A comprehensive look at commodity volatility forecasting

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Abstract

This research combines recent advances in the Realized Volatility (RV) literature and three specific commodity futures factors to improve the forecasts of commodity volatility. The three forecasting variables are the term structure slope, the time to maturity and a measure of supply and demand uncertainty. I first assess these variables' empirical contribution to commodity futures volatility, in adding them in RV forecast models. First in the univariate HAR-RV of Corsi (2009) and second in the multivariate VAR-RV of Andersen, Bollerslev, Diebold, and Labys (2003). The long-term memory of assets RV justifies the former, whereas the "financialization" of commodities and the resulting commodity connectedness, supports the latter. I evaluate the out of sample validity of these forecast models and propose one risk management application. Hence, this research is important for both economic understanding and risk management purposes.

JEL classification: C53, C58,

Keywords: Realized volatility, Samuelson effect, term structure, supply and demand, commodity futures, derivatives, risk management, financial econometrics.

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1. Introduction

I use three economic variable of commodity volatility, the slope of the term structure (proxy for inventories), the time to maturity (Samuelson hypothesis) and uncertainty indices (uncertainty resolution). Previous research use squared returns as proxy for unconditional volatility or (G)ARCH family models for the conditional volatility and test whether the inclusion of volatility factors have additional forecast power. Instead, Andersen, Bollerslev, and Meddahi (2005) show that using the RV as a proxy for the latent volatility in the left side of the equation considerably improves the forecast power. In using the RV as a finer proxy for the true latent volatility, I disentangle the validity of these hypotheses through the various forecast power of the proxy variables. I test whether the variable inclusion improves two existing advanced RV forecasting methods. The target models are the univariate HAR-RV of Corsi (2009), and the multivariate VAR-RV of Andersen et al. (2003). The inclusion of commodity specific factors in a HAR-RV is similar to Haugom, Langeland, Molnar, and Westgaard (2014).[1] Using a VAR is particularly appropriate to extract the volatility commonalities, as it allows the use of information shared across futures contracts. This new model improves the explanatory power, in the light of the recent findings on financialization of commodities. Indeed Masters (2008), Tang and Xiong (2012) and Singleton (2014) predict or document the increase of volatility or return commonalities in commodity markets, due to the massive, parallel entry of long only funds tracking popular indices such as the SP-GSCI or the BCOM. Later in the estimation, I also include commodity indices tracking dummy coded "one" if the commodity is part of a popular index. Despite my sample is too recent to test for the effect of the addition of a particular commodity to an index, I am able to test for index inclusion features in cross-section. Finally, I control for the out of sample validity of these two extended RV forecasts and find that results hold.

The remainder of the article proceeds as follow, in the next section I present the literature review on RV, stylized fact of assets volatility in general and of commodity futures in particular. In section three, I present the research design, including the data organization and models to test. In section four, I discuss the empirical results and in section five, I introduce one potential application of RV for the estimation of optimal hedge ratio with commodity futures. Section six concludes.

[1]Haugom et al., 2014 use an HAR-RV, extended with implied volatility, and additional forecasting variables (including one of the variable of the present article, the term structure slope, a proxy for the inventory effect) on the crude oil contract.

2. Literature review

2.1. Realized volatility

There are three families in volatility modeling. The first is the group of stochastic volatility such as the Heston (1993) model, where the volatility follows a diffusion process and possibly jumps (see, e.g. Bates, 1996). The second is the (G)ARCH type of Engle (1982) and Bollerslev (1986), which models volatility conditional on past realizations. Finally, the approach of Andersen and Bollerslev (1998a,b) use the RV, estimated with data sampled at high frequency, as a proxy for the integrated volatility (IV). Andersen and Bollerslev (1998a) show that, as the frequency increases towards infinity, the estimation tends to the IV. For instance, Andersen, Bollerslev, and Diebold (2007) summarize the theory with the following formula.[2] Let the RV be,

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} \left(r^2_{\Delta, t+j\times\Delta} \right), \tag{1}$$

where $1/\Delta$ is the number of observations in one day and r^2_Δ represents the squared returns sampled at the frequency Δ. Then, as $\Delta \to 0$,

$$RV_{t+1}(\Delta) \to \int_t^{t+1} \sigma^2(s)\, ds + \sum_{t<s\leq t+1} \kappa^2(s) \tag{2}$$

The right hand side of the equation is the IV from the stochastic volatility formulation, with a diffusion process σ and with discrete jumps of size κ. The IV converges to the increment of the diffusion process as the sampling frequency tends to infinity. I denote RV the squared root of the realized variance expressed in eq.(1) and eq.(2). In theory, the ideal frequency for the IV estimation through RV should be the tick or some sub-tick period to keep evenly spread measurements.[3] In practice however, because of market microstructure noise, the "optimal" sampling (and the most common choice) is the five-minute intervals (see, e.g., Liu, Patton, and Sheppard, 2015). This new proxy for volatility vastly improves volatility modeling. These improvements include jumps (see, e.g., Andersen, Benzoni, and Lund, 2002 and Andersen et al., 2007), realized bi-power variation (Barndorff-Nielsen and Shephard, 2004), realized semi-variances, (Barndorff-Nielsen, Kinnebrock, and Shephard, 2008) or realized covariance, allowing for the estimations of more stable betas in the CAPM or APT frameworks (Andersen, Bollerslev, Diebold, and Wu, 2006). Second, the inclusion of

[2]See also Comte and Renault (1998) and Meddahi (2002).

[3]Which is required e.g., in the presence of Epps effects (Epps, 1979) to correct for non-synchronous observations.

this parameter-free measure enhance the out of sample volatility forecasting, over (G)ARCH and SV models. With Mincer-Zarnowitz type regressions (Mincer and Zarnowitz, 1969), the empirical research shows that a VAR model using lagged-RV performs overall better than advanced GARCH models to forecast n-days ahead daily squared returns (Andersen et al., 2003). In addition, the replacement of the dependent variable, the daily volatility, by its smoother intraday RV reciprocal, vastly improves the R^2 of all forecast models (see, e.g., Andersen and Bollerslev, 1998b and Andersen et al., 2005).

Since the publication of these results, the literature considerably expanded the RV based models. In particular, the Heterogeneous AutoRegressive RV (HAR-RV, Corsi, 2009), uses past RV sampled over different periods as predictors. In a similar fashion, Engle (2002b) and Hansen and Lunde (2011) augment the GARCH with an exogenous RV variable. Closely, Shephard and Sheppard (2010) replace the GARCH AR term with its realized counterpart in the High-frEquency-bAsed VolatilitY (HEAVY) model. Finally, Ghysels, Santa-Clara, and Valkanov (2004) use a Mixed Data Sampling (MIDAS) model that averages the RV computed with the same frequency, but at different time.

2.2. Common volatility property of asset returns

Unconditionally, asset returns sampled at daily frequency are leptokurtic, display a negative skewness and have a standard deviation that greatly dominates their mean.[4] Moreover, while returns are almost serially uncorrelated, the squared or absolute returns are highly predictable and clustered over time. Asset returns' volatility also follow mean reverting processes. The heavy tail property is considered e.g., in GARCH models by Nelson (1991) with alternative distributions such as the Generalized Error Distribution. Again, beyond the high kurtosis, the Extreme Value Theory helps to model extreme events such as the 1987 US stock price drop, using alternative classes of distributions. Also, the Fractional Integrated GARCH of Baillie, Bollerslev, and Mikkelsen (1996) and the HAR-RV of Corsi (2009), account for the important long-term persistence of volatility. In addition, almost all assets (and stocks in particular) experience an asymmetric volatility reaction depending on the sign of the returns. The causes for this phenomenon also named "leverage effect" (Black, 1976) is controversial (see, e.g., Engle, 2004 and Bollerslev, Litvinova, and Tauchen, 2006). Asymmetric models take the effect into account, in particular the Exponential-GARCH of Nelson (1991) and the GJR-GARCH of Glosten, Jagannathan, and Runkle (1993).

Next, the intraday volatility of assets displays strong repeating diurnal patterns because of microstructure and volume effects (Andersen and Bollerslev, 1997). Other peculiarities

[4] For instance, the daily returns on the SP-500 during the period from 1990 to 2008 displays a standard deviation more than 40 times higher than its mean, a kurtosis of eight and a skewness of -0.26.

and tackling research include the following non-exhaustive list. A GARCH-X extended with cross-sectional volatility (Hwang and Satchell, 2005), the inclusion of Markov Switching regimes (Hamilton, 1989) in GARCH such as the MS-GARCH of Cai (1994), or the volume and news arrival effects, considered in the Mixture of Distribution Hypothesis of Clark (1973) and based on SV models.

2.3. Commodity futures volatility

Despite many commonalities between commodity markets and other asset markets, some stylized facts of commodity futures differ. A first noticeable discrepancy is the inverse asymmetric reactions between commodity futures price and volatility or "inverse leverage effect", arising from shocks on inventories. In the theory of storage, Working (1933), Kaldor (1939) and Brennan (1958) predict this behavior, arising from the non-negativity constraints of inventories. When the resources are scarce, the market becomes inelastic, and a decrease in one unit of inventory leads to a dramatic price upward revision. This effect is documented in Ng and Pirrong (1994), Carpantier (2010), Carpantier and Dufays (2012) and Carpantier and Samkharadze (2012). A second, and related, discrepancy with respect to other assets comes from the distribution properties: commodities have a positive skewness, and contrary to stock returns, this skewness also shows up at the individual level (see, e.g., Gorton and Rouwenhorst, 2006).

To my knowledge, the first economic variable of volatility in (storable[5]) commodity futures markets comes from the theory of storage and implies an inverse, convex relationship between inventories and volatility. More recent versions of the theory of storage in equilibrium (e.g., Deaton and Laroque, 1992) also predict this relationship, which is confirmed empirically by Fama and French (1988), Ng and Pirrong (1994), Geman and Nguyen (2005), Geman and Ohana (2009) and Carpantier and Samkharadze (2012) among many others. Interestingly, Kogan, Livdan, and Yaron (2009) extend the prediction to a non-monotonic convex relationship between volatility and inventories, because of investment constraints. They confirm the existence of this "v-shape" function as well as Haugom et al. (2014). This implies that both low and high inventory states lead to high volatility.

The second economic theory of volatility in commodity futures, more controversial, is the Samuelson (1965, 1976) hypothesis, which predicts an inverse relationship between time to maturity and volatility. Despite many empirical tests, the Samuelson effect is still uncertain. On the one hand, Rutledge (1976) and Grammatikos and Saunders (1986) reject the hypothesis, on the other hand Milonas (1986) and Galloway and Kolb (1996) find support

[5]The research shows that intangible commodities like electricity or those whose exchange value is higher than their use value such as gold or silver behave more like the rest of financial assets.

in all commodities, but not in financial futures. In addition, Bessembinder, Coughenour, Seguin, and Monroe-Smoller (1996) develop a model in support of the Samuelson hypothesis, if the spot price has negative covariance with the slope of the term structure. This implies a temporary price change, which is more likely to occur in real assets than in financial assets. Indeed, the empirical tests strongly reject the Samuelson hypothesis, with the futures on NIKKEI (Chen, Duan, and Hung, 2000) and with the SP-500 (Moosa and Bollen, 2001), whereas Bessembinder et al. (1996) find empirical evidences in the commodity futures markets.

The third hypothesis (see, Anderson and Danthine, 1983), does not contradict Samuelson and links volatility to the uncertainty resolution ought to occur seasonally, for instance at the end of a crop when the supply is publicly known. Anderson (1985), Khoury and Yourougou (1993) and Galloway and Kolb (1996) find a seasonal volatility effect in parallel with support to the Samuelson hypothesis. The literature tackles the uncertainty resolution with seasonal effects since it is a convenient proxy for both supply and demand patterns in commodities. For instance, the natural gas term structure has a strong seasonal component due to the demand rising every winter in the North American market, whereas the agricultural products have a supply volatility component arising for instance from the US Department of Agriculture figures during the crop season. Other uncertainty resolution variables might be at play, however.

3. Methodology and hypotheses

3.1. Data

3.1.1. Data collection

From the Barchart API,[6] I download five minutes continuous prices at close[7] for the first two m consecutive maturities of commodity futures contracts from May 6, 2008 until January 18, 2019. I choose a cross-section of $C = 9$ contracts evenly spread in three different subgroups: i) energy, ii) agriculture and metals.[8]. I display the specificities of the contracts in Table A. Contrary to daily computations of futures' price change that must correct for the regular contract expirations, intraday volatility computation implies no need to roll the position from the nearest (hereafter nearby) to the second nearest (hereafter first deferred)

[6] https://www.barchart.com/

[7] Despite their higher frequency, intraday data also have "open", "high", "low", "close" and "trading volume" information, as with daily data.

[8] Energy: crude oil, natural gas and heating oil. Agriculture: wheat, corn and soybean. Metals: copper, gold and silver. Softs: sugar, coffee and cocoa.

contract, the day prior to maturity. However, following the standard approach in commodity futures research I roll the contracts onto the first deferred contract before the actual maturity.[9] I choose the roll date as the settlement time of the 10^{th} business day before maturity. The data includes a time-stamp and the maturity date of each contract. I report descriptive statistics at daily and five-minute sampling frequency in the Panel A of Table 1.

[Insert Table 1 here]

3.1.2. Data preparation

I compute five-minute log price changes for each futures contract available as $r^m_{c,t,j} = f^m_{c,t,j+1} - f^m_{c,t,j}$. The subscripts c, t and j stand for commodity, day, and time[10] of the observation, respectively. The superscript m indicates the maturity of the contract. I compute the RV as $RV_t = \sqrt{\sum_{j=1}^{1/\Delta} r^2_{t,\Delta\times j}}$ with $1/\Delta$ equal to the number of five minutes observations available given the market open hours of each contract. I present summary statistics for daily RV measures sampled at 5-minute intervals in the Panel B of Table 1. A large strand of literature on theory of storage highlights the relation between the slope of the term structure and inventories. Because this relationship finds strong empirical support (see, e.g., Gorton, Hayashi, and Rouwenhorst, 2012), I use this proxy for inventories. I normalize the slope of the term structure to take in account the maturity gap differences across contracts. For instance, the raw sugar contract NY#11 has five maturities per year on March, May, July, October and December, while the Brent crude oil contract has one per month. With the normalization, I correct this maturity mismatch. Hence, the slope of the term structure is defined as $SL^m_{c,t} = \frac{f^{m+1}_{c,t} - f^m_{c,t}}{\delta(m)} \times 250$ where $\delta(m)$ indicates the time difference in days between the two consecutive maturities. I annualize the variable for ease of interpretation. To test for the two alternative hypotheses of monotonic decreasing relationship (theory of storage) and the "v-shape" (Kogan et al., 2009), I use two different specifications. The first is just the continuous slope; the second is the measure of Haugom et al. (2014) who halve the slope measure, conditional on its sign. Formally it is, $SL^+_{c,t} = max\left(SL_{c,t}, 0\right)$ and $SL^-_{c,t} = min\left(SL_{c,t}, 0\right)$.

I compute the time to maturity in days for each daily observation, crossing the time stamp with the contract maturity information available in the full ticker. I compute the time to maturity both in calendar and business days for robustness and find virtually no differences in the results. However, my first assumption is that the latent information exists

[9] Previous research justify this procedure by the possible market squeezes and thinly traded contracts during the period immediately preceding the maturity.

[10] In terms of amount of five-minute periods elapsed for each day, i.e., $j = 1, 2, \ldots, 288$.

even when futures are not traded. Hence, I articulate the variable around calendar days. Similarly, to capture the seasonal effects, in zero-intercept regressions, I use 12 dummies variable coded one during a specific month and zero otherwise.

3.2. Methodology

3.2.1. Preliminary tests of commodity volatility factors

I begin with a univariate test of the specific commodity volatility factors alone as explanatory variables of the realized volatility.

$$RV_{c,t} = \alpha_{1,c} \times SL_{c,t-1} + \alpha_{2,c} \times DTM_{c,t-1} + \sum_{i=3}^{i=14} \alpha_{i,c} \times DM_{i,t-1} + \epsilon_{c,t}, \tag{3}$$

Where $SL_{c,t-1}$ is the slope, $DTM_{c,t-1}$ stands for the days to maturity, $DM_{i,t-1}$ are the 12 dummy variables set at one for each month. To disentangle the potential asymetric effect of the slope, I test the measure of Haugom et al. (2014) in a second setting,

$$RV_{c,t} = \alpha^{+}_{1,c} \times SL^{+}_{c,t-1} + \alpha^{-}_{1,c} \times SL^{-}_{c,t-1} + \alpha_{2,c} \times DTM_{c,t-1} + \sum_{i=3}^{i=14} \alpha_{i,c} \times DM_{i,t-1} + \epsilon_{c,t}, \tag{4}$$

where $SL^{+}_{c,t-1}$ and $SL^{-}_{c,t-1}$ stand for the conditional term structure. I report the results in Table 2. In Panel A, I show the results for the eq. 3 and in Panel B for the eq. 4. In both regressions, there is a clear evidence for a positive relationship between the days to maturity and volatility, which goes against the Samuelson (Samuelson, 1965, 1976) hypothesis. The seasonal pattern is very strong for each of the commodities, with at least one dummy parameter positively significant except for the Crude Oil contract in the second setting. The following results detail this pattern further. Finally, in the first setting, the slope coefficient appears significant at the 1% level in six of the nine contracts, but with varying signs. In the second setting however, the patterns are perfectly consistent, as Haugom et al. (2014) show, what matters is the magnitude of the slope and not its sign. The backwardation (contango) stands for a negative (positive) slope and hence, all coefficients are negative (positive) and significant. For the remainder of the study, I therefore choose to keep this conditional slope setting.

[Insert Table 2 here]

3.2.2. *Univariate tests: HAR-RV and commodity factors*

I start with the following extended HAR-RV model,

$$RV_{c,t} = \beta_{1,c} \times RV^{D}_{c,t-1} + \beta_{2,c} \times RV^{W}_{c,t-1} + \beta_{3,c} \times RV^{M}_{c,t-1} + \boldsymbol{\beta_{v,c}} \times \boldsymbol{X_{c,t-1}} + \epsilon_{c,t}, \tag{5}$$

where the superscripts, D, W and M stands for the estimation timespan of the RV of a day, a week and a month, respectively. $\boldsymbol{X_{c,t-1}}$ represents the volatility determinant vectors, slope, days to maturity and seasonality dummies, stacked into a matrix; and $\boldsymbol{\beta_{v,c}}$ is the associated vector of coefficients with v from 4 to 15.

3.2.3. *Multivariate test: VAR-HAR-RV and commodity factors*

Subsequently I test a multivariate model as the inclusion of the lagged RV of other commodity contracts helps to decompose the commodity futures volatility. In fact, Andersen et al. (2003) and Andersen et al. (2005) use a VAR-RV model and reach enhanced results. In this research, I assume that a VAR-RV improves the forecast power due to the financialization-induced connectedness.[11] Since I have both common and idiosyncratic variables in the system, I jointly estimate the nine equations with Seemingly Unrelated Regressions (SUR) procedure,

$$RV_{c,t} = \gamma_{1,c} \times RV^{agriculture}_{c,t-1} + \gamma_{2,c} \times RV^{energy}_{c,t-1} + \gamma_{3,c} \times RV^{metal}_{c,t-1} + \boldsymbol{\gamma_{v,c}} \times \boldsymbol{Z_{c,t-1}} + \epsilon_{c,t}, \tag{6}$$

where $RV^{sector}_{c,t-1}$ stands for the realized volatility average over the three sectors, agriculture, energy and metal. $\boldsymbol{Z_{c,t-1}}$ includes all explanatory variables of eq. 5 and v goes from 4 to 18. In this formulation, I estimate $RV^{sector}_{c,t-1}$ over one day but I drop the superscript D. I use sector index of commodity RV instead of a complete VAR system to limit the number of parameters to estimate and assuming that the average volatility of each sector is a valid proxy.[12].

[11] All contracts of the studies are constituents of the two most popular indices SP-GSCI and BCOM, with the exception of the copper contract traded on COMEX that is only part of BCOM. The SP-GSCI uses the copper contract traded on the LME instead.

[12] Robustness checks with the full VAR show virtually no differences. Tables are available upon request

3.3. *Out of sample tests*

I test the validity of the parameters estimated in eq.(5) and eq.(6) on one day ahead period. I use a rolling training window of 500 days, which I assume being a good trade-off between accuracy at capturing all the effects, in particular the seasonal pattern, and flexibility in adapting faster to new market volatility features. I run the regression for each day of the eight remaining years and collect the predicted value for the next day. I then compare the accuracy of the forecast series versus a simple AR(1) RV model and a GARCH(1,1) computed on the daily returns, using actual daily RV as the dependant variable. To test the out of sample forecast power I use first a Mincer-Zarnowitz (Mincer and Zarnowitz, 1969) type regressions, similar to e.g., Andersen and Bollerslev (1998b),

$$RV_{c,t} = \psi_0 + \psi_1 \times \widehat{RV_{c,t}} + \epsilon_{c,t}, \tag{7}$$

where $\widehat{RV_{c,t}}$ is the estimate of the RV given the parameters of eq.(5) and eq.(6) applied on the training sample. If the out of sample prediction is not biased, I expect the parameters ψ_0 and ψ_1 to be not significantly different from zero and one, respectively. I use a F-statistic to test that these two parameters are jointly equal to zero and one. Andersen and Bollerslev (1998b) explain that the R^2 is typically low in forecasts using daily squared returns as dependent variable, whereas the use of intraday RV considerably improves the explanatory power of all models. Second, and jointly with the Mincer-Zarnowitz regression, I compute the mean squared error (MSE) and mean absolute error (MAE) of each of the models tested. In a second step, I directly compare the accuracy of the three forecasting models, using the modified Diebold-Mariano Diebold and Mariano (1995) test of Harvey, Leybourne, and Newbold (1997).

4. Empirical results

In Table 3 I display the results of the commodity volatility factors augmented HAR-RV model and in Appendix B I display the complete results including all coefficients for all month dummies.

[Insert Table 3 here]

In Table 4 I report the results of the multivariate HAR-RV model and in appendix C I display the complete results including all coefficients for all month dummies.

[Insert Table 4 here]
[Insert Figure 1 here]

In Table 5 I report the out of sample Tests. In the Panel A I report the absolute performance of the univariate, multivariate and of a GARCH(1,1) models. The measures are a F-statistic of Mincer-Zarnowitz regressions, the MAE and MSE. In Panel B, I report the F-statistics and p-values of the modified Diebold-Mariano test of relative performance in the context of absolute and squared errors.

[Insert Table 5 here]

Finally, in Figure 1, I plot the multivariate HAR-RV model out of sample forecast series versus the actual RV series for each of the nine commodities.

5. Risk management application

One way to compute the optimal hedge ratio (OHR), that is, the amount of futures contracts relative to a position in the underlying is (see, Ederington, 1979),

$$OHR_t = \frac{\mathrm{Cov}\left(r^s_{c,t}, r^f_{c,t}\right)}{\mathrm{Var}\left(r^f_{c,t}\right)} \tag{8}$$

This measure is also the slope parameter in a regression of the spot on the futures price, estimated for instance on a prior window of observations. Baillie and Myers (1991) propose a bivariate GARCH (BGARCH) to improve the OHR estimation and to obtain parameters that follow more closely the time-varying properties of the distribution of returns. One main issue with this procedure is that the estimation of multivariate GARCH models is complex (see, e.g., Bollerslev, 1990; Engle, 2002a and Bauwens, Laurent, and Rombouts, 2006).

I propose an alternative estimation of the OHR, inspired by the realized beta of Andersen et al. (2006) in the CAPM framework. I define the realized OHR (ROHR) as,

$$ROHR_t = \frac{\mathrm{RCov}\left(r^s_{c,t}, r^f_{c,t}\right)}{\mathrm{RVar}\left(r^f_{c,t}\right)}, \tag{9}$$

where RCov and RVar stand for the realized covariance and the realized variances respectively. Moreover, because forecasts of these parameter-free measures are available, it is possible to compute them forward, using the predicted values of the eq.(5) or eq.(6) model as follow,

$$\widehat{ROHR}_t = \frac{\widehat{\mathrm{RCov}_\mathrm{t}}\left(r^s_{c,t}, r^f_{c,t}\right)}{\widehat{\mathrm{RVar}_\mathrm{t}}\left(r^f_{c,t}\right)}, \tag{10}$$

However in preliminary tests I find that despite very high in-sample explanatory power for RCov factors (the HAR-RCov and multivariate HAR-RCov), the out-of-sample power for RCov is extremely low.[13]. Hence I choose a restricted but more efficient model, relating directly the ROHR with the RV factors. The model becomes,

$$ROHR_{c,t} = \theta_{1,c} \times RV^{agriculture}_{c,t-1} + \theta_{2,c} \times RV^{energy}_{c,t-1} + \theta_{3,c} \times RV^{metal}_{c,t-1} + \boldsymbol{\theta_{v,c}} \times \boldsymbol{Z_{c,t-1}} + \epsilon_{c,t}, \quad (11)$$

where the independent variables are the same as in eq. (6).

I display the in-sample results in 6 and the complete results with each dummy variables in the Appendix D. I display the out-of-sample test in 7 and I plot in Figure 2 the predicted and actual ROHR.

[Insert Table 6 here]
[Insert Figure 2 here]

6. Conclusion

In combining traditional economic theories of commodity futures market volatility and recent advances in the estimation of the true latent volatility, this research improves commodity volatility modeling. I use two models of RV. One uses the long-term properties of volatility common to all asset classes. The second builds on the recent evidences of commodity connectedness and considers the volatility information shared in the cross-section of commodity contracts. These new volatility-modeling settings for commodities have immediate practical applications. In particular, I introduce one utilization of RV and RV models in the context of optimal hedge ratios estimation, which is of importance for both financial and industrial commodity sectors.

[13] The tables for in- and out-of-sample modeling are available upon request

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Table 1: **descriptive statistics of daily returns and realized volatility on futures contracts**

In Panel A, I report the annualized mean, standard deviation, skewness, kurtosis of the daily returns on the nearby futures contracts of the nine selected commodities. I also report the proportion of days during which the corresponding contract was in contango, the AR(1) coefficient of squared returns and its t-statistic. In Panel B, I report the mean of the daily realized-volatility with five-minutes sampling for the nearby and first deferred contracts as well as the mean daily realized covariance. I also report the AR(1) coefficient and its t-statistic for the nearby contract realized volatility. The period is from May 2008 to January 2019.

Panel A: daily returns of nearby contracts

Ticker (contract)	Mean %	σ %	Skewness	Kurtosis	Contango %	AR(1) %	t-statistic
C (corn)	−6.91	27.86	0.00	0.02	14	2.63	1.36
CL (WTI crude oil)	−20.96	34.99	0.00	0.02	14	28.76	15.56
GC (gold)	4.58	17.00	−0.01	0.02	12	18.21	9.41
HG (copper)	−1.35	25.01	0.00	0.02	35	30.93	16.79
HO (heating oil)	−5.24	29.48	0.00	0.02	22	17.63	9.28
NG (natural gas)	−36.53	42.44	0.01	0.01	12	18.18	9.58
S (soybeans)	9.52	22.66	−0.01	0.02	43	3.65	1.89
SI (silver)	3.77	30.60	−0.02	0.02	8	18.40	9.48
W (wheat)	−12.93	30.47	0.00	0.01	3	8.88	4.59

Panel B: daily realized-volatility

Ticker (Contract)	Nearby %	First deferred %	Realized Covariance $\times 10^{-4}$	AR(1) nearby	t-statistic
C (corn)	1.73	1.65	3.12	0.42	24.02
CL (WTI crude oil)	1.99	5.09	4.99	0.81	72.13
GC (gold)	1.01	1.00	1.15	0.60	38.32
HG (copper)	1.50	1.49	2.43	0.64	42.52
HO (heating oil)	1.79	1.74	3.34	0.78	64.31
NG (natural gas)	2.72	2.54	7.56	0.48	27.97
S (soybeans)	1.41	1.36	2.08	0.43	24.86
SI (silver)	1.77	1.72	2.99	0.48	27.80
W (wheat)	1.94	1.85	3.67	0.48	27.89

Table 2: **Univariate regressions of commodity volatility factors on daily RV**

In Panel A, I report the coefficients of the nine univariate regressions of daily RV in date t sampled at five minutes interval on commodity volatility factors in date $t-1$ that are, (i) the log term structure, i.e., the log price difference between the nearby and first deferred contracts, scaled by the number of days between their two maturities; (ii) the time to maturity in days and (iii) 12 dummy vectors set to one for each different month from January to December. The model is eq. 3 In Panel B, I repeat the test with the conditional term structure of Haugom et al. (2014). The model is, eq. 4 and $SL^{+}_{c,t} = max\,(SL_{c,t}, 0)$ and $SL^{-}_{c,t} = min\,(SL_{c,t}, 0)$, with $SL_{c,t} = \frac{f^{m+1}_{c,t} - f^{m}_{c,t}}{\delta(m)} \times 250$. I indicate by an abbreviation the month of highest positive significance, and leave empty when no month is significantly positive. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
	$RV_{c,t}$								
	Panel A: continuous term structure								
term structure	0.23	11.82***	−12.79***	8.84***	5.97***	2.56***	0.26	−1.12	6.14***
	(0.59)	(37.76)	(−6.06)	(6.32)	(11.00)	(11.05)	(1.05)	(−0.58)	(8.43)
days to maturity $\times 10^{-7}$	37.60***	14.54***	24.16***	37.50***	20.08***	29.09***	26.09***	42.36***	27.01***
	(23.56)	(8.88)	(25.20)	(24.78)	(12.85)	(12.92)	(20.80)	(22.59)	(20.74)
seasonality (+)	Jul***	Sep***	Sep***	Oct***	Feb***	Jan***	Aug***	Nov***	Aug***
Observations	2,653	2,657	2,639	2,648	2,657	2,657	2,653	2,580	2,653
Adjusted R^2	0.80	0.85	0.79	0.77	0.82	0.83	0.81	0.75	0.88
	Panel B: conditional term structure								
contango	6.09***	12.34***	31.34***	36.95***	17.82***	4.31***	12.79***	10.54***	6.60***
	(4.57)	(37.53)	(9.68)	(12.74)	(22.51)	(17.48)	(6.02)	(4.25)	(8.71)
backwardation	−1.29**	3.38*	−47.00***	−6.85***	−5.54***	−5.51***	−0.32	−26.51***	−13.64
	(−2.52)	(1.95)	(−16.72)	(−3.46)	(−7.12)	(−10.04)	(−1.21)	(−6.75)	(−1.51)
days to maturity $\times 10^{-7}$	37.72***	15.86***	24.37***	35.76***	15.84***	28.73***	28.55***	40.98***	27.05***
	(23.72)	(9.60)	(26.82)	(24.03)	(10.72)	(13.37)	(21.74)	(21.97)	(20.79)
seasonality (+)	Jul***	Nov***	Oct***	Oct***	Feb***	Jan***	Aug***	Nov***	Aug***
Observations	2,653	2,657	2,639	2,648	2,657	2,657	2,653	2,580	2,653
Adjusted R^2	0.80	0.85	0.81	0.78	0.84	0.85	0.81	0.76	0.88

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3: **Univariate regressions of daily RV on HAR-RV extended with commodity volatility factors**

I report the coefficients of the nine univariate regressions of daily RV in date t sampled at five minutes interval on commodity volatility factors in date $t-1$ that are, (i) the halved log term structure, i.e., the log price difference between the nearby and first deferred contracts, scaled by the number of days between their two maturities differentiated when in backwardation or in contango; (ii) the days to maturity and (iii) 12 dummy vectors set to one for each different month from January to December. The model is eq. 5. I indicate by an abbreviation the month of highest positive significance, and leave empty when no month is significantly positive. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
					$RV_{c,t}$				
$RV^{D}_{c,t-1}$	0.22***	0.29***	0.14***	0.08***	0.27***	0.10***	0.14***	0.13***	0.13***
	(9.77)	(12.46)	(6.38)	(3.56)	(11.91)	(4.15)	(6.15)	(5.66)	(5.79)
$RV^{W}_{c,t-1}$	0.07**	0.31***	0.17***	0.39***	0.23***	0.24***	0.08**	0.24***	0.28***
	(2.13)	(8.50)	(4.12)	(8.95)	(6.71)	(6.55)	(2.30)	(5.77)	(7.26)
$RV^{M}_{c,t-1}$	0.06	0.30***	0.46***	0.38***	0.38***	0.30***	0.39***	0.32***	0.28***
	(1.46)	(8.28)	(11.51)	(9.26)	(12.11)	(7.40)	(9.43)	(7.26)	(6.28)
contango	4.67***	0.98***	14.26***	5.43**	2.89***	1.18***	6.76***	5.22**	1.34*
	(3.61)	(2.98)	(5.25)	(2.33)	(5.19)	(4.45)	(3.40)	(2.38)	(1.83)
backwardation	−0.84*	−0.62	−32.81***	−1.72	−1.88***	−2.80***	0.20	−17.04***	1.86
	(−1.69)	(−0.53)	(−13.87)	(−1.14)	(−3.77)	(−5.47)	(0.81)	(−4.90)	(0.23)
days to maturity $\times 10^{-7}$	24.48***	1.45	4.60***	4.96***	1.24	9.50***	11.03***	10.97***	7.78***
	(11.86)	(1.26)	(4.86)	(3.68)	(1.27)	(4.42)	(7.42)	(5.28)	(5.26)
seasonality (+)	Jun***	Nov***	Feb***	Oct***	Nov***	Jan***	Jul***	Nov***	Jun***
Observations	2,649	2,657	2,623	2,639	2,657	2,657	2,649	2,528	2,649
Adjusted R^2	0.81	0.93	0.87	0.87	0.93	0.87	0.84	0.81	0.90

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4: **Multivariate regressions of daily RV on HAR-RV extended with commodity volatility factors**

I report the coefficients of the nine multivariate regressions of daily RV in date t sampled at five minutes interval on commodity volatility factors in date $t-1$ that are, (i) the halved log term structure, i.e., the log price difference between the nearby and first deferred contracts, scaled by the number of days between their two maturities differentiated when in backwardation or in contango; (ii) the days to maturity and (iii) 12 dummy vectors set to one for each different month from January to December. The model is eq. 6. I indicate by an abbreviation the month of highest positive significance, and leave empty when no month is significantly positive. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
					$RV_{c,t}$				
$RV_{t-1}^{D,agriculture}$	0.29***	0.03*	−0.01	0.01	0.02	−0.01	0.05*	0.04	0.01
	(5.67)	(1.84)	(−0.95)	(0.73)	(1.00)	(−0.42)	(1.71)	(1.28)	(0.20)
$RV_{t-1}^{D,energy}$	0.04*	−0.01	0.01	0.01	−0.01	0.10**	0.02	−0.05**	0.01
	(1.88)	(−0.34)	(0.75)	(0.46)	(−0.60)	(2.37)	(1.47)	(−2.18)	(0.56)
$RV_{t-1}^{D,metal}$	0.12***	0.10***	0.06***	0.11***	0.07***	0.02	0.08***	0.28***	0.06***
	(4.66)	(5.19)	(3.43)	(4.26)	(4.65)	(0.74)	(4.11)	(4.74)	(3.20)
$RV_{c,t-1}^{D}$	0.01	0.26***	0.09***	0.03	0.26***	0.04	0.09***	−0.00	0.12***
	(0.17)	(9.93)	(3.56)	(0.95)	(10.05)	(1.40)	(2.86)	(−0.08)	(4.11)
$RV_{c,t-1}^{W}$	0.05	0.31***	0.16***	0.37***	0.22***	0.24***	0.07**	0.22***	0.28***
	(1.53)	(8.50)	(4.09)	(8.43)	(6.38)	(6.49)	(2.05)	(5.33)	(7.09)
$RV_{c,t-1}^{M}$	0.01	0.28***	0.43***	0.38***	0.38***	0.28***	0.36***	0.27***	0.25***
	(0.16)	(7.73)	(10.60)	(9.26)	(11.98)	(6.80)	(8.55)	(5.98)	(5.68)
contango	2.13	1.40***	13.81***	5.63**	3.38***	1.33***	4.78**	4.34**	1.48**
	(1.61)	(4.19)	(5.08)	(2.44)	(5.96)	(4.90)	(2.37)	(1.97)	(2.02)
backwardation	−0.92*	−0.07	−32.99***	−1.11	−2.06***	−3.01***	0.07	−16.00***	2.15
	(−1.86)	(−0.06)	(−13.97)	(−0.74)	(−4.13)	(−5.82)	(0.28)	(−4.57)	(0.26)
days to maturity $\times 10^{-7}$	20.27***	−2.29*	4.33***	3.13**	−1.27	8.37***	7.66***	9.95***	6.16***
	(9.63)	(−1.72)	(4.43)	(2.21)	(−1.13)	(3.41)	(4.73)	(4.68)	(3.95)
seasonality (+)	Jul***	Nov***	Aug***	Oct*	Jan***	Jan***	Jul***	Nov***	Jun***
Observations	2,649	2,653	2,619	2,635	2,653	2,653	2,649	2,524	2,649
Adjusted R^2	0.82	0.93	0.87	0.87	0.94	0.87	0.84	0.81	0.90

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: **Out of sample forecasting accuracy and model comparison**

In Panel A, I report the F-statistic of a Mincer-Zarnowitz regression where the dependent variable is the actual daily realized volatility and the independent variable are the one day ahead predicted value for volatility from the three models: (i) a GARCH(1,1) on the daily return, (ii) The extended univariate HAR-RV, and (iii) the multivariate HAR-RV. The F-statistic is on the joint significance that the intercept and the coefficient of the regression are of zero and one, respectively. I also report the mean absolute error (MAE) and mean squared error (MSE) of the models. In panel B, I report the statistics of the modified Diebold-Mariano Diebold and Mariano (1995) test of Harvey et al. (1997). The first (second) setting tests whether the univariate (multivariate) extended HAR-RV performs better than the variance forecast of the GARCH(1,1). For each setting, I report the F-statistic and its p-value of the test for a MAE (1) and MSE (2).

	C	CL	GC	HG	HO	NG	S	SI	W
	Panel A: Mincer-Zarnowitz F-test, MSE and MAE								
GARCH(1,1)									
MZ F-test	260.83	48.64	65.56	41.70	61.01	34.29	335.54	29.09	49.78
MAE %	0.52	0.40	0.29	0.42	0.34	0.67	0.43	0.59	0.47
MSE $\times 10^{-4}$	0.86	0.32	0.16	0.34	0.29	1.34	0.43	0.71	0.47
Ex-HAR-RV									
MZ F-test	19.57	6.85	16.57	13.35	14.32	7.62	20.16	6.08	7.87
MAE %	0.40	0.30	0.26	0.37	0.27	0.54	0.32	0.52	0.39
MSE $\times 10^{-4}$	0.41	0.21	0.14	0.27	0.18	0.86	0.28	0.58	0.39
Multi-HAR-RV									
MZ F-test	26.86	5.04	10.01	15.12	12.46	7.92	28.11	8.90	8.10
MAE %	0.39	0.30	0.26	0.37	0.27	0.54	0.32	0.53	0.40
MSE $\times 10^{-4}$	0.38	0.21	0.14	0.25	0.18	0.86	0.27	0.59	0.40
	Panel B: Modified Diebold-Marino test								
Ex-HAR-RV > GARCH(1,1)									
HLN-DM(1) F-test	6.89	7.85	2.42	3.03	4.10	6.26	−1.39	0.80	3.39
	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.92]	[0.21]	[0.00]
HLN-DM(2) F-test	3.71	5.74	1.98	1.99	2.90	4.73	0.06	0.45	−0.04
	[0.00]	[0.00]	[0.02]	[0.02]	[0.00]	[0.00]	[0.48]	[0.33]	[0.52]
Multi-HAR-RV > GARCH(1,1)									
HLN-DM(1) F-test	8.32	7.80	2.20	3.01	4.34	5.98	−1.01	0.28	3.17
	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.84]	[0.39]	[0.00]
HLN-DM(2) F-test	4.07	5.66	1.75	1.90	3.02	4.60	0.11	0.28	−0.21
	[0.00]	[0.00]	[0.04]	[0.03]	[0.00]	[0.00]	[0.46]	[0.39]	[0.58]

Table 6: **Multivariate realized beta (optimal hedging ratio) modeling with RV factors**

I report the coefficients of the nine multivariate SUR estimations of daily realized beta (or optimal hedging ratio) in date t, computed with the five-minute sampling realized covariance of the nearby and first deferred contracts, divided by the RV of the nearby contract. The independent variables are the three sector index factors of RV, the HAR-RV variables and the commodity volatility factors in date $t-1$. The model is eq. 11. I indicate by an abbreviation the month of highest positive significance, and leave empty when no month is significantly positive. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
					$ROHR_{c,t}$				
$RV^{D,agriculture}_{t-1}$	-29.66^{***}	4.97^{***}	7.67	-1.92	-4.64	-1.01	1.51	-1.44	-0.37
	(-2.77)	(2.89)	(1.25)	(-0.22)	(-1.48)	(-0.30)	(0.32)	(-0.11)	(-0.09)
$RV^{D,energy}_{t-1}$	-8.82^{**}	-1.23	-8.03	-19.26^{**}	-3.36	-16.62^{***}	1.11	-19.22^{*}	-6.45^{*}
	(-2.44)	(-0.69)	(-1.53)	(-2.45)	(-1.07)	(-3.45)	(0.33)	(-1.72)	(-1.92)
$RV^{D,metal}_{t-1}$	11.74^{*}	3.74	16.75	-26.34	-4.82	-7.87	11.55^{*}	-15.67	-4.64
	(1.88)	(1.43)	(1.17)	(-1.60)	(-1.01)	(-1.56)	(1.95)	(-0.33)	(-0.80)
$RV^{D}_{c,t-1}$	11.14^{**}	1.10	-83.62^{***}	2.78	2.61	4.45^{**}	-8.95	21.79	13.69^{**}
	(2.45)	(0.66)	(-3.18)	(0.16)	(0.55)	(2.05)	(-1.53)	(1.01)	(2.08)
$RV^{W}_{c,t-1}$	5.97	-0.14	-8.58	31.20	-12.95	4.14	53.06^{***}	-51.41^{**}	-0.58
	(1.24)	(-0.04)	(-0.20)	(0.99)	(-1.29)	(1.18)	(4.34)	(-2.23)	(-0.04)
$RV^{M}_{c,t-1}$	2.74	-1.82	85.13^{*}	19.62	28.12^{***}	-18.77^{***}	-120.73^{***}	52.71^{*}	15.39
	(0.31)	(-0.42)	(1.73)	(0.67)	(2.69)	(-3.72)	(-6.68)	(1.71)	(0.90)
contango $\times 10^{1}$	-13.82^{***}	-4.54^{***}	-84.76^{***}	-64.97^{***}	-3.84^{***}	-2.11^{***}	15.08^{***}	-74.00^{***}	-6.92^{***}
	(-6.57)	(-13.27)	(-6.47)	(-6.10)	(-3.53)	(-7.50)	(3.79)	(-7.37)	(-4.91)
backwardation $\times 10^{1}$	7.79^{***}	15.01^{***}	4.11	42.45^{***}	13.88^{***}	6.67^{***}	6.00^{***}	126.20^{***}	21.59
	(9.92)	(13.74)	(0.36)	(6.12)	(13.87)	(12.72)	(12.14)	(7.90)	(1.36)
days to maturity $\times 10^{-6}$	43.93^{***}	-11.61^{***}	-4.78	-5.54	-24.95^{***}	-4.85^{**}	-1.15	7.52	-0.16
	(15.13)	(-10.22)	(-1.13)	(-0.93)	(-12.13)	(-2.14)	(-0.40)	(0.86)	(-0.06)
seasonality (+)	Apr***	Oct***	Aug***	Oct***	Aug**	Aug***	Jun***	Oct***	Apr**
Observations	2,648	2,653	2,619	2,635	2,653	2,653	2,648	2,523	2,648
Adjusted R^2	0.97	1.00	0.94	0.87	0.99	0.98	0.98	0.73	0.98

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: **Out of sample forecasting accuracy of RBeta (ROHR) and model comparison**

In Panel A, I report the F-statistic of a Mincer-Zarnowitz regression where the dependent variable is the actual daily ROHR and the independent variable are the one day ahead ROHR predicted value from the two models of (5) and (6) (univariate and multivariate HAR respectively on the ROHR. The F-statistic is on the joint significance that the intercept and the coefficient of the regression are of zero and one, respectively. I also report the mean absolute error (MAE) and mean squared error (MSE) of the models. In panel B, I report the statistics of the modified Diebold-Mariano Diebold and Mariano (1995) test of Harvey et al. (1997). The null is that the univariate HAR does not perform better than the multivariate model. I report the F-statistic and its p-value of the test for a MAE (1) and MSE (2).

	C	CL	GC	HG	HO	NG	S	SI	W
	Panel A: Mincer-Zarnowitz F-test, MSE and MAE								
AR-RBeta									
MZ F-test	795.76	850.63	4,528.65	5,887.81	329.89	758.36	1,585.90	1,674.76	1,116.11
MAE $\times 10^2$	6.99	2.45	8.97	14.75	4.65	5.62	6.23	17.34	6.97
MSE $\times 10^3$	5.62	0.80	3.13	6.15	5.35	4.89	3.66	21.06	6.10
Multi-HAR-RBeta									
MZ F-test	63.50	63.51	17.79	62.12	104.60	71.31	34.32	22.02	57.59
MAE $\times 10^2$	9.95	3.43	14.80	22.89	6.06	6.76	8.72	27.70	8.33
MSE $\times 10^3$	19.93	2.81	39.28	86.76	6.69	11.92	16.90	172.67	17.40
	Panel B: Modified Diebold-Marino tests								
AR-RBeta > Multi-HAR-RBeta									
HLN-DM(1) F-test	21.39	25.57	44.14	43.31	21.10	18.37	22.33	31.91	16.27
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
HLN-DM(2) F-test	9.76	6.84	23.44	17.56	8.43	6.18	7.82	5.78	4.96
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Fig. 1: **Realized volatility and multivariate HAR out of sample forecast**

I plot the five-minutes sampling daily realized volatility and the out of sample forecast of the multivariate HAR model. I use a rolling window of 500 days to estimate the next day predicted volatility, using the multivariate HAR model estimated with SUR. The period is May 2008 - January 2019. The blue line is the realized and the red, the forecast series. The ordinate unit is percentage and the commodity symbol is displayed above each plot.

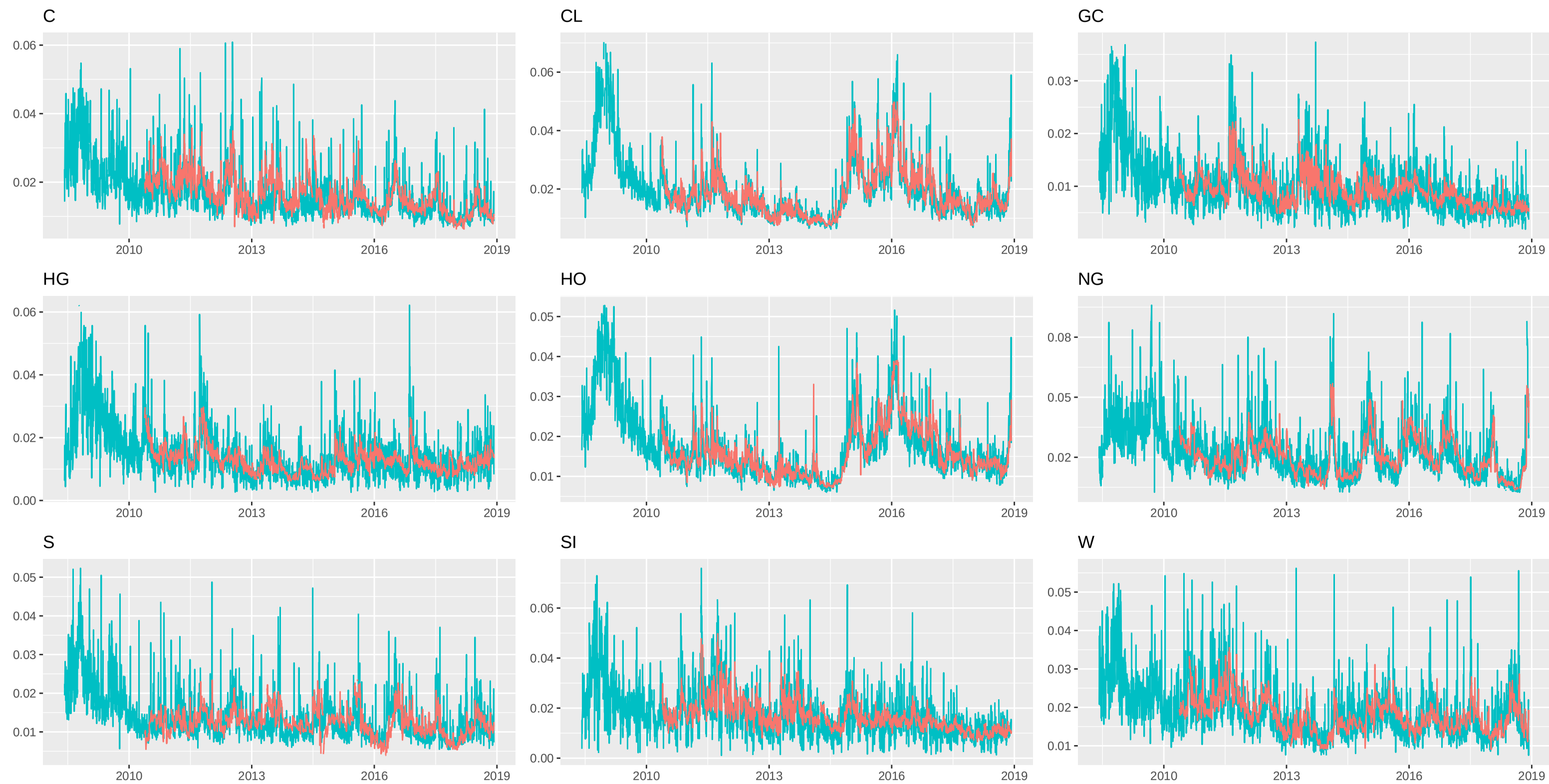
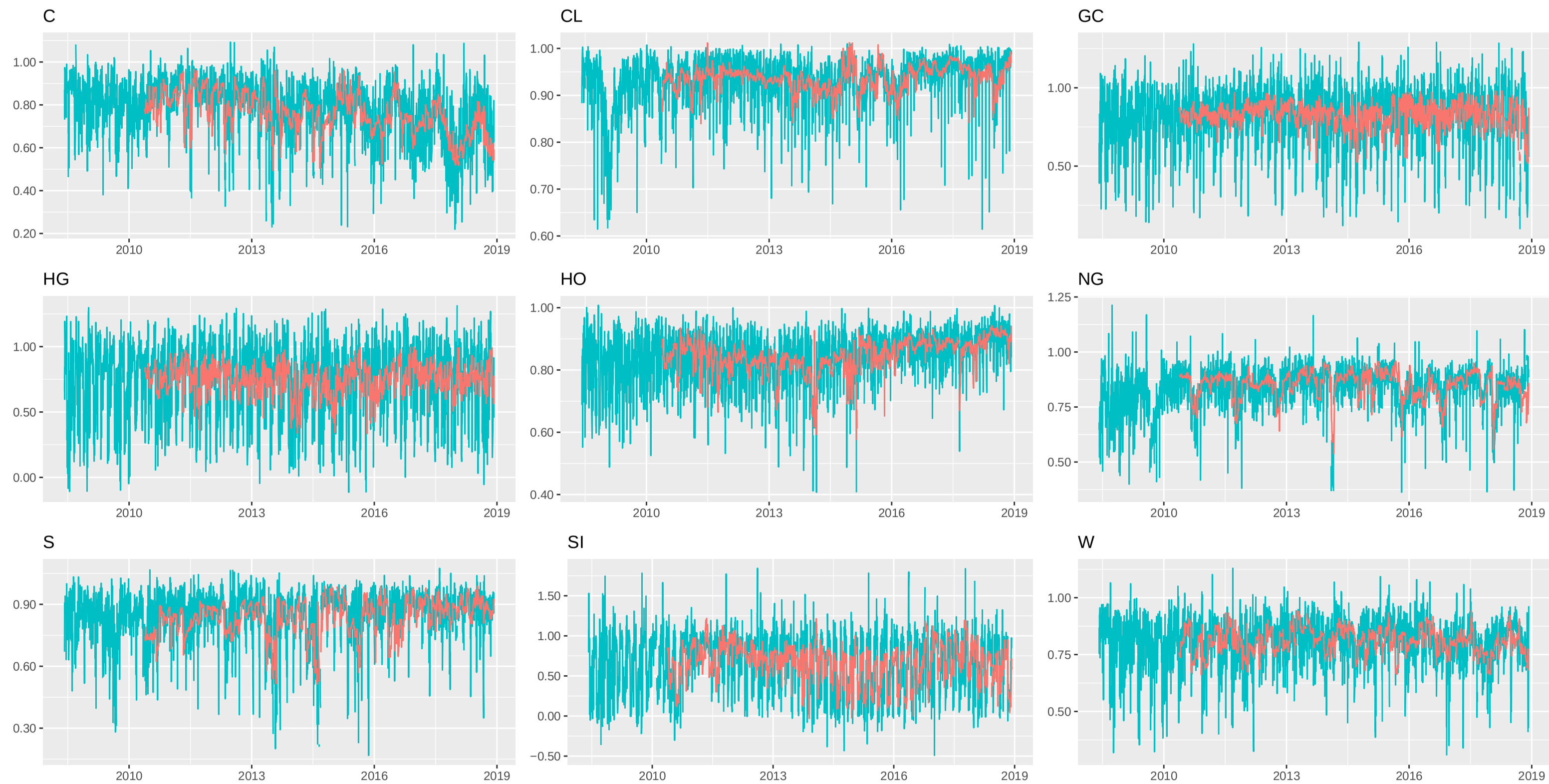


Fig. 2: **Realized beta and multivariate HAR out of sample forecast**

I plot the five-minutes sampling daily realized beta, computed with daily realized volatility and covariances and the out of sample forecast of the multivariate HAR model. I use a rolling window of 500 days to estimate the next day predicted volatility, using the multivariate HAR model estimated with SUR. The period is May 2008 - January 2019. The blue line is the realized and the red, the forecast series. The ordinate is in raw unit and the commodity symbol is displayed above each plot.



Appendix A.

Table 8: **Description of futures contracts**

I report the specifications of the futures contracts written on the nine selected commodities. The specifications include their trading venues, ticker, underlying commodities and unit. I also report their maturity months with the appropriate code letter.

Ticker	Trading venue	Underlying	Unit	Maturity
C	CBT	Corn	bu (5,000)	HKNUZ
CL	NYMEX/ICE	WTI crude oil	bbl (1,000)	FGHJKMNQUVXZ
GC	CMX	Gold	oz (100)	GJMQVZ
HG	COMEX	Copper	lb (25,000)	FGHJKMNQUVXZ
HO	NYMEX	Heating oil	gal (42,000)	FGHJKMNQUVXZ
NG	NYMEX/ICE	Natural gas	MMBtu (10,000)	FGHJKMNQUVXZ
S	CBT	Soybeans	bu (5,000)	FHKNQUX
SI	CMX	Silver	oz (5,000)	FHKNUZ
W	CBT	Chicago wheat	bu (5,000)	HKNUZ

Maturity month code: F = January, G = February, H = Mars, J = April, K = May, M = June, N = July, Q = August, U = September, V = October, X = November, Z = December.

Appendix B.

Table 9: **Univariate regressions of daily RV on HAR-RV extended with commodity volatility factors**

I report the coefficients of the nine univariate regressions of daily RV in date t sampled at five minutes interval on commodity volatility factors in date $t-1$ that are, (i) the halved log term structure, i.e., the log price difference between the nearby and first deferred contracts, scaled by the number of days between their two maturities differentiated when in backwardation or in contango; (ii) the days to maturity and (iii) 12 dummy vectors set to one for each different month from February to December. The model is eq. 5. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
	$RV_{c,t}$								
$RV^D_{c,t-1}$	0.22***	0.29***	0.14***	0.08***	0.27***	0.10***	0.14***	0.13***	0.13***
	(9.77)	(12.46)	(6.38)	(3.56)	(11.91)	(4.15)	(6.15)	(5.66)	(5.79)
$RV^W_{c,t-1}$	0.07**	0.31***	0.17***	0.39***	0.23***	0.24***	0.08**	0.24***	0.28***
	(2.13)	(8.50)	(4.12)	(8.95)	(6.71)	(6.55)	(2.30)	(5.77)	(7.26)
$RV^M_{c,t-1}$	0.06	0.30***	0.46***	0.38***	0.38***	0.30***	0.39***	0.32***	0.28***
	(1.46)	(8.28)	(11.51)	(9.26)	(12.11)	(7.40)	(9.43)	(7.26)	(6.28)
contango	4.67***	0.98***	14.26***	5.43**	2.89***	1.18***	6.76***	5.22**	1.34*
	(3.61)	(2.98)	(5.25)	(2.33)	(5.19)	(4.45)	(3.40)	(2.38)	(1.83)
backwardation	−0.84*	−0.62	−32.81***	−1.72	−1.88***	−2.80***	0.20	−17.04***	1.86
	(−1.69)	(−0.53)	(−13.87)	(−1.14)	(−3.77)	(−5.47)	(0.81)	(−4.90)	(0.23)
days to maturity $\times 10^{-7}$	24.48***	1.45	4.60***	4.96***	1.24	9.50***	11.03***	10.97***	7.78***
	(11.86)	(1.26)	(4.86)	(3.68)	(1.27)	(4.42)	(7.42)	(5.28)	(5.26)
Jan $\times 10^{-4}$	40.41***	17.39***	8.42**	8.40	16.66***	92.71***	22.42***	12.31	30.03***
	(4.85)	(3.24)	(2.34)	(1.54)	(3.58)	(8.26)	(3.69)	(1.51)	(4.62)
Feb $\times 10^{-4}$	29.33***	10.97**	10.26***	0.99	13.14***	61.91***	15.34**	20.67***	31.73***
	(3.58)	(1.98)	(2.84)	(0.18)	(2.68)	(5.47)	(2.50)	(2.62)	(4.90)
Mar $\times 10^{-4}$	43.52***	1.58	2.15	1.01	−9.80**	48.10***	20.56***	9.45	37.86***
	(5.61)	(0.29)	(0.62)	(0.20)	(−2.17)	(4.79)	(3.54)	(1.27)	(5.88)
Apr $\times 10^{-4}$	41.40***	8.13	8.05**	4.53	3.55	52.41***	13.20**	29.83***	33.02***
	(5.16)	(1.57)	(2.32)	(0.87)	(0.82)	(5.45)	(2.22)	(3.82)	(5.03)
May $\times 10^{-4}$	55.86***	12.54**	3.25	7.90	11.09***	56.60***	24.50***	21.17***	36.56***
	(6.79)	(2.40)	(0.91)	(1.53)	(2.60)	(5.96)	(4.30)	(2.70)	(5.44)
Jun $\times 10^{-4}$	70.02***	11.31**	6.31*	1.29	9.33**	55.74***	30.08***	15.90**	48.02***
	(8.61)	(2.16)	(1.88)	(0.26)	(2.24)	(5.78)	(5.30)	(2.11)	(7.21)
Jul $\times 10^{-4}$	82.69***	11.49**	−1.22	0.06	4.86	56.11***	40.70***	12.16*	34.72***
	(8.36)	(2.12)	(−0.36)	(0.01)	(1.17)	(5.87)	(6.28)	(1.67)	(4.74)
Aug $\times 10^{-4}$	58.29***	14.53***	9.34***	2.54	9.24**	57.09***	34.61***	22.94***	43.61***
	(6.63)	(2.77)	(2.82)	(0.51)	(2.27)	(6.28)	(5.31)	(3.00)	(6.10)
Sep $\times 10^{-4}$	59.57***	16.18***	7.62**	8.00	9.92**	56.23***	19.81***	28.94***	35.34***
	(7.06)	(2.96)	(2.13)	(1.59)	(2.34)	(5.99)	(3.01)	(3.79)	(5.09)
Oct $\times 10^{-4}$	56.83***	12.80**	2.55	14.23***	8.61**	51.43***	24.10***	11.34	32.37***
	(6.68)	(2.43)	(0.73)	(2.84)	(2.09)	(5.46)	(3.82)	(1.43)	(4.89)
Nov $\times 10^{-4}$	46.21***	23.85***	6.20*	4.79	17.84***	66.70***	24.01***	30.05***	33.31***
	(5.59)	(4.57)	(1.76)	(0.91)	(4.21)	(6.31)	(3.86)	(3.91)	(5.18)
Dec $\times 10^{-4}$	38.45***	7.82	3.19	−11.67**	4.92	68.88***	13.12**	11.92	25.48***
	(4.63)	(1.47)	(0.89)	(−2.17)	(1.10)	(6.46)	(2.14)	(1.50)	(3.82)
Observations	2,649	2,657	2,623	2,639	2,657	2,657	2,649	2,528	2,649
Adjusted R^2	0.81	0.93	0.87	0.87	0.93	0.87	0.84	0.81	0.90
Note:									*p<0.1; **p<0.05; ***p<0.01

Appendix C.

Table 10: **Multivariate regressions of daily RV on HAR-RV extended with commodity volatility factors**

I report the coefficients of the nine multivariate regressions of daily RV in date t sampled at five minutes interval on commodity volatility factors in date $t-1$ that are, (i) the halved log term structure, i.e., the log price difference between the nearby and first deferred contracts, scaled by the number of days between their two maturities differentiated when in backwardation or in contango; (ii) the days to maturity and (iii) 12 dummy vectors set to one for each different month from January to December. The model is eq. 6. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
					$RV_{c,t}$				
$RV^{D,agriculture}_{t-1}$	0.29***	0.03*	−0.01	0.01	0.02	−0.01	0.05*	0.04	0.01
	(5.67)	(1.84)	(−0.95)	(0.73)	(1.00)	(−0.42)	(1.71)	(1.28)	(0.20)
$RV^{D,energy}_{t-1}$	0.04*	−0.01	0.01	0.01	−0.01	0.10**	0.02	−0.05**	0.01
	(1.88)	(−0.34)	(0.75)	(0.46)	(−0.60)	(2.37)	(1.47)	(−2.18)	(0.56)
$RV^{D,metal}_{t-1}$	0.12***	0.10***	0.06***	0.11***	0.07***	0.02	0.08***	0.28***	0.06***
	(4.66)	(5.19)	(3.43)	(4.26)	(4.65)	(0.74)	(4.11)	(4.74)	(3.20)
$RV^{D}_{c,t-1}$	0.01	0.26***	0.09***	0.03	0.26***	0.04	0.09***	−0.00	0.12***
	(0.17)	(9.93)	(3.56)	(0.95)	(10.05)	(1.40)	(2.86)	(−0.08)	(4.11)
$RV^{W}_{c,t-1}$	0.05	0.31***	0.16***	0.37***	0.22***	0.24***	0.07**	0.22***	0.28***
	(1.53)	(8.50)	(4.09)	(8.43)	(6.38)	(6.49)	(2.05)	(5.33)	(7.09)
$RV^{M}_{c,t-1}$	0.01	0.28***	0.43***	0.38***	0.38***	0.28***	0.36***	0.27***	0.25***
	(0.16)	(7.73)	(10.60)	(9.26)	(11.98)	(6.80)	(8.55)	(5.98)	(5.68)
contango	2.13	1.40***	13.81***	5.63**	3.38***	1.33***	4.78**	4.34**	1.48**
	(1.61)	(4.19)	(5.08)	(2.44)	(5.96)	(4.90)	(2.37)	(1.97)	(2.02)
backwardation	−0.92*	−0.07	−32.99***	−1.11	−2.06***	−3.01***	0.07	−16.00***	2.15
	(−1.86)	(−0.06)	(−13.97)	(−0.74)	(−4.13)	(−5.82)	(0.28)	(−4.57)	(0.26)
days to maturity $\times 10^{-7}$	20.27***	−2.29*	4.33***	3.13**	−1.27	8.37***	7.66***	9.95***	6.16***
	(9.63)	(−1.72)	(4.43)	(2.21)	(−1.13)	(3.41)	(4.73)	(4.68)	(3.95)
Jan $\times 10^{-4}$	26.11***	15.19***	7.62*	3.34	15.94***	90.10***	16.94***	16.32*	27.60***
	(3.03)	(2.66)	(1.93)	(0.56)	(3.27)	(7.91)	(2.63)	(1.83)	(4.05)
Feb $\times 10^{-4}$	14.16*	10.16*	9.85**	−2.98	13.33***	58.61***	10.35	24.02***	29.46***
	(1.66)	(1.75)	(2.50)	(−0.50)	(2.65)	(5.12)	(1.59)	(2.76)	(4.34)
Mar $\times 10^{-4}$	28.97***	0.09	1.99	−2.52	−10.54**	46.01***	15.94***	11.07	36.85***
	(3.62)	(0.02)	(0.54)	(−0.46)	(−2.29)	(4.49)	(2.64)	(1.38)	(5.61)
Apr $\times 10^{-4}$	28.98***	4.84	7.78**	0.08	1.80	50.49***	8.83	30.26***	31.99***
	(3.56)	(0.89)	(2.11)	(0.01)	(0.40)	(5.09)	(1.44)	(3.63)	(4.82)
May $\times 10^{-4}$	40.61***	8.29	2.13	2.90	8.18*	54.30***	17.45***	23.12***	34.80***
	(4.83)	(1.51)	(0.56)	(0.53)	(1.83)	(5.47)	(2.92)	(2.77)	(5.11)
Jun $\times 10^{-4}$	58.43***	9.48*	6.41*	−2.73	7.85*	54.94***	24.63***	17.12**	47.59***
	(7.05)	(1.72)	(1.77)	(−0.51)	(1.80)	(5.52)	(4.12)	(2.12)	(7.05)
Jul $\times 10^{-4}$	77.37***	7.70	−0.86	−6.11	1.99	55.81***	36.21***	11.20	35.64***
	(7.71)	(1.29)	(−0.22)	(−1.06)	(0.43)	(5.33)	(5.26)	(1.32)	(4.75)
Aug $\times 10^{-4}$	43.36***	10.92*	9.52***	−2.89	6.34	55.16***	31.44***	23.35***	43.63***
	(4.76)	(1.94)	(2.59)	(−0.53)	(1.43)	(5.64)	(4.68)	(2.76)	(6.03)
Sep $\times 10^{-4}$	44.60***	12.13**	7.10*	0.51	6.35	52.21***	16.02**	30.67***	33.59***
	(5.19)	(2.10)	(1.85)	(0.09)	(1.41)	(5.23)	(2.38)	(3.70)	(4.76)
Oct $\times 10^{-4}$	44.40***	8.40	2.26	9.01*	4.97	47.23***	19.73***	11.37	30.01***
	(5.18)	(1.50)	(0.60)	(1.66)	(1.13)	(4.68)	(3.05)	(1.35)	(4.44)
Nov $\times 10^{-4}$	30.48***	19.74***	4.81	−0.39	14.61***	64.99***	18.79***	32.46***	30.09***
	(3.60)	(3.56)	(1.26)	(−0.07)	(3.24)	(5.97)	(2.93)	(3.90)	(4.53)
Dec $\times 10^{-4}$	24.64***	5.51	3.02	−16.67***	3.37	66.54***	8.01	16.40*	23.57***
	(2.90)	(0.99)	(0.79)	(−2.90)	(0.73)	(6.13)	(1.25)	(1.92)	(3.44)
Observations	2,649	2,653	2,619	2,635	2,653	2,653	2,649	2,524	2,649
Adjusted R^2	0.82	0.93	0.87	0.87	0.94	0.87	0.84	0.81	0.90

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Appendix D.

Table 11: **Multivariate realized beta (optimal hedging ratio) modeling with RV factors**

I report the coefficients of the nine multivariate SUR estimations of daily realized beta (or optimal hedging ratio) in date t, computed with the five-minute sampling realized covariance of the nearby and first deferred contracts, divided by the RV of the nearby contract. The independent variables are the three sector index factors of RV, the HAR-RV variables and the commodity volatility factors in date $t-1$. The model is eq. 11. I modify the magnitude of the variables by a power of 10 to improve the coefficients readability. The t-statistics are in parenthesis. The study period is from May 2008 to January 2019.

	(C)	(CL)	(GC)	(HG)	(HO)	(NG)	(S)	(SI)	(W)
					$ROHR_{c,t}$				
$RV_{t-1}^{D,agriculture}$	-29.66^{***}	4.97^{***}	7.67	−1.92	−4.64	−1.01	1.51	−1.44	−0.37
	(−2.77)	(2.89)	(1.25)	(−0.22)	(−1.48)	(−0.30)	(0.32)	(−0.11)	(−0.09)
$RV_{t-1}^{D,energy}$	-8.82^{**}	−1.23	−8.03	-19.26^{**}	−3.36	-16.62^{***}	1.11	-19.22^{*}	-6.45^{*}
	(−2.44)	(−0.69)	(−1.53)	(−2.45)	(−1.07)	(−3.45)	(0.33)	(−1.72)	(−1.92)
$RV_{t-1}^{D,metal}$	11.74^{*}	3.74	16.75	−26.34	−4.82	−7.87	11.55^{*}	−15.67	−4.64
	(1.88)	(1.43)	(1.17)	(−1.60)	(−1.01)	(−1.56)	(1.95)	(−0.33)	(−0.80)
$RV_{c,t-1}^{D}$	11.14^{**}	1.10	-83.62^{***}	2.78	2.61	4.45^{**}	−8.95	21.79	13.69^{**}
	(2.45)	(0.66)	(−3.18)	(0.16)	(0.55)	(2.05)	(−1.53)	(1.01)	(2.08)
$RV_{c,t-1}^{W}$	5.97	−0.14	−8.58	31.20	−12.95	4.14	53.06^{***}	-51.41^{**}	−0.58
	(1.24)	(−0.04)	(−0.20)	(0.99)	(−1.29)	(1.18)	(4.34)	(−2.23)	(−0.04)
$RV_{c,t-1}^{M}$	2.74	−1.82	85.13^{*}	19.62	28.12^{***}	-18.77^{***}	-120.73^{***}	52.71^{*}	15.39
	(0.31)	(−0.42)	(1.73)	(0.67)	(2.69)	(−3.72)	(−6.68)	(1.71)	(0.90)
contango $\times 10^{1}$	-13.82^{***}	-4.54^{***}	-84.76^{***}	-64.97^{***}	-3.84^{***}	-2.11^{***}	15.08^{***}	-74.00^{***}	-6.92^{***}
	(−6.57)	(−13.27)	(−6.47)	(−6.10)	(−3.53)	(−7.50)	(3.79)	(−7.37)	(−4.91)
backwardation $\times 10^{1}$	7.79^{***}	15.01^{***}	4.11	42.45^{***}	13.88^{***}	6.67^{***}	6.00^{***}	126.20^{***}	21.59
	(9.92)	(13.74)	(0.36)	(6.12)	(13.87)	(12.72)	(12.14)	(7.90)	(1.36)
days to maturity $\times 10^{-6}$	43.93^{***}	-11.61^{***}	−4.78	−5.54	-24.95^{***}	-4.85^{**}	−1.15	7.52	−0.16
	(15.13)	(−10.22)	(−1.13)	(−0.93)	(−12.13)	(−2.14)	(−0.40)	(0.86)	(−0.06)
Jan $\times 10^{-1}$	7.09^{***}	9.74^{***}	8.60^{***}	8.87^{***}	9.08^{***}	9.16^{***}	8.40^{***}	9.12^{***}	7.91^{***}
	(56.81)	(207.88)	(51.48)	(35.50)	(105.67)	(97.34)	(73.06)	(25.99)	(71.57)
Feb $\times 10^{-1}$	7.37^{***}	9.68^{***}	8.79^{***}	8.04^{***}	8.81^{***}	8.58^{***}	8.96^{***}	5.62^{***}	8.37^{***}
	(58.78)	(200.19)	(52.88)	(31.84)	(98.43)	(90.32)	(76.28)	(16.15)	(74.24)
Mar $\times 10^{-1}$	7.17^{***}	9.72^{***}	8.27^{***}	7.98^{***}	8.86^{***}	9.02^{***}	8.37^{***}	8.28^{***}	8.22^{***}
	(61.00)	(217.96)	(51.79)	(34.75)	(107.92)	(105.36)	(75.71)	(25.80)	(77.28)
Apr $\times 10^{-1}$	7.89^{***}	9.71^{***}	8.64^{***}	8.40^{***}	9.09^{***}	8.94^{***}	9.10^{***}	5.41^{***}	8.75^{***}
	(66.57)	(214.05)	(54.05)	(36.04)	(112.48)	(103.69)	(81.49)	(15.81)	(81.38)
May $\times 10^{-1}$	7.25^{***}	9.64^{***}	8.24^{***}	8.43^{***}	9.17^{***}	8.94^{***}	8.05^{***}	8.60^{***}	8.23^{***}
	(60.76)	(217.78)	(49.83)	(37.04)	(115.46)	(105.20)	(76.88)	(26.45)	(74.87)
Jun $\times 10^{-1}$	7.82^{***}	9.82^{***}	8.53^{***}	8.30^{***}	9.07^{***}	8.96^{***}	8.73^{***}	5.62^{***}	8.57^{***}
	(65.31)	(225.57)	(54.63)	(36.99)	(116.57)	(107.58)	(82.84)	(17.21)	(77.79)
Jul $\times 10^{-1}$	7.51^{***}	9.73^{***}	8.50^{***}	7.37^{***}	8.95^{***}	8.97^{***}	8.69^{***}	7.94^{***}	8.65^{***}
	(58.22)	(210.87)	(52.84)	(32.49)	(112.00)	(105.49)	(76.48)	(24.61)	(77.02)
Aug $\times 10^{-1}$	8.04^{***}	9.67^{***}	8.71^{***}	8.43^{***}	9.05^{***}	9.04^{***}	8.38^{***}	4.83^{***}	8.57^{***}
	(66.26)	(221.16)	(56.77)	(37.81)	(117.98)	(111.02)	(77.53)	(14.47)	(78.48)
Sep $\times 10^{-1}$	7.04^{***}	9.74^{***}	7.77^{***}	7.26^{***}	9.04^{***}	8.83^{***}	8.08^{***}	7.24^{***}	7.71^{***}
	(58.10)	(217.84)	(47.55)	(32.07)	(114.60)	(99.20)	(70.86)	(22.58)	(71.37)
Oct $\times 10^{-1}$	7.27^{***}	9.74^{***}	8.72^{***}	8.29^{***}	8.95^{***}	8.31^{***}	8.99^{***}	8.79^{***}	8.03^{***}
	(60.96)	(229.73)	(56.70)	(38.11)	(117.39)	(91.69)	(80.52)	(27.48)	(77.34)
Nov $\times 10^{-1}$	7.35^{***}	9.70^{***}	7.89^{***}	8.13^{***}	8.99^{***}	8.65^{***}	8.52^{***}	5.08^{***}	8.30^{***}
	(61.22)	(225.43)	(49.20)	(35.91)	(114.29)	(100.13)	(76.06)	(15.71)	(79.52)
Dec $\times 10^{-1}$	6.76^{***}	9.66^{***}	8.59^{***}	7.29^{***}	8.94^{***}	8.97^{***}	9.01^{***}	7.69^{***}	7.55^{***}
	(54.91)	(213.97)	(53.44)	(31.09)	(108.71)	(101.59)	(78.70)	(23.15)	(69.37)
Observations	2,648	2,653	2,619	2,635	2,653	2,653	2,648	2,523	2,648
Adjusted R^2	0.97	1.00	0.94	0.87	0.99	0.98	0.98	0.73	0.98

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$