Optimal Contracting in a Principal-Agent-Subagent Model *

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ABSTRACT

This paper expands the classic principal-agent model and introduces a new continuous-time principal-agent-subagent model, in which the administrative manager has interpersonal authority over the productive employee. The two optimal contracts, (i) between the investors and the manager and (ii) between the manager and the employee, are uniquely determined in subgame perfect Nash equilibrium, which equates the manager's continuation value to her maximal profit from hiring the employee. When shirking is a more severe agency problem than stealing, the investors benefit from this three-layer hierarchy as they have limited liability, provide fewer minimum incentives, and their agency costs get mitigated. The manager, getting paid earlier, is self-motivated to supervise the employee, and it is suboptimal for them to collude in shirking. That twenty-year personnel data from a medium-sized firm shows that the agency costs are reduced when more work is conducted by hiring employees—rather than managers—lends support to my model.

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1 Introduction

Contractual relations are the essence of the firm (Jensen and Meckling, 1976). Conflicts of interests exist when different stakeholders have different utility maximization problems, and compensation contracts are designed to dissipate the divergence of interests. Despite the importance of contracts in solving agency issues, the absence of data has limited the existing empirical finance research on *executive* compensation studies, and mathematical complexity has limited the theoretical finance research on a *principal-agent* (PA) framework in which investors hire an entrepreneur or manager to operate the business. Non-executive or *productive employee* is almost silent in the context of agency problems. The analysis presented here represents a significant departure from the previous literature in that I introduce a *principal-agent-subagent* (PAS) model in which the administrative manager has interpersonal authority over the productive employee.[1] Under this three-layer hierarchy environment, the manager is hired by the investors to operate and administer the firm, and once the manager finds a profitable investment project, she hires and supervises an employee to carry out production activities. The PAS model contractual relations are described in Figure 1 (a).

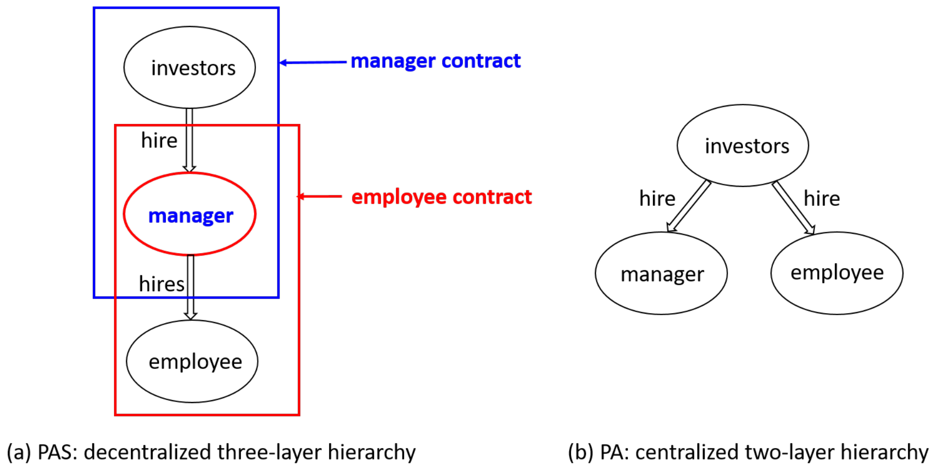

Figure 1: PAS (left panel) and PA (right panel) contractual relations

For each project, two contracts are involved: a *manager contract* that the investors commit to the manager, and an *employee contract* that the manager writes to the employee. There is no direct contractual relation between the investors and the employee. I address the following questions: (i) What are the optimal contracts under this environment? (ii) Does hiring the employee affect incentives provisions in the optimal contracts? (iii) Is the optimal manager contract under this environment different from the one under the PA setting?

[1] Van Den Steen defines interpersonal authority as (i) the manager tells the employees what to do, (ii) the employees tend to act in accord with these instructions; and (iii) they often do so against their own beliefs or immediate preferences (which distinguishes authority from advice).

The importance of "management" in a large corporation is widely documented.[2] However, existing dynamic contracting literature is reluctant to differentiate the manager's administrative role from the productive role, largely due to the difficulty of modeling cognitive capacities of the manager. For instance, typical continuous-time agency models use an ambiguous word "effort" which determines the growth of the outputs (e.g., He, 2009; Sannikov, 2008). My PAS framework offers one way to examine different roles by assigning "administrative" to the manager and "productive" to the employee.[3] The growth of the cash flows requires the employee to carry out production activities under the manager's supervision. Specifically, the manager (i) *makes decisions* about making investments and contracting with employees on behalf of the investors; (ii) *processes information* including collecting production information about the investments and generating performance reports to the investors; and (iii) has *interpersonal authority*, i.e., supervises and compensates employees based on their performance.[4]

Although authority is typically decentralized in an organization,[5] existing dynamic contracting literature focuses on a centralized PA framework, largely relying on the assumption of "complete contracts" where the Revelation Principle applies (Myerson 1982). Specifically, a PAS setting (decentralized three-layer hierarchy) can be replicated by a PA setting (centralized two-layer hierarchy) (Mookherjee 2013). Figure 1 illustrates and compares the PAS model with the PA model. However, risk aversion, participation constraint, and collusion of the manager and the employee potentially violate the conditions to apply the Revelation Principle (e.g., He, 2012; Sannikov 2008). By adopting the PAS model, this paper makes the first attempt to study the dynamic agency model in a decentralized framework, in which the manager has interpersonal authority over the employee.

Existing dynamic contracting literature studies the PA setting mostly under moral hazard. Specifically, the manager may be (i) *shirking* (He 2009), causing a reduction of the growth of the project, and/or (ii) *stealing* (DeMarzo and Sannikov 2006), causing a diversion of the earnings. However, this literature typically focuses on characterizing an optimal manager contract which resolves only one hidden action as either results in a reduction of the investors' profit, and treats the productive employee as an input (like a capital asset) which the manager transform into profit.[6] This treatment is insufficient in the PAS framework as it fails to recognize the moral hazard problem from the employee, *shirking*. In the PAS setting, although the productive employee neither observes the cash flows of the project nor has the access to the firm's internal

[2] Alfred Marshall has listed "management" as a fourth critical factor of production, besides land, labor, and capital.

[3] I examine the case in which the manager also carries out real production in the extension, see Section 6.2.

[4] In line with views of Alfred Marshall, Chandler (1962), Van Den Steen (2010), Mookherjee (2013), etc. about the executives' responsibilities

[5] Williamson (1967) finds it is impossible for a firm to growth without hierarchies.

[6] For instance, DeMarzo and Sannikov (2006) find that shirking is equivalent to stealing cash flows at a fixed rate.

funds, the growth of the project also fades when the employee stops providing costly effort. In addition, it is necessary to differentiate the two hidden actions from the manager in this three-layer hierarchy, as shirking from her is equivalent to stealing cash flows at a fixed rate only when the employee is working. Relative to the existing literature, this paper highlights how different hidden actions affect the agency problem.

My PAS model is constructed based on the seminal work introduced by Sannikov using martingale representation theorem in a continuous-time framework. Specifically, the manager contract adopts DeMarzo and Sannikov's (2006) and the employee contract adopts Sannikov's (2008). This framework is analytically convenient for several reasons. First, there is a unique state variable, *continuation value*, in each contract, which summarizes the action and compensation, and how they are affected by the project's cash flows. Second, it allows me to characterize each optimal contract in terms of ordinary differential equation (ODE) which can be numerically solved. Third, using Feynman-Kac formula, it allows me to conduct comparative static analyses and compare the optimal manager contract in my PAS framework to the one under the PA setting.

Most importantly, it allows me to solve the optimal contracts in subgame perfect Nash equilibrium (SPNE) using backward induction. My PAS framework, a dynamic game with imperfect information, inevitably complicates the PA framework. I first fix the manager's actions and solve the optimal employee contract, and then solve the optimal manager contract based on the employee's induced action. This helps me rule out noncredible Nash equilibrium (NE) which is suboptimal for all players.

I find the following main novel results. First, there is a unique SPNE solution in which both the manager and employee are working and the manager is reporting the cash flows truthfully. In this PAS setting, optimal contracts have the same stopping time. The manager's associations with both contracts and different roles in each provide a link between the two contracts. Specifically, in SPNE, the manager's continuation value in the optimal manager contract is equal to her highest profit in the optimal employee contract. The manager's continuation value is bounded below due to participation constraint, which also makes her profit bounded below. The manager's profit is bounded above due to its concavity nature, which also makes her continuation value bounded above. As a result, the employee's continuation value and the investors' profit are also bounded below and above in my PAS framework. This is unlike DeMarzo and Sannikov (2006) where the investors have unlimited liability, and also unlike the bang-bang contracts in Innes (1990) where limited liability is ex-ante.

Second, unlike in a PA setting (e.g., He, 2009; Sannikov, 2006), the employee's minimum incentives to work, although motivated by the manager, depend on both the employee's and manager's effort. The more the manager contributes to the project, the fewer incentives she needs to provide to the employee.

Third, in my PAS framework, the manager's minimum incentives actually come from two directions: (a) incentives to work come from self-motivation and (b) incentives to truth-telling come from the investors. This is different from DeMarzo and Sannikov (2006) and Shim (2011) in which both are incentivized by the investors. The manager's self-motivation to supervise the employee is a direct consequence of her administrative role. The manager needs to work in order to provide enough incentives to induce the employee to work. When the manager is shirking, the employee, as a result of lacking enough incentives, will be shirking, too. It is suboptimal since the employee will not only not contribute to the growth of the project but also share the positive cash flows with the manager.

Fourth and most importantly, the investors' agency costs in the PAS model is *less than* the agency costs in the PA model (DeMarzo and Sannikov, 2006). If the investors hire the manager and employee at the same level, they need to commit a contract to each of them individually as both have different maximization problems which might reduce the investors' profit. By granting interpersonal authority to the manager, the investors only need to contract with the manager and the agency costs occurred due to the employee's conflicts of interest have been partially transferred to the manager. It is now on the manager's shoulders to bear the agency costs from the employee's divergence of interests.[7] *Agency costs mitigation* is one reason of introducing hierarchies inside a firm.

Agency costs mitigation in my PAS framework also comes from resolving one manager's moral hazard problem, *shirking*. When shirking is a more severe agency problem than stealing, the investors benefit from this three-layer hierarchy as the manager is self-motivated to work and they do not need to provide extra incentives to induce her to work. The alliterative "stealing and shirking" are both costs to the investors and so it is correct to lump them together as agency costs. When shirking is no longer a moral hazard problem, agency costs reduces automatically.

Fifth, unlike DeMarzo and Sannikov's (2006) and He's (2009) where it is optimal for the manager to shirk after a history of good outputs, it is suboptimal in the PAS model for either the employee or the manager to shirk. This noncredible NE is automatically ruled out in backward induction.

I also conduct empirical tests to further provide evidence to my findings. The data consists twenty-year personnel records including all employees' salaries of a medium-sized U.S. firm. I use two alternative efficiency ratios, the expense ratio and the asset utilization ratio (Ang. Cole, and Lin 2000), as measures of agency costs. I use the salary ratio (employees' salaries over total salaries) as an explanatory variable. An increase in the salary ratio indicates more work are conducted by hiring employees instead of managers to finish, representing an adoption of a Principal-Agent-Subagent model over a Principal-Agent model. Tests are statistically

[7] In this paper, I am comparing the case with a hierarchy versus without. In reality, the optimal number of hierarchical levels are industry specific and are firm unique, should be more of empirical research interests.

significant with correct signs of coefficients, proving empirically that agency costs reduce under a Principal-Agent-Subagent framework.

My paper contributes the literature in many aspects. First, I offer an explanation to the wide existence of hierarchies from the agency theory. Second, according to my knowledge, this paper is the first attempt to expand the dynamic PA model by adding one more player, Subagent, without introducing additional constraints or frictions. Third, this paper helps complete the studies on contract theory. It is also the first paper, according to my knowledge, trying to examine the connections between two contracts. Fourth, this paper also contributes the continuous-time finance literature.

I present the PAS model in Section 2, derive the subgame perfect Nash equilibrium optimal contracts in Section 3, and discuss the optimal contracts' properties in Section 4. Section 5 considers an empirical study on agency costs, and Section 6 considers the model's extensions. Finally, Section 7 concludes the paper. Proofs are in the Appendix.

2 Principal-Agent-Subagent Model

To offer a clearer reference, throughout the paper, I use "they" as a pronoun for investors, "she" for manager, and "he" for employee.

My basic framework expands the classic Principal-Agent model with a Subagent, in which the risk-neutral investors[8] of an infinitely lived firm hire a risk-neutral[9] manager to run the firm. In line with views of Alfred Marshall, Chandler (1962), Van Den Steen (2010), Mookherjee (2013), etc. about the executives responsibilities: the manager (i) *makes decisions* about making investments and contracting with employees on behalf of the investors; (ii) *processes information* including collecting production information about the investments and generating performance reports to the investors; and (iii) has *interpersonal authority*, i.e., supervises and compensates employees based on their performance.

The manager has found a profitable investment opportunity that, if funded with an initial investment of $K > 0$,[10] would generate a *cumulative* cash fow X_t per unit of time, which evolves according to an arithmatic Brownian motion

$$dX_t = \mu dt + \sigma dB_t,$$

where $B = \{B_t, \mathcal{F}_t; 0 \leq t < \infty\}$ is a standard Brownian motion in a complete probability

[8]We can think of the investors have well-diversified portfolios, which essentially make them risk-neutral.

[9]This manager is risk-neutral but effectively risk-averse, as the manager has a linear vN-M utility of salary on the positive half-line, while $-\infty$ below zero.

[10]I would not discuss how the firm fund itself as it is not the main focus of this paper. For reference, please refer to DeMarzo, Fishman, He, and Wang (2012).

space $(\Omega, \mathcal{F}, \mathcal{P})$. The manager hires and supervises a risk-averse employee to carry out real production.

To capture the manager's and employee's each contribution to the project, I assume

$$dX_t = m_t e_t dt + \sigma dB_t, \tag{1}$$

where $m_t \in \{0, 1\}$ and $e_t \in \{0, \mu\}$ are the manager's and employee's *binary* choice of effort, "shirking" and "working," respectively. Here, $m_t = 1$ means the manager is supervising the employee and $e_t = \mu$ means the employee is working on the production, at time t. The assumptions capture the manager's unique administrative role in supervising the activities of the productive employee, which also require both the manager and employee to work at the same time to achieve the expected return μ from the project at time t. There is an effort cost of $\phi > 0$ and $h > 0$ for the manager and employee, respectively, from working.[11] Only the manager observes the cash flows X_t, neither the employee nor the investors do. The manager reports cash flows $\hat{X}_t \leq X_t$ to the investors, and receives a fraction $\lambda \in (0, 1)$ of the cash flows she steals, $\lambda(dX_t - d\hat{X}_t)$, at time t.[12] These assumptions capture the manager's unique administrative role in processing information.

Below is a summary of the Principal-Agent-Subagent model assumptions:

	Investors	**Manager**	**Employee**
Pronoun	"they"	"she"	"he"
Risk attitude	risk neutral	risk neutral but effectively risk averse	risk averse with utility function
Effort choice	–	$m_t = 1$ working $m_t = 0$ shirking	$e_t = \mu$ working $e_t = 0$ shirking
Effort cost	–	$\phi > 0$ working 0 shirking	$h > 0$ working 0 shirkig
Cash flows	unobservable	Observes, reports $\hat{X}_t$, receives $\lambda(dX_t - d\hat{X}_t)$	unobservable

Table 1: Principal-Agent-Subagent model

For simplicity, I assume the investors, manager, and employee are equally patient, and all of them discount future cash flows at the market interest rate $r > 0$. When both the manager and employee are working and the manager is not stealing the earnings at all time, the project's

[11] Everything remains the same if I extend the binary effort choice and effort cost to a compact set whenever the ratio of effort cost to the mean of the project remains the same.

[12] I assume $X_t - \hat{X}_t$ is Lipschitz-continuous so the manager steals only at a bounded rate.

first-best value (risk-sharing) is

$$E_t\left[\int_t^{\infty} e^{-r(s-t)}dX_s\right] = \frac{\mu}{r}.$$

The manager and employee start to work on this project together at time $t = 0$. In light of Spear and Wang's (2005) work about optimal termination of contracts, the project output has a deterministic impact on the employee's career because of his productive role. When the output is too bad, the employee is *fired* due to the employee's poor performance as the manager finds him to poor to punish effectively; when the output is too good, the employee is *laid off* due to the manager's participation constraint as the manager finds him too expensive to motivate continuously. In light of Chandler's (1962) notes about most top executives' non-involvement in actual production activities, the manager's association with the project is terminated *only* when the project is liquidated. Unlike the employee, the manager will not be fired by the investors just based on the performance of this project. Though still incumbent, the manager shifts her focus to other administrative duties or tries to find another investment opportunity. I assume the firm has multiple projects running and the other projects are doing well enough to offset the temporary loss from this project. Upon liquidation, the investors recover an inefficient liquidation value of $L < \frac{\mu}{r}$, while the manager and employee get nothing. Obviously, the manager will be fired if there is a series of bad performance, such as project liquidations, which make her either below the investors' expectation or make the firm close to bankruptcy. However, this is not the focus of this paper.

Both the manager and employee have no initial wealth, and they cannot save, either. The outside option for each is assumed to be 0. As the investors can always choose not to invest the project, they will only commit to contracts that generate non-negative profit for them. Similarly, the manager can always choose not to hire anyone, so she will only contract with an employee who can bring non-negative profit for her.

2.1 The formulation of the investors' problem

Denote $\Pi_m \equiv \{\{u\}, \tau_m\}$ as the manager contract that the investors commit to the manager, in which τ_m corresponds to the project liquidation time suppose the employee is still incumbent, and $\{u_t, 0 \le t \le \tau_m\}$ is the salary the investors pay to the manager. I say the manager's effort process $\{m_t, 0 \le t \le \tau_m\}$ and the reported cash flow $\{\hat{X}_t, 0 \le t \le \tau_m\}$ are incentive compatible with respect to salary $\{u_t, 0 \le t \le \tau_m\}$ if they maximize the manager's total expected profit until the project's liquidation time τ_m, denoted by $\mathbb{IC}_m$.

The investors' problem is to offer an incentive compatible contract to the manager that maximizes their profit

$$I_0 = \max_{\Pi_m \in \mathbb{IC}_m} E\left[\int_0^{\tau_m} e^{-rt}(d\hat{X}_t - u_t dt) + e^{-r\tau_m} L\right]$$

subject to giving the manager a specific value of $W_0^m > 0$

$$W_0^m = \max_{\substack{m=\{m_t \in \{0,1\}: 0 \le t \le \tau_m\} \\ d\hat{X}=\{d\hat{X}_t \in [0, dX_t]: 0 \le t \le \tau_m\}}} E\left[\int_0^{\tau_m} e^{-rt}\Big(\lambda(dX_t - d\hat{X}_t) + (u_t - c_t - \phi m_t)dt\Big)\right]$$

and the manager's participation constraint for all $0 \le t \le \tau_m$

$$E_t\left[\int_t^{\tau_m} e^{-r(s-t)}\Big(\lambda(dX_s - d\hat{X}_s) + (u_s - c_s - \phi m_s)ds\Big)\right] \ge 0.$$

2.2 The formulation of the manager's problem

Similarly, denote $\Pi_e \equiv \{\{c\}, \tau_e\}$ as the employee contract that the manager writes to the employee, in which τ_e corresponds to the employee's firing or layoff time conditioning on the project keeps running, and $\{c_t, 0 \le t \le \tau_e\}$ is the consumption the employee receives from the manager. I say the employee's effort process $\{e_t, 0 \le t \le \tau_e\}$ is incentive compatible with respect to consumption $\{c_t, 0 \le t < \tau_e\}$ if it maximizes the employee's total expected utility until the employee's firing or layoff time τ_e, denoted by $\mathbb{IC}_e$.

The manager's problem is to offer an incentive compatible contract to the employee that maximizes her profit

$$M_0 = \max_{\Pi_e \in \mathbb{IC}_e} E\left[\int_0^{\tau_e} e^{-rt}\Big(\lambda(dX_t - d\hat{X}_t) + (u_t - c_t - \phi m_t)dt\Big)\right]$$

subject to giving the employee a specific value of $W_0^e > 0$

$$W_0^e = \max_{e=\{e_t \in \{0,\mu\}: 0 \le t \le \tau_e\}} E\left[\int_0^{\tau_e} e^{-rt}\Big(u(c_t) - \frac{h}{\mu}e_t\Big)dt + e^{-r\tau_e} W_{\tau_e}^e\right]$$

and the employee's participation constraint for all $0 \le t \le \tau_e$

$$E_t\left[\int_t^{\tau_e} e^{-r(s-t)}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + e^{-r(\tau_e - t)} W_{\tau_e}^e\right] \ge 0.$$

Remark. Here, I impose no extra condition on W_0^m or W_0^e, except bounded below by zero due to participation constraint, as I make no assumption about the market. This Principal-Agent-Subagent model can be tailored to any types of market systems simply by varying W_0. For example, W_0^m corresponds to the value which maximizes the investors' profit for a competitive market of managers (He 2009).

3 Model Solution

To solve the Principal-Agent-Subagent model, I use backward induction: I first take the manager's effort choice and reporting amount as given, derive the optimal employee compensation contract, and then solve the optimal manager compensation contract based on the employee's optimal effort choice. I further explore the Nash equilibrium results by the end of this section.

3.1 Optimal Employee Compensation Contract

To solve for the optimal employee contract, I first fix the manager's effort strategy $m = \{m_t\}$ and the amount of stealing $\hat{X} = \{\hat{X}_t\}$ as the employee has no control over his manager's effort choice and the amount she is diverting. I also assume the manager is always supervising him, so $m_t = 1$ for all t, and in section 4.1.4, I discuss the case when the manager is shirking. I further assume that the manager tells the truth at all time, hence $\hat{X}_t = X_t$ for all t. In section 4.1.5, I discuss the case when the manager is under-reporting.

Therefore (1) becomes

$$dX_t = e_t dt + \sigma dB_t$$

and the manager's problem becomes

$$\max_{\Pi_e \in \mathbb{IC}_e} E\left[\int_0^{\tau_e} e^{-rt}(u_t - c_t - \phi)dt\right].$$

Later in the optimal manager contract, I will show under what conditions the manager is induced to work and report the true earnings.[13]

3.1.1 Employee's continuation payoff and incentive compatibility

Fix an arbitrary consumption process $c = \{c_t\}$ and an effort strategy $e = \{e_t\}$, which may or may not be optimal for the employee given c. The employee's continuation value, his expected future payoff from (c, e) after time t to firing or layoff time τ_e, is

$$W_t^e(c,e) = E_t^e\left[\int_t^{\tau_e} e^{-r(s-t)}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + e^{-r(\tau_e - t)}W_{\tau_e}^e\right], \tag{2}$$

where E^e denotes the expectation under the probability measure Q^e induced by the employee's strategy e. Based on the Martingale Representation Theorem, the following proposition ex-

[13]Note: Employee contract makes sense only if there is a manager contract, therefore $\tau_e \leq \tau_m$, aka the project is still running. This will be further discussed later in this section.

presses the evolution of W_t^e in terms of the cash flows path $\{X_s : 0 \le s \le t\}$, and provides a sufficient and necessary condition to induce the employee to work.

Proposition 1. *There exists a progressively measurable process $\theta^e = \{\theta_t^e, \mathcal{F}_t; 0 \le t \le \tau_e\}$ in $\mathcal{L}^2$ such that* [14]

$$dW_t^e = \Big(rW_t^e - u(c_t) + \frac{h}{\mu}e_t\Big)dt + \theta_t^e\big(dX_t - e_t dt\big) \tag{3}$$

for all $t \in [0, \tau_e]$, and working is optimal for the employee if and only if

$$\theta_t^e \ge \frac{h}{\mu} \tag{4}$$

almost everywhere for all $t \in [0, \tau_e]$. $\frac{h}{\mu}$ represents the minimum incentives required to motivate the employee to work.

Proposition 1 states that θ^e is the sensitivity of the employee's continuation value to cash flows. When he is working, $e_t = \mu$, $dX_t - \mu dt$ has mean 0, and $\big(rW_t^e - u(c_t) + h\big)dt$ is the drift of the employee's continuation value, which grows at the interest rate r and falls due to the consumption utility $u(c)$ and effort cost h. Later in the optimal employee contract, working is optimal in the employment interval $(0, W_p^e)$ and the employee is fired or laid off whenever W^e hits 0 or W_p^e. To motivate the employee to work, the instantaneous volatility of his continuation payoff, θ^e, must be larger than $\frac{h}{\mu}$. To see this, if he chooses to shirk, he gains a private benefit of hdt (no effort cost) but loses $\mu\theta^e dt$ in compensation (dX_t becomes driftless under shirking). Thus he will work if and only if $\mu\theta^e \ge h$, or $\theta^e \ge \frac{h}{\mu}$. Since it is costly to expose him to risk, later in the optimal employee constract θ^e is set at the minimal level that induces him to work, $\theta^e = \frac{h}{\mu}$.

3.1.2 Optimality equation and boundary conditions

Now I use the dynamic programming approach to determine the most profitable way for the manager to deliver the employee any value W^e. W^e is the unique state variable in this contract which determines the employee's consumption $c(W^e)$, effort choice $e(W^e)$, and how W_t^e changes by X_t. Consider the highest profit $M(W^e)$ that the manager can derive when she delivers to the employee with value W^e. Function $M(W^e)$ together with the optimal choices of $e(W^e) = \mu$ and $c(W^e)$ satisfy the Hamilton-Jacobi-Bellman (HJB) equation

$$rM(W^e) = \max_c \; u - c - \phi + M'(W^e)\Big(rW^e - u(c) + h\Big) + \frac{M''(W^e)}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2. \tag{5}$$

[14] $\mathcal{L}^2$ **space**: A process θ is in $\mathcal{L}^2$ if $E\Big[\int_0^t \theta_s^2 ds\Big] < \infty$ for all $t \in [0, \infty)$.

This optimality equation plays a key role in analyzing the optimal employee contract, which states that the manager is maximizing the expected current flow of profit $u - c - \phi$ plus the expected change of future profit due to the drift and volatility of the employee's continuation value.

The optimal choice of consumption maximizes

$$-c - M'(W^e)u(c)$$

There are potentially two different phases in the employee's life-cycle: vesting and vested period [15]. During the vesting period, the employee must wait in order to be compensated while putting effort. Phases are entirely determined by his continuation value, and not necessarily in chronological order, therefore reversible. It is completely possible that the employee receives consumption from the manager now but gets nothing in the next second, and vice versa.

(a) Vesting Period: $(0, W_v^e]$. The employee's consumption is 0.

Intuitively, consumption zero is optimal because it maximizes the drift of W_t^e away from the inefficient low firing point.

(b) Vested Period: (W_v^e, W_p^e). The employee's consumption satisfies

$$u'(c) = -\frac{1}{M'(W^e)} > 0.$$

Intuitively, $1/u'(c)$ and $-M'(W^e)$ are the marginal costs of giving the employee value through current consumption and through his continuation payoff, respectively. Those marginal costs must be equal under the optimal contract, except in the vesting interval.

Three boundary conditions are required to pin down a solution to the optimality equation (5) and layoff boundary W_p^e. The manager fires the employee when W^e hits 0 or lays him off when W^e reaches W_p^e ($W_p^e > 0$). Let τ_e be the first hitting time at which either $W^e = 0$ or $W^e = W_p^e$.

(a) Firing boundary at $W^e = 0$:

$$M(0) = 0 \tag{6}$$

Once $W^e = 0$, θ^e from Proposition 1 is 0, therefore the manager cannot provide enough incentives to induce the employee to work, so the dynamic of W^e becomes

$$dW_t^e = -u(c_t)dt \leq 0,$$

[15]Vesting is an issue in conjunction with employer contributions to an employee stock option plan, or to a retirement plan such as a 401(k), annuity or pension plan.

and no matter how much positive consumption the manager can deliver to the employee, his continuation value will decrease further below zero.[16] The employee is too poor to be punished effectively, hence he will be fired by the manager and gets nothing.

(b) Layoff boundary and smooth pasting conditions at $W^e = W^e_p$:

$$M(W^e_p) = 0 \tag{7}$$

$$M'(W^e_p) = \frac{c+\phi}{rW^e_p - u(c)} \tag{8}$$

Because the manager's highest profit from hiring the employee is a *concave* function with a straightly positive initial slope starting from $W^e = 0$[17], it is optimal for the manager to lay off the employee when his continuation payoff becomes sufficiently large. The layoff boundary W^e_p is defined as the point at which the manager's payoff hits 0 and beyond which will be negative from continuing hiring the employee. The smooth pasting condition at W^e_p is defined at which the employee exerts 0 effort[18]. When the employee is laid off, he immediately receives a positive consumption as a severance pay from the manager. Layoff is different from firing although in both scenarios the employee is no longer associated with the firm. The severance consumption is also defined in the smooth pasting condition (8).

Layoff happens due to the income effect: when the flow of payments to the employee is large enough, it costs the manager too much to compensate him for his effort, so it is optimal to allow effort 0. When the employee gets richer, the monetary cost of delivering utility to him rises indefinitely (since $u'(c) \to 0$ as $c \to \infty$), while the utility cost of output stays the same (either 0 or μ).

Another important reason, which distinguishes the Principal-Agent-Subagent model from the classic Principal-Agent model, is that in Nash equilibrium, the manager's highest profit from the optimal employee contract equals to her continuation value from the optimal manager contract. Since the manager's continuation is bounded below by zero due to participation constraint, her maximized payoff from the employee must be nonnegative, as well. This is verified in Section 4.5. However, this type of constraint does not exist in the Principal-Agent model.

3.1.3 Optimal employee contract

Theorem 1 summarizes the optimal employee compensation contract in the Principal-Agent-Subagent model.

[16] Later in the this section, I will show at the firing point, the manager's salary u is zero, too.

[17] Concavity of the manager's payoff function is proved in the Appendix.

[18] Later in the this section, I will show at the layoff boundary, the manager's salary u is zero.

Theorem 1. ***Optimal Employee Compensation Contract***
The unique concave manager's value function M that satisfies the ODE (5) with boundary (6) and (7) and smooth-pasting conditions (8) characterizes any optimal employee contract with positive profit to the manager. There are three cases for employee's starting value $W_0^e > 0$:

- *case 1: $0 < W_0^e \le W_v^e$: this employee is considered to be underqualified, hence the manager would hire him with a vesting period.*
- *case 2: $W_v^e < W_0^e < W_p^e$: this employee is considered to be qualified, hence the manager would hire him with a vested period.*
- *case 3: $W_0^e \ge W_p^e$: this employee is considered to be overqualified, hence the manager would choose not to hire him.*

If $W_0^e \in [0, W_p^e]$, then the optimal employee contract attains profit $M(W_0^e)$. Such a contract is based on the employee's continuation value as a state variable, which starts at W_0^e and evolves according to

$$dW_t^e = \big(rW_t^e - u(c_t) + h\big)dt + \frac{h}{\mu}\big(dX_t - \mu dt\big)$$

under consumption $c_t = c(W_t^e)$ and effort $e_t = \mu$, until the firing or layoff time τ_e. Firing occurs when W_t^e hits 0 for the first time, and after that, the employee gets zero consumption and puts zero effort. Layoff happens when W_t^e hits $W_p^e > 0$ for the first time. After being laid off, the employee gets a lump-sum consumption of c and puts effort 0. In either case, the employee is no longer associated with the project or the firm.

A typical form of the value function $M(W^e)$, together with W_v^e and W_p^e are shown in Figure 2.

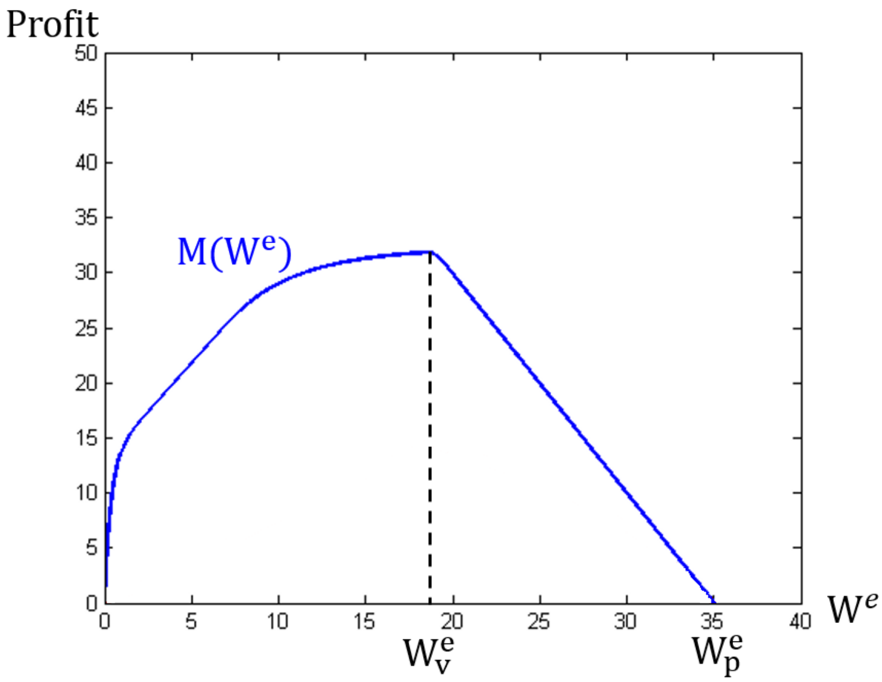

Figure 2: For $\mu = 10$, $r = 0.1$, $\sigma = 5$, $h = 3$, $\phi = 4$, $\lambda = 1$, $L = 25$, $u(c) = 4\sqrt{c}$, $c \in [0, 1]$

Note My solving for the optimal employee contract relies on two crucial assumptions: (1) I assume the manager is supervising the employee all the time, so $m_t = 1$ for all $0 \leq t \leq \tau_e$; (2) I also assume she is always reporting the entire earnings to the investors, hence $\hat{X}_t = X_t$ for all $0 \leq t \leq \tau_e$. Before jumping to solve the optimal manager contract, I relax each assumption and reexamine the optimal employee contract.

3.1.4 Optimal employee contract when the manager is shirking

When the manager is shirking, the project cash flow follows

$$dX_t = \sigma dB_t$$

regardless of the employee's effort choice. The manager's problem, therefore, becomes

$$\max_{\Pi_e \in \mathbb{IC}_e} E\left[\int_0^{\tau_e} e^{-rt}(u_t - c_t)dt\right]$$

and the dynamic of the employee's continuation payoff (Proposition 1) becomes

$$\begin{aligned} dW_t^e &= \left(rW_t^e - u(c_t) + \frac{h}{\mu}e_t\right)dt + \theta_t^e dX_t \\ &= \left(rW_t^e - u(c_t) + \frac{h}{\mu}e_t\right)dt + \theta_t^e \sigma dB_t^e. \end{aligned}$$

However, the manager cannot induce the employee to work, or the minimum incentives required to motivate working is ∞. The following proposition summarizes the result.

Proposition 2. *Working is suboptimal for the employee when the manager is shirking.*

The intuition for this proposition is straightforward. The project requires both the manager and employee to work together, even though the employee is the only one carrying out real production and the manager's role is only to supervise the productive employee. The project stops growing if either of them is not working. When the manager is shirking, the output dX_t becomes driftless whenever the employee is working or shirking. If the employee chooses to shirk, he gains a private benefit of hdt from not putting effort but loses nothing in compensation. Thus the manager needs to work in order to provide enough incentives to induce the employee to work. If the manager is shirking, the employee will be shirking, too.

3.1.5 Optimal employee contract when the manager is stealing

Now I relax the second assumption and "allow" the manager to secretly divert cash flows for her own benefit. Below proposition summarizes the findings.

Proposition 3. *It is suboptimal for the manager to steal, even though she can still provide enough incentives to induce the employee to work.*

To see this, without loss of generality, I assume

$$dX_t - d\hat{X}_t = a_t dt$$

where a_t is progressively adapted to $\mathcal{B}^e_{(\cdot)}$. The manager's problem, therefore, becomes

$$\max_{\substack{\Pi_e \in \mathbb{IC}_e \\ a=\{a_t: 0 \le dX_t - d\hat{X}_t = a_t dt\}}} E\left[\int_0^{\tau_e} e^{-rt}\big(\lambda a_t + u_t - c_t - \phi\big)dt\right],$$

hence the HJB equation in Section 4.1.2 becomes

$$rM(W^e) = \max_{c,a}\,(\lambda a + u - c - \phi) + M'(W^e)\Big(rW^e - u(c) + h\Big) + \frac{M''(W^e)}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2.$$

The optimal choice of the stealing amounts should maximize λa and, since $\lambda > 0$, the manager will essentially choose to steal the entire earnings from the project and report 0 to the investors. As the investors get nothing left, this problem can now be viewed as a classic Principal-Agent problem in which the manager is essentially owning the firm and being the real "principal." However, this is inefficient. The intuition for this is that, since $\lambda < 1$, there are dead-weight costs of concealing and diverting cash flows.[19] The investors could always increase their profit through improving the manager contract by making direct payments to the manager, in which there are no dead-weight costs, instead of "allowing" her to steal. In this case, the manger's value remains the same, while the investors' profit increases from saving the unnecessary costs.

So far, I solve the optimal employee contract and derive the condition under which working is optimal for the employee. In the following subsection, I derive the optimal manager contract and explore under what conditions working and truth-telling are incentive compatible for the manager.

3.2 Optimal Manager Compensation Contract

In section 4.1.4, I have shown that when the manager is not supervising the employee, the employee has no incentives to work, either. Therefore the investors need to design a contract that motivates the manager to work. In section 4.1.5, I have shown that when the manager is allowed to steal, she has incentives to essentially take over the entire project and become the real "owner." This is inefficient, suboptimal and definitely not what the investors want, hence an optimal manager contract should also induce the manager to report the true performance of the project to the investors. However, I first show in this section that, when the manager has interpersonal authority over the employee, the manager is self-motivated to work and the investors do not need to provide extra monetary incentives to her under the Principal-Agent-Subagent model. This is a key difference against the classic Principal-Agent framework in which there are no hierarchies among agents.

3.2.1 Manager is self-motivated to work with interpersonal authority

In the classic Principal-Agent model without a hierarchy between agents, each of the manager and employee has two types of moral hazard problems, shirking and stealing, both of which are costly for the investors. The investors couldn't tell either of the hidden actions and have to incentivize each to take the desired actions in order to maximize their profit. Alchian and Demsetz (1972) have come up with a solution that the manager undertakes a specialized duty—to *supervise* the other and has the authority to expel or replace him for unsatisfied performance.

After granting the administrative manager with interpersonal authority over the employee, the investors' agency costs are mitigated as (1) there is no direct contractual relation between the investors and employee, and the investors do not need to sacrifice their profit to induce the employee to work; (2) the employee has no incentives to steal as he has no access to the

[19] DeMarzo and Sannikov (2006) have shown that the manager pays a proportional cost $(1-\lambda)$ to the stealing amounts, even if being put back into the project again.

project's cash flows due to the presence of a hierarchy; (3) the manager is self-motivated to work, e.g. supervising the employee, and the investors do not need to provide extra incentives to motivate the manager to work.

(1) and (2) follow directly from the definition of interpersonal authority, and Proposition 4 in the following states (3) and explores the property.

Proposition 4. *The administrative manager, who has interpersonal authority over the productive employee, is self-motivated to supervise him.*

Proof of this proposition is quite straightforward. I have shown in Proposition 2 that once the manager is not working, the employee will be shirking, too. A direct consequence is that if the manager decides not to pay effort to this project, hiring an employee for her is essentially a pure cost. The manager needs to compensate the employee based as she has interpersonal authority over him. Since the employee's consumption c is nonnegative, when the manager is shirking, hiring the employee will not only not increase the salary for the manager (the project is not growing), but also share the nonnegative part of earnings with her. Therefore, it is suboptimal for the manager to shirk due to the hierarchy.

This is the biggest advantage of introducing hierarchies among agents which makes the those, who have interpersonal authority over others, self-motivated to work. Without a hierarchy, since effort is not costless, the manager would choose shirking over working if she wasn't got enough compensation from the investors. With a hierarchy, even though the manager still has the temptation to shirk, the benefits she receives from supervising and motivating the employee to work is larger than shirking. I show in Section 7.3 that it is suboptimal for the manager to collude with the employee to shirk. After granting interpersonal authority to the manager, the investors know she is self-motivated to work and, therefore, do not need to provide extra incentives to stimulate her. These saved incentives provision actually mitigate the investors' agency costs, compared to a Principal-Agent model in which there isn't a hierarchy between two agents. This is further discussed in Section 5.

In the Principal-Agent-Subagent framework, the only moral hazard problem the investors need to worry about is the possibility that the manager might steal from the project or underreport the true earnings to them. This is in agreement with Roe's (2005) summary about corporate government that the executives' incentives problem is usually not the executives are lazy and do not work hard enough. Corporate executives put in remarkably long hours of very intense effort. A more severe problem is they take actions other than maximizing the investors' profit, which I define as "stealing" in a broad sense.[20]

[20] DeMarzo and Sannikov (2006) treat shirking as stealing. However, this is true only if $\lambda = \frac{\phi}{\mu}$. Since λ is assumed to be less than 1, and if the manager's effort cost is larger than the drift of the dynamics of cash flow, then $\lambda < \frac{\phi}{\mu}$. Only making the sensitivity larger than λ is not enough to guarantee the manager to work, and

Therefore, throughout the rest of this section, under the Principal-Agent-Subagent framework and the optimal employee contract which I derive previously, it makes sense to assume the dynamic of the cash flow X_t evolves according to

$$dX_t = \mu dt + \sigma dB_t,$$

and the investors' problem becomes

$$\max_{\Pi_m \in \mathbb{IC}_m} E\bigg[\int_0^{\tau_m} e^{-rt}(d\hat{X}_t - u_t dt) + e^{-r\tau_m} L\bigg].$$

I discuss the optimal manager contract when the manager is shirking in Section 7.3.[21]

3.2.2 Manager's continuation payoff and incentive compatibility

Fix an arbitrary salary process $u = \{u_t : 0 \le t \le \tau_m\}$, an effort strategy $m = \{m_t = 1 : 0 \le t \le \tau_m\}$ and an amount of reporting $\hat{X} = \{\hat{X}_t : 0 \le t \le \tau_m\}$, which may or may not be optimal for the manager given u. The employee's consumption process $c = \{c_t : 0 \le t \le \tau_m\}$ has been optimally determined by the optimal employee contract in Section 4.1. Define the manager's continuation value $W_t^m(u, 1, \hat{X})$ after a history of reports $(\hat{X}_s, 0 \le s \le t)$ to be the total expected payoff the manager receives, from transfers and termination utility, if she tells the truth after time t to liquidation time τ_m, to be:

$$W_t^m(u, 1, \hat{X}) = E_t^m\bigg[\int_t^{\tau_m} e^{-r(s-t)}(u_s - c_s - \phi)ds\bigg], \tag{9}$$

where E^m denotes the expectation under the probability measure Q^m induced by the manager's report $\hat{X}$. Based on the Martingale Representation Theorem, the following proposition expresses the evolution of W_t^m in terms of the cash flows path $\{X_s : 0 \le s \le t\}$, and provides a sufficient and necessary condition to induce the manager to report truth earnings.

Proposition 5. *There exists a progressively measurable process $\theta^m = \{\theta_t^m, \mathcal{F}_t; 0 \le t \le \tau_m\}$ in $\mathcal{L}^2$ such that*

$$dW_t^m = \big(rW_t^m - u_t + c_t + \phi\big)dt + \theta_t^m\big(d\hat{X}_t - \mu dt\big) \tag{10}$$

we need the sensitivity to be larger than $\frac{\phi}{\mu}$ as well. This will only happen in the classic Principal-Agent model without hierarchies, but it will not happen in a Principal-Agent-Subagent model, as the agent is self-motivated to work.

[21]Note: Proposition 5 also implicitly implies that $\tau_m \le \tau_e$, since the manager can choose not to hire the employee. This will be further discussed in Section 7.3.

for all $t \in [0, \tau_m]$, and truth-telling is incentive compatible if and only if

$$\theta_t^m \geq \lambda \tag{11}$$

almost everywhere for all $t \in [0, \tau_m]$. λ represents the minimum incentives required to motivate the manager to tell the truth.

Proposition 5 states that θ^m is the sensitivity of the manager's continuation value toward her reports. When she is not stealing, $\hat{X} = X$, $dX_t - \mu dt$ has mean 0, and $\big(rW_t^m - u_t + c_t + \phi\big)dt$ is the drift of the manager's continuation value, which grows at the interest rate r and falls due to her salary, the employee's consumption, and effort cost ϕ. The manager has incentives not to steal earnings if she gets at least λ of continuation value for each reported dollar, that is, if $\theta_t^m \geq \lambda$. If this condition holds for all t, then the manager's payoff will always integrate to less than her continuation value if she deviates. If this condition fails on a set of positive measure, the manager can obtain at least a little bit more than her continuation value if she underreports earnings when $\theta_t^m < \lambda$. Given the concavity of I, which is proved in the Appendix, $\theta^m = \lambda$ is optimal.

3.2.3 Optimality equation and boundary conditions

Now I use the dynamic programming approach to determine the most profitable way for the investors to deliver the manager any value W^m. W^m is the unique state variable in this contract which determines the manager's salary $u(W^m)$, reporting amounts $\hat{X}(W^m)$, and how W_t^m changes by X_t. Consider the highest profit $I(W^m)$ that the investors can derive when they deliver the manager value W^m. Function $I(W^m)$ together with the optimal choices of $\hat{X}(W^m) = X$ and $u(W^e)$ satisfy the Hamilton-Jacobi-Bellman (HJB) equation

$$rI(W^m) = \max_u \; \mu - u + I'(W^m)\Big(rW^m - u + c + \phi\Big) + \frac{I''(W^m)}{2}\lambda^2\sigma^2.$$

This optimality equation plays a key role in analyzing the optimal manager contract, which states that the investors are maximizing the expected current flow of profit $\mu - u$ plus the expected change of future profit due to the drift and volatility of the manager's continuation value.

The optimal choice of salary maximizes

$$-u - I'(W^m)u.$$

Similar to the employee, there are also two different phases in the manager's life-cycle:

vesting and vested period. Define W_s^m as the lowest value such that

$$I'(W_s^m) = -1.$$

(a) Vesting Period: $(0, W_s^m]$. The manager's salary is 0.

Intuitively, salary zero is optimal because it maximizes the drift of W_t^m away from the inefficient low liquidation point.

(b) Vested Period: $(W_s^m, W_*^m]$. The manager's salary is

$$u = W^m - W_s^m,$$

where $W_*^m < \infty$.

Intuitively, because the investors have the option to provide the manager with W^m by paying a lump-sum transfer of $u > 0$ and moving to the optimal contract with salary $W^m - u$, $I'(W^m) \geq I(W^m - u) - u$. This implies that $I'(W^m) \geq -1$ for all W^m; that is, the marginal cost of compensating the manager can never exceed the cost of an immediate transfer. These transfers, and the option to terminate, keep the manager's continuation value between 0 and W_s^m.

W_*^m is the maximum value that W^m can reach, which also makes investors' profit bounded below[22]. The existence of W_*^m is guaranteed in the Nash equilibrium of optimal manager and employee contracts. This boundedness property does not exist in a classic Principal-Agent model without hierarchies. For instance, DeMarzo and Sannikov (2006) introduce a Principal-Agent setting in which the agent's payoff W, corresponding to W^m in my paper, is unbounded above. Figure 3 below is presented in their paper which is a plot of the principal's payoff.

[22] This is because a continuous function on a bounded interval is also bounded.

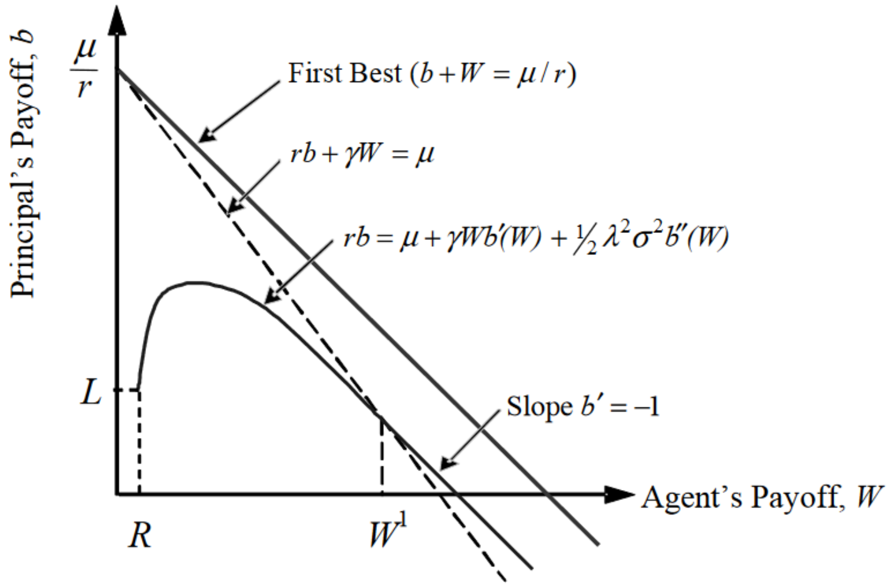

Figure 3: DeMarzo and Sannikov (2006) Figure 1.

Since the agent's payoff is unbounded above, it is possible that with the presence of positive and big shocks, W becomes so large that makes the investors' payoff significantly below zero. However, the investors', either outside equityholders or outside debtholders, liabilities should be limited to the amounts they invest, and this project is funded with an initial investment of $K > 0$, which is finite. Therefore, at least one contractual option, a boundary condition, or a value constraint must be implemented onto the classic Principal-Agent model in order to avoid the investors' unlimited liability problem. However, this problem does not exist in a Principal-Agent-Subagent model which equips limited liability on the investors naturally. I further explore this point in Nash equilibrium later.

Therefore, within the range $[0, W_s^m]$, the investors' value function satisfies the following second-order ordinary differential equation:

$$rI(W^m) = \mu + I'(W^m)\Big(rW^m + c + \phi\Big) + \frac{I''(W^m)}{2}\lambda^2\sigma^2. \tag{12}$$

Three boundary conditions are required to pin down a solution to the optimality equation (12) and the payment boundary W_s^m.

(a) Liquidation boundary at $W^m = 0$:

$$I(0) = L. \tag{13}$$

Similar to the firing boundary of $M(W^e)$, once $W^m = 0$, θ^m from Proposition 5 is 0, the investors must terminate the contract and liquidate the project as they cannot provide enough incentives to induce the manager to report truth earnings.

(b) Payment boundary with smooth-pasting and super-contacting conditions at W^m_s:

$$I'(W^m_s) = -1, \tag{14}$$

$$I''(W^m_s) = 0, \tag{15}$$

where (14) and (15) are equivalent to

$$rI(W^m_s) + rW^m_s = \mu - c - \phi.$$

These payment boundary conditions have a natural interpretation: It is beneficial to postpone payment to the manager by making W^m_s larger because doing so reduces the risk of early termination. Postponing payment is sensible until the payment boundary W^m_s, when the investors and manager's required expected returns exhaust the available expected cash flows minus employee's consumption and manager's effort cost.

3.2.4 Optimal manager contract

Theorem 2 summarizes the optimal manager compensation contract in the Principal-Agent-Subagent model.

Theorem 2. ***Optimal Manager Compensation Contract***
The unique concave investors' value function I that satisfies the ODE (12) with boundary (13), smooth-pasting (14) and super-contacting conditions (15) characterizes any optimal manager contract with positive profit to the investors. There are two cases for manager's starting value $W^m_0 > 0$*:*

- *case 1:* $0 < W^m_0 \leq W^m_s$*: if the manager can hire an underqualified or qualified employee, the investors will make this investment with a postponed payment to the manager.*

- *case 2:* $W^m_s < W^m_0 \leq W^m_*$*: if the manager can hire an underqualified or qualified employee, the investors will make this investment with an immediate payment* $W^m_0 - W^m_s$ *to the manager.*

If $W^m_0 \in [0, W^m_s]$*, then the optimal contract attains profit* $I(W^m_0)$*. Such a contract is based on the manager's continuation value as a state variable, which starts at* W^m_0 *and evolves according to*

$$dW^m_t = \big(rW^m_t + c_t + \phi\big)dt + \lambda\big(dX_t - \mu dt\big)$$

under payments $u_t = 0$*, effort* $m_t = 1$*, and reports* $\hat{X}_t = X_t$ *until the project's liquidation time* τ_m*. Payment occurs whenever* $W^m > W^m_s$ *and the investors will distribute all the accrued increment* $W^m - W^m_s$ *to the manager in order to pull down her continuation level back to*

W_s^m. W_s^m here is not an absorbing state, which means that even though the current manager's continuation value is above W_s^m, after dragging it back to W_s^m, it still has chance to hit zero which will lead to liquidation of the project.[23] *Liquidation occurs when W_t^m hits 0 for the first time, and after that, the employee is either fired or retired, the manager, though still hired, shifts her focus on other duties, and the investors get liquidation value L.*

A typical form of the value function $I(W^m)$, together with W_s^m and W_*^m are shown in Figure 4.

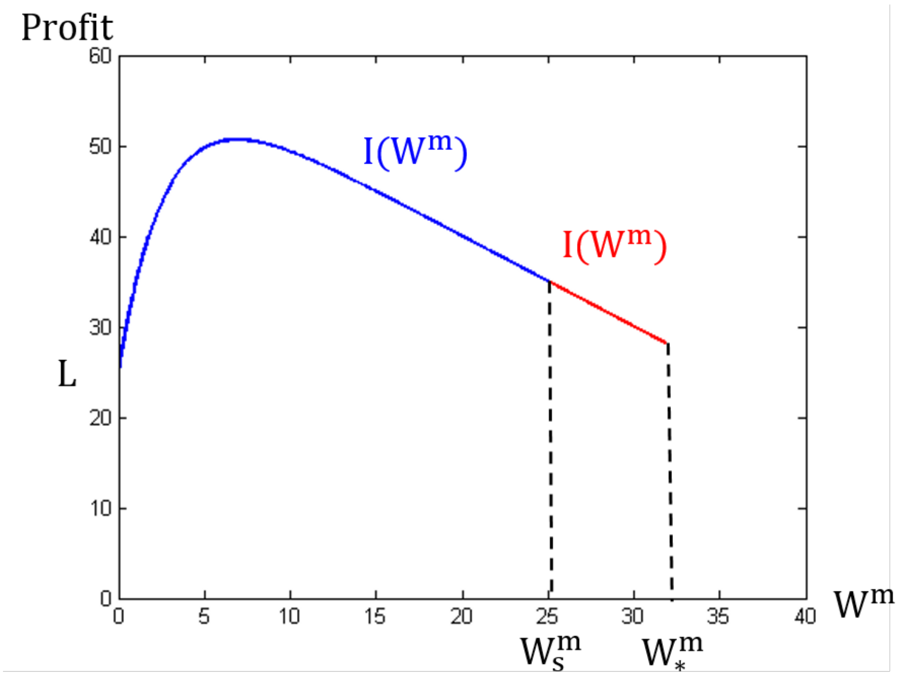

Figure 4: For $\mu = 10$, $r = 0.1$, $\sigma = 5$, $h = 3$, $\phi = 4$, $\lambda = 1$, $L = 25$, $u(c) = 4\sqrt{c}$, $c = 0$

Below is a summary of the optimal contracts:

[23] If the cumulative earnings follows a geometric brownian motion, then W_s^m is an absorbing state. See He (2009) for details.

	Optimal Manager Contract	**Optimal Employee Contract**
Players	investors and manager	manager and employee
Notation	$\Pi_m \equiv \{\{u\}, \tau_m\}$	$\Pi_e \equiv \{\{c\}, \tau_e\}$
Incentive compatibility	$m = 1$: manager is working $\hat{X} = X$: manager is truth-telling	$e = \mu$: employee is working
State variable dynamics	W^m: manager's continuation value $dW_t^m = \big(rW_t^m + c_t + \phi\big)dt + \lambda\sigma dB_t$	W^e: employee's continuation value $dW_t^e = \big(rW_t^e - u(c_t) + h\big)dt + \frac{h}{\mu}\sigma dB_t$
Optimality equation	I: investors' profit $rI(W^m) = \mu + I'(W^m)\big(rW^m + c + \phi\big) + \frac{I''(W^m)}{2}\lambda^2\sigma^2$	M: manager's profit $rM(W^e) = u - c - \phi + M'(W^e)\big(rW^e - u(c) + h\big) + \frac{M''(W^e)}{2}\left(\frac{h}{\mu}\right)^2\sigma^2$
Boundary conditions	liquidation: $I(0) = L$ payment: $I'(W_s^m) = -1$, $I''(W_s^m) = 0$	firing: $M(0) = 0$ retirement: $M(W_p^e) = 0$, $M'(W_p^e) = \frac{c+\phi}{rW_p^e - u(c)}$
Compensation	$u = \max\{W^m - W_s^m, 0\}$	$c = \max\{0, i(-\frac{1}{M'(W^e)})\}$ [24]

Table 2: Principal-Agent-Subagent model solution

So far, I solve the optimal manager contract and optimal employee contract individually and I derive the Nash equilibrium in the next section.

3.3 Nash Equilibrium of Optimal Contracting

Figure 5 demonstrates the contractual relations of the Principal-Agent-Subagent model.

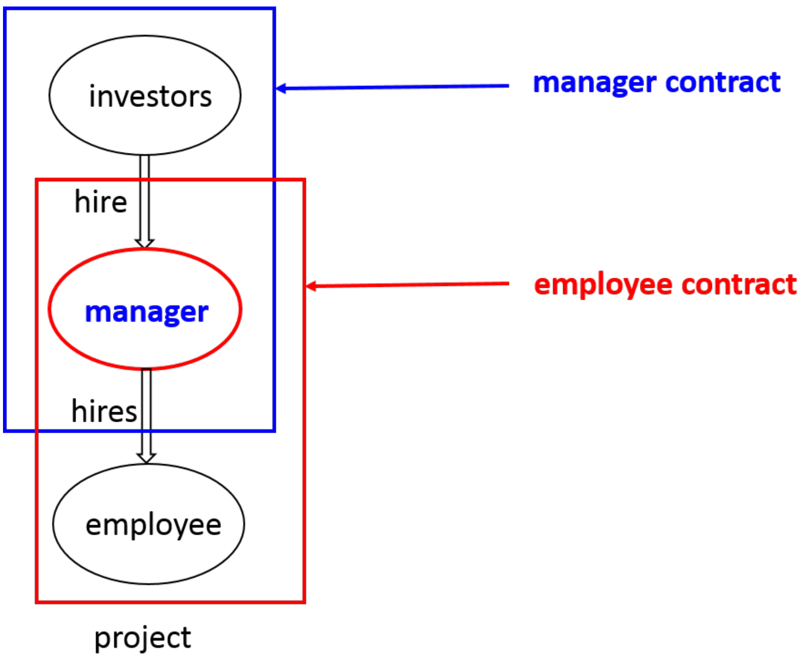

Figure 5: Principal-Agent-Subagent model

Three players are associated with the Principal-Agent-Subagent model: (1) investors, (2)

[24] i is the inverse of the marginal utility function $u'(c)$, where $u'(x) = y \leftrightarrow i(y) = x$.

manager, and (3) employee; two contracts are involved in the Principal-Agent-Subagent model, (1) manager contract, and (2) employee contract, in which the manager is associated with both manager and employee contracts. In the manager contract, the manager is hired, gets paid, and is motivated to tell the truth by the investors, and the manager hires, pays the employee, and motivates him to work in the employee contract. There is no direct contractual relation between the investors and the employee.

The manager's associations with both contracts and different roles in each contract provide a link between the optimal manager and optimal employee contracts. Theorem 3 summarizes the findings.

Theorem 3. ***Nash Equilibrium of Optimal Contracting***
Under optimal contracting of both manager and employee contracts,

(a) optimal manager contract and optimal employee contract have the same stopping time:

$$\tau_m = \tau_e \equiv \tau; \tag{16}$$

(b) the manager's continuation value from optimal manager contract equals to her payoff from optimal employee contract at all time:

$$M_t = W_t^m, \tag{17}$$

for all $t \in [0, \tau]$.

Theorem 3(a) states that under optimal contracting, the project's liquidation time corresponds to the employee's firing or layoff time. The proof of it is trivial. The project needs two people to finish and has positive expected return only when both the manager and employee are working. Later in Section 7.3, I show it is suboptimal to allow the manager or employee to shirk in the Principal-Agent-Subagent model. If either shirks, it is better for the investors to liquidate the project.

Under optimal contracting, continuation values W^e (2) and W^m (9) are both under the same probability measure, which also make their evolutions dW_t^e (3) and dW_t^m (10), and the optimality equations $M(W^e)$ (5) and $I(W^m)$ (12) all defined under this probability measure. The proof of Theorem 3(b) uses Feynman-Kac formula, and $M(W^e)$ can be written in its probabilistic representation

$$M(W^e) = E^{W_0^e = W^e}\left[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\right],$$

where, intuitively, the manager's value function $M(W^e)$ from hiring the employee equals to her salary $u_t dt$, minus the consumption paid to the employee $c_t dt$, and her effort cost ϕdt until the optimal stopping time *tau*, all discounted by the same discount rate r. This is equivalent to the manager's continuation value

$$W_0^m = E\left[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\right],$$

in Nash equilibrium of the Principal-Agent-Subagent model.

The intuition of Theorem 3 is that although the Principal-Agent-Subagent model has two contracts Π_e and Π_m, these two contracts are working as a whole under optimal contracting: the existence of one contract relies on the existence of the other since this project requires the employee and manager to work together. In Nash equilibrium, upon liquidation $\tau_m = \tau$, the manager's continuation value $W_\tau^m = 0$, which is equal to her payoff $M(W_\tau^e) = 0$ from firing or laying off the employee $\tau_e = \tau$. At each point in time before τ, the required return of the manager's payoff from optimal employee contract $rM(W^e)$ is equal to the instantaneous payoff from her salary u minus employee's consumption c minus effort cost ϕ plus the capital gains from changes of the employee continuation value W^e. Once adding from time 0 to τ, the manager's profit $M(W^e)$ from optimal employee contract is essentially the same as her continuation value W_0^m from optimal manager contract.

Using Theorem 3, I revisit Theorem 1 and 2, and Figure 6 shows the relations between W^e and W^m in Nash equilibrium.[25]

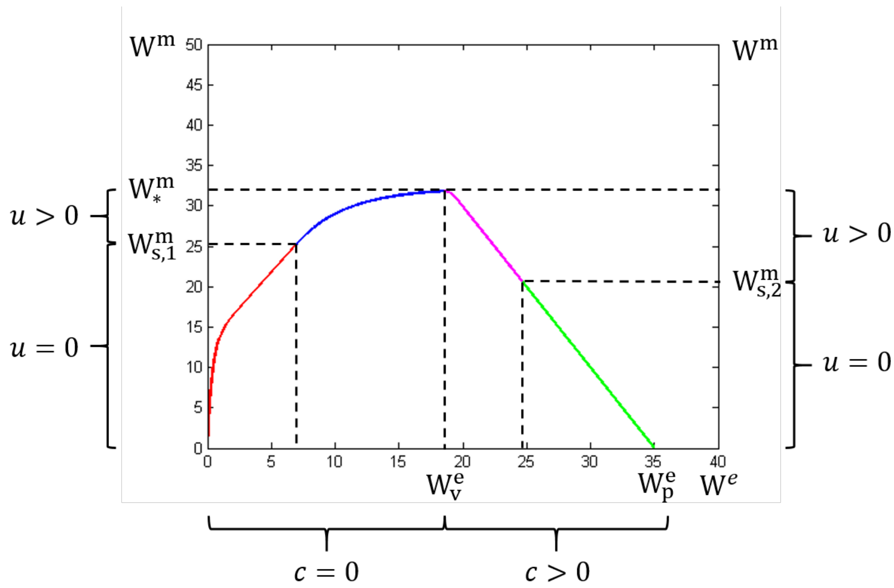

Figure 6: Optimal Principa-Agent-Subagent model

Figure 6 indicates that optimal Principal-Agent-Subagent model can be divided into 4 regions according to the manager and employee's compensations:

[25]Same parameters as Figure 2 and 4: $\mu = 10$, $r = 0.1$, $\sigma = 5$, $h = 3$, $\phi = 4$, $\lambda = 1$, $L = 25$, $u(c) = 4\sqrt{c}$, $c = 0$

(1) Manager's salary $u = 0$ and employee's consumption $c = 0$:
This region corresponds to the *red line* in Figure 6. The intuition is that the employee's continuation value W^e is too small, which also makes the manager's continuation value W^m small. It is optimal for the manager not to compensate the employee to avoid the inefficient low firing point and it is also optimal for the investors not to compensate the manager to avoid inefficient liquidation.

(2) Manager's salary $u > 0$ but employee's consumption $c = 0$:
This region corresponds to the *blue line* in Figure 6. Although the value of W^e has made W^m large enough for the manager to receive compensation from the investors, it is still too small for the employee to be compensated from the manager. Deferring his compensation will help avoid early ending of the project.

(3) Manager's salary $u > 0$ and employee's consumption $c > 0$:
This region corresponds to the *pink line* in Figure 6, where the value of W^e is good enough, not too small or large, for both the manager and employee to be compensated.

(4) Manager's salary $u = 0$ but employee's consumption $c > 0$:
This region corresponds to the *green line* in Figure 6. In this region, W^e is too large, that making the manager's profit, which equals to W^e, too small. Although the employee still gets paid by the manager, the investors will defer the manager's compensation to avoid the continuous growing of W^e which forces the manager to lay off the employee resulting in inefficient liquidation.

As the investors' value function $I(W^m)$ is also a function of W^e, Figure 7 plots both the investors' and manager's payoffs as a function of W^e.

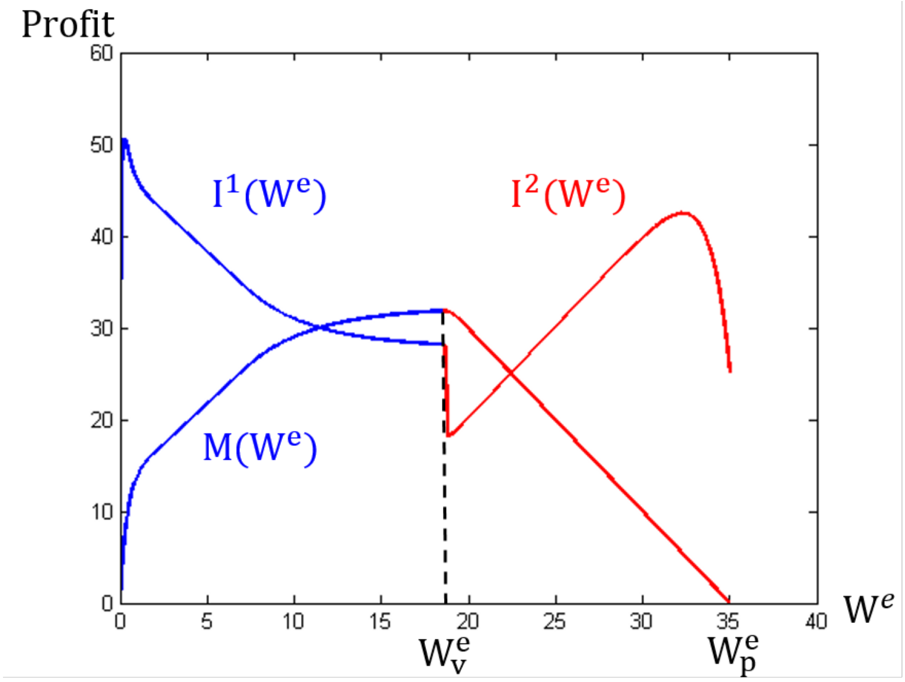

Figure 7: Optimal Principal-Agent-Subagent model

The manager's payoff value $M(W^e)$ is a concave function of W^e. Starting from $W^e = 0$, $M(W^e)$ first increases with respect to W^e until it reaches the maximal point W^e_v. After that, $M(W^e)$ decreases down to 0, where $W^e = W^e_p$. Therefore, there should be two investors' value functions for the increasing and decreasing parts of $M(W^e)$, respectively. During the increasing interval $[0, W^e_v]$, $I(W^e)$ corresponds to zero employee's consumption, and during the decreasing interval $[W^e_v, W^e_p]$, $I(W^e)$ corresponds to positive employee's consumption.

4 Model Properties

In this section, I explore the Nash equilibrium properties of the Principal-Agent-Subagent model and make a comparison to the classic Principal-Agent model without a hierarchy.

4.1 Comparative statics analyses

Before making a comparison, I first make comparative static analyses for the maximal investors' and manager's profits, manager's and employee's continuation values, and optimal manager and employee contracts' boundaries with respect to model parameters: project's expected return and volatility, employee's and manager's effort costs, diverting earnings' cost, and liquidation value. Table 3 summarizes the results.[26]

[26]Here, I do not show comparative static results with respect to r, because most of them are ambiguous. For instance, there are two offsetting effects on the payment boundary W^m_s. When r increases, the benefit

	$\partial M(W^e)/$	$\partial W^e/$	$\partial W^e_p/$	$\partial I(W^m)/$	$\partial W^m/$	$\partial W^m_s/$
$\partial\mu$	+	?	0	+	0	?
$\partial\sigma^2$	−	?	0	−	?	+
∂h	?	?	0	0	0	0
$\partial\phi$	−	0	0	?	+	?
$\partial\lambda$	0	0	0	−	?	+
∂L	0	0	0	+	0	−

Table 3: Comparative Statics for the Optimal Principal-Agent-Subagent model

Most of the results are intuitive, and some of them are worth paying attention to, such as $\frac{\partial I(W^m)}{\partial\lambda} < 0$, $\frac{\partial W^m_s}{\partial\lambda} > 0$. The investors' highest payoff from the manager contract $I(W^m)$ increases when the minimum incentives required for truth-telling λ decreases, and the payment boundary W^m_s decreases when λ decreases. These two partial derivatives actually highlight the important property of the Principal-Agent-Subagent model, which differentiate it from the classic Principal-Agent model without a hierarchy. The manager's and investors' concave payoff functions correspond to their risk averse attitude, which are captured by $\frac{\partial M(W^e)}{\partial\sigma^2} < 0$ and $\frac{\partial I(W^m)}{\partial\sigma^2} < 0$. A larger σ represents a higher risk of the project which reduces their profits. I further discuss these issues in the following section.

4.2 Principal-Agent model without a Hierarchy

In order to make a comparison between Principal-Agent-Subagent model and Principal-Agent model, I first modify DeMarzo and Sannikov's Principal-Agent model (2006) in which the investors hire two agents directly, instead of hiring one agent and letting this agent hires another, to carry out the production.

The project is exactly the same as before, and to make things easier, I assume *homogeneous* agents. To capture each agent's contribution, I assume the cumulative cash flow X follows

$$dX_t = a^1_t a^2_t dt + \sigma dB_t,$$

where $a_t \in \{0, \sqrt{\mu}\}$ is each agent effort choice with effort cost $\phi > 0$. Each agent observes cash flow $\frac{X}{2}$ and reports cash flow $\frac{\hat{X}}{2}$ to the investors, respectively. The rest of the assumptions are exactly the same as in Section 3, and I use superscripts 1 and 2 to represent each agent separately.

Below is a summary of the Principal-Agent model without a Hierarchy.

from exchanging the relative consumption timings is smaller; investors should pay cash later, i.e., a larger W^m_s. However, the cost from future terminations is also reduced due to a larger discounting effect, which makes investors less worried about inefficient turnovers, and thus lowers W^m_s. As a result, the overall effect is ambiguous.

	Investors	**Agent 1**	**Agent 2**
Risk attitude	risk neutral	risk neutral but effectively risk averse	risk neutral but effectively risk averse
Effort choice	–	$a_t^1 = \sqrt{\mu}$ working $a_t^1 = 0$ shirking	$a_t^2 = \sqrt{\mu}$ working $a_t^2 = 0$ shirking
Effort cost	–	$\phi > 0$ working 0 shirking	$\phi > 0$ working 0 shirkig
Cash flows	unobservable	Observes, reports $\frac{\hat{X}}{2}$, receives $\lambda(\frac{dX_t}{2} - \frac{d\hat{X}_t}{2})$	Observes, reports $\frac{\hat{X}}{2}$, receives $\lambda(\frac{dX_t}{2} - \frac{d\hat{X}_t}{2})$

Table 4: Principal-Agent model without a Hierarchy

4.2.1 The formulation of the investors' problem

The investors commit to each agent with an agent compensation contract. Similar to the manager contract, denote them by $\Pi_1 \equiv \{\{u^1\}, \tau\}$ and $\Pi_2 \equiv \{\{u^2\}, \tau\}$ and denote $\mathbb{IC}_1$ and $\mathbb{IC}_2$ as the incentive compatible contract, respectively.

The investors' problem is to offer an incentive contract to each agent respectively that maximizes their profit

$$I_0^a = \max_{\substack{\Pi_m^1 \in \mathbb{IC}_m^1 \\ \Pi_m^2 \in \mathbb{IC}_m^2}} E\left[\int_0^\tau e^{-rt}(d\hat{X}_t^1 - u_t^1 dt + d\hat{X}_t^2 - u_t^2 dt) + e^{-r\tau} L\right]$$

subject to giving each agent a specific value of $W_0^1 > 0$ and $W_0^2 > 0$

$$W_0^1 = \max_{\substack{a^1 = \{a_t^1 \in \{0, \sqrt{\mu}\}: 0 \le t \le \tau\} \\ d\hat{X}^1 = \{d\hat{X}_t^1 \in [0, dX_t^1]: 0 \le t \le \tau\}}} E\left[\int_0^\tau e^{-rt}\Big(\lambda(dX_t^1 - d\hat{X}_t^1) + (u_t^1 - \frac{\phi}{\sqrt{\mu}} a_t^1) dt\Big)\right]$$

$$W_0^2 = \max_{\substack{a^2 = \{a_t^2 \in \{0, \sqrt{\mu}\}: 0 \le t \le \tau\} \\ d\hat{X}^2 = \{d\hat{X}_t^2 \in [0, dX_t^2]: 0 \le t \le \tau\}}} E\left[\int_0^\tau e^{-rt}\Big(\lambda(dX_t^2 - d\hat{X}_t^2) + (u_t^2 - \frac{\phi}{\sqrt{\mu}} a_t^2) dt\Big)\right]$$

and each agent's participation constraint for all $0 \le t \le \tau$

$$E_t\left[\int_t^\tau e^{-r(s-t)}\Big(\lambda(dX_s^1 - d\hat{X}_s^1) + (u_s^1 - \frac{\phi}{\sqrt{\mu}} a_s^1) ds\Big)\right] \ge 0$$

$$E_t\left[\int_t^\tau e^{-r(s-t)}\Big(\lambda(dX_s^2 - d\hat{X}_s^2) + (u_s^2 - \frac{\phi}{\sqrt{\mu}} a_s^2) ds\Big)\right] \ge 0.$$

4.2.2 Agency costs

Theorem 4 highlights the advantages of a Principal-Agent-Subagent model over a Principal-Agent model without a hierarchy.

Theorem 4. *In Nash equilibrium of the Principal-Agent-Subagent model, investors' agency costs can be mitigated, compared to the classic Principal-Agent model without a hierarchy.*

(1) When shirking is a more severe agency problem than stealing, investors should choose a Principal-Agent-Subagent model over a Principal-Agent model in order to mitigate agency costs. The manager would also benefit from a Principal-Agent-Subagent model as she would get paid earlier.

(2) When stealing is a more severe agency problem than shirking, although agency costs stay the same, investors should also choose a Principal-Agent-Subagent model over a Principal-Agent model if the investors have limited budget to cover losses, since a Principal-Agent-Subagent model naturally provides upper and lower bounds for both the manager's and employee's continuation values, which makes the investors' profit always bounded below. Investors' limited liabilities constraint does not exist in a Principal-Agent model.

Figure 8 illustrates investors' agency costs mitigation and limited liabilities, and the manager's earlier payment boundary.

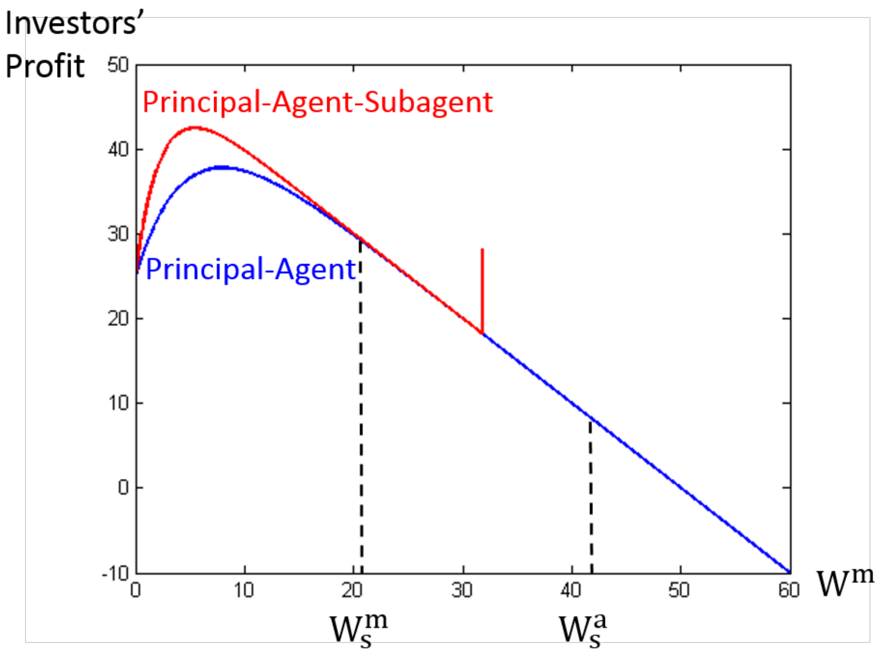

Figure 8: Principal-Agent-Subagent model vs. Principal-Agent model

Figure 8 shows that the investors receive a higher profit, which is equivalent to lower agency costs, in the Principal-Agent-Subagent model. At the same time, the manager would also benefit from the Principal-Agent-Subagent model as she would be compensated by the investors earlier. Earlier payment boundary could also be viewed as the investors' confidence of less likely occurrence of inefficient liquidation. This can be also interpreted as a result of agency costs reduction.

Why can a Principal-Agent-Subagent model eliminate agency costs, compared to the classic Principal-Agent model without a hierarchy? The reasons are as follows. Agency costs arise when there are hidden actions from the manager. The hidden actions in both models come in two varieties: shirking and stealing. Shirking happens when the manager is not contributing to the project and stealing occurs when the reported outputs to the investors do not equal to real outputs.

In the first case, "shirking," the manager could be hands-off the project, either because she is lazy and simply interested in overinvestments of free cash flows, or more plausibly because the project selection is a mistake or the manager chooses a project that is beyond her ability. For instance, the failure of Google Glass in 2015 was more a reason for a series mistakes, such as overprice, ugly design, unclear functions, or simply a terrible idea of the product itself, instead of the managers being lazy or not working hard enough. In both the Principal-Agent-Subagent and Principal-Agent models, those are all captured by effort cost ϕ, in which a high ϕ could make a positive investors' profit impossible.

In the second case, "stealing," the amounts of cash flows received by the investors do not equal to the outputs of the project, either because the manager secretly diverts earnings from the project into her own hands, or more plausibly because she inefficient uses of the earnings. For example, the manager consumes executive perquisites, such as purchasing an excessively fancy office space or reimbursing some of her personal expenses. In both the Principal-Agent-Subagent and Principal-Agent models, those are all captured by diverting benefit factor λ, in which a high λ would significantly lower the investors' profit.

The alliterative "stealing and shirking" are both costs to the investors and so it is correct to lump them together as costs, *agency costs*. The investors, therefore, need to design a compensation contract and provide enough incentives to induce the manager to work and report full earnings of the project to them. However, under a Principal-Agent-Subagent model, the manager is self-motivated to work as she has interpersonal authority over the employee. Proposition 2 states that when the manager is shirking, the employee will be shirking, too, and Proposition 4 implies that a shirking employee for the manager is simply a pure cost. He shares the positive profit of the project with the manager without increasing her compensation. It is suboptimal to hire a shirking employee so the manager is self-motivated to supervise him in order to induce him to work. In this case, after introducing a hierarchy between the manager and employee, the investors do not need to provide extra monetary incentives to induce the manager to work. These saved incentives mitigate the agency costs.

5 Empirical tests

In this section, I analyze twenty years of personnel data from one firm and test my result that agency costs can be mitigated in a Principal-Agent-Subagent model compared to a Principal-Agent model without hierarchies.

The dataset contains *confidential* personnel records for all employees of a medium-sized U.S. firm in a service industry over the years 1969-1988. There are 68,437 employee-years of data, and each observation contains an employee ID number, job title, salary, and bonus. Baker, Gibbs, and Holmstrom (1993) have assigned levels to each employee, according to flows between job titles. For instance, if a job was fed primarily by people from level n, and fed people into level $n+2$, it was assumed to be at level $n+1$. Figure 9 represents the results of the level-assignment process.

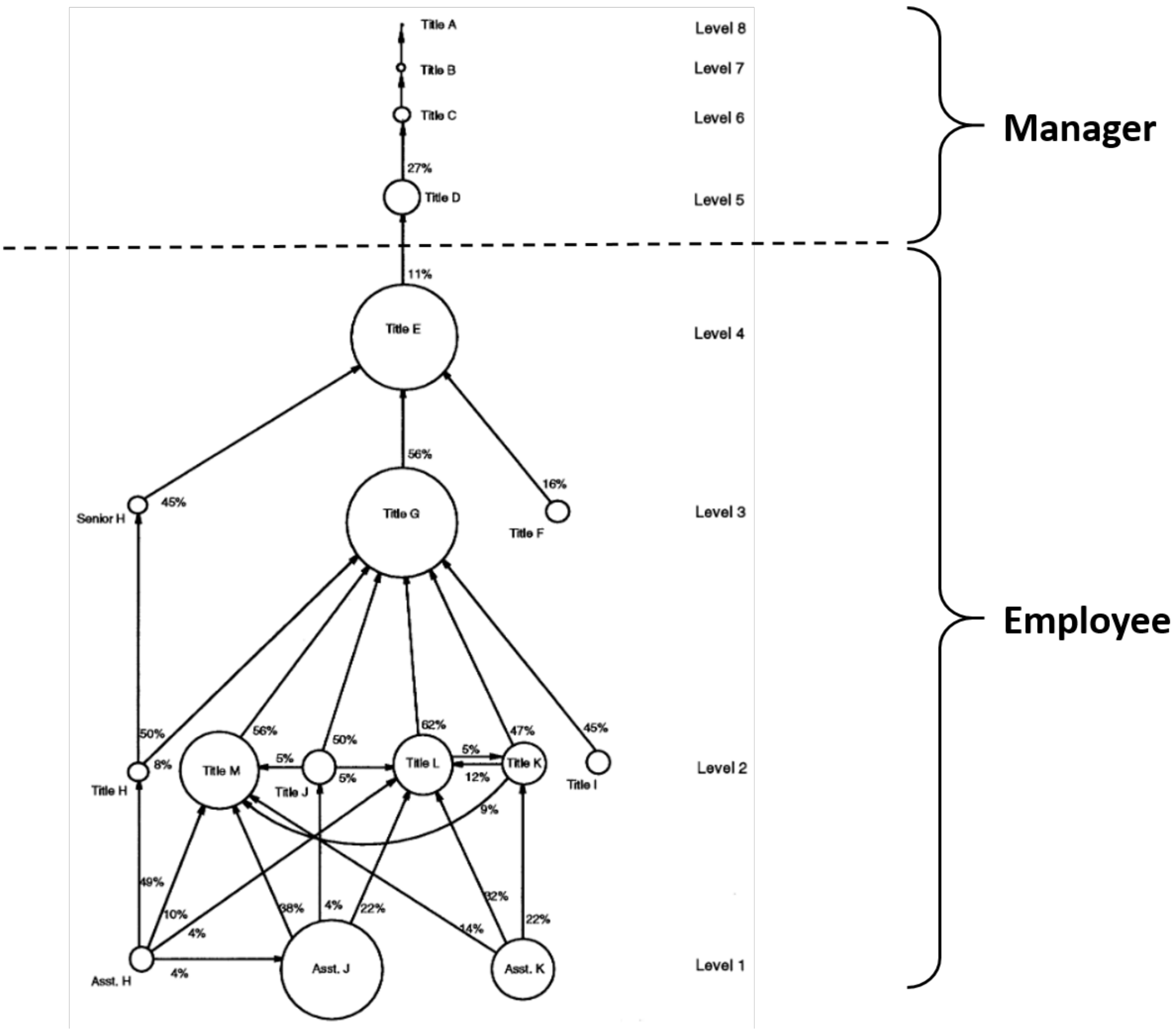

Figure 9: Hierarchies: level-assignment

This hierarchical structure is quite simple and stable throughout the twenty years. To fit the Principal-Agent-Subagent framework, I group level 5 to level 8 as "manager" and level 1 to 4 as "employee." There are two reasons why I group them in this way: (1) level 1 to 4 personnel are mostly in production and selling positions, while level 5 to 8 are mostly in administrative and management positions; (2) level 1 to 4, in general, do not have access to cash flows of the firm, while level 5 to 8 have more or less opportunities. The first reason is consistent with the model that the productive employee is hired by the administrative manager, and the second reason is consistent with the assumption that earnings are observable only to the manager.

To measure agency costs of the firm, I use two alternative efficiency ratios that frequently appear in the accounting and financial economics literature:

1. the expense ratio, which is selling, general and administrative expenses (SG&A) scaled by annual sales

2. the asset utilization ratio, which is annual sales divided by total assets

The first ratio is a measure of how effectively the firm's management controls non-production related costs, including excessive perquisite consumption, and some other administrative expenses. A growth of the expense ratio means that the growth of SG&A expenses is larger than the growth of sales revenue, which indicates that the manager is inefficiently using of cash flows. This is exactly the definition of agency costs. The second ratio is a measure of how effectively the firm's management deploys its assets. In contrast to the expense ratio, agency costs are inversely related to the asset utilization ratio. A declining of the asset utilization ratio means that the growth if sales revenue is less than the growth of the firm's total assets, which indicates that the manager is purchasing unproductive assets. Agency costs, in this case, are higher due to manager's unnecessary perquisites.

I use the salary ratio, which is total employee's salary over total salaries (employee's plus manager's salaries), as the explanatory variable for the agency costs. The salary ratio of zero means a pure Principal-Agent model. An increase in the salary ratio indicates more work are conducted by hiring employees instead of managers to finish, representing an adoption of a Principal-Agent-Subagent model over a Principal-Agent model without hierarchies.

Table 5 presents the results from univariate regressions analyzing agency costs as measured by the expense ratio and asset utilization ratio, respectively.

	Expense Ratio	**Asset Utilization Ratio**
Intercept	3.83***	-2.17**
	(2.93)	(-2.34)
Salary Ratio	-3.92 **	2.43**
	(-2.79)	(2.44)
Adjusted R^2	0.26	0.21

Table 5: Empirical tests

The expenses ratio is statistically negatively related to salary ratio means that when new work is conducted by hiring an employee instead of a manager to finish, agency costs reduces. The asset utilization ratio is statistically positively related to salary ratio indicates that when a new job is carried out by hiring an employee instead of a manager to finish, agency costs reduce. Both two empirical tests confirm the theoretical part of my model and agree that a Principal-Agent-Subagent model mitigates agency costs compared to a Principal-Agent model without hierarchies.

6 Extensions

6.1 Replacement v.s. Layoff

In this subsection, I modify the original employee contractual environment where the manager replaces a new employee before laying off the current one. In the original employee contracting problem, Section 4.1, firing and layoff are the only contractual options. Firing corresponds to the employee's continuation value being zero and layoff corresponds to his continuation value being too high. In either case, the manager's payoff is zero and Nash equilibrium makes her continuation value equal to zero, too. This results in the investors liquidating the project.

Alternatively, I could allow the manager to replace the employee before reaching the layoff boundary, which leaves the manager with a positive profit, equivalent to a positive manager's continuation value in Nash equilibrium. This results in the investors continuing running the project. Once reaching the replacement boundary, the manager hires a new employee to replace the old one. I assume the new and old employee are identical. For instance, both contribute the same work to the project, share the same utility function, and have the same effort cost.

Everything else follows directly from Section 4, except the layoff boundary now changes to replacement boundary.

$$M(W_h^e) = a$$

$$M'(W_h^e) = \frac{ra + c + \phi}{rW_h^e - u(c)}$$

A typical form of the value function $M(W^e)$ and together with W_v^e and W_h^e, are shown in Figure 10.

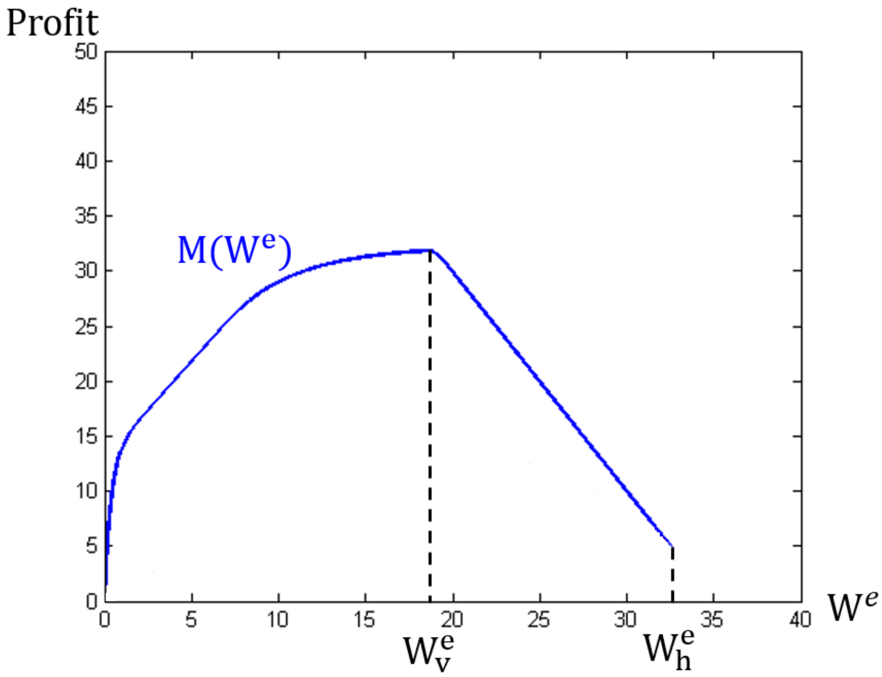

Figure 10: Replacement with $a = 5$

The manager would pick a new employee with a starting value *less* than W_h^e as a replacement for the old employee. Optimal manager contracting problem stays the same as before. Under optimal contracting, inefficient liquidation happens only when the employee's continuation value falls down to zero. Replacement would not trigger a liquidation.

6.2 Delegation v.s. Supervision

In this subsection, I modify the manager's role in the firm from supervision into delegation. In Section 3, I assume the manager's contribution to the project is (1) decision making, (2) information processing, and (3) interpersonal authority, which she hires and supervises a productive employee. Now, suppose instead of supervision, the manager also carries real production. To capture the manager's and employee's each contribution to the project, I assume $\mu_1\mu_2 = \mu$ and

$$dX_t = m_t e_t dt + \sigma dB_t$$

where $m_t \in \{0, \mu_1\}$ and $e_t \in \{0, \mu_2\}$ are the manager's and employee's *binary* effort choice, respectively. The employee's continuation value W_t^e becomes

$$W_t^e(c, e) = E_t^e\Big[\int_t^{\tau_e} e^{-r(s-t)}\Big(u(c_s) - \frac{h}{\mu_2}e_s\Big)ds + e^{-r(\tau_e - t)}W_{\tau_e}^e\Big],$$

and the dynamic of dW_t^e follows

$$dW_t^e = \Big(rW_t^e - u(c_t) + \frac{h}{\mu_2}e_t\Big)dt + \theta_t^e\big(dX_t - \mu_1 e_t dt\big).$$

However, the minimum incentives required to motivate the employee to work stays the same as in Proposition 1, which is $\frac{h}{\mu}$. Everything else follows exactly the same as Section 4, including the manager's and investors' optimality equations and boundary conditions.

The manager's delegation or supervision role surprisely does not matter in the optimal contracts of the Principal-Agent-Subagent model. The reason is because the minimum incentives required to motivate the employee to work does not change when the manager contributes more than supervision to the project. When the employee chooses to shirk, dX_t becomes driftless no matter how much the manager contributes, hence he still loses $\mu\theta^e dt$ in compensation even though he contributes less in the project.

6.3 Collusion between Manager and Employee

Proposition 1 assumes that implementing high effort at all times is optimal. Because the reduction in cash flows due to shirking is bounded—unlike the case of diversion—it may be optimal to stop providing incentives and allow the employee to shirk after some good histories of output, while still hired by the manager.[27] Specifically, when the employee shirks his payoff would not need to depend on cash flows, so the employee's continuation value W^e would evolve according to

$$dW_t^e = \begin{cases} \big(rW_t^e - u(c_t) + h\big)dt + \frac{h}{\mu}\big(dX_t - \mu dt\big) & \text{if the employee is working} \\ \big(rW_t^e - u(c_t)\big)dt & \text{if the employee is shirking} \end{cases}$$

I have shown in Section 5.1 that $\frac{\partial M(W^e)}{\partial \sigma^2} < 0$. Because the manager's payoff function is concave, this reduction in the volatility of W_t^e could be beneficial. For that not to be the case, and for the high effort working to remain optimal, it must be that for all W^e, the manager's payoff rate from having the employee shirk would be less than that under the existing contract:

$$rM(W^e) \geq u - c + M'(W^e)\big(rW^e - u(c)\big).$$

Hence, the employee's continuation value and the manager's payoff if allowing the employee to shirk are given by

$$W^{e,s} = \frac{u(c)}{r},$$

[27] This is different from the firing and layoff boundaries, since at either point, the employee is out.

and

$$M^{e,s} = \frac{u-c}{r}.$$

However, as I have derived in Section 4.1, each employee's continuation value W^e is associated with a unique consumption c, where $c = \max\{0, i(-\frac{1}{M'(W^e)})\}$ maximizes the manager's payoff. These two equations agree only at the firing boundary. Hence, in general, it is not optimal for the employee to shirk under the Principal-Agent-Subagent model.

This is different from the Principal-Agent models introduced by DeMarzo and Sannikov (2006) or He (2009). In their framework, shiring is optimal, under additional assumptions of the parameters.

I have showed in Section 4.1.5 that the manager could not provide enough incentives to induce the employee to work if she is shirking. Since $\frac{\partial I(W^m)}{\partial \sigma^2} < 0$ and similar logic from the previous subsection, it might be possible that there exists an optimal manager contract when both manager and employee are shirking. The manager's continuation values would evolve according to

$$dW_t^m = \begin{cases} \big(rW_t^m + c_t + \phi\big)dt + \lambda\big(dX_t - \mu dt\big) & \text{if both are working} \\ \big(rW_t^m + c_t\big)dt + \lambda\sigma dB_t & \text{if the manager is shirking} \end{cases}$$

Similarly, for high effort to remain optimal, it must be that for all W_t^m, the investors' payoff rate from having the manager shirk would be less than that under the existing contract:

$$rI(W^m) \geq I'(W^m)(rW^m + c) + \frac{I''(W^m)}{2}\lambda^2\sigma^2,$$

and one necessary condition needed to be satisifed is that

$$rI(W^m) \geq I'(W^m)(rW^m + c).$$

Hence the manager's continuation value and the investors' payoff when the manager is shirking are given by

$$W^{m,s} = -\frac{c}{r},$$

and

$$I(W^{m,s}) = 0.$$

Since the continuation value of the manager is always nonnegative by the participation constraint, the above equations will hold only when $c = 0$, which implies $W^m = 0$, corresponding to the project liquidation condition. Therefore, it is not optimal for both the manager and employee to shirk. This subhsection is also in harmony with Section 4.2.1 in which the manager is self-motivated to work if she chooses to hire an employee.

Section 7.3 highlight another important property about the Principal-Agent-Subagent model: it is suboptimal to allow the manager to collude with the employee to shirk. This property does not exist in the classic Principal-Agent model.

7 Conclusions

This paper initiates the first step in continuous-time finance to expanding the classic Principal-Agent model by introducing a new Principal-Agent-Subagent model in which the manager has interpersonal authority over the employee, instead of the investors hiring another manager. This type of hierarchy among agents exists in all types of corporations in the real world but was not carefully studied by finance researchers. Existing corporate finance literature has studied extensively on Principal-Agent model by introducing additional constraints or frictions but never expanding the model by introducing a hierarchy. My Principal-Agent-Subagent completes the literature by studying the optimal contracts and incentives provision among each player.

There are two contracts in the Principal-Agent-Subagent model: (1) manager contract in which the investors commit to the manager, (2) employee contract in which the manager writes to the employee. The manager plays an important role in both contracts, and Nash equilibrium equates the manager's continuation value from the manager contract to her maximal profit from the employee contract. The manager not only receives incentives from the investors but also provides incentives to the employee. The minimum incentives required to motivate the employee to work depends on both employee's and manager's effort, and the employee is shirking whenever the manager is shirking. The manager's incentives to work and report the truth earnings are coming from two different ways separately: (1) incentives to work comes from self-motivation, (2) incentives of truth-telling comes from the investors.

Compared to a classic Principal-Agent model without hierarchies, the Principal-Agent-Subagent model naturally provides upper and lower bounds for both manager's and employee's continuation values, so the investors' value is always bounded below, indicating a natural limited liability constraint on the investors. In general, it is not optimal for either the manager or the employee to shirk under the Principal-Agent-Subagent framework, which is different from the Principal-Agent model where shirking is optimal after a history of good outputs. The biggest advantage of introducing a hierarchy among agents is that one of the manager's hidden actions, shirking, no longer exists in the Principal-Agent-Subagent model, since she is self-motivated to work. As the investors do not need to provide extra incentives to induce the manager to work, these saved monetary incentives help mitigate the agency costs. Reducing agency costs is probably the most important reason why there are hierarchies inside the firms, why there is only one

CEO in any corporations, and why it is the CEO's job to hire subordinate executives instead of shareholders. Empirical tests have also provided support to my Principal-Agent-Subagent model.

As the manager and employee have different compensation structures, one way to continue this study is to implement the manager and employee contracts with a capital ownership (inside debt, inside equity, and cash). Another interesting direction is to explore possible employee career options' (promotion and demotion) impacts on firm performance. Future research can also examine how the firm fund the project and the optimal investment strategy with the present of hierarchies.

Appendix

Proof of Introduction

Hiring an employee expands the firm

Suppose the cumulative earnings with only one employee (aka manager) follow $dX_t^m = \mu dt + \sigma dB_t$. Assume there are n homogeneous employees who can also carry out the same production as the manager independently[28], then earnings with n employees follow $dX_t^e = n\mu dt + n\sigma dB_t$. The firm expands as $E[X_t^e] > E[X_t^m]$[29].

Hiring an employee minimizes the risk

Suppose there are two types of employees and both generate the same expected return and volatility of cash flows, $dX_t^1 = \mu dt + \sigma dB_t^1$ and $dX_t^2 = \mu dt + \sigma dB_t^2$ respectively. The difference between these two employees lies in the correlation $\rho < 1$, so we could define $dB_t^2 = \rho dB_t^1 + \sqrt{1-\rho^2} dB_t^*$ where B^* is orthogonal to B^1. If the investors hire heterogeneous employees, within the investment boundary, the cumulative earnings will follow $dX_t = 2\mu dt + \sigma\sqrt{2(1+\rho)} dB_t^1$, where $\sqrt{2(1+\rho)} < 2$.

[28] I also assume that there exists a $N < \infty$ such that as long as $n < N$, the expected earnings from these n employees is proportional to μ and the volatility is also proportional to σ. N could be interpreted as the up limit size of one investment opportunity.

[29] This is according to Comparative Theorem from Karatzas and Shreve. One typical example is Uber. More Uber drivers on the streets, higher revenue Uber generates.

Proof of Proposition 1

The employee's total expected payoff from (c, e) given the information at time t is

$$\begin{aligned} V_t^e &= E_t^e\Big[\int_0^{\tau_e} e^{-rs}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + e^{-r\tau_e}W_{\tau_e}^e\Big] \\ &= \int_0^t e^{-rs}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + E_t^e\Big[\int_t^{\tau_e} e^{-rs}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + e^{-r\tau_e}W_{\tau_e}^e\Big] \\ &= \int_0^t e^{-rs}\Big(u(c_s) - \frac{h}{\mu}e_s\Big)ds + e^{-rt}W_t^e \end{aligned}$$

is a Q^e-martingale. Assuming that the filtration $\{\mathcal{F}_t\}$ satisfies the usual conditions, the Q^e-martingale V^e must have a right-continuous with left limits modification by theorem 1.3.13 of Karatzas and Shreve (1991). Then by the Martingale Representation Theorem (KS, p182), we get the representation

$$V_t^e = V_0^e + \int_0^t e^{-rs}\sigma\theta_s^e dB_s^e$$

for $0 \leq t \leq \tau_e$, where

$$dB_t^e = \frac{1}{\sigma}\Big(dX_t - e_t dt\Big)$$

is a Brownian increment under Q^e and the factor $e^{-rs}\sigma$ that multiplies θ_s^e is just a convenient rescaling. Differentiating with respect to t we find that

$$\begin{aligned} dV_t^e &= e^{-rt}\sigma\theta_t^e dB_t^e \\ &= -re^{-rt}W_t^e dt + e^{-rt}dW_t^e + e^{-rt}\Big(u(c_t) - \frac{h}{\mu}e_t\Big)dt \end{aligned}$$

therefore

$$dW_t^e = \Big(rW_t^e - u(c_t) + \frac{h}{\mu}e_t\Big)dt + \theta_t^e\Big(dX_t - e_t dt\Big).$$

Next, let us compare the cases of working versus shirking from t to τ_e. Define by

$$V_t^w = \int_0^t e^{-rs}\Big(u(c_s) - h\Big)ds + e^{-rt}W_t^e(c, \mu)$$

the time t expectation of the employee's total payoff if he was working before time t, and plans to work after time t. Define by

$$V_t^s = \int_0^t e^{-rs}u(c_s)ds + e^{-rt}W_t^e(c, \mu)$$

the time t expectation of the employee's total payoff if he was shirking before time t, and plans to work after time t. Let us identify the drift of the process V_t^s under the probability measure

Q^s, where

$$\sigma B_t^w = \sigma B_t^s - \int_0^t \mu ds = \sigma B_t^s - \mu t$$

We have

$$\begin{aligned}
dV_t^s &= e^{-rt}u(c_t)dt - re^{-rt}W_t^e(c,\mu)dt + e^{-rt}dW_t^e(c,\mu) \\
&= e^{-rt}u(c_t)dt - re^{-rt}W_t^e(c,\mu)dt + e^{-rt}\Big((rW_t^e(c,\mu) - u(c_t) + h)dt + \theta_t^w \sigma dB_t^w\Big) \\
&= e^{-rt}hdt + e^{-rt}\theta_t^w \sigma dB_t^w \\
&= e^{-rt}hdt + e^{-rt}\theta_t^w(\sigma dB_t^s - \mu dt) \\
&= e^{-rt}(h - \theta_t^w \mu)dt + e^{-rt}\theta_t^w \sigma dB_t^s
\end{aligned}$$

If (4) does not hold on a set of positive measure, then the drift of V_t^s under Q^s is non-negative and positive on a set of positive measure. Thus, there exists a time $t > 0$ such that

$$W_0^e(c,0) = E^s[V_t^s] > V_0^s = W_0^e(c,\mu).$$

Because the employee gets utility $E^s[V_t^s]$ if he chooses shirking until time t and then switches to working, then working is suboptimal.
Suppose (4) holds for working, then V_t^s is a Q^s-supermartingale for shirking.
Moreover, since the process $W^e(c,e)$ is bounded from below, we can add

$$V_\infty^s = \int_0^\infty e^{-rs}u(c_s)ds$$

as the last element of the supermartingale V^s. Therefore

$$W_0^e(c,\mu) = V_0^s \geq E^s[V_\infty^s] = W_0^e(c,0)$$

so working is at least as good as shirking. ■

Proof of Theorem 1

We conjecture an optimal contract using the HJB equation. To show that HJB equation has an appropriate solution, we start by investigating its regularity properties (Lemma 1, 2, and 3) in order to prove existence and uniqueness of an appropriate solution. From that solution, Lemma 4 conjectures an optimal contract.

To ensure regularity, we start with a version of the HJB equation in which consumption is bounded from above by the level $\bar{c}$ such that $u'(\bar{c}) = \gamma$.

Lemma 1 proves continuity and concavity properties of the solutions to (5).

Lemma 1

The solutions to the HJB equation (5) exist and are unique and continuous in initial condition $M(W^e)$ and $M'(W^e)$. Moreover, initial conditions with $M''(W^e) < 0$ result in a concave solution.

Proof. The proof is similar to Sannikov Lemma 2. ■

Lemma 2 proves monotonicity properties of the phase diagram of solution.

Lemma 2

Consider two solutions M and $\hat{M}$ of equation (5) that satisfy $M(W^e) = \hat{M}(W^e)$ and $M'(W^e) < \hat{M}'(W^e)$. Then $M'(W^{e\prime}) < \hat{M}'(W^{e\prime})$ for all $W^{e\prime}$. Therefore, $M(W^{e\prime}) < \hat{M}(W^{e\prime})$ for all $W^{e\prime} > W^e$ and $M(W^{e\prime}) > \hat{M}(W^{e\prime})$ for all $W^{e\prime} < W^e$.

Proof. The proof is similar to Sannikov Lemma 2. ■

The optimal contract is constructed from a specific solution of the HJB equation, which satisfies $M(W^e) \geq M_p(W^e)$ for all $W \in [0, W_p^e]$ and boundary conditions

$$M(0) = 0 \qquad M(W_p^e) = M_p(W_p^e) = 0 \qquad M'(W_p^e) = M_p'(W_p^e)$$

for some $W_p^e \in (0, W_p^{e*}]$. Lemma 3 shows that there is a unique function with these properties.

Lemma 3

There exists a unique function $M \geq M_p$ that solves the HJB equation (5) and satisfies the boundary and smooth-pasting conditions (6), (7) and (8) for some $W_p^e \in (0, W_p^{e*}]$.

Proof. The proof is similar to Sannikov Lemma 3. ■

Lemma 4

Consider the unique solution $M(W^e) \geq M_p(W^e)$ of equation (5) that satisfies conditions (6), (7), and (8) for some $W_p^e \in (0, W_p^{e*}]$. Let $c : [0, W_p^e] \to [0, \bar{c}]$ be the maximizer in (5), and $\theta : [0, W_p^e] \to [0, \bar{\theta}]$ be the minimum incentives needed. For any starting condition $W_0^e \in [0, W_p^e]$, there is a unique in the sense of probability law weak solution to equation

$$\begin{aligned} dW_t^e &= \Big(rW_t^e - u(c(W_t^e)) + \frac{h}{\mu}e(W_t^e)\Big)dt + \theta(W_t^e)\Big(dX_t - e(W_t^e)dt\Big) \\ &= \Big(rW_t^e - u(c(W_t^e)) + \frac{h}{\mu}e(W_t^e)\Big)dt + \theta(W_t^e)\sigma dB_t^{e(W_t^e)} \end{aligned}$$

until the time $\tau_e = \inf\{t : W_t^e = 0 \text{ or } W_p^e\}$. The contract (c, e) defined by

$$\begin{aligned}
c_t &= 0, \qquad\quad e_t = \mu, \text{ for } t \in [0, \tau_e) \text{ and } 0 < W_t^e \le W_v^e \\
c_t &= c(W_t^e), e_t = \mu, \text{ for } t \in [0, \tau_e) \text{ and } W_v^e < W_t^e < W_p^e \\
c_t &= 0, \qquad\quad e_t = 0, \text{ for } t = \tau_e \text{ and } W_t^e = 0 \\
c_t &= c(W_p^e), e_t = 0, \text{ for } t = \tau_e \text{ and } W_t^e = W_p^e
\end{aligned}$$

is incentive compatible, and it initially has value W_0^e to the employee and profit $M(W_0^e)$ to the manager. The value of W_o^e is determined by the relative bargaining powers between the manager and the employee.

Proof. There is a weak solution of (5) unique in the sense of probability law by theorem 5.5.15 (p.341) from KS because the volatility is bounded above 0 by $\bar{\theta}\sigma$ (nondegeneracy) and the drift and volatility of W^e are bounded on $[0, W_p^e]$ (local integrability). Define $W_t^e = W_{\tau_e}^e$ for $t > \tau_e$, and let us show that $W_t^e = W_t^e(c, e)$, where $W_t^e(c, e)$ is the employee's true continuation value in the contract (c, e).

From the representation of $W_t^e(c, e)$ in Proposition 1, we have [30]

$$d(W_t^e(c,e) - W_t^e) = r(W_t^e(c,e) - W_t^e)dt + (\theta_t^e - \theta(W_t^e))\sigma dB_t^e$$

$$\Downarrow$$

$$E_t[W_{t+s}^e(c,e) - W_{t+s}^e] = e^{rs}(W_t^e(c,e) - W_t^e)$$

Note that $E_t[W_{t+s}^e(c,e) - W_{t+s}^e]$ must remain bounded, because both W^e (by 0 and W_p^e) and $W^e(c,e)$ (since c is bounded) are bounded. We conclude that $W_t^e = W_t^e(c,e)$ for all $t \ge 0$, and in particular, the employee gets value $W_0^e = W_0^e(c,e)$ from the entire contract. Also, the contract (c, e) is incentive compatible, since $(e_t, \theta_t^e) \in \{0, \mu\} \times [0, \bar{\theta}]$ for all t.

To see that the manager gets profit $M(W_0^e)$, consider

$$G_t^e(\Pi_e) = \int_0^t e^{-rs}(u_s - c_s - \phi)ds + e^{-rt}M(W_t^e)$$

By Ito's lemma, the drift of $G_t^e(\Pi_e)$ is

$$e^{-rt}\bigg(\Big(u_t - c_t - \phi - rM(W_t^e)\Big) + M'(W_t^e)\Big(rW_t^e - u(c_t) + \frac{h}{\mu}e_t\Big) + \frac{1}{2}M''(W_t^e)(\theta_t^e)^2\sigma^2\bigg).$$

[30] When one constructs a weak solution W_t^e, one may need a filtration bigger than that generated by underlying the Brownian motion. Then, while generally the Martingale Representation Theorem (used in Proposition 1 to represent $W_t^e(c,e)$) may fail, it holds on the minimal filtration that contains the Brownian motion and the solution if the solution is unique in the sense of probability law (see Jacod and Yor, 1977).

The value of this expression is 0 before time τ_e by the HJB equation. Therefore, $G_t^e(\Pi_e)$ is a bounded martingale until τ_e and the manager's profit from the entire contract is

$$E\Big[\int_0^{\tau_e} e^{-rs}(u_s - c_s - \phi)ds + e^{-r\tau_e} M_p(W_{\tau_e}^e)\Big] = E[G_{\tau_e}^e(\Pi_e)] = G_0^e(\Pi_e) = M(W_0^e)$$

since $M(W_{\tau_e}^e) = M_p(W_{\tau_e}^e)$. ■

Our last step is to verify that the contract presented in Lemma 4 is optimal. We start with a lemma that bounds from above the manager's profit from contracts that give the employee a value higher than W_p^{e*}.

Lemma 5

The profit from any contract (c, e) with an employee's value $W_0^e \geq W_p^{e*}$ is at most $M_p(W_0^e)$.

Proof. Define c by $u(c) = rW_0^e$, and from section 4.1.2, $c \geq c_t$ for all t. Concavity of the utility function $u(\cdot)$ implies that

$$u(c_t) \leq u(c) + u'(c)(c_t - c)$$

We have

$$\begin{aligned}
W_0^e &= E\Big[\int_0^{\tau_e} e^{-rt}\Big(u(c_t) - \frac{h}{\mu}e_t\Big)dt + e^{-r\tau_e}W_{\tau_e}^e\Big] \\
&\leq E\Big[\int_0^{\tau_e} e^{-rt}\Big(u(c) + u'(c)(c_t - c) - \frac{h}{\mu}e_t\Big)dt + e^{-r\tau_e}W_{\tau_e}^e\Big] \\
&\leq E\Big[\int_0^{\tau_e} e^{-rt}u(c)dt + e^{-r\tau_e}W_{\tau_e}^e\Big] + E\Big[\int_0^{\tau_e} e^{-rt}\Big(u'(c)(c_t - c) - \frac{h}{\mu}e_t\Big)dt\Big] \\
&\leq -\frac{e^{-r\tau_e}u(c)}{r} + \frac{u(c)}{r} + \frac{e^{-r\tau_e}u(c)}{r} \\
&= \frac{u(c)}{r}
\end{aligned}$$

The last inequality follow from $E\big[\int_0^{\tau_e} e^{-rt}\big(u'(c)(c_t - c) - \frac{h}{\mu}e_t\big)dt\big] \leq 0$, since $u'(c) \geq 0$ while $c_t \leq c$. Therefore the profit from this contract is at most $M_p(W_0^e)$. ■

Lemma 6

Consider a concave function M of the HJB equation. Any incentive compatible employee contract Π_e achieves profit of at most $M(W_0^e(c, e))$.

Proof. The concavity of M implies that

$$\min_{c\in[0,\infty)} M(W^e) + \frac{c}{r} - M'(W^e)(W^e - \frac{u(c)}{r})$$
$$= \min_{W_p^e \in \left[0, \frac{u(\infty)}{r}\right)} M(W^e) - M(M_p^e) - M'(W^e)(W^e - W_p^e) \geq 0$$

therefore $-c - rM(W^e) + M'(W^e)(rW^e - u(c)) \leq 0$

Denote the employee's continuation value by $W_t^e = W_t^e(c, e)$, which is represented by (3) using the process θ_t^e. By Lemma 5, the profit is at most $M_p(W_0^e) \leq M(W_0^e)$ if $W_0^e \geq W_p^{e*}$. If $W_0^e \in [0, W_p^{e*}]$, let us show the drift of $G_t^e(\Pi_e)$ is always nonpositive. If $e_t = \mu$, then the drift is nonpositive followed by equation (3). If $e_t = 0$, then the concavity of M implies that the drift of $G_t^e(\Pi_e)$ is nonpositive.

It follows that $G_t^e(\Pi_e)$ is a bounded supermartingale until the stopping time τ_e' (possible ∞), where τ_e' is defined by the time when W_t^e reaches W_p^{e*}. At time τ_e, the manager's future profit is less than or equal to $M_p(W_p^{e*}) \leq M(W_p^{e*})$ by Lemma 5. Therefore, the manager's expected profit at time 0 is less than or equal to

$$E^e\left[\int_0^{\tau_e'} e^{-rt}(u_s - c_s - \phi)ds + e^{-r\tau_e'}M(W_{\tau_e'}^e)\right] = E^e[G_{\tau_e'}^e(\Pi_e)] \leq G_0^e(\Pi_e) = M(W_0^e). \quad \blacksquare$$

Proof of Proposition 2

Let us compare the cases of working versus shirking from t to τ_e. Define by

$$V_t^w = \int_0^t e^{-rs}\Big(u(c_s) - h\Big)ds + e^{-rt}W_t^e(c, \mu)$$

the time t expectation of the employee's total payoff if he was working before time t, and plans to work after time t. Define by

$$V_t^s = \int_0^t e^{-rs}u(c_s)ds + e^{-rt}W_t^e(c, \mu)$$

the time t expectation of the employee's total payoff if he was shirking before time t, and plans to working after time t. Let us identify the drift of the process V_t^s under the probability measure Q^s, where

$$\sigma B_t^w = \sigma B_t^s$$

We have

$$
\begin{aligned}
dV_t^s &= e^{-rt}u(c_t)dt - re^{-rt}W_t^e(c,\mu)dt + e^{-rt}dW_t^e(c,\mu) \\
&= e^{-rt}u(c_t)dt - re^{-rt}W_t^e(c,\mu)dt + e^{-rt}\Big((rW_t^e(c,\mu) - u(c_t) + h)dt + \theta_t^w\sigma dB_t^w\Big) \\
&= e^{-rt}hdt + e^{-rt}\theta_t^w\sigma dB_t^w \\
&= e^{-rt}hdt + e^{-rt}\theta_t^w\sigma dB_t^s
\end{aligned}
$$

Since $h > 0$ by definition, then the drift of V_t^s under Q^s is nonnegative and positive on a set of positive measure. Thus, there exists a time $t > 0$ such that

$$W_0^e(c,0) = E^s[V_t^s] > V_0^s = W_0^e(c,\mu).$$

Because the employee gets utility $E^s[V_t^s]$ if he chooses shirking until time t and then switches to working, then working is suboptimal. ■

Proof of Proposition 3

Proposition 1 remains the same when I allow the manager to divert cash flows and the only thing changes is the HJB equation (5). In addition, the optimal choice of consumption is not affected either. Therefore the manager can still provide enough incentives to the employee even though the manager is stealing earnings.

Consider any incentive compatible manager and employee contracts $\{\{c_t\},\{u_t\},\tau_e\}$, where $\hat{X}_t < X_t$. To prove the proposition, I will show that there are new incentive compatible manager and employee contracts, which give the same consumption to the employee, the same payoff to the manager, but greater profit to the investors, under which the manager reports cash flows truthfully. The contracts are $\{\{c_t\},\{u_t'\},\tau_e'\}$, where $\hat{X}_t = X_t$. Note that the manager's payoff contains two parts: the salary from the investors and the stealing amounts from the project, and what I mean the same payoff is that the sum of the two are equal, although each part might be different.

Without loss of generality, I assume $\forall t$, where $0 \le t \le \tau_e$

$$\lambda(dX_t - d\hat{X}_t) + u_t = u_t'.$$

Since the employee receives the same consumption at each point under the two contracts, therefore

$$\tau_e = \tau_e'.$$

Incentive compatibility of both manager and employee contracts are guaranteed because both the continuation values of the manager and employee are the same. The incentive compatible conditions for employee and manager contracts are derived in Proposition 1 and Proposition 5, which each only depends on W^e and W^m, respectively.

The employee has the same continuation value under the old and new contract,

$$W_0^e = E\Big[\int_0^{\tau_e} e^{-rt}\big(u(c_t) - h\big)dt + e^{-r\tau_e} W_{\tau_e}^e\Big] = W_0^{e\prime},$$

since his consumption c_t remains the same.

The manager has the same continuation value under the old and new contracts,

$$\begin{aligned} W_0^m &= E\Big[\int_0^{\tau_e} e^{-rt}\Big(\lambda(dX_t - d\hat{X}_t) + (u_t - c_t - \phi)dt\Big) + e^{-r\tau_e} M_p(W_{\tau_e}^e)\Big] \\ &= E\Big[\int_0^{\tau_e} e^{-rt}\big(u_t' - c_t - \phi\big)dt + e^{-r\tau_e} M_p(W_{\tau_e}^e)\Big] \\ &= W_0^{m\prime}, \end{aligned}$$

since her payoff remains the same.

However, the investors have greater profit under the new contract,

$$\begin{aligned} b_0 &= E\Big[\int_0^{\tau_m} e^{-rt}(d\hat{X}_t - u_t dt) + e^{-r\tau_m} L\Big] \\ &= E\Big[\int_0^{\tau_m} e^{-rt}(\lambda d\hat{X}_t + (1-\lambda)d\hat{X}_t - u_t dt) + e^{-r\tau_m} L\Big] \\ &= E\Big[\int_0^{\tau_m} e^{-rt}(\lambda dX_t + (1-\lambda)d\hat{X}_t - u_t' dt) + e^{-r\tau_m} L\Big] \\ &< E\Big[\int_0^{\tau_m} e^{-rt}(\lambda dX_t + (1-\lambda)dX_t - u_t' dt) + e^{-r\tau_m} L\Big] \\ &= E\Big[\int_0^{\tau_m} e^{-rt}(dX_t - u_t' dt) + e^{-r\tau_m} L\Big] \\ &= b_0' \end{aligned}$$

since $\lambda(dX_t - d\hat{X}_t) + u_t = u_t'$, $\lambda < 1$, and $\tau_e = \tau_m$, which I show in Section 4. ■

Proof of Proposition 4

If the manager is shirking, by Proposition 3, the employee will be shirking, too. The dynamic of earnings becomes

$$dX_t = \sigma dB_t.$$

For the rest of the proof, please refer to derivation in Section 7.3. ■

Proof of Proposition 5

Note that $W_t^m(u, 1, \hat{X})$ is also the manager's continuation value if $\hat{X}_s, 0 \le s \le t$, were the true cash flows and the manager reported truthfully. Therefore, without loss of generality we can prove (9) for the case in which the manager truthfully reports $\hat{X} = X$. In that case,

$$\begin{aligned} V_t^m &= E_t^m\left[\int_0^{\tau_m} e^{-rs}(u_s - c_s - \phi)ds\right] \\ &= \int_0^t e^{-rs}(u_s - c_s - \phi)ds + E_t^m\left[\int_t^{\tau_m} e^{-rs}(u_s - c_s - \phi)ds\right] \\ &= \int_0^t e^{-rs}(u_s - c_s - \phi)ds + e^{-rt}W_t^m \end{aligned}$$

is a Q^m-martingale and by the martingale representation theorem there is a process θ^m such that

$$dV_t^m = e^{-rt}\theta_t^m(dX_t - \mu dt)$$

where $dX_t - \mu dt$ is a multiple of the standard Brownian motion. Differentiating with respect to t, we find

$$\begin{aligned} dV_t^m &= e^{-rt}\theta_t^m(dX_t - \mu dt) \\ &= -re^{-rt}W_t^m dt + e^{-rt}dW_t^m + e^{-rt}(u_t - c_t - \phi)dt \end{aligned}$$

therefore

$$dW_t^m = \big(rW_t^m - u_t + c_t + \phi\big)dt + \theta_t^m\big(dX_t - \mu dt\big).$$

If the manager steals $dX_t - d\hat{X}_t$ at time t, she gains immediate income of $\lambda(dX_t - d\hat{X}_t)$ but loses $\theta_t^m(dX_t - d\hat{X}_t)$ in future expected payoff. Therefore, the payoff from reporting strategy $\hat{X}$ gives the manager the payoff of

$$W_0^m + E\left[\int_0^{\tau_m} e^{-rt}\lambda(dX_t - d\hat{X}_t) - \int_0^{\tau_m} e^{-rt}\theta_t^m(dX_t - d\hat{X}_t)\right], \qquad (\star)$$

where W_0^m denotes the manager's payoff under truth-telling. We see that if $\theta_t^m \ge \lambda$ for all t then $(\star)$ is maximized when the manager chooses $d\hat{X}_t = dX_t$, since she cannot overreport cash flows. If $\theta_t^m < \lambda$ on a set of positive measure, then the manager is better off underreporting on this set than always telling the truth.[31] ■

[31] For example, the manager can report $d\hat{X}_t = dX_t - dt$ when $\theta^m < \lambda$ and tell the truth when $\theta \ge \lambda$. Because the probability measures over paths of X and $\hat{X}$ are equivalent, $\theta^m < \lambda$ on a set of positive measure and the

Proof of Theorem 2

First, let us verify that function I defined in the proposition is concave. Note that $I'(W^m) \geq -1$ and $rI(W^m) + rW^m < \mu - c - \phi$ imply $I'' < 0$. Therefore, the the left of W_s^m, with boundary conditions $I'(W_s^m) = -1$ and $rI(W_s^m) + rW^m = \mu - c - \phi$, function I enters the region where it is concave. Moreover, it stays concave because a concave function can never exit this region (this can be seen geometrically).

Next, let us prove that I represents the investors' optimal profit, which is achieved by the contract outlined in the Theorem. Define

$$G_t^m(\Pi_m) = \int_0^t e^{-rs}\big(dX_s - u_s ds\big) + e^{-rt} I(W_t^m)$$

Under an arbitrary incentive compatible contract, W_t^m evolves according to

$$dW_t^m = \big(rW_t^m - u_t + c_t + \phi\big)dt + \theta_t^m \sigma dB_t^m$$

Then, from Ito's lemma,

$$\begin{aligned} e^{rt} dG_t^m = & \big(\mu + I'(W_t^m)(rW_t^m + c_t + \phi) + \frac{1}{2} I''(W_t^m)(\theta_t^m)^2\sigma^2 - rI(W_t^m)\big) \\ & - \big(1 + I'(W_t^m)\big)dt \\ & + \big(1 + I'(W_t^m)\theta_t^m\big)\sigma dB_t^m \end{aligned}$$

The first term is ≤ 0 from equation (11) and the second term is also ≤ 0 since $I'(W^m) \geq -1$, G_t^m is a supermartingale. It is a martingale if and only if $\theta_t^m = \lambda$ and $W_t^m \leq W_s^m$ for $t > 0$ and u_t is increasing only when $W_t^m > W_s^m$.

We can now evaluate the investors' payoff for an arbitrary incentive compatible contract. Note that $I(W_{\tau_m}^m) = L$. For all $t < \infty$,

$$\begin{aligned} & E\Big[\int_0^{\tau_m} e^{-rs}(dX_s - u_s ds) + e^{-r\tau_m} L\Big] \\ = & E\Big[G_{t\wedge\tau_m}^m + 1_{t\leq\tau_m}\Big(\int_t^{\tau_m} e^{-rs}(dX_s - u_s ds) + e^{-r\tau_m} L - e^{-rt} I(W_t^m)\Big)\Big] \\ = & E[G_{t\wedge\tau_m}^m] + e^{-rt} E\Big[1_{t\leq\tau_m}\Big(E_t\Big[\int_t^{\tau_m} e^{-r(s-t)}(dX_s - u_s ds) + e^{-r(\tau_m - t)} L\Big] - I(W_t^m)\Big)\Big] \end{aligned}$$

Since

$$E[G_{t\wedge\tau_m}^m] \leq G_0^m = I(W_0^m)$$

manager will gain from this deviation.

$$E_t\Big[\int_t^{\tau_m} e^{-r(s-t)}(dX_s - u_s ds) + e^{-r(\tau_m - t)}L \le \frac{\mu}{r} - W_t^m = \text{ first-best}$$

Therefore, letting $t \to \infty$,

$$E\Big[\int_0^{\tau_m} e^{-rs}(dX_s - u_s ds) + e^{-r\tau_m}L\Big] \le I(W_0^m).$$

Finally, for a contract that satisfies the conditions of the Theorem, G_t^m is a martingale until time τ_m because $I'(W_t^m)$ stays bounded. Therefore, the payoff $I(W_0^m)$ is achieved with equality. ■

Proof of Theorem 3

$\tau_e \le \tau_m$ is trivial to see. When the project is liquidated, there is no point for the manager to keep hiring a non-productive employee. To show that $\tau_m \le \tau_e$, please refer to Section 7.3.

According to Theorem 1 and 2:

$$dW_t^e = (rW_t^e - u(c_t) + h)dt + \frac{h}{\mu}\sigma dB_t^e$$
$$dW_t^m = (rW_t^m - u_t + c_t + \phi)dt + \lambda\sigma dB_t^m$$

Although, the first SDE is under measure Q^e where $e = \mu$, and the second is under measure Q^m where $\hat{X} = X$, the first SDE is actually under the implicitly assumption of $m = 1$ and $\hat{X} = X$ and the second is under the assumption of $e = \mu$ and $m = 1$. Hence the above SDEs share essentially the same Brownian component. I define $dB_t = dB_t^e = dB_t^m$ for the sake of simplicity. Therefore, under optimal contracting of both employee and executive, all the expectations and dynamics are induced under the same underlying

$$dX_t = \mu dt + \sigma dB_t$$

Since $\tau_e = \tau_m = \tau$, the manager's highest payoff from optimal employee contract is

$$M_t = E_t\Big[\int_t^{\tau} e^{-r(s-t)}\Big((u_s - c_s - \phi)ds\Big)\Big]$$

and the manager's continuation value from optimal manager contract is

$$W_t^m = E_t\Big[\int_t^{\tau} e^{-r(s-t)}\Big((u_s - c_s - \phi)ds\Big)\Big].$$

Since both expectations are under the same probability measure, hence

$$M_t = W_t^m \qquad \forall t \in [0, \tau]. \quad \blacksquare$$

Proof of Section 5.1

In light of the Feynman-Kac formula, $M(W^e)$ can be written in its probabilistic representation

$$M(W^e) = E^{W_0^e = W^e}\left[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\right], \tag{$\star$ 1}$$

where the process $\{W^e\}$ evolves according to

$$dW_t^e = \big(rW_t^e - u(c_t) + h\big)dt + \frac{h}{\mu}\sigma dB_t. \tag{$\star$ 2}$$

Similarly, $I(W^m)$ can be written in its probabilistic representation

$$I(W^m) = E^{W_0^m = W^m}\left[\int_0^\tau e^{-rt}\mu dt + e^{-r\tau}L - \int_0^\tau e^{-rt}u_t dt\right], \tag{$\star$ 3}$$

where the process $\{W^m\}$ evolves according to

$$dW_t^m = \big(rW_t^m + c_t + \phi\big)dt + \lambda\sigma dB_t - u_t dt, \tag{$\star$ 4}$$

and $u_t dt$ is a process that reflects W_t^m at W_s^m.

Lemma A

Suppose that W_t^e and W_t^m evolve as according to

$$dW_t^e = (rW_t^e - u(c_t) + h)dt + \frac{h}{\mu}(dX_t - \mu dt)$$

$$dW_t^m = \big(rW_t^m + c_t + \phi\big)dt + \lambda\big(dX_t - \mu dt\big)$$

on the interval $[0, W_p^e]$ and $[0, W_s^m]$ respectively until time τ. Then

$$W_t^e = e^{rt}W_0^e - \int_0^t e^{-r(s-t)}\big(u(c_s) - h\big)ds + \int_0^t e^{-r(s-t)}\frac{h}{\mu}\sigma dB_s$$

$$W_t^m = e^{rt}W_0^m + \int_0^t e^{-r(s-t)}\big(c_s + \phi\big)ds + \int_0^t e^{-r(s-t)}\lambda\sigma dB_s$$

In addition, the same function $M : [0, W_p^e] \to \mathbb{R}$ both solves

$$rM(W^e) = u - c - \phi + M'(W^e)\Big(rW^e - u(c) + h\Big) + \frac{M''(W^e)}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2$$

with boundary conditions $M(0) = M(W_p^e) = 0$, and satisfies

$$M(W^e) = E^{W_0^e = W^e}\Bigg[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\Bigg].$$

Similarly, the same function $I : [0, W_s^m] \to \mathbb{R}$ both solves

$$rI(W^m) = \mu + I'(W^m)\Big(rW^m + c + \phi\Big) + \frac{I''(W^m)}{2}\lambda^2\sigma^2$$

with boundary conditions $I(0) = L$ and $I'(W_s^m) = -1$, and satisfies

$$I(W^m) = E^{W_0^m = W^m}\Bigg[\int_0^\tau e^{-rt}\mu dt + e^{-r\tau}L - \int_0^\tau e^{-rt}u_t dt\Bigg].$$

Proof. To solve for the above SDEs, first take $d(e^{-rt}W_t^e)$ and $d(e^{-rt}W_t^m)$, using Ito's Lemma, we get

$$d(e^{-rt}W_t^e) = -re^{-rt}W_t^e dt + e^{-rt}\big(rW_t^e - u(c_t) + h\big)dt + e^{-rt}\frac{h}{\mu}\sigma dB_t$$
$$d(e^{-rt}W_t^m) = -re^{-rt}W_t^m dt + e^{-rt}\big(rW_t^m + c_t + \phi\big)dt + e^{-rt}\lambda\sigma dB_t$$

Integrate both sides from 0 to t, we get the Lemma.

Suppose M solves (5), and let us show that it satisfies ($\star$ 1). Define

$$H_t^m = \int_0^t e^{-rs}(u_s - c_s - \phi)ds + e^{-rt}M(W_t^e)$$

Then, using Ito's lemma,

$$e^{rt}dH_t^m = \Bigg(u_t - c_t - \phi + M'(W_t^e)\Big(rW_t^e - u(c_t) + h\Big) + \frac{M''(W_t^e)}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2 - rM(W_t^e)\Bigg)dt$$
$$+ M'(W_t^e)\frac{h}{\mu}\sigma dB_t$$

From equation (5), H^m is a martingale. Because M is bounded, H^m is a martingale until time τ, so that

$$M(W_0^e) = H_0^m = E[H_\tau^m] = E\Bigg[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\Bigg].$$

Similarly, suppose I solves (12), and let us show that it satisfies ($\star$ 3). Define

$$H_t^i = \int_0^t e^{-rs}\mu ds - \int_0^t e^{-rs}u_s ds + e^{-rt}I(W_t^m)$$

Then, using Ito's lemma,

$$\begin{aligned} e^{rt}dH_t^i =& \Big(\mu + I'(W_t^m)\Big(rW_t^m + c_t + \phi\Big) + \frac{I''(W_t^m)}{2}\lambda^2\sigma^2 - rI(W_t^m)\Big)dt \\ &- \big(1 + I'(W_t^m)\big)u_t dt \\ &+ M'(W_t^e)\frac{h}{\mu}\sigma dB_t \end{aligned}$$

From equation (12), condition $I'(W_s^m) = -1$, and the fact that u_t takes positive value only when $W_t^m = W_s^m$, H^i is a martingale. Because I is bounded, H^i is a martingale until time τ, so that

$$I(W_0^m) = H_0^i = E[H_\tau^i] = E\Big[\int_0^\tau e^{-rt}\mu dt + e^{-r\tau}L - \int_0^\tau e^{-rt}u_t dt\Big].\blacksquare$$

Lemma B

For any $\theta \in \{\mu, \sigma^2, h, \phi, \lambda, L\}$ and denote by $M_\theta(W^e)$ and $I_\theta(W^m)$ the manager's and investors' value function for that parameter value. Then

$$\frac{\partial W_t^e}{\partial \theta} = -\int_0^t e^{-r(s-t)}\frac{\partial\big(u(c_s) - h\big)}{\partial\theta}ds + \int_0^t e^{-r(s-t)}\frac{\partial\big(\frac{h}{\mu}\sigma\big)}{\partial\theta}dB_s$$

$$\frac{\partial W_t^m}{\partial \theta} = \int_0^t e^{-r(s-t)}\frac{\partial\big(c_s + \phi\big)}{\partial\theta}ds + \int_0^t e^{-r(s-t)}\frac{\partial(\lambda\sigma)}{\partial\theta}dB_s$$

$$\begin{aligned}\frac{\partial M_\theta(W^e)}{\partial\theta} =& E^{W_0^e = W^e}\Bigg[\int_0^\tau e^{-rt}\Bigg(\frac{\partial(u - c - \phi)}{\partial\theta} + \frac{\partial(rW^e - u(c) + h)}{\partial\theta}M_\theta'(W^e) \\ &+ \frac{1}{2}\frac{\partial(\frac{h}{\mu})^2\sigma^2}{\partial\theta}M_\theta''(W^e)\Bigg)dt\Bigg]\end{aligned}$$

$$\begin{aligned}\frac{\partial I_\theta(W^m)}{\partial\theta} =& E^{W_0^m = W^m}\Bigg[\int_0^\tau e^{-rt}\Bigg(\frac{\partial\mu}{\partial\theta} + \frac{\partial(rW^m + c + \phi)}{\partial\theta}I_\theta'(W^m) \\ &+ \frac{1}{2}\frac{\partial\lambda^2\sigma^2}{\partial\theta}I_\theta''(W^m)\Bigg)dt + e^{-r\tau}\frac{\partial L}{\partial\theta}\Bigg]\end{aligned}$$

$$\frac{\partial W_p^e}{\partial \theta} = 0$$

$$\frac{\partial W_s^m}{\partial \theta} = -\frac{\partial I_\theta(W_s^m)}{\partial \theta} + \frac{\partial\big(\frac{\mu-c-\phi}{r}\big)}{\partial \theta}$$

Proof. Consider a value of W_p^e and a corresponding incentive compatible contract of Proposition 1, that is one in which the employee is laid off whenever W_t^e hit W_p^e. Then the manager's profit under this contract is

$$M_{\theta,W_p^e}(W^e) = E^{W_0^e=W^e}\bigg[\int_0^\tau e^{-rt}(u_t - c_t - \phi)dt\bigg].$$

By Lemma A, $M_{\theta,W_p^e}(W^e)$ solves equation

$$rM_{\theta,W_p^e}(W^e) = u - c - \phi + \Big(rW^e - u(c) + h\Big)M'_{\theta,W_p^e}(W^e) + \frac{1}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2 M''_{\theta,W_p^e}(W^e)$$

with boundary conditions $M_{\theta,W_p^e}(0) = M_{\theta,W_p^e}(W_p^e) = 0$. Denote by $W_p^e(\theta)$ the choice of W_p^e that maximizes the manager's profit $M_{\theta,W_p^e}(W_0^e)$ for a given value of parameter θ. Then $M_\theta(W^e) = M_{\theta,W_p^e(\theta)}(W^e)$. By the Envelope Theorem,

$$\frac{\partial M_\theta(W^e)}{\partial \theta} = \frac{\partial M_{\theta,W_p^e}(W^e)}{\partial \theta}|_{W_p^e=W_p^e(\theta)}.$$

Differentiating above equation with respect to θ at $W_p^e = W_p^e(\theta)$ and using the Envelope Theorem, we find that $\frac{\partial M_\theta(W^e)}{\partial \theta}$ satisfies the equation

$$\begin{aligned} r\frac{\partial M_\theta(W^e)}{\partial \theta} =& \frac{\partial(u-c-\phi)}{\partial \theta} + \frac{\partial(rW^e - u(c) + h)}{\partial \theta}M'_\theta(W^e) + (rW^e - u(c) + h)\frac{\partial}{\partial W^e}\frac{\partial M_\theta(W^e)}{\partial \theta} \\ &+ \frac{1}{2}\frac{\partial(\frac{h}{\mu})^2\sigma^2}{\partial \theta}M''_\theta(W^e) + \frac{1}{2}\Big(\frac{h}{\mu}\Big)^2\sigma^2\frac{\partial^2}{\partial (W^e)^2}\frac{\partial M_\theta(W^e)}{\partial \theta} \end{aligned}$$

with boundary conditions $\frac{\partial M_\theta(0)}{\partial \theta} = \frac{\partial M_\theta(W_p^e)}{\partial \theta} = \frac{\partial M_p(W_\tau^e)}{\partial \theta}$ and $\frac{\partial}{\partial W^e}\frac{\partial M_\theta(W_p^e)}{\partial \theta} = 0$. The lemma then follows from Lemma A.

Similarly, consider a value of W_s^m and a corresponding incentive compatible contract of Proposition 5, that is one in which the process u reflects W^m at W_s^m. Then the investors' profit under this contract is

$$I_{\theta,W_s^m}(W^m) = E^{W_0^m=W^m}\bigg[\int_0^\tau e^{-rt}\mu dt + e^{-r\tau}L - \int_0^\tau e^{-rt}u_t dt\bigg].$$

By Lemma A, $I_{\theta,W_s^m}(W^m)$ solves equation

$$rI_{\theta,W_s^m}(W^m) = \mu + \Big(rW^m + c + \phi\Big) I'_{\theta,W_s^m}(W^m) + \frac{1}{2}\lambda^2\sigma^2 I''_{\theta,W_s^m}(W^m)$$

with boundary conditions $I_{\theta,W_s^m}(0) = L$ and $I'_{\theta,W_s^m}(W_s^m) = -1$. Denote by $W_s^m(\theta)$ the choice of W_s^m that maximizes the investors' profit $I_{\theta,W_s^m}(W_0^m)$ for a given value of parameter θ. Then $I_\theta(W^m) = I_{\theta,W_s^m(\theta)}(W^m)$. By the Envelope Theorem,

$$\frac{\partial I_\theta(W^m)}{\partial\theta} = \frac{\partial I_{\theta,W_s^m}(W^m)}{\partial\theta}|_{W_s^m=W_s^m(\theta)}.$$

Differentiating above equation with respect to θ at $W_s^m = W_s^m(\theta)$ and using the Envelope Theorem, we find that $\frac{\partial I_\theta(W^m)}{\partial\theta}$ satisfies the equation

$$\begin{aligned} r\frac{\partial I_\theta(W^m)}{\partial\theta} =& \frac{\partial\mu}{\partial\theta} + \frac{\partial(rW^m + c + \phi)}{\partial\theta} I'_\theta(W^m) + (rW^m + c + \phi)\frac{\partial}{\partial W^m}\frac{\partial I_\theta(W^m)}{\partial\theta} \\ &+ \frac{1}{2}\frac{\partial\lambda^2\sigma^2}{\partial\theta} I''_\theta(W^m) + \frac{1}{2}\lambda^2\sigma^2\frac{\partial^2}{\partial(W^m)^2}\frac{\partial I_\theta(W^m)}{\partial\theta} \end{aligned}$$

with boundary conditions $\frac{\partial I_\theta(0)}{\partial\theta} = \frac{\partial L}{\partial\theta}$ and $\frac{\partial}{\partial W^m}\frac{\partial I_\theta(W_s^m)}{\partial\theta} = 0$. The lemma then follows from Lemma A.

To find the effect of the parameters on W_p^e and W_s^m, we need to differentiate

$$M'_\theta(W_p^e) = \frac{c}{rW_p^e - u(c)}$$

and

$$I_\theta(W_s^m) = -W_s^m + \frac{\mu - c - \phi}{r}$$

with respect to θ, and the Lemma follows. ■

$$\frac{\partial W_t^e}{\partial\mu} = \int_0^t e^{-r(s-t)}\Big(-\frac{h}{\mu^2}\sigma\Big)dB_s \ ?\ 0$$

$$\frac{\partial W_t^e}{\partial\sigma} = \int_0^t e^{-r(s-t)}\Big(\frac{h}{\mu}\Big)dB_s \ ?\ 0$$

$$\frac{\partial W_t^e}{\partial h} = \frac{e^{rt}}{r} - \frac{1}{r} + \int_0^t e^{-r(s-t)}\Big(\frac{\sigma}{\mu}\Big)dB_s \ ?\ 0$$

$$\frac{\partial W_t^e}{\partial\phi} = \frac{\partial W_t^e}{\partial\lambda} = \frac{\partial W_t^e}{\partial L} = 0$$

$$\frac{\partial W_t^m}{\partial\sigma} = \int_0^t e^{-r(s-t)}\lambda dB_s \ ?\ 0$$

$$\frac{\partial W_t^m}{\partial \phi} = \int_0^t e^{-r(s-t)} ds \ > \ 0$$

$$\frac{\partial W_t^m}{\partial \lambda} = \int_0^t e^{-r(s-t)} \sigma dB_s \ ? \ 0$$

$$\frac{\partial W_t^m}{\partial \mu} = \frac{\partial W_t^m}{\partial h} = \frac{\partial W_t^m}{\partial L} = 0$$

$$\frac{\partial M_\theta(W^e)}{\partial \mu} = E^{W_0^e=W^e}\left[\int_0^\tau e^{-rt}\left(- \frac{h^2\sigma^2}{\mu^3} M_\theta''(W^e)\right) dt\right] \ > \ 0$$

$$\frac{\partial M_\theta(W^e)}{\partial \sigma^2} = E^{W_0^e=W^e}\left[\int_0^\tau e^{-rt}\left(\frac{1}{2}\Big(\frac{h}{\mu}\Big)^2 M_\theta''(W^e)\right) dt\right] \ < \ 0$$

$$\frac{\partial M_\theta(W^e)}{\partial h} = E^{W_0^e=W^e}\left[\int_0^\tau e^{-rt}\left(M_\theta'(W^e) + \frac{h\sigma^2}{\mu^2} M_\theta''(W^e)\right) dt\right] \ ? \ 0$$

$$\frac{\partial M_\theta(W^e)}{\partial \phi} = E^{W_0^e=W^e}\left[\int_0^\tau e^{-rt}(-1) dt\right] \ < \ 0$$

$$\frac{\partial M_\theta(W^e)}{\partial \lambda} = \frac{\partial M_\theta(W^e)}{\partial L} = 0$$

$$\frac{\partial I_\theta(W^m)}{\partial \mu} = E^{W_0^m=W^m}\left[\int_0^\tau e^{-rt} dt\right] \ > \ 0$$

$$\frac{\partial I_\theta(W^m)}{\partial \sigma^2} = E^{W_0^m=W^m}\left[\int_0^\tau e^{-rt} \lambda^2 \sigma I_\theta''(W^m) dt\right] \ < \ 0$$

$$\frac{\partial I_\theta(W^m)}{\partial h} = 0$$

$$\frac{\partial I_\theta(W^m)}{\partial \phi} = E^{W_0^m=W^m}\left[\int_0^\tau e^{-rt} I_\theta'(W^m) dt\right] \ ? \ 0$$

$$\frac{\partial I_\theta(W^m)}{\partial \lambda} = E^{W_0^m=W^m}\left[\int_0^\tau e^{-rt} \lambda \sigma^2 I_\theta''(W^m) dt\right] \ < \ 0$$

$$\frac{\partial I_\theta(W^m)}{\partial \lambda} = E^{W_0^m=W^m}\left[e^{-r\tau}\right] \ > \ 0$$

$$\frac{\partial W_s^m}{\partial \mu} = -E^{W_0^m=W^m}\left[\int_0^\tau e^{-rt} dt\right] + \frac{1}{r} = E^{W_0^m=W^m}\left[\frac{2 - e^\tau}{r}\right] \ ? \ 0$$

$$\frac{\partial W_s^m}{\partial \sigma^2} = -\frac{\partial I_\theta(W^m)}{\partial \sigma^2} \; > \; 0$$
$$\frac{\partial W_s^m}{\partial h} = -\frac{\partial I_\theta(W^m)}{\partial h} \; = \; 0$$
$$\frac{\partial W_s^m}{\partial \phi} = -E^{W_0^m = W^m}\left[\int_0^\tau e^{-rt} I_\theta'(W^m) dt\right] - \frac{1}{r} \; ? \; 0$$
$$\frac{\partial W_s^m}{\partial \lambda} = -\frac{\partial I_\theta(W^m)}{\partial \lambda} \; > \; 0$$
$$\frac{\partial W_s^m}{\partial L} = -\frac{\partial I_\theta(W^m)}{\partial L} \; < \; 0.$$

Proof of Theorem 4

Let's assume the project's liquidation time between the Principal-Agent and the Principal-Agent-Subagent model is the same. Rewrite the investors' problem:

$$\begin{aligned} I_0^a &= \max_{\substack{\Pi_m^1 \in \mathbb{IC}_m^1 \\ \Pi_m^2 \in \mathbb{IC}_m^2}} E\left[\int_0^\tau e^{-rt}(d\hat{X}_t^1 + d\hat{X}_t^2 - u_t^1 dt - u_t^2 dt) + e^{-r\tau} L\right] \\ &= \max_{\Pi_m^a \in \mathbb{IC}_m^a} E\left[\int_0^\tau e^{-rt}(d\hat{X}_t^a - u_t^a dt) + e^{-r\tau} L\right] \end{aligned}$$

where $\hat{X}_t^a \equiv \hat{X}_t^1 + \hat{X}_t^2$, $u_t^a \equiv u_t^1 + u_t^2$, and $\Pi_a \equiv \{\{u^a\}, \tau\}$ is the Optimal Total Compensation Contract.

$\mathbb{IC}_a$: the first agent's effort process $\{a_t^1, 0 \leq t \leq \tau\}$ and the reported cash flow $\{\hat{X}_t^1, 0 \leq t \leq \tau\}$ are incentive compatible with respect to his salary $\{u_t^1, 0 \leq t \leq \tau\}$ if it maximizes the first agent's total expected profit until his exit time τ, and the second agent's effort process $\{a_t^2, 0 \leq t \leq \tau\}$ and the reported cash flow $\{\hat{X}_t^2, 0 \leq t \leq \tau\}$ are incentive compatible with respect to his salary $\{u_t^2, 0 \leq t \leq \tau\}$ if it maximizes the second agent's total expected profit until his exit time τ.

Subject to giving either or both agents a total specific value of $W_0^a \geq 0$

$$W_0^a = \max_{\substack{a^1 = \{a_t^1 \in \{0, \sqrt{\mu}\} : 0 \leq t \leq \tau\} \\ a^2 = \{a_t^2 \in \{0, \sqrt{\mu}\} : 0 \leq t \leq \tau\} \\ d\hat{X}^1 = \{d\hat{X}_t^1 \in [0, dX_t^1] : 0 \leq t \leq \tau\} \\ d\hat{X}^2 = \{d\hat{X}_t^2 \in [0, dX_t^2] : 0 \leq t \leq \tau\}}} E\left[\int_0^\tau e^{-rt}\Big(\lambda(dX_t^1 - d\hat{X}_t^1) + \lambda(dX_t^2 - d\hat{X}_t^2) + (u_t^1 - \frac{\phi}{\sqrt{\mu}} a_t^1 + u_t^2 - \frac{\phi}{\sqrt{\mu}} a_t^2) dt\Big)\right]$$

Total continuation value: fix an arbitrary salary process $u^i = \{u_t^i : 0 \leq t \leq \tau\}$, an effort strategy $a^i = \{a_t^i = \sqrt{\mu} : 0 \leq t \leq \tau\}$ and, an amount of reporting $\hat{X}^i = \{\hat{X}_t^i : 0 \leq t \leq \tau\}$, which may or may not be optimal for agent i given u^i, where $i = \{1, 2\}$. Define the total continuation value $W^a_t(u^1, u^2, a^1, a^2, \hat{X}^1, \hat{X}^2)$ after a history of reports $(\hat{X}^a_s, 0 \le s \le t)$ to be the total expected payoff either or both agents receive, from transfers, if both agents tell the truth after time t to liquidation time τ, to be:

$$W^a_t(u^1, u^2, a^1, a^2, \hat{X}^1, \hat{X}^2) = E^a_t\Big[\int_t^\tau e^{-r(s-t)}(u^1_s - \frac{\phi}{\sqrt{\mu}}a^1_s + u^2_s - \frac{\phi}{\sqrt{\mu}}a^2_s)ds\Big]$$

where E^a denotes the expectation under the probability measure Q^a induced by both agents' strategy a^1 and a^2.

Similar to Proposition 1 and 5, at any moment of time $0 \le t \le \tau$, there is a sensitivity θ^a_t of both agents' continuation value toward their reports such that

$$dW^a_t = \big(rW^a_t - u^a_t + \frac{\phi}{\sqrt{\mu}}a^1_t + \frac{\phi}{\sqrt{\mu}}a^2_t\big)dt + \theta^a_t\big(dX_t - a^1_t a^2_t dt\big)$$

Proposition 1 implies that working is optimal if and only if

$$\theta^a_t \ge \frac{\phi}{\sqrt{\mu}}, \quad \forall 0 \le t \le \tau$$

Proposition 5 implies truth-telling is incentive compatiable if and only if

$$\theta^a_t \ge \lambda, \quad \forall 0 \le t \le \tau$$

Therefore, in order for the investors to induce both agents to work and tell the truth, the minimal level of incentives required is

$$\theta^a_t \ge \lambda_a = \max\{\frac{\phi}{\sqrt{\mu}}, \lambda\}, \quad \forall 0 \le t \le \tau$$

Similar to Section 4.2, the investors' value function satisfies the following second-order ordinary differential equation:

$$rI(W^a) = \mu + I'(W^a)\Big(rW^a + \phi + \phi\Big) + \frac{I''(W^a)}{2}\lambda_a^2\sigma^2, \quad 0 \le W^a \le W^a_s,$$

with

$$I(W^a) = -W^a + \frac{\mu - \phi - \phi}{r}, \quad W^a > W^a_s.$$

In Section 5.1, I have showed that

$$\frac{\partial I(W)}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial W^m_s}{\partial \lambda} > 0$$

therefore, if $\lambda_a > \lambda$, $I(W^a) < I(W^m)$ and $W_s^a > W_s^m$. ■

References

[1] Alchian, Armen, and Harold Demsetz, 1972, "Production, Information Costs, and Economic Organization," *American Economic Review*, 62, 777-797.

[2] Anderson, Ronald W., M. Cecilia Bustamante, Stephane Guibaud, and Mihail Zervos, 2017, "Agency, Firm Growth, and Managerial Turnover," *Journal of Finance*.

[3] Ang, James S., Rebel A. Cole, and James Wuh Lin, 2000, "Agency Costs and Ownership Structure," *Journal of Finance*, February, 55(1), 81-106.

[4] Baker, Geroge, Michael Gibbs, and Bengt Holmstrom, 1993, "Hierarchies and compensation, a case study," *European Economic Review*, 37, 366-378.

[5] Baker, Geroge, Michael Gibbs, and Bengt Holmstrom, 1994, "The Internal Economics of the Firm: Evidence from Personnel Data," *Quarterly Journal of Economics*, November, 881-919.

[6] Baliga, Sandeep, and Thomas Sjostrom, 1998, "Decentralization and Collusion," *Journal of Economic Theory*, 83, 196-232.

[7] Calvo, Guillermo A., and Stanislaw Wellisz, 1978, "Supervision, Loss of Control, and the Optimum Size of the Firm," *Journal of Political Economy*, 86, 943-952.

[8] Chandler, Alfred, 1962, *Strategy and Structure*, Cambridge, MA: MIT Press.

[9] Chandler, Alfred, 1977, *The Visible Hand*, Cambridge, MA: Harvard University Press.

[10] DeMarzo, Peter, Michael Fishman, Zhiguo He, and Neng Wang, 2012, "Dynamic Agency and q Theory of Investment," *Journal of Finance*, 67, 2295-2340.

[11] DeMarzo, Peter, and Yuliy Sannikov, 2006, "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model," *Journal of Finance*, 61, 2681-2724.

[12] Detemple, Jerome, Suresh Govindaraj, and Mark Loewenstein, 2001, "Optimal contracts and intertemporal incentives with hidden actions," Working paper, Boston University.

[13] Hackbarth, Dirk, and David C. Mauer, 2012, "Optimal Priority Structure, Capital Structure, and Investment," *Review of Financial Studies*, 25(3), 747-796.

[14] He, Zhiguo, 2009, "Optimal Executive Compensation when Firm Size Follows Geometric Brownian Motion," *Review of Financial Studies*, 22, 859-892.

[15] Innes, Robert D, 1990, "Limited liability and incentive contracting with ex-ante action choices," *Journal of Economic Theory*, 52(1), 45-67.

[16] Jensen, Michael C., and William H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305-360.

[17] Karatzas, Ioannis, and Steven E. Shreve, 1991, "Brownian Motion and Stochastic Calculus," *Springer-Verlag: New York.*

[18] Milgrom, Paul, and John Robers, 1992, "Economics, Organization, and Management," *Prentice-Hall International, Inc.*

[19] Mookherjee, Dilip, 2013, "Incentices in Hierarchies," *Handbook of Organizational Economics.*

[20] Myerson, Roger B., 1982, "Optimal Coordination Mechanisms in Generalized Principal Agent Problems," *Journal of Mathematical Economics*, 10, 67-81.

[21] Roe, Mark J., 2005, "The Institutions of Corporate Governance," *Handbook of New Institutional Economics*, 371-399.

[22] Leland, Hayne E., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49(4), 1213-1252.

[23] Van Den Steen, Eric., 2010, "Interpersonal Authority in a Theory of The Firm," *American Economic Review*, 100(1), 466-490.

[24] Sannikov, Yuliy, 2008, "A Continuous-Time Version of the Principal-Agent Problem," *Review of Economic Studies*, 75, 957-984.

[25] Shim, Ilhyock, 2011, "Dynamic Prudential Regulation: Is Prompt Corrective Action Optimal?" *Journal of Money, Credit and Banking*, 43(8), 16251661.

[26] Spear, Stephen E., and Cheng Wang, 2005, "When to fire a CEO: optimal termination in dynamic contracts," *Journal of Economic Theory*, 120, 239-256.

[27] Williamson, Oliver, 1967, "Hierarchical Control and Optimal Firm Size," *Journal of Political Economy*, 123-138.g

[28] Zame, William R., 2007, "Incentives, Contracts, and Markets: A General Equilibrium Theory of Firms," *Econometrica*, 75(5), 1453-1500.