Distress Risk: An Accelerated Failure Time Survival Analysis Approach

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Abstract

This paper explores the performance of an accelerated failure time (AFT) survival model in predicting corporate bankruptcies. AFT models provide an alternative to the proportional hazard model that allows for non-monotonic hazard functions with respect to time and does not assume proportionality. We show that at a quarterly frequency, the AFT model outperforms comparative models from literature in terms of both in-sample and out-of-sample bankruptcy prediction accuracy across all evaluation metrics. Furthermore the AFT specification generates a parsimonious prediction model and models the survival time directly and thus provides more intuitive summary statistics.

1. Introduction

Bankruptcy is an important corporate event as it impacts an array of stakeholders associated with the distressed entity. The recent global macroeconomic and financial events have reinvigorated interest in bankruptcy and default prediction (Bagliano and Morana 2014) and have also raised questions as to whether more accurate credit risk quantification could have moderated the extent and impact of the global financial crisis (Milne 2014). The failings of financial institutions in quantifying bankruptcy probabilities and credit exposure became particularly evident through the sheer magnitude of losses incurred by financial institutions and can partly be attributed to the frequency of model forecast horizons. Typical credit risk models, such as the JP Morgan CreditMetrics model, mostly forecast expected credit losses over short horizons and while this leads to an increase in the predictive power of the forecasts, frequencies that are too short have limited utility as you end up predicting bankruptcy essentially as it occurs (Campbell et al. 2008). On the other hand, typical accounting bankruptcy models estimate bankruptcies over longer horizons (see for example, Shumway (2001), Chava and Jarrow (2004), Lando and Nielsen (2010), among others) and while these advance predictions are instructive, they are prone to significant forecast errors.

To generate longer term predictions of financial distress accounting models frequently employ a discrete-time logistic hazard model (for example, Shumway, 2001; Chava and Jarrow, 2004 and Campbell et al., 2008). Shumway (2001) demonstrated that hazard models are superior to their static counterparts as they are dynamic in nature, implicitly considering all information up to a point in time. They utilise more data and, therefore, produce more precise parameter estimates, leading to more accurate forecasts. Further, hazard models implicitly account for the duration dependence of default. However, commonly applied proportional cox models have unrealistic assumptions that may not hold when predicting bankruptcies. Notably, the assumption that every firm has an identically shaped hazard function and that the proportion of each firm's hazard is unchanged throughout time, is a particularly limiting assumption if significant heterogeneity exists between firms.

Motivated by the significant mispricing of credit risk in the lead up to the global financial crisis (GFC), we consider an alternative survival model specification known as an accelerated failure time (AFT) survival model to predict corporate bankruptcy.[1] AFT models allow for non-monotonic hazard functions with respect to time and do not assume proportionality. A firm's hazard of bankruptcy thus becomes a function of time allowing for hazards to increase in the short-term during a firm's establishment and growth phase and decrease in the long-term as a firm reaches maturity. Consequently, we expect that an AFT survival model specification will be more appropriate for forecasting corporate bankruptcies and will prove superior to alternative specifications. To test this hypothesis, we compare the in-sample and out-of-sample three month ahead forecast accuracy of four widely used bankruptcy models from literature against their AFT specification. A three month forecast horizon is chosen to match the

[1] AFT models originated from medicine statistics; see for example Wei (1992).

reporting frequency of most listed firms and should provide sufficient time to identify the onset of financial distress yet ensure the prediction is accurate and still useful. To the best of the authors' knowledge, AFT models have not yet been widely applied to bankruptcy prediction with most survival models following the discrete-time logistic hazard model proposed by Shumway (2001).

Given our AFT model specification, forecast horizon, and ongoing contention of predictors of default within the literature, we examine the significance of a range of accounting and market-based variables in our model capturing a firm's profitability, solvency, and liquidity. The validity of accounting ratios has been refuted by studies such as Agarwal and Taffler (2008) and Hillegeist et al. (2004). Furthermore, while accounting variables capturing a firm's leverage and profitability are generally found to be significant, accounting variables capturing a firm's liquidity are less prominent; the model proposed by Shumway (2001), for example, does not contain a liquidity variable. More recently, prediction models have focussed on market-based and macroeconomic covariates (Das et al. 2007 and Duffie et al. (2009), however, there is compelling evidence from Beaver et al. (2005), Bharath and Shumway (2008) and Campbell et al. (2008) that accounting ratios continue to hold predictive power in well-specified failure prediction models containing both market-based and macroeconomic covariates.

We make the following contributions to literature. Firstly, we demonstrate that AFT specification is superior in prediction accuracy when benchmarked against the commonly applied discrete-time logistic hazard model using the variable specifications of Altman (1968), Zmijewski (1984), Shumway (2001) and Campbell et al. (2008). For example, the discrete-time logistic model of Shumway (2001) is found to have a discriminatory power of 0.82, as quantified by the area under the Receiver Operating Characteristic (ROC) curve, compared to an AFT specification of 0.94. Similarly the AFT model has an out of sample accuracy of 0.89 compared to 0.85 for the Campbell et al (2008) model. While previous studies have tended to focus on a single accuracy measure, (for example, Shumway (2001) focused on the forecast accuracy of decile rankings, while Campbell et al. (2008) relied on McFadden's R^2), we use a range of evaluation measures, including the area under the ROC curve and the Schwarz Bayesian Information (BIC) Criterion, to demonstrate the superiority of AFT models in both in-sample and out-of sample tests.

Secondly, after examining numerous accounting and market-based variables, we construct a new parsimonious five variable AFT bankruptcy prediction model and demonstrate its superiority over the Shumway (2001) and Campbell et al. (2008) model in in-sample and out-of-sample evaluations. Out-of-sample the AFT model achieves a ROC value 0.8948 compared to 0.8735 and 0.8598 for the Shumway (2001) and Campbell et al. (2008), respectively. We find that both accounting and market-based variables are important predictors of bankruptcy in line with previous literature. In particular, we show that net income over total assets and total liabilities over total assets are significant, however, a commonly used covariate, stock price volatility, is not significant after the inclusion of working capital ratio, which captures a firm's short-term liquidity.

This paper structured as follows. Section 2 reviews the literature relating to bankruptcy prediction, including the various models and conflicting views of accounting, market and economic variables. Section 3 details the methodology and Section 4 describes the dataset. Section 5 discusses the findings of results and contrasts this against the existing findings within the literature. Finally, Section 6 concludes.

2. Bankruptcy Prediction Literature

Early bankruptcy models employed a range of company ratios to predict corporate failure. Beaver (1966) examined thirty most popular ratios and found that the 'cash flow to debt' ratio held the most explanatory power. Altman (1968) extended the traditional ratio analysis with more rigorous multivariate discriminant analysis (MDA) framework and identified five variables capturing profitability, liquidity and leverage to be significant predictors of default.

Further extensions yielded the Altman (1977) seven variable commercial Zeta model using firm's return on assets, debt service coverage, profitability, liquidity, leverage and size. Dambolena and Khoury (1980) found that firms which enter bankruptcy demonstrate ratio instability in the years prior to bankruptcy and introduced the stability of financial ratios as explanatory variables in the derivation of the discriminant function. Zmijewski (1984), however, argued that discriminant techniques which involve a pair-sample design where bankrupt firms are matched to similar non-bankrupt firms leads to overstating of the importance of covariates as the sample bankruptcy rate does not reflect the market bankruptcy rate. Further, he claimed that the matching process is flawed and arbitrary. Ohlson (1980) overcame some of limiting features of the MDA approach with a logit model and confirmed that nine accounting variables relating to firm size, leverage, profitability and liquidity held explanatory power. Similarly, Zmijewski (1984) adopted a probit specification, which assumes normality rather than a logistic distribution.

Shumway (2001) demonstrated that early bankruptcy studies were biased by their static nature and suggested an alternative technique, known as survival analysis, to predict bankruptcy. He proposed a discrete-time hazard model to predict a firm's bankruptcy using both accounting and market-based variables. Survival analysis, particularly the Proportional Cox model and discrete-time logistic hazard model, have since been routinely applied (Chava and Jarrow 2004; Campbell et al. 2008; Bharath and Shumway 2008). Hazard models provide more precise estimates as they are dynamic in nature and account for more historical information. He found that market-based variables of excess returns and volatility estimated over the past year and the relative size of the firm to the market to be significant predictors of failure. He further demonstrated that a number of accounting ratios used in Altman (1968) and Zmijewski (1984) were no longer significant in a multi-period hazard model specification. Similarly, Chava and Jarrow (2004) combined the market-based variables suggested by Shumway (2001) and established that accounting variables only marginally add to the predictive power of models containing only market-based variables. Concerns about high misclassification rates (Begley et al. 1996), the going-concern nature of financial statement preparation (Hillegeist et al. 2004) and accounting statement manipulation (Agarwal and Taffler 2008) have also questioned the validity of accounting variables in bankruptcy prediction.

However, a more recent study by Campbell et al. (2008) examined the predictive ability of both accounting and market-based covariates and found that the optimal specification varies with the forecast horizon, with market-based covariates dominating over short forecast horizons. They found net income to the market value of assets, total liabilities to the market value of assets along with one year average excess returns, three month daily volatility, the natural logarithm of firm size to the S&P500 market capitalisation, cash to the market value of assets, the market-to-book ratio and the firm's stock price to be statistically significant covariates in predicting financial distress. Similarly, Bharath and Shumway (2008) used a combination of accounting and market-based variables within a hazard model specification and found the face value of debt to be an additional predictor of failure. The use of both accounting and market-based variables in failure prediction models is also supported by Tinoco and Wilson (2013) who find that the two variable types complement each other in failure prediction models.

In addition to accounting and market-based variables a number of studies have proposed the inclusion of macroeconomic variables within bankruptcy prediction models to explain the clustering of defaults observed during crises. Das et al. (2007) found there was evidence of default clustering beyond that predicted by models while Duffie et al. (2009) provided evidence towards a contagion hypothesis. Both papers found Merton's distance-to-default to be a significant predictor of default. However, Lando and Nielsen (2010) were unable to find evidence of default contagion using their model specification after replicating and confirming the results of Das et al. (2007). They argued that while the default of firms can cause the hazard intensity of other firms to increase it does not induce instant default. The studies of Bharath and Shumway (2008) and Campbell et al. (2008) also contradicted the findings presented by Das et al. (2007) and Duffie et al. (2009) and found that in well-specified models consisting of both accounting and market-based variables, the distance-to-default measure is not significant. This suggests that any evidence of contagion above that predicted by the models in previous studies may be the result of missing covariates. This is supported by Figlewski et al. (2012) who examined the influence of numerous macroeconomic variables on credit rating changes and the transition to default and found that once credit rating history and financial market conditions were accounted for, macroeconomic variables held no significance in their model. Similarly, Tang and Yan (2010) demonstrated that the majority of default risk is explained by firm-level determinants and that macroeconomic factors improved the fit only marginally.

While debate on the most suitable statistical technique and the most appropriate covariates to include in the model to forecast bankruptcy is on-going, a frequently used opposing class of bankruptcy and default prediction models, known as structural models, emerged from the seminal papers of Black and Scholes (1973) and Merton (1974). The contingent claims framework for valuing corporate liabilities they developed recognises a firm's equity as a call option on the value of the firm's assets which allows for continual evolution of default risk as equity is updated. Subsequent studies have since proposed a numbers of extensions (see, for example, Black and Cox (1976), Geske (1977), Vasicek (1984), Kim et al. (1993), Longstaff and Schwartz (1995)). However, despite the various extensions, a calibration of a number of structural models proposed by literature carried out by Huang and Huang (2003) showed that structural model produce credit spreads well below what is observed in the market. Further, Jarrow and Protter (2004) argue that the framework of structural models implies that a firm's default time becomes entirely predictable.

3. Methodology

3.1. Survival Analysis

We use the survival analysis technique to model corporate bankruptcies. For corporate bankruptcies, firms are at risk of going bankrupt essentially as soon as they incorporate, and therefore incorporation would signal the beginning of the 'at risk' stage within survival analysis. In modelling the bankruptcy of listed companies, the 'at risk' stage must be amended to the time companies first listed on the stock exchange as this represents the time from which accounting and market-variables can be observed. This is, therefore, a limitation of modelling listed companies, as firms may be in different stages of their company life cycle when they list on an exchange.

Survival analysis originated in medical and biology fields but there are a number of features that make it suitable for modelling corporate bankruptcies. Shumway (2001) points out that in contrast to static models that bias their results by collecting data for a finite and subjective number of years in the lead up to the bankruptcy event, survival models are dynamic in nature as they track characteristics over the entire life of the company. Further, survival models are able to handle left and right censoring and truncation of data, unlike comparative ordinary least squares and binary regression techniques[2].

Survival models are commonly referred to as hazard models because the model output is interpreted as the probability of experiencing the hazard over a specific time frame. The dependent variables in survival models are jointly an indicator variable, signalling if the subject experienced the event, as well as survival time, *t*. The survivor function is given by:

$$S(t, X) = 1 - F(t, X) \tag{1}$$

Where $F(t, X)$ is the cumulative density function (CDF). The expression given by (1), is equivalently:

$$S(t, X) = Pr(T > t) \tag{2}$$

Therefore, the CDF is defined as:

$$F(t, X) = Pr(T < t) \tag{3}$$

[2] Left censoring is where a subject already experienced the event prior to the beginning of the study, while right censoring is where a subject stops being observed prior to experiencing the event. Left truncation is where the subject was already 'at risk' of experiencing the event prior to the beginning of the study.

The continuous time hazard function is the conditional probability of experiencing the event during time *t* given survival up to time *t*, this is given by:

$$h(t,X) = \frac{f(t,X)}{S(t,X)} \tag{4}$$

Where $f(t,X)$ is the probability density function of the CDF given by (3) and can be interpreted as the unconditional instantaneous probability of experiencing the event during time *t*. Parametric survival analysis relies on making distributional assumptions on the error term, while semi-parametric survival analysis only makes distributional assumptions on the model covariates. Parameters of the hazard function are estimated using maximum likelihood.

The hazard function, $h(t,X)$, is known as the conditional failure rate or instantaneous rate of failure, with units $1/t$. The hazard function can vary over time and it is the underlying process that determines the shape of the hazard function, therefore, the distributional assumptions that give rise to the shape of the hazard function are particularly important. The shape of the hazard function is synonymous with the accumulation of risk overtime. The most commonly applied survival analysis model is the Cox Proportional Hazard model, which is a semiparametric model where the distributional form of survivor function is not specified. It is a 'proportional hazard' model because it assumes that the shape of each subject's hazard is identical and that hazard ratios are constant over time (Huang and Friedman 2009). This may be a particularly limiting assumption if significant heterogeneity exists between subjects.

Consequently, we estimate an AFT model as it does not rely on a proportional hazard assumption. On the other hand, AFT models are parametric models that require making distributional assumptions on the survivor function, however, a unique property of AFT models is that they can allow for non-monotonic hazard functions with respect to time. This may make it appropriate for predicting corporate failures because, for example, a firm's hazard may be linked to the stages of the company life cycle. The risk of bankruptcy may accumulate during the establishment and growth phases as the company seeks to gain market share and deal with the pressure of expansion, but it is likely to decrease as the company matures. We assume a log-logistic distribution for the AFT survival model, as this is one of the few distributions that does not have a proportional hazard interpretation. Further, logistic regressions have been estimated in literature to approximate a discrete time, semi-parametric hazard model (Shumway 2001, Chava and Jarrow 2004 and Campbell et al. 2008). Following the success of these models, the very similar log-logistic distribution should also be appropriate. A log-logistic AFT survival model allows for both monotonic and non-monotonic hazard functions with respect to time. The shape of the log-logistic AFT survival model will be determined by the estimated gamma parameter.

3.2. Accelerated Failure Time Model

AFT models are assumed to follow a linear relationship between the log survival time, t_j, and the covariates, x_j:

$$\ln(t_j) = x_j\hat{\beta}_x + \varepsilon_j \tag{5}$$

Where $\hat{\beta}$ is a vector of coefficients, x_j is a matrix of covariates that could contain both firm-specific variables and macroeconomic factors and ε_j is an error term. It is important to understand how the time scaling parameters work and the interpretation may initially seem counter intuitive. AFT models assume a distribution for the quantity τ_j:

$$\tau_j = \exp(-x_j\beta_x) \cdot t_j \tag{6}$$

From (6) it is evident that $\exp(-x_j\beta_x)$ acts as an acceleration parameter that interacts with failure time, t_j, therefore:

- If $\exp(-x_j\beta_x)$=1, time passes at the normal rate.
- If $\exp(-x_j\beta_x)$>1, time passes more quickly and thereby shortens survival time.
- If $\exp(-x_j\beta_x)$<1, time passes more slowly and thereby lengthens survival time.

Further, (6) can be rearranged to give:

$$\ln(t_j) = x_j\beta_x + \ln(\tau_j) \tag{7}$$

Where $\ln(\tau_j)$ is a random quantity with a distribution determined by the assumed distribution of τ_j. In estimating the models detailed in subsequent sections, it is assumed that τ_j follows a log-logistic distribution and, therefore, $\ln(\tau_j)$ follows logistic distribution. This assumption is consistent with the literature. For example, logistic distributions were assumed as early as Ohlson (1980) and applied in a survival model setting by Shumway (2001), Chava and Jarrow (2004) and Campbell et al. (2008).

Interpreting the coefficients of AFT models is more intuitive than other commonly applied survival models such as proportional hazard models. From (7) it can be shown that the effect of a unit increase in x_j increases with t_j. For example, if $\beta_1 = 0.5$, then a one-unit increase in $x_1$increases the expected value of $\ln(t)$ by 0.5. Therefore, it can easily be shown that the impact of x_1 has an accelerated impact with regards to $\ln(t)$. For example, at $t = 1$, the predicted time of failure, for a one-unit increase in x_1, is $\exp(0.5) \times 1 = 1.65$. Further, in AFT models the exponentiated coefficients, $\exp(\beta_j)$ can be interpreted as time ratios. For example, consider the case where there is a one-unit increase in x_1, then the change in expected failure time is given by $\exp(\beta_1)$. This is demonstrated below:

$$t_j = \exp(\beta_1 \cdot x_1) \cdot \tau_j \tag{8}$$

$$t_j{}^* = \exp\{\beta_1 \cdot (x_1 + 1)\} \cdot \tau_j \quad (9)$$

Therefore, the ratio of $t_j{}^*$ to t_j and, therefore, $E(t_j{}^*)$ to $E(t_j)$, is simply $\exp(\beta_1)$. In subsequent sections, the model estimates will be reported as time ratios. We investigate two alternative specifications of (7):

$$\ln(t_j) = x_j\beta_x + \ln(\tau_j) \quad (10)$$

$$\ln(t_j) = x_j\beta_x + z_j\delta_z + \ln(\tau_j) \quad (11)$$

Where x_j is a matrix of firm-specific accounting variables and z_j is a matrix of firm-specific market-based variables.

Following the criticisms put forward by Wang (2004), we do not use an automatic procedure such as principal component analysis for determining the combination of covariates that enter the model specification. Specifically, while such procedures may maximise the explanatory power of the model they do not necessarily choose the most informative variables from an economic perspective. An in-sample estimation and holdout sample evaluation approach is also adopted to assess the accuracy of survival models.

3.3. *Evaluation Methods*

We use the area under the Receiver Operating Characteristic (ROC) Curve (AUC), the Schwarz-Bayes Information Criterion (BIC), McFadden's pseudo-R^2 coefficient, decile accuracy assessment and the Brier Score (BS to identify the most accurate and parsimonious survival model specification. The evaluation methods are motivated by literature to ensure a consistent comparison with previous models.

Following Chava and Jarrow (2004) and Tinoco and Wilson (2013), the AUC is used as the primary method to compare the accuracy of models. Tinoco and Wilson (2003) argue that the AUC is the most appropriate measure of determining the real performance of hazard models as it provides a graphical representation of a model's trade-off between a true positive rate and false positive rate. Altman (1968) outlines the importance of both types of misclassification arguing that while incorrectly classifying a firm that goes bankrupt as non-bankrupt is important, incorrectly classifying a non-bankrupt firm as bankrupt is also important as this represents a missed investment opportunity. The AUC is calculated to identify which model is better on average at distinguishing between bankrupt and non-bankrupt firms. An AUC of 1.0 represents perfect classification, while an AUC of 0.5 represents the accuracy of a random guess.

The BIC was proposed by Schwarz (1978) as a method of ranking models relative to each other by penalising models according to their degrees of freedom, thereby favouring parsimonious specifications. A similar statistic known as the Akaike Information Criterion (AIC) also exists, however, the BIC more heavily penalises the number of parameters within a model.

As statistical methods that rely on maximum likelihood to estimate parameters, do not have R^2 as a goodness of fit measure, a number of pseudo R^2 statistics have been developed due to the ease of interpretation. Following Campbell et al. (2008), we use the McFadden's pseudo-R^2 coefficient defined as 1 - (L_1/L_0) where L_1 and L_0 are is the log-likelihood of the estimated model and the null model, which contains only a constant term, respectively. This is a relative performance measure that enables comparative models to be ranked against each other.

Following those of Shumway (2001) and Chava and Jarrow (2004), a number of models were compared on how well they could distinguish between groups of financially distressed firms by assessing the accuracy of decile rankings. This involves sorting firms into deciles based on the probability of bankruptcy estimates from each model. The actual number of bankruptcies are then calculated for each decile and converted into percentages. Models that perform well should have large proportions of actual bankruptcies in the deciles representing the highest level of bankruptcy risk and low proportions of actual bankruptcies in groups representing the safest firms, from a financial distress perspective.

Lastly, Brier (1950) proposed the BS as a means of verification for probability forecasts of events. The BS is given by:

$$BS = \frac{1}{N}\sum_{j=1}^{N}(a_j - p_j)^2 \tag{12}$$

Where a_j is the actual outcome for observation j and p_j is the predicted probability for observation j.

4. Data

4.1. Bankruptcy Data

Our sample of active and delisted firms between 1980 and 2014 is obtained from the Compustat/CRSP database. Consistent with previous literature we exclude financial and real estate firms with a Global Industry Classification Standard (GICS) code from 4010 to 4040, as they have significantly different economic drivers that would influence bankruptcy.[3] We distinguish active and delisted firms and classify 'bankrupt firms' as delisted firms that underwent bankruptcy or liquidation according to both the Compustat and CRSP classification. The corresponding codes are '02' and '03' (Compustat variable 'DLRSN') and '400-499', '572', '574' and '560' (CRSP Delisting Code). Companies that delisted for other reasons are classified as 'exit firms' and will be treated as censored observations within the survival model. This yields a total of 546 bankruptcy observations, 8,664 other exit firms and an average of 3,874 active firms for each year over the 1980-2014 period, a total of 544,250 quarterly firm observations. We use the Compustat database to identify the delisting dates and take the CRSP delisting dates if Compustat has not classified that particular firm as having entered bankruptcy or liquidation. The delist dates implied from CRSP and Compustat are then cross checked against the UCLA Law Bankruptcy database.

[3] See for example Shumway (2001), Chava and Jarrow (2004) and Campbell et al. (2008), among others.

The number of firms in each category for each year is reported in Table 1. The sample begins with a low default rate gradually rising to a peak of 1.69% in 1991, coinciding with the early 1990's recession. Post 1991, the default rate declines steadily until 2006, just before the GFC, when it reaches its lowest point since 1980. The average default rate across the entire sample is 0.39%. Interestingly, we do not observe a distinctive peak in the default rate around the Dot-com bubble or the GFC. There are potentially a number of reasons for this. Firstly, we exclude financial and insurance companies which may impact on the number of bankruptcies observed throughout the GFC. Secondly, as Compustat classifies bankruptcy based on the date of legal outcome rather than the date of filing for bankruptcy, some firms that filed for bankruptcy following the GFC may still be involved in legal proceedings.[4]

4.2. Accounting Variables

We collect six accounting variables at a quarterly frequency to test their explanatory power in predicting corporate bankruptcies. A downside of a quarterly frequency is that a number of observations are lost from firms that do not report quarterly results. This, however, is a trade-off for the increased accuracy of using quarterly statements over annual statements.[5] Accounting variables are lagged by a quarter to avoid look-ahead bias. Further, to minimise the impact of outliers and to ensure a concave and continuous maximum likelihood function exists, all variables are winsorised at the first and ninety-ninth percentile, following the approach of Campbell et al. (2008) and Shumway (2001). The accounting variables collected are intended to measure a firm's liquidity, profitability and leverage as these are factors expected to influence bankruptcy according to literature. The accounting ratios are:

- Working Capital Ratio (WCR) = Working Capital / Total Assets
- Net Income to Total Assets (NITA) = Annualise Net Income[6] / Total Assets
- Return on Equity (ROE) = Annualise Net Income / Average Total Equity[7]
- Net Profit Margin (NPM) = Annualise Net Income / Annualised Total Revenue
- Asset Turnover Ratio (ATOR) = Annualised Revenue / Average Total Assets
- Total Liabilities to Total Assets (TLTA) = Total Liabilities / Total Assets

Accounting variables measuring a firm's liquidity (Quick Ratio and Working Capital Ratio) and profitability (Net Profit Margin, Net Income to Total Assets and Return on Equity) are expected to have an inverse relationship with financial distress. The variables that describe a firm's leverage (Total Liabilities to Total Assets) are expected to be positively related with financial distress. The distributional

[4] For example, Lehman Brothers filed for Chapter 11 bankruptcy protection on 15th September 2008 yet the bankruptcy proceedings are still ongoing (Wiggins et al. 2014).

[5] Provided financial distress is not correlated with the frequency of reporting, this should not impact results. While there is not an intuitive link between reporting frequency and financial distress, it could be argued that reporting quarterly increases the cost of compliance and that firms with a higher likelihood of distress, therefore, may not report quarterly.

[6] Cumulative revenue and income are calculated for each quarter in the financial year and then the cumulative balance is annualised.

[7] Average balances are calculated as the average between the current quarter and previous quarter

properties of the six accounting variables for active, bankrupt and exit firms are outlined in Table 2. The table shows considerable variation across the different groups. As expected, active firms have the most financially healthy ratios demonstrating lower levels of leverage, greater profitability and greater liquidity. Conversely, defaulted firms have high leverage, low profitability and low liquidity, in line with expectations. Other firm exits also demonstrate properties that are different to those of the active firm group. Compared to the active firm group, other firm exits demonstrate lower profitability and liquidity and higher leverage, though not to the extent of the group of bankrupt firms. The relatively lower level of variation between other exit firms and active firms may impact on the ancillary investigation using a broad exit indicator as the event of interest.

4.3. Market-Based Variables

Following Campbell et al. (2008) among others, we also include market-based variables because they are able to incorporate current information much faster than accounting information and are forward looking in nature. Further, accounting information is already outdated by the time it is released (Agarwal and Taffler 2008).

We collect the daily prices for our sample of firms and calculate the daily log returns. In addition, daily returns of the S&P500 value-weighted index are similarly collected. Using individual firm returns and market returns, rolling three month returns, volatility, excess returns and volatility of excess returns are calculated. Excess returns (EX_RET) and volatility (SIGMA) are calculated to isolate the idiosyncratic component of a firm's performance. The rolling 3-month average daily returns and volatility are then annualised, consistent with the methodology of Campbell et al. (2008), for ease of interpretation.

In addition, the natural logarithm of each firm's market capitalisation of equity were also calculated as firm size is expected to be a significant predictor of financial distress. Following Campbell et al. (2008), the market capitalisation was then substituted for the book value of equity collected from Compustat to recreate market-based accounting ratios where the book value of total assets is equal to the market capitalisation of the firm plus the book value of total liabilities. Campbell et al. (2008) argued that the market-based accounting ratios hold greater predictive power than the book-value based ratios. Market-based variables were also winsorised at the first and ninety-ninth percentiles.

The descriptive statistics of the market-based firm specific variables are detailed in table 3. The mean market capitalisation of the sample is US $1,435 million, while the median market capitalisation US $153 million, indicating the distribution is considerably positively skewed. There is also a notable difference between the mean and median size of active firms versus bankrupt firms. The mean and median size of bankrupt firms, in the year they went bankrupt, was US $13.9 million and US $3.1 million, respectively. In contrast, the mean and median size of active firms is US $1,448.8 million and US $155.9 million, respectively. Bankrupt firms also have lower returns and higher standard deviation of returns. The average annualised quarterly return for bankrupt firms is -25.2%, compared to that of 7.3% for active firms. These firms also have annualised daily standard deviation, estimated over the past quarter, of 101.7% compared to 61.9% for active firms. Table 3 also displays the constructed market-based accounting variables. They demonstrate the same patterns described earlier for the accounting variables in table 2; however, these variables will arguably contain more timely information through the market value of equity. This is particularly applicable for firms experiencing financial distress as their equity value is likely to be discounted in the lead up to bankruptcy.

Table 4 reports the correlation coefficients of the accounting (Panel A) and market-based (Panel B) variables, respectively. The level of correlation between variables was considered when developing each model to ensure no multicollinearity. We see high levels of correlation between WCR and TLTA, NITA and ROE as well as NITA and NPM. The high and negative correlation between WCR and TLTA demonstrates that there is an inverse relationship between liquidity and gearing within the sample, which is as expected. Further, the high correlation between NITA, ROE and NPM is not surprising as these are all profitability metrics.

5. Results

5.1. AFT Model Estimation and Comparison

We use the models of Altman (1968), Zmijewski (1984), Shumway (2001) and Campbell et al. (2008) as benchmark models throughout this study. They are estimated using the conditional multi-period logit hazard model specification following that of Shumway (2001) and Campbell et al. (2008) and then compared to the AFT specification. While direct comparison of coefficients is not possible between the two specifications, the area under the ROC curve and McFadden's R^2 helps to distinguish between the in-sample accuracy of each model. The comparison with Shumway (2001) and Campbell et al. (2008) can be found in Table 5. The results of Altman (1968) and Zmijewski (1984) are in the appendix.[8]

All covariates enter the logistic hazard and AFT specifications with the expected sign for both the Shumway (2001) and Campbell et al. (2008) models. There is, however, a different interpretation of coefficients between the two models. The dependent variable in the logistic hazard model is binary, with the bankruptcy event equal to one. Therefore, variables with positive coefficients increase the likelihood of experiencing the event and vice versa for negative coefficients. In contrast, the dependent variable in the AFT model is log survival time. Therefore, positive coefficients increase the survival time which therefore decreases the likelihood of going bankrupt. For example, the net income to total assets variable, NITA, enters the Shumway (2001) logistic hazard model with a negative sign, consequently, increases in NITA reduce the likelihood of default. Similarly, NITA enters the AFT specification of the Shumway (2001) model with a positive coefficient and, therefore, increases survival time which implies it reduces the likelihood of default.

[8] These models have comparatively lower accuracy than the models of Shumway (2001) and Campbell et al. (2008), hence, for brevity the results are presented in the appendix.

By looking at the significance and value of the AFT parameter LN_GAMMA from the comparative models, we see that the AFT specification is appropriate. The exponentiated AFT parameter describes the shape of the hazard function and the coefficient for both the Shumway (2001) and Campbell et al. (2008) models is 0.87 ($e^{-0.14}$), which is less than 1, indicating that a firm's hazard increases with low values of time and then decreases as time increases. LN_GAMMA is significant at the 0.01 level across all comparative models. The fact that the AFT parameter results in a non-monotonic hazard function supports the use of AFT survival models, particularly because a monotonically decreasing hazard function is attainable through other survival model specifications. These results seem intuitive as they suggest firms have higher likelihoods of financial distress during early years, potentially coinciding with their establishment and growth phases of the company life cycle, after which the likelihood of default decreases as the firm matures. All chi-square test statistics are calculated using robust standard errors.

The significance of variables are largely similar between the two specifications, however, there are two exceptions. The cash to market value of assets (CASHMTA) variable of the Campbell et al. (2008) model is significant at a 0.001 level in the logistic specification, however, it is not statistically significant within the AFT specification. The CASHMTA variable is attempting to capture a firm's liquidity which is important for the short-term solvency of a firm, however, its lack of significance is likely due to multicollinearity among covariates. The denominator of the CASHMTA variable contains information regarding the market value of equity. Information also containing the market value of equity can similarly be found in the MRK_TLTA, EX_RET, SIGMA, RELSIZE, MB and PRICE variables as some proportion of the variability of these measures are all linked to the firm's market value of equity. In addition, we find that in our dataset the PRICE variable is not significant in the logistic specification, and only weakly significant in the AFT specification. In contrast, Campbell et al. (2008) find both of these variables to be highly statistically significant within their sample that spans the 1963 to 2003 period.

Finally, in terms of in-sample accuracy, the area under the ROC curve (AUC), increases marginally from 0.9353 to 0.9378 from the logistic to AFT specification in the Shumway (2001) model and from 0.9317 to 0.9379 in the Campbell. et al. (2008) model. Overall, the evidence supports the application of the AFT models for forecasting bankruptcy and we show that the AFT framework outperforms the already high levels of accuracy of models developed under an alternative discrete-time logistic specification. The success of the AFT framework is likely due to the flexibility of non-monotonic hazard functions with respect to time, even in the face of the restrictive parametric distributional assumptions.

5.2. Accounting & Market Variable Model Estimation

We estimate a number of models to find the most accurate predictors of bankruptcy given the alternative AFT specification being analysed. Variables are judgementally selected based on their contribution to the in-sample accuracy of the model, whilst also being mindful of the economic significance of the variables and multicollinearity with other covariates. Table 6 reports log-logistic AFT results for six models containing both accounting and market-based variables. The coefficients reported are the exponentiated coefficients and are interpreted as time ratios that either accelerate or decelerate survival time and, therefore, have the opposite effect on the hazard function. Coefficients less than one decrease survival time while coefficients greater than one increase survival time. For example, the coefficient of TLTA in model (6) can be interpreted as a unit increase in TLTA, decreases survival time by 23.0% compared to the baseline survival time. Conversely a unit increase in EX_RET increases survival time by 163.0% compared to the baseline survival time. As these are the exponentiated coefficients, negative chi-square test statistics are observed on coefficients less than one, indicating negative unadjusted coefficients.

With the exception of the quick ratio variable, QR, in model (1) all variables enter the survival model with the expected effect. The unexpected sign associated with the QR ratio is likely a result of overfitting. In an unreported model with QR as a single covariate, it enters the specification with a positive coefficient. Model (1) highlights that NITA is a more significant predictor of bankruptcy than ROE and NPM and that WCR better captures a firm's liquidity than QR. Model (2) is the most accurate and parsimonious accounting variable-only model; it contains TLTA, WCR, ATOR and NITA as covariates. These variables collectively capture the entire range of a firm's dynamics including leverage, liquidity, asset efficiency and profitability. Given the difference in magnitude of the coefficients, whilst also considering the variability of the underlying variable, NITA has a larger effect on increasing survival time compared to WCR, while TLTA decreases survival time more than ATOR.

Models (3) to (6) investigate how the optimal specification changes through the introduction of market-based variables. In model (3), EX_RET and SIGMA enter the specification with all variables remaining significant at the highest level. The coefficients of the accounting variables change slightly between models (2) and (3), indicating that EX_RET and SIGMA are better at capturing some of the effects previously captured by these variables as well as also capturing additional effects. As expected, EX_RET has a coefficient greater than one meaning that increases in excess returns, increase a firm's survival time while, SIGMA has a coefficient of less than one, meaning that increases in volatility decreases a firm's survival time. Firm size, LN_MCAP, as measured by the natural logarithm of the firm's market capitalisation, is introduced in model (4) and is highly statistically significant. As expected, firm size is positively related to survival time indicating that survival times are longer for larger firms. SIGMA and ATOR lose their significance once LN_MCAP is introduced, leading to the most accurate and parsimonious specification in model (6). A significant improvement in fit is evident from the BIC which reduces from 2,819.74 in model (2) to 1,839.39 in model (6). Further, the area under the ROC curve increases from 0.86 in model (2) to 0.94 in model (6), while the McFadden's R^2 statistic improves from 0.31 to 0.56.

Model (6) represents the most accurate and parsimonious AFT specification containing a combination of both accounting and market-based variables. Evident from AFT model development in Table 6, the accounting variables of TLTA, WCR and NITA hold their significance after the inclusion of a number of market-based variables. As demonstrated by the results of models (5) and (6), it is interesting to find that SIGMA loses its significance to WCR and LN_MCAP; however, this is likely due to multicollinearity with a large proportion of SIGMA accounted for by the variation in EX_RET and LN_MCAP.

In all models except model (3), the AFT parameter, LN_GAMMA, enters the specification with statistical significance suggesting that the AFT specification is appropriate. There are varying results with how each model accelerates or decelerates a firm's hazard. The exponentiated LN_GAMMA coefficient for models (1) to (2) are greater than 1, indicating that a firm's hazard monotonically decreases with time, measured by firm age. However, models (3) to (6) have exponentiated LN_GAMMA coefficients of less than 1, indicating that a firm's hazard increases with low values of firm age and then decreases as firm age increases. This result was similarly found in the AFT specification of the Shumway (2001) and Campbell et al. (2008) models and, as previously mentioned, this suggests that young firms have a higher risk of bankruptcy, which decreases as the firm matures.

A comparison with existing literature reveals that, consistent with the findings of Shumway (2001), Campbell et al. (2008) and Tian et al. (2015), the most accurate model consists of both accounting and market-based variables. The accounting variables of NITA and TLTA are significant predictors of default across all studies. While Shumway (2001) does not find an accounting variable capturing a firm's liquidity to be significant, Campbell et al. (2008) finds CASHMTA to be significant, although Table 5 shows CASHMTA is not significant under an AFT specification. In contrast, we find that WCR was is more significant than the quick ratio and current ratio. Further, in unreported results, the RELSIZE variable used by Shumway (2001) and Campbell et al. (2008) is not significant under an AFT specification after the inclusion of LN_MCAP. Finally, the most interesting conflict is that SIGMA is not significant after the inclusion of WCR and LN_MCAP. As mentioned previously, this is likely due to much of the variation in SIGMA being captured by EX_RET and LN_MCAP. However, as indicated by Campbell et al. (2008) and Tian et al. (2015), optimal variable choice is a function of the forecast horizon with market-based variables being more appropriate for short-run forecasts. Therefore, some of the discrepancies between the current research and the comparison models may be a result of different sampling frequencies. Overall, we find Model (6) to be the most accurate accounting and market-based AFT model, and as such, this is the model we refer to hereafter. The model contains five covariates in contrast to the eight variable model fitted by Campbell et al. (2008).

Next, we compare our AFT model to the models of Shumway (2001) and Campbell et al. (2008) in terms of their ability to sort firms into categories of increasing risk. Table 7 reports the percentage accuracies of actual bankruptcies for each decile of risk. Decile 1 represents the safest group of firms, from a financial distress perspective, while decile 10 contains the riskiest group of firms. Each model performs well in distinguishing firms into groups of risk. The first 5 deciles of risk account for roughly 2.5% of actual bankruptcies, while the 10^{th} decile accounts for 82.0-84.0% of actual bankruptcies. Our

AFT model has the highest accuracy in the 10th decile, but it is marginally less accurate overall in the middle group of deciles.

Motivated by Campbell et al. (2008), we also investigate the predictability of accounting ratios using the ‘market value of assets’. Table 8 contrasts the results of our AFT model when the accounting ratios are calculated using the market value of assets as opposed to the book value of assets. While there is only a slight reduction in the McFadden’s R^2 statistic, from 0.56 to 0.55, the BIC deteriorates from 1,839.39 to 1,871.55, indicating that a book value of assets is preferred to market value of assets. Although these results are inconsistent with the findings of Campbell et al. (2008), this is likely due to the same reasons that SIGMA loses its significance; there is higher correlation between the covariates as a result of the fluctuations in equity value being captured in all market-based variables. Further, the result is consistent with the findings of Welch (2004) and Graham and Harvey (2001); firms rarely react to changes in their capital structure as a result of fluctuations in their market value of equity and, therefore, book leverage may be more appropriate for bankruptcy prediction as it is more stable. Consequently, the results point to the book value model being a more accurate and parsimonious AFT survival model.

Next, we investigate the stability of coefficients using subsamples to gauge whether the bankruptcy risk dynamics are time varying. First, the subsample is partitioned such that 50% of bankruptcies occur in each sample. That is, into periods spanning the Q1 1980 to Q2 1992 and Q3 1993 to Q4 2014. The second subsample is partitioned by time, such that each sample reflects half of the entire time period considered. This results in samples spanning the Q1 1980 to Q2 1997 and Q3 1997 to Q4 2014 periods. The results are reported in Table 9. The table shows that the AUC is broadly similar across the sub-samples and the coefficients are comparable despite the vastly different composition of active and bankrupt firms between column 3 & 4. All coefficient signs are consistent with the full sample, and most coefficients are highly significant. The LN_GAMMA parameter is less than one and also highly significant across all sub-samples. Overall, our results indicating that the AFT model specification is robust to the choice of time period.

5.3. Out-of-sample Forecast Evaluation

The accuracy assessment in the previous sections is entirely based on an in-sample fit which does not necessarily translate into high forecast accuracy. Box and Jenkins (1970) stress the importance for model adequacy and parsimony for suitable forecasting models. While it must be acknowledged that every forecasting model is miss-specified, ensuring model adequacy and parsimony will result in a model that is robust to the miss-specification (Allen and Fildes 2001). This is supported by Gilbert (1995) who demonstrates that the prediction error resulting from parameter estimation is greater than the prediction error from an approximate parsimonious model.

To compare forecast accuracy, the parameters of each model are estimated using the in-sample period until Q4 2005. One step ahead quarterly forecast estimates are then obtained for each active firm over the holdout sample covering the 2006-2014 period and each models’ AUC calculated. The AUC is the most appropriate measure of forecast accuracy assessment as it is concerned with discriminating firms into bankrupt and non-bankrupt categories, while the other measures considered are useful for comparing model fit. The forecast accuracy of our AFT model along with the models of Shumway (2001) and Campbell et al. (2008) is reported in Table 10.

Consistent with the argument of Box and Jenkins (1970), the model of Shumway (2001) significantly outperforms that of Campbell et al. (2008) in forecast ability due to a more parsimonious specification, despite being only marginally worse in terms of in-sample fit. In support with the in-sample results, our AFT model outperforms both models in terms of forecast accuracy. Furthermore, the AFT model, as well as the Shumway (2001) model, perform comparatively well relative to their in-sample fit. It is expected that there is a deterioration in forecast accuracy compared to in-sample fit, however, both models experience only a mild deterioration in accuracy. This is particularly impressive, as the model parameters are estimated using data available by Q4 2005 and the forecast assessment period covers the turbulent economic periods of the GFC and the European Financial Crisis. Further, the holdout sample spans fairly period of 9 years, or 36 quarters. By comparison, the Campbell et al. (2008) model experiences a significant drop in accuracy from an AUC of 0.9379 in the in-sample period, between 1980 and 2014, to 0.8598 in the holdout sample, between 2006 and 2014.

To gauge the overall fit of the predicted probabilities, relative to the actual observed bankruptcy rate of the sample, the quarterly bankruptcy rate is plotted against the model implied bankruptcy rate. Figure 1 compares the actual quarterly bankruptcy rates in the sample with estimated bankruptcy rates implied by our AFT model. Actual sample bankruptcy rates are calculated as the number of bankruptcies divided by the number of active companies in that quarter. Estimated bankruptcy rates are calculated as the average of the bankruptcy probabilities estimated in the quarter. As shown by figure 1, the aggregated predicted bankruptcy probability tracks the trend in the actual bankruptcy rate relatively well, however, the model also goes through periods of overestimating and underestimating the bankruptcy rate and this is particularly observable over the 1989-1992 period, where the estimated bankruptcy rate is much lower than the actual bankruptcy rate.[9]

6. Conclusion

We examine the application of accelerated failure time (AFT) survival models for predicting corporate bankruptcies. The AFT survival model specification was proposed given the contention within the literature regarding the most appropriate method for bankruptcy prediction. By a similar motivation, quarterly sampling frequency was used to overcome issues with other broadly used frequencies within the literature. Bankruptcy prediction models typically have one-year sampling frequencies, which is arguably too long to provide a timely evolution of bankruptcy risk, while structural models focus on much

[9] Whilst this suggests that there may be omitted macroeconomic factors which shift the aggregate level of the bankruptcy rate, in an unreported model, we find that the inclusion of a number of macroeconomic variables from literature does not improve the accuracy of the model.

short frequencies, such as daily and weekly. Further, the model also focused on one-period forecasts to allow for rolling window forecasts to ensure parameter estimates were kept up to date.

As a consequence, it was shown that the AFT model was superior to a range of comparative models from literature. By comparing the accuracy of AFT models against the widely used discrete-time logistic hazard model, it was shown that the AFT specification was superior to the leading models of Shumway (2001) and Campbell et al. (2008) in terms of both in-sample fit and out-of-sample forecast accuracy. Further, a superior AFT model was fit from the new dataset and sampling frequency at hand. It was shown that the widely used stock market return volatility variable was inferior to an accounting variable capturing a firm’s liquidity; working capital ratio.

In summary, this thesis makes three contributions to the literature. First, the appropriateness of the AFT survival model specification was demonstrated from its superiority to commonly applied methods within literature. Second, while the majority of bankruptcy studies focus on a single accuracy evaluation method, a range of in-sample and out-of-sample evaluation methods were used. By doing so, it was shown that previously used measures that favoured in-sample accuracy performed poorly during out-of-sample assessments, given out-of-sample accuracy favours parsimonious specifications. Specifically, the combination of the BIC, which heavily penalises covariates in a model, and the area under the ROC curve are suitable accuracy assessment measures.

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Table 1 – Sample of Active, Bankrupt and Exit Firms per Year

Total number of active firms, bankruptcies and other firm exits for each year. The active number of firms is calculated at the end of the calendar year as the sum of all firms still active at that point in time.

Year	Active Firms	Bankruptcies	%	Other Firm Exits	%
1980	1,485	-	0.00%	7	0.47%
1981	2,571	1	0.04%	9	0.35%
1982	2,818	6	0.21%	10	0.35%
1983	3,217	4	0.12%	40	1.24%
1984	3,481	19	0.55%	62	1.78%
1985	3,630	22	0.61%	170	4.68%
1986	3,789	32	0.84%	302	7.97%
1987	3,947	16	0.41%	298	7.55%
1988	3,829	37	0.97%	347	9.06%
1989	3,736	49	1.31%	300	8.03%
1990	3,707	51	1.38%	225	6.07%
1991	3,781	64	1.69%	194	5.13%
1992	3,964	40	1.01%	204	5.15%
1993	4,304	23	0.53%	178	4.14%
1994	4,549	21	0.46%	260	5.72%
1995	4,699	19	0.40%	325	6.92%
1996	5,065	22	0.43%	310	6.12%
1997	5,119	25	0.49%	456	8.91%
1998	4,893	13	0.27%	552	11.28%
1999	4,754	12	0.25%	585	12.31%
2000	4,696	9	0.19%	566	12.05%
2001	4,350	11	0.25%	463	10.64%
2002	4,125	16	0.39%	309	7.49%
2003	3,963	7	0.18%	248	6.26%
2004	3,975	3	0.08%	197	4.96%
2005	3,926	6	0.15%	256	6.52%
2006	3,891	1	0.03%	264	6.78%
2007	3,839	2	0.05%	281	7.32%
2008	3,740	1	0.03%	180	4.81%
2009	3,624	5	0.14%	173	4.77%
2010	3,598	2	0.06%	187	5.20%
2011	3,580	4	0.11%	191	5.34%
2012	3,580	-	0.00%	164	4.58%
2013	3,683	3	0.08%	173	4.70%
2014	3,673	-	0.00%	178	4.85%
Total		546		8,664	

Table 2 – Accounting Variable Summary Statistics

The descriptive statistics for the Working Capital Ratio (WCR), Net Income to Total Assets (NITA), Return on Equity (ROE), Net Profit Margin (NPM), Asset Turnover Ratio (ATOR) and Total Liabilities to Total Assets (TLTA) for the full sample covering 1980-2014. As all variables are winsorised at the first and ninety-ninth percentile the reported min and max values across the panels may be identical. The variation in the observation numbers is due to the availability of Compustat accounting information.

Variable	*WCR*	*NITA*	*ROE*	*NPM*	*ATOR*	*TLTA*
Panel A: Active Firms						
Mean	0.268	-0.015	-0.005	-0.385	1.105	0.482
Median	0.244	0.035	0.082	0.033	0.955	0.483
Std. Dev.	0.252	0.209	0.694	2.321	0.832	0.245
Min	-0.360	-1.127	-3.882	-19.699	0.000	0.044
Max	0.876	0.336	3.272	0.575	4.479	1.295
Observations	521,574	509,182	508,867	516,295	509,233	531,234
Panel B: Default Firms						
Mean	-0.016	-0.321	-0.234	-1.027	1.244	0.816
Median	-0.061	-0.219	-0.108	-0.213	1.014	0.877
Std. Dev.	0.337	0.365	2.105	2.976	1.094	0.365
Min	-0.360	-1.127	-3.882	-19.699	0.000	0.044
Max	0.876	0.336	3.272	0.575	4.479	1.295
Observations	524	526	526	510	525	546
Panel C: Other Firm Exits						
Mean	0.193	-0.111	-0.107	-0.617	1.101	0.549
Median	0.179	0.004	0.042	0.005	0.933	0.524
Std. Dev.	0.294	0.304	1.190	2.577	0.859	0.304
Min	-0.360	-1.127	-3.882	-19.699	0.000	0.044
Max	0.876	0.336	3.272	0.575	4.479	1.295
Observations	8,489	8,419	8,414	8,300	8,413	8,642

Table 3 – Summary Statistics for Firm-Specific Market Variables

The descriptive statistics for the natural logarithm of market capitalisation of equity (LN_MCAP), the annualised value of the gross average daily returns over the past quarter in excess of the value-weighted S&P500 return (EX_RET), the annualised daily volatility of returns over the past quarter (SIGMA) and a number of accounting variables described earlier utilising the 'market value of assets'. As all variables are winsorised at the first and ninety-ninth percentile the reported min and max values across the panels may be identical. The variation in the observation numbers is due to the availability of Compustat accounting information.

Variable	*LN_MCAP*	*EX_RET*	*SIGMA*	*M_WCR*	*M_NITA*	*M_ROE*	*M_ATOR*	*M_TLTA*
Panel A: Active Firms								
Mean	1,449.8	-0.055	0.619	0.183	-0.004	-0.017	0.822	0.374
Median	155.9	-0.071	0.560	0.137	0.025	0.040	0.616	0.338
Std. Dev.	4,384.6	0.357	0.295	0.207	0.114	0.344	0.756	0.255
Min	1.5	-0.811	0.128	-0.236	-0.633	-2.244	0.000	0.010
Max	31,987.9	2.029	1.680	1.001	0.210	1.053	3.991	0.964
Observations	534,753	534,881	534,875	521,574	521,946	524,754	521,999	531,234
Panel B: Default Firms								
Mean	13.9	-0.368	1.017	0.030	-0.220	-1.025	1.072	0.676
Median	3.1	-0.496	0.962	-0.044	-0.176	-0.824	0.819	0.798
Std. Dev.	33.5	0.512	0.410	0.310	0.227	0.991	0.986	0.301
Min	1.5	-0.811	0.128	-0.236	-0.633	-2.244	0.000	0.013
Max	309.7	2.029	1.680	1.001	0.210	1.053	3.991	0.964
Observations	546	545	546	524	529	529	528	546
Panel C: Exit Firms								
Mean	721.6	-0.051	0.744	0.139	-0.068	-0.235	0.818	0.404
Median	72.7	-0.117	0.674	0.101	0.003	0.004	0.591	0.361
Std. Dev.	2,631.1	0.559	0.362	0.239	0.180	0.623	0.779	0.268
Min	1.5	-0.811	0.128	-0.236	-0.633	-2.244	0.000	0.010
Max	31,987.9	2.029	1.680	1.001	0.210	1.053	3.991	0.964
Observations	8,664	8,663	8,664	8,489	8,456	8,464	8,451	8,642

Table 4 – Correlation Matrix

Correlation matrices of accounting and market-based variables. Correlations are calculated ignoring the cross-sectional and time-series behaviour of the panel dataset.

Panel A: Accounting Variables

Variables	WCR	NITA	ROE	NPM	ATOR	TLTA
WCR	1.00					
NITA	0.00	1.00				
ROE	-0.02	0.46	1.00			
NPM	-0.13	0.53	0.23	1.00		
ATOR	0.03	0.22	0.11	0.22	1.00	
TLTA	-0.61	-0.08	0.02	0.08	0.14	1.00

Panel B: Market-based Variables

Variables	LN_MCAP	EX_RET	SIGMA	M_WCR	M_NITA	M_ROE	M_ATOR	M_TLTA
LN_MCAP	1.00							
EX_RET	0.05	1.00						
SIGMA	-0.25	-0.12	1.00					
M_WCR	-0.18	-0.03	0.08	1.00				
M_NITA	0.11	0.13	-0.40	-0.02	1.00			
M_ROE	0.07	0.11	-0.32	0.03	0.79	1.00		
M_ATOR	-0.13	-0.02	-0.08	0.30	0.13	0.07	1.00	
M_TLTA	-0.02	-0.06	-0.14	-0.11	0.01	-0.07	0.43	1.00

Table 5 – Logit and AFT Specifications of the Shumway (2001) & Campbell et al. (2008) Models

The parameter estimates for the logistic and AFT specifications of the Shumway (2001) and Campbell et al. (2008) models estimated across the full-sample covering 1980-2014. McFadden's R^2 is calculated following Campbell et al. (2008). The logistic regressions ran here were random effects panel data models. When non-panel data logistic regressions were ran, the McFadden's R^2 is similar to those reported in Campbell et al. (2008), at roughly 0.32. The chi-square test statistics are in parentheses, with *p<0.05, **p<0.01, ***p<0.001 representing the levels of significance.

	Shumway (2001)		Campbell et al. (2008)	
	Logistic Specification	AFT Specification	Logistic Specification	AFT Specification
NITA	-0.75	1.68	-2.10	3.13
	(-4.13)***	(6.64)***	(-12.42)***	(8.81)***
TLTA	4.02	-3.75		
	(15.14)***	(-13.35)***		
MRK_TLTA			3.14	-3.28
			(11.04)***	(-9.89)***
EX_RET	-0.62	0.93	-0.74	0.86
	(-4.61)***	(4.10)***	(-5.14)***	(3.73)***
SIGMA	0.74	-0.62	0.82	-0.75
	(3.93)***	(-2.78)**	(4.92)***	(-3.61)***
RELSIZE	-1.36	1.08	-0.88	0.90
	(-13.87)***	(14.33)***	(-11.93)***	(11.24)***
CASHMTA			-3.03	1.47
			(-7.11)***	(1.90)
MB			-0.06	0.13
			(-0.87)	(1.56)
PRICE			-1.31	1.29
			(-2.40)*	(2.08)*
Constant	-29.82	23.22	-17.96	16.77
	(-17.41)***	(21.01)***	(-8.66)***	(8.35)***
LN_SIGMA	2.01		1.23	
	(10.31)***		(5.38)***	
LN_GAMMA		-0.14		-0.14
		(-2.90)**		(-2.58)**
BIC	5,973	2,134	5,990	2,172
Psd. Log-Likelihood	-2,947	-1,015	-2,936	-1,014
Wald Chi2 p-value	0.00	0.00	0.00	0.00
McFadden's R^2	0.02	0.51	0.01	0.50
ROC Area	0.9353	0.9378	0.9317	0.9379
ROC Sig	0.0000	0.0000	0.0000	0.0000
Brier Score	0.0010	0.0010	0.0010	0.0010

Table 6 – AFT Accounting and Market-based Models

The parameter estimates for six alternative AFT models fit using the full-sample covering 1980-2014. The models examine the effectiveness of accounting and market-based variables. The model building resulted in a parsimonious specification provided by (6) consisting of both accounting and market-based variables. The chi-square test statistics are in parentheses, with *p<0.05, **p<0.01, ***p<0.001 representing the levels of significance.

	(1)	(2)	(3)	(4)	(5)	(6)
TLTA	0.03	0.03	0.05	0.23	0.15	0.23
	(-7.99)***	(-8.19)***	(-8.21)***	(-9.49)***	(-13.03)***	(-9.85)***
QR	0.74					
	(-5.52)***					
WCR	81.74	40.91	13.06	2.00		2.04
	(8.15)***	(7.14)***	(6.39)***	(4.05)***		(4.38)***
NPM	0.82					
	(-3.24)**					
ATOR	0.70	0.72	0.74	1.01		
	(-4.22)***	(-4.04)***	(-4.01)***	(0.29)		
ROE	1.23					
	(2.73)**					
NITA	1080.38	985.52	28.02	2.15	2.24	2.23
	(9.46)***	(9.56)***	(7.75)***	(6.22)***	(6.86)***	(6.65)***
EX_RET			4.90	1.60	1.61	1.63
			(5.75)***	(3.82)***	(3.90)***	(3.86)***
SIGMA			0.10	0.81	0.78	
			(-10.01)***	(-1.80)	(-2.16)*	
LN_MCAP				1.89	1.94	1.89
				(20.62)***	(20.99)***	(20.99)***
Constant	17343.68	16575.82	54471.35	146.55	204.98	120.06
	(25.63)***	(26.05)***	(27.96)***	(28.93)***	(33.05)***	(39.63)***
LN_GAMMA	1.15	1.18	0.98	0.50	0.50	0.49
	(2.60)**	(2.97)**	(-0.43)	(-14.37)***	(-14.75)***	(-15.08)***
No. Firms	12,248	12,417	12,416	12,416	12,660	12,434
No. Defaults	496	500	499	499	518	500
BIC	2,775.86	2,819.74	2,537.16	1,859.27	1,903.58	1,839.39
Psd. Log-Likelihood	-1,322.41	-1,363.97	-1,209.57	-864.07	-899.26	-867.24
Wald Chi2 p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
McFadden's R^2	0.3247	0.3109	0.3865	0.5599	0.5581	0.5601
ROC Area	0.8594	0.8567	0.8868	0.9440	0.9444	0.9434
ROC Sig	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Brier Score	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Table 7 – Decile Accuracy Assessment

In-sample accuracy of the two comparison models reported against our AFT model. The numbers are in percentages and reflect the proportion of bankruptcies in each decile according to the sorting of probability of failure estimates from each model.

Decile	Shumway	Campbell et al.	AFT Model
1 - 5	2.3	2.3	2.4
6	1.5	1.4	2.2
7	2.9	2.5	2.4
8	3.5	3.3	3.8
9	7.1	7.4	5.8
10	82.6	83.0	83.4

Table 8 – Book Value versus Market Value of Assets

The parameter estimates for our AFT model using the 'Book Value of Assets' and the 'Market Value of Assets' proposed by Campbell et al. (2008). Models were estimated using the full-sample covering 1980-2014. The chi-square test statistics are in parentheses, with *p<0.05, **p<0.01, ***p<0.001 representing the levels of significance.

	AFT Model	
	Book Value of Assets	*Market Value of Assets*
TLTA	0.23	
	(-9.85)***	
MRK_TLTA		0.23
		(-9.27)***
WCR	2.03	
	(4.38)***	
MRK_WCR		3.46
		(6.41)***
NITA	2.23	
	(6.65)***	
MRK_NITA		6.82
		(8.02)***
EX_RET	1.63	1.60
	(3.86)***	(3.32)***
LN_MCAP	1.90	1.88
	(20.99)***	(20.19)***
Constant	120.30	114.43
	(39.63)***	(34.45)***
LN_GAMMA	0.49	0.52
	(-15.08)***	(-13.91)***
No. Firms	12,434	12,438
No. Defaults	500	494
BIC	1,839.39	1,871.55
Psd. Log-Likelihood	-867.24	-883.23
Wald Chi2 p-value	0.0000	0.0000
Pseudo R^2	0.5601	0.5520
ROC Area	0.9434	0.9429
ROC Sig	0.0000	0.0000
Brier Score	0.0010	0.0010

Table 9 – Parameter Stability Tests

Columns 1 & 2 contrast the stability of parameter estimates over two time periods; 1980 Q1 – 1991 Q2 and 1991 Q3 – 2014 Q4 while columns 3 & 4 contrast the stability of parameter estimates between 1980 Q1 – 1997 Q2 and 1997 Q3 – 2014 Q4. As the data in columns 3 & 4 is partitioned according to time, there are significant differences in the number of defaults between each time period.

AFT Model	1	2	3	4
	Q1 1980 - Q2 1991	Q3 1992 - Q4 2014	Q1 1980 - Q2 1997	Q3 1997 – Q4 2014
TLTA	0.35	0.25	0.32	0.41
	(-6.71)***	(-7.07)***	(-8.82)***	(-3.15)**
WCR	1.89	1.65	1.67	1.84
	(3.99)***	(2.23)*	(3.85)***	(1.81)
NITA	1.67	2.27	1.99	1.78
	(3.97)***	(5.43)***	(6.38)***	(2.84)**
EX_RET	1.33	1.95	1.39	2.34
	(2.53)*	(3.59)***	(3.37)***	(2.56)*
LN_MCAP	1.52	1.82	1.60	1.53
	(9.76)***	(14.43)***	(14.88)***	(6.25)***
Constant	87.77	152.08	99.79	231.58
	(36.12)***	(30.17)***	(44.32)***	(19.98)***
LN_GAMMA	0.35	0.49	0.36	0.49
	(-14.03)***	(-9.75)***	(-17.60)***	(-4.81)***
No. Firm	5,375	10,523	8,286	8,844
No. Defaults	243	267	399	101
BIC	875.98	1045.53	1237.03	601.14
Psd. Log-Likelihood	-390.88	-471.49	-569.18	-250.56
Wald Chi2 p-value	0.0000	0.0000	0.0000	0.0000
McFadden's R^2	0.5424	0.5308	0.5997	0.4062
ROC Area	0.9380	0.9425	0.9400	0.9347
ROC Sig	0.0000	0.0000	0.0000	0.0000
Brier Score	0.0010	0.0010	0.0010	0.0010

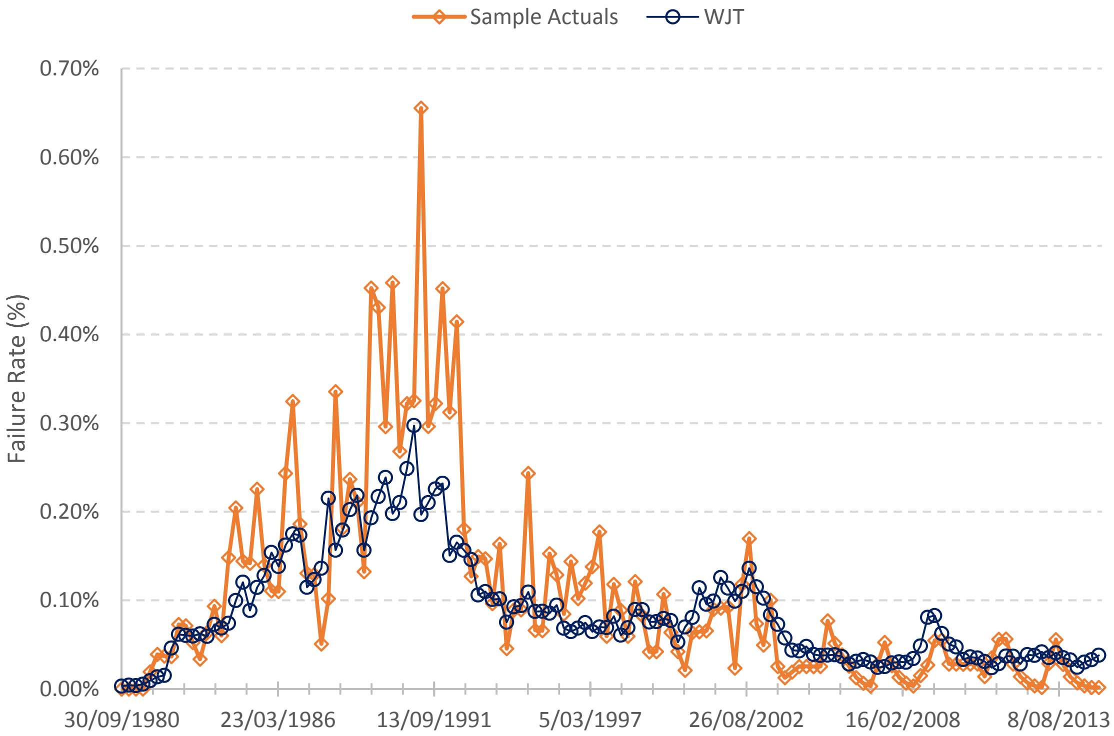
Table 10 - Forecast Accuracy

The forecast accuracy of the models of Shumway (2001), Campbell et al. (2008) and the best in-sample specification of our AFT model. Parameter estimates are obtained as at Q4 2005 and forecast accuracy is assessed over the 2006-2014 period. The chi-square test statistics are in parentheses, with *p<0.05, **p<0.01, ***p<0.001 representing the levels of significance. Parameters are presented as time ratios.

	Shumway (2001)	Campbell et al. (2008)	AFT Model
NITA	4.94	9.43	2.22
	(7.20)***	(7.69)***	(6.87)***
TLTA	0.04	0.06	0.25
	(-12.75)***	(-10.14)***	(-9.80)***
RELSIZE	2.43	2.09	
	(13.23)***	(10.95)***	
EX_RET	2.30	2.22	1.60
	(4.23)***	(4.21)***	(3.98)***
SIGMA	0.65	0.59	
	(-2.29)*	(-2.92)**	
CASHMTA		1.03	
		(0.04)	
MB		1.36	
		(3.48)***	
PRICE		3.10	
		(1.91)	
WCR			1.86
			(4.05)***
LN_MCAP			1.75
			(18.42)***
Constant	4.22E+08	1.95E+06	121.92
	(19.95)***	(7.67)***	(41.53)***
LN_GAMMA	0.73	0.71	0.45
	(-5.99)***	(-6.23)***	(-16.54)***
ROC Area	0.8735	0.8598	0.8948
ROC Sig	0.0000	0.0000	0.0000
Brier Score	0.0001	0.0001	0.0001

Figure 1 – Aggregate Predicted Failure Probabilities

The predicted failure probabilities of our AFT model vs the actual sample failure rate.



Appendix

The parameter estimates for the logistic and AFT specifications of the Altman (1968) and Zmijewski (1984) models estimated across the full-sample covering 1980-2014. The variables are defined as follows: working capital to total assets (WCR), retained earnings to total assets (RETA), market equity to total liabilities (MCAP_TL), sales to total assets (SALES_TA) and earnings before interest and taxes to total assets (EBIT_TA), ratio of net income to total assets (NITA), the ratio of total liabilities to total assets (TLTA), and the ratio of current assets to current liabilities (CR). The chi-square test statistics are in parentheses, with *p<0.05, **p<0.01, ***p<0.001 representing the levels of significance.

	Altman (1968)		Zmijewski (1984)	
	Logistic Specification	AFT Specification	Logistic Specification	AFT Specification
WCR	-3.64	4.50		
	(-14.38)***	(7.49)***		
RETA	-2.90	8.07		
	(-19.17)***	(7.97)***		
MCAP_TL	-0.14	0.17		
	(-6.37)***	(2.22)*		
SALES_TA	0.40	-0.58		
	(6.35)***	(-7.16)***		
EBIT_TA	-0.45	0.64		
	(-3.38)***	(2.69)**		
NITA			-2.50	8.65
			(-16.65)***	(5.92)***
TLTA			3.96	-6.72
			(17.09)***	(-10.24)***
CR			0.00	-0.02
			(-0.15)	(-0.27)
Constant	-8.00	7.16	-11.47	12.25
	(-33.23)***	(35.02)***	(-33.86)***	(18.12)***
LN_SIGMA	1.20		1.59	
	(6.51)***		(12.20)***	
LN_GAMMA		0.15		0.32
		(2.50)*		(4.44)***
BIC	6,575	2,768	6,925	2,908
Psd. Log-Likelihood	-3,248	-1,331	-3,436	-1,415
Wald Chi2 p-value	0.00	0.00	0.00	0.00
McFadden's R^2	0.01	0.31	0.01	0.29
ROC Area	0.8500	0.8686	0.8271	0.8524
ROC Sig	0.0000	0.0000	0.0000	0.0000
Brier Score	0.0010	0.0010	0.0010	0.0010