following [Han \(2008\)](#) and [Ramachandran and Tayal \(2021\)](#), I use relative open interest, measured as open interest scaled by the number of shares outstanding or stock trading volume, as a proxy for option demand<sup>4</sup>. In Table 5, we can observe that the option demands decrease in delta fluctuations for both call options and put options. The results are consistent with our Hypothesis 2, the delta fluctuations lower the option demands and push down the option prices and have higher subsequent option returns.

[Insert Table 5 about here]

### 5.3. Possible Explanations

We should note that the demand-based option pricing only explains why buying demand pushes up the option prices; however, it does not address why we observe varying levels of net buying demand (net selling pressure) and option returns in response to different delta fluctuations. In the section, we discuss some possible explanations for the option return predictability and the buy-sell imbalance in delta fluctuations.

Firstly, a rational explanation is not suitable for our analysis. If all option investors are rational and with the same degree of risk-aversion, the positive relationship between option returns and delta fluctuations would imply that option buyers undertake higher risks, brought by delta fluctuations, than option sellers. But as discussed in Section 4.2, the delta-fluctuations introduce equivalent risks for both option buyers and sellers.

Secondly, a plausible explanation, as discussed in Section 3 and Section 5.1.1, is that option buyers are generally risk-averse investors, while option sellers tend to exhibit more “risk-seeking” behavior. Selling options involves the possibility of incurring unlimited losses that can exceed the option premiums, whereas buying options offers limited risk exposure. Consequently, “risk-seeking” investors are more inclined to sell options. Risk-averse investors, on the other hand, demand higher return premiums to compensate for their aversion to risk. This divergence in risk perceptions between option buyers and sellers contributes to the mispricing observed in options. Risk-averse buyers charge higher premiums to account for the perceived risks, while sellers, who

---

<sup>4</sup> We can also calculate the customer net buying demand as  $(\text{Open buy} - \text{Open sell}) / (\text{Open buy} + \text{Open sell})$ . The results are also consistent with our hypotheses.

may be less conscious of the risks involved, contribute to the mispricing. As a result, the asymmetry in risk perceptions plays a significant role in the mispricing exhibited in options.

Third, the financial friction explanation is implausible as well. Since there are margin requirements for option sellers, leading to greater financial restrictions for them compared to option buyers, if these restrictions were binding, sellers would typically tend to charge higher compensation than buyers, which contradicts our previous findings.

Overall, the explanation of “risk-seeking” option sellers aligns with all of our findings. In the next section, we will examine under which scenarios the return patterns become more prominent.

#### *5.4. Time-Serial and Cross-Section Variations in Option Mispricing and Limit to Arbitrage*

In our hypothesis 3.1 and 3.2, we assume (1) during periods of higher market sentiment and uncertainty, “risk-seeking” investors tend to exhibit more bias, and the market conditions make it costlier to arbitrage, and (2) when underlying stocks face greater limits to arbitrage, correcting mispricing becomes more challenging. Considering that the positive relationship between delta fluctuations and option returns can be attributed to behavioral-driven option mispricing, we hypothesize that the pattern of delta fluctuations and option returns becomes more pronounced in these aforementioned situations.

##### *5.4.1. Market Sentiment and Uncertainty*

[De Long, et al. \(1990\)](#) suggest that noise traders are subject to the influence of sentiment. [Baker and Wurgler \(2006\)](#) show that high sentiment makes arbitrage harder. [Stambaugh, et al. \(2012\)](#) indicate that mispricing should be stronger following high sentiment. Inspired by the previous literature, we hypothesize the relationship between delta fluctuations and option returns, driven by behavioral bias, are stronger during periods of higher sentiment. In Table 6, the sample periods are classified as low-median-high sentiment subperiods proxied by *SENT\_DUMMY* based on market sentiment index ([Baker and Wurgler, 2006](#)) and then we interact them with delta fluctuations. As we can observed, the interaction terms are significantly positive in all specifications.

[Insert Table 6 about here]

The VIX is the market expectation of future volatility for S&P500 index, which is also called the “investor fear gauge”. The higher the VIX, the greater the fear ([Whaley, 2000](#)). The VIX

also reflects stock market uncertainty, also related to the arbitrage cost. In Table 7, the sample periods are classified as low-median-high VIX subperiods proxied by *VIX\_DUMMY* and then we interact them with delta fluctuations. Similarly, the interaction terms are significantly positive in all specifications.

During the periods with high sentiment and market uncertainty, the risk perception differences between option buyers and option sellers are more prominent and the mispricing caused by biased option sellers are harder to be corrected, so the delta fluctuation's effects on returns are stronger in such circumstances.

[Insert Table 7 about here]

#### 5.4.2. *Stock Characteristics*

In the section, I conduct several independent double sorting tests to examine how the underlying stock characteristics could affect the delta fluctuations effects. Every month, options are independently sorted on the stock characteristics into three groups, and then are sorted into quintiles by their delta fluctuations. For each of the three groups, we long options with the most volatile deltas and short options with the most stable deltas. We then can compare the long-short spreads across different characteristic group to see how the underlying stock characteristics affect the relationship between delta fluctuations and option returns. On the one hand, we can verify that that delta-fluctuation effects cannot be fully absorbed by the stock characteristics that have been examined in the previous literature. On the other hand, we can examine our hypothesis 3.2 and explore how limits to arbitrage would affect the option mispricing.

Following [Lam and Wei \(2011\)](#), we roughly classify our stock characteristic into three types of limits to arbitrage measurements: arbitrage risk (Idiosyncratic volatility; Volatility of volatility risk; Gamma risk; Lottery preference); information uncertainty (Opinion dispersion; Analyst coverage); transaction costs (Stock size; Stock illiquidity). In Table 8, we can observe the High-Low spreads in all columns are significant, which means that the delta fluctuations effects cannot be fully explained by those characteristics. On the other hand, the High-Low spreads increase in all the proxies for limits to arbitrage.

The results are aligned with our previous findings and further confirm our hypotheses on option mispricing – “risk-seeking” option sellers generate the option mispricing, and the presence

of limits to arbitrage impedes the correction of such mispricing, which enhances the option return predictability based on delta fluctuations.

[Insert Table 8 about here]

## 6. Conclusions

Option deltas are approximate (inverse) probabilities of profit for option (sellers)buyers and volatile deltas make both parties harder to implement their strategies. If deltas fluctuate with higher uncertainty, no matter for call and put options, they will bring about the same risks for both the option buyers and sellers. Ideally, if option buyers and sells perceive and react to the delta fluctuation risks the same way, we won't observe any option pricing variations in different levels of delta volatility.

In our empirical analysis, at the end of each month, for each optional stock, we select a pair of options (one call and one call) that are closest to at-the-money with about 50 days to maturity. We construct monthly measure of delta fluctuations by average daily absolute delta changes for the selected options. We then compare the delta-hedged option returns for the following month. By portfolio sorting tests, we find option returns monotonically increase in delta fluctuations with a significant profitability of the long-short strategy. To establish the robustness of our results, we employ alternative measures of option expensiveness and control for various stock and option characteristics in the Fama-Macbeth regressions. In all cases, our findings remain consistent. Additionally, we further examine the relationship between delta fluctuations and option demands and find option demands decreasing in delta fluctuations, which is consistent with our previous results. The lower demand from option buyers decreases the option prices and lead to higher option returns.

As investors have the choice of buying or selling options, one might question why option buyers lead in pricing delta fluctuations. A plausible explanation is that the “risk-seeking” investors are more inclined to sell options due to the inherent riskier nature of selling options. Consequently, these “risk-seeking” option sellers may be less aware of associated risks, leading to option mispricing and predictable returns in options. Our further exploration of time-series and cross-sectional variations in such mispricing has found that, with higher limits to arbitrage, option mispricing becomes more prominent.

In summary, our study uncovers a strong positive relationship between delta fluctuations and option returns. We attribute this predictability to the mispricing of options, resulting from the asymmetric risk perceptions between option buyers and sellers. Additionally, we observe that with higher limits to arbitrage, the return predictability becomes more pronounced, as it becomes more challenging for arbitrageurs to correct the mispricing in options.

## References

- Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov, 2013, Anomalies and financial distress, *Journal of Financial Economics* 108, 139-159.
- Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *The Journal of Finance* 61, 1645-1680.
- Bakshi, Gurdip, Charles Cao, and Zhiwu Chen, 1997, Empirical performance of alternative option pricing models, *The Journal of Finance* 52, 2003-2049.
- Bakshi, Gurdip, and Nikunj Kapadia, 2003, Delta-hedged gains and the negative market volatility risk premium, *The Review of Financial Studies* 16, 527-566.
- Bali, Turan G, Heiner Beckmeyer, Mathis Mörke, and Florian Weigert, 2023, Option return predictability with machine learning and big data, *The Review of Financial Studies* hhad017.
- Bali, Turan G, and Scott Murray, 2013, Does risk-neutral skewness predict the cross-section of equity option portfolio returns?, *Journal of Financial and Quantitative Analysis* 48, 1145-1171.
- Barberis, Nicholas, Lawrence J Jin, and Baolian Wang, 2021, Prospect theory and stock market anomalies, *The Journal of Finance* 76, 2639-2687.
- Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of political economy* 81, 637-654.
- Bollen, Nicolas PB, and Robert E Whaley, 2004, Does net buying pressure affect the shape of implied volatility functions?, *The Journal of Finance* 59, 711-753.
- Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected stock returns and variance risk premia, *The Review of Financial Studies* 22, 4463-4492.
- Boyer, Brian H, and Keith Vorkink, 2014, Stock options as lotteries, *The Journal of Finance* 69, 1485-1527.
- Britten-Jones, Mark, and Anthony Neuberger, 2000, Option prices, implied price processes, and stochastic volatility, *The Journal of Finance* 55, 839-866.
- Buraschi, Andrea, and Alexei Jiltsov, 2006, Model uncertainty and option markets with heterogeneous beliefs, *The Journal of Finance* 61, 2841-2897.
- Byun, Suk-Joon, and Da-Hea Kim, 2016, Gambling preference and individual equity option returns, *Journal of Financial Economics* 122, 155-174.
- Cao, Jie, and Bing Han, 2013, Cross section of option returns and idiosyncratic stock volatility, *Journal of Financial Economics* 108, 231-249.
- Carhart, Mark M, 1997, On persistence in mutual fund performance, *The Journal of Finance* 52, 57-82.
- Choy, Siu Kai, and Jason Wei, 2020, Liquidity risk and expected option returns, *Journal of Banking & Finance* 111, 105700.
- Choy, Siu Kai, and Jason Wei, 2022, Investor attention and option returns, *Management Science*.
- Christoffersen, Peter, Ruslan Goyenko, Kris Jacobs, and Mehdi Karoui, 2018, Illiquidity premia in the equity options market, *The Review of Financial Studies* 31, 811-851.
- De Long, J Bradford, Andrei Shleifer, Lawrence H Summers, and Robert J Waldmann, 1990, Noise trader risk in financial markets, *Journal of political Economy* 98, 703-738.
- Edelen, Roger M, Ozgur S Ince, and Gregory B Kadlec, 2016, Institutional investors and stock return anomalies, *Journal of Financial Economics* 119, 472-488.
- Eisdorfer, Assaf, Ronnie Sadka, and Alexei Zhdanov, 2022, Maturity driven mispricing of options, *Journal of Financial and Quantitative Analysis* 57, 514-542.



- Engelberg, Joseph, R David McLean, and Jeffrey Pontiff, 2018, Anomalies and news, *The Journal of Finance* 73, 1971-2001.
- Fama, Eugene F, and Kenneth R French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of financial economics* 33, 3-56.
- Franke, Günter, Richard C Stapleton, and Marti G Subrahmanyam, 1998, Who buys and who sells options: The role of options in an economy with background risk, *journal of economic theory* 82, 89-109.
- Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M Poteshman, 2008, Demand-based option pricing, *The Review of Financial Studies* 22, 4259-4299.
- Goyal, Amit, and Alessio Saretto, 2009, Cross-section of option returns and volatility, *Journal of Financial Economics* 94, 310-326.
- Green, T Clifton, and Stephen Figlewski, 1999, Market risk and model risk for a financial institution writing options, *The Journal of Finance* 54, 1465-1499.
- Han, Bing, 2008, Investor sentiment and option prices, *The Review of Financial Studies* 21, 387-414.
- Hirshleifer, David, Kewei Hou, and Siew Hong Teoh, 2012, The accrual anomaly: Risk or mispricing?, *Management Science* 58, 320-335.
- Hirshleifer, David, and Danling Jiang, 2010, A financing-based misvaluation factor and the cross-section of expected returns, *The Review of Financial Studies* 23, 3401-3436.
- Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *The Review of Financial Studies* 28, 650-705.
- Huang, Darien, Christian Schlag, Ivan Shaliastovich, and Julian Thimme, 2019, Volatility-of-volatility risk, *Journal of Financial and Quantitative Analysis* 54, 2423-2452.
- Hull, John C, 2003. *Options futures and other derivatives* (Pearson Education India).
- Jones, Christopher S, and Joshua Shemesh, 2018, Option mispricing around nontrading periods, *The Journal of Finance* 73, 861-900.
- Kuchler, Theresa, and Michaela Pagel, 2021, Sticking to your plan: The role of present bias for credit card paydown, *Journal of Financial Economics* 139, 359-388.
- Lam, FY Eric C, and KC John Wei, 2011, Limits-to-arbitrage, investment frictions, and the asset growth anomaly, *Journal of Financial Economics* 102, 127-149.
- Leland, Hayne E, 1980, Who should buy portfolio insurance?, *The Journal of Finance* 35, 581-594.
- Liu, Jun, and Francis A Longstaff, 2004, Losing money on arbitrage: Optimal dynamic portfolio choice in markets with arbitrage opportunities, *Review of Financial studies* 611-641.
- Meier, Stephan, and Charles Sprenger, 2010, Present-biased preferences and credit card borrowing, *American Economic Journal: Applied Economics* 2, 193-210.
- Muravyev, Dmitriy, 2016, Order flow and expected option returns, *The Journal of Finance* 71, 673-708.
- Muravyev, Dmitriy, and Xuechuan Charles Ni, 2020, Why do option returns change sign from day to night?, *Journal of Financial Economics* 136, 219-238.
- O'Donoghue, Ted, and Matthew Rabin, 1999, Doing it now or later, *American economic review* 89, 103-124.
- O'Donoghue, Ted, and Matthew Rabin, 2015, Present bias: Lessons learned and to be learned, *American Economic Review* 105, 273-279.
- Poteshman, Allen M, 2001, Underreaction, overreaction, and increasing misreaction to information in the options market, *The Journal of Finance* 56, 851-876.

- Ramachandran, Lakshmi Shankar, and Jitendra Tayal, 2021, Mispricing, short-sale constraints, and the cross-section of option returns, *Journal of Financial Economics* 141, 297-321.
- Shleifer, Andrei, and Robert W Vishny, 1997, The limits of arbitrage, *The Journal of finance* 52, 35-55.
- Stambaugh, Robert F, Jianfeng Yu, and Yu Yuan, 2012, The short of it: Investor sentiment and anomalies, *Journal of financial economics* 104, 288-302.
- Tian, Meng, and Liuren Wu, 2021, Limits of arbitrage and primary risk taking in derivative securities, *Available at SSRN 3779350*.
- Whaley, Robert E, 2000, The investor fear gauge, *Journal of portfolio management* 26, 12.
- Zhan, Xintong, Bing Han, Jie Cao, and Qing Tong, 2022, Option return predictability, *The Review of Financial Studies* 35, 1394-1442.

**Table 1: Summary Statistics**

This table reports the descriptive statistics of key variables mentioned in the paper. The descriptive statistics include the sample mean, 10<sup>th</sup> percentile, 25<sup>th</sup> percentile, median, 75<sup>th</sup> percentile, 90<sup>th</sup> percentile, standard deviation, and the number of non-missing observations for the corresponding variable. The sample period that is from January 1996 to October 2021. Delta hedged return refers to daily-rebalanced delta hedged option return till monthend. DHR w/o rebalancing refers to delta hedged option return without daily rebalancing till monthend. Delta hedged strategy is that, for each stock at the end of the previous month, I buy one contract of call option against a short position of delta shares of the underlying stock. The position is held for one month. For the daily rebalanced returns, the delta-hedges are rebalanced daily. For the returns without daily rebalancing, I calculate the buy-and-hold returns.  $avg|\Delta\delta|$  is the monthly average of daily absolute delta changes. VRP1 and VRP2 are alternative measures for option expensiveness which reflect the difference between the ex-ante implied volatility and realized volatility.  $\frac{Open\ interest}{Shares\ outstanding}$  and  $\frac{Open\ interest}{Stock\ trading\ volume}$  are option open interest scaled by underlying stock shares outstanding and trading volumes.

VARIABLES	N	mean	sd	p5	p25	p50	p75	p95
Panel A: Call options								
Delta hedged return	236226	-0.01	0.055	-0.072	-0.026	-0.009	0.006	0.044
DHR w/o rebalancing	236226	-0.031	0.056	-0.965	-0.108	-0.052	-0.029	-0.01
$avg \Delta\delta $	236226	0.041	0.015	0.021	0.031	0.039	0.049	0.069
VRP1	236226	0.053	0.233	-0.205	0.006	0.043	0.111	0.374
VRP2	236226	0.097	0.356	-0.482	-0.12	0.1	0.321	0.657
$\frac{Open\ interest}{Shares\ outstanding}$	236226	7.548	40.354	0.065	0.525	1.973	6.544	30.882
$\frac{Open\ interest}{Stock\ trading\ volume}$	236226	0.92	3.266	0.007	0.058	0.219	0.753	3.66
Panel B: Put options								
Delta hedged return	236226	-0.013	0.045	-0.063	-0.025	-0.011	0.002	0.034
DHR w/o rebalancing	236226	-0.028	0.046	-0.898	-0.086	-0.043	-0.025	-0.009
$avg \Delta\delta $	236226	0.042	0.016	0.021	0.031	0.04	0.05	0.072
VRP1	236226	0.053	0.233	-0.205	0.006	0.043	0.111	0.374
VRP2	236226	0.103	0.357	-0.472	-0.113	0.107	0.326	0.662
$\frac{Open\ interest}{Shares\ outstanding}$	236226	4.791	17.87	0.033	0.276	1.007	3.532	19.354
$\frac{Open\ interest}{Stock\ trading\ volume}$	236226	0.538	2.486	0.004	0.032	0.116	0.397	2.082

**Table 2: Portfolio sorting: Delta fluctuations and delta-hedged option returns**

This table reports the relationship between option delta fluctuations and delta-hedged option returns. The sample period is from January 1996 to December 2021. For each option, we use the average daily absolute delta changes as our proxy for monthly delta fluctuations. At the end of each month, we select the nearest at-the-money option pair (one call and one put) with about 50 days to maturity for each stock and then sort all call (put) options into quintiles based on their monthly delta fluctuations. After portfolio formation, for each group, we calculate the average delta hedged call(put) returns FFC4 alpha adjusted by Fama and French (1993)-Carhart (1997) four factors, q5 alpha adjusted by Hou, et al. (2015) factors. Group 1 is the portfolio group with the lowest delta fluctuations and group 5 is the portfolio group with the highest delta fluctuations. High-Low is the profit from the long-short strategy which is, for each month, we long the most volatile delta group and short the most stable delta group. Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively. Monthly SR is the monthly Sharpe ratio for the long-short strategy.

Panel A: delta-hedged option returns with daily rebalancing

Option delta fluctuation groups	Call options			Put options		
	Average return	FFC4 alpha	q5 alpha	Average return	FFC4 alpha	q5 alpha
1 (Low)	-1.76%	-1.77%	-1.80%	-2.01%	-2.00%	-1.97%
2	-0.96%	-0.95%	-0.94%	-1.44%	-1.42%	-1.40%
3	-0.76%	-0.74%	-0.77%	-1.25%	-1.24%	-1.21%
4	-0.49%	-0.47%	-0.48%	-1.10%	-1.09%	-1.08%
5 (High)	-0.44%	-0.42%	-0.46%	-1.01%	-0.99%	-0.97%
High- Low	1.32%***	1.35%***	1.34%***	1.00%***	1.01%***	1.00%***
t-value	(20.74)	(20.73)	(19.43)	(22.70)	(22.29)	(20.55)
Monthly SR	1.18	1.18	1.10	1.29	1.27	1.17

Panel B: delta-hedged option returns without daily rebalancing

Option delta fluctuation groups	Call options			Put options		
	Average return	FFC4 alpha	q5 alpha	Average return	FFC4 alpha	q5 alpha
1 (Low)	-3.90%	-3.95%	-3.91%	-3.31%	-3.35%	-3.32%
2	-3.31%	-3.36%	-3.27%	-2.73%	-2.77%	-2.72%
3	-3.15%	-3.19%	-3.12%	-2.56%	-2.60%	-2.54%
4	-3.04%	-3.07%	-2.99%	-2.51%	-2.54%	-2.49%
5 (High)	-3.10%	-3.14%	-3.06%	-2.53%	-2.56%	-2.51%
High- Low	0.80%***	0.81%***	0.85%***	0.78%***	0.79%***	0.81%***
t-value	(14.49)	(14.40)	(14.35)	(19.22)	(19.53)	(18.79)
Monthly SR	0.82	0.82	0.82	1.09	1.11	1.07

**Table 3: Alternative measures for option expensiveness: Variance risk premium**

This table reports the relationship between option delta fluctuations and option variance risk premium. The sample period is from January 1996 to December 2021. For each option, we use the average daily absolute delta changes as our proxy for monthly delta fluctuations. At the end of each month, we select the nearest at-the-money option pair (one call and one put) with about 50 days to maturity for each stock and then sort all call (put) options into quintiles based on their monthly delta fluctuations. We construct two types of variance risk premium for each option and then compared the average variance risk premiums across different bins. The VPR1 is based on the option implied volatility(Black and Scholes, 1973) minus the historical realized volatility and the VPR2 is based on the Model-Free implied volatility(Britten-Jones and Neuberger, 2000) minus the ex-post realized volatility, both of which can reflect the option expensiveness. Group 1 is the portfolio group with the lowest delta fluctuations and group 5 is the portfolio group with the highest delta fluctuations. High-Low is the difference between the variance risk premiums of group 5 and group 1. Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

Option delta fluctuation groups	Call options		Put options	
	VRP1	VRP2	VRP1	VRP2
1 (Low)	8.31%	13.67%	8.14%	15.50%
2	5.10%	9.34%	5.08%	10.44%
3	4.02%	8.27%	3.92%	8.62%
4	3.14%	7.35%	3.07%	8.07%
5 (High)	3.07%	7.72%	3.43%	7.98%
High- Low	-5.24%***	-5.95%***	-4.70%***	-7.53%***
t-value	(-16.35)	(-14.95)	(-14.85)	(-20.15)

**Table 4: Fama-Macbeth regressions: Controlling for stock and option characteristics**

This table reports the results of Fama-Macbeth regressions, which presents the relationship between option delta fluctuations and delta-hedged option returns after controlling for several stock and option characteristics. The sample period is from January 1996 to December 2021. Panel A reports the results of call options and Panel B reports those of put options. For both call options and put options, we have four specifications: regressing next-month delta-hedged option returns on delta fluctuations without any control variables in column (1); with both stock and option characteristics control variables in column (2); with only option characteristics control variables in column (3); with only stock characteristics control variables in column (4). The option characteristics control variables include: the idiosyncratic volatility (*IVOL*), the difference between implied volatility and historical volatility ( $HV - IV$ ), the volatility of volatility ( $avg|\Delta IV|$ ), the model-free implied skewness and kurtosis ( $MFIS, MFIK$ ), the option percentage bid-ask spread ( $Option\ spread\ %$ ), the average daily open interest scaled by stock shares outstanding ( $avg \frac{option\ open\ interest}{stock\ shrou}$ ), the average daily delta and gamma ( $avgdelta, avggamma$ ); the option vega scaled by stock price ( $\frac{vega}{stock\ price}$ ), the option theta ( $theta$ ). The stock characteristics control variables include: the idiosyncratic volatility (*IVOL*), the difference between implied volatility and historical volatility ( $HV - IV$ ), the logarithm of stock price, size, book-to-market ratio and Amihud illiquid measure ( $\log(stockprc), \log(size), \log(BM)$  and  $\log(illiquid)$ ), the past 12-1 return momentum (*Momentum*), the gross profits scaled by total assets (*Profit*), the net stock issuance in the past year (*ISSUE\_1Y*), the lottery-like characteristics (*Max5*), the analyst forecast dispersions (*Opinion Dispersion*). Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Call options			
(1)		(2)	
No control	Delta hedged return	Control for both option and stock characteristics	Delta hedged return
$avg \Delta delta $	0.271*** (13.07)	$avg \Delta delta $	0.109*** (6.25)
Control for stocks	No	Control for stocks	Yes
Control for options	No	Control for options	Yes
Observations	226,301	Observations	197,397
Number of months	310	Number of months	310
Adjusted R-squared	0.008	Adjusted R-squared	0.170
(3)		(4)	
Control for option related characteristics	Delta hedged return	Control for stock related characteristics	Delta hedged return
$avg \Delta delta $	0.146*** (10.26)	$avg \Delta delta $	0.134*** (7.50)
<i>IVOL</i>	-0.050***	<i>IVOL</i>	-0.059***

	(-21.06)		(-24.90)
<i>HV – IV</i>	0.034***	<i>HV – IV</i>	0.036***
	(18.01)		(18.60)
<i>avg ΔIV </i>	-0.123***	<i>log (stockprc)</i>	0.011***
	(-7.03)		(8.09)
<i>MFIS</i>	0.001	<i>log (size)</i>	-0.003***
	(0.65)		(-7.56)
<i>MFIK</i>	-0.002***	<i>log (BM)</i>	0.000
	(-3.48)		(0.29)
<i>Option spread %</i>	0.024***	<i>log(Illiquid)</i>	-0.002***
	(6.70)		(-4.35)
<i>avg</i> $\frac{\text{option open interest}}{\text{stock shrout}}$	-0.000***	<i>Momentum</i>	0.002***
	(-4.56)		(3.12)
<i>avgdelta</i>	0.019***	<i>Profit</i>	0.004***
	(9.02)		(6.02)
<i>avggamma</i>	0.162***	<i>ISSUE_1Y</i>	-0.003***
	(9.94)		(-2.60)
$\frac{\text{vega}}{\text{stock price}}$	-2.179***	<i>Max5</i>	-0.042**
	(-15.04)		(-2.36)
<i>theta</i>	-0.001***	<i>Opinion Dispersion</i>	-0.016
	(-5.08)		(-1.45)
<i>Intercept</i>	-0.008***	<i>Intercept</i>	-0.016***
	(-3.27)		(-3.92)
Control for stocks	No	Control for stocks	Yes
Control for options	Yes	Control for options	No
Observations	226,301	Observations	197,397
Number of months	310	Number of months	310
Adjusted R-squared	0.160	Adjusted R-squared	0.144

Panel B: Put options

	(1)		(2)
No control	Delta hedged return	Control for both option and stock characteristics	Delta hedged return
<i>avg Δdelta </i>	0.202*** (14.67)	<i>avg Δdelta </i>	0.139*** (12.44)
Control for stocks	No	Control for stocks	Yes
Control for options	No	Control for options	Yes

Observations	226,301	Observations	197,397
Number of months	310	Number of months	310
Adjusted R-squared	0.008	Adjusted R-squared	0.142
	(3)		(4)
Control for option related characteristics	Delta hedged return	Control for stock related characteristics	Delta hedged return
<i>avg</i>   $\Delta\delta$	0.165*** (11.98)	<i>avg</i>   $\Delta\delta$	0.161*** (11.78)
<i>IVOL</i>	-0.033*** (-22.00)	<i>IVOL</i>	-0.058*** (-22.72)
<i>HV - IV</i>	0.026*** (19.76)	<i>HV - IV</i>	0.037*** (19.09)
<i>avg</i>   $\Delta IV$	-0.119*** (-10.21)	$\log(\text{stockprc})$	-0.006*** (-4.67)
<i>MFIS</i>	0.002*** (3.13)	$\log(\text{size})$	-0.002*** (-6.90)
<i>MFIK</i>	-0.001** (-2.00)	$\log(BM)$	0.000 (1.02)
<i>Option spread %</i>	0.007** (2.47)	<i>Log(Illquid)</i>	-0.002*** (-7.23)
<i>avg</i> $\frac{\text{option open interest}}{\text{stock shrou}}$	-0.000** (-2.37)	<i>Momentum</i>	-0.001* (-1.82)
<i>avgdelta</i>	-0.007*** (-2.94)	<i>Profit</i>	0.002*** (4.38)
<i>avggamma</i>	0.041*** (4.35)	<i>ISSUE_1Y</i>	0.000 (0.28)
$\frac{\text{vega}}{\text{stock price}}$	-1.249*** (-11.92)	<i>Max5</i>	-0.076*** (-5.73)
<i>theta</i>	0.001*** (5.89)	<i>Opinion Dispersion</i>	-0.031*** (-3.66)
<i>Intercept</i>	0.006** (2.22)	<i>Intercept</i>	0.027*** (6.81)
Control for stocks	No	Control for stocks	Yes
Control for options	Yes	Control for options	No
Observations	226,301	Observations	197,397
Number of months	310	Number of months	310
Adjusted R-squared	0.138	Adjusted R-squared	0.114



**Table 5: Delta fluctuations and option demands**

This table reports the relationship between option delta fluctuations and option demands. The sample period is from January 1996 to December 2021. For each option, we use the average daily absolute delta changes as our proxy for monthly delta fluctuations. At the end of each month, we select the nearest at-the-money option pair (one call and one put) with about 50 days to maturity for each stock and then sort all call (put) options into quintiles based on their monthly delta fluctuations. We then compare the next-month average daily option demands. I use relative open interest, measured as open interest scaled by the number of shares outstanding or stock trading volume, as a proxy for option demand. Group 1 is the portfolio group with the lowest delta fluctuations and group 5 is the portfolio group with the highest delta fluctuations. High-Low is the difference between the option demands of group 5 and group 1. Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

## Panel A: Call options

Option delta fluctuation groups	$\frac{\text{Open interest}}{\text{Shares outstanding}}$	$\frac{\text{Open interest}}{\text{Stock trading volume}}$
1 (Low)	13.49	1.362
2	11.84	1.214
3	11.06	1.128
4	9.95	0.972
5 (High)	8.71	0.762
High- Low	-4.78***	-0.600***
t-value	(-9.20)	(-21.40)

## Panel B: Put options

Option delta fluctuation groups	$\frac{\text{Open interest}}{\text{Shares outstanding}}$	$\frac{\text{Open interest}}{\text{Stock trading volume}}$
1 (Low)	9.74	0.904
2	7.79	0.753
3	7.05	0.667
4	6.33	0.579
5 (High)	5.40	0.473
High- Low	-4.34***	-0.431***
t-value	(-13.99)	(-23.78)

**Table 6: Fama-Macbeth regressions: Interacting with market sentiment index**

This table reports how market sentiment affect relationship between delta fluctuations and delta-hedged option returns. The sample periods are classified as low-median-high sentiment subperiods proxied by *SENT\_DUMMY* based on market sentiment index (Baker and Wurgler, 2006), and then we interact them with delta fluctuations. The *SENT\_DUMMY* equals 0 in low sentiment, equals 1 in median sentiment, and equals 2 in high sentiment. The control variables are the same as Table 4. Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

## Panel A: Call options

VARIABLES	(1) Delta hedged return	(2) Delta hedged return	(3) Delta hedged return	(4) Delta hedged return
<i>avg</i>   $\Delta\delta$	0.112*** (7.44)	0.068*** (6.28)	0.059*** (4.84)	0.044*** (3.67)
<i>avg</i>   $\Delta\delta$   $\times$ <i>SENT_DUMMY</i>	0.107*** (7.82)	0.055*** (6.57)	0.053*** (5.68)	0.047*** (5.31)
<i>SENT_DUMMY</i>	-0.010*** (-8.10)	-0.002** (-2.34)	-0.007*** (-3.07)	-0.011*** (-3.08)
Control for stocks	No	No	Yes	Yes
Control for options	No	Yes	No	Yes
Observations	226,301	226,301	197,397	197,397
Number of months	310	310	310	310
Adjusted R-squared	0.005	0.158	0.141	0.167

## Panel B: Put options

VARIABLES	(1) Delta hedged return	(2) Delta hedged return	(3) Delta hedged return	(4) Delta hedged return
<i>avg</i>   $\Delta\delta$	0.081*** (6.45)	0.062*** (6.25)	0.063*** (5.89)	0.050*** (5.67)
<i>avg</i>   $\Delta\delta$   $\times$ <i>SENT_DUMMY</i>	0.075*** (8.06)	0.062*** (7.12)	0.059*** (7.46)	0.056*** (7.96)
<i>SENT_DUMMY</i>	-0.011*** (-9.36)	0.003* (1.95)	0.012*** (5.83)	0.012*** (3.96)
Control for stocks	No	No	Yes	Yes
Control for options	No	Yes	No	Yes
Observations	226,301	226,301	197,397	197,397
Number of months	310	310	310	310
Adjusted R-squared	0.005	0.136	0.111	0.139

**Table 7: Fama-Macbeth regressions: Interacting with VIX market fear index**

This table reports how market fear condition affect the relationship between delta fluctuations and delta-hedged option returns. The sample periods are classified as low-median-high market fear subperiods proxied by *VIX\_DUMMY* based on the market expectation of future volatility for S&P500 index and then we interact them with delta fluctuations. The *VIX\_DUMMY* equals 0 in low VIX, equals 1 in median VIX, and equals 2 in high VIX. The control variables are the same as Table 4. Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

## Panel A: Call options

VARIABLES	(1) Delta hedged return	(2) Delta hedged return	(3) Delta hedged return	(4) Delta hedged return
<i>avg</i>   $\Delta$ delta	0.095*** (7.51)	0.055*** (5.94)	0.044*** (5.18)	0.038*** (3.82)
<i>avg</i>   $\Delta$ delta  $\times$ <i>VIX_DUMMY</i>	0.113*** (7.97)	0.057*** (6.70)	0.053*** (5.15)	0.042*** (4.70)
<i>VIX_DUMMY</i>	-0.011*** (-9.08)	-0.003** (-2.03)	-0.006*** (-2.97)	-0.013*** (-3.85)
Control for stocks	No	No	Yes	Yes
Control for options	No	Yes	No	Yes
Observations	226,301	226,301	197,397	197,397
Number of months	310	310	310	310
Adjusted R-squared	0.005	0.158	0.141	0.167

## Panel B: Put options

VARIABLES	(1) Delta hedged return	(2) Delta hedged return	(3) Delta hedged return	(4) Delta hedged return
<i>avg</i>   $\Delta$ delta	0.080*** (8.50)	0.062*** (7.52)	0.061*** (6.93)	0.056*** (7.23)
<i>avg</i>   $\Delta$ delta  $\times$ <i>VIX_DUMMY</i>	0.075*** (8.00)	0.066*** (7.52)	0.060*** (7.16)	0.052*** (7.14)
<i>VIX_DUMMY</i>	-0.012*** (-9.68)	0.002 (1.62)	0.013*** (5.46)	0.009*** (2.92)
Control for stocks	No	No	Yes	Yes
Control for options	No	Yes	No	Yes
Observations	226,301	226,301	197,397	197,397
Number of months	310	310	310	310
Adjusted R-squared	0.005	0.136	0.111	0.139

**Table 8: Double sorting: Cross-sectional variations in the delta fluctuation - return relationships**

This table reports the independent sorting results that show the cross-sectional variations in the relationship between option delta fluctuations and delta-hedged option returns, accounting for several stock characteristics. Every month, options are independently sorted on the stock characteristics into three groups, and then are sorted into quintiles by their delta fluctuations. For each of the three groups, we long options with the most volatile deltas and short options with the most stable deltas. We then can compare the long-short spreads across different characteristic group to see how the underlying stock characteristics affect the relationship between delta fluctuations and option returns. We roughly classify our stock characteristic into four types of limits to arbitrage measurements: arbitrage risk (Idiosyncratic volatility; Volatility of volatility risk; Gamma risk; Lottery preference); information uncertainty (Opinion dispersion; Analyst coverage); transaction costs (Stock size; Stock illiquidity). Newey and West (1987) t-statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Stock IVOL (1-3)						
Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-0.48%	-1.29%	-3.06%	-1.20%	-1.66%	-2.86%
2	-0.11%	-0.65%	-1.99%	-1.01%	-1.22%	-2.04%
3	0.04%	-0.53%	-1.69%	-0.86%	-1.11%	-1.76%
4	0.13%	-0.30%	-1.31%	-0.67%	-0.95%	-1.66%
5 (High)	0.22%	-0.23%	-1.29%	-0.64%	-0.80%	-1.51%
High- Low	0.71%***	1.06%***	1.77%***	0.56%***	0.86%***	1.35%***
t-value	(7.46)	(9.59)	(14.88)	(7.17)	(10.04)	(15.47)
Panel B: Volatility of volatility risk (1-3)						
Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-1.35%	-1.46%	-2.30%	-1.74%	-1.92%	-2.24%
2	-0.76%	-0.90%	-1.07%	-1.37%	-1.39%	-1.48%
3	-0.64%	-0.70%	-0.79%	-1.30%	-1.19%	-1.23%
4	-0.47%	-0.43%	-0.50%	-1.10%	-1.06%	-1.06%
5 (High)	-0.44%	-0.39%	-0.46%	-0.98%	-1.00%	-0.97%
High- Low	0.91%***	1.07%***	1.84%***	0.76%***	0.92%***	1.27%***
t-value	(10.15)	(10.47)	(15.95)	(10.10)	(11.65)	(13.42)
Panel C: Gamma risk (1-3)						
	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)

Option delta fluctuation (1-5)						
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-0.89%	-1.58%	-2.42%	-2.14%	-1.64%	-2.13%
2	-0.15%	-0.98%	-1.65%	-1.67%	-1.15%	-1.47%
3	-0.12%	-0.73%	-1.36%	-1.50%	-0.96%	-1.21%
4	0.01%	-0.51%	-1.02%	-1.40%	-0.76%	-1.03%
5 (High)	0.01%	-0.44%	-0.87%	-1.33%	-0.68%	-0.94%
High- Low	0.90%***	1.14%***	1.55%***	0.81%***	0.96%***	1.19%***
t-value	(6.96)	(12.28)	(17.48)	(7.47)	(14.01)	(16.59)

Panel D: Lottery preference (1-3)

Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-0.80%	-1.83%	-3.58%	-1.32%	-2.00%	-3.57%
2	-0.19%	-0.73%	-2.35%	-0.92%	-1.32%	-2.45%
3	0.05%	-0.42%	-1.82%	-0.73%	-1.04%	-1.98%
4	0.13%	-0.10%	-1.20%	-0.59%	-0.81%	-1.62%
5 (High)	0.22%	0.03%	-0.96%	-0.53%	-0.62%	-1.37%
High- Low	1.02%***	1.85%***	2.62%***	0.79%***	1.39%***	2.20%***
t-value	(9.81)	(11.87)	(16.34)	(8.88)	(13.01)	(18.97)

Panel E: Opinion dispersion (1-3)

Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-0.45%	-1.26%	-2.64%	-1.53%	-1.58%	-2.40%
2	-0.09%	-0.65%	-1.61%	-1.27%	-1.19%	-1.62%
3	-0.09%	-0.59%	-1.21%	-1.12%	-1.04%	-1.39%
4	0.08%	-0.32%	-0.89%	-0.96%	-0.96%	-1.24%
5 (High)	0.04%	-0.27%	-0.79%	-0.85%	-0.84%	-1.16%
High- Low	0.49%***	1.00%***	1.85%***	0.68%***	0.74%***	1.24%***
t-value	(4.40)	(9.36)	(16.75)	(6.38)	(9.08)	(15.34)

Panel F: Analyst coverage (1-3)

	Call options			Put options		
--	--------------	--	--	-------------	--	--

Option delta fluctuation (1-5)						
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-2.49%	-1.31%	-0.63%	-2.47%	-1.67%	-1.33%
2	-1.54%	-0.72%	-0.27%	-1.76%	-1.24%	-1.17%
3	-1.18%	-0.60%	-0.26%	-1.43%	-1.19%	-1.04%
4	-0.89%	-0.34%	-0.12%	-1.33%	-0.98%	-0.87%
5 (High)	-0.83%	-0.27%	-0.05%	-1.12%	-0.91%	-0.89%
High- Low	1.67%***	1.05%***	0.58%***	1.35%***	0.76%***	0.44%***
t-value	(15.24)	(10.36)	(5.06)	(17.01)	(9.09)	(5.19)

Panel G: Stock size (1-3)

Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-3.22%	-1.03%	-0.02%	-2.79%	-1.62%	-1.12%
2	-2.09%	-0.72%	0.11%	-1.98%	-1.30%	-1.01%
3	-1.83%	-0.55%	0.13%	-1.66%	-1.18%	-0.92%
4	-1.49%	-0.34%	0.18%	-1.46%	-1.04%	-0.80%
5 (High)	-1.47%	-0.27%	0.26%	-1.35%	-0.89%	-0.74%
High- Low	1.75%***	0.76%***	0.27%***	1.44%***	0.72%***	0.38%***
t-value	(18.13)	(8.64)	(2.92)	(18.50)	(7.90)	(4.27)

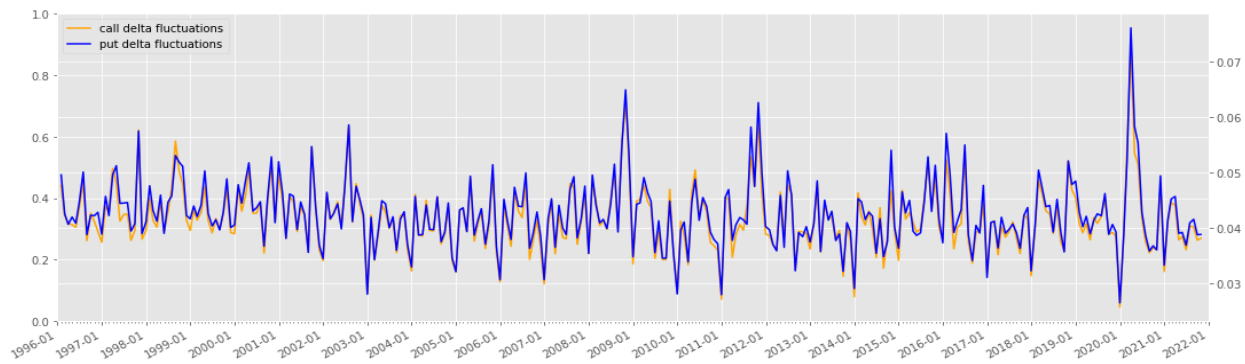
Panel H: Stock illiquidity (1-3)

Option delta fluctuation (1-5)	Call options			Put options		
	1(Low)	2(Median)	3(High)	1(Low)	2(Median)	3(High)
1 (Low)	-0.14%	-1.16%	-3.12%	-1.15%	-1.66%	-2.79%
2	0.05%	-0.77%	-2.00%	-1.05%	-1.33%	-1.91%
3	0.07%	-0.58%	-1.74%	-0.97%	-1.22%	-1.58%
4	0.12%	-0.41%	-1.33%	-0.87%	-1.03%	-1.37%
5 (High)	0.20%	-0.35%	-1.27%	-0.82%	-0.94%	-1.21%
High- Low	0.34%***	0.82%***	1.84%***	0.33%***	0.73%***	1.58%***
t-value	(3.67)	(8.47)	(17.34)	(4.22)	(8.36)	(19.72)

## Appendix:

**Figure 1: The time-series distribution of the cross-sectional mean delta fluctuations**

The figure shows the time-series distribution of the cross-sectional average of the delta fluctuations  $avg|\Delta\delta|$  for both call options and put options.



**Simulation: How delta fluctuations affect the moneyness for buyers and sellers**

The diagram shows the skewness of ATM option's delta difference between option buyers and sellers in simulation repeated for 1 million times.

