FINANCIALLY CONSTRAINED INDEX FUTURES ARBITRAGE

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ABSTRACT. We develop two models for index futures arbitrage that take the financing constraints faced by real-world arbitrageurs into account. Our models predict that the price of an index futures contract and the value of its underlying index should deviate further from their theoretical cost-of-carry relationship when (a) the contract has a long time to go before expiry, and (b) volatility is high. The fact that these predictions enjoy considerable empirical support highlights the importance of financing constraints for explaining index futures mispricing.

1. INTRODUCTION

In theory, the price of an index futures contract and the value of its underlying index should obey a simple cost-of-carry relationship that depends only on the dividend yield of the index and the return on cash. Violations of this relationship should offer straightforward opportunities to arbitrageurs, and should therefore be extremely rare. In reality, however, mispriced index futures are commonplace. Their prevalence seems to be a brazen violation of the law of one price and a striking failure of the arbitrage assumptions at the heart of financial economics. With that in mind, it is unsurprising that index futures mispricing has attracted considerable attention in the academic finance literature.

Modest and Sundaresan (1983) rationalised limited deviations of index futures prices from their theoretical cost-of-carry values. They noted that transaction costs allow the price of a futures contract to fluctuate with impunity inside a band around its theoretical price, since the round-trip costs of initiating and liquidating an arbitrage trade would exceed any potential profit in that case. However, their reasoning implies

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that the width of the non-arbitrageable band should be independent of the maturity of the contract.

MacKinlay and Ramaswamy (1988) argued that the width of the non-arbitrageable band around the theoretical price of an index futures contract should increase with its maturity, for three reasons. First, there is a greater risk that unanticipated changes in dividends will erode the profits of arbitrage strategies involving long-dated index futures contracts. Second, the cost of financing unanticipated mark-to-market cash flows increases with the maturity of the futures leg of an index futures arbitrage strategy. Finally, the cost of rebalancing the basket of stocks used to track the index is higher for index futures arbitrage strategies involving longer-dated futures contracts, due to larger tracking errors. As a result, they predicted larger gaps between observed and theoretical index futures prices for longer-dated contracts. This prediction was confirmed empirically for S&P 500 index futures.

The positive dependence of S&P 500 index futures mispricing on contract maturity has been documented by several other studies, including Bhatt and Cakici (1990) and Switzer et al. (2000). It has also been documented in index futures markets outside the U.S. For example, Yadav and Pope (1990, 1994) observed that absolute differences between theoretical and observed prices of FTSE 100 index futures in the U.K. are positively related to contract maturity, while Bühler and Kempf (1995) found that maturity was a significant determinant of mispricing for German DAX index futures.[1] Similarly, Fung and Draper (1999) and Draper and Fung (2003) showed that contract maturity is an important determinant of mispricing for Hong Kong Hang Seng index futures contracts.

A second empirical feature of index futures mispricing concerns the dependence of the gap between theoretical and observed futures prices on volatility. Numerous studies have documented a positive relationship between index futures mispricing and

[1]As an aside, the DAX is a very narrow index, comprising only 30 liquid German blue chips. Moreover, since it is a total return index, its performance is based on the assumption that dividends are reinvested in the index portfolio. Arbitrageurs wishing to exploit mispriced DAX index futures are thus not exposed to meaningful tracking error or dividend risk. This casts doubt on two of the explanations for the positive maturity dependence of index futures mispricing proffered by MacKinlay and Ramaswamy (1988).

volatility, including Merrick (1987), Hill et al. (1988), Kawaller et al. (1990), Stoll and Whaley (1990), Chan (1992), Brailsford and Hodgson (1997), Fung and Draper (1999), Gay and Jung (1999), Draper and Fung (2003), Richie et al. (2008) and Cummings and Frino (2011).

The usual interpretation for the volatility dependence of index futures mispricing is that futures contracts impound new information more rapidly than their underlying indices, so that futures prices quickly diverge from their theoretical cost-of-carry values during periods of high volatility. However, Tu et al. (2016) showed the gap between the price of an index futures contract and its fair value is positively related to the expected future volatility over the life of the contract, which suggests that there may be more to the volatility dependence of index futures mispricing than differences in efficiency between index futures markets and underlying equity markets.

This paper contributes to the literature on index futures arbitrage by showing that the empirical features of index futures mispricing described above can be explained parsimoniously in terms of the financing constraints encountered by arbitrageurs. In particular, we demonstrate that those features are the inevitable consequence of two simple facts: (a) arbitrageurs periodically experience negative cash flows (in the form of margin calls and collateral payments) during the life of an arbitrage trade; and (b) arbitrageurs only have access to limited supplies of capital.

In Section 2 we present a baseline model of index futures arbitrage, in which an arbitrageur with a limited stock of trading capital initiates an index futures arbitrage position at some time when the futures contract is trading away from its theoretical fair value. If his capital is sufficient to fund the margin calls and collateral payments that occur during the life of the strategy, he unwinds the position when the futures contract expires, realising a profit. However, if his capital is insufficient, the position is unwound involuntarily when he can no longer fund margin calls and collateral payments, and his capital is lost.

The crucial innovation of our model is the introduction of financing risk, which is the risk of being closed out involuntarily due to an inability to fund margin calls and collateral payments. In Section 3, which examines the stylised features of our baseline model, we demonstrate that this innovation explains a lot. In particular, we show that the arbitrageur will only initiate an arbitrage trade involving a long-dated futures contract if the gap between the futures price and the fair value of the contract is large. A similar observation applies to the situation when volatility is high. This is because the maturity of a futures contract and its volatility both increase the financing risk borne by arbitrageurs. Our baseline model therefore captures the most important empirical features of index futures mispricing documented in the literature surveyed above.

A limitation of our baseline model is that, in reality, an index futures arbitrageur does not have to wait until maturity to close out their position. They have the option to unwind their position prior to maturity should the mispricing move in their favour. Indeed, this option can add significant value to a prospective arbitrage opportunity. Such an early unwinding option was first introduced by Brennan and Schwartz (1988), and many subsequent models of index futures arbitrage have also included this option (see e.g, Brennan and Schwartz 1990, Dai et al. 2011, Duffie 1990). Therefore, in Section 4, we extend our baseline model in a similar vein. Formally, this gives rise to an optimal stopping problem, which we convert into a free-boundary problem that can be solved numerically.

In Section 5 we examine the stylised features of our extended model, which includes the option to unwind early. We find that, although the model is more sophisticated than the baseline model, it captures the same empirical features of index futures mispricing. In other words, the inclusion of financing constraints still induce a minimum level of mispricing below which an arbitrageur would not entertain a potential arbitrage opportunity, even with the option to unwind the position early. Importantly, the minimium mispricing level is found, once more, to be an increasing function of both the futures contract's time to maturity and volatility.

Section 6 summaries our results and offers a few brief conclusions. Proofs are given in the appendix.

2. A Baseline Model

Let S_t be the value of an equity index at time t and let F_t be the contemporaneous price of a futures contract on the index with expiry date $T > t$. Let D_t denote the present value at time t of the dividends paid by the basket of stocks in the index over the time interval $[t, T]$, and let P_t denote the price at time t of a unit par discount bond maturing at time T. The theoretical *fair value* of the futures contract is then given by $\hat{F}_t := \frac{S_t - D_t}{P_t}$.

We refer to the difference

$$X_t := F_t - \hat{F}_t = F_t - \frac{S_t - D_t}{P_t}$$

between the futures price and the fair value of the futures contract as the *arbitrage spread* of the contract. In an ideal market without frictions or constraints, a textbook arbitrage strategy would allow a risk-free profit to be made with zero initial investment whenever the arbitrage spread is non-zero. To wit, if $X_t > 0$, an arbitrageur could take a short position in the futures contract, short-sell the discount bond and use the proceeds to purchase the basket of stocks in the index, with all dividends reinvested in the discount bound. On the other hand, if $X_t < 0$, he could take a long futures position, short-sell the basket of stocks in the index and invest the proceeds in the discount bond, with that investment used to fund dividend payments. In both cases, the trade would terminate upon expiry of the futures contract, with the cash flow from the futures position used to settle the stock and bond positions. Since the price of the futures contract and its fair value are guaranteed to converge to the index value at expiry, implying that $X_T = 0$, a profit of $|X_t|$ would be realised at expiry in either case.

A major shortcoming of the naive strategy outlined above is its failure to account for the real-world financing constraints that make index futures arbitrage a risky venture in practice. These constraints stem from the interim payments required during the life of an arbitrage trade, before any profit can be realised. On the futures leg, an arbitrageur will receive margin calls whenever the futures price moves against his position. In the case when the strategy involves borrowing and buying the basket of stocks, the stock will typically be used as collateral for a margin loan. A fall in the stock price will then trigger a call for additional collateral. Alternatively, when the stock is borrowed and sold short, the arbitrageur will be required to post collateral with the lender. In that case, a significant increase in the stock price will also precipitate a call for additional collateral. In practice, any increase in the arbitrage spread after a trade has been initiated will result in a call for additional margin or collateral, possibly on both legs of the transaction. Since no arbitrageur has an unlimited supply of capital, the accumulation of margin calls and collateral payments could exhaust their financial reserves before they have an opportunity to realise a profit. In such an event, they could be forced to unwind an ultimately profitable arbitrage position at a considerable loss.[2]

In keeping with the theoretical literature on index futures arbitrage, we regard the arbitrage spread X_t of an index futures contract as an exogenous stochastic process. Brownian bridge-type models are a popular choice for this purpose (see e.g., Brennan and Schwartz 1988, 1990, Dai et al. 2011, Duffie 1990). We follow suit by modelling the arbitrage spread as a Brownian bridge determined by the SDE

$$\mathrm{d}X_t = -\frac{X_t}{T-t}\,\mathrm{d}t + \sigma\,\mathrm{d}B_t, \tag{2.1}$$

where $\sigma > 0$ is the instantaneous standard deviation of the spread and B_t is a standard one-dimensional Brownian motion. This model captures two essential features of the arbitrage spread. First, since the drift term in (2.1) is negligible when $t \ll T$, it follows that X_t behaves like a driftless Brownian motion in that case. The economic implication is that the gap between the price of a futures contract and its fair value can be large when the remaining time to expiry is large, and can remain that way for an extended period of time. Second, since $-1/(T-t) \to -\infty$ as $t \to T$, the drift term in (2.1) becomes arbitrarily large in absolute value as $t \to T$, whenever $X_t \neq 0$, and with the opposite sign to that of X_t. It therefore follows that $X_t \to 0$ as $t \to T$. Economically, this captures the fact that the futures price and the index value must converge as

[2]This is a large part of the story of the failure of Long-Term Capital Management (Lowenstein 2000).

the expiry date of the contract approaches, resulting in an arbitrage spread of zero at expiry.

In order to model the impact of financing constraints on index futures arbitrage, we consider a financially constrained arbitrageur who wants to exploit the opportunities presented by (2.1). His preferences are governed by a CARA utility function

$$U(z) := \frac{1}{\gamma}(1 - \mathrm{e}^{-\gamma z}),$$

where $\gamma > 0$ quantifies his risk-aversion.[3] We suppose that he initiates an arbitrage trade at some time $t < T$, when the arbitrage spread $X_t \geq 0$ is non-negative.[4] This is done by taking a short position in the futures contract, short-selling the discount bond and purchasing the basket of stocks in the index. We assume that the arbitrageur intends to unwind the position at time T, when the futures contract expires.[5] In the textbook scenario without financing constraints, he would make a guaranteed profit of $X_t - X_T = X_t \geq 0$. However, in our model he only has a limited amount of capital $c > 0$ to fund margin calls. This means that the arbitrage trade is no longer risk-free, since the arbitrageur must contend with the possibility that the arbitrage spread could increase beyond $X_t + c$ at some time prior to expiry. Since each increase in the arbitrage spread (due to an increase in the futures price, a decrease in the value of the basket of stocks, or both) precipitates in a call for an equivalent amount of collateral, his capital c will have been consumed by margin calls by the time the arbitrage spread reaches the *liquidity threshold* $X_t + c$, and his position will be unwound involuntarily.

Based on the analysis above, the arbitrage will be unwound (either voluntarily or involuntarily) at time ζ_t, where

$$\zeta_t := \inf\{s \in [t, T] \,|\, X_s = X_t + c\}.^6$$

[3]Note that the arbitrageur is risk-averse if $\gamma > 0$ and becomes risk-neutral in the limit as $\gamma \to 0$.

[4]Note that the symmetry of (2.1) implies that $-X_t$ is also a Brownian bridge. Hence, the analysis that follows can be applied equally well to the case when $X_t \leq 0$. We will focus exclusively on non-negative arbitrage spreads, with the understanding that doing so results in no loss of generality.

[5]In reality, index futures arbitrage positions are rarely held all the way through to the expiry of the futures contract, since there are usually better opportunities to unwind the position prior to expiry. We address this limitation of our baseline model in Section 4.

[6]In the event that $X_s < X_t + c$, for all $s \in [t, T]$, this definition implies that $\zeta_t = T$.

There are two cases to consider. First, if $\zeta_t = T$, then the arbitrage spread does not reach the liquidity threshold prior to expiry and the arbitrageur unwinds his position voluntarily on the expiry date T, realising a profit of X_t. On the other hand, if $\zeta_t < T$, then the arbitrage spread does reach the liquidity threshold before expiry and the arbitrageur's position is unwound involuntarily at time ζ_t, realising a loss of c. By combining these two cases, we obtain the following expression for the value of an arbitrage trade initiated at time $t < T$ when the arbitrage spread is $X_t \geq 0$:

$$
\begin{aligned}
V(t,x) &:= \mathsf{E}_{t,x}\left(\mathbf{1}_{\{\zeta_t = T\}} \mathrm{e}^{-\rho(T-t)} U(x) + \mathbf{1}_{\{\zeta_t < T\}} \mathrm{e}^{-\rho(\zeta_t - t)} U(-c)\right) \\
&= \mathrm{e}^{-\rho(T-t)} U(x) - \mathsf{E}_{t,x}\left(\mathbf{1}_{\{\zeta_t < T\}} \mathrm{e}^{-\rho(T-t)} U(x) - \mathbf{1}_{\{\zeta_t < T\}} \mathrm{e}^{-\rho(\zeta_t - t)} U(-c)\right) \\
&= \underbrace{\mathrm{e}^{-\rho(T-t)} U(x)}_{V_\infty(t,x)} - \underbrace{\left(\mathrm{e}^{-\rho(T-t)} U(x) \mathsf{P}_{t,x}(\zeta_t < T) - U(-c) \mathsf{E}_{t,x}\left(\mathbf{1}_{\{\zeta_t < T\}} \mathrm{e}^{-\rho(\zeta_t - t)}\right)\right)}_{C(t,x)},
\end{aligned}
\tag{2.2}
$$

where $\rho > 0$ is a discount factor that reflects the arbitrageur's cost of capital, $\mathsf{P}_{t,x}$ is the probability measure under which $X_t = x \geq 0$, and $\mathsf{E}_{t,x}$ is the corresponding expected value operator. Note that the first term $V_\infty(t,x)$ in (2.2) is the value of the unconstrained arbitrage opportunity, while the second term $C(t,x)$ can be interpreted as a penalty charge due to the financing constraint imposed by having a limited supply of capital to fund margin calls and collateral payments.

Using Jeanblanc et al. (2009, Proposition 4.3.5.3), we obtain the following expression for the probability that the arbitrage spread reaches the liquidity threshold before the futures contract expires, given that $X_t = x \geq 0$:

$$
\mathsf{P}_{t,x}(\zeta_t < T) = \mathrm{e}^{-\frac{2c(x+c)}{\sigma^2(T-t)}}. \tag{2.3}
$$

Furthermore, in Appendix A we demonstrate that

$$
\mathsf{E}_{t,x}\left(\mathbf{1}_{\{\zeta_t < T\}} \mathrm{e}^{-\rho(\zeta_t - t)}\right) = G(t,x), \tag{2.4}
$$

with

$$
\begin{aligned}
G(t,x) \coloneqq \frac{1}{2\sigma\sqrt{T-t}} \int_0^1 \frac{\mathrm{e}^{-\rho(T-t)z}}{\big(z(1-z)\big)^{3/2}} \Bigg((zx+c)\mathrm{e}^{-\frac{2c(x+c)}{\sigma^2(T-t)}} \phi\Bigg(\frac{z(x+2c)-c}{\sigma\sqrt{(T-t)z(1-z)}} \Bigg) \\
- \big(z(x+2c)-c\big)\phi\Bigg(\frac{zx+c}{\sigma\sqrt{(T-t)z(1-z)}} \Bigg) \Bigg) \,\mathrm{d}z,
\end{aligned} \tag{2.5}
$$

where $\phi(y) \coloneqq \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-y^2/2}$ is the standard normal probability density function. Using these expressions, (2.2) can be written as

$$
V(t,x) = \underbrace{\mathrm{e}^{-\rho(T-t)}U(x)}_{V_\infty(t,x)} - \underbrace{\Big(U(x)\mathrm{e}^{-\rho(T-t)}\mathrm{e}^{-\frac{2c(x+c)}{\sigma^2(T-t)}} - U(-c)G(t,x) \Big)}_{C(t,x)}. \tag{2.6}
$$

Since (2.5) cannot be evaluated analytically, we cannot obtain an explicit formula for the value function (2.6). Nevertheless, standard numerical integration techniques allow for quick and accurate numerical approximation of the function.

Given $t < T$ and $x \geq 0$, observe that $U(x) \geq 0$, $U(-c) < 0$ and $G(t,x) > 0$, which ensure that $C(t,x) > 0$. Hence, $V(t,x) < V_\infty(t,x)$, which is to say that the risk of capital depletion unambiguously reduces the value of any arbitrage opportunity. Also note that $C(t,x) \to 0$ as $c \to \infty$, whence $V(t,x) \to V_\infty(t,x)$ as $c \to \infty$. Economically, this means that the risk of capital depletion becomes insignificant if the arbitrageur is very well capitalised, since he will almost always be able to fund margin calls and collateral payments. In that case, the penalty charge is essentially zero and the value of the arbitrage opportunity is close to the value of the unconstrained opportunity.

The possibility of capital exhaustion makes index futures arbitrage a risky strategy. As a result, the arbitrageur may choose not to exploit arbitrage opportunities under certain conditions. We assume that he will attempt to exploit a non-negative arbitrage spread if the value of the opportunity, given by (2.6), exceeds the value $U(0) = 0$ of doing nothing. However, he will disregard any arbitrage opportunity for which the value of the spread does not exceed the value of doing nothing. The first condition can be expressed mathematically as $V(t,x) > 0$, while the second condition can be expressed as $V(t,x) \leq 0$. Based on these assumptions, we can partition $[0,T] \times \mathbb{R}_+$

into a *trade region* $\mathcal{T}$ and a *no-trade region* $\mathcal{N}$, given by

$$\mathcal{T} := \{(t,x) \in [0,T] \times \mathbb{R}_+ \,|\, V(t,x) > 0\}$$

$$\mathcal{N} := \{(t,x) \in [0,T] \times \mathbb{R}_+ \,|\, V(t,x) \leq 0\}.$$

The arbitrageur will initiate an arbitrage trade at time t if the observed spread $X_t \geq 0$ satisfies $(t, X_t) \in \mathcal{T}$. On the other hand, he will do nothing if $(t, X_t) \in \mathcal{N}$.

Inspection of (2.6) reveals that $V(t, 0) < 0$, for all $t < T$, while it follows from the dominated convergence theorem that $V(t,x) \to \infty$ as $x \to \infty$. In addition, $V(t,x)$ increases monotonically as $x \to \infty$, for all $t < T$, by virtue of the properties of the Brownian bridge (2.1). Consequently, the trade region and the no-trade region are separated by a *no-trade frontier* $[0,T] \ni t \mapsto x^*(t)$, with the property that $(t,x) \in \mathcal{T}$ if $x > x^*(t)$ and $(t,x) \in \mathcal{N}$ if $x \leq x^*(t)$. The frontier is implicitly determined by the equation $V(t, x^*(t)) = 0$, which is equivalent to

$$1 - \mathrm{e}^{-\frac{2c(x^*(t)+c)}{\sigma^2(T-t)}} = -\frac{U(-c)}{U(x^*(t))}\mathrm{e}^{\rho(T-t)}G(t, x^*(t)). \tag{2.7}$$

Although this equation cannot be solved explicitly, standard numerical techniques yield quick and accurate answers.

3. Analysis of the Baseline Model

The primary insight from our model is that margin calls and collateral requirements can make index futures arbitrage a very risky strategy, since arbitrageurs have limited supplies of capital. If an arbitrageur runs out of capital to fund those cash flows, their position is unwound involuntarily, which usually crystallises a significant loss. Consequently, the attractiveness of an arbitrage opportunity should depend on the level of risk-aversion of a would-be arbitrageur.

The dependence of the value function (2.6) on the arbitrageur's level of risk aversion is illustrated in Figure 3. Panel (a) plots the value function against the size of the arbitrage spread, for a fixed time to expiry and different levels of risk aversion, while Panel (b) plots the value function against the time to expiry of the futures contract, for a fixed arbitrage spread and different levels of risk aversion. The solid curve in both panels illustrates the value function for a risk-neutral arbitrageur, while the dashed curves illustrate the cases for risk-averse arbitrageurs. Both panels show that higher levels of risk-aversion correspond with lower values for an arbitrage opportunity, all else being equal, since a more risk-averse arbitrageur attaches a larger penalty charge to the possibility of exhausting their capital on margin calls.

As expected, Panel (a) confirms that the value of an arbitrage opportunity increases with the size of the arbitrage spread. However, the relationship becomes increasingly concave as risk-aversion increases. This means that large arbitrage spreads are not much more attractive than moderate spreads, for risk-averse arbitrageurs. In Panel (a) we also see that every curve intercepts the horizontal axis at some positive arbitrage spread, which is the value of the no-trade frontier for that level of risk-aversion and time to expiry. This point shifts to the right as risk-aversion increases, indicated that more risk-averse arbitrageurs require a larger spread before they are willing to initiate an index futures arbitrage trade.

Panel (b) shows that the value of a given arbitrage spread is a decreasing function of the time to expiry of the futures contract. This is because an arbitrage position involving a long-dated futures contract is riskier than an otherwise identical position involving a short-dated contract. To understand why, note that the drift term in (2.1) is smaller (assuming that the arbitrage spread is positive) in the case of the long-dated contract. Since the drift term provides the force that pins the arbitrage spread to zero at expiry, it is more likely that the spread will reach the liquidity threshold before expiry, in a case of the long-dated contract. In Panel (b) we also see that there is always some time to expiry beyond which the value of the arbitrage opportunity is negative. That is to say, for a given arbitrage spread, there is a maximum time to expiry, beyond which an arbitrageur loses interest in the opportunity. As expected, less risk-averse arbitrageurs are willing to exploit longer-dated arbitrage opportunities, since they are more tolerant of financing risk. However, even risk-neutral arbitrageurs are unwilling to exploit a given arbitrage spread beyond a certain time to expiry.

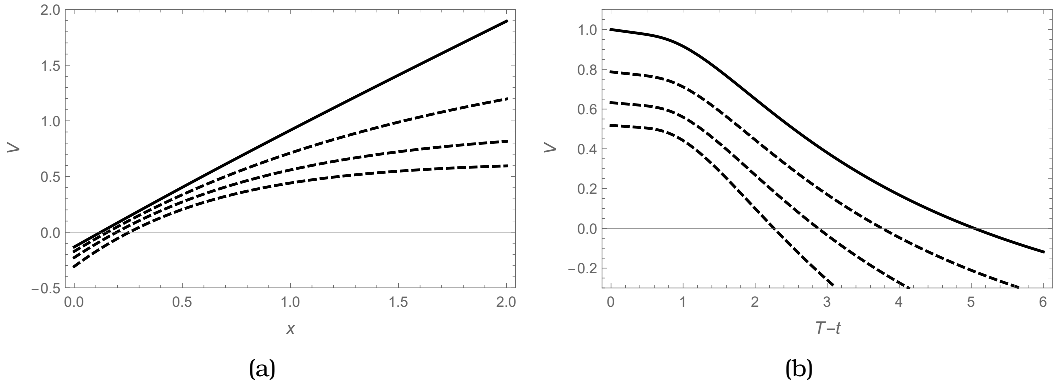

FIGURE 1. The behaviour of the value function for different levels of risk-aversion. Panel (a) plots the function $x \mapsto V(t,x)$, for a fixed time to expiry $T-t=1$. Panel (b) plots the function $T-t \mapsto V(t,x)$, for a fixed arbitrage spread $x=1$. The solid curves in both panels depict the risk-neutral case when $\gamma \to 0$, while the dashed curves depict the risk-averse cases when $\gamma = 0.5, 1, 1.5$, with higher levels of risk-aversion corresponding to lower values for the value function. The remaining parameter values are $c=\sigma=1$ and $\rho=0.05$.

Figure 3 illustrates the behaviour of the no-trade frontier, which is determined by solving (2.7). Each panel illustrates the relationship between the frontier and the time to expiry of the futures contract, while varying one of the remaining parameters. Panel (a) demonstrates the impact of varying the arbitrageur's initial capital, while Panels (b)–(d) do the same for the volatility of the arbitrage spread, the discount rate and the arbitrageur's level of risk aversion. The solid curve in each panel depicts the same base case, corresponding to a set of default parameter values.

Overall, the evidence in Figure 3 points to an unambiguous positive dependence of the no-trade frontier on the time to expiry of the futures contract. This dependence is robust to changes in the values of the model parameters. Economically, it stems from the fact that index futures arbitrage involving long-dated futures contracts results in elevated levels of financing risk. Consequently, arbitrageurs are only willing to exploit the opportunities presented by long-dated futures contracts when the arbitrage spread is sufficiently large to compensate for the higher penalty charge due to an increased likelihood of capital depletion. This provides a straightforward explanation for the widely documented positive dependence of index futures mispricing on time to expiry (see e.g., Bhatt and Cakici 1990, Bühler and Kempf 1995, Draper and Fung 2003, Fung and Draper 1999, MacKinlay and Ramaswamy 1988, Switzer et al. 2000, Yadav and Pope 1990, 1994).

Panel (a) shows that the no-trade frontier is negatively related to the arbitrageur's initial capital. The economic explanation is that a lower level of capital increases the financing risk of an index futures arbitrage trade, which in turn increases the penalty charge associated with that risk. An arbitrageur with less capital to fund margin calls and collateral payments will require a larger arbitrage spread as compensation, before being willing to initiate a trade.

In Panel (b), we see that the no-trade frontier is positively related to the volatility of the arbitrage spread. Once again, this can be explained in terms of financing risk, since a higher volatility increases the probability that the spread will reach the arbitrageur's liquidity threshold prior to the expiry of the futures contract. Since his position will be unwound automatically if that happens, resulting in a substantial loss, it follows that an increase in the spread volatility will increase the arbitrageur's financing risk, and therefore also the penalty charge associated with that risk. He will thus require a larger arbitrage spread as compensation, before being willing to exploit mispriced futures contract. This analysis provides a parsimonious explanation for the well-documented positive dependence of index futures mispricing on volatility (see e.g., Brailsford and Hodgson 1997, Chan 1992, Cummings and Frino 2011, Draper and Fung 2003, Fung and Draper 1999, Gay and Jung 1999, Hill et al. 1988, Kawaller et al. 1990, Merrick 1987, Richie et al. 2008, Stoll and Whaley 1990).

Panel (c) indicates that an increase in the discount rate marginally reduces the value of the no-trade frontier, which means that smaller arbitrage spreads are slightly more attractive, but the impact is not significant. Finally, as expected, Panel (d) shows that an increase in the arbitrageur's risk aversion has a dramatic effect on the no-trade frontier, increasing its value substantially. The economic interpretation is that a more risk-averse arbitrageur will require a higher arbitrage spread to offset the financing risk

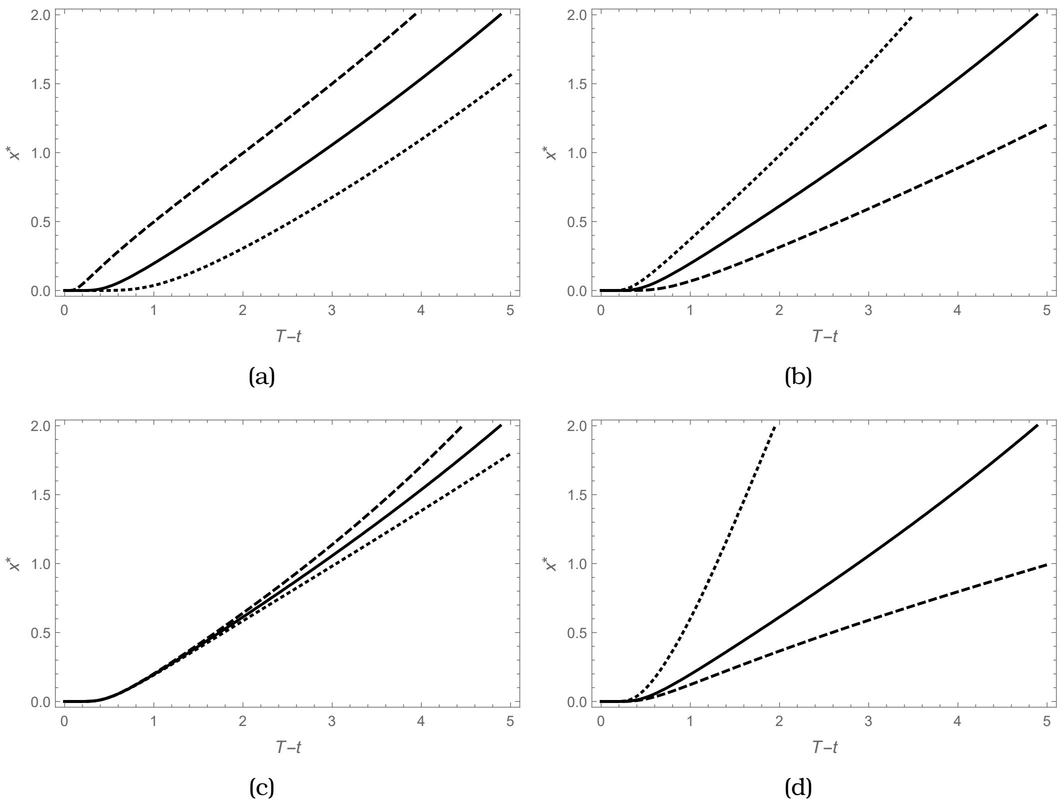

FIGURE 2. The behaviour of the no-trade frontier as a function of time to expiry. The figure in each panel plots the function $T - t \mapsto x^*(T - t)$, for different parameter values, with the solid line in each figure depicting the case when $x = c = \sigma = \gamma = 1$. Panel (a) illustrates the impact of the arbitrageur's initial capital c on the no-trade frontier, with the upper dashed curve corresponding to the case when $c = 0.5$ and the lower dotted curve corresponding to the case when $c = 1.5$. Panel (b) illustrates the impact of the volatility σ of the arbitrage spread on the no-trade frontier, with the lower dashed curve corresponding to the case when $\sigma = 0.8$ and the upper dotted curve corresponding to the case when $\sigma = 1.2$. Panel (c) illustrates the impact of the discount rate ρ on the no-trade frontier, with the upper dashed curve corresponding to the case when $\rho = 0$ and the lower dotted curve corresponding to the case when $\rho = 0.1$. Panel (d) illustrates the impact of the arbitrageur's risk aversion γ on the no-trade frontier, with the lower dashed curve corresponding to the case when $\gamma \to 0$ and the lower dotted curve corresponding to the case when $\gamma = 2$.

due to margin calls and collateral payments, before being willing to initiate an index futures arbitrage trade, all else being equal.

4. A Model with Optimal Unwinding

Our assumption that the arbitrageur chooses to unwind his position only when the futures contract expires (unless it is unwound beforehand because the arbitrage spread reaches his liquidity threshold) is a limitation of the baseline model in Section 2. In reality, it may be preferable for him to unwind before the contract expires. For example, if the arbitrage spread reaches zero prior to expiry, then the impact of discounting means that unwinding then is certainly better than waiting until expiry. But there may be an even better time to unwind, since if the trade is initiated when the arbitrage spread is $X_t \geq 0$, it could overshoot the origin at some random time τ, where $\tau \in [t, T]$, which is to say that $X_\tau < 0$. Unwinding at that time would yield a larger profit $X_t - X_\tau > X_t$ than unwinding when the spread first reaches zero.

Following Brennan and Schwartz (1988, 1990), Duffie (1990), and Dai et al. (2011), we extend the baseline model by allowing the arbitrageur to unwind an index futures arbitrage position before the futures contract matures. Assuming, as before, that he initiates the trade at time $t < T$, when the arbitrage spread is $X_t \geq 0$, this amounts to choosing a stopping time $\tau \in [t, T]$ at which to unwind the position. As before, there are two cases to consider. First, if $\zeta_t \geq \tau$, then the arbitrage spread does not reach the liquidity threshold before the arbitrageur chooses to unwind his position, and he unwinds voluntarily at time τ, realising a profit of $X_t - X_\tau$. On the other hand, if $\zeta_t < \tau$, then the arbitrage spread does reach the liquidity threshold before the arbitrageur chooses to unwind, and his position is unwound involuntarily at time ζ_t, realising a loss of c. By combining these two cases and allowing the arbitrageur to choose the optimal time to unwind his position, we obtain the following expression for the value of an arbitrage trade initiated at time $t < T$ when the arbitrage spread is $X_t = x \geq 0$:

$$\widehat{V}(t,x) := \sup_{\tau \in \mathfrak{S}_{t,T}} \mathsf{E}_{t,x}\Big(\mathbf{1}_{\{\zeta_t \geq \tau\}} \mathrm{e}^{-\rho(\tau - t)} U(x - X_\tau) + \mathbf{1}_{\{\zeta_t < \tau\}} \mathrm{e}^{-\rho(\zeta_t - t)} U(-c)\Big), \tag{4.1}$$

where $\mathfrak{S}_{t,T}$ denotes the set of all stopping times taking values in the interval $[t, T]$.

In order to evaluate (4.1), we begin by postulating the existence of a lower threshold $[0,T] \ni t \mapsto x_*(t) \leq 0$, with the property that it is optimal for the arbitrageur to unwind his position when the arbitrage spread first reaches that threshold. That is to say, the optimal unwinding time is

$$\tau_* := \inf\{s \in [t,T] \,|\, X_s = x_*(s)\}.$$

Next, using standard results from optimal stopping theory (see e.g., Peskir and Shiryaev 2006), the solution to the optimal stopping problem (4.1) must also solve the following free-boundary problem:

$$\begin{aligned}
&\frac{\partial \widetilde{V}}{\partial t}(t,z) - \frac{z}{T-t}\frac{\partial \widetilde{V}}{\partial z}(t,z) \\
&\qquad + \frac{1}{2}\frac{\partial^2 \widetilde{V}}{\partial z^2}(t,z) - \rho \widetilde{V}(t,z) = 0 && \text{if } (t,z) \in [0,T) \times \big(x_*(t), x+c\big); && \text{(4.2a)}\\
&\widetilde{V}(t,z) = U(x-z) && \text{if } (t,z) \in [0,T] \times \big(-\infty, x_*(t)\big]; && \text{(4.2b)}\\
&\frac{\partial \widetilde{V}}{\partial z}\big(t, x_*(t)\big) = U'\big(x - x_*(t)\big) && \text{if } t \in [0,T); && \text{(4.2c)}\\
&\widetilde{V}(t, x+c) = U(-c) && \text{if } t \in [0,T); && \text{(4.2d)}\\
&\widetilde{V}(T,z) = U(z). && && \text{(4.2e)}
\end{aligned}$$

The function $[0,T] \times \mathbb{R} \ni (t,z) \mapsto \widetilde{V}(t,z)$ and the free boundary $[0,T] \ni t \mapsto x_*(t)$ are determined by solving this system of equations numerically. Finally, we obtain the value function (4.1) by setting $\widehat{V}(t,x) := \widetilde{V}(t,z)|_{z=x}$.

5. Analysis of the Model with Optimal Unwinding

Extending the baseline model to include the option of unwinding an index futures arbitrage trade before the futures contract expires has some interesting consequences, although it does not fundamentally alter the positive dependencies of the no-trade frontier on contract maturity and volatility. The most significant consequence is that arbitrage opportunities become much more attractive. In particular, the freedom to unwind early induces the arbitrageur to initiate arbitrage positions when the arbitrage spread is much smaller than he would be willing to countenance if he had to hold those positions for their full term. There are two reasons. First, being able to close-out a position at any time allows the arbitrageur to manage the financial risk resulting from margin calls and collateral payments. Second, it allows him to derive an additional profit from instances when the arbitrage spread overshoots the origin.

Figure 5 illustrates the dependence of the value (4.1) of an arbitrage position on the arbitrage spread and the time to expiry of the futures contract, for different levels of risk aversion, if early unwinding is possible. For comparison, the dependencies if the value (2.6) without early unwinding and the value without the financing constraint are considered as well. The solid curves depict the value function with the early unwinding option, while the dashed and dotted curves depict the value functions without the option and without the financing constraint, respectively. As expected, we observe that the value functions with the early unwinding option (solid lines) and without the financing constraint (dotted lines) always exceed the value function with the financing constraint but without the freedom to unwind early (dashed lines). This is because the freedom to close-out an arbitrage trade before expiry enhances its value, for the reasons described above, while the presence of financing risk decreases its value. We also see that the relationship between the solid and dotted lines is inconsistent, implying that the tradeoff between the benefit of being able to unwind early and the benefit of not being constrained by margin calls and collateral payments can resolve in either direction.

Panels (a) and (b) show that the value of an arbitrage opportunity increases with the size of the arbitrage spread, irrespective risk aversion. However, the dependence is approximately linear for the risk-neutral arbitrageur and significantly concave for his risk-averse counterpart. In particular, the risk-averse arbitrageur derives almost no marginal value from an increase in the arbitrage spread beyond a certain point, which implies that very large spreads are not significantly more attractive than smaller ones.

Note that the dotted curves in Panels (a) and (b) intersect the origin, indicating that the value of a zero arbitrage spread is zero, in the case of a textbook arbitrage trade without financing constraints. However, the dashed curves assume negative values as the arbitrage spread approaches zero, which means that the arbitrageur in our baseline model will reject small arbitrage opportunities which do not compensate him for the financing risk arising from margin calls and collateral payments. The solid curves, by contrast, exhibit the surprising feature of being positively valued when the arbitrage spread is zero. This means that the freedom to unwind early confers a positive value on an arbitrage spread of zero. Economically, this is because an arbitrage trade initiated when the spread is zero can be unwound for a profit if the spread ever becomes negative before the futures contract expires, and the probability of that happening is positive.

The curves in Panels (c) and (d) show the value of a given fixed arbitrage spread is a decreasing function of the time to expiry of the futures contract, irrespective of the arbitrageur's risk aversion or the ability to unwind early. However, for very short maturities, we note the value of the strategy with the early unwinding option (solid curves) significantly exceeds the values of the other two strategies (dashed and dotted curves). This is because of the dominance of the drift term in (2.1) when the time to expiry is very small, which causes any deviation of the arbitrage spread from zero to be corrected quickly. With discretionary unwinding, the arbitrageur can profit from such deviations.

The solid curves in Figure 5 illustrate the dependence of the value (4.1) of an arbitrage position with the option to unwind early on the volatility of the arbitrage spread and the arbitrageur's initial capital, for different levels of risk aversion. The dependencies of the value (2.6) without early unwinding (dashed curves) and the value without the financing constraint (dotted curves) are illustrated as well.

Panels (a) and (b) show that the value of an arbitrage trade in our baseline model with financing risk but no early unwinding option decreases monotonically as volatility increases, eventually becoming negative, with a substantially stronger rate of decline in the case of a risk-averse arbitrageur. This is because an increase in volatility increases the probability of the arbitrage spread reaching the liquidity threshold. In the case of the arbitrage position with the early unwinding option, the relationship

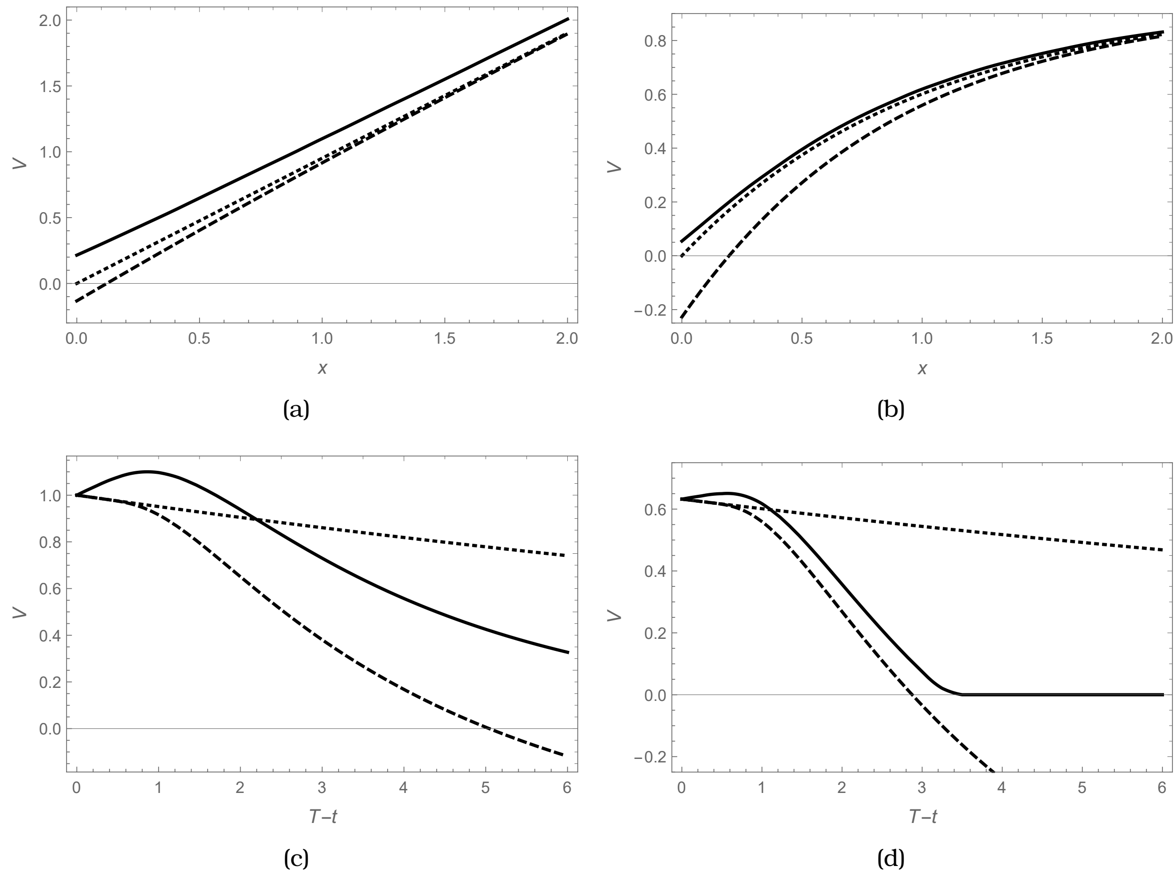

FIGURE 3. The behaviour of the value functions for different levels of risk aversion. The solid curve in each panel represents the value $\widehat{V}(t,x)$ of the financially constrained arbitrage trade with the option to unwind early, the dashed curve represents the value $V(t,x)$ of the financially constrained arbitrage trade without the option to unwind early, and the dotted curve represents the value $V_\infty(t,x)$ of the textbook unconstrained arbitrage strategy. Panels (a) and (c) describe the risk-neutral scenario ($\gamma \to 0$), while Panels (b) and (d) describe a risk-averse scenario ($\gamma = 1$). Panels (a) and (b) illustrate the dependence of the value of an arbitrage trade on the arbitrage spread, under the two risk-aversion scenarios, with the values of the remaining parameters given by $T - t = \sigma = c = 1$ and $\rho = 0.05$. Panels (c) and (d) illustrate dependence of the value of the arbitrage trade on time to expiry, with the values of the remaining parameters given by $x = \sigma = c = 1$ and $\rho = 0.05$.

is non-monotonic, increasing before decreasing. The explanation is that an increase in volatility also increases the probability that the arbitrage spread will overshoot the origin, giving the arbitrageur an opportunity close out the position for a larger profit than the initial spread. For moderate volatilities, this effect dominates the increased financing risk, but the situation is reversed for very high volatilities.

In Panels (c) and (d), we observe that the value of an arbitrage trade is an increasing function of the arbitrageur's initial capital, with risk-aversion not having a significant effect on the relationship. Moreover, the freedom to unwind early yields a relatively modest increase in value, relative to the value for the baseline model. For both the baseline model and the extended model with optional unwinding, the positive relationship between the value of an arbitrage tade and the arbitrageur's capital follows because an increase in initial capital decreases the financial risk due to margin calls and collateral payments.

Figure 5 illustrates the dependence of the no-trade frontiers for the baseline model (dashed curves) and the extended model (solid curves) on time to maturity. The grey-out curves in each panel represent a base case with default parameter values. The dark curves in Panels (a) and (b) illustrate the impact of varying risk-aversion, while leaving the values of the remaining parameters unchanged. The dark curves in Panels (c) and (d) and Panels (e) and (f) do the same for initial capital and volatility, respectively. We see that the no-trade frontiers for both models are increasing functions of time to expiry, which agrees with the empirical evidence on the relationship between index futures mispricing and contract maturity. Moreover, since the positive maturity dependence of the no-trade frontier is unaffected by variations in the model parameters, this empirical prediction of our models is very robust.

Note that Figure 5 illustrates a very interesting consequence of the early unwinding option, which we briefly mentioned before. In each panel, we see that the solid curve starts at zero at some positive time to expiry, while for shorter times to expiry it does not exist. In other words, the no-trade frontier for the extended model does not exist when the time to expiry of the futures contract is very short. Economically, this means that the arbitrageur in our model is always willing to initiate an arbitrage trade involving a very short dated futures contract, even if the contract is priced correctly and the arbitrage spread is zero. This is because the financial risk of an arbitrage position is small in the case of a very short dated contract, since the dominance of the drift term in (2.1) ensures that there is very little chance of the arbitrage spread reaching the

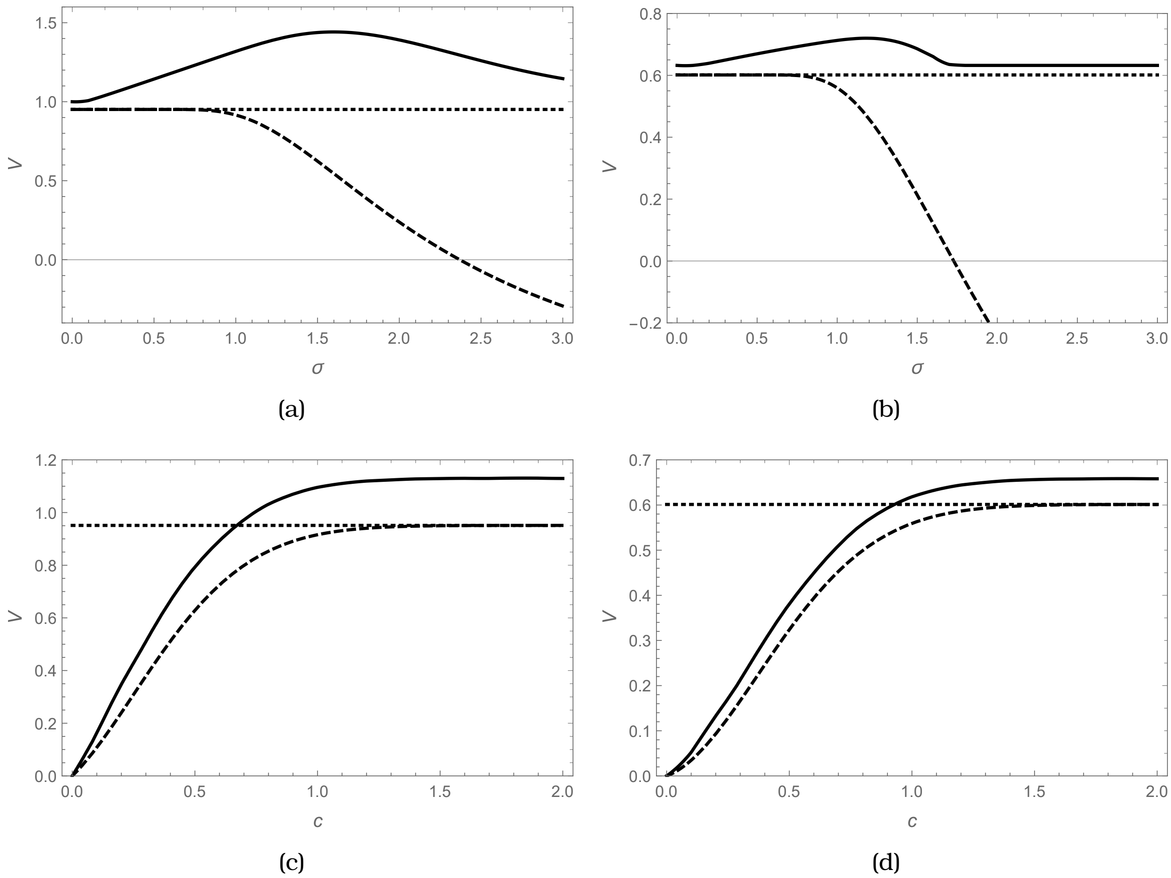

FIGURE 4. The behaviour of the value functions for different levels of risk aversion. The solid curve in each panel represents the value $\widehat{V}(t,x)$ of the financially constrained arbitrage trade with the option to unwind early, the dashed curve represents the value $V(t,x)$ of the financially constrained arbitrage trade without the option to unwind early, and the dotted curve represents the value $V_\infty(t,x)$ of the textbook unconstrained arbitrage strategy. Panels (a) and (c) describe the risk-neutral scenario ($\gamma \to 0$), while Panels (b) and (d) describe a risk-averse scenario ($\gamma = 1$). Panels (a) and (b) illustrate the dependence of the value of an arbitrage trade on the arbitrage spread, under the two risk-aversion scenarios, with the values of the remaining parameters given by $T - t = x = c = 1$ and $\rho = 0.05$. Panels (c) and (d) illustrate dependence of the value of the arbitrage trade on time to expiry, with the values of the remaining parameters given by $T - t = x = \sigma = 1$ and $\rho = 0.05$.

liquidity frontier before the contract expires. However, initiating an arbitrage position under such circumstances allows the arbitrageur to profit from any short-lived deviations of the arbitrage spread from zero, if he has the option to unwind at any time. Of course, no arbitrageur would do this in practice, since any small profit they may make from discretionary unwinding would be consumed by transaction costs.

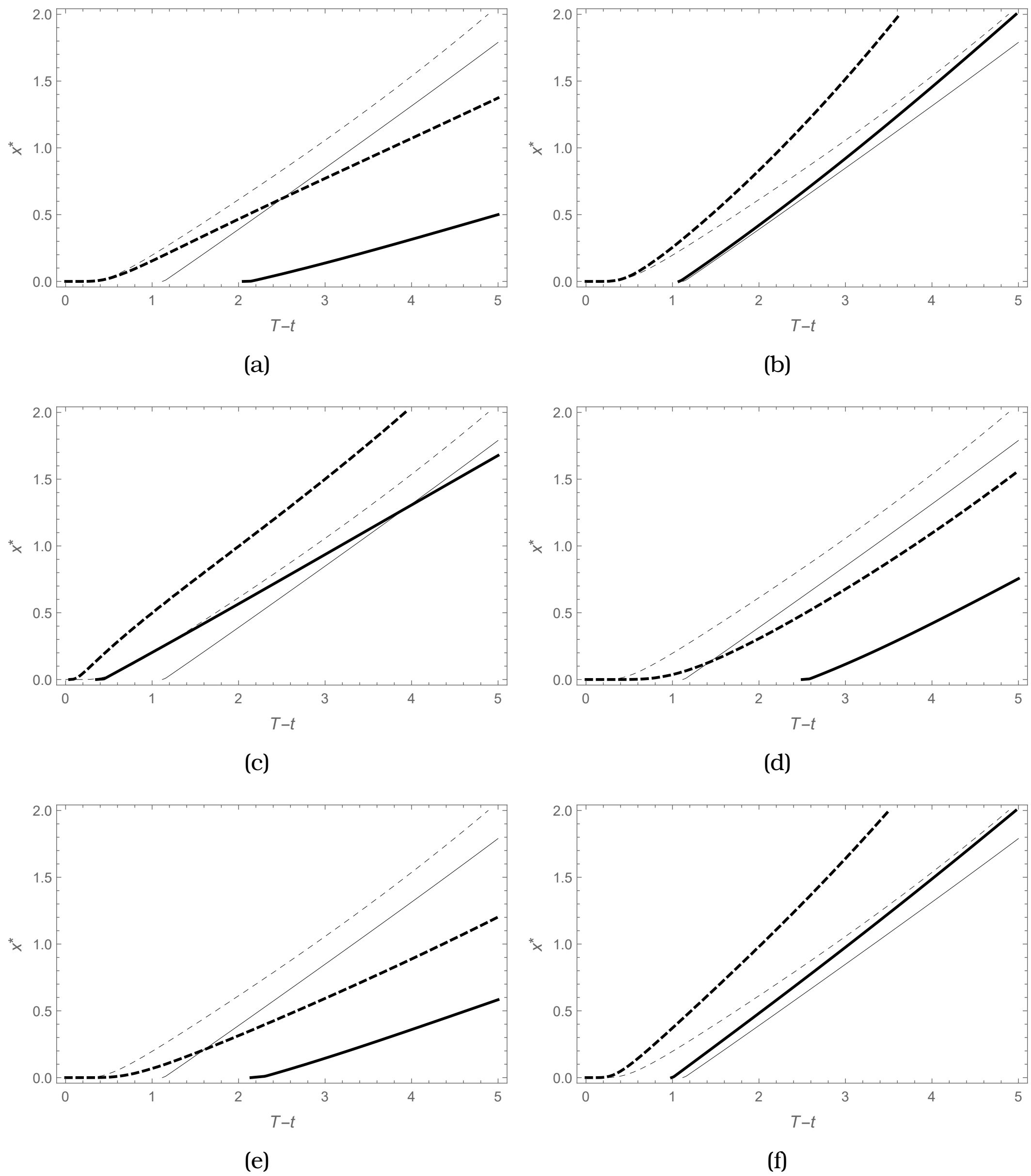

FIGURE 5. The behaviour of the no-trade frontier as a function of time to expiry. The figures in each panel plot the function $T - t \mapsto x^*(t)$, for different parameter values. The dashed curves in all panels depict the no-trade frontier for the baseline model, while the solid curves depict the no-trade frontier for the extended model with early unwinding. The greyed-out solid and dashed curves correspond to the default parameter values $x = \sigma = c = \gamma = 1$ and $\rho = 0.05$, which are presented for easy comparison. Panels (a) and (b) present a low risk-aversion scenario and a high risk-aversion scenario, respectively, with the remaining parameters assigned their default values. Panels (c) and (d) present a low capital scenario and a high capital scenario, respectively, with the remaining parameters assigned their default values. Panels (e) and (f) present a low volatility scenario and a high volatility scenario, respectively, with the remaining parameters assigned their default values.

Recall that the optimal unwinding threshold $x_*(t)$, for $t \in [0, T]$, specifies the value of the arbitrage spread at which it is optimal for the arbitrageur to unwind an index futures arbitrage position in our extended model. The dependence of this threshold on time to expiry is analysed in Figure 5, with Panel (a) illustrating the effect of varying the arbitrageur's initial capital and Panel (b) illustrating the effect of varying his risk aversion. For an arbitrage position initiated when the futures spread is non-negative, the curves indicate that the optimal level to close out the position is always negative, so long as the time to maturity of the futures contract is not too large. This agrees with our intuition that the early unwinding option gives the arbitrageur an opportunity to profit from the arbitrage spread overshooting its equilibrium level of zero. Hence, under normal circumstances, it is sub-optimal to unwind an arbitrage position when the spread is non-negative. Interestingly, however, the curves in Figure 5 also reveal that it is optimal to unwind an arbitrage position involving a very long-dated futures contract when the arbitrage spread is positive. This is because, in the case of arbitrage positions involving very long-dated futures contracts, the probability of the arbitrage spread reaching the liquidity threshold is very high, which is to say that the financing risk of the arbitrage trade is very high. For this reason, the arbitrageur should be keen to unwind earlier, rather than later.

6. Conclusions

In this paper, we developed two stylized models for determining the value of a classical index futures arbitrage. The important innovation of these models was the introduction of a financing constraint, in the form of a limited supply of capital.

Adding such constraints allowed us to parsimoniously explain two robust empirical features of index futures mispricing. Specifically, that the level of observed mispricing is larger for longer dated futures contracts and also during times of higher volatility. Put simply, limited capital introduces risk into traditional arbitrage opportunities which reduce their appeal and decrease arbitrage demand (allowing prices to deviate further from their traditional cost-of-carry pricing relationship). Importantly, the risk

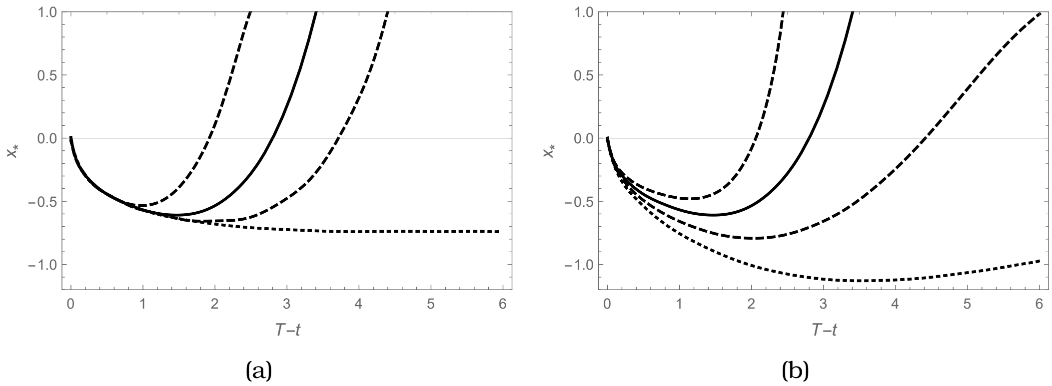

FIGURE 6. The behaviour of the optimal unwinding threshold as a function of time to expiry. The figures in each panel plot the function $T - t \mapsto x_*(t)$, which determines the arbitrage spread at which it is optimal to unwind an index futures arbitrage position. The solid curves in both panels correspond to the base case with $x = \sigma = c = \gamma = 1$ and $\rho = 0.05$. The dashed curves in Panel (a) illustrate the optimal unwinding threshold for $c = 0.5, 1.5$, while the dotted curve plots the threshold for the unconstrained case when $c \to \infty$. The dashed curves in Panel (b) illustrate the optimal unwinding threshold for $\gamma = 0.5, 1.5$, while the dotted curves plots the threshold for the risk-neutral case when $\gamma \to 0$.

of the capital constraint binding increases when both time to maturity and volatility increases, providing an intuitive link between these two stylized facts.

While financing constraints reduce the attractiveness of an index futures arbitrage, the option to unwind the arbitrage early *increases* its value. Indeed, the value of this embedded option is also increasing in time to maturity and volatility. It is for this reason that we extended our baseline model to explicitly include the countervailing effects of the early unwinding option. However, the qualitative feature of our baseline model (without the option) held, indicating that collateral constraints are the dominating factor.

APPENDIX A. PROOF OF EQUATION (2.4)

Given $t < T$ and $x \geq 0$, we obtain

$$\begin{aligned}\mathsf{P}_{t,x}(\zeta_t \leq t+u) &= \mathsf{P}_{t,x}\Big(\sup_{s\in[t,t+u]} X_s \geq x + c\Big)\\ &= \mathsf{P}\Big(\sup_{s\in[0,u]} (x+\sigma\tilde{B}_s) \geq x + c \,\Big|\, x + \sigma\tilde{B}_{T-t} = 0\Big)\end{aligned}$$

$$= \mathsf{P}\Big(\sup_{s\in[0,u]} \tilde{B}_s \geq \frac{c}{\sigma} \,\Big|\, B_{T-t} = -\frac{x}{\sigma}\Big),$$

where $\tilde{B}_s$ is a standard one-dimensional Brownian motion. This expression can be evaluated with the help of Beghin and Orsingher (1999, Theorem 2.1), yielding

$$\mathsf{P}_{t,x}(\zeta_t \leq t+u) = \mathrm{e}^{-\frac{2c(x+c)}{\sigma^2(T-t)}}\,\Phi\Big(\frac{(x+2c)u - c(T-t)}{\sigma\sqrt{(T-t)(T-t-u)u}}\Big)+1-\Phi\Big(\frac{c(T-t)+xu}{\sigma\sqrt{(T-t)(T-t-u)u}}\Big),$$

Where $\Phi(\,\cdot\,)$ is the standard normal cumulative distribution function. Since

$$\mathsf{E}_{t,x}\Big(\mathbf{1}_{\{\zeta_t<T\}}\mathrm{e}^{-\rho(\zeta_t-t)}\Big) = \int_t^T \mathrm{e}^{-\rho u}\mathsf{P}_{t,x}(\zeta_t \leq t+\mathrm{d}u),$$

we get (2.4) and (2.5) by differentiating the previous expression with respect to u and performing the substitution $z := {}^{u}\!/_{(T-t)}$.

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