Futures Contract Collateralization and its Implications

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Abstract

Defining a futures return as the rate of change of futures prices, as done in many empirical studies, implicitly implies that a futures contract is fully collateralized. We adjust futures' returns to explicitly account for holding the minimum margin (collateral) and the return to this collateral. Different collateral choices of the futures affect the dynamic properties of returns to futures contracts and modify their risk profile. In our empirical study, we document these discrepancies under full and partial collateralization. The discrepancy is minimal except when the futures prices and minimum margins are volatile. Our findings broadly verify the common belief that commodity futures serve as a good asset class for diversification purposes.

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1 Introduction

It is commonly accepted that commodity futures provide an attractive asset class to add to a portfolio because they are not highly correlated with equities and among themselves (e.g., see Gorton and Rouwenhorst (2006), Erb and Harvey (2006), Geman and Kharoubi (2008), Büyükşahin, Haige and Robe (2010), Chong and Miffre (2010), Daskalakia and Skiadopoulos (2011), Bhardwaj, Gorton and Rouwenhorst (2015)). This belief is based on

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studying time series properties of futures prices (and returns) that implicitly assume the futures contracts are fully collateralized and where the returns on the required margins are ignored.[1] This is a serious omission because futures contracts exist precisely because they provide an exchange traded contract for investing in a commodity that is highly levered. Ignoring both the leverage and the return to the margin accounts could possibly bias the return moments and correlations. These biases may, in principle, revise our beliefs with respect to the attractiveness of commodity futures as an alternative asset class. The purpose of this paper is to investigate the time series properties of commodity futures returns explicitly incorporating both leverage and margin account returns to determine what difference, if any, these complications make.

In the existing literature, futures price returns are defined as the rate of change of futures prices; the role of margins (collateral) is not considered. Gorton and Rouwenhorst (2006) collected monthly returns over 1959-2004 and documented a number of stylized facts about commodity futures (e.g., expected returns and Sharpe ratio). A related study is Erb and Harvey (2006), who study the statistical properties of monthly returns (e.g., return correlations among futures) for a wide range of commodity futures in 1982-2004. Gorton, Hayashi and Rouwenhorst (2007) carry out time series analysis on monthly futures data in 1970-2006. Anderson, Bianchi and Goldberg (2014) analyze the monthly futures returns over a much longer sample spanning 1929-2013. Hamilton and Wu (2015) study the predictability of expected futures return and its relationship with notional values of investment held by index funds.

Using futures price returns implicitly assumes full collateralization. This is convenient because futures prices are then the only data input needed to compute such a return. From a practical perspective, it side-steps the technical difficulty of formally defining a futures return. Note that the value of a futures contract is always zero at the end of each period due to marking-to-market; hence, futures returns are technically undefined. The return on a futures contract is only well defined if the base, the non-zero initial futures price, is interpreted as the collateral held in the margin account.

Despite its convenience, the full collateral assumption is unrealistic. It neglects the fact that the minimum margin changes over time and can be correlated with the trading of futures.[2] In this paper, we allow for a choice of the futures collateralization and examine

[1]This convention is commonly adopted in the literature; see Gorton and Rouwenhorst (2006, Appendix A), and Erb and Harvey (2006, Note 1)

[2]A strand of literature studies the relationship between minimal margin and futures trading. Hartzmark

how this affects the statistical analysis of futures' returns. We generalize the definition of futures return (known as the *adjusted futures return*) to correctly reflect the change in value of marked-to-market futures contracts under partial collateralization. We define partial collateralization as holding the less than full collateral. Additionally, the adjusted return accounts for the interest accrued on the collateral and any changes in the collateral due to marking-to-market.

The adjusted returns under partial collateralization serve as the basis for our theoretical and empirical studies. We examine the dynamic properties of a self-financing portfolio of futures, and find that full collateralization leads to a constant inflow/outflow of capital. In addition, we examine the difference in the statistical properties of futures returns under full and partial collateralization. Different statistical measures are compared, including the standard deviation, correlation, and market beta. We find that returns on the collateral itself and stochastic interest rates contribute to the discrepancy between multi-period futures returns under different choices of collateralization. The discrepancy tends to be larger when the sampling frequency of futures return is lower.

Our empirical study estimates futures contract returns under full collateral and partial collateral, where partial collateral is defined as holding the maximum leverage (minimal margin). We show that ignoring the adjustments due to changing minimal margin requirements can lead to a bias in the sample statistics during volatile periods when margin requirements change rapidly. The bias becomes more substantial when the sampling frequency is lower relative to the updating frequency of margins. In many occasions, a shift from full to partial collateralization leads to a significant adjustment in return correlations. The adjustments become large when futures prices and minimal margins are volatile (e.g., during the 2008 GFC and the crash of crude oil futures in 2020). As an implication for portfolio diversification, the common belief that commodity futures serve as an attractive asset class remains valid, except in volatile markets, even after the consideration of margins and the interest earned on collateral.

The rest of the paper is organized as follows. Section 2 introduces the problem and clarifies the definition of a futures return under partial collateralization. Section 3 establishes theoretical results on the impact of collateral choice on a portfolio's return and capital.

(1986) endogenizes futures' margins and analyzes how this may lead to a change of investors' composition and the market performance of futures. More recently, Hedegaard (2014) examines the effect of margin changes on market liquidity and volatility. Abruzzo and Park (2014) finds that futures price volatility has an asymmetric effect on margins.

Section 4 discusses how partial collateralization changes the statistical properties of futures' returns. Section 5 reports the results of our empirical study. Section 6 concludes. The technical proofs and supplementary results are collected in the Appendix.

2 The Problem

The purpose of this section is to understand, theoretically, the impact that margins have on a futures contract's return.

2.1 Definition of a Futures Return

For ease of analysis, we adopt a discrete time setting $t = 0, 1, ..., T$. Consider a futures contract with delivery date T. The futures price F_t is set at at the start of period $[t, t+1)$ $(t = 0, 1, ..., T-1)$ such that the futures contract has zero value.[3] Let V_t denote the time t value of the futures contract.

At the start of period $[t, t+1)$, as noted, the futures contract starts out with zero value, i.e., $V_t = 0$. At the end of each period $[t, t+1)$, the contract is *marked-to-market*: the contract holder receives $F_{t+1} - F_t$, and the contract value reverts again to zero. Consequently, futures contracts have value $V_t \equiv 0$ for all $t = 0, 1, ..., T-1$.

For example, if one goes long a futures contract at time t, the initial value of the position is $V_t = 0$. The value plus cash flow at time $t+1$ is $V_{t+1} + F_{t+1} - F_t$. And, the "return" on a long position in a futures contract is:

$$\frac{V_{t+1} + F_{t+1} - F_t - V_t}{V_t}.$$

Due to marking-to-market, the denominator is 0; therefore, the return is not well-defined.

The problem is how to define a return on a long position in a futures contract. The solution is that, in practice (and economic theory), futures contracts require the posting of collateral/margin to guarantee performance.

Let's consider the return to a futures contract including the collateral posted. At the start of a period $[t, t+1)$, collateral C_t is posted. The collateral is then invested in a default-free interest-bearing securities (the money market account, or mma) until the futures contract is sold or expires. We assume that the collateral is positive $(C_t > 0)$ and

[3] Since the delivery date will be fixed in what follows, we do not need to include the delivery date T as an argument in the futures price. Note that $F_T = S_T$, the spot price at the delivery date.

exogenously determined by the exchange. In addition, we assume that the mma earns interest at a risk-free interest rate r_t.[4]

Due to the collateral, one can consider an *augmented portfolio* consisting of a long position in a futures contract *plus collateral*. The augmented portfolio is worth

$$V_t + C_t$$

at time t; and

$$V_{t+1} + F_{t+1} - F_t + C_t(1 + r_t)$$

at time $t+1$. The return on augmented portfolio over the period $(t, t+1]$ is therefore

$$\begin{aligned} a_{t+1} &= \frac{V_{t+1} + F_{t+1} - F_t + C_t(1 + r_t) - (V_t + C_t)}{V_t + C_t} \\ &= \frac{F_{t+1} - F_t + C_t(1 + r_t) - C_t}{C_t} \\ &= \frac{F_{t+1} - F_t}{C_t} + r_t \\ &= \frac{|F_t|}{C_t} \left(\frac{F_{t+1} - F_t}{|F_t|} \right) + r_t. \end{aligned} \tag{1}$$

Note that a_{t+1} is always well-defined since $C_t > 0$.

On the other hand, the rate of change of futures prices over $(t, t+1]$ is given by

$$b_{t+1} := \frac{F_{t+1} - F_t}{|F_t|}. \tag{2}$$

In what follows, we will refer to a_{t+1} as the *futures return adjusted for holding partial collateral* (or the *adjusted futures return*), and b_{t+1} as the *futures return under full collateral* (plus interest rate).[5]

Define the collateral ratio $\pi_t := \frac{C_t}{|F_t|}$ over $[t, t+1)$. It satisfies $0 < \pi_t \leq 1$. The leverage ratio is given by π_t^{-1}. From (1)-(2) and by the definition of π_t, we deduce the relation

[4] This interest rate is often the LIBOR rate less some haircut.

[5] Observe that $a_{t+1} = b_{t+1} + r_t$ when $\pi_t = 1$. Because the stochastic variation of interest rate is negligible compared to that of b_{t+1} as revealed by our empirical analysis, we will simply drop the qualifier "plus interest rate" while referring to b_{t+1}.

between adjusted and unadjusted futures returns,

$$a_{t+1} = \frac{1}{\pi_t} b_{t+1} + r_t. \tag{3}$$

2.2 Issues with Full Collateral

As noted above, the academic literature investigating futures contract returns (e.g., see Gorton and Rouwenhorst [10], p. 6), sets $C_t = F_t$, which is called "fully or 100% collateralized." This simplification has numerous problems.

1. First, a futures minimal margin is not 100%, it is typically a small percent. Futures contracts are an attractive alternative to buying and storing a commodity precisely because of this implied leverage when posting the minimal margin. Ignoring margin distorts the true returns investors obtain when investing in commodities.

2. Total collateral is related to the commodity's volatility due to the manner in which exchanges determine the minimal margins. For example, letting π_t denote the minimal margin, a first approximation is given by $\pi_t = c_0 + c_1 \sigma_{t-1}$ where σ_{t-1} is the conditional standard deviation at time $t-1$. This implies, of course, that the size of a commodity's minimal collateral is correlated with changes in the commodity's volatility and market conditions, implying that a commodity's adjusted futures return moments differ from unadjusted returns.

3. The commodity's minimal margin differs across commodities (and even delivery dates on a commodity). This difference induces differences in the adjusted futures contract return's correlations across commodities and with equities as compared to the unadjusted returns.

4. Last, the margin held affects a commodity futures adjusted return's distribution and, hence, its risk. Given expression (3), we have that

$$\frac{E_t(a_{t+1}) - r_t}{\sigma_t(a_{t+1})} = \frac{\frac{1}{\pi_t} E_t\left(b_{t+1}\right)}{\frac{1}{\pi_t} \sigma_t\left(b_{t+1}\right)} = \frac{E_t\left(b_{t+1}\right)}{\sigma_t\left(b_{t+1}\right)}.$$

We see that the Sharpe ratio is modified due to margins because the numerator changes. Including the margin held reduces the adjusted futures return's Sharpe ratio relative to the fully collateralized return. In addition, we see that

$$P_t(a_{t+1} \leq \theta) = P_t\left(b_{t+1} \leq [\theta - r_t]\,\pi_t\right)$$

for a given θ. Note that, for the same threshold, the tail probability of the adjusted return is larger than the tail probability of the unadjusted return, because $[\theta - r_t]\,\pi_t$ is generally smaller than θ. This implies that the probability of a large drawdown increases when margin is considered. Of course, due to the increased leverage, the probability of a larger return also increases.

3 Portfolio Considerations

When constructing a portfolio, we want to invest a fixed amount of capital at the beginning, and neither add nor remove any capital until the strategy is terminated, at some future time T. This is called a *self-financing* trading strategy. The purpose of this section is to study the implications of different margins across commodities on the portfolio's return and capital. Two facts are shown:

1. For a fully collateralized futures trading strategy, capital does not remain unchanged. And, as a consequence, the number of futures contracts held must be modified across time to keep the trading strategy self-financing.

2. Partially collateralized futures trading strategies differ from fully collateralized futures trading strategies due to the different return relations given by expression (1). This induces different portfolio variances and Sharpe ratios.

3.1 Some Notations

Let us define some notations:

- K_t = the capital at time $t \in [0, 1, ..., T]$. K_0 is given.
- $F_i(t)$ = the time t futures price for the i^{th} commodity where $i = 1, 2..., n$.
- $N_i(t)$ = the number of futures contracts of the i^{th} commodity held at time t (negative holdings are possible).
- $C_i(t)$ = collateral deposited *per futures contract* for the i^{th} commodity at time t.

- $C_i(t)\,|N_i(t)|$ = total collateral deposited for the i^{th} commodity at time t. Note that the absolute value of the holdings is utilized because positive collateral must be held for negative positions.

- $\frac{C_i(t)|N_i(t)|}{K_0} = w_i(t)$ = the percentage of capital allocated to the i^{th} commodity at time t where

$$\sum_{i=1}^{n} w_i(t) = 1.$$

  Note that $w_i(t) \geq 0$.

For an equally weighted portfolio, set $N_i(t) = \frac{K_0}{nC_i(t)} \geq 0$, to generate $w_i(t) = \frac{1}{n}$. Define the collateral ratio as $0 < \pi_i(t) \leq 1$. Then, the collateral is given by

$$C_i(t) = \pi_i(t)\,|F_i(t)|\,.$$

3.2 Time Dynamics

This section discusses the time dynamics of the portfolio.

1. Time 0. Choose $N_i(0)$ for $i = 1, ..., n$ such that

$$\sum_{i=1}^{n} |N_i(0)|\,C_i(0) = K_0. \tag{4}$$

   This expression allocates the initial capital across the different commodities based on the collateral requirements.

2. Enter time 1. The capital is now given by

$$K_1 = \sum_{i=1}^{n} N_i(0)[F_i(1) - F_i(0)] + r_0 K_0 + K_0 \tag{5}$$

   Note that the first term is the change in value of the futures contracts. The second two terms correspond to the value of the capital at time 1. It equals the initial capital plus interest earned (recall that the summed collateral equals total capital).

   This implies

$$K_1 - K_0 = \sum_{i=1}^{n} N_i(0)[F_i(1) - F_i(0)] + r_0 K_0$$

   Note that the capital changes at time 1.

We next want to express the return on the portfolio as a weighted average of the returns on the individual futures positions. This can be done as follows. The return on the portfolio is:

$$\begin{aligned}\frac{K_1 - K_0}{K_0} &= \sum_{i=1}^{n} \frac{|N_i(0)|\, C_i(0)}{K_0} sign(N_i(0)) \left[\frac{F_i(1) - F_i(0)}{C_i(0)}\right] + r_0 \\ &= \sum_{i=1}^{n} w_i(0) \left[sign(N_i(0)) \frac{F_i(1) - F_i(0)}{C_i(0)}\right] + r_0.\end{aligned}$$

The term $sign(N_i(0))$ adjusts for the position being positive or negative.

We can write this as

$$\begin{aligned}\frac{K_1 - K_0}{K_0} &= \sum_{i=1}^{n} w_i(0) \left[\frac{F_i(0)}{C_i(0)} sign(N_i(0)) \frac{F_i(1) - F_i(0)}{F_i(0)}\right] + r_0 \\ &= \sum_{i=1}^{n} w_i(0) \left[\frac{1}{\pi_i(0)} sign(N_i(0)) \frac{F_i(1) - F_i(0)}{F_i(0)}\right] + r_0. \qquad (6)\end{aligned}$$

This implies the variance and covariances of the returns are:

$$\begin{aligned}\sigma_0^2 \left(\frac{K_1 - K_0}{K_0}\right) &= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i(0) w_j(0) \frac{1}{\pi_i(0)} \frac{1}{\pi_j(0)} \cdot \\ & Cov_0 \left(sign(N_i(0)) \frac{F_i(1) - F_i(0)}{F_i(0)}, sign(N_j(0)) \frac{F_j(1) - F_j(0)}{F_j(0)}\right).\end{aligned}$$

Note that if $\frac{1}{\pi_i(0)} \neq 1$, then the *variance of the adjusted portfolio's return is different from the fully collateralized position's variance.*

$$\frac{E_0 \left(\frac{K_1 - K_0}{K_0}\right) - r_0}{\sigma_0 \left(\frac{K_1 - K_0}{K_0}\right)} = \frac{\sum_{i=1}^{n} w_i(0) \frac{1}{\pi_i(0)} sign(N_i(0)) E_0 \left(\frac{F_i(1) - F_i(0)}{F_i(0)}\right)}{\sum_{i,j=1}^{n} \frac{w_i(0)}{\pi_i(0)} \frac{w_j(0)}{\pi_j(0)} Cov_0 \left(sign(N_i(0)) \frac{F_i(1) - F_i(0)}{F_i(0)}, sign(N_j(0)) \frac{F_j(1) - F_j(0)}{F_j(0)}\right)}.$$

The Sharpe ratio can change due to the different $\pi_i(0)$ across i.

3. Rebalance time 1. Choose $N_i(1)$ for $i = 1, ..., n$ such that

$$\sum_{i=1}^{n} |N_i(1)|\, C_i(1) = K_1.$$

In this rebalancing, if $K_1 > K_0$, and the collateral remains unchanged ($C_i(1) = C_i(0)$), then *the number of futures contracts held must change to keep the portfolio* *capital fully utilized.*

4. Enter time 2.

$$K_2 = \sum\nolimits_{i=1}^{n} N_i(1)[F_i(2) - F_i(1)] + r_1 K_1 + K_1$$

$$\frac{K_2 - K_1}{K_1} = \sum\nolimits_{i=1}^{n} w_i(1) \frac{1}{\pi_i(1)} \left[sign(N_i(1)) \frac{F_i(2) - F_i(1)}{F_i(1)} + r_1 \right]$$

5. So forth ....

Remark 1 *(Full Collateral) Let $\pi_i(t) = 1$ for all t. Then we have*

$$\sum\nolimits_{i=1}^{n} N_i(0) F_i(0) = K_0.$$

And it follows that

$$\begin{aligned} K_1 &= \sum\nolimits_{i=1}^{n} N_i(0)[F_i(1) - F_i(0)] + r_0 K_0 + K_0 \\ &= \sum\nolimits_{i=1}^{n} N_i(0) F_i(1) + r_0 K_0 \\ &\neq K_0. \end{aligned}$$

Note that for a fully collateralized futures trading strategy, capital can be either gained or lost. In general, it does not stay unchanged. The portfolio's return is

$$\frac{K_1 - K_0}{K_0} = \sum\nolimits_{i=1}^{n} w_i(0) \left[sign(N_i(0)) \frac{F_i(1) - F_i(0)}{F_i(0)} + r_0 \right].$$

Remark 2 *For an equally weighted portfolio, set $N_i(0) = \frac{K_0}{nC_i(0)} \geq 0$. The portfolio's return is*

$$\frac{K_1 - K_0}{K_0} = \sum\nolimits_{i=1}^{n} \frac{1}{n} \frac{1}{\pi_i(0)} \left[\frac{F_i(1) - F_i(0)}{F_i(0)} + r_0 \right].$$

4 Statistical Analysis

The purpose of this section is to derive various statistical relationship between a_{t+1} and b_{t+1} that we can test empirically. We first discuss the probabilistic structure of our model. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t>0}, P)$ denote a filtered probability space. The quantities F_t, C_t, π_t and r_t are known at the start of period $[t, t+1)$; i.e., they are $\mathcal{F}_t$-measurable. Consequently, a_{t+1} and b_{t+1} are $\mathcal{F}_{t+1}$-measurable.

4.1 One-period Moments

We will start with one-period conditional moments. Let $E_t[\cdot]$ denote the conditional expectation given $\mathcal{F}_t$. We first observe that, conditional on $\mathcal{F}_t$, the futures returns under partial and full collateral are linearly related. This follows from (3), implying that their conditional moments are equal after a suitable transformation. For example, leveraging does not affect the conditional kurtosis of futures returns because kurtosis is a normalized measure:

$$\kappa_t^a = \frac{E_t[(a_{t+1} - \mu_{at})^4]}{\sigma_{at}^4} = \frac{\pi_t^{-4} E_t[(b_{t+1} - \mu_{bt})^4]}{\pi_t^{-4}\sigma_{bt}^4} = \kappa_t^b,$$

where $\mu_{st} = E_t(s_{t+1})$ and $\sigma_{st}^2 = Var_t(s_{t+1})$ for $s = a, b$. The same equivalence applies to the conditional skewness. Note that all of these normalized measures are unit-free (i.e., independent of the unit of measurement).

The effect of partial collateral is seen in conditional *centralized* moments (which do not take the normalized form). For example, the conditional variance of futures return is magnified by the squared leverage ratio π_t^{-2}:

$$Var_t(a_{t+1}) = \pi_t^{-2} Var_t(b_{t+1}). \tag{7}$$

Similarly, the conditional kurtosis is inflated by π_t^{-4}:

$$E_t[(a_{t+1} - \mu_{at})^4] = \pi_t^{-4} E_t[(b_{t+1} - \mu_{bt})^4],$$

Equalities hold *iff* full collateral is assumed (i.e., $\pi_t = 1$).

4.1.1 Cross Asset Correlations

The cross correlations of futures returns are important in understanding diversification in portfolios. Let W denote a risky asset and w_t denote its return. Define $\rho_t^{bw} := corr_t(b_{t+1}, w_{t+1})$, the conditional correlation between the returns on futures under full collateral and the returns on asset W.

We can compute the one-step ahead conditional correlation between the adjusted futures returns and asset W. Using expression (3),

$$\begin{aligned}
\rho_t^{aw} &= corr_t(a_{t+1}, w_{t+1}) \\
&= corr_t(\pi_t^{-1} b_{t+1} + r_t, w_{t+1}) \\
&= corr_t(\pi_t^{-1} b_{t+1}, w_{t+1}) \\
&= corr_t(b_{t+1}, w_{t+1}) \\
&= \rho_t^{bw}. \qquad (8)
\end{aligned}$$

The third equality follows from the $\mathcal{F}_t$-measurability of r_t. The fourth equality holds because correlation is normalized; hence, the presence of margins does not alter correlations with futures contracts.

Now, suppose the other asset is a different futures contract. Let a_t' denote its adjusted return (with collateral ratio π_t'), and let b_t' denote its return under full collateral. A similar argument shows that the conditional correlation remains the same, i.e.,

$$\begin{aligned}
\rho_t^{aa'} &= corr_t(a_{t+1}, a'_{t+1}) \\
&= corr_t(\frac{1}{\pi_t} b_{t+1} + r_{t+1}, \frac{1}{\pi_t'} b'_{t+1} + r_{t+1}) \\
&= corr_t(\frac{1}{\pi_t} b_{t+1}, \frac{1}{\pi_t'} b'_{t+1}) \\
&= corr_t(b_{t+1}, b'_{t+1}) \\
&= \rho_t^{bb'}.
\end{aligned}$$

4.1.2 Market Betas

Market betas are useful for understanding how commodity futures returns correlate with an aggregate equity index. Let m_t denote the return of market portfolio M.

The one-period ahead conditional market beta of the adjusted futures return denoted

β_t^a is given by

$$\begin{aligned}\beta_t^a &= \frac{Cov_t(a_{t+1}, m_{t+1})}{Var_t(m_{t+1})} \\ &= \frac{Cov_t(\pi_t^{-1}b_{t+1} + r_t, m_{t+1})}{Var_t(m_{t+1})} \\ &= \frac{\pi_t^{-1}Cov_t(b_{t+1}, m_{t+1})}{Var_t(m_{t+1})} \\ &= \pi_t^{-1}\beta_t^b,\end{aligned}$$

where $\beta_t^b := \frac{Cov_t(b_{t+1}, m_{t+1})}{Var_t(m_{t+1})}$ represents the conditional market beta of commodity futures under full collateral. The third equality follows from the $\mathcal{F}_t$-measurability of r_t.

We thus see that the conditional market beta of futures contracts are magnified by the leverage ratio. The return on the margin account, the interest rate, plays no role in the calculation of the one-period conditional adjusted futures market beta.

4.2 Multi-period Analysis

We now proceed to the multi-period analysis by studying the statistical properties of futures return over a time horizon $k > 1$. Let:

- $a_{t,t+k}$ denote the return on the augmented portfolio over $(t, t+k]$. It is $\mathcal{F}_{t+k}$-measurable.
- $b_{t,t+k} := \frac{F_{t+k} - F_t}{|F_t|}$ denote the rate of change of futures prices over $(t, t+k]$. It is $\mathcal{F}_{t+k}$-measurable.

  $$b_{t,t+k} := \frac{(F_{t+k} - F_{t+k-1}) + (F_{t+k-1} - F_{t+k-2}) + \cdots + (F_{t+1} - F_t)}{|F_t|}$$

  Note the telescoping sum in the numerator.
- $r_{t,t+k} := (1 + r_t)(1 + r_{t+1}) \cdots (1 + r_{t+k-1}) - 1$ denote the interest rate (i.e., return on the mma) over $[t, t+k)$. Note that $r_{t,t+k}$ is $\mathcal{F}_{t+k-1}$-measurable, hence it is stochastic as of time t.

Suppose we are at time t. Here, π_t is known, while $a_{t,t+k}$, $b_{t,t+k}$ and $r_{t,t+k}$ are random. Note that they reduce to the notations in the previous section when $k = 1$ ($a_{t+1} = a_{t,t+1}$, $b_{t+1} = b_{t,t+1}$ and $r_t = r_{t,t+1}$).

We now generalize (1) to the k-period horizon. Note that the change in the futures contract is adjusted each period by the change in the collateral over the next period. We obtain the following relationship between $a_{t,t+k}$ and $b_{t,t+k}$:

$$a_{t,t+k} = \frac{F_{t+k} - F_t + \{\cdots [(C_t(1+r_t) + \Delta C_{t+1})(1+r_{t+1}) + \Delta C_{t+2}] \cdots + \Delta C_{t+k-1}\}(1+r_{t+k-1}) - C_t}{C_t}$$
$$= \frac{|F_t|}{C_t}\left(\frac{F_{t+k} - F_t}{|F_t|}\right) + r_{t,t+k} + \frac{\Delta C_{t+1}(1+r_{t+1}) \cdots (1+r_{t+k-1}) + \cdots + \Delta C_{t+k-1}(1+r_{t+k-1})}{C_t},$$

where $\Delta C_t = C_{t+1} - C_t$. Define

$$c_{t,t+k} := \frac{1}{C_t} \sum_{i=1}^{k-1} \Delta C_{t+i} \prod_{j=i}^{k-1}(1+r_{t+j}) = \frac{1}{C_t} \sum_{i=1}^{k-1} \Delta C_{t+i}(1+r_{t+i,t+k-1}).$$

Using the previously defined notations, we arrive at

$$a_{t,t+k} = \frac{1}{\pi_t} b_{t,t+k} + r_{t,t+k} + c_{t,t+k} \tag{9}$$

4.2.1 The Variance of Futures Returns

This section compares the variance of futures returns under partial and full collateral. In contrast to the one-period horizon setting, a formal analysis needs to acknowledge both the effect of leveraging and the stochastic nature of interest rates that span multiple periods.

Going forward k periods (where $k > 1$), the conditional variance of futures return under partial collateral is given by

$$\sigma_t^2(a_{t,t+k}) = \sigma_t^2\left(\frac{1}{\pi_t} b_{t,t+k} + r_{t,t+k} + c_{t,t+k}\right)$$
$$= \frac{1}{\pi_t^2}\sigma_t^2(b_{t,t+k}) + \sigma_t^2(r_{t,t+k} + c_{t,t+k}) + \frac{2}{\pi_t} Cov_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k}). \tag{10}$$

Under mild conditions, futures returns adjusted for partial collateral have a larger variance than that under full collateral. This is stated in the following proposition.

Proposition 1 *Suppose* $\pi_t < 1$ *and* $\sigma_t(r_{t,t+k} + c_{t,t+k}) < (1 - \pi_t)\sigma_t(\frac{1}{\pi_t} b_{t,t+k})$. *Then* $\sigma_t^2(a_{t,t+k}) \geq \sigma_t^2(b_{t,t+k})$ *for* $k > 1$.

The conditions assume that the futures are levered and that the variation of the leveraged futures returns is large relative to the variation of the returns on the mma and the collateral. These are likely to be satisfied in practice because the required collateral is often a small proportion of the futures price.

Compared to the one-period analysis in (7), we see that both $\sigma_t^2(a_{t,t+k})$ and $\frac{1}{\pi_t^2}\sigma_t^2(b_{t,t+k})$ are generally different because of the last two terms of (10)). To interpret the discrepancy, we may express (10) as

$$\sigma_t^2(a_{t,t+k}) = \frac{1}{\pi_t^2}\sigma_t^2(b_{t,t+k})\left[1 + x_t^2 + 2x_t\rho_{t,t+k}^{b,rc}\right],$$

where $x_t := \frac{\sigma_t(r_{t,t+k}+c_{t,t+k})}{\sigma_t(\frac{1}{\pi_t}b_{t,t+k})}$ and $\rho_{t,t+k}^{b,rc} := cor_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k})$. This shows that the source of the difference is x_t and $\rho_{t,t+k}^{b,rc}$; i.e., the stochastic variation of the multi-period returns on the mma, the collateral $(r_{t,t+k} + c_{t,t+k})$, and their correlations with futures returns under full collateral $(b_{t,t+k})$. The discrepancy only vanishes (hence recovering the result of one-period analysis (7)) if the interest rate and return on the collateral has zero stochastic variation relative to the leveraged futures return (i.e., $x_t = 0$). We will show later in our empirical analysis that this discrepancy varies over time and becomes significant during certain time periods.

Example 1 *As an illustration, assume that* $\pi_t = 0.1$ *(10% collateral ratio),* $k = 5$ *(weekly horizon for daily data), and that the conditional standard deviations are* $\sigma_t(r_{t,t+5}+c_{t,t+5}) = 0.1$ *and* $\sigma_t(b_{t,t+5}) = 0.1$*. This gives the lower bound of the conditional covariance:* $Cov_t(b_{t,t+5}, r_{t,t+5} + c_{t,t+5}) > -\sigma_t(r_{t,t+5} + c_{t,t+5})\sigma_t(b_{t,t+5}) > -0.01$*. It follows from (10) that the conditional variance of the adjusted futures return has the lower bound:* $\sigma_t(a_{t,t+5}) > [10^2(0.01) + 0.01 + 2(10)(-0.01)]^{1/2} = 0.9$*. This is substantially larger than* $\sigma_t(b_{t,t+5}) = 0.1$*. On the other hand, the adjusted futures return has a standard deviation* $\sigma_t(\frac{1}{\pi_t}b_{t,t+5}) = 1$*, which is contained within this range.*

4.2.2 Cross Asset Correlations

Suppose we have another risky asset W with return $w_{t,t+k}$ over period $(t, t + k]$, where $k > 1$. Let us evaluate the conditional correlation of k-step returns between the augmented portfolio and asset W. Define $x_t := \frac{\sigma_t(r_{t,t+k}+c_{t,t+k})}{\sigma_t(\frac{1}{\pi_t}b_{t,t+k})}$. By simple algebra, we obtain

$$\begin{aligned}\rho^{a,w}_{t,t+k} &= corr_t(a_{t,t+k}, w_{t,t+k})\\ &= corr_t(\frac{1}{\pi_t}b_{t,t+k} + r_{t,t+k} + c_{t,t+k}, w_{t,t+k})\\ &= \frac{\frac{1}{\pi_t}Cov_t(b_{t,t+k}, w_{t,t+k}) + Cov_t(r_{t,t+k} + c_{t,t+k}, w_{t,t+k})}{\sqrt{\frac{1}{\pi_t^2}\sigma_t^2(b_{t,t+k}) + \sigma_t^2(r_{t,t+k} + c_{t,t+k}) + \frac{2}{\pi_t}Cov_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k})}\sigma_t(w_{t,t+k})}\\ &= \frac{\rho^{b,w}_{t,t+k} + \rho^{w,rc}_{t,t+k} \cdot x_t}{\sqrt{1 + x_t^2 + \frac{2}{\pi_t}\rho^{b,rc}_{t,t+k} \cdot x_t}},\end{aligned}$$

where $\rho^{a,w}_{t,t+k} := corr_t(a_{t,t+k}, w_{t,t+k})$, $\rho^{b,w}_{t,t+k} := corr_t(b_{t,t+k}, w_{t,t+k})$, $\rho^{w,rc}_{t,t+k} := corr_t(w_{t,t+k}, r_{t,t+k} + c_{t,t+k})$, and $\rho^{b,rc}_{t,t+k} := corr_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k})$.

Compared to the one-period result (8), the conditional correlations $\rho^{a,w}_{t,t+k}$ and $\rho^{b,w}_{t,t+k}$ are not the same due to the presence of the multi-period interest rate and the return on collateral. The two correlations are more similar when the returns on the mma and collateral have a smaller variation relative to the futures returns under full collateral (i.e., when x_t approaches zero).

Suppose W is another futures contract. Let $a'_{t,t+k}$ denote its adjusted return over period $(t, t+k]$ with the collateral ratio π'_t at time t. Similarly, let $b'_{t,t+k}$ denote the futures return under full collateral.

Denote $\rho^{a,a'}_{t,t+k}$, the k-step ahead conditional correlation between the adjusted returns $a_{t,t+k}$ and $a'_{t,t+k}$. Define $x_t := \frac{\sigma_t(r_{t,t+k}+c_{t,t+k})}{\sigma_t(\frac{1}{\pi_t}b_{t,t+k})}$ and $x'_t := \frac{\sigma_t(r_{t,t+k}+c'_{t,t+k})}{\sigma_t(\frac{1}{\pi'_t}b'_{t,t+k})}$. By simple algebra,

$$\begin{aligned}\rho^{a,a'}_{t,t+k} &= corr_t(a_{t,t+k}, a'_{t,t+k})\\ &= corr_t(\frac{1}{\pi_t}b_{t,t+k} + r_{t,t+k} + c_{t,t+k}, \frac{1}{\pi'_t}b'_{t,t+k} + r_{t,t+k} + c'_{t,t+k})\\ &= \frac{\rho^{b,b'}_{t,t+k} + \rho^{b,rc'}_{t,t+k} \cdot x'_t + \rho^{b',rc}_{t,t+k} \cdot x_t + \rho^{rc,rc'}_{t,t+k} \cdot x_t \cdot x'_t}{\sqrt{1 + x_t^2 + 2\rho^{b,rc}_{t,t+k} \cdot x_t}\sqrt{1 + (x'_t)^2 + 2\rho^{b',rc'}_{t,t+k} \cdot x'_t}}.\end{aligned}$$

Again, we see that $\rho^{a,a'}_{t,t+k}$ and $\rho^{b,b'}_{t,t+k}$ are different due to the presence of the multi-period interest rate and the returns on the collateral. The difference is smaller when the returns on the mma and collateral have a smaller stochastic variation relative to the futures returns under full collateral (i.e., both x_t and x'_t approach zero).

4.2.3 Market Betas

Let $m_{t,t+k}$ denote the k-period return of the market portfolio M. Define $\beta^b_{t,t+k} := \frac{Cov_t(b_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})}$ to be the conditional market beta for the k-period returns on the futures under full collateral. Let $\beta^r_{t,t+k}$ denote the conditional market beta for the k-period interest rate $r_{t,t+k}$, and $\beta^c_{t,t+k}$ denote the conditional market beta for the k-period returns on collateral $c_{t,t+k}$. Note that, for $k > 1$, $\beta^r_{t,t+k}$ and $\beta^c_{t,t+k}$ are generally non-zero due to the stochastic nature of $r_{t,t+k}$ and $c_{t,t+k}$ as of time t.

Going forward k periods, where $k > 1$, we evaluate the conditional market beta of the adjusted return:

$$\begin{aligned}
\beta^a_{t,t+k} &= \frac{Cov_t(a_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})} \\
&= \frac{Cov_t(\frac{1}{\pi_t} b_{t,t+k} + r_{t,t+k} + c_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})} \\
&= \frac{\frac{1}{\pi_t} Cov_t(b_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})} + \frac{Cov_t(r_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})} + \frac{Cov_t(c_{t,t+k}, m_{t,t+k})}{Var_t(m_{t,t+k})} \\
&= \frac{1}{\pi_t} \beta^b_{t,t+k} + \beta^r_{t,t+k} + \beta^c_{t,t+k}.
\end{aligned}$$

As seen, the risk of the augmented portfolio depends on the risk of the adjusted futures return, the risk of the stochastic interest rate, and the risk of collateral changes, where all risks are measured relative to the market portfolio.

5 Empirical Analysis

This section provides our empirical estimation of the futures return moments and correlations.

5.1 Data

Our sample consists of futures prices spanning January 2003 to May 2021. Using daily close prices obtained from Datastream, we compute the simple weekly returns (Wednesday-to-

Wednesday).[6] The interest rate is taken to be the effective Federal funds rate released by St. Louis Fed.

For our baseline analysis, we consider five front month futures (i.e., futures contracts that are closest to expiration): corn (C), wheat (W), gold (GC), the British pound (BP), and WTI crude oil (CL). In terms of trading volume these are among the most representative futures in their respective categories. We include other commodities subsequently. In addition, we collect historical margins and futures contract specifications from the CME Group. This enables us to compute the minimal margins.[7] Table 1 reports the summary statistics.

We compare adjusted and unadjusted futures contract returns. For the adjusted returns, we assume that partial collateralization corresponds to holding the maximum leverage possible (π_t is defined to be the minimal margin percent), implying the minimal margin is posted. To better understand the dynamics of these returns, we plot the daily time series of futures prices (Figure 1), the standard deviation of futures returns computed over a one-year rolling window (Figure 2), and the leverage ratios determined by the minimum margins required by the exchange (Figure 3; $1/\pi_t$). The standard deviation of futures returns exhibits substantial variation over time. Due to the leverage effect, the standard deviation of adjusted futures returns (in red) dominates those of the unadjusted futures returns (in blue) and collateral returns (in yellow). The leverage ratio tends to be lower when futures prices fluctuate more, e.g., the minimum collateral is at a record high during March 2020 when crude oil futures experiences a sharp fall in price.

5.2 The Correlation Analysis

Figure 4 displays the time series plots of correlations between different pairs of futures returns (computed using the previous year's data), in the form of a matrix. Each subplot contains both the return correlations under full collateral (raw correlation; in blue) and the return correlations adjusted for maximum leveraging (adjusted correlation; in red). Unlike unconditional correlations (which remain close to zero for many pairs; see Table 1), the rolling correlations vary quite substantially over time. The futures prices tend to be

[6] Multi-day returns are used to examine the role of stochastic interest rates in affecting the correlations of adjusted futures returns. See Section 3.

[7] Historical data of minimum margins (minimum performance bond requirements) are available for corn, wheat and British pound futures over the entire sample period (January 2003 to May 2021). The margin data for gold and WTI crude oil futures are available over a shorter time span (January 2009 to May 2021). For the latter futures, the missing leverage ratios over earlier dates are replaced with the time series average.

more highly correlated during 2009-2010 and 2020-21. The fully collateralized and adjusted return correlations are mostly parallel with occasional deviations.

The plots also highlight the episodes with sizable correlation adjustments for partial collateralization (green if adjustment > 0.05; orange if < -0.05). Many of these episodes occur after the crash of the crude oil futures price in March 2020, when the fully collateralized return correlations increase sharply for almost all pairs (except for C-W). The correlations become higher for all pairs involving CL after accounting for partial collateralization. Table 2 lists the time when each future pair achieves maximum conditional correlation adjustment across the whole sample. For example, in 2020-21, GC-CL, BP-CL and C-CL experienced large adjustments in correlation by more than 50%.

To investigate the underlying reasons for correlation adjustments, we decompose the adjustment according to the sources of the difference. The difference in correlations, denoted D, can be broken down as follows:

$$D := corr(a, a') - corr(b, b') = D_1 + D_2 + D_3.$$

Each component is interpreted below (see Lemma A1 in the Appendix for the exact definition):

1. D_1 represents the contribution to the correlation adjustment due to leveraging.
2. D_2 measures the variation in the changes of the collateral and how they co-move with leveraged returns across the futures.
3. D_3 measures the variation of interest rates and how they co-move with leveraged returns and changes in collateral.

Figure 5 plots the time series of these components. The leverage effect (D_1; in red) contributes almost all of the correlation adjustment. The rest of the adjustments are mainly due to the changes in collateral and its association with the leveraged returns (D_2; in blue). The effect of interest rates and its association with leveraged returns (as captured by D_3; in black) is negligible. The last observation is expected because its time series variation is dominated by that of the adjusted futures returns.[8]

[8] In our sample of weekly data, the standard deviation of unadjusted futures returns is more than 50 times of the standard deviation of interest rates; see Table 1, Panel A.

We examine the major contributor D_1 by further breaking it down into interpretable sub-components. See Lemma A2 and the entailing discussion in the Appendix. These sub-components are displayed in Figure A1. Among the cases in which the correlation adjustment is noticeable, many of them are driven by the interaction between the futures return and the leverage ratio (D_{11}; in purple). Another important determinant is the time variation of the leverage ratio and its (linear and quadratic) association with futures returns (D_{12}; in black). These two factors dominate the effects of the product terms (D_{13} and D_{14}; in yellow and green).

5.3 Robustness Tests

All of the commodity futures in the above analysis expire in a month's time. It is interesting to see whether the correlation adjustments change with the futures maturity. We repeat the above correlation analysis for futures expiring in six months. The time series of correlations (Figures A2) appear qualitatively similar. The correlation adjustments are visible for similar time periods. There are some notable differences, however: e.g., unlike the front-month futures, the correlations of longer-maturity futures during 2020-21 are adjusted downward after accounting for leveraging. The decomposition (Figures A3) reveals that D_1 and D_2 are smaller in magnitude as the futures' maturity increases. This indicates, for longer-term futures, a weaker interaction of the co-movement in futures returns and their leverage ratios, less volatile leverage ratios over time, and a weaker link between leverage ratios and futures returns. Consistent with this finding, we observe a decline in the maximum adjustment to correlations across the futures pairs as a commodity futures' maturity increases (Table A1, Panels B and C).

We next study the impact of sampling frequency on return correlations. Using daily futures returns, we obtain the correlations using a one-year rolling window (Figure A4). The correlation series becomes noisier in comparison to weekly return correlations (Figure 4). This is due to the different stylized facts of futures returns as the sampling frequency changes. Furthermore, correlation adjustments for leveraging are visible, and the larger adjustments tend to occur over similar time periods. Compared to weekly return correlations, the maximum adjustments are generally smaller in absolute value with some slight variation in different time periods and the adjustment direction (Table A1, Panel A). The sources of the correlation adjustments are qualitatively the same as in the benchmark case.

Our benchmark results are based on five commodity futures. We now extend the study to 13 futures series. Due to the large number of futures pairs involved, we do not show the time series return correlation plots. We report only the maximum correlation adjustments for all pairs of futures (Table A2). While there are more heterogeneous ranges of time periods in which the correlation adjustment achieves its maximum, many of them occur in 2009-10 in the aftermath of the global financial crisis, or after March 2020 when oil prices dropped sharply.

As our final check, we examine the robustness of our analysis with respect to the roll-over effect of futures. In our previous analysis, the price series of front-month futures are obtained from the futures contract with the nearest maturity, i.e., it regularly "rolls over" to the next nearest futures contract as the current month's contract expires. This leads to unwanted discontinuities of the time-to-maturity at a futures expiration date.

To obtain a time series of futures prices with a maturity fixed, a cubic spline is fitted to the cross section of futures prices for various maturities on any given day. This yields the estimated futures price as a function of maturity for each day. This price series is hypothetical because it may not come from any traded futures contract, although it is associated with a portfolio of futures with different maturities. Table A3 compares the dynamics of the return series obtained by this method (smoothed returns, denoted r_s) with that of the return series obtained by rolling over expiring futures contracts (roll-over returns, denoted r). The difference in their fluctuations as measured by their standard deviation is small for all futures in our sample. This suggests that the results of our correlation analysis are unlikely to be driven by the roll-over of futures. The result is somewhat expected because, from what we observe in the empirical data, the temporal variation of futures prices dominates the basis (defined as the rate of change of futures prices across consecutive maturities; see, e.g., Gorton, Hayashi and Rouwenhorst (2007) [9], footnote 6). Our result is consistent with Carchano and Pardo (2009) [14], which recommends the least complex method to roll over futures contracts.

6 Conclusion

In time series analysis of commodity futures data, it is common practice to define a futures return as the rate of change of futures prices. An implicit assumption underlying this definition is that of full collateralization. This paper relaxes this assumption by modifying the definition of futures return to allow for partial collateralization. The modified definition explicitly accounts for leveraging and the return on collateral in the margin account. This leads to nontrivial changes in the stylized facts of futures' returns.

We explore the implications of different collateralization choices on portfolios. In particular, we compare full collateralization versus holding the maximum possible leverage (minimum margin), which we call partial collateralization. Considering the minimum margin implies that the input capital needs to be constantly adjusted so that the portfolio remains self-financing. Furthermore, a shift from full to partial collateralization can result in a different futures trading strategy due to a change in the portfolio's variance, Sharpe ratio, market beta and return correlation.

Our empirical findings indicate that correlations on futures returns are minimally affected by partial collateralization. The affect is widened when futures prices and minimum margin are more volatile, e.g., in the aftermath of the 2008 GFC and after the crash of crude oil futures in March 2020. This broadly verifies the common belief that commodity futures serve as a good asset class to diversify a portfolio, except during market turbulence. The impact tends to be larger as the sampling frequency of futures returns is lower.

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Appendix

A1 Supplementary Results

Lemma A1: We have

$$corr(a,a') - corr(b,b') = D_1 + D_2 + D_3,$$

where D_i $(i = 1,2,3)$ are given as follows:

$$\begin{aligned}
D_1 &:= \frac{Cov(\frac{1}{\pi}b, \frac{1}{\pi'}b')}{\sigma(a)\sigma(a')} - \frac{Cov(b,b')}{\sigma(b)\sigma(b')},\\
D_2 &:= \frac{Cov(\frac{1}{\pi}b, c') + Cov(c, \frac{1}{\pi'}b') + Cov(c,c')}{\sigma(a)\sigma(a')},\\
D_3 &:= \frac{Cov(\frac{1}{\pi}b, r) + Cov(r, \frac{1}{\pi'}b') + Var(r)}{\sigma(a)\sigma(a')}.
\end{aligned}$$

Lemma A2: Assume that $E(b) = E(b') = 0$. Then we have

$$corr(\frac{1}{\pi}b, \frac{1}{\pi'}b') - corr(b,b') = D_{11} + D_{12} + D_{13} - D_{14},$$

where D_{1i} $(i = 1,2,3,4)$ are given as follows:

$$\begin{aligned}
D_{11} &:= \frac{1}{\sigma(a)\sigma(a')} Cov(\frac{1}{\pi}\frac{1}{\pi'}, bb'),\\
D_{12} &:= \frac{1}{\sigma(a)\sigma(a')} \left[E(\frac{1}{\pi})E(\frac{1}{\pi'}) - \frac{\sigma(\frac{1}{\pi}b)\sigma(\frac{1}{\pi'}b')}{\sigma(b)\sigma(b')} \right] Cov(b,b'),\\
D_{13} &:= \frac{1}{\sigma(a)\sigma(a')} Cov(\frac{1}{\pi}, \frac{1}{\pi'}) Cov(b,b'),\\
D_{14} &:= \frac{1}{\sigma(a)\sigma(a')} Cov(\frac{1}{\pi}, b) Cov(\frac{1}{\pi'}, b').
\end{aligned}$$

Each of the sub-components D_{1i} $(i = 1,2,3,4)$ bears an interpretation (up to scaling by the standard deviations of leveraged returns), summarized below:

1. $D_{11} := \frac{1}{\sigma(a)\sigma(a')} Cov(\ell\ell', bb')$ measures the interaction between the co-movement of leverage ratios and that of futures returns.

2. $D_{12} := \frac{1}{\sigma(a)\sigma(a')}\left[E(\ell)E(\ell') - \frac{\sigma(\ell b)\sigma(\ell' b')}{\sigma(b)\sigma(b')}\right] Cov(b, b')$ captures the degree of time variation of the leverage ratios (as measured by their standard deviation), and the linear/quadratic association between leverage ratio and futures returns (as measured by $Cov(\ell, b)$ and $Cov(\ell^2, b^2)$).[9]

3. $D_{13} := \frac{1}{\sigma(a)\sigma(a')} Cov(\ell, \ell')Cov(b, b')$ is the scaled product of the covariance between leverage ratios (over futures) and the covariance of futures returns (over futures).

4. $D_{14} := \frac{1}{\sigma(a)\sigma(a')} Cov(\ell, b)Cov(\ell', b')$ is the scaled product of the covariances between futures returns and leverage ratio.

A2 Technical Proofs

A2.1 Proof of Proposition 1

If $\sigma_t(r_{t,t+k} + c_{t,t+k}) = 0$, then $\pi_t < 1$ implies that $\sigma_t^2(a_{t,t+k}) = \sigma_t^2(\frac{1}{\pi_t} b_{t,t+k}) > \sigma_t^2(b_{t,t+k})$. From now on, we assume that $\sigma_t(r_{t,t+k} + c_{t,t+k}) > 0$.

Suppose the contrary, i.e., $\sigma_t^2(a_{t,t+k}) < \sigma_t^2(b_{t,t+k})$. Applying this inequality to (10) yields

$$\begin{aligned}
&\sigma_t^2(b_{t,t+k}) > \sigma_t^2(a_{t,t+k}) = \frac{1}{\pi_t^2}\sigma_t^2(b_{t,t+k}) + \sigma_t^2(r_{t,t+k} + c_{t,t+k}) + \frac{2}{\pi_t} Cov_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k}) \\
\Longrightarrow\ & Cov_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k}) < -\frac{\pi_t}{2}\sigma_t^2(r_{t,t+k} + c_{t,t+k}) - \frac{\pi_t}{2}\left(\frac{1}{\pi_t^2} - 1\right)\sigma_t^2(b_{t,t+k}) \\
\Longrightarrow\ & \frac{Cov_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k})}{\sigma_t(b_{t,t+k})\sigma_t(r_{t,t+k} + c_{t,t+k})} < -\frac{\pi_t}{2}\left[\frac{\sigma_t(r_{t,t+k} + c_{t,t+k})}{\sigma_t(b_{t,t+k})} + \left(\frac{1}{\pi_t^2} - 1\right)\frac{\sigma_t(b_{t,t+k})}{\sigma_t(r_{t,t+k} + c_{t,t+k})}\right] \\
\Longrightarrow\ & cor_t(b_{t,t+k}, r_{t,t+k} + c_{t,t+k}) < -\frac{1}{2}\left[\frac{\sigma_t(r_{t,t+k} + c_{t,t+k})}{\sigma_t(\frac{1}{\pi_t} b_{t,t+k})} + \left(1 - \pi_t^2\right)\frac{\sigma_t(\frac{1}{\pi_t} b_{t,t+k})}{\sigma_t(r_{t,t+k} + c_{t,t+k})}\right].
\end{aligned} \tag{11}$$

[9] To see this, suppose the futures return b has mean zero and are independent of the leverage ratio ℓ. We thus have

$$\begin{aligned}
\sigma^2(\ell b) &= Cov(\ell^2, b^2) + \{\sigma^2(\ell) + [E(\ell)]^2\}\{\sigma^2(b) + [E(b)]^2\} - [E(\ell)E(b) + Cov(\ell, b)]^2 \\
&= \{\sigma^2(\ell) + [E(\ell)]^2\}\sigma^2(b).
\end{aligned}$$

This implies that

$$\frac{\sigma(\ell b)\sigma(\ell' b')}{\sigma(b)\sigma(b')} = \sqrt{\sigma^2(\ell) + [E(\ell)]^2}\sqrt{\sigma^2(\ell') + [E(\ell')]^2}.$$

Substituting into the expression of D_2, we deduce that $D_2 = 0$ if in addition the leverage ratios are constant over time.

Consider the quadratic function:

$$f(x) = x^2 - 2x + \left(1 - \pi^2\right).$$

The solutions of the inequality $f(x) > 0$ are $x < 1 - \pi$ and $x > 1 + \pi$.

The assumed conditions ensure that $x_t := \frac{\sigma_t(r_{t,t+k}+c_{t,t+k})}{\sigma_t(\frac{1}{\pi_t}b_{t,t+k})} < 1 - \pi_t$. We deduce that

$$x_t^2 - 2x_t + \left(1 - \pi_t^2\right) > 0. \tag{12}$$

Since $x_t > 0$ by assumption, we can divide both sides of (12) by x_t, and obtain

$$\begin{aligned} x_t - 2 + \left(1 - \pi_t^2\right)\frac{1}{x_t} > 0 \\ \Longrightarrow x_t + \left(1 - \pi_t^2\right)\frac{1}{x_t} > 2 \\ \Longrightarrow \frac{1}{2}\left[x_t + \left(1 - \pi_t^2\right)\frac{1}{x_t}\right] > 1. \end{aligned}$$

Combining with inequality (11) yields $\rho_t(b_{t,t+k}, r_{t,t+k}) < -1$. We thus arrive at a contradiction (for correlation must be at least as large as -1). The stated claim follows immediately.

A2.2 Proof of Lemma A1

Given two distinct futures, let b and b' denote their returns under full leverage, a and a' denote the returns of the augmented portfolio. We let $\ell = \frac{1}{\pi}$ and $\ell' = \frac{1}{\pi'}$ denote the leverage ratios of the two futures, c and c' denote their changes in collateral over k period, and let r be the common interest rate (return on mma). We then have $a = \ell b + c + r$ and similarly for a'. For notational simplicity, we drop the time subscripts, and all the moments below are viewed as unconditional moments.

We may then decompose the correlation adjustment as follows

$$
\begin{aligned}
D &= corr(a,a') - corr(b,b') \\
&= \frac{Cov(a,a')}{\sigma(a)\sigma(a')} - \frac{Cov(b,b')}{\sigma(b)\sigma(b')} \\
&= \frac{Cov(\ell b + c + r, \ell' b' + c' + r)}{\sigma(a)\sigma(a')} - \frac{Cov(b,b')}{\sigma(b)\sigma(b')} \\
&= \left[\frac{Cov(\ell b, \ell' b')}{\sigma(a)\sigma(a')} - \frac{Cov(b,b')}{\sigma(b)\sigma(b')}\right] + \frac{Cov(\ell b, c') + Cov(c, \ell' b') + Cov(c,c')}{\sigma(a)\sigma(a')} \\
&+ \frac{Cov(\ell b, r) + Cov(r, \ell' b') + Var(r)}{\sigma(a)\sigma(a')} \\
&=: D_1 + D_2 + D_3.
\end{aligned}
$$

A2.3 Proof of Lemma A2

Using the same notation as in Lemma A1, the first component of correlation adjustment (D_1) is expressed as follows:

$$D_1 = \frac{Cov(\ell b, \ell' b')}{\sigma(a)\sigma(a')} - \frac{Cov(b,b')}{\sigma(b)\sigma(b')}. \tag{13}$$

The covariance term in the first fraction is given by

$$Cov(\ell b, \ell' b') = E(\ell b \ell' b') - E(\ell b)E(\ell' b'). \tag{14}$$

On the other hand, we have

$$Cov(\ell\ell', bb') = E(\ell\ell' bb') - E(\ell\ell')E(bb'). \tag{15}$$

Eliminating $E(\ell b \ell' b')$ in (14) and (15) yields

$$Cov(\ell b, \ell' b') = Cov(\ell\ell', bb') + E(\ell\ell')E(bb') - E(\ell b)E(\ell' b').$$

Subsituting into (13), and using the assumption that $E(b) = E(b') = 0$, we obtain

$$
\begin{aligned}
&corr(a, a') - corr(b, b') \\
=&\frac{Cov(\ell\ell', bb') + E(\ell\ell')E(bb') - E(\ell b)E(\ell' b')}{\sigma(a)\sigma(a')} - \frac{Cov(b, b')}{\sigma(b)\sigma(b')} \\
=&\frac{1}{\sigma(a)\sigma(a')}Cov(\ell\ell', bb') + \frac{1}{\sigma(a)\sigma(a')}[Cov(\ell, \ell') + E(\ell)E(\ell')]Cov(b, b') \\
&- \frac{1}{\sigma(a)\sigma(a')}E(\ell b)E(\ell' b') - \frac{1}{\sigma(a)\sigma(a')}\frac{\sigma(a)\sigma(a')}{\sigma(b)\sigma(b')}Cov(b, b') \\
=&\frac{1}{\sigma(a)\sigma(a')}Cov(\ell\ell', bb') + \frac{1}{\sigma(a)\sigma(a')}\left[E(\ell)E(\ell') - \frac{\sigma(a)\sigma(a')}{\sigma(b)\sigma(b')}\right]Cov(b, b') \\
&+ \frac{1}{\sigma(a)\sigma(a')}Cov(\ell, \ell')Cov(b, b') - \frac{1}{\sigma(a)\sigma(a')}E(\ell b)E(\ell' b') \\
=&D_{11} + D_{12} + D_{13} - D_{14},
\end{aligned}
$$

as claimed.

Figures and Tables

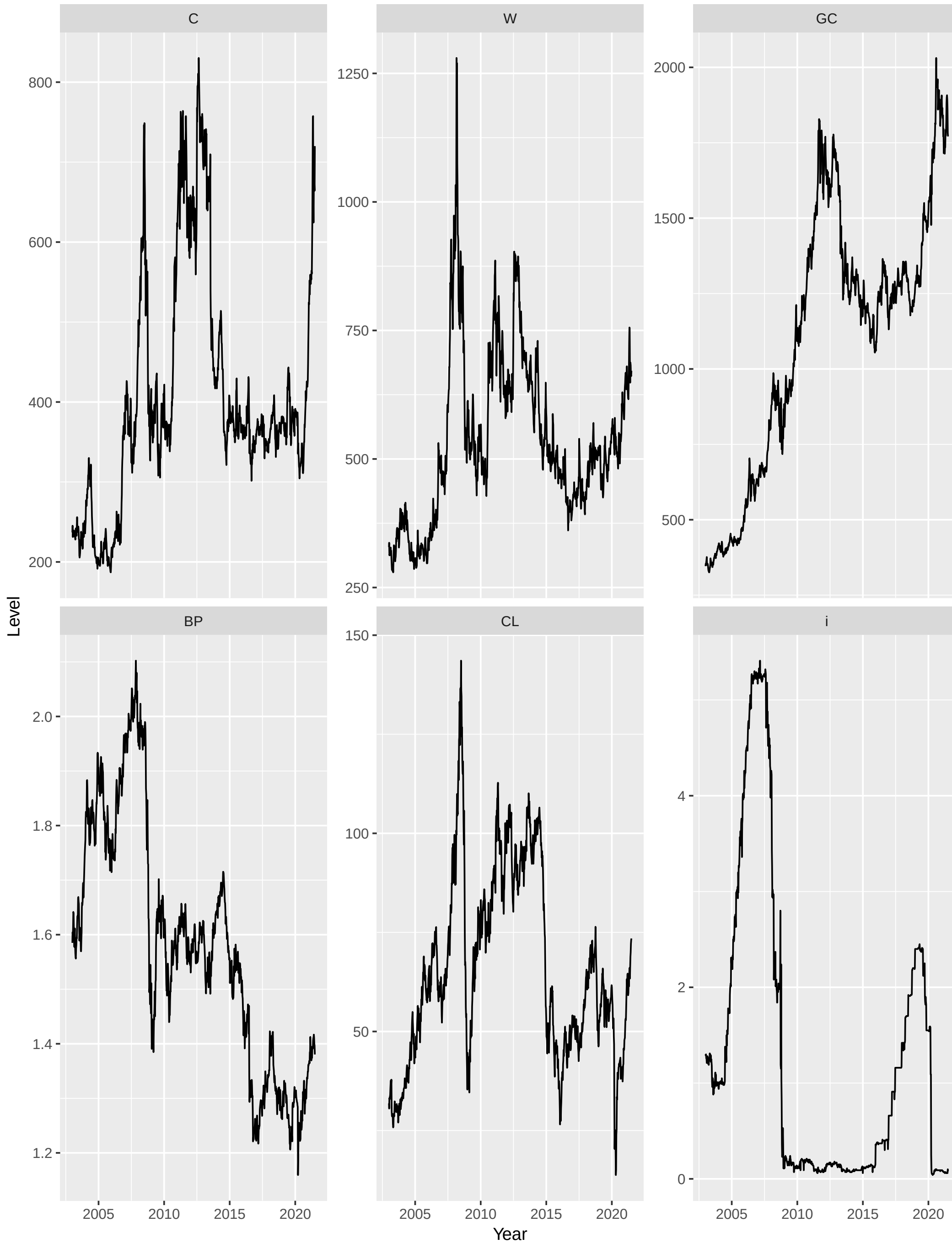

Figure 1: Futures prices and interest rate.

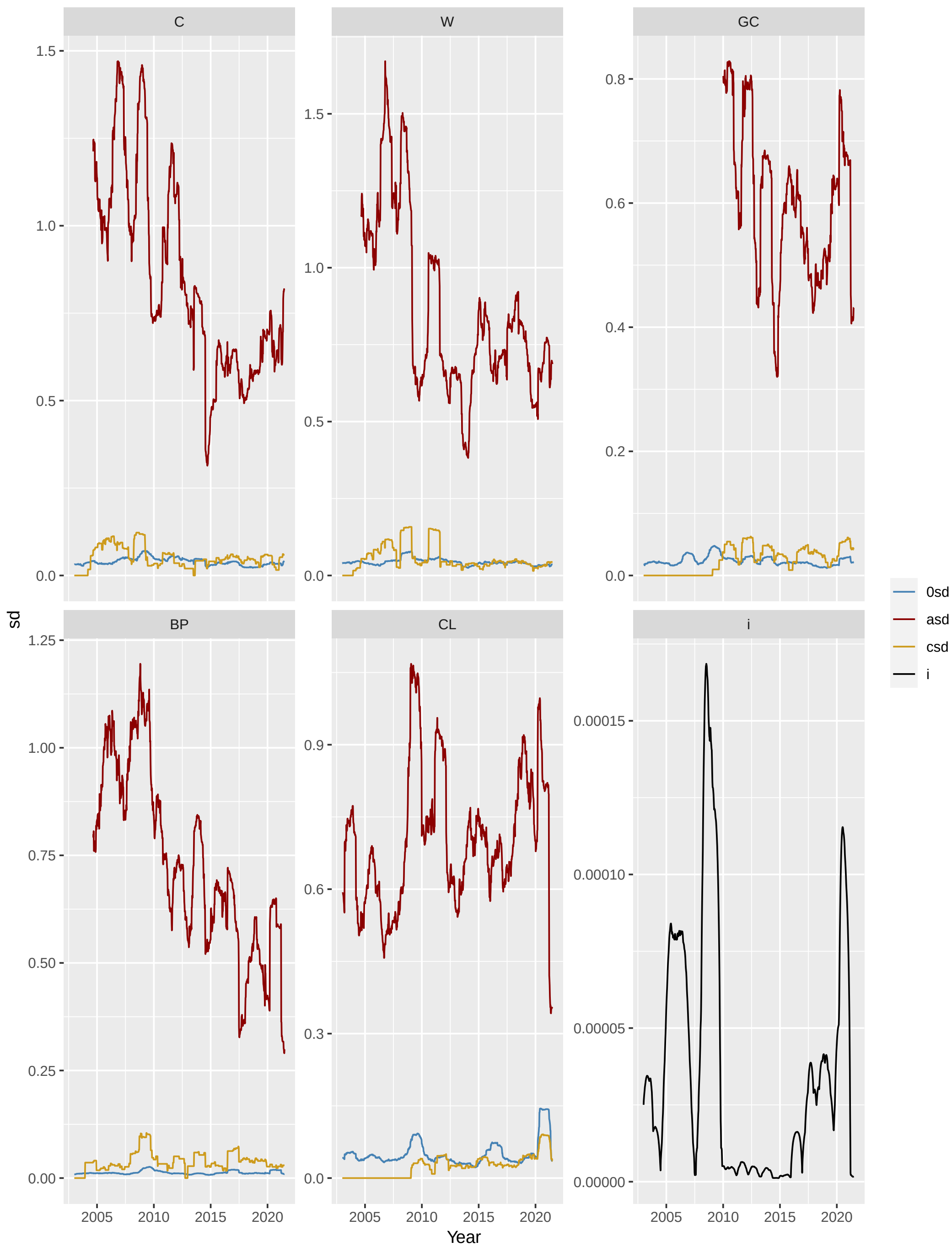

Figure 2: Standard deviation of futures returns and interest rate.

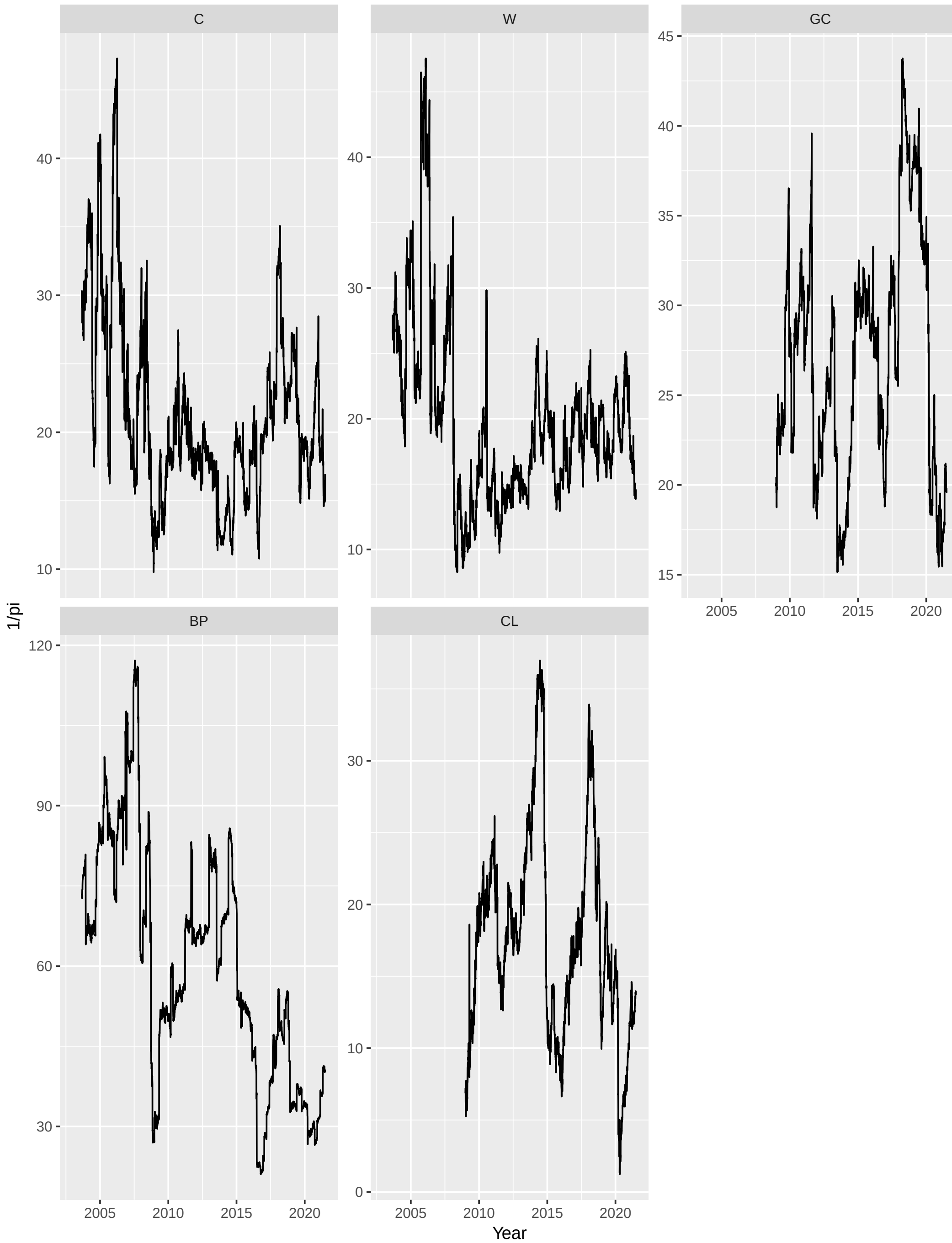

Figure 3: Leverage ratio of futures.

Figure 4: Time series of correlations (futures maturing in one month).

Figure 5: Correlation decomposition (futures maturing in one month).

Table 1: Summary statistics

Panel A: Returns under full collateralization

	Mean	sd	Skewness	Kurtosis		Correlation C	W	GC	BP	CL	SPX
C	0.0018	0.042	-0.032	5.951	C						
W	0.0018	0.0448	0.616	4.782	W	0.581					
GC	0.0020	0.0249	-0.396	6.117	GC	0.192	0.166				
BP	-0.0001	0.0133	-0.796	8.680	BP	0.169	0.171	0.323			
CL	0.0024	0.0546	0.912	20.919	CL	0.204	0.134	0.228	0.255		
SPX	0.0017	0.0221	-0.951	10.281	SPX	0.176	0.153	0.137	0.338	0.325	
FFR	0.0002	0.0002	1.333	3.627	FFR	0.043	0.057	0.050	0.015	0.005	-0.033

Panel B: Returns under partial collateralization

	Mean	sd	Skewness	Kurtosis		Correlation C	W	GC	BP	CL
C	0.0549	0.9118	0.446	6.339	C					
W	0.0491	0.9183	1.005	6.633	W	0.580				
GC	0.0401	0.6102	-0.392	6.307	GC	0.153	0.102			
BP	0.0006	0.7552	-0.217	4.478	BP	0.113	0.147	0.234		
CL	0.0128	0.7167	-0.176	5.266	CL	0.223	0.123	0.211	0.297	
					SPX	0.195	0.168	0.132	0.375	0.413

Panel C: Returns on collateral

	Mean	sd	Skewness	Kurtosis		Correlation C	W	GC	BP	CL
C	0.0042	0.0603	2.561	28.285	C					
W	0.0034	0.0690	6.697	102.357	W	0.256				
GC	0.0013	0.0331	2.716	28.886	GC	0.013	0.014			
BP	0.0014	0.0442	3.627	49.138	BP	0.015	0.058	0.112		
CL	0.0002	0.0339	1.716	24.665	CL	0.006	0.032	0.124	0.037	

Note: Summary statistics are reported for the following front-month futures: corn (C), wheat (W); gold (GC); British Pound (BP); and WTI crude oil (CL). SPX: spot S&P 500. FFR: Federal funds rate. Sample period is Jan 2003 - May 2021. All series are Wednesday-to-Wednesday weekly returns.

Table 2: Maximum absolute correlation adjustments

Pair	Date	Correlation	Adjusted correlation	Difference	% difference
BP-CL	2021-03-10	0.347	0.649	0.301	87%
C-BP	2008-12-17	0.426	0.283	-0.143	-34%
C-CL	2020-04-01	0.305	0.117	-0.189	-62%
C-GC	2010-01-20	0.251	0.342	0.091	36%
C-W	2011-07-13	0.699	0.537	-0.162	-23%
GC-BP	2017-06-07	-0.181	-0.321	-0.140	-77%
GC-CL	2020-04-08	0.572	0.245	-0.327	-57%
W-BP	2009-06-10	0.350	0.233	-0.118	-34%
W-CL	2008-12-10	0.280	0.163	-0.116	-42%
W-GC	2010-01-27	0.334	0.420	0.087	26%

Note: Included in the table are the maximum absolute correlation adjustment (fifth column) in the sample for each pair of futures. The correlation adjustment is equal to the correlation of leveraged futures returns (fourth column) minus the correlation of returns under full collateral (third column). All returns are on a weekly basis. All correlations are computed over a one-year rolling window. Date (second column) refers to the right endpoint of the rolling window over which absolute correlation adjustment achieves its maximum. Sample period is Jan 2003 - May 2021.

Appendix Figures and Tables

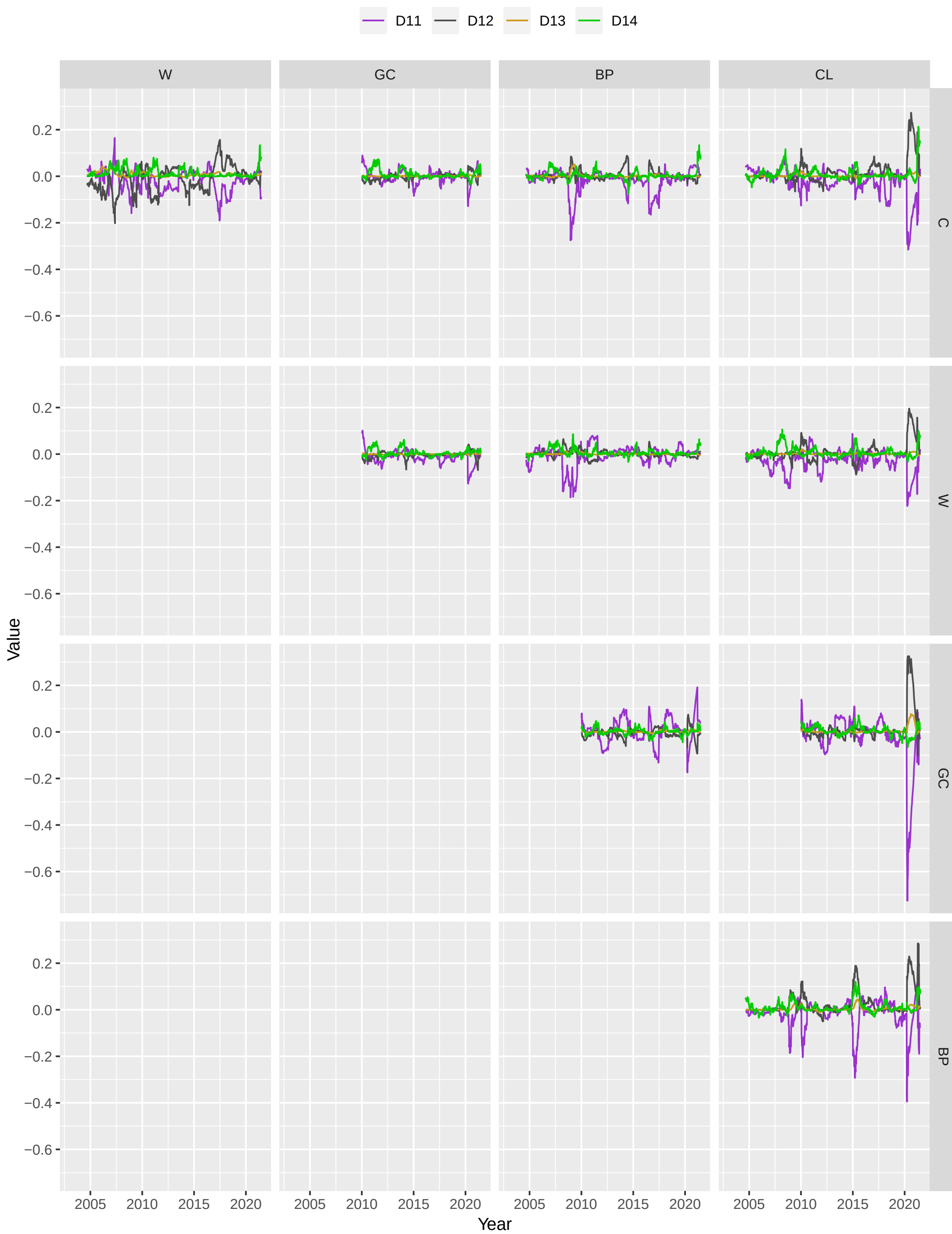

Figure 1: Decomposition of D_1 (futures maturing in one month).

Figure 2: Time series of correlations (futures maturing in six months).

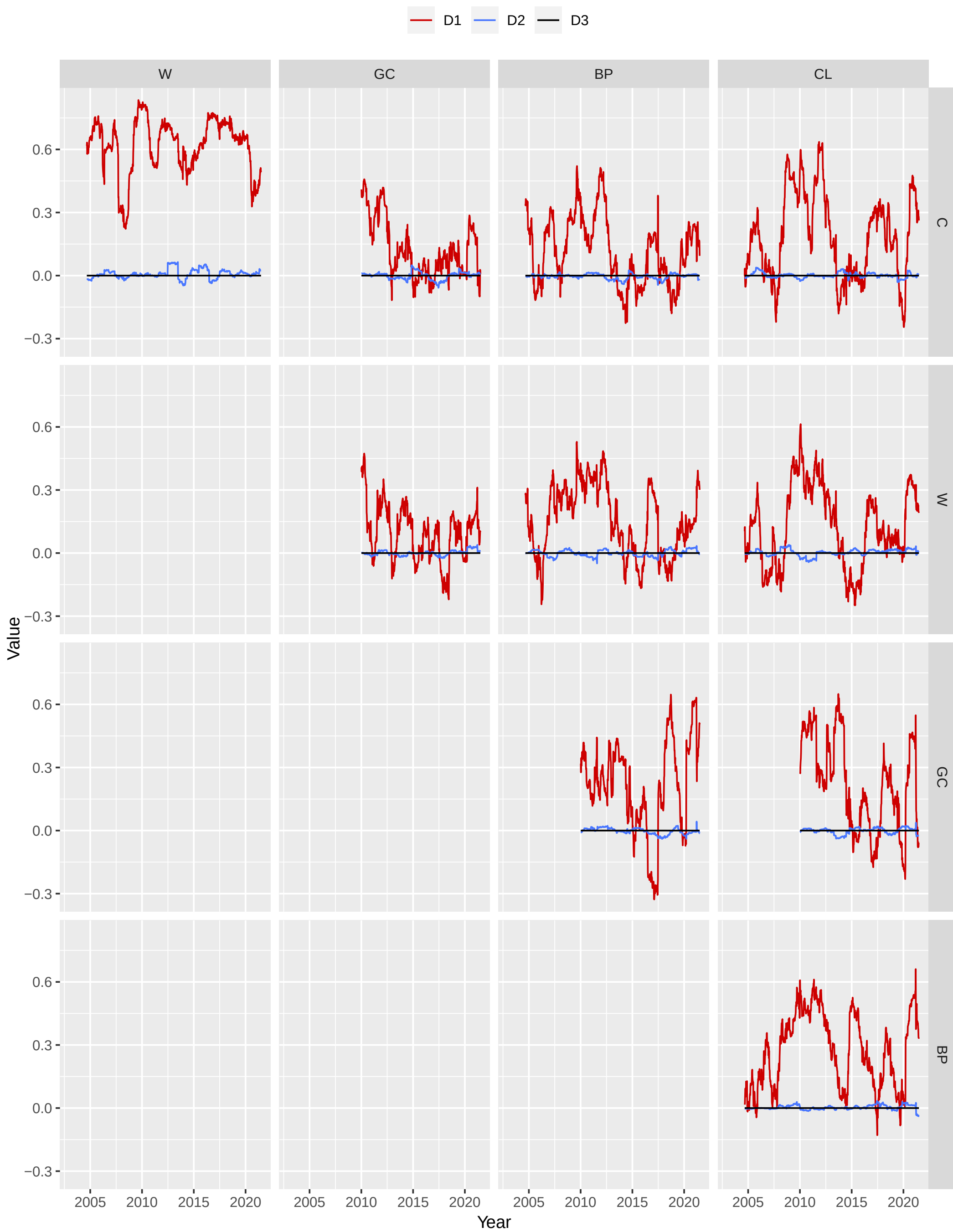

Figure 3: Correlation decomposition (futures maturing in six months).

Figure 4: Time series of correlations (daily futures return).

Table A1: Maximum absolute correlation adjustments - robustness checks

Pair	Date	Correlation	Adjusted correlation	Difference	% difference
Panel A: Daily returns					
BP-CL	2021-04-20	0.211	-0.060	-0.271	-129%
C-BP	2008-12-17	0.406	0.307	-0.099	-24%
C-CL	2021-04-20	0.177	0.041	-0.136	-77%
C-GC	2010-01-19	0.208	0.269	0.062	30%
C-W	2008-11-06	0.596	0.493	-0.104	-17%
GC-BP	2017-06-05	-0.117	-0.216	-0.099	-85%
GC-CL	2021-04-20	0.089	-0.077	-0.166	-186%
W-BP	2008-10-29	0.288	0.158	-0.130	-45%
W-CL	2008-11-24	0.409	0.309	-0.100	-25%
W-GC	2008-06-09	0.253	0.166	-0.087	-34%
Panel B: Futures maturing in 6 months					
BP-CL	2020-03-18	0.470	0.315	-0.155	-33%
C-BP	2017-06-07	0.232	0.099	-0.133	-57%
C-CL	2020-05-20	0.287	0.114	-0.173	-60%
C-GC	2020-05-13	0.238	0.156	-0.082	-35%
C-W	2011-07-13	0.683	0.548	-0.135	-20%
GC-BP	2020-03-18	0.500	0.361	-0.140	-28%
GC-CL	2020-04-08	0.516	0.237	-0.279	-54%
W-BP	2009-06-10	0.364	0.233	-0.131	-36%
W-CL	2007-01-10	0.008	-0.106	-0.114	-1423%
W-GC	2010-01-27	0.314	0.409	0.095	30%
Panel C: Futures maturing in a year					
BP-CL	2021-05-19	0.484	0.361	-0.123	-26%
C-BP	2016-09-14	0.287	0.170	-0.117	-41%
C-CL	2020-05-20	0.359	0.259	-0.100	-28%
C-GC	2012-08-01	0.213	0.129	-0.084	-39%
C-W	2011-04-06	0.768	0.626	-0.142	-19%
GC-BP	2020-03-18	0.468	0.318	-0.150	-32%
GC-CL	2020-04-08	0.463	0.232	-0.230	-50%
W-BP	2017-06-14	0.185	0.093	-0.092	-50%
W-CL	2007-01-10	0.049	-0.073	-0.122	-248%
W-GC	2010-01-27	0.317	0.411	0.094	30%

Note: Included in the table are the maximum absolute correlation adjustment (fifth column) in the sample for each pair of futures. The correlation adjustment is equal to the correlation of leveraged futures returns (fourth column) minus the correlation of returns under full collateral (third column). All returns are on a weekly basis. All correlations are computed over a one-year rolling window. Date (second column) refers to the right endpoint of the rolling window over which absolute correlation adjustment achieves its maximum. Sample period is Jan 2003 - May 2021.

Table A2: Maximum absolute correlation adjustments, more commodity futures

Pair	Date	Corr	Adj corr	Diff	% diff	Pair	Date	Corr	Adj corr	Diff	% diff
BO-BP	2010-03-17	0.439	0.327	-0.112	-25%	GC-TY	2012-01-25	0.122	0.280	0.157	129%
BO-CL	2021-04-28	0.057	0.204	0.147	257%	HG-BP	2010-03-03	0.597	0.470	-0.126	-21%
BO-FV	2010-06-09	-0.460	-0.230	0.230	50%	HG-CL	2016-11-02	0.377	0.137	-0.240	-64%
BO-GC	2020-04-08	0.436	0.252	-0.184	-42%	HG-FV	2010-06-02	-0.442	-0.264	0.178	40%
BO-HG	2010-05-12	0.631	0.469	-0.161	-26%	HG-JY	2021-03-10	0.237	-0.307	-0.544	-230%
BO-JY	2014-02-12	-0.302	0.155	0.457	151%	HG-NG	2013-06-05	-0.066	-0.161	-0.095	-144%
BO-NG	2019-05-08	-0.128	-0.022	0.106	83%	HG-TU	2010-01-27	-0.268	-0.407	-0.138	-52%
BO-TU	2021-03-03	-0.068	-0.191	-0.124	-183%	HG-TY	2010-04-28	-0.266	-0.102	0.164	62%
BO-TY	2020-09-09	-0.017	-0.167	-0.151	-900%	JY-CL	2021-03-17	-0.284	0.171	0.455	160%
BP-CL	2021-03-10	0.347	0.649	0.301	87%	JY-FV	2017-09-27	0.854	-0.166	-1.020	-119%
BP-FV	2018-02-07	0.096	-0.017	-0.113	-117%	JY-NG	2010-12-01	-0.316	0.131	0.447	141%
BP-JY	2006-01-11	0.637	-0.149	-0.786	-123%	JY-TU	2017-09-27	0.758	-0.134	-0.892	-118%
BP-NG	2019-05-08	0.109	0.007	-0.102	-94%	JY-TY	2007-08-15	0.744	-0.284	-1.028	-138%
BP-TU	2015-06-03	0.208	0.068	-0.140	-67%	NG-FV	2010-11-17	-0.093	-0.234	-0.141	-152%
BP-TY	2009-09-16	-0.151	-0.287	-0.136	-90%	NG-TU	2021-03-24	-0.104	0.024	0.128	123%
C-BO	2021-03-31	0.111	0.229	0.118	107%	NG-TY	2018-02-14	0.008	0.097	0.089	1083%
C-BP	2008-12-17	0.426	0.283	-0.143	-34%	S-BO	2018-03-21	0.447	0.357	-0.091	-20%
C-CL	2020-04-01	0.305	0.117	-0.189	-62%	S-BP	2018-02-14	0.194	0.276	0.082	42%
C-FV	2010-07-14	-0.369	-0.134	0.234	64%	S-CL	2020-08-26	0.436	0.630	0.195	45%
C-GC	2010-01-20	0.251	0.342	0.091	36%	S-FV	2010-07-14	-0.408	-0.314	0.093	23%
C-HG	2010-01-13	0.205	0.314	0.109	53%	S-GC	2018-05-23	0.062	0.162	0.100	160%
C-JY	2020-03-18	0.261	-0.173	-0.435	-166%	S-HG	2010-07-28	0.464	0.343	-0.121	-26%
C-NG	2019-03-13	-0.071	0.018	0.089	125%	S-JY	2021-03-17	-0.299	0.357	0.656	220%
C-S	2018-04-04	0.524	0.435	-0.089	-17%	S-NG	2019-05-01	0.041	0.136	0.096	235%
C-TU	2009-12-02	-0.010	-0.140	-0.130	-1287%	S-TU	2021-03-10	0.258	0.122	-0.135	-53%
C-TY	2008-10-15	0.021	-0.102	-0.123	-586%	S-TY	2020-10-14	0.214	0.130	-0.084	-39%
C-W	2011-07-13	0.699	0.537	-0.162	-23%	TU-FV	2010-10-20	0.908	0.623	-0.284	-31%
CL-FV	2020-03-25	-0.015	-0.283	-0.268	-1840%	TU-TY	2021-03-24	0.724	0.622	-0.102	-14%
CL-NG	2020-05-06	0.244	0.405	0.161	66%	W-BO	2011-05-11	0.668	0.576	-0.092	-14%
CL-TU	2020-05-20	-0.180	-0.455	-0.275	-152%	W-BP	2009-06-10	0.350	0.233	-0.118	-34%
CL-TY	2020-03-25	0.091	-0.217	-0.308	-339%	W-CL	2011-12-21	0.200	0.088	-0.112	-56%
FV-TY	2010-01-06	0.893	0.616	-0.277	-31%	W-FV	2010-06-09	-0.352	-0.126	0.226	64%
GC-BP	2017-06-07	-0.181	-0.321	-0.140	-77%	W-GC	2010-01-27	0.334	0.420	0.087	26%
GC-CL	2020-04-08	0.572	0.245	-0.327	-57%	W-HG	2011-06-08	0.566	0.468	-0.098	-17%
GC-FV	2010-01-06	-0.203	-0.040	0.163	80%	W-JY	2020-07-01	-0.500	0.128	0.628	126%
GC-HG	2012-04-04	0.443	0.261	-0.182	-41%	W-NG	2010-07-14	0.191	0.116	-0.075	-39%
GC-JY	2017-11-01	0.774	-0.374	-1.148	-148%	W-S	2011-01-12	0.558	0.431	-0.127	-23%
GC-NG	2020-04-08	0.291	0.174	-0.117	-40%	W-TU	2012-02-22	-0.205	-0.063	0.142	69%
GC-TU	2012-03-14	0.248	0.350	0.102	41%	W-TY	2011-06-15	-0.340	-0.181	0.159	47%

Note: Included in the table are the maximum absolute correlation adjustment (fifth column) in the sample for each pair of futures. The correlation adjustment is equal to the correlation of leveraged futures returns (fourth column) minus the correlation of returns under full collateral (third column). All returns are on a weekly basis. All correlations are computed over a one-year rolling window. Date (second column) refers to the right endpoint of the rolling window over which absolute correlation adjustment achieves its maximum. Sample period is Jan 2003 - May 2021. BO: soybean oil. BP: British Pound. C: corn. CL: WTI crude oil. GC: gold. HG: copper. JY: Japanese Yen. NG: Henry Hub natural gas. S: soybean. TU: 2-year US Treasury note. FV: 5-year US Treasury note. TY: 10-year US Treasury note. W: wheat.

Table A3: Standard deviations of roll-over and smoothed returns

	maturity: 1 month		maturity: 6 months	
	sd(r)	sd(r_s)	sd(r)	sd(r_s)
Panel A: Jan 2003 - May 2021				
C	0.042	0.040	0.031	0.037
W	0.045	0.050	0.033	0.039
GC	0.025	0.025	0.025	0.025
BP	0.013	0.013	0.013	0.013
CL	0.057	0.055	0.043	0.043
Panel B: Jan 2003 - Dec 2019				
C	0.042	0.041	0.031	0.037
W	0.045	0.051	0.034	0.040
GC	0.025	0.025	0.025	0.025
BP	0.013	0.013	0.013	0.013
CL	0.048	0.046	0.040	0.040

Note: r : futures returns obtained by rolling over expiring contracts. r_s : futures returns obtained by cubic spline. All returns are Wednesday-to-Wednesday weekly returns.