Limited Stock Market Participation and Conditional Consumption Asset Pricing

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Abstract

Conditional consumption asset pricing has had limited success empirically - the implied risk aversion ranges from -3000 to 2000 in Nagel and Singleton (2011) and -250 to 600 in Roussanov (2014). We develop an equilibrium model where heterogeneous investors optimally choose to exit or enter the market. Non-financial income in conjunction with a constraint gives rise to state-dependent market participation, resulting in limited risk-sharing among remaining shareholders and hence a reasonable required price of risk. Our model also shows why previous empirical tests assuming full market participation can imply large or even negative risk aversion. We conduct an empirical test of our theory using the Consumer Expenditure data. Our conditional test shows that only a reasonable boundary of risk aversion (e.g., 4 to 40) is enough to explain the dynamic of equity premium.

JEL Classification: *G11, G12, G17*

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1 Introduction

The standard conditional consumption-based asset pricing model implies that equity premium at each time is determined by the amount of consumption risk and the required compensation per unit of consumption risk at each time[1]. However, the empirical evidence largely has failed to offer support for this theory. First, when the price of risk is assumed to be time-invariant[2], the empirically implied risk aversion is unreasonably high or negative[3]. Second, when the conditional price of risk is estimated, the counter-cyclical variation in the implied price of risk is found to be dramatic, reaching large positive values during bad states and large negative values during good states[4]. The empirical boundaries in the extant literature cannot be rationalized given reasonable risk aversion coefficient[5] and positive risk-return trade-off.

In this article, we argue that one reason for the empirical failure of the conditional consumption asset pricing theory stems from relying on models that assume full participation in the stock market. It is stockholders aggregate risk aversion and consumption that should mainly affect stock valuations. The evidence on limited market participation in the U.S. households has been well documented in the literature[6]. The recent 2016 Survey of Consumer Finances documents only 29.7% of the U.S. households hold either a stock or a mutual fund directly (60.2% when indirect holdings are accounted for[7]). Furthermore, the group of stock

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[1]That is, $E_t[R^e_{t+1}] = \underbrace{\gamma_t}_{\textit{Price of risk}} \cdot \underbrace{Cov_t(R^e_{t+1}, \Delta C_{t+1}/C_t)}_{\textit{Quantity of risk}}$
where R^e_{t+1} is the excess stock returns, $\frac{\Delta C_{t+1}}{C_t}$ is consumption growth, and γ_t is the relative risk aversion.

[2]In this specification, the observed large risk aversion is linked to the large risk aversion from the unconditional test (Mehra and Prescott (1985)) because a large risk aversion obtained by OLS is attributable to low covariance. For unconditional consumption-based asset pricing test, a better measure for the consumption risk is suggested for potential resolution (See Mankiw and Zeldes (1991), Aït-Sahalia et al. (2004), Parker and Julliard (2005), Jagannathan and Wang (2007), Malloy et al. (2009), Savov (2011), and Kroencke (2017) for example).

[3]Using different specifications, Attanasio (1991) documents 168, 201, 259, and 286. Ferson and Harvey (1993) documents 42, 49, 80, 99, 169, and 184. Duffee (2005) documents -237, -181, -168, ..., -31, and -28.

[4]Duffee (2005) documents ranges from -88 to -4 and -91 to 1, -200 to 600 in Sarkar and Zhang (2009), -3000 to 2000 in Nagel and Singleton (2011) and -250 to 600 in Roussanov (2014)

[5]Conine et al. (2017) document that the range of reasonable constant risk aversion estimated directly from the past studies during the period from 1970 to 2014 is from 0.6 to 10

[6]See Mankiw and Zeldes (1991) and Gomes and Michaelides (2008) among others.

[7]This is based on variable name 'HEQUITY' in Survey of Consumer Finances.

market participants is found to be time-varying (Vissing-Jorgensen (2002)). The aim of this paper is to explore the implication of time-varying market participation for conditional consumption-based asset pricing both theoretically and empirically.

We develop a general equilibrium model for an economy populated by N investors having power utility of consumption. Investors differ in their risk aversion[8]. The stockholders continuously trade in two securities - a riskless bond and a risky asset, whereas the non-stockholders trade only in a riskless bond. All investors receive stochastic non-financial income (which we refer to also as labor income or endowment) that is positively correlated with aggregate dividend. The introduction of labor income in conjunction with short selling constraints gives rise to endogenous entry or exit of stock market by investors given their level of risk aversion. We solve in closed form for optimal investment policies, consumption choices and asset prices. In doing so, and in contrast to the CARA setup[9], our model shows the importance of non-financial income on optimal consumption, investment policies, equilibrium asset moments and decisions to exit or enter the stock market .

The main contribution of this paper is to present a novel consumption-based asset pricing model (C-CAPM) for both market and an individual stock. The equity premium is given by the covariance between the *stockholders'* consumption growth and stock returns multiplied by the consumption-weighted harmonic mean of *stockholders'* risk aversions[10]. The implications of our conditional consumption asset pricing equation can be summarized as follows. First, as for the level, the price of risk is lower than the one generated in the full market participation case. This is because we find that the remaining stockholders are less risk-averse than non-stockholders. This finding also leads to limited risk-sharing across the remaining stockholders which, in turn, generates higher quantity of risk in our economy compared to the full participation case.

Second, as for the dynamics, when the stock valuations are high (low), more risk-averse

[8]There can be heterogeneity of risk aversion among institutional investors given different regulations, investment horizon or characteristics.

[9]See Appendix A for more details.

[10]That is, $E_t[R^e_{k,t+1}] = \frac{\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{\gamma_i}} Cov_t(R^e_{k,t+1}, \frac{\Delta \sum_{i=1}^{h_t^*} C^*_{i,t+1}}{\sum_{i=1}^{h_t^*} C^*_{i,t}})$ $\forall k = m, 1, 2, ..., K$ for either market $k = m$ or individual stocks $k = 1, .., K$ in discrete time expression, see **Proposition 4** and Appendix C.

investors than existing stockholders enter (leave) the market. We find that this entry (exit) of investors with high risk aversion raises (lowers) the harmonic mean of stockholders' risk aversion beyond the opposite effect of changes in the cross-sectional distribution of shareholders' consumption. Thus, our model generates time-varying aggregate risk aversion mainly through limited market participation along the suggestion in Brunnermeier and Nagel (2008).

Third, we find that the conditional covariance is counter-cyclical because more heterogeneous investors enter (exit) the market and hence risk-sharing is improved (worsened). This is in contrast to the pro-cyclical covariance generated in Duffee (2005) where only *the composition effect* drives the time-variation in the covariance term given the full market participation in his setup. We find that endogenous limited market participation dominates the composition effect in the dynamics of the covariance which, in turn, relaxes the required dramatic counter-cyclical variation in the price of risk.

Given our CRRA preferences, we find that the sensitivity of optimal consumption to labor is not unity and hence investors invest a part of labor income in financial assets. As a result, we show that fluctuations in labor income affects the equilibrium stock price, returns and volatility. Labor income shock is also necessary for generating limited market participation. In our economy, investors face uninsurable labor income shock assumed to be *positively* correlated with unexpected stock returns. Then, when there is a negative labor income shock, this could be the moment that stock price crashes. To hedge against this unexpected labor income shock, the optimal stock holding includes an intertemporal hedging demand term in addition to the first term proportional to the mean-variance efficient portfolio. Then, for investors with sufficiently high risk aversion the intertemporal hedging term dominates the mean variance term and hence optimally hold a negative position in the stock.

Our model generates positive cross-sectional relation between the optimal consumption and risk aversion - more risk-averse investors consume more given the same level of wealth. This is different from the prediction in a heterogeneous economy without labor income. Understanding this difference boils down to comparing consumption smoothing and precautionary saving demands between these two economies. The magnitude of the precautionary saving demand is mainly determined by the quantity of consumption risk. First, the heterogeneity of investors in our incomplete market setting improves the risk-sharing which in turn reduces the quantity of consumption risk, whereas in the complete market, the risk-sharing is always perfect and the quantity of consumption risk does not vary considerably with the degree of heterogeneity. Second, the inclusion of non-financial income in our setup further lowers the quantity of consumption risk by generating an imperfect correlation between consumption growth and stock returns, which is equal to 1 in the complete market setting. Taken together, the consumption smoothing demand dominates the precautionary saving demand in our economy while it is not the case in other complete market economies[11].

This finding has a direct implication on the non-stationary cross-sectional distribution of consumption (or severe inequality) across heterogeneous investors discussed in Chan and Kogan (2002) and Cvitanić et al. (2012). Since we find that the least risk-averse investor consumes the least at the beginning, when wealth is equal across investors, we show by means of simulation, that it takes an astronomical amount of time for the least risk-averse investor to asymptotically dominate the others in terms of consumption[12].

Finally, we conduct an empirical test of our theory. Specifically, we explore the implication of time-varying market participation using the Consumer Expenditure data. Our main results are summarized as follows. First, in the time-series regression with the assumption of time-invariant price of risk, we confirm that the covariance of aggregate (full participation) consumption growth with stock returns cannot predict the future excess stock returns and the result implies a negative price of risk, similar to Duffee (2005). However, the test using only stockholders' consumption growth implies a reasonable positive price of risk level of 20. Second, we allow the price of risk to vary as a function of a commonly used state variables. Using aggregate consumption, we also confirm the dramatic counter-cyclical variation in the implied price of risk. However, if only the consumption of stockholders is used, we find that the time-varying implied price of risk is less counter-cyclical with reasonable boundary. Third, we also impose the price of risk measures motivated by our theory. We show that our equation produces reasonable range of time-varying price of risk (e.g., 4 to 40). Finally, we

[11] A detailed discussion is provided in Appendix F.

[12] Our simulation result shows that the consumption (wealth) of the least risk-averse investor accounts for 0.05% (5%) of the total consumption (the entire market wealth) in 50 years. With this speed, the linear extrapolation predicts it takes 138,876 years (1,037 years) for the least risk-averse investor to fully dominate the others. If the exponential extrapolation is used for the consumption, it takes 334 years.

find that our price of risk measures rise when a higher proportion of households invests in the stock market, consistent with our theory.

The rest of the paper is organized as follows: Section 2 reviews the literature which could be skipped by informed readers. Section 3 discusses the economic setup and solves the optimization problems. Section 4 solves and examines the equilibrium. Section 5 simulates the model. Section 6 empirically tests the implications of our paper. Section 7 concludes.

2 Literature review

Several studies have theoretically examined the limited stock market participation to explain broad asset pricing features. One class of these studies[13] *exogenously* specifies a group of investors excluded from the stock market. Basak and Cuoco (1998) examine the equity premium in an economy where less risk-averse investor is the only stockholder out of two investors. Guvenen (2009) considers a real business cycle model with two investors who differ in their EIS. The investor with higher EIS is the only stockholder. Since there is no dynamics in market entry or exit in this class of models, these models do not derive implications of time-varying stock market participation for asset pricing which is the main contribution of this article. Moreover, our article shows that, allowing market participation to be determined by individuals' optimal choice makes it harder to explain the equity premium. This is because as long as the equity premium is sufficiently high, the non-stockholders are willing to enter the market. This entry of more risk-averse investors decreases the equilibrium equity premium given the improved risk-sharing.

The other class of studies[14] more realistically *endogenizes* the stock market participation. Even though the market participation is determined by individuals' optimization at each time in this class of papers, neither of these authors examines the implication of state-dependent stock market participation as in our paper. This is because the main focus of these papers are to examine either the unconditional asset moments, participation rate, or investors' life-cycle

[13] See Basak and Cuoco (1998), Guo (2004), Polkovnichenk (2004), and Guvenen (2009), for example, among others.

[14] See Allen and Gale (1994), Williamson (1994), Constantinides et al. (2002), Haliassos and Michaelides (2003), Cao et al. (2005), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), and Fagereng et al. (2017), among others.

behavior.

Our paper belongs to the large literature on the heterogeneous investors[15]. Chan and Kogan (2002), for example, examine an economy with heterogeneous investors and show that changes in the cross-sectional distribution of wealth among the heterogeneous agents leads to counter-cyclical variation in Sharpe ratio, without assuming time-varying individual risk aversion as in Campbell and Cochrane (1999). Our paper also generates time-varying risk aversion but with a different mechanism. More specifically, in addition to the time-variation in cross-sectional distribution of wealth, the optimal exit and entry of investors constitute another channel of time variation. We show that this latter channel affects the price of risk in opposite direction to the way time-variation in wealth distribution does. These counterbalancing channels render the market price of risk less counter-cyclical (to even pro-cyclical) compared to Chan and Kogan (2002). In contrast with the extant literature that relies on high counter-cyclical price of risk, our theory relies on counter-cyclical risk-sharing with moderate price of risk to match the observed counter-cyclicality in equity risk premium.

Our paper is also related to the work investigating labor income risk. Christensen et al. (2012) is the first to solve the equilibrium of an economy with labor income in closed form using CARA preferences. However, there is no stochastic dynamics in this economy because neither CARA investors take into account wealth nor invest labor income in the financial assets (A description of this economy is in Appendix A). Thus, we consider CRRA investors. In general, there is no closed-form solution for the maximization problems associated with CRRA investors in the presence of labor income risk which is not perfectly correlated with stock returns. Using only one shock for the stock dynamics, Koo (1998) provides the optimal solution in closed-form where financial wealth is always non-negative. Building upon Koo (1998), our paper is the first to present, in closed form, expressions for optimal policies and asset prices in a general equilibrium with heterogeneous investors facing labor income risks. We verify that our solution holds exactly without assuming sufficient liquidity in the case where the correlation between stock returns and labor income is perfect.

[15]See Constantinides and Duffie (1996), Basak and Cuoco (1998), Chan and Kogan (2002), Bhamra and Uppal (2009) Chen et al. (2012), Cvitanić et al. (2012), Chabakauri (2013), and Bhamra and Uppal (2014), for example, among others.

Our paper is also related to studies that examine consumption and stock volatilities. Our model helps match the consumption volatility as well as stock volatility fairly well. First, in the case where there is no labor income, the consumption volatility is identical to dividend volatility. By introducing labor income, the consumption volatility can be even lower than labor income volatility, as in the data. Second, with regards to stock volatility, numerous studies examined the excess volatility puzzle[16]. The mechanism in our model differs from these studies. In addition to effect of labor shock, once heterogeneity is introduced, the equilibrium stock volatility is a function of consumption and wealth distribution. We find that the higher wealth inequality relative to consumption inequality, the greater gap between stock volatility and dividend volatility. Given that, as discussed above, risk-averse investors consumer less in the beginning when wealth is equal, the speed of consumption inequality is less than the dispersion of wealth in our economy, hence generating plausible excess volatility[17].

3 The Economy

3.1 The basic setup

We consider a continuous pure-exchange economy over the infinite time horizon. The uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Ω is the set of all possible states. $\mathcal{F} = \{\mathcal{F}_t\}_{t \in \tau}$ is the filtration that represents the investors' information available at time t where $\tau \in [0, \infty)$. The probability measure $\mathbb{P}$ is defined on $(\Omega, \mathcal{F}_\infty)$ where $\mathcal{F}_\infty = \bigcup_{t=0}^{\infty} \mathcal{F}_t$, represents the investors' common beliefs. The filtration $\mathcal{F}$ is generated by two-dimensional standard Brownian motion $W = [W_d, W_y]$. All stochastic processes introduced in the remainder of the paper are assumed to be adapted to $\mathcal{F}_t$.

The stockholders continuously trade in two securities - a riskless bond and a risky asset, whereas the non-stockholders trade only in a riskless bond. A riskless bond price at time t is denoted by B_t and net supply of the bond is zero. The initial bond price is normalized to unity $B_0 = 1$. Therefore, the bond price follows the dynamics: $\frac{dB_t}{B_t} = r_t dt$ where the

[16] See Shiller (1981), LeRoy and Porter (1981), Keim and Stambaugh (1986), Campbell and Shiller (1988), Cochrane (1992), Hodrick (1992), Gabaix (2012) and Wachter (2013) among others.

[17] A detailed discussion is provided in Appendix G.

parameter r_t denotes the risk-free rate. A risky asset is in unit net supply and a claim to a continuous exogenous dividend D_t that follows Geometric Brownian Motion (GBM): $\frac{dD_t}{D_t} = \mu_d dt + \sigma_d dW_{d,t}$ where $\mu_d > 0$ is the expected dividend growth rate, and $\sigma_d > 0$ is the dividend growth volatility. Moreover, the equilibrium price dynamics of risky asset has the form[18] :

$$\frac{dS_t + D_t dt}{S_t} = \mu_{s,t} dt + \sigma_{s,t}^d dW_{d,t} + \sigma_{s,t}^y dW_{y,t} \tag{1}$$

where $\mu_{s,t}, \sigma_{s,t}^d$, and $\sigma_{s,t}^y > 0$ and $dW_{d,t} dW_{y,t} = \rho dt, \rho > 0$. The risk-free rate r_t, the expected stock returns $\mu_{s,t}$, and the stock volatility $\sigma_{s,t} = \sqrt{(\sigma_{s,t}^d)^2 + (\sigma_{s,t}^y)^2 + 2\rho\sigma_{s,t}^d \sigma_{s,t}^y}$ are to be endogenously determined in equilibrium.

The economy is populated by infinitely lived N investors all having time-separable power utility of consumption. They differ in the coefficient of relative risk aversion. Since this paper aims to examine the conditional asset pricing, the power utility is chosen over the CARA utility whose economy does generate rich stochastic dynamics. The assumption of power utility for individual investors is justified by the recent work of Brunnermeier and Nagel (2008).

Investor i is maximizing $\forall t \in [0, \infty)$

$$\mathbb{E}_t[\int_t^\infty \frac{e^{-\delta s} C_{i,s}^{1-\gamma_i}}{1-\gamma_i} ds] \tag{2}$$

For investors $i = 1, ..., N$ whose risk aversion coefficient is $\gamma_1, ..., \gamma_N$, respectively, with $0 < \gamma_1 < ... < \gamma_N$. $C_t \in \mathbb{R}^+$ is one perishable consumption good that serves as the numéraire. $\delta > 0$ is the subjective time preference rate. $\mathbb{E}_t$ denotes the expectation taken at time t. All investors receive, for simplicity, the same level of stochastic exogenous non-financial income (labor income) Y_t that evolves as :

$$\frac{dY_t}{Y_t} = \mu_y dt + \sigma_y dW_{y,t} \tag{3}$$

where $\mu_y > 0$ is the expected labor income growth rate $\sigma_y > 0$ is the labor income growth volatility.

[18]in contrast to Koo (1998) who uses one shock, this conjecture for the equilibrium stock price dynamics is confirmed in **Proposition 1** which shows that CRRA investors invest a part of labor income in financial assets, and thus, labor income shock affects the equilibrium stock price eventually. This is not the case for CARA investors. For more details, see Appendix A.

3.2 The individual investor's problem

In this section, we study the individual investors' utility maximization problem in search of the optimal portfolio and consumption choice. We consider a stockholder's and non-stockholder's optimization separately. A stockholder's financial wealth evolves according to

$$\begin{aligned} dX_{i,t} &= \pi_{i,t}(\frac{dS_t + D_t dt}{S_t}) + (X_{i,t} - \pi_{i,t}) r_t dt + (Y_t - C_{i,t})dt \\ &= [\pi_{i,t}(\mu_{s,t} - r_t) + r_t X_{i,t} + Y_t - C_{i,t}]dt + \pi_{i,t}\sigma^d_{s,t} dW_{d,t} + \pi_{i,t}\sigma^y_{s,t} dW_{y,t} \end{aligned} \tag{4}$$

$\forall i = 1, 2, ..., h_t$ where *the dollar amount of stock holding* (not fraction of wealth) $\pi_{i,t}$ satisfies $\int_0^t \pi^2_{i,s} ds < \infty$ and h_t is index for the cut-off stock'h'older who distinguishes the stockholders from the non-stockholders. It also denotes the number of stockholders at each time. Similarly, a non-stockholder's financial wealth evolves according to

$$dX_{i,t} = [r_t X_{i,t} + Y_t - C_{i,t}]dt \tag{5}$$

$\forall i = h_t + 1, ..., N$. The value function V is defined by

$$V_{i,t}(x, y) = \max_{(c_{i,t}, \pi_{i,t}) \in \mathcal{A}} E_t[\int_t^\infty \frac{e^{-\delta s} C_s^{1-\gamma_i}}{1 - \gamma_i} ds] \tag{6}$$

$\forall i = 1, 2, ..., N$, $\forall t \in [0, \infty)$ and $X^{(c,\pi)}_{i,t} = x$, and $Y_t = y$.

The investors maximize the lifetime sum of expected utility in (2) subject to the labor income process (3) and the wealth dynamics (4) or (5). The closed form solution for this maximization problem does not exist in general[19]. However, assuming as in Koo (1998), that borrowing constraint ($X_{i,t} \geq 0 \;\; \forall t \in [0, \infty)$) never binds with sufficient liquidity (i.e., $X_{i,t}/Y_t \to \infty$). We solve the maximization problem in closed form in our setting. Moreover, this condition prevents the investors with relatively low risk aversion from being the non-stockholders, which generate one key feature of economy: stockholders are more risk-averse than non-stockholders for any point of time.

The following proposition shows the optimal consumption and investment as functions of asset parameters. Since the asset parameters are also functions of the consumption and

[19]The closed form solution for the stockholders' optimization problem exists if stock returns are perfectly correlated with labor income growth.

investment as shall be shown in Section 4, they are jointly determined in the equilibrium.

Proposition 1. *The investors' optimal consumption, stock holdings, and the wealth dynamics are given by*

$$C^*_{i,t} = (r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \;\; \forall i = 1, 2, ..., h_t \tag{7}$$

$$C^*_{i,t} = (r_t + \frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y}) \;\; \forall i = h_t + 1, ..., N \tag{8}$$

$$\pi^*_{i,t} = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_t\sigma_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y} \;\; \forall i = 1, 2, ..., h_t \tag{9}$$

$$\begin{aligned} dX_{i,t} = {} & [(\frac{\lambda_t^2}{\gamma_i} - \frac{\delta - r_t}{\gamma_i} + \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{\mu_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}]dt \\ & + \pi^*_{i,t}\sigma^d_{s,t}dW_{d,t} + \pi^*_{i,t}\sigma^y_{s,t}dW_{y,t} \;\; \forall i = 1, 2, ..., h_t \end{aligned} \tag{10}$$

$$dX_{i,t} = [(-\frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y}) - \frac{\mu_y Y_t}{r_t - \mu_y}]dt \;\; \forall i = h_t + 1, ..., N \tag{11}$$

$\forall t \in [0, \infty)$

where $\rho_t \equiv Corr_t(\sigma^d_{s,t}dW_{d,t} + \sigma^y_{s,t}dW_{y,t}, \sigma_y dW_{y,t}) = \frac{\sigma^d_{s,t}\rho + \sigma^y_{s,t}}{\sigma_{s,t}} = \frac{\sigma^d_{s,t}\rho + \sigma^y_{s,t}}{\sqrt{(\sigma^d_{s,t})^2 + (\sigma^y_{s,t})^2 + 2\rho\sigma^d_{s,t}\sigma^y_{s,t}}}$[20] and λ_t is the Sharpe ratio

Proof : See Appendix B.1

First, concerning the optimal consumption, note that the sensitivity of the optimal consumption to labor income ($\partial C^*_{i,t}(X_{i,t}, Y_t)/\partial Y_t$) is not unity, different from CARA utility case[21]. This means the investors invest a part of labor income in the financial assets, and hence fluctuations in labor income eventually affects the equilibrium stock price.

Second, the non-stockholders' financial wealth in (11) is deterministic, whereas the stockholders' financial wealth in (10) is stochastic. This implies that the consumption of stockholders in (7), is more volatile and more correlated with stock returns than that of non-stockholders in (8), consistent with the empirical finding (Mankiw and Zeldes (1991)).

Third, as illustrated in Figure 1, our model generates positive cross-sectional relation be-

[20] ρ_t is the correlation between stock returns and labor income growth. When the correlation between dividend shock and labor income shock is perfect ($\rho = 1$), ρ_t is also equal to 1.

[21] For more details, see Appendix A.2.

tween the optimal consumption and risk aversion - more risk-averse investors consume more given the same level of wealth. This finding is opposite to the prediction in a heterogeneous complete economy without labor income. In Appendix F we derive the derivative of optimal consumption with respect to risk aversion and by arranging its terms, it is straightforward to notice that understanding this difference boils down the magnitude of consumption smoothing and precautionary saving demands between these two economies. The magnitude of the precautionary saving demand is mainly driven by the quantity of consumption risk. First, the heterogeneity of investors in our incomplete market setting improves the risk-sharing which in turn reduces the quantity of consumption risk, whereas in the complete market, the risk-sharing is always perfect and the degree of heterogeneity does not change the quantity of consumption risk substantially. Second, the inclusion of non-financial income in our setup further lowers the quantity of consumption risk by generating an imperfect correlation between consumption growth and stock returns, which is equal to 1 in the complete market setting. Because these two effects, the precautionary saving demand in our economy is lower than in the complete market, the consumption smoothing demand is more likely to dominate the precautionary saving demand in our economy while it is not the case in other complete market economies. By making Sharpe ratio equal to zero in equation (F.1) for non-stockholders, it is also straightforward to notice that optimal consumption is monotonic with respect to risk aversion as well as higher than non-stockholders. See Appendix F for further discussion.

This positive relation between the consumption and risk aversion has an important implication. Since the least risk-averse investor consumes the least at the beginning, it takes astronomical[22] amount of time for the least risk-averse investor to asymptotically dominates the others in terms of consumption as the wealth of the least risk-averse investor accounts for the higher proportion of the market over time[23]. Through this channel, our model can reduce the impact of non-stationary cross-sectional redistribution of consumption on our

[22]Our simulation result shows that the consumption (wealth) of least risk-averse investor accounts for 0.05% (5%) of the total consumption (the entire market wealth) in 50 years. With this speed, the linear extrapolation predicts it takes 138,876 years (1,037 years) for the least risk-averse investor to fully dominate the others. If the exponential extrapolation is used for the consumption, it takes 334 years.

[23]This is consistent with the empirical findings that market participation generates wealth inequality (e.g., Favilukis (2013) and Gabaix et al. (2016).)

economy without assuming the 'catching up with the Joneses' preferences as in Chan and Kogan (2002).

Regarding the optimal stock holding, and unlike CARA, the optimal stock holding for CRRA investor is a function of the financial wealth and labor income. more importantly, the optimal stock holding $\pi^*_{i,t}$ has the intertemporal hedging demand term in addition to the first term which is proportional to the mean-variance efficient portfolio. It is straightforward to see from equation (9) that without labor income, the optimal stock holding is always non-negative and therefore every investor is a stockholder as long as the equity premium is positive. Also, it is worth emphasizing that only first term is inversely associated with the relative risk aversion γ_i. Therefore, the optimal stock holding is monotonically decreasing with risk aversion and for investors with relatively high risk aversion can have a negative optimal stock holding $\pi^*_{i,t} < 0$[24]. The economic intuition is when labor income innovations are positively correlated with unexpected stock returns $\rho_t > 0$, an employed investor with sufficiently high risk aversion hedges the consumption against an unexpected decrease in income by short-selling the risky asset. With a negative holding, the investor's portfolio will pay off when labor income unexpectedly falls, providing a hedge. However, we impose a short-selling constraint and therefore, the investors whose optimal holding is negative sub-optimally have zero position[25]

In addition, rearranging the optimal stock holding equation shows what determines the sign of stock holding. By rearranging terms in (9), the condition under which the stock holding is positive is simply equivalent to

$$\frac{X_{i,t}}{Y_t}\lambda_t(r_t + \rho_t\sigma_y\lambda_t - \mu_y) + \lambda_t - \gamma_i\rho_t\sigma_y > 0 \tag{12}$$

This shows that the state variable which mainly drives the time-varying market participation is the ratio of financial wealth to labor income. When the financial wealth is sufficiently

[24] This finding is consistent with Koo (1998), Heaton and Lucas (2000), and Viceira (2001).

[25] In the literature, other mechanisms to generate the limited market participation are considered as follows. Fixed setup or transaction costs: Allen and Gale (1994), Williamson (1994), Heaton and Lucas (1996), Haliassos and Michaelides (2003), Alan (2006), Fagereng et al. (2017); Life-cycle model: Constantinides et al. (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017); Model uncertainty: Cao et al. (2005); Borrowing constraint: Allen and Gale (1994), Heaton and Lucas (1996), Constantinides et al. (2002), Haliassos and Michaelides (2003), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017).

high during the good states, then an investor has more liquidity to enter the market. As for labor income, the higher labor income level, the higher labor income risk (given the GBM assumption), and the stronger motive to hedge the labor income risk by short-selling the stock. Equation (12) also shows that the higher risk aversion an investor has, the less likely the investor has a positive stock holding.

4 Equilibrium

This section discusses the equilibrium of the model. Section 4.1 defines the equilibrium. Section 4.2 examines the characteristics of equilibrium asset parameters in this economy. Section 4.3 describes how the cut-off stockholder is determined in equilibrium. Finally, Section 4.4 studies the consumption risk in equilibrium.

4.1 Description of the equilibrium

Definition 1. *An equilibrium is a set of processes* $\{r_t, \mu_{s,t}, \sigma_{s,t}\}$ *and consumption and investment policies* $\{C^*_{i,t}, \pi^*_{i,t}\}_{i\in 1,\ldots,h_t}$ *and* $\{C^*_{i,t}\}_{i\in h_t+1,\ldots,N}$ *which maximize the sum of life time expected utility (2) for each investor and satisfy the securities market-clearing conditions:*

1. Stock market clears: $$\sum_{i=1}^{h_t} \pi^*_{i,t} = S_t \quad \forall t \in [0,\infty) \tag{13}$$

2. Bond market clears: $$\sum_{i=1}^{N} X_{i,t} - \sum_{i=1}^{h_t} \pi^*_{i,t} = \underbrace{\sum_{i=1}^{h_t} X_{i,t} - S_t}_{\textit{Demand by stockholders}} + \underbrace{\sum_{i=h_t+1}^{N} X_{i,t}}_{\textit{by non-stockholders}} = 0 \quad \forall t \in [0,\infty) \tag{14}$$

The stock is in unit supply, and hence the stock market clearing condition is represented by (13). The bond is in zero supply. Since $\sum_{i=1}^{N} X_{i,t}$ represents the total financial wealth of all investors invested in both the stock and the bond, $\sum_{i=1}^{N} X_{i,t} - S_t$ represents the total demand on the bond. Thus, the zero supply bond market clearing condition is represented by $\sum_{i=1}^{N} X_{i,t} - S_t = 0$. We can decompose this into the amount of bond owned by stockholders $\sum_{i=1}^{h_t} X_{i,t} - S_t$ and by non-stockholders $\sum_{i=h_t+1}^{N} X_{i,t}$. Given the initial condition that $\sum_{i=1}^{h_0} X_{i,0} - S_0 + \sum_{i=h_0+1}^{N} X_{i,0} = 0$, it suffices to satisfy the following equation for the bond market clearing:

$$\sum_{i=1}^{h_t} dX_{i,t} - dS_t + \sum_{i=h_t+1}^{N} dX_{i,t} = 0 \quad \forall t \in [0, \infty) \tag{15}$$

Lemma 1. *The equation (15) together with the stock market clearing condition (13) implies the consumption clearing condition :*

$$\sum_{i=1}^{h_t} C^*_{i,t} + \sum_{i=h_t+1}^{N} C^*_{i,t} = N \cdot Y_t + D_t \tag{16}$$

Proof : See Appendix B.2

4.2 Derivation of the equilibrium

For an incomplete market where labor income is partially correlated with the stock, a Martingale approach has not been developed yet[26]. Therefore, to the best of our knowledge , it is not possible to specify the state price density and solve the equilibrium by maximizing the 'social planner's welfare function as it is done in the existing studies on heterogeneous investors[27]. Notwithstanding, we can solve for the general equilibrium without specifying the SDF (Stochastic Discount Factor) based on the optimal consumption and portfolio choice obtained by solving for the HJB (Hamilton-Jacobi-Bellman) equation. The equilibrium is derived in five steps. First, from the stock market clearing condition (13), the equation for the equilibrium Sharpe ratio is obtained. Second, by matching the deterministic terms of the dynamics of both left and right hand side of (16), the equation for the equilibrium risk-free rate is obtained. Third, by matching the diffusion terms of the dynamics of (16), the equation(s) for the equilibrium stock volatility are obtained. Fourth, from the consumption clearing condition (16), the optimal consumption in (7) and (8), and the market clearing condition in (13) and (14), the closed form solution for the equilibrium stock price is computed. Fifth, in a Nash equilibrium setting, searching for the investor who distinguishes

[26] He and Pearson (1991a) and He and Pearson (1991b) develop a martingale approach for a dynamic consumption-portfolio problem with incomplete markets and short-sale constraint. However, they do not consider non-financial income. Although He and Pagès (1993) develop a martingale approach for the economy with labor income and borrowing constraints, the labor income risk does not constitute an additional source of uncertainty in their paper. We investigated the martingale approach but we failed to find a solution.

[27] See Basak and Cuoco (1998), Chan and Kogan (2002), Bhamra and Uppal (2009) Chen et al. (2012), Cvitanić et al. (2012), Chabakauri (2013), and Bhamra and Uppal (2014), for example, among others.

the stockholders from the non-stockholders in such a way as to preclude any optimal deviation from the stockholders to non-stockholders, and vice versa, the cut-off stockholder (h_t^*) and the equilibrium endogenous asset parameters are finally determined. **Proposition 2** summarizes the set of equations for the equilibrium and stock price.

Proposition 2. *In equilibrium, defined by* ***Definition 1****, the set of equations for the Sharpe ratio λ_t, the risk-free rate r_t, the stock volatility $\sigma_{s,t}$ and the stock price are given by:*[28]

$$\lambda_t = (\sigma_{s,t}\sum_{i=1}^{N} X_{i,t} + \rho_t\sigma_y g(\boldsymbol{\theta}_t)Y_t h_t)(\sum_{i=1}^{h_t}\frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i})^{-1} \tag{17}$$

$$r_t = \delta + (\mu_d D_t + \mu_y N\cdot Y_t)(\sum_{i=1}^{N}\frac{C^*_{i,t}}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}(\sum_{i=1}^{N}\frac{C^*_{i,t}}{\gamma_i})^{-1}\sum_{i=1}^{h_t}\frac{C^*_{i,t}}{\gamma_i}(1+\frac{1}{\gamma_i}) \tag{18}$$

$$\begin{aligned}\sigma_{s,t} = &(\sum_{i=1}^{N} X_{i,t})^{-1}[(\sum_{i=1}^{h_t}\frac{C^*_{i,t}}{\gamma_i})^{-1}(\sigma_d D_t\frac{\sigma_{s,t}}{\sigma^d_{s,t}} \\ &+\sum_{i=1}^{h_t}\frac{\rho_t\sigma_y C^*_{i,t}}{X_{i,t}/Y_t g(\boldsymbol{\theta}_t)+1})(\sum_{i=1}^{h_t}\frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i}) - \rho_t\sigma_y g(\boldsymbol{\theta}_t)Y_t h_t]\end{aligned} \tag{19}$$

$$\begin{aligned}S_t =&\frac{D_t}{r_t} + Y_t[\frac{N}{r_t} - \frac{h_t + \frac{1}{r_t}\sum_{i=1}^{h_t}(\frac{\delta-r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})}{r_t+\rho_t\sigma_y\lambda_t-\mu_y} - \frac{N-h_t+\frac{1}{r_t}\sum_{i=h_t+1}^{N}(\frac{\delta-r_t}{\gamma_i})}{r_t-\mu_y}] \\ &+\frac{1}{r_t}[(\mu_d D_t+\mu_y N\cdot Y_t)(\sum_{i=1}^{N}\frac{C^*_{i,t}}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}(\sum_{i=1}^{N}\frac{C^*_{i,t}}{\gamma_i})^{-1}\sum_{i=1}^{h_t}\frac{C^*_{i,t}}{\gamma_i}(1+\frac{1}{\gamma_i})]\sum_{i=1}^{N}\frac{X_{i,t}}{\gamma_i} \\ &-\frac{\lambda_t^2}{2r_t}\sum_{i=1}^{h_t}(\frac{\gamma_i-1}{\gamma_i^2})X_{i,t}\end{aligned} \tag{20}$$

$\forall t \in [0,\infty)$ where $g(\boldsymbol{\theta}_t) \equiv \frac{1}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}$[29], $\boldsymbol{\theta}_t \equiv [r_t, \rho_t, \sigma_y, \lambda_t, \mu_y]$

Proof : See Appendix B.3.

Except for the equilibrium stock price, the endogenous asset parameters, as a function of the cut-off stockholder, are obtained by computationally solving for the four unknowns

[28] There are two equations for equilibrium $\sigma_{s,t}$ because there are two diffusion terms in the process of (16) and two parameters ($\sigma^d_{s,t}, \sigma^y_{s,t}$) for stock volatility. **Proposition 2** shows only the first equation for expositional convenience. For more details, see Appendix B.3

[29] $g(\boldsymbol{\theta}_t)Y_t$ is the certainty equivalent present value (CEPV), which is the minimal amount of wealth that any investor requires in order for her to permanently give up her *status quo* (with wealth $X_{i,t}$, and labor income process under incomplete market and subject to borrowing constraints) and live with no labor income thereafter.

$(\lambda_t, r_t, \sigma^d_{s,t}, \sigma^y_{s,t})$ from the set of four equations (B.60). To understand the role of labor income, it is useful to compare the endogenous asset parameters of the current model with the simpler case where there is no labor income. We study each endogenous asset parameter and compare how it differs from the case where labor income does not exist. We also show how our equilibrium equations reduces to known equations in nested economies (no-labor income and no heterogeneity).

Sharpe ratio

From the Sharpe ratio, we can derive the equilibrium equity premium ($\mu^e_{s,t} = \lambda_t \sigma_{s,t}$).

$$\mu^e_{s,t} = \lambda_t \sigma_{s,t} = \frac{\sum_{i=1}^N X_{i,t}}{\sum_{i=1}^{h_t} \frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i}} \sigma^2_{s,t} + \frac{g(\boldsymbol{\theta}_t)Y_t h_t}{\sum_{i=1}^{h_t} \frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i}} Cov_t(\frac{dS_t}{S_t}, \frac{dY_t}{Y_t}) \tag{21}$$

Note that this equation is the same as in Roussanov (2014). Hence, our paper provides an economic foundation and the derivation underlying this expression.

If we consider the case where the labor income does not exist (17)[30], the market is complete and only the market risk component remains[31]: $\lambda_t = (\sum_{i=1}^N \frac{X_{i,t}}{\sum_{i=1}^N X_{i,t}} \frac{1}{\gamma_i})^{-1} \sigma_{s,t}$. By substituting for the stock volatility $\sigma_{s,t}$ in this case[32], $\lambda_t = (\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sigma_d$. The only time-variation in the Sharpe ratio in this perfect market comes from the cross-sectional distribution of consumption. As Chan and Kogan (2002) points out, the Sharpe ratio is counter-cyclical even without assuming time-varying individual risk aversion (Campbell and Cochrane (1999)). The mechanism is as follows. The relatively risk-tolerant investors hold a higher proportion of their wealth in stock. Thus, at times when the stock market declines, the consumption of relatively risk-tolerant investors decreases more than others. This makes the consumption-weighted *harmonic* mean of risk aversion $(\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i})^{-1}$ tilted towards relatively high risk-averse investors, increasing the Sharpe ratio. In addition to this time-varying distribution of consumption, the Sharpe ratio in our case with labor income, equation (17), has another source of time-variation - market participation- which counterbalance the

[30] This economy is studied in heterogeneous agent literature. See Chan and Kogan (2002), Bhamra and Uppal (2009), Cvitanić et al. (2012), Bhamra and Uppal (2014) and Cochrane (2017) for example.

[31] In unreported results, we derive the endogenous asset parameters in this case using the Martingale approach and show that it is the same as forcing labor income to zero in our expression.

[32] $\sigma_{s,t} = (\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sum_{i=1}^N \frac{X_{i,t}}{\sum_{i=1}^N X_{i,t}} \frac{1}{\gamma_i} \sigma_d$

consumption redistribution effect.

In addition to the above channels, Labor income also influences the Sharpe ratio through risk-sharing. Without labor income risk-sharing is perfect, hence introducing more heterogeneous investors into this economy only changes the average risk aversion. In our economy the risk-sharing is imperfect and hence the risk-sharing is improved with more heterogeneous investors, decreasing the Sharpe ratio. More detailed discussion is provided in Appendix F.

Finally, if we further turn off the heterogeneity, our models reduces to the traditional single investor economy (Lucas (1978)) and the Sharpe ratio is simply $\lambda_t = \gamma\sigma_d$.

Risk-free rate

The equilibrium risk-free rate in (18) has the standard components: the subjective discount rate δ, the consumption smoothing demand $(\mu_d D_t + \mu_y N \cdot Y_t)(\sum_{i=1}^N \frac{C^*_{i,t}}{\gamma_i})^{-1}$, and the precautionary saving demand $-\frac{\lambda_t^2}{2}(\sum_{i=1}^N \frac{C^*_{i,t}}{\gamma_i})^{-1}\sum_{i=1}^{h_t} \frac{C^*_{i,t}}{\gamma_i}(1+\frac{1}{\gamma_i})$. First, the consumption smoothing term is the expected aggregate consumption growth[33] multiplied by aggregate average risk aversion: $\frac{\mu_d D_t + \mu_y N\cdot Y_t}{\sum_{i=1}^N C^*_{i,t}} \cdot (\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}}\frac{1}{\gamma_i})^{-1}$. Note that the expected aggregate consumption growth is time-varying in our economy while without labor income, the expected aggregate consumption growth is simply expected dividend. Second, the precautionary saving demand term is a function of the Sharpe ratio and the stockholders' consumption. Therefore, if every investor is a non-stockholder, the precautionary saving demand is zero in our model. When we consider the economy without labor income, the precautionary saving term reduces to $-\frac{\sigma_d^2}{2}(\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}}\frac{1}{\gamma_i})^{-3}\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}}\frac{1}{\gamma_i}(1+\frac{1}{\gamma_i})$. Finally, by shutting down both heterogeneity and labor income, we recover the expression of the risk-free rate in the simplest representative economy is $r_t = \delta + \mu_d\gamma - \frac{\sigma_d^2}{2}\gamma(1+\gamma)$.

Stock volatility

The equilibrium stock price in (1) has two Brownian motions given the power preference. Accordingly, the total stock volatility is represented by two volatility parameters (i.e., $\sigma_{s,t} = \sqrt{(\sigma^d_{s,t})^2 + (\sigma^y_{s,t})^2 + 2\rho\sigma^d_{s,t}\sigma^y_{s,t}}$). The level of $\sigma^d_{s,t}$ and $\sigma^y_{s,t}$ are closely related to the parameters values of σ_d and σ_y. Since the labor income volatility σ_y contributes to total stock volatility

[33] That is $\frac{1}{dt}E_t[\frac{d\sum_{i=1}^N C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}}] = \frac{\mu_d D_t + \mu_y N\cdot Y_t}{\sum_{i=1}^N C^*_{i,t}}$

in addition to the dividend volatility σ_d, our model provides the setting under which labor income risk plays a role in explaining the excess volatility puzzle[34]. The role of the labor income volatility on the stock volatility is discussed in details in Appendix G.

If we turn off the labor income ($Y_t = 0$). Then, the stock volatility is represented by one parameter (i.e., $\sigma_{s,t} = \sigma^d_{s,t}$) and its value is $\sigma_{s,t} = \sigma_d(\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sum_{i=1}^N \frac{X_{i,t}}{\sum_{i=1}^N X_{i,t}} \frac{1}{\gamma_i}$. This equation shows that the excess volatility can also be generated under the complete market[35] if the cross-sectional wealth distribution is more tilted towards risk-tolerant investors than consumption distribution (i.e., $\sum_{i=1}^N \frac{X_{i,t}}{\sum_{i=1}^N X_{i,t}} \frac{1}{\gamma_i} > \sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i}$). Bonsang et al. (2005) indeed documents empirically that consumption is more evenly distributed than wealth. As discussed in the Section 3, we find that in our economy more risk-averse investors consume more than others and it takes a long time for the consumption redistribution to be tilted toward the least risk-averse investor. This finding explains why the inequality holds in our setting for a long time.

Stock price

We solve for the equilibrium stock price in closed form as follows. On one hand, from the optimal consumption in (7) and (8), we have the relation between the optimal consumption and financial wealth. The consumption clearing condition in (16) shows that the aggregate consumption should be the sum of the dividend level and the aggregate labor income. Using these latter three equations give the relation between the aggregate financial wealth level and both dividend and labor income. On the other hand, relying on the asset market clearing conditions in (13) and (14), we show that the equilibrium stock price is equal to the aggregate financial wealth. Using these two expressions together, we obtain the closed form solution for the equilibrium stock price without relying on the SDF in equation (20).

The first term D_t/r_t is the stock price when $\mu_d = \sigma_d = Y_t = 0$. The second term shows the effect of labor income on the stock price. The expected dividend and labor income growth

[34] See Shiller (1981), LeRoy and Porter (1981), Keim and Stambaugh (1986), Campbell and Shiller (1988), Cochrane (1992), and Hodrick (1992), for example, among others.

[35] Other theoretical papers in the heterogeneous investor literature also suggest that heterogeneity 'may' lead to excess volatility. See Bhamra and Uppal (2009), Cvitanić et al. (2012) and Bhamra and Uppal (2014), for example.

appear to be positively associated with the stock price as shown in the third term. However, an increase in these quantities can also lead to the decrease in the stock price due to the increase in the risk-free rate (discount rate channel). A positive shock to labor income leads to an increase in financial wealth given the CRRA preference. Since the aggregate financial wealth is equal to the equilibrium stock price - an increase in the financial wealth drives up the equilibrium stock price. Since non-stockholders affect the stock price through this channel, the stock price also depends on non-stockholders-related terms. Finally, the Sharpe ratio appears to be negatively associated with the stock price as shown in the third and last terms. However, an increase in the Sharpe ratio also leads to a decrease in the risk-free rate through precautionary saving motive, driving up the stock price.

Without labor income, the stock price is $S_t = \frac{D_t}{r_t} + \frac{1}{r_t}\sum_{i=1}^{N}\frac{X_{i,t}}{\gamma_i}[\mu_d(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\sum_{i=1}^{N}C_{i,t}^*}\frac{1}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}((\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1}\sum_{i=1}^{N}\frac{C_{i,t}}{\gamma_i}(1+\frac{1}{\gamma_i}) + \frac{\gamma_i - 1}{\gamma_i})]$. If we further simplify the economy by considering a representative investor, the equilibrium stock price is $S_t = \frac{D_t}{r_t - \mu_d + \gamma\sigma_d^2} = \frac{D_t}{\delta + \mu_d(\gamma-1) - \frac{\sigma_d^2}{2}\gamma(\gamma-1)}$, the same as in the existing studies (e.g., Cvitanić et al. (2012)). If there is no uncertainty on dividend stream ($\sigma_d = 0$), the equilibrium stock price becomes the one in the Gordon's dividend model ($S_t = \frac{D_t}{r_t - \mu_d}$).

4.3 Cut-off Stockholder

We turn to the problem of determining the cut-off stockholder (h_t). As shown in **Proposition 2**, all endogenous asset parameters are a function of the cut-off stockholder ($\lambda_t(h_t), r_t(h_t), \sigma_{s,t}(h_t)$). Also, since each investor's optimal stock holding is a function of these endogenous asset parameters $\pi_{i,t}^*(\lambda_t, r_t, \sigma_{s,t})$, the optimal holding is also a function of the cut-off stockholder $\pi_{i,t}^*(h_t)$. Hence an investor i's decision to be a cut-off stockholder ($h_t = i$) changes not only i's optimal stock holding but also every other agent' optimal stock holding. In this nature of the problem, we further restrict the equilibrium to the Nash Equilibrium to preclude each investor from optimally deviating from a stockholder to non-stockholder or vice versa, given the cut-off stockholder (h_t).

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Definition 2. *An equilibrium is a set of processes* $\{r_t(h_t^*), \mu_{s,t}(h_t^*), \sigma_{s,t}(h_t^*)\}$ *and consumption* *and investment policies* $\{C^*_{i,t}(h^*_t), \pi^*_{i,t}(h^*_t)\}_{i\in 1,\ldots,h^*_t}$ *and* $\{C^*_{i,t}(h^*_t)\}_{i\in h^*_t+1,\ldots,N}$ *which maximize the sum of life time expected utility (2) for each investor and satisfy the securities market-clearing conditions (13) and (14) such that short-selling is not allowed and* h^*_t *satisfies the following.*

$$1.\ \pi^*_{i,t}(h_t; h_t = h^*_t) \geq 0 \qquad \forall i = 1, ..., h^*_t \tag{22}$$

$$2.\ \pi^*_{i,t}(h_t; h_t = i) < 0 \qquad \forall i = h^*_t + 1, ..., N \tag{23}$$

The first condition in (22) states that given the cut-off stockholder $h_t = h^*_t$, the investors who are less risk-averse than the investor h^*_t have positive stock holding and therefore, they remain in the stock market. The second condition in (23) guarantees that when an investor who is more risk-averse than the investor h^*_t enters the stock market and becomes the cut-off stockholder, her optimal stock holding is negative and therefore, she cannot be a stockholder given short-selling constraint. **Proposition 3** shows how h^*_t who satisfies the **Definition 2** can be determined.

Proposition 3. *The investor* h^*_t *is*

$$h^*_t \equiv \arg\min_i \pi^*_{i,t}(h_t; h_t = i) \ \text{ s.t. } \pi^*_{i,t}(h_t; h_t = i) > 0 \tag{24}$$

By the monotonicity of $\pi^*_{i,t}(h_t; h_t = h^*_t)$ with respect to i, h^*_t defined as in **Proposition 3** satisfies the first condition in (22). Also, By the monotonicity of $\pi^*_{i,t}(h_t; h_t = i)$ with respect to i, h^*_t satisfies the second condition in (23). Figure 2 also visually confirms that h^*_t in **Proposition 3** guarantees no-deviation. Consequently, given the short-selling constraint, the investors who are more risk-averse than the investor h^*_t leave the stock market. However, since the optimal stock holding is time-varying $\pi^*_{i,t}$, non-stockholders can be a stockholder at different point in time.

4.4 Consumption risk

In the canonical consumption-based asset pricing model with a representative investor, under no-arbitrage assumption, the conditional equity premium is the quantity of consumption risk measured by the conditional covariance between stock returns and consumption growth multiplied by the price of consumption risk represented by risk aversion[36].

$$E_t[dR_t^e] = \underbrace{\gamma_t}_{\textit{Price of risk}} \cdot \underbrace{Cov_t(dR_t^e, dC_t^*/C_t^*)}_{\textit{Quantity of risk}} \tag{25}$$

where $dR_t^e \equiv \frac{dS_t + D_t dt}{S_t} - r_t dt$ is the total instantaneous excess equity return, $\frac{dC_t^*}{C_t^*}$ is the consumption growth, and γ_t ($\equiv -\frac{C_t^* u''(C_t^*)}{u'(C_t^*)}$) is the coefficient of relative risk aversion. The price of risk measures the required compensation for one unit of risk. If the representative investor has the power utility, γ_t is constant over time ($\gamma_t = \gamma$). By contrast, in habit preferences (Constantinides (1990) and Campbell and Cochrane (1999)), γ_t is time-varying risk aversion. The following proposition shows the equilibrium equity premium in the economy in which stock market participation is time-varying[37].

Proposition 4. *In an economy where market participation is time-varying, the equilibrium equity premium is given by*

$$E_t[dR_t^e] = \underbrace{\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}}_{\textit{Price of risk}} \cdot \underbrace{Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*})}_{\textit{Quantity of risk}} \tag{26}$$

Proof : See Appendix B.5

Proposition 4 shows that among all investors, it is the consumption of stockholders ($\forall i = 1, ..., h_t^*$) which determines the equity premium and the consumption of non-stockholders ($\forall i = h_t^* + 1, ..., N$) does not affect the equity premium directly. Moreover, in this economy, time-varying market participation h_t^* is one of the sources of time-variation in both the price and quantity of risk.

The quantity of risk is proportional to the degree of imperfect risk-sharing among stockholders at each time. The higher the number of *heterogeneous* stockholders -good times-, the higher risk-sharing and hence lower quantity of risk. This mechanism generates a countercyclical quantity of risk in our economy. This is in contrast to the empirical findings assuming the full market participation (See Duffee (2005), Santos and Veronesi (2006), and Sarkar and

[36]For proof, see Appendix B.4
[37]For individual stocks, see Appendix C.

Zhang (2009)[38]). The reason for pro-cyclical covariance between the aggregate consumption growth and stock returns in their paper is follows. During good times, the proportion of stockholders increases which in turn makes aggregate consumption more correlated with stock returns[39]. While in our economy it is the time for which risk-sharing is reduced.

In regards to the price of risk, it is derived as the consumption-weighted harmonic mean of stockholders' risk aversion $\sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}$. The following Corollary describes one of its properties.

Corollary 1. *The price of consumption risk is positively associated with* h_t^*

Since stockholders are less risk-averse than non-stockholders, the non-stockholder who enters the stock market is more risk-averse than the existing stockholders. Therefore, the entry of an investor (increase in h_t^*), holding the consumption redistribution constant, leads to the increase in the (harmonic) mean of stockholders' risk aversion. In addition, if the stock market participation is pro-cyclical, the stockholders' average risk aversion is likely to be even pro-cyclical.

To summarize, with time-varying market participation, a low level of the price of risk, a high level of the quantity of risk, and a less counter-cyclical or even pro-cyclical variation in the price of risk are generated. **Corollary 2** shows how the equity premium is associated with the covariance between aggregate consumption growth and stock returns. The expression in this corollary helps understand, from the lenses of our model, the high levels of price of risk as well negative risk return trade-off generated in the extant literature when full participation was tested.

Corollary 2. *In an economy where market participation is time-varying, the association*

[38]Duffee (2005) and Sarkar and Zhang (2009) empirically find that the conditional covariance between aggregate consumption and stock returns is positively associated with the stock market wealth-to-consumption ratio. Santos and Veronesi (2006) theoretically predicts that the conditional covariance between aggregate consumption and stock returns is negatively associated with labor-to-consumption ratio, which is consistent with the positive association with the covariance and the stock market wealth-to-labor ratio in a setting under which the source of consumption is only financial wealth and labor income.

[39]Another interpretation of this procyclicality was discussed in Duffee (2005). When stock market wealth accounts for larger proportion of consumption, the change in consumption becomes more sensitive to the change in stock market wealth. Therefore, when the ratio of stock market wealth to labor income is high -good times- the covariance between aggregate consumption growth and stock returns becomes high.

between the equilibrium equity premium and the conditional covariance of the aggregate consumption growth with stock returns is given by

$$E_t[dR_t^e]$$
$$= \frac{\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*}) - \frac{1}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} \sum_{i=h_t^*+1}^{N} \frac{C_{i,t}^* \sigma_y(\rho\sigma_{s,t}^d + \sigma_{s,t}^y)}{X_{i,t}(r_t - \mu_y)/Y_t + 1} \tag{27}$$

Proof : See Appendix B.5

The previous empirical tests[40] of the conditional consumption-based asset pricing have modeled the equity premium as follows.

$$E_t[dR_t^e] = \alpha + \Gamma_t Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*}) \tag{28}$$

By equating (27) with equation (28), the implied price of risk in the empirical studies using both stock 'h'older and 'n'on-stockholders' consumption can be recovered as

$$\hat{\Gamma}_t^{HN} \equiv \frac{\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} - \frac{\sum_{i=h_t^*+1}^N \frac{C_{i,t}^* \sigma_y(\rho\sigma_{s,t}^d + \sigma_{s,t}^y)}{X_{i,t}(r_t-\mu_y)/Y_t+1} + \alpha}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i} Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*})} \equiv \frac{\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} - a_t \tag{29}$$

By contrast, the price of risk using the stockholders' consumption only is $\hat{\Gamma}_t^H \equiv \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$.

Note that, contrary to $\hat{\Gamma}_t^H$, $\hat{\Gamma}_t^{HN}$ could be negatively associated with h_t^* unless the second term a_t varies considerably as a function of h_t^*. This is because, in the first term, a new entry of investors (increase in h_t^*) raises only the denominator. This negative relation can explain why the price of risk in the empirical studies assuming full participation is so countercyclical. The conjecture of negative relation will be tested by the simulation in Section 5. In addition, (29) shows that why $\hat{\Gamma}_t^{HN}$ could be negative. If OLS intercept α is sufficiently large positive, $\hat{\Gamma}_t^{HN}$ can be negative as observed in the empirical findings. This suggests a possibility that the negative price of risk from the data could be due to the full market participation assumption that leads to a poor measure of consumption risk.

[40] For example, Duffee (2005) considers the time-varying price of risk Γ_t with the aggregate consumption $\sum_{i=1}^N C_{i,t}^*$.

5 Simulation

To simulate the model, we map the current theoretical economy into the United States. The continuous model is discretized and simulated in monthly time increments. To choose parameter values, the U.S. dividend and labor income data from 1960 to 2015 are used and for a similar time span, a total of 500 months is considered in the simulation. The annualized parameter values in the simulation are reported in Table 1 and a detailed description of the data source is in Appendix D. Section 5.1 studies how the equilibrium asset parameters vary depending on the different cut-off stockholders. In Section 5.2, we study the dynamics of stock market participation. In Section 5.3, we investigate the dynamics of consumption risk. Finally, in Section 5.4, the unconditional asset moments are examined.

5.1 The Asset Parameters with Cut-off Stockholder

In this section, we examine how the endogenous asset parameters $(\lambda_t, \sigma_{s,t}, r_t)$ vary as function of different cut-off with different risk aversion $h_0 = i$ at given time to understand the effect of entry of investors. Figure 3 shows the result for the investors from $i = 3$ to $i = 30$ at time 0 as an example. Panel A and B plot the Sharpe ratio and equity premium, respectively[41]. Note that they are not monotonic in the entry of higher risk-averse investor. When the least risk-averse investor is the only stockholder, the highest Sharpe ratio and equity premium are attained because there should be substantial compensation for bearing the market risk alone. As more investors enter the market, the Sharpe ratio and equity premium keep decreasing. However, at some point, they are turning to increasing as the investors who want to optimally short-sell the stock enter the market. This is because a higher compensation is required in the market in response to increasing selling demand. Since we impose short-selling constraint, the cut-off stockholder is h_t^* who has the lowest positive stock holding. This finding implies that allowing market participation to be determined by individuals' optimal choice makes it harder to explain the equity premium. As long as the equity premium is sufficiently high, the non-stockholders are willing to enter the market. This entry of more risk-averse investors decreases the equilibrium, whereas in a setting where the

[41] To save space, the result for the stock volatility is not reported. It has the same pattern as the Sharpe ratio and equity premium.

limited market participation is exogenous, a high equity premium can be attained because no matter how high the equity premium is, the non-stockholder cannot enter the market, thus not decreasing the equity premium.

Panel C decomposes the equity premium into the quantity and price of risk as given in **Proposition 4**. This shows that the price of risk with limited market participation ($i = 12$) is lower than one in full market participation ($i = 30$). This is because under limited market participation, the remaining stockholders are less risk-averse than non-stockholders and thus they do not require a huge compensation for the risk. Also, due to limited risk-sharing across the remaining stockholders, the quantity of risk is higher than the one in full market participation. This result also provides another intuition of why the equity premium is increasing at some point. The increasing price of risk dominates the decreasing quantity of risk at h_t^*.

Panel D plots the risk-free rate with different cut-off stockholder ($h_t = i$). As studied in Section 4.2, the market participation affects the risk-free rate only through precautionary saving motive. The precautionary saving term depends on the Sharpe ratio and the stockholders' aggregate consumption[42]. The precautionary saving motive initially decreases due to decreasing consumption risk with better risk-sharing. But, at some point, increasing the stockholders' consumption level drives up the the precautionary saving term, decreasing the risk-free rate.

5.2 The Dynamics of Stock market participation

In this section, we discuss the behavior of time-varying stock market participation. At first, in order to examine the general distribution of stock market participation across time, we simulate 10,000 economies at a monthly frequency. Panel A of Figure 4 presents the simulated kernel distribution of the cut-off stockholder h_t^*. It shows the symmetric distribution of h_t^* with mean near 12th agent. Second, we study how stock market participation varies over time depending on the economic states represented by the stock market wealth-

[42]If the economy is populated only by non-stockholders, the precautionary saving terms do not exist in this model.

to-(aggregate) labor ratio $\frac{S_t}{N \cdot Y_t}$[43]. Panel B of Figure 4 plots one sample path of time-variation of the stock market participation h_t^* in association with the stock market wealth-to-labor ratio $\frac{S_t}{N \cdot Y_t}$. Notably, it shows that they are strongly positively correlated each other[44]. The economic intuition for positive relation is as follows. A positive shock to the stock price relative to labor income shock induces investors to invest in the stock market more due to sufficient liquidity and also reduced labor income risk. This leads a non-stockholder who is more risk-averse than the existing stockholders to invest in the stock.

5.3 The Dynamics of Consumption Risk

In this section, we test how both the price and quantity of risk behave along with the economic state if the aggregate consumption is used to measure the consumption risk in an economy where the stock market participation is limited. To compare this case with the one using the only consumption of stockholders, we test following hypothesis.

Hypothesis 1 : $\hat{\Gamma}_t^{HN} = \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} - a_t$ is negatively associated with $\frac{S_t}{N \cdot Y_t}$ and h_t because market participation changes only denominator.

Hypothesis 2 : $\hat{\Gamma}_t^{H} = \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$ is positively associated with $\frac{S_t}{N \cdot Y_t}$ and h_t by **Corollary 1**.

Hypothesis 3 : $Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*})$ is positively associated with $\frac{S_t}{N \cdot Y_t}$ and h_t, due to the composition effect for $\frac{S_t}{N \cdot Y_t}$ and, as for h_t, due to the larger proportion of stockholders whose consumption is strongly correlated with stock returns.

Hypothesis 4 : $Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*})$ is negatively associated with $\frac{S_t}{N \cdot Y_t}$ and h_t due to higher risk-sharing with more stockholders.

We run the pooled OLS panel regression with 100 simulated economies. Panel A of Table 2 reports the regression of the price of risk on $\frac{S_t}{N \cdot Y_t}$ or h_t. The dependent variable in the first and second row of the table is the $\hat{\Gamma}_t^{HN}$ when both stockholders' and non-stockholders consumption (denoted by HN) are used. The dependent variable in the third and last row of the table is $\hat{\Gamma}_t^{H}$. The result shows that $\hat{\Gamma}_t^{HN}$ has a counter-cyclical variation, similar to the

[43]We use this ratio as a state variable because of its role in determining the market participation as demonstrated in (12). Also, the literature uses the ratio or similarly the stock market wealth-to-consumption ratio as a state variable (See Koo (1998), Lettau and Ludvigson (2001), Duffee (2005), Roussanov (2014), Wang et al. (2016))

[44]31.58% on average for 10,000 simulations

implied price of risk in the existing studies[45], confirming **Hypothesis 1**. Contrary to this, $\hat{\Gamma}_t^H$ is pro-cyclical, confirming **Hypothesis 2**[46]. This result implies that an increase in the market participation level during good states leads to the higher stockholders' average risk aversion.

Also, Panel B shows that the quantity of risk measured by the aggregate consumption is pro-cyclical, replicating the composition effect of Duffee (2005). By contrast, when it comes to the quantity of risk measured by the stockholders' consumption, it is counter-cyclical due to higher risk-sharing with more participation h_t when the stock valuations $\frac{S_t}{N \cdot Y_t}$ are high, confirming **Hypothesis 4**. To summarize, the result shows that if the aggregate consumption is used in an economy where stock market participation is time-varying, the quantity of risk is pro-cyclical and hence requires a huge counter-cyclical variation in the price of risk to account for the time-variation in the equity premium as in the current empirical studies. However, the counter-cyclical quantity of risk measured only by stockholders' consumption relaxes the required dramatic counter-cyclical variation in the price of risk

5.4 The Unconditional of Asset Prices

In this section, we study the unconditional moments based on the simulated data. First, we examine how the consumption of stockholders differs from the aggregate consumption. Table 3 presents the standard deviation, covariance, and implied price of risk given the equity premium 4.44% the model generates. As in the previous empirical finding (Mankiw and Zeldes (1991)), the consumption of stockholders is more volatile and correlated with stock returns than that of the aggregate investors, implying the low level of the price of risk ($\hat{\Gamma}$). This simulation result provides a part of the explanation for why the previous empirical studies document the high level of the implied price of risk using the aggregate consumption.

Second, we compare the unconditional moments of consumption growth and stock returns which the model generates to the corresponding empirical moments for the U.S. Table 3 reports the result. Consumption is endogenous in the model, and therefore it is of particular interest to produce empirically plausible moments for consumption growth. Panel A presents

[45] Duffee (2005), Sarkar and Zhang (2009), and Roussanov (2014)

[46] Although our model generates the pro-cyclical price of risk with CRRA investors, the result with habit-forming utility function would be a less counter-cyclical variation in the price of risk.

the unconditional consumption growth moments observed in the data and the model. In a theoretical model, without labor income, the consumption is equal to dividend stream, and therefore the moments of consumption growth should be equal to those of dividend growth. However, the empirically observed moments of dividend growth (4% and 9%) are much higher than those of consumption growth (1.8% and 1.37%). In our model, with labor income as an additional source of consumption, the moments of consumption growth have the reasonable values because the moments of labor income growth (1% and 3%) reduces the moments of consumption growth.

Panel B reports the unconditional moments of the equity premium, stock volatility, and correlation between stock returns and aggregate consumption growth. First, the average equity premium the present model generates is 4.44% that compares to 5.98% in the data. We further decompose the equity premium into the price of risk $\sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}$ and the quantity of risk $Cov(dR_t^e, \frac{\Delta \sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*})$. When it comes to the quantity of risk, the model generates the slightly higher quantity by 0.14%. As for the stock volatility, its value is higher than the volatility of its fundamental - dividend volatility. As studied in Section 4, this is because a heterogeneity and labor volatility account for the stock volatility in the excess of dividend volatility. Finally, we compute the the average market participation rate. The model generates around 40% market participation rate which is higher than 30% market participation.

6 Empirical test

In this section, we empirically test our conditional consumption asset pricing model using micro-level household data from the Consumer Expenditure Survey (CE) for the period 1996-2015. The CE data provide monthly data on expenditure, income, and demographic characteristics of the sample households in the United States. Throughout the section, our aim is to compare the empirical result using consumption of stockholders only to the ones using aggregate consumption as in the extant literature. The aggregate consumption including both stock'h'olders and 'n'on-stockholders' consumption is denoted by HN and the consumption of stockholders is denoted by H. we describe our data briefly in the next section but we leave a a detailed description of the data to Appendix D.

6.1 Specification

We mainly follow Duffee (2005) for the empirical specification. We first model excess stock returns and consumption growth as a function of respective set of instruments (First stage regression). The forecasting regressions for excess stock returns and consumption growth are given by

$$r_t^e = X'_{r^e,t-1}\beta_{r^e} + \epsilon_{r^e,t}, \qquad \Delta c_{j,t} = X'_{c_j,t-1}\beta_{c_j} + \epsilon_{\Delta c_j,t} \qquad \forall j = HN, H \tag{30}$$

where r_t^e denotes the log change in real per capita total market value for NYSE/Amex/Nasdaq minus 1-month real U.S. T-bill, $\Delta c_{j,t}$ denotes the log change in real per capita consumption, β_{r^e} and β_{c_j} are parameter vectors and the vectors $X_{r^e,t-1}$ and $X_{c_j,t-1}$ are instrument variables for excess stock returns and consumption growth, respectively. The product of the residuals from the first stage regression is the ex-post covariance between excess stock returns and consumption growth.

$$Cov^*(r_t^e, \Delta c_{j,t}) = \hat{\epsilon}_{r^e,t}\hat{\epsilon}_{\Delta c_j,t} \ \forall j = HN, H \tag{31}$$

To capture the conditional covariance which is the expected covariance given the conditional information available to investors at time $t-1$, the ex-post covariance is projected on a set of instruments X_{t-1} in the second stage regression.

$$Cov^*(r_t^e, \Delta c_{j,t}) = X'_{t-1}\beta_j + \epsilon_{j,t} \qquad \forall j = HN, H \tag{32}$$

Finally, a realized excess stock returns corrected for the Jensen's inequality is regressed on the conditional covariances obtained from the second stage regression.

$$r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,j} + [b_{1,j} + b_{2,j}p_{t-1}]\widehat{Cov}_{t-1}(r_t^e, \Delta c_{j,t}) + u_{j,t} \qquad \forall j = HN, H \tag{33}$$

$\hat{\epsilon}^2_{r^e,t}$ is the term to capture $Var(r_t^e)$ for the Jensen's inequality correction. p_{t-1} is an observable proxy for the price of risk. $\widehat{Cov}_{t-1}(r_t^e, \Delta c_{j,t})$ denotes the fitted value of second stage regression. In the end, $\hat{\Gamma}_{j,t} \equiv \hat{b}_{1,j} + \hat{b}_{2,j}p_t$ represents the empirically implied price of risk. In testing the above equation, we first consider the time-invariant price of risk case where $b_{2,j} = 0$. We also examine time-varying price of risk case where $b_{2,j} \neq 0$. In this case, $b_{1,j}$ allows for capturing the value of price of risk when the observable proxy is zero ($p_{t-1} = 0$).

Finally, to directly test the equilibrium equity premium equation in **Proposition 4**, we consider the specification of $b_{1,H} = 0$ with $p_t = \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$ as prescribed by theory. Throughout the empirical analysis, we use OLS with Newey and West (1987) robust standard error.

6.2 Instrument variables

We cannot observe the true conditional excess stock returns, consumption growth and conditional covariance which investors had at each point of time. Therefore, it is essential to infer the conditional expectation using the instrument variables that are most likely to be an element of investors' information set. First, we run a 'Kitchen-Sink' regression based on fourteen variables in Welch and Goyal (2008) and Rapach and Zhou (2013). Based on the explanatory power of those variables, we select the the specification which fits the empirical model the best. For ($X_{r^e,t-1}$) we select log dividend-price ratio ($logD/P_{t-1}$), Book-to-market ratio (BM_{t-1}), Net equity expansion ($NTIS_{t-1}$), and Long-term yield (LTY_{t-1}). As for the instruments of consumption growth ($X_{c,t-1}$), given the autocorrelation of monthly growth rate, we use the lagged consumption growth for month t through $t-3$ ($\Delta c_{j,t-1}$, $\Delta c_{j,t-2}$, $\Delta c_{j,t-3}$ $\forall j = HN, H$). Following our theoretical derivation of optimal consumption we also include the lagged variable of 1-month T-bill (r_{t-1}) as an instrument for consumption growth. For the instruments of the conditional covariance (X_{t-1}), the stock market wealth-to-consumption ratio ($M/C_{j,t-1}$ $\forall j = HN, H$) is considered in light of the composition effect (Duffee (2005), Lustig and Nieuwerburgh (2008), and Lustig et al. (2013)). Also, the log dividend-price ratio ($logD/P_{t-1}$) is included to capture the economic states which the composition effect cannot pick up. Moreover, following Duffee (2005), we include the lagged variable of ex-post covariance ($\hat{\epsilon}_{r^e,t-1}\hat{\epsilon}_{\Delta c_j,t-1}$ $\forall j = HN, H$). We also consider the volatility of stock returns and consumption growth ($\sqrt{\hat{\epsilon}^2_{r^e,t}}$ and $\sqrt{\hat{\epsilon}^2_{\Delta c_j,t}}$) in a sense that the covariance is the product of these two volatilities and correlation.

For the instruments for the p_{t-1}, we consider the stock market wealth-to-consumption ratio as in (Duffee (2005)), the the log dividend-price ratio and surplus consumption as measured in Wachter (2002)[47]:

$$s_t \equiv \frac{1-0.96}{1-0.96^{40}} \sum_{j=0}^{40} 0.96^j \Delta c_{t-j} \tag{34}$$

In order to directly test **Proposition 4**, we still need to proxy for relative risk aversion in the implied price of risk. We use two proxies. The first is to assume that the relative risk aversion is inversely associated with financial wealth invested in a stock ($\gamma_i \propto \frac{1}{W_i}$) based on the literature[48]. Theoretically, the more risk-averse the investor is, the less amount of wealth invested in stock. The second proxy is to assume that the relative risk aversion is inversely associated with the probability of having a positive financial wealth ($\gamma_i \propto \frac{1}{Prob(\tilde{W}_i > 0)}$). Based on these two assumptions, we construct two measures for the model-implied price of risk $\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\hat{\gamma}_{m,i}}}$. $\quad \forall m = 1, 2$

$$\text{where} \quad \hat{\gamma}_{1,i} = \frac{k}{W_i}, \quad \hat{\gamma}_{2,i} = \frac{k}{\Phi(X_{i,CE}' \hat{\beta}_{SCF})}, \quad k \text{ is any constant.} \tag{35}$$

In CE survey, a household is asked the total value of stocks, mutual funds, and bonds in his/her fifth interview. Thus, we can obtain one numerical value for one household and we convert it to real dollar value using September 2010 dollars for $\hat{\gamma}_{1,i}$. as a result, we can only identify 'likely' stockholders in CE because the question is based on responses to the combined holdings of 'Stocks, mutual funds, and bonds'. By contrast, in the Survey of Consumer Finances (SCF) we can accurately identify who owns stocks and mutual funds. We estimate a Probit regression of whether a household owns stock on a set of observable characteristics known to affect stock-holdings[49]. The estimates of the coefficients from the Probit model in the SCF are applied to the CE data to calculate the probability of being a stockholder for each household in our CE sample. Since the observable households characteristics $X_{i,CE}$ are time-invariant variable, we can obtain the time-invariant probability of being a stockholder

[47]Since CE data are based on sample households, it is limited to accurately represent the habit level and economic states. Therefore, in measuring the stock market wealth-to-consumption ratio and surplus consumption, we rely on the consumption (nondurables and services) from the total U.S consumption data (NIPA; National Income and Product Accounts) as in Duffee (2005).

[48]See King and Leape (1998), Riley and Chow (1992), Donkers et al. (2001), Guiso and Paiella (2008), and Bucciol and Miniaci (2011), for example, among others.

[49]A detailed description of SCF data, a set of characteristics, and Probit regression results are in Appendix E.

for each household and construct a proxy of time-invariant risk aversion $\hat{\gamma}_{2,i}$. Note that the constant k in $\hat{\gamma}_{1,i}$ and $\hat{\gamma}_{2,i}$ does not affect the estimated price in our following regression.

$$r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,m} + b_{2,m}\frac{\sum_{i=1}^{h^*_{t-1}} C^*_{i,t-1}}{\sum_{i=1}^{h^*_{t-1}} \frac{C^*_{i,t-1}}{\hat{\gamma}_{i,m}}}\widehat{Cov}_{t-1}(r_t^e, \Delta c_{H,t}) + u_{m,t} \quad \forall m = 1, 2 \tag{36}$$

After constructing the measures, we simply examine two things. First, we calculate the correlation between the two measures $\left(\frac{\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\hat{\gamma}_{1,i}}}, \frac{\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\hat{\gamma}_{2,i}}}\right)$ and the correlation coefficient is 0.463, implying that they contain similar information. Second, we also test whether two measures for the model-implied price of risk are positively associated with the time-varying stock market as in the model. For this end, we separately regress two measures on time-varying stock market participation captured by the participation rate at each point of time. Table 4 reports the result. Consistent with our **Corollary 1**, the model-implied price of risk is positively associated with time-varying stock market participation at 1% significance level. This indicates that empirically when a higher proportion of households invests in the stock market, the average stockholders' risk aversion captured as in this paper increases[50].

6.3 Empirical Findings

Table 5 reports the first and second stage regressions to obtain the conditional covariance. Panel A shows low $AdjR^2$ of predicting stock returns compared to that of consumption growth as in the literature on return prediction. Although not reported, all selected instruments are significant at the conventional level. Note that the 1-month T-bill is highly significant in predicting future consumption growth consistent with our theory. Different from Panel A, the results in Panel B show that time-variation in ex-post covariance between consumption growth and excess stock returns are not well explained by the set of instruments. As Duffee (2005) points out, it is difficult to capture time-variation in conditional covariance.

Based on the constructed ex-post conditional covariance between excess stock returns and consumption growth, we first calculate the unconditional covariance. The result is

[50]Moreover, unreported regression of h_t^* on M/C_{t-1} shows that its coefficient is 0.021 and statistically significant at 5 percent level using T-statistic based on Newey and West (1987). This implies, consistent with our theory, that many households choose to enter the market in response to the favorable state of the market.

reported in Table 6. The consumption growth of stockholders is more volatile and more correlated with excess stock returns than that of aggregate investors by more than twice. This higher covariance of stockholders' consumption with excess stock returns leads to the more plausible price of risk (22.16) than the one calculated by the aggregate consumption (49.74). This results shed lights on why the previous studies find extremely high price of risk unconditionally (Ferson and Harvey (1993) and Duffee (2005))[51].

Turing to a conditional test, we assess whether the conditional covariance using our setting can improve over the previous tests in predicting excess stock returns with positive and reasonable level of the implied price of risk. First, Table 7 reports the result with the assumption of the time-invariant price of risk (i.e., $r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,j} + b_{1,j}\widehat{Cov}_{t-1}(r_t^e, \Delta c_{j,t}) + u_{j,t}$). The coefficient on conditional covariance constructed by the aggregate consumption (HN) is not significant and negative, similar to the result in Duffee (2005). By contrast, if the only consumption of stockholders is considered, first, the statistical relation between risk-return improves. Also, most importantly, its coefficient, which is the implied price of risk ($\hat{\Gamma}$) in this specification, has nonnegative and plausible value (19.73). This result shows a positive risk-return trade-off and also support for our limited participation consumption-based asset pricing model.

Panel A of Table 8 reports the regression results with the assumption of time-varying price of risk (i.e., $r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,j} + [b_{1,j} + b_{2,j}p_{t-1}]\widehat{Cov}_{t-1}(r_t^e, \Delta c_{j,t}) + u_{j,t}$) with the stock market wealth-to-consumption as a proxy for p_{t-1}. The implied price of risk ranges from -137 to 100 when the aggregate consumption is used, whereas it ranges from -19 to 58 using the consumption of stockholders only. Although 7.59% of the sample produces a negative price of risk, in this case, this compares to 66.2% of the sample when the aggregate consumption is used. Most notably, the magnitude of the coefficient for H is lower than the one using the aggregate consumption case HN and the coefficient is indistinguishable from zero. This result implies the statistical evidence of the counter-cyclical time-variation in the price of risk is very weak when the consumption of stockholders only is used to measure the consumption

[51] Using NIPA dataset instead,Ferson and Harvey (1993) find the implied coefficients of price of risk are 147.493, 318.265, 39.828, and 193.748 for non-seasonally adjusted services, seasonally adjusted services, non-seasonally adjusted nondurables, and seasonally adjusted nondurables, respectively in their Table 3. Also, Duffee (2005) documents 160 as the implied price of risk.

risk. If time-variation in the price of risk is mainly driven by time-varying individual risk aversion, we expect the statistical significant counter-cyclical variation in the price of risk.

The quite similar results are obtained in Panel B and C using the surplus consumption and log dividend-to-price ratio, respectively. However, the implied price of risk is never negative in these two panels. To summarize the results in this table, conditional covariance constructed by the consumption of stockholders produces a plausible price of risk and counter-cyclicality is not supported compared to the the one using the aggregate consumption. to explore teh countercyclicality a bit more, Figure 5 depicts the estimated time-varying price of risk $\hat{\Gamma}_{j,t}$ as a linear function of the stock market wealth-to-consumption ratio $(\hat{b}_{1,j} + \hat{b}_{2,j} p_t)$ based on the result in Panel A of Table 8. The shaded area is rescaled kernel density of the stock market wealth-to-consumption. Both HN and H cases have the negative slope with respect to the stock market wealth-to-consumption ratio, implying the counter-cyclical price of risk. However, when the consumption of stockholders is used, the slope is less steep. This shows that the implied price of risk is less counter-cyclical when the consumption of stockholders is only used than the one using the aggregate consumption.

Table 9 shows the result of (36). Since the model-implied price of risk $\frac{\sum_{i=1}^{h^*_{t-1}} C^*_{i,t-1}}{\sum_{i=1}^{h^*_{t-1}} \frac{C^*_{i,t-1}}{\hat{\gamma}_i}}$ is only for stockholders, the aggregate consumption (HN) is not considered. As predicted in theory, all coefficients are positive and significant at the conventional level. Compared to the coefficients in Table 8, these are more significant. Also, the implied price of risk has reasonable boundary. Overall, this empirical result supports time-varying market participation as viable extension to conditional consumption asset pricing.

7 Conclusion

This paper shows that an equilibrium model with time-varying market participation can offer support for conditional consumption asset pricing. Heterogeneous investors facing labor income risk in the presence of short-selling constraint optimally choose to enter or exit the market. This mechanism generates counter-cyclical quantity of risk due to time-varying risk-sharing, relaxing the required dramatic counter-cyclical variation in the price of risk. In addition, our model explains why an implausibly high or negative risk aversion can be obtained in the empirical studies assuming full market participation. We also empirically test our theory using Consumer Expenditure data. Using the stockholders' consumption considerably lowers the counter-cyclical variation in the price of risk, and positive risk return tradeoff is always supported by the data in contrast to using full participation case.

Our model can be extended in several directions for future research. We rely on heterogeneity on risk aversion, labor income risk, and short-selling constraint to generate the limited market participation. Considering other frictions [52] or another source of heterogeneity[53] can further improve the predictability results of stock returns as well as generate more realistic time-varying market participation. We also argue that while we rely on power utility for deriving the basic setup, a better improvement of our results for matching the levels of the asset moments may be achieved if combining recursive utilities with heterogeneity in both EIS and risk aversion.

[52] Fixed setup or transaction costs: Allen and Gale (1994), Williamson (1994), Heaton and Lucas (1996), Haliassos and Michaelides (2003), Alan (2006), Fagereng et al. (2017); Life-cycle model: Constantinides et al. (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017); Model uncertainty: Cao et al. (2005); Borrowing constraint: Allen and Gale (1994), Heaton and Lucas (1996), Constantinides et al. (2002), Haliassos and Michaelides (2003), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017).

[53] EIS: Gomes and Michaelides (2008), Guvenen (2009); Belief: Cao et al. (2005), Cvitanić et al. (2012), Bhamra and Uppal (2014); time preference rate: Cvitanić et al. (2012), Bhamra and Uppal (2014); age: Constantinides et al. (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017)

References

Aït-Sahalia, Y., PARKER, J. A., YOGO, M., 2004. Luxury Goods and the Equity Premium. Journal of Finance 59, 2959–3004.

Alan, S., 2006. Entry Costs and Stock Market Participation Over the Life Cycle. Working paper .

Allen, F., Gale, D., 1994. Limited Market Participation and Volatility of Asset Prices. American Economic Review 84, 933–955.

Attanasio, O. P., 1991. Risk, Time-Varying Second Moments and Market Efficiency. Review of Economic Studies 58, 479–494.

Attanasio, O. P., Weber, G., 1995. Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. Journal of Political Economy 103, 1121–1157.

Basak, S., Cuoco, D., 1998. An Equilibrium Model with Restricted Stock Market Participation. Review of Financial Studies 11, 309–341.

Bhamra, H. S., Uppal, R., 2009. The Effect of Introducing a Non-Redundant Derivative on the Volatility of Stock-Market Returns When Agents Differ in Risk Aversion. Review of Financial Studies 22, 2303–2330.

Bhamra, H. S., Uppal, R., 2014. Asset Prices with Heterogeneity in Preferences and Beliefs. Review of Financial Studies 27, 519–580.

Bonsang, E., Perelman, S., den Bosch, K. V., 2005. Income, Wealth and Consumption Inequality - First Results from the Survey of Health, Ageing and Retirement in Europe. Mannheim Research Institute for the Economics of Aging pp. 325–331.

Brunnermeier, M. K., Nagel, S., 2008. Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-evidence on Individuals. American Economic Review 98, 713–736.

Bucciol, A., Miniaci, R., 2011. Household Portfolios and Implicit Risk Preference. Review of Economics and Statistics 93, 1235–1250.

Campbell, J. Y., Cochrane, J. H., 1999. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. Journal of Political Economy 107, 205–251.

Campbell, J. Y., Shiller, R. J., 1988. The dividend-price ratio and expectations of future dividends and discount factors. Review of Financial Studies 1, 195–228.

Cao, H. H., Wang, T., Zhang, H. H., 2005. Model Uncertainty, Limited Market Participation, and Asset Prices. Review of Financial Studies 18, 1219–1251.

Chabakauri, G., 2013. Dynamic Equilibrium with Two Stocks, Heterogeneous Investors, and Portfolio Constraints. Review of Financial Studies 26, 3104–3141.

Chan, Y. L., Kogan, L., 2002. Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices. Journal of Political Economy 110, 1255–1285.

Chen, H., Joslin, S., Tran, N.-K., 2012. Rare Disasters and Risk Sharing with Heterogeneous Beliefs. Review of Financial Studies 25, 2189–2224.

Christensen, P. O., Larsen, K., Munk, C., 2012. Equilibrium in Securities Markets with Heterogeneous Investors and Unspanned Income Risk. Journal of Economic Theory 147, 1035–1063.

Cochrane, J. H., 1992. Explaining the variance of price-dividend ratios. Review of Financial Studies 5, 243–280.

Cochrane, J. H., 2017. Macro-Finance. Review of Finance 21, 945–985.

Conine, T. E., McDonald, M. B., Tamarkin, M., 2017. Estimation of relative risk aversion across time. Applied Economics 49, 2117–2124.

Constantinides, G. M., 1990. Habit formation: a resolution of the equity premium puzzle. Journal of Political Economy 72, 519–543.

Constantinides, G. M., Donaldson, J. B., Mehra, R., 2002. Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle. Quarterly Journal of Economics 117, 269–296.

Constantinides, G. M., Duffie, D., 1996. Asset Pricing with Heterogeneous Consumers. Journal of Political Economy 104, 219–240.

Cvitanić, J., Jouini, E., Malamud, S., Napp, C., 2012. Financial Markets Equilibrium with Heterogeneous Agents. Review of Finance 16, 285–321.

Donkers, B., Melenberg, B., Soest, A. V., 2001. Estimating Risk Attitudes using Lotteries: A Large Sample Approach. Journal of Risk and Uncertainty 22, 165–195.

Duffee, G. R., 2005. Time Variation in the Covariance between Stock Returns and Consumption Growth. Journal of Finance 60, 1673–1712.

Fagereng, A., Gottlieb, C., Guiso, L., 2017. Asset Market Participation and Portfolio Choice over the Life-Cycle. Journal of Finance 72, 705–750.

Favilukis, J., 2013. Inequality, stock market participation, and the equity premium. Journal of Financial Economics 107, 740–759.

Ferson, W. E., Harvey, C. R., 1993. Seasonality and Heteroskedasticity in Consumption-Based Asset Pricing: An Analysis of Linear Models. Research in Finance 11, 1–35.

Gabaix, X., 2012. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. Quarterly Journal of Economics 127, 645–700.

Gabaix, X., Lasry, J.-M., Lions, P.-L., Moll, B., 2016. The Dynamics of Inequality. Econometrica 84, 2071–2111.

Gomes, F., Michaelides, A., 2005. Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence. Journal of Finance 60, 869–904.

Gomes, F., Michaelides, A., 2008. Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. Review of Financial Studies 21, 415–448.

Guiso, L., Paiella, M., 2008. Risk Aversion, Wealth, and Background Risk. Journal of the European Economic Association 6, 1109–1150.

Guo, H., 2004. Limited Stock Market Participation and Asset Prices in a Dynamic Economy. Journal of Financial and Quantitative Analysis 39, 495–516.

Guvenen, F., 2009. A Parsimonious Macroeconomic Model for Asset Pricing. Econometrica 77, 1711–1750.

Haliassos, M., Michaelides, A., 2003. Portfolio choice and liquidity constraint. International Economic Review 44, 143–177.

He, H., Pagès, H. F., 1993. Labor income, borrowing constraints, and equilibrium asset prices. Economic Theory 3, 663–696.

He, H., Pearson, N. D., 1991a. Consumption and Portfolio Policies with Incomplete Markets and Short-sale Constraints: The finite Dimensional Case. Mathematical Finance 1, 1–10.

He, H., Pearson, N. D., 1991b. Consumption and Portfolio Policies with Incomplete Markets and Short-sale Constraints: The Infinite Dimensional Case. Journal of Economic Theory 54, 259–304.

Heaton, J., Lucas, D., 1996. Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing. Journal of Political Economy 104, 443–487.

Heaton, J., Lucas, D., 2000. Portfolio choice in the presence of background risk. Economic Journal 110.

Hodrick, R. J., 1992. Dividend yields and expected stock returns: Alternative procedures for inference and measurement. Review of Financial Studies 5, 357–386.

Jagannathan, R., Wang, Y., 2007. Lazy Investors, Discretionary Consumption,and the Cross-Section of Stock Returns. Journal of Finance 62, 1623–1661.

Keim, D. B., Stambaugh, R. F., 1986. Predicting returns in the stock and bond markets. Journal of Financial Economics 17, 357–390.

King, M. A., Leape, J. I., 1998. Wealth and portfolio composition: Theory and evidence. Journal of Public Economics 69, 155–193.

Koo, H. K., 1998. Consumption and Portfolio Selection with Labor Income: A Continuous Time Approach. Mathematical Finance 8, 49–65.

Kroencke, T. A. K. T. A., 2017. Asset Pricing without Garbage. Journal of Finance 72, 47–98.

LeRoy, S. F., Porter, R. D., 1981. Stock Price Volatility: Tests Based on Implied Variance Bounds. Econometrica 49, 97–113.

Lettau, M., Ludvigson, S., 2001. Consumption, Aggregate Wealth, and Expected Stock Returns. Journal of Finance 56, 815–849.

Lucas, R. E., 1978. Asset Prices in an Exchange Economy. Econometrica 46, 1429–1445.

Lustig, H., Nieuwerburgh, S. V., 2008. The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street. Review of Financial Studies 21, 2097–2137.

Lustig, H., Nieuwerburgh, S. V., Verdelhan, A., 2013. The Wealth-Consumption Ratio. Review of Asset Pricing Studies 3, 38–94.

Malloy, C. J., Moskowitz, T. J., Vissing-Jorgensen, A., 2009. Long-Run Stockholder Consumption Risk and Asset Returns. Journal of Finance 64, 2427–2479.

Mankiw, N. G., Zeldes, S. P., 1991. The Consumption of Stockholders and Non-Stockholders. Journal of Financial Economics 29, 97–112.

Mehra, R., Prescott, E. C., 1985. The Equity Premium: A Puzzle. Jounral of Monetary Economics 15, 146–161.

Nagel, S., Singleton, K. J., 2011. Estimation and Evaluation of Conditional Asset Pricing Models. Journal of Finance 66, 873–909.

Newey, W. K., West, K. D., 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. Econometrica 55, 703–708.

Parker, J. A., Julliard, C., 2005. Consumption Risk and the Cross Section of Expected Returns. Journal of Political Economy 113, 185–222.

Polkovnichenk, V., 2004. Limited stock market participation and the equity premium. Finance Research Letters 1, 24–34.

Rapach, D. E., Zhou, G., 2013. Forecasting stock returns. in:Elliott, G., Timmermann A., Handbook of Economic Forecasting 2A, 328–383.

Riley, W. B. J., Chow, K. V., 1992. Asset Allocation and Individual Risk Aversion. Financial Analysts Journal 48, 32–37.

Roussanov, N., 2014. Composition of Wealth, Conditioning Information, and the Cross-Section of Stock Returns. Journal of Financial Economics 111, 352–380.

Santos, T., Veronesi, P., 2006. Labor Income and Predictable Stock Returns. of Financial Studies 19, 1 – 44.

Sarkar, A., Zhang, L., 2009. Time varying consumption covariance and dynamics of the equity premium: Evidence from the G7 countries. Journal of Empirical Finance 16, 613 – 631.

Savov, A., 2011. Asset Pricing with Garbage. Journal of Finance 66, 177–201.

Shiller, R. J., 1981. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? American Economic Review 71, 421–436.

Viceira, L. M., 2001. Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income. Journal of Finance 56, 433 – 470.

Vissing-Jorgensen, A., 2002. Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures. National Bureau of Economic Research working paper .

Wachter, J. A., 2002. Habit formation and returns on bonds and stocks. Working paper FIN01-024, Stern School of Business, NYU 69.

Wachter, J. A., 2013. Can Time-varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? Journal of Finance 68, 987–1035.

Wang, C., Wang, N., Yang, J., 2016. Optimal consumption and savings with stochastic income and recursive utility. Journal of Economic Theory 165, 292–331.

Welch, I., Goyal, A., 2008. A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. Review of Financial Studies 21, 1455–1508.

Williamson, S. D., 1994. Liquidity and market participation. Journal of Economic Dynamics and Control 18, 629–670.

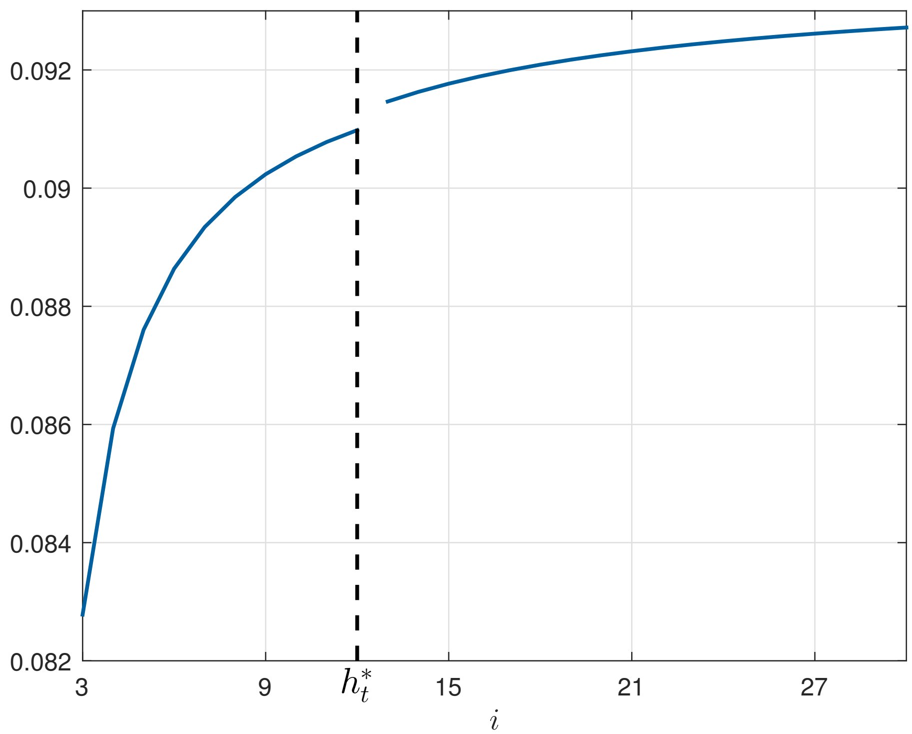

Figure 1: **Variation of optimal consumption across stockholders**
This figure plots the cross-sectional variation of the optimal consumptions at time 0 ($t = 0$). The cut-off stockholder h_t^* is 12th stockholder $i = 12$. Therefore, the stockholders range from the first stockholder to 12th stockholder and non-stockholders range from 13th to the last stockholder (30th).

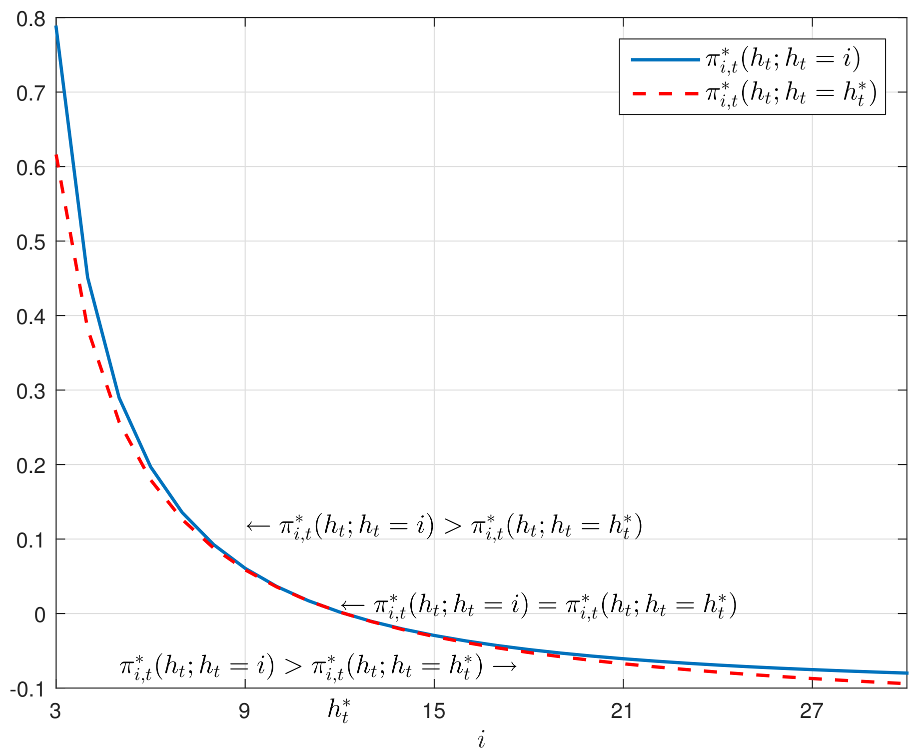

Figure 2: **Optimal stock holdings across stockholders**

This figure plots the cross-sectional variation of the optimal stock holdings at time 0 ($t = 0$). The optimal stock holding depends on the cut-off stockholder h_t. The solid line is the optimal stock holdings of each stockholder i when each one believes she is the cut-off stockholder ($h_t = i$). The dashed line is the optimal stock holdings of each stockholder when all stockholders know the true cut-off stockholder ($h_t = h_t^*$). The cut-off stockholder h_t^* is 12th stockholder $i = 12$. Therefore, the stockholders from the first stockholder to 12th stockholder and non-stockholders range from 13th to the last stockholder (30th).

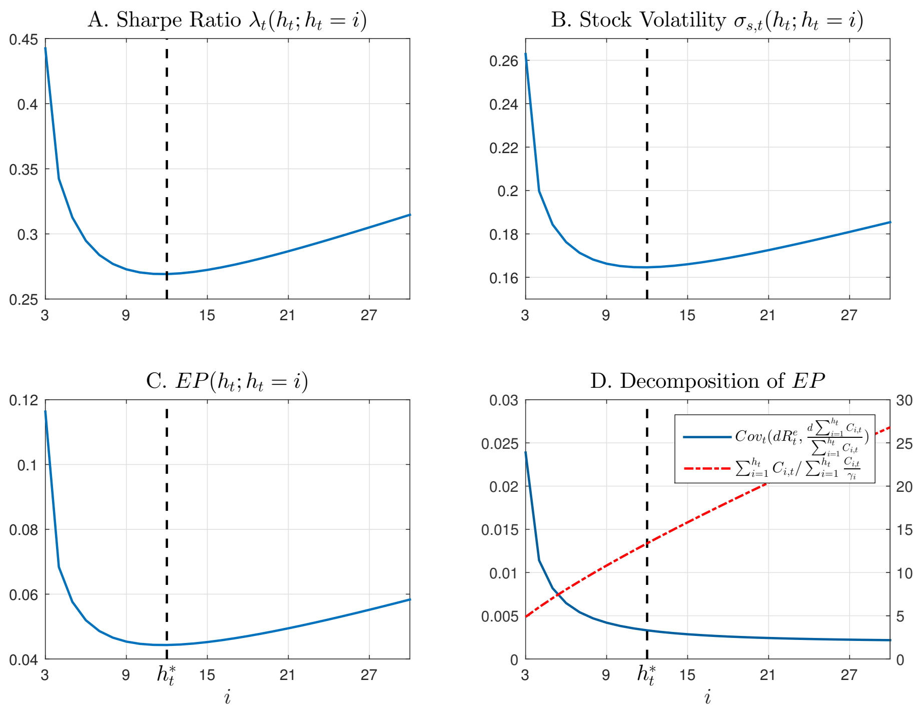

Figure 3: **Asset moments as a function of the cut-off stockholder**
This figure plots the variation of the asset parameters when each stockholder from $i = 3$ to $i = 30$ is the cut-off stockholder at time 0 ($t = 0$). Panel A illustrates the Sharpe ratio, Panel B is the stock volatility. Panel C is the equity premium. Panel D is the decomposition of equity premium into the quantity of risk and price of risk.

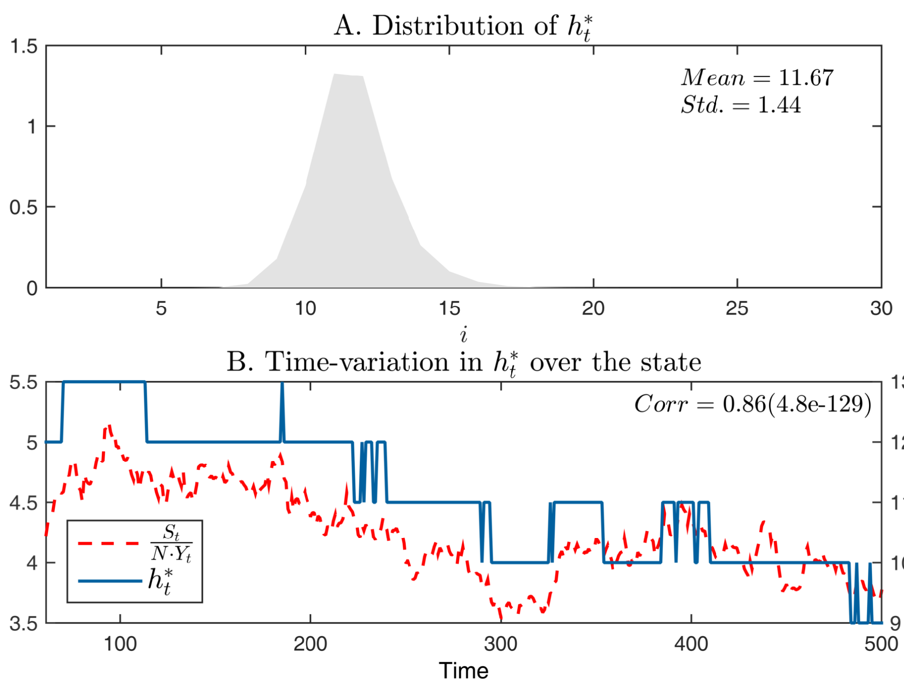

Figure 4: **Distribution and time-variation of the cut-off stockholder** h_t^*
Panel A presents the simulated kernel distribution of the cut-off stockholder h_t^*. Mean value and standard deviation of the entire sample are reported in northwest. Panel B presents one sample path of time-variation in the cut-off stockholder h_t^* in response to the stock market wealth-to-labor ratio $\frac{S_t}{N \cdot Y_t}$. The left (right) y-axis represents the value for $\frac{S_t}{N \cdot Y_t}$ (h_t^*). Correlation coefficient reports the sample correlation between two variables and the p-value for the null hypothesis of zero correlation is reported in the parenthesis. For this analysis, we simulate 10,000 sample paths of the model economy. Each path consists of 500 monthly observations. The first 60 observations (5 years) are discarded to reduce the dependence on initial condition.

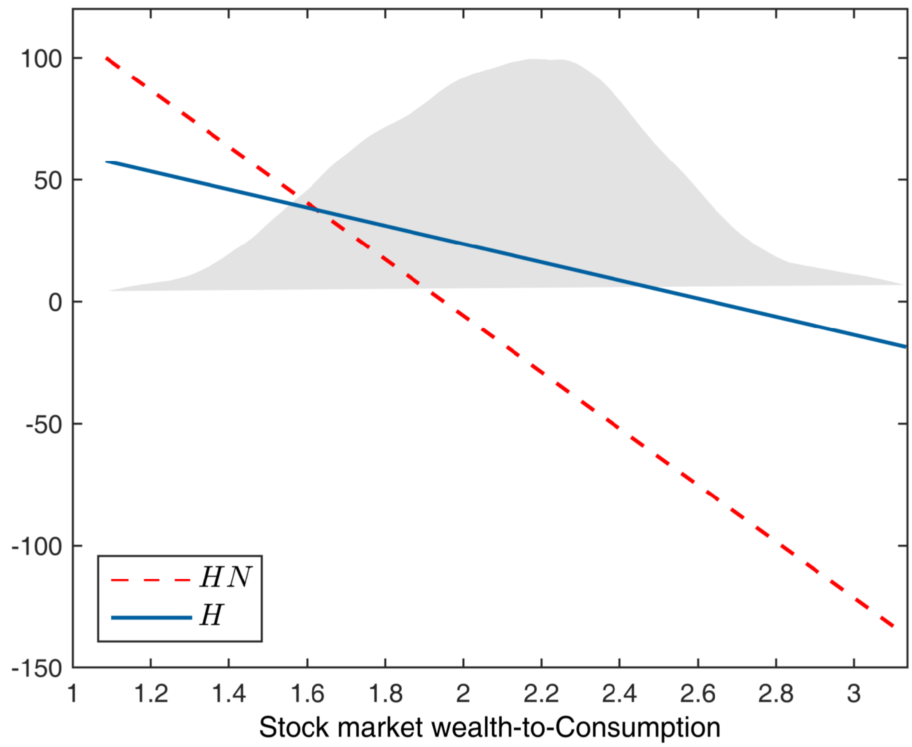

Figure 5: **Conditional Price of Risk using Stock market wealth-to-Consumption ratio**

This figure depicts the estimated price of risk implied by the regression of excess stock returns on the conditional covariance between the consumption growth and excess stock returns together with a time-varying price of risk as a function of the stock market wealth-to-consumption ratio. The dashed line shows the price of risk when the aggregate households' consumption including both stock'h'olders and 'n'on-stockholders (HN) is used. The solid line is the price of risk when the consumption of stockholders (H) is used. Rescaled kernel density of the conditioning variable is shaded in the background.

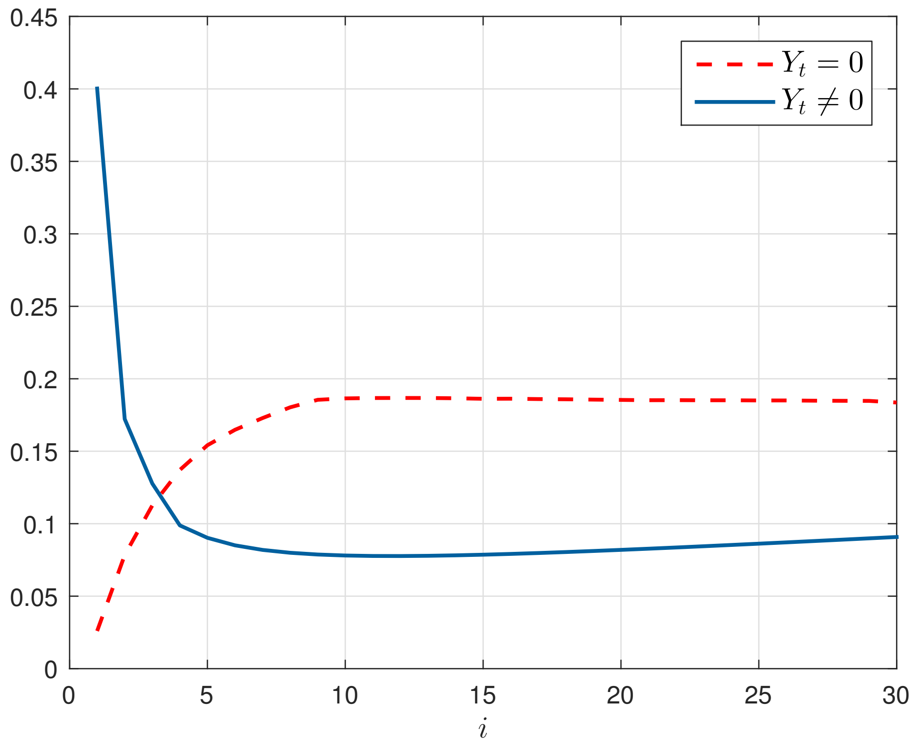

Figure 6: **Sharpe ratio as a function of the cut-off stockholder**

As in Figure 3, the Sharpe ratio is illustrated as a function of the cut-off stockholder from $i = 3$ to $i = 30$. The dashed line is the result for the case where investors do not face the labor income risk under the complete market. The solid line is the result for the case where investors face with the labor income risk under the incomplete market (The sample plot in Panel A in Figure 3. from $i = 3$). The same parameter values as in Table 1 are used for both cases.

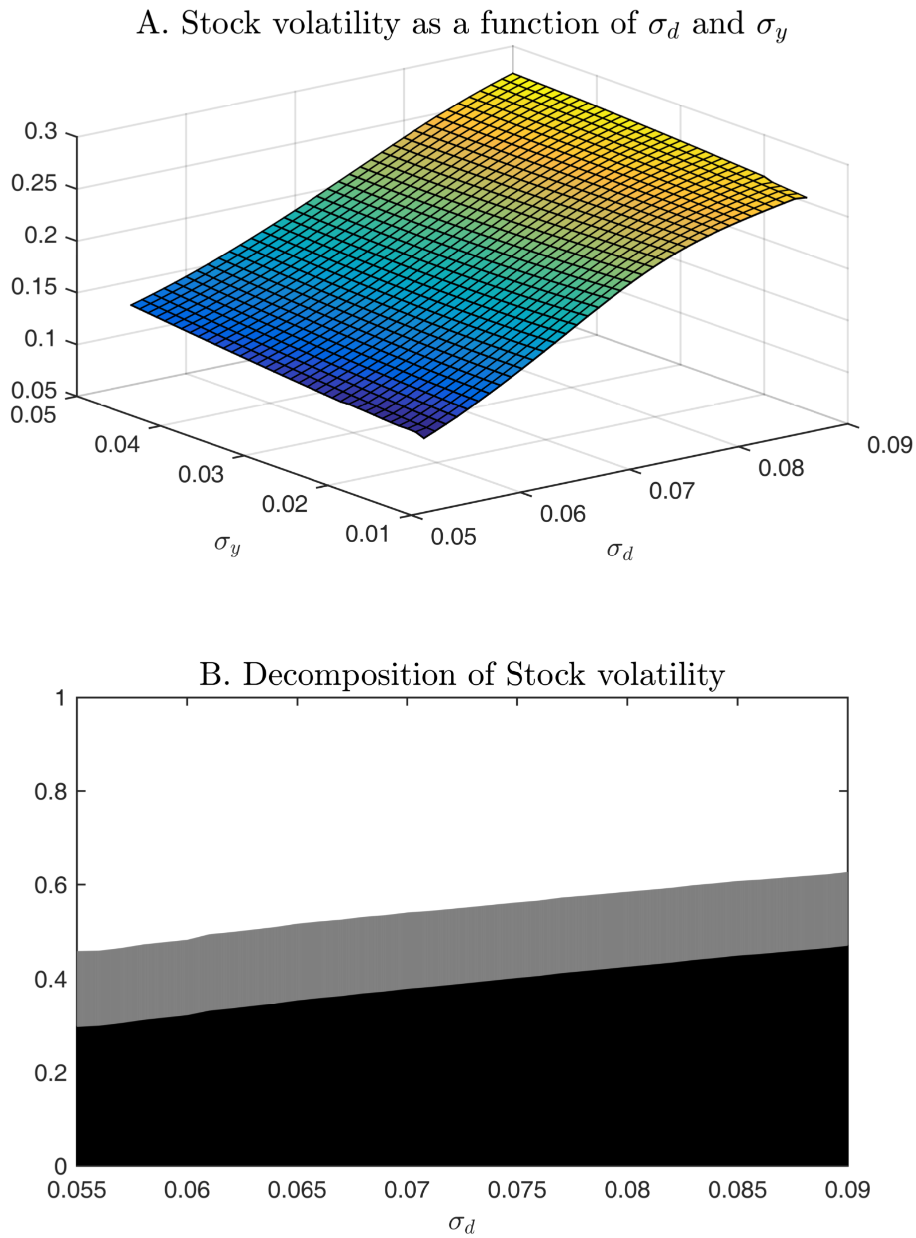

Figure 7: **Stock volatility as a function of dividend and labor income volatility** In Panel A, stock volatility is plotted as a function of dividend volatility σ_d and labor income volatility σ_y. The range of σ_d is from 5.5% to 9% and the range of σ_y is from 1.5% to 5%. In Panel B, for each value of dividend volatility, the stock volatility is decomposed into dividend volatility and labor income volatility. The black area denotes the proportion of dividend volatility in stock volatility. The gray area denotes the proportion of labor income volatility. The white area and gray area together denote the excess volatility.

Table 1: Model Parameters

Table 1 presents the annualized model parameters used to simulate the model. The moments of Dividend and labor income are chosen based on the U.S. real data from 1960 to 2015. Dividend and labor income growth are calculated by log change in real per capita and first moments of growth are corrected for Jensen's inequality. A detailed description of the data is Appendix D. We use the same boundary of risk aversion from 1 to 100 as in Chan and Kogan (2002).

Parameter	Symbol	Value
Panel A: Dividend and Labor income parameters		
Dividend growth mean	μ_d	0.04
Dividend growth volatility	σ_d	0.09
Labor income growth mean	μ_y	0.01
Labor income growth volatility	σ_y	0.03
$Corr(dW_{d,t}, dW_{y,t})$	ρ	0.4
Panel B: Investor-related parameters		
Subjective time preference	δ	0.01
Lowest risk aversion coefficient	γ_1	1
Highest risk aversion coefficient	γ_N	100
Number of investors	N	30
Panel C: Initial value		
Initial dividend stream	D_0	$0.05 \times N$
Initial labor income	Y_0	1

Table 2: Dynamics of Consumption risk

Table 2 reports the pooled OLS panel regression of the price or quantity of risk on $\frac{S_t}{N \cdot Y_t}$ or h_t^* based on the simulated data. We simulate 100 sample paths of the economy. Each path constitutes the individual of the panel and consists of 500 monthly observations. Parameter values for the simulation are in Table 1. The first 60 observations (5 years) are discarded to reduce the dependence on the initial condition. HN denotes the aggregate consumption including both stock'h'olders and 'n'on-stockholders' consumption. H denotes the consumption of stock'h'olders. h_t^* denotes the cut-off stock'h'older. $\sum_{i=1}^{N}$ and $\sum_{i=1}^{h_t^*}$ denote the summation over all investors and stockholders, respectively. For convenience, all variables are normalized to have zero mean and unit standard deviation. T-statistics based on Newey and West (1987) are in brackets and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Consumption	Dependent var.	$\frac{S_t}{N \cdot Y_t}$	h_t^*	Constant	R^2	N
Panel A: The Price of consumption risk						
HN	$\hat{\Gamma}_t^{HN} = \sum_{i=1}^{N} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i} - a_t$	-0.486***		5.1×10^{-7}	0.072	44,000
		(-58.37)		(0.00)		
HN	$\hat{\Gamma}_t^{HN} = \sum_{i=1}^{N} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i} - a_t$		-0.062***	7.6×10^{-7}	4.9×10^{-4}	44,000
			(-4.65)	(0.00)		
H	$\hat{\Gamma}_t^{H} = \sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}$	0.099***		5.3×10^{-6}	0.002	44,000
		(10.39)		(0.00)		
H	$\hat{\Gamma}_t^{H} = \sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}$		0.743***	3.9×10^{-6}	0.198	44,000
			(104.22)	(0.00)		
Panel B: The Quantity of consumption risk						
HN	$Cov_t(dR_t^e, d\sum_{i=1}^{N} C_{i,t}^* / \sum_{i=1}^{N} C_{i,t}^*)$	0.514***		-0.002	0.065	44,000
		(55.19)		(-0.22)		
HN	$Cov_t(dR_t^e, d\sum_{i=1}^{N} C_{i,t}^* / \sum_{i=1}^{N} C_{i,t}^*)$		0.052***	-0.002	0.001	44,000
			(6.54)	(-0.19)		
H	$Cov_t(dR_t^e, d\sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} C_{i,t}^*)$	-0.081***		-0.001	0.002	44,000
		(-8.92)		(-0.11)		
H	$Cov_t(dR_t^e, d\sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} C_{i,t}^*)$		-0.627***	-0.001	0.172	44,000
			(-95.73)	(-0.14)		

Table 3: Simulated Unconditional Moments of Consumption growth and Stock Returns

Table 3 presents the annualized consumption and stock returns moments. Target values for the moments are obtained by the U.S. data from 1960 to 2015. To estimate the model-implied unconditional moments of consumption growth and stock returns, 10,000 sample paths of the model are simulated. Each path consists of 500 monthly observations. The first 60 observations (5 years) are discarded to reduce the dependence on the initial condition. The mean value of each moment is calculated across time and paths. The aggregate consumption data are from NIPA (National Income and Product Account). For excess stock returns R^e, the simple return on CRSP value-weighted NYSE/Amex/Nasdaq minus 1-month U.S. T-bill is used. Consumptions are in the end of September 2010 dollars. h_t^* denotes the cut-off stock'h'older. $\sum_{i=1}^{N}$ and $\sum_{i=1}^{h_t^*}$ denote the summation over all investors and stockholders, respectively.

Input/Moment	U.S. data	Model
Panel A: Consumption moments (%)		
$E[\frac{\Delta \sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}]$	1.80	1.11
$\sigma[\frac{\Delta \sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}]$	1.37	2.79
$Cov(R^e, \frac{\Delta \sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*})$	0.04	0.19
$Corr(R^e, \frac{\Delta \sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*})$	17.06	38.12
Panel B: Stock returns moments (%)		
$E[R^e]$	5.98	4.44
$Cov(R^e, \frac{\Delta \sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*})$	0.20[1]	0.34
$\sum_{i=1}^{h_t^*} C_{i,t}^* / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}$	30.15[2]	13.02
$\sigma[R^e]$	15.34	17.89
Panel C: Market Participation rate (%)		
$E[\frac{h_t}{N}]$	29.7[3]	38.91

[1] Due to data availability of stockholders' consumption, conditional covariance of consumption growth stockholders with stock returns is obtained based on the data from 1996 to 2015. A detailed description of the way we calculate ex post conditional covariance is in Section 6.

[2] This is obtained by dividing unconditional equity premium $E[R^e]$ by unconditional ex post conditional covariance $Cov(R^e, \frac{\Delta \sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*})$.

[3] This is from 2016 SCF. The market participation rate is 60.2% when indirect holdings are accounted for.

Table 4: Regressions of the model-implied price of risk on time-varying market participation

Table 4 reports the regression of model-implied price of consumption risk on time-varying market participation based on the households survey data by Consumer Expenditure (CE).

$$\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\hat{\gamma}_{m,i}}} = b_0 + b_1(\frac{h^*}{N})_t + u_t \quad \forall m = 1, 2$$

h_t^* denotes the cut-off stock'h'older. $\sum_{i=1}^{h_t^*}$ denotes the summation over all stockholders. $(\frac{h^*}{N})_t$ is the proportion of stockholders in the total sample households. In order to capture the individual relative risk aversion γ_i, we construct two measures. The first measure uses the total value of wealth invested in stock and the second measure uses the probability of being a stockholder based on the coefficients obtained from the Probit regression using the SCF (The Survey of Consumer Finances) data.

$$\hat{\gamma}_{1,i} = \frac{k}{W_i} \quad \hat{\gamma}_{2,i} = \frac{k}{\Phi(X'_{i,CE}\hat{\beta}_{SCF})}$$

All data are in monthly frequency and September 2010 dollars. The sample is 238 observations from April 1996 through December 2015. A detailed description of data and Probit regression result are in Appendix D and E, respectively. T-statistics based on Newey and West (1987) are in brackets and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Relative risk aversion	b_1	Constant	R^2
$\hat{\gamma}_{1,i}$	0.897***	-0.007	0.335
	(10.86)	(-0.61)	
$\hat{\gamma}_{2,i}$	0.731***	1.011***	0.045
	(3.33)	(28.87)	

Table 5: Predicting Covariances between Stock Returns and Consumption Growth

Table 5 reports the regression of excess stock returns and consumption growth on the respective set of instruments (Panel A) and the regression of ex post covariance on the set instruments (Panel B) based on the households survey data by Consumer Expenditure (CE).

$$r_t^e = X'_{r^e,t-1}\beta_{r^e} + \epsilon_{r^e,t}, \qquad \Delta c_{j,t} = X'_{c_j,t-1}\beta_{c_j} + \epsilon_{\Delta c_j,t} \qquad Cov^*(r_t^e, \Delta c_{j,t}) = X'_{t-1}\beta_j + \epsilon_{j,t} \;\; \forall j = HN, H$$

The definition of a variable is as follows. Log change in real per capita total market value for NYSE/Amex/Nasdaq minus 1-month real U.S. T-bill (r_t^e), log change in real per capita aggregate consumption including both stock'h'olders and 'n'on-stockholders ($\Delta c_{HN,t}$), log change in real per capita consumption of stock'h'olders ($\Delta c_{H,t}$), log dividend-price ratio ($logD/P_t$), Book-to-market ratio (BM_t), Net equity expansion ($NTIS_t$), Long-term yield (LTY_t), and stock market wealth-to-consumption ratio ($M/C_{i,t}$). The ex post covariance is the product of the residuals from the first stage regression (i.e., $Cov^*(r_t^e, \Delta c_{j,t}) = \hat{\epsilon}_{r^e,t-1}\hat{\epsilon}_{\Delta c_j,t-1}$). All data are in monthly frequency and September 2010 dollars. The sample is 238 observations from April 1996 through December 2015. A detailed description of data is in Appendix D.

Dependent variable	Instruments	*Adj* R^2
Panel A: 1st stage regression		
r_t^e	$logD/P_{t-1}$, BM_{t-1}, $NTIS_{t-1}$, LTY_{t-1}	0.04
$\Delta c_{HN,t}$	$\Delta c_{HN,t-1}$, $\Delta c_{HN,t-2}$, $\Delta c_{HN,t-3}$, r_{t-1}	0.42
$\Delta c_{H,t}$	$\Delta c_{H,t-1}$, $\Delta c_{H,t-2}$, $\Delta c_{H,t-3}$, r_{t-1}	0.24
Panel B: 2nd stage regression		
$\hat{\epsilon}_{r^e,t}\hat{\epsilon}_{\Delta c_{HN},t}$	$\hat{\epsilon}_{r^e,t-1}\hat{\epsilon}_{\Delta c_{HN},t-1}$, $M/C_{HN,t-1}$, $logD/P_{t-1}$, $\sqrt{\epsilon^2_{r^e,t-1}}$, $\sqrt{\hat{\epsilon}^2_{\Delta c_{HN},t-1}}$	0.02
$\hat{\epsilon}_{r^e,t}\hat{\epsilon}_{\Delta c_H,t}$	$\hat{\epsilon}_{r^e,t-1}\hat{\epsilon}_{\Delta c_H,t-1}$, $M/C_{H,t-1}$, $logD/P_{t-1}$, $\sqrt{\epsilon^2_{r^e,t-1}}$, $\sqrt{\hat{\epsilon}^2_{\Delta c_H,t-1}}$	0.01

Table 6: Unconditional moments of Consumption Growth

Table 6 reports annualized standard deviation, covariances, correlations between log excess stock return and log change in real per capita consumption based on the households survey data by Consumer Expenditure (CE). Unconditional mean excess return is mean of Jensen's inequality corrected log change in real per capita total market value for NYSE/Amex/Nasdaq minus 1-month U.S. T-bill $\frac{1}{T}\sum_{t=1}^{T} r_t^e + \frac{1}{2T}\sum_{t=1}^{T}\hat{\epsilon}^2_{r^e,t}$ (4.39%). The unconditional standard deviation is calculated by $\sqrt{\frac{1}{T}\sum_{t=1}^{T}\hat{\epsilon}^2_{\Delta c_j,t}}\ \forall j = HN, H$. The unconditional covariance is $\frac{1}{T}\sum_{t=1}^{T}\hat{\epsilon}_{r^e,t}\hat{\epsilon}_{\Delta c_j,t}\ \forall j = HN, H$. $\hat{\epsilon}_{\Delta c_j,t}$ and $\hat{\epsilon}_{r^e,t}$ are obtained from the 1st stage regression as reported in Table 5. The implied price of risk ($\hat{\Gamma}_j$) is calculated by dividing unconditional excess returns by covariance. $\Delta c_{HN,t}$ denote log change in real per capita aggregate consumption including both stock'h'olders and 'n'on-stockholders. $\Delta c_{H,t}$ denote log change in real per capita consumption of stock'h'olders. All data are in monthly frequency and September 2010 dollars. The sample is 238 observations from April 1996 through December 2015. A detailed description of data is in Appendix D.

Consumption ($j =$)	Standard Deviation (%)	Covariance (%)	$\hat{\Gamma}$
HN	6.28	0.09	49.74 ($= \hat{\Gamma}^F$)
H	13.79	0.20	22.16 ($= \hat{\Gamma}^T$)

Table 7: Regressions of Stock Returns on the Conditional Covariance

Table 7 reports the regression of excess equity returns on the conditional covariance with time-invariant price of risk based on the households survey data by Consumer Expenditure (CE).

$$r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,j} + b_{1,j}\widehat{Cov}_{t-1}(r_t^e, \Delta c_{j,t}) + u_{j,t} \quad \forall j = HN, H$$

The table reports the regression results with different sets of instruments for ex post covariance. r_t^e denotes the log change in real per capita total market value for NYSE/Amex/Nasdaq minus 1-month U.S. T-bill. $\Delta c_{HN,t}$ denote log change in real per capita aggregate consumption including both stock'h'olders and 'n'on-stockholders. $\Delta c_{H,t}$ denote log change in real per capita consumption of stock'h'olders. All data are in monthly frequency and September 2010 dollars. The sample is 238 observations from April 1996 through December 2015. A detailed description of data is in Appendix D. T-statistics based on Newey and West (1987) are in brackets and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Consumption ($j=$)	$b_{1,j}$ ($=\hat{\Gamma}_j$)	Constant	R^2
HN	-14.688	0.005	0.002
	(-0.75)	(1.38)	
H	19.728*	0.001	0.01
	(1.73)	(0.15)	

Table 8: Nonlinear Regressions of Stock Returns on the Conditional Covariance

Table 8 reports the regression of excess equity returns on the conditional covariance with time-varying price of risk based on the households survey data by Consumer Expenditure (CE).

$$r^e_t + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,j} + [b_{1,j} + b_{2,j}p_{t-1}]\widehat{Cov}_{t-1}(r^e_t, \Delta c_{j,t}) + u_{j,t} \quad \forall j = HN, H$$

The table reports the regression results with different sets of instruments for ex post covariance and proxy for p_{t-1}. Panel A uses stock market wealth-to-consumption, Panel B uses Surplus consumption, and Panel C uses dividend-to-price ratio for p_{t-1}. r^e_t denotes the log change in real per capita total market value for NYSE/Amex/Nasdaq minus 1-month U.S. T-bill. $\hat{\Gamma}_t$ denotes the implied time-varying price of risk which is measured by $\hat{b}_{j,1} + \hat{b}_{j,2}p_{t-1}$. $Prob(\hat{\Gamma}_t < 0)$ calculates the proportion of the negative implied price of consumption risk in the entire time-series sample. $\Delta c_{HN,t}$ denote log change in real per capita aggregate consumption including both stock'h'olders and 'n'on-stockholders. $\Delta c_{H,t}$ denote log change in real per capita consumption of stock'h'olders. All data are in monthly frequency and September 2010 dollars. The sample is 238 observations from April 1996 through December 2015. A detailed description of data is in Appendix D. T-statistics based on Newey and West (1987) are in brackets and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Consumption ($j =$)	$b_{1,j}$	$b_{2,j}$	Constant	$Adj\ R^2$	$\hat{\Gamma}_{j,t}$	$Prob(\hat{\Gamma}_{j,t} < 0)$ (%)
Panel A ($p_t =$) : Stock market wealth-to-Consumption ratio						
HN	225.541**	-115.660**	0.006*	0.03	$[-137, 100]_t$	66.2
	(2.23)	(-2.55)	(1.66)			
H	98.158	-37.274	-1.0×10^{-4}	0.01	$[-19, 58]_t$	7.59
	(1.39)	(-1.22)	(-0.03)			
Panel B ($p_t =$) : Surplus consumption						
HN	40.026	-14.7×10^3*	0.005	0.02	$[-76, 72]_t$	67.1
	(1.06)	(-1.89)	(1.38)			
H	33.809	-3.9×10^3	-3.1×10^{-5}	0.01	$[3, 42]_t$	0
	(1.30)	(-0.77)	(-0.01)			
Panel C ($p_t =$) : Log dividend-to-Price ratio						
HN	499.960	127.737	0.005	0.01	$[-78, 81]_t$	71.3
	(1.56)	(1.62)	(1.39)			
H	167.817	36.496	5.3×10^{-7}	0.01	$[3, 48]_t$	0
	(0.62)	(0.56)	(0.00)			

Table 9: Nonlinear Regressions of Stock Returns on the Conditional Covariance

As a direct test of **Proposition 4**, the following regression equation is considered based on the households survey data by Consumer Expenditure (CE).

$$r_t^e + \frac{1}{2}\hat{\epsilon}^2_{r^e,t} = b_{0,m} + b_{2,m}\frac{\sum_{i=1}^{h^*_{t-1}} C^*_{i,t-1}}{\sum_{i=1}^{h^*_{t-1}} \frac{C^*_{i,t-1}}{\hat{\gamma}_{m,i}}}\widehat{Cov}_{t-1}(r_t^e, \Delta c_{H,t}) + u_{m,t} \quad \forall m = 1, 2$$

In order to capture the individual relative risk aversion γ_i, we construct two measures. The first measure uses the total value of wealth invested in stock (Panel A) and the second measure uses the probability of being a stockholder based on the coefficients obtained from the Probit regression using the SCF (The Survey of Consumer Finances) data (Panel B). $\hat{\Gamma}_{H,t}$ denotes the implied time-varying price of risk which is measured by $\hat{b}_{2,H}\sum_{i=1}^{h^*_{t-1}} C^*_{i,t-1}/\sum_{i=1}^{h^*_{t-1}} \frac{C^*_{i,t-1}}{\hat{\gamma}_{m,i}}$. $Prob(\hat{\Gamma}_t < 0)$ calculates the proportion of the negative implied price of consumption risk in the entire time-series sample. A detailed description of data and Probit regression result are in Appendix D and E. T-statistics based on Newey and West (1987) are in brackets and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

$$\hat{\gamma}_{1,i} = \frac{k}{W_i} \quad \hat{\gamma}_{2,i} = \frac{k}{\Phi(X'_{i,CE}\hat{\beta}_{SCF})}$$

Consumption ($j =$)	$b_{2,m}$	Constant	R^2	$\hat{\Gamma}_{H,t}$	$Prob(\hat{\Gamma}_{H,t} < 0)$ (%)
Panel A: Harmonic measure using Wealth $\hat{\gamma}_{1,i}$					
H	121.065**	0.001	0.02	$[4, 40]_t$	0
	(2.35)	(0.36)			
Panel B: Harmonic measure using the Probability of Stockholder $\hat{\gamma}_{2,i}$					
H	16.974*	0.001	0.01	$[14, 24]_t$	0
	(1.72)	(0.18)			

Table 10: Probit Regression of Stock Ownership

Table 10 reports the Probit regression of whether household owns stock or not on the observable characteristics. The SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, and 2013. The dependent variable takes one if a household has positive holding either in stock (hstocks=1) or mutual funds excluding MMMFs (hnmmf=1) otherwise zero. The regressors are age of household (age), age squared (age^2), an indicator for race not being white/Caucasian (race=1), the number of kids (kids), an *highschool* indicator for at least 12 but less than 16 years of education for head of household (educ>11 and educ>16), an *college* indicator for 16 or more years of education (educ >16), the log of real total household income before taxes (income), the log of real dollar amount in checking and savings account (log(checking+saving)) (set to zero if checking and savings = 0), and indicator for checking and savings account = 0, an indicator for dividend income (X5709=1), and year dummies. Robust standard errors are used for z-statistic and statistical significance at the 10%, 5%, and 1% levels are denoted by *, **, ***, respectively. The second column reports the estimated coefficient and the third column reports the z-statistic.

Variable	Coefficient	Z-statistic
age	0.004**	2.57
age^2	-3.5×10^{-5}***	-2.77
$1_{i \in nonwhite}$	-0.346***	-32.26
$kids$	-0.039***	-11.82
$1_{i \in highschool}$	-0.118***	-14.81
$1_{i \in college}$	0.199***	19.19
$log(income)$	0.256***	69.00
$log(chk + saving)$	0.088***	38.76
$1_{chk+saving=0}$	0.486***	22.34
$1_{Div>0}$	1.426***	160.71
1_{1992}	-4.4×10^{-4}	-0.03
1_{1995}	0.077***	4.73
1_{1998}	0.293***	18.27
1_{2001}	0.304***	19.00
1_{2004}	0.182***	11.17
1_{2007}	0.040**	2.41
1_{2010}	-0.078**	-5.11
1_{2013}	-0.181**	-11.42
$Cons$	-4.492**	-96.11
Number of Obs.		206,106
Pseudo R^2		0.409

Appendix A. CARA investors case

In this section, we study the economy populated by heterogeneous CARA(Constant Absolute Risk Aversion) investors based on the setting in Christensen et al. (2012). While Christensen et al. (2012) studies the full participation case with finite time horizon and idiosyncratic labor income, we solve the equilibrium of an economy where there are non-stockholders which arises from the short-selling constraint in an infinite time horizon. Also as in the main section, we consider the systematic labor income.

A.1 The basic setup

As in the main section, there is a single riskless bond such that $\frac{dB_t}{B_t} = r_t dt$ in zero net supply and risky asset in unit net supply, which is a claim to a dividend D_t that follows Arithmetic Brownian Motion

$$dD_t = \mu_d dt + \sigma_d dW_t \tag{A.1}$$

The equilibrium stock price dynamics has the following form :

$$dS_t = (S_t r_t + \mu^e_{s,t} - D_t)dt + \sigma_{s,t} dW_{d,t} \tag{A.2}$$

where $\mu^e_{s,t}$ denotes the total expected excess return over the risk-free rate and $\sigma_{s,t}$ is the (absolute) price volatility. Thus ratio $\lambda_{s,t} = \mu^e_{s,t}/\sigma_{s,t}$ is the Sharpe ratio. The economy is populated by infinitely lived N (types of) investors and all having exponential utility with different risk aversion. Investor i is maximizing $\forall t \in [0, \infty)$

$$\mathbb{E}_t[\int_t^\infty -e^{-a_i C_{i,s}} ds] \tag{A.3}$$

$\forall i = 1, 2, ..., h_t, ..., N$ whose absolute risk aversion coefficient is $a_1, a_2, ..., a_{h_t}, ..., a_N$, respectively, with $0 < a_1 < a_2 < ... < a_{h_t} < ... < a_N$. All stockholders receive the same level (systematic) of stochastic exogenous income Y_t that evolves as :

$$dY_t = \mu_y dt + \sigma_y dW_{y,t} \tag{A.4}$$

where $dW_{d,t} dW_{y,t} = \rho dt$.

A.2 The individual investor's problem

The stockholders and non-stockholders financial wealth dynamics are

$$dX_{i,t} = [r_t X_{i,t} + \pi_{i,t}\mu^e_{s,t} + Y_t - C_{i,t}]dt + \pi_{i,t}\sigma_{i,t} dW_{d,t} \quad \forall i = 1, 2, ..., h_t \tag{A.5}$$

$$dX_{i,t} = [r_t X_{i,t} + Y_t - C_{i,t}]dt \quad \forall i = h_t + 1, ..., N \tag{A.6}$$

,respectively where $\pi_{i,t}$ represents the number of units of the risky asset owned by the investor at time t. With the value function $V_{i,t}(x,y) = \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} E_t[\int_t^\infty -e^{-a_i C_{i,s}} ds]$. The HJB equations are

$$\begin{aligned} 0 = &\max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} -e^{-a_i C_{i,t}} - \delta V + [\pi_{i,t}\mu^e_{s,t} + r_t X_{i,t} + Y_t - C_{i,t}]V_x + \frac{1}{2}\pi^2_{i,t} V_{xx}\sigma^2_{s,t} \\ &+ \mu_y V_y + \frac{1}{2}\sigma^2_y V_{yy} + \pi_{i,t}\rho\sigma_{s,t} Y_t \sigma_y V_{xy} \quad \forall i = 1,2,...,h_t \end{aligned} \tag{A.7}$$

$$\begin{aligned} 0 = &\max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} -e^{-a_i C_{i,t}} - \delta V + [r_t X_{i,t} + Y_t - C_{i,t}]V_x + \mu_y V_y + \frac{1}{2}\sigma^2_y V_{yy} \\ &\forall i = h_t + 1, ..., N \end{aligned} \tag{A.8}$$

Under mild integrability conditions (Christensen et al. (2012)), the solution for this maximization problem exists. The investors' optimal consumption, portfolio policy, and in turn the wealth dynamics are

$$\begin{aligned} C^*_{i,t} = &r_t X_{i,t} + Y_t + \frac{1}{a_i r_t}(\delta - r_t - a_i\rho\sigma_y\lambda_{s,t} + \frac{\lambda^2_{s,t}}{2} + \mu_y a_i - \frac{\sigma^2_y a^2_i(1-\rho^2)}{2}) \\ &\forall i = 1,2,...,h_t \end{aligned} \tag{A.9}$$

$$C^*_{i,t} = r_t X_{i,t} + Y_t + \frac{1}{a_i r_t}(\delta - r_t + \mu_y a_i - \frac{\sigma^2_y a^2_i}{2}) \quad \forall i = h_t + 1, ..., N \tag{A.10}$$

$$\pi^*_{i,t} = \frac{\lambda_{s,t}}{a_i r_t \sigma_{s,t}} - \frac{\rho\sigma_y}{r_t\sigma_{s,t}} \quad \forall i = 1,2,...,h_t \tag{A.11}$$

$$\begin{aligned} dX_{i,t} = &\frac{1}{a_i r_t}[-(\delta - r_t) + \frac{\lambda^2_{s,t}}{2} - \mu_y a_i + \frac{\sigma^2_y a^2_i(1-\rho^2)}{2}]dt + \pi^*_{i,t}\sigma_{s,t} dW_{d,t} \\ &\forall i = 1,2,...,h_t \end{aligned} \tag{A.12}$$

$$dX_{i,t} = \frac{1}{a_i r_t}[-(\delta - r_t) - \mu_y a_i + \frac{\sigma^2_y a^2_i}{2}]dt \quad \forall i = h_t + 1, ..., N \tag{A.13}$$

A.3 Equilibrium

From the stock market clearing condition, the Sharpe ratio is identified. Also, by matching terms from the dynamics of the consumption clearing condition equation ($\sum_{i=1}^N C^*_{i,t} =$

$D_t + N \cdot Y_t$), the equilibrium risk-free rate and stock volatility are determined. They are given by

$$\lambda_{s,t} = (\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sigma_d + \rho\sigma_y h_t) \tag{A.14}$$

$$\begin{aligned} r_t =& \delta + (\mu_d + \mu_y N)(\sum_{i=1}^{N} \frac{1}{a_i})^{-1} - \frac{\sigma_d^2}{2}(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sum_{i=1}^{N} \frac{1}{a_i})^{-1} - \sigma_d\rho\sigma_y h_t(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sum_{i=1}^{N} \frac{1}{a_i})^{-1} \\ & - \frac{\sigma_y^2}{2}(\sum_{i=1}^{N} \frac{1}{a_i})^{-1}[(\sum_{i=1}^{h_t} \frac{1}{a_i})(1-\rho^2) + \sum_{i=h_t+1}^{N} \frac{1}{a_i} + \rho^2 h_t^2(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}] \end{aligned} \tag{A.15}$$

$$\sigma_{s,t} = \frac{\sigma_d}{r_t} \tag{A.16}$$

$$\begin{aligned} S_t =& \frac{D_t}{r_t} + \frac{\sigma_y^2}{2r_t^2}[\sum_{i=1}^{h_t} a_i(1-\rho^2) + \sum_{i=h_t+1}^{N} a_i] - \frac{\lambda_{s,t}^2}{2r_t^2}\sum_{i=1}^{h_t}\frac{1}{a_i} \\ & - \frac{(\mu_y - \rho\sigma_y\lambda_{s,t})h_t}{r_t^2} - \frac{\mu_y(N-h_t)}{r_t^2} - \frac{(\delta - r_t)\sum_{i=1}^{N}\frac{1}{a_i}}{r_t^2} \end{aligned} \tag{A.17}$$

Since the sensitivity of the optimal consumption to labor income is unity as in (A.9) and (A.10), labor income shock does not affect the equilibrium stock price. Also, most importantly, contrary to power utility case, financial wealth ($X_{i,t}$) and labor income (Y_t) no-longer play a role as a state variable. Thus, neither does financial nor labor income shock change the optimal unit demand for stock $\pi_{s,t}^*$ or any asset parameters ($\lambda_{s,t}, r_t, \sigma_{s,t}$). Therefore, together with the fact that the only time-varying terms which determine the asset parameters are h_t and accordingly N_t^h, the cut-off stockholder who distinguishes stockholders from non-stockholders is the same for all horizons ($h_t = h_0 \ \ \forall t > 0$), and in turn, all asset parameters are time-constant ($\lambda_{s,t} = \lambda_{s,0}, r_t = r_0, \sigma_{s,t} = \sigma_{s,0} \ \ \forall t > 0$). Also, the equilibrium stock price varies only through the cash flow shock (change in D_t).

To summarize, since there is no wealth effect in CARA investor case, there is no stochastic dynamics in this economy, and hence it is impossible to study conditional asset pricing using CARA preference. Even though dynamics can be generated as in Christensen et al. (2012) by considering the finite horizon, this dynamics is only deterministic, and therefore, perfectly predictable at any point in time.

Appendix B. Proofs

B.1 Proof of Proposition 1

The following proof is based on Koo (1998). To ensure the existence of an optimal policy, we impose the following well-posedness conditions as in Koo (1998) and Wang et al. (2016).

$$\textit{Condition 1}: \delta > (1-\gamma_i)(r_t + \frac{\lambda_t^2}{2\gamma_i}) \qquad \forall t \geq 0 \ \ \textit{and} \ \ i = 1, 2, ..., N \tag{B.1}$$

where λ_t denotes the Sharpe ratio.

$$\textit{Condition 2}: r_t - \mu_y > 0 \qquad \forall t \geq 0 \tag{B.2}$$

$$\textit{Condition 3}: \delta - \mu_y(1-\gamma_i) + \frac{\sigma_y^2 \gamma_i (1-\gamma_i)}{2} > 0 \qquad \forall t \geq 0 \ \ \textit{and} \ \ i = 1, 2, ..., N \tag{B.3}$$

The Hamilton-Jacobi-Bellman (HJB) equation associated with the problem (4) for a stockholder i is

$$\begin{aligned} 0 = & \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} \frac{C_{i,t}^{1-\gamma_i}}{1-\gamma_i} - \delta V + [\pi_{i,t}(\mu_{s,t} - r_t) + r_t X_{i,t} + Y_t - C_{i,t}]V_x \\ & + \frac{1}{2}\pi_{i,t}^2 V_{xx}((\sigma_{s,t}^d)^2 + (\sigma_{s,t}^y)^2 + 2\rho\sigma_{s,t}^d\sigma_{s,t}^y) + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} + \pi_{i,t}(\rho\sigma_{s,t}^d + \sigma_{s,t}^y) Y_t \sigma_y V_{xy} \\ & \forall i = 1, 2, ..., h_t \end{aligned} \tag{B.4}$$

This can be re-written as

$$\begin{aligned} 0 = & \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} \frac{C_{i,t}^{1-\gamma_i}}{1-\gamma_i} - \delta V + [\pi_{i,t}(\mu_{s,t} - r_t) + r_t X_{i,t} + Y_t - C_{i,t}]V_x \\ & + \frac{1}{2}\pi_{i,t}^2 V_{xx}\sigma_{s,t}^2 + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} + \pi_{i,t}\rho_t\sigma_{s,t} Y_t \sigma_y V_{xy} \\ & \forall i = 1, 2, ..., h_t \end{aligned} \tag{B.5}$$

where $\rho_t \equiv Corr_t(\sigma_{s,t}^d dW_{d,t} + \sigma_{s,t}^y dW_{y,t}, dW_{y,t}) = \frac{\sigma_{s,t}^d \rho + \sigma_{s,t}^y}{\sigma_{s,t}}$ If $\sigma_{s,t}^y = 0$, $\sigma_{s,t} = \sigma_{s,t}^d$ and $\rho_t = \rho$. Then, the optimal consumption and portfolio are given by

$$C_{i,t} = V_x^{-\frac{1}{\gamma_i}} \qquad\qquad \pi_{i,t} = -\frac{(\mu_{s,t} - r_t)V_x}{V_{xx}\sigma_{s,t}^2} - \frac{\rho_t\sigma_{s,t}Y_t\sigma_y V_{xy}}{V_{xx}\sigma_{s,t}^2} \tag{B.6}$$

By substituting these expressions into the HJB equation, we obtain the following PDE,

$$\begin{aligned} 0 = & \frac{\gamma_i V_x^{-\frac{1-\gamma_i}{\gamma_i}}}{1-\gamma_i} - \delta V - \frac{(\mu_{s,t} - r_t)^2 V_x^2}{2V_{xx}\sigma_{s,t}^2} + (r_t X_{i,t} + Y_t)V_x + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} \\ & - \frac{(\mu_{s,t} - r_t)V_x\rho_t Y_t \sigma_y V_{xy}}{V_{xx}\sigma_{s,t}} - \frac{\rho_t^2 Y_t^2 \sigma_y^2 V_{xy}^2}{2V_{xx}} \end{aligned} \tag{B.7}$$

We define two following functions for the value function. p is defined as

$$p \equiv \frac{V_y(x,y)}{V_x(x,y)} \qquad \forall x>0, y>0 \tag{B.8}$$

Due to homogeneity of value function, p is a continuous function of the financial wealth to income ratio $z \equiv \frac{x}{y}$ for $x>0$ and $y>0$. q is defined as

$$q \equiv \frac{(1-\gamma_i)V(x,y)}{(x+p(z)y)^{1-\gamma_i}} \tag{B.9}$$

By homogeneity of value function, q is also a continuous function of z. Then, the value function is given by

$$V(x,y) = \frac{q(z)}{1-\gamma_i}(x+p(z)y)^{1-\gamma_i} \qquad \forall x>0, y>0 \tag{B.10}$$

Define $\phi(z) \equiv V(z,1)$. By homogeneity,

$$V(x,y) = V(y\cdot z, y) = y^{1-\gamma_i}V(z,1) = y^{1-\gamma_i}\phi(z) \tag{B.11}$$

$$V_x(x,y) = y^{1-\gamma_i}\phi'(z)y^{-1} = y^{-\gamma_i}\phi'(z) \tag{B.12}$$

$$V_y(x,y) = (1-\gamma_i)\phi(z)y^{-\gamma_i} - \phi'(z)zy^{-\gamma_i} \tag{B.13}$$

Therefore,

$$p(z) = \frac{V_y(x,y)}{V_x(x,y)} = (1-\gamma_i)\frac{\phi(z)}{\phi'(z)} - z \tag{B.14}$$

By rearranging term,

$$\phi'(z) = (1-\gamma_i)\frac{\phi(z)}{p(z)+z} = (1-\gamma_i)\frac{q(z)(z+p(z))^{1-\gamma_i}/(1-\gamma_i)}{p(z)+z} = q(z)(p(z)+z)^{-\gamma_i} \tag{B.15}$$

By substituting this into (77) and (78),

$$\begin{aligned}
&V_x(x,y) = y^{-\gamma_i}q(z)(p(z)+z)^{-\gamma_i} \\
&V_{xx}(x,y) = -q(z)y^{-\gamma_i-1}(z+p(z))^{-1-\gamma_i}(\gamma_i+p'(z)) \\
&V_y(x,y) = q(z)p(z)(z+p(z))^{-\gamma_i}y^{-\gamma_i} \\
&V_{yy}(x,y) = -y^{-\gamma_i-1}(z+p(z))^{-\gamma_i-1}q(z)(\gamma_i p(z)^2 + z^2p'(z)) \\
&V_{xy}(x,y) = -y^{-\gamma_i-1}q(z)(z+p(z))^{-\gamma_i-1}(\gamma_i p(z) - zp'(z))
\end{aligned} \tag{B.16}$$

Then, the optimal consumption is expressed by

$$C_{i,t} = q(z)^{-\frac{1}{\gamma_i}}(x + p(z)y) \tag{B.17}$$

$$\pi_{i,t} = \frac{(\mu_{s,t} - r_t)(x + p(z)y)}{\sigma_{s,t}^2(\gamma_i + p'(z))} - \frac{\rho_t \sigma_y}{\sigma_{s,t}} \frac{\gamma_i y p(z) - x p'(z)}{\gamma_i + p'(z)} \tag{B.18}$$

Then HJB equation can be re-written as

$$\begin{aligned} 0 = {} & [\frac{\gamma_i (p(z) + z)^2 q(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \delta \frac{(z + p(z))^2}{1 - \gamma_i} + (r_t z + 1)(z + p(z)) \\ & + \mu_y p(z)(z + p(z)) - \frac{1}{2}\sigma_y^2(\gamma_i p(z)^2)]\gamma_i + \frac{\lambda_t^2 (p(z) + z)^2}{2} - \lambda_t \rho_t \sigma_y \gamma_i p(z)(z + p(z)) \\ & + \frac{\rho_t^2 \sigma_y^2 (\gamma_i p(z))^2}{2} + p'(z)A \end{aligned} \tag{B.19}$$

where $A \equiv \frac{\gamma_i (p(z)+z)^2 q(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \delta \frac{(z+p(z))^2}{1-\gamma_i} + (r_t z + 1)(z + p(z)) + \mu_y p(z)(z + p(z)) - \frac{1}{2}\sigma_y^2(\gamma_i p(z)^2 + z^2 p'(z)) + \lambda_t \rho_t \sigma_y z(z + p(z)) + \frac{\rho_t^2 \sigma_y^2 (-2\gamma_i p(z) z + z^2 p'(z))}{2}$

Each term can be factorized by the order of z.

$$\begin{aligned} 0 = {} & z^2[\frac{\gamma_i^2 q(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \frac{\delta \gamma_i}{1 - \gamma_i} + r_t \gamma_i + \frac{\lambda_t^2}{2}] \\ & + z[\frac{\gamma_i^2 2p(z) q(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \delta \gamma_i \frac{2p(z)}{1 - \gamma_i} + r_t p(z) \gamma_i + \gamma_i + \mu_y p(z) \gamma_i + \lambda_t^2 p(z) - \lambda_t \rho_t \sigma_y \gamma_i p(z)] \\ & + z o(z) + p'(z)A \end{aligned} \tag{B.20}$$

where $o(z)$ is a function such that $\lim_{z \to \infty} \frac{o(z)}{z} = 0$

After dividing all terms by z^2, as $z \to \infty$ goes to infinity, because of $\lim_{z \to \infty} p'(z) = 0$ (Koo (1998)) and $\lim_{z \to \infty} \frac{o(z)}{z} = 0$.

$$\begin{aligned} 0 = {} & [\frac{\gamma_i^2 q^*(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \frac{\delta \gamma_i}{1 - \gamma_i} + r_t \gamma_i + \frac{\lambda_t^2}{2}] \\ & + \frac{1}{z}[\frac{\gamma_i^2 2p^*(z) q^*(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \delta \gamma_i \frac{2p^*(z)}{1 - \gamma_i} + r_t p^*(z) \gamma_i + \gamma_i + \mu_y p^*(z) \gamma_i + \lambda_t^2 p^*(z) - \lambda_t \rho_t \sigma_y \gamma_i p^*(z)] \end{aligned} \tag{B.21}$$

The above PDE can be solved by $q^*(z)$ and $p^*(z)$ satisfying the following equations.

$$\begin{aligned} 0 &= \frac{\gamma_i^2 q^*(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \frac{\delta \gamma_i}{1 - \gamma_i} + r_t \gamma_i + \frac{\lambda_t^2}{2} \\ 0 &= \frac{\gamma_i^2 2p^*(z) q^*(z)^{-\frac{1}{\gamma_i}}}{1 - \gamma_i} - \delta \gamma_i \frac{2p^*(z)}{1 - \gamma_i} + r_t p^*(z) \gamma_i + \gamma_i + \mu_y p^*(z) \gamma_i + \lambda_t^2 p^*(z) - \lambda_t \rho_t \sigma_y \gamma_i p^*(z) \end{aligned} \tag{B.22}$$

We obtain the following function to solve the PDE under $z \to \infty$.

$$\lim_{z\to\infty} q(z) = q^*(z) = (r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})^{-\gamma_i} \tag{B.23}$$

$$\lim_{z\to\infty} p(z) = p^*(z) = \frac{1}{r_t + \rho_t\sigma_y\lambda_t - \mu_y} \tag{B.24}$$

Then, the value function under $z \to \infty$ is

$$V^*(x,y) = \frac{(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})^{-\gamma_i}}{1-\gamma_i}(x + \frac{y}{r_t + \rho_t\sigma_y\lambda_t - \mu_y})^{1-\gamma_i} \qquad \forall x > 0, y > 0 \tag{B.25}$$

Based on the value function, the optimal consumption and stock-holding for stockholder i is

$$\begin{aligned}
&C_{i,t}^* = (r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y})\\
&\pi_{i,t}^* = \frac{\lambda_t}{\gamma_i\sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_t\sigma_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}\\
&\forall X_{i,t} > 0, Y_t > 0, i = 1, 2, ..., h_t
\end{aligned} \tag{B.26}$$

The financial wealth dynamics of a stockholder is

$$\begin{aligned}
dX_{i,t} &= [\pi_{i,t}^*(\mu_{s,t} - r_t) + r_t X_{i,t} + Y_t - C_{i,t}^*]dt + \pi_{i,t}^*\sigma_{s,t}^d dW_{dt} + \pi_{i,t}^*\sigma_{s,t}^y dW_{y,t}\\
&= [\frac{\lambda_t^2}{\gamma_i}(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{\lambda_t\rho_t\sigma_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y} + r_t X_{i,t} + Y_t\\
&- (r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y})]dt + \pi_{i,t}^*\sigma_{s,t}^d dW_{dt} + \pi_{i,t}^*\sigma_{s,t}^y dW_{y,t}\\
&= [(\frac{\lambda_t^2}{\gamma_i} - \frac{\delta - r_t}{\gamma_i} + \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{\mu_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}]dt\\
&+ \pi_{i,t}^*\sigma_{s,t}^d dW_{dt} + \pi_{i,t}^*\sigma_{s,t}^y dW_{y,t}
\end{aligned} \tag{B.27}$$

In the same way, let us consider the non-stockholder's problem. Non-stockholder's HJB equation is given by

$$0 = \max_{(c_{i,t})\in\mathcal{A}} \frac{C_{i,t}^{1-\gamma_i}}{1-\gamma_i} - \delta V + (r_t X_{i,t} + Y_t - C_{i,t})V_x + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} \quad \forall i = h_t + 1, 2, ..., N \tag{B.28}$$

By substituting the optimal consumption, the HJB equation can be re-written as

$$0 = \frac{\gamma_i V_x^{-\frac{1-\gamma_i}{\gamma_i}}}{1-\gamma_i} - \delta V + (r_t X_{i,t} + Y_t)V_x + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} \quad i = h_t + 1, 2, ..., N \tag{B.29}$$

The above HJB can re-written by using the same functions and notation as before,

$$\begin{aligned}0 &= \frac{\gamma_i y^{1-\gamma_i} q(z)^{-\frac{1-\gamma_i}{\gamma_i}} (p(z)+z)^{1-\gamma_i}}{1-\gamma_i} - \delta \frac{y^{1-\gamma_i} q(z)(p(z)+z)^{1-\gamma_i}}{1-\gamma_i} \\ &+ (r_t z + 1) y^{1-\gamma_i} q(z)(p(z)+z)^{-\gamma_i} + \mu_y y q(z) p(z)(z+p(z))^{-\gamma_i} y^{-\gamma_i} \\ &- \frac{1}{2}\sigma_y^2 y^2 y^{-\gamma_i - 1}(z+p(z))^{-\gamma_i - 1} q(z)(\gamma_i p(z)^2 + z^2 p'(z))\end{aligned} \tag{B.30}$$

Again, each term can be factorized by the order of z.

$$\begin{aligned}0 &= z^2 [\frac{\gamma_i q(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{\delta}{1-\gamma_i} + r_t] + z[\frac{2p(z)\gamma_i q(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{2p(z)\delta}{1-\gamma_i} + r_t p(z) + 1 + \mu_y p(z)] \\ &- z o(z) + p'(z)\frac{\sigma_y^2 z^2}{2}\end{aligned} \tag{B.31}$$

where $o(z)$ is a function such that $\lim_{z\to\infty} \frac{o(z)}{z} = 0$.

$$\begin{aligned}0 &= [\frac{\gamma_i q(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{\delta}{1-\gamma_i} + r_t] + \frac{1}{z}[\frac{2p(z)\gamma_i q(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{2p(z)\delta}{1-\gamma_i} + r_t p(z) + 1 + \mu_y p(z)] \\ &+ \frac{o(z)}{z} + \frac{p'(z)}{z^2}\frac{\sigma_y^2 z^2}{2}\end{aligned} \tag{B.32}$$

After dividing all terms by z^2, as $z \to \infty$ goes to infinity, because of $\lim_{z\to\infty} p'(z) = 0$ and $\lim_{z\to\infty} \frac{o(z)}{z} = 0$

$$0 = [\frac{\gamma_i q^*(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{\delta}{1-\gamma_i} + r_t] + \frac{1}{z}[\frac{2p^*(z)\gamma_i q^*(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{2p^*(z)\delta}{1-\gamma_i} + r_t p^*(z) + 1 + \mu_y p^*(z)] \tag{B.33}$$

The above PDE can be solved by $q^*(z)$ and $p^*(z)$ satisfying the following equations.

$$\begin{aligned}0 &= \frac{\gamma_i q^*(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{\delta}{1-\gamma_i} + r_t \\ 0 &= \frac{2p^*(z)\gamma_i q^*(z)^{-\frac{1}{\gamma_i}}}{1-\gamma_i} - \frac{2p^*(z)\delta}{1-\gamma_i} + r_t p^*(z) + 1 + \mu_y p^*(z)\end{aligned} \tag{B.34}$$

$$\lim_{z\to\infty} q(z) = q^*(z) = (r_t + \frac{\delta - r_t}{\gamma_i})^{-\gamma_i} \tag{B.35}$$

$$\lim_{z\to\infty} p(z) = p^*(z) = \frac{1}{r_t - \mu_y} \tag{B.36}$$

Then, the optimal consumption for a non-stockholder is

$$C^*_{i,t} = (r_t + \frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y}) \quad \forall X_{i,t} > 0, Y_t > 0, i = h_t + 1, ..., N \tag{B.37}$$

As in the same way for the stockholders, the financial wealth dynamics of a non-stockholder is

$$dX_{i,t} = [(-\frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y}) - \frac{\mu_y Y_t}{r_t - \mu_y}]dt \quad \forall X_{i,t} > 0, Y_t > 0, i = h_t + 1, ..., N \tag{B.38}$$

B.2 Proof of Lemma 1

Bond market clearing condition is

$$\sum_{i=1}^{h_t} X_{i,t} - S_t + \sum_{i=h_t+1}^{N} X_{i,t} = 0 \tag{B.39}$$

This can be achieved by $\sum_{i=1}^{h_0} X_{i,0} - S_0 + \sum_{i=h_0+1}^{N} X_{i,0} = 0$ and $d\sum_{i=1}^{h_t} X_{i,t} - dS_t + d\sum_{i=h_t+1}^{N} X_{i,t} = 0$. Thus, consider the following dynamics of bond market clearing condition.

$$\begin{aligned}
& d\sum_{i=1}^{h_t} X_{i,t} - dS_t + d\sum_{i=h_t+1}^{N} X_{i,t} \\
= & \sum_{i=1}^{h_t} [\pi^*_{i,t}(\mu_{s,t} - r_t) + r_t X_{i,t} + Y_t - C^*_{i,t}]dt + \sigma^d_{s,t}\sum_{i=1}^{h_t} \pi^*_{i,t} dW_{d,t} + \sigma^y_{s,t}\sum_{i=1}^{h_t} \pi^*_{i,t} dW_{y,t} \\
& - (\mu_{s,t} S_t - D_t)dt - S_t\sigma^d_{s,t} dW_{d,t} - S_t \sigma^y_{s,t} dW_{y,t} \\
& + \sum_{i=h_t+1}^{N} [r_t X_{i,t} + Y_t - C^*_{i,t}]dt = 0
\end{aligned} \tag{B.40}$$

Stock market clearing condition is $\sum_{i=1}^{h_t} \pi^*_{i,t} = S_t$ and bond market clearing condition implies $\sum_{i=1}^{h_t} X_{i,t} + \sum_{i=h_t+1}^{N} X_{i,t} = S_t$. Applying these equations to (B.40) and rearranging terms yield,

$$\sum_{i=1}^{h_t} C^*_{i,t} + \sum_{i=h_t+1}^{N} C^*_{i,t} = N \cdot Y_t + D_t \tag{B.41}$$

B.3 Proof of Proposition 2

1. Sharpe ratio
From above, the optimal stock holding is

$$\pi^*_{i,t} = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_t\sigma_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}$$
$$\forall X_{i,t} > 0, Y_t > 0, i = 1, 2, ..., h_t \tag{B.42}$$

Considering the stock market clearing condition,

$$\sum_{i=1}^{h_t}(\frac{\lambda_t}{\gamma_i\sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_t\sigma_y Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) = \sum_{i=1}^{N} X_{i,t} \tag{B.43}$$

This provides the equation for the Sharpe ratio. For expositional convenience, we can define $g(\boldsymbol{\theta}_t) \equiv \frac{1}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}$.

$$\sum_{i=1}^{h_t}(\frac{\lambda_t}{\gamma_i\sigma_{s,t}}(X_{i,t} + g(\boldsymbol{\theta}_t)Y_t)) - \frac{h_t\rho_t\sigma_y Y_t g(\boldsymbol{\theta}_t)}{\sigma_{s,t}} = \sum_{i=1}^{N} X_{i,t} \tag{B.44}$$

$$\lambda_t = \frac{\sigma_{s,t}\sum_{i=1}^{N} X_{i,t} + h_t\rho_t\sigma_y Y_t g(\boldsymbol{\theta}_t)}{\sum_{i=1}^{h_t}\frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i}} \tag{B.45}$$

2. Risk-free rate
From the consumption clearing condition,

$$\sum_{i=1}^{h_t} C^*_{i,t} + \sum_{i=h_t+1}^{N} C^*_{i,t} = N \cdot Y_t + D_t \tag{B.46}$$

Dynamics of the above equation is

$$\begin{aligned}
&\sum_{i=1}^{h_t}(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})([(\frac{\lambda_t^2}{\gamma_i} - \frac{\delta - r_t}{\gamma_i} + \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \\
&- \frac{\mu_y Y_t}{r_t + \rho_t\mu_y\lambda_t - \mu_y}]dt + \pi^*_{i,t}\sigma^d_{s,t}dW_{dt} + \pi^*_{i,t}\sigma^y_{s,t}dW_{y,t} + \frac{Y_t\mu_y dt + Y_t\sigma_y dW_{y,t}}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \\
&+ \sum_{i=h_t+1}^{N}(r_t + \frac{\delta - r_t}{\gamma_i})([(-\frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y}) - \frac{\mu_y Y_t}{r_t - \mu_y}]dt + \frac{Y_t\mu_y dt + Y_t\sigma_y dW_{y,t}}{r_t - \mu_y}) \\
&= N \cdot Y_t\mu_y dt + N \cdot Y_t\sigma_y dW_{y,t} + D_t\mu_d dt + D_t\sigma_d dW_{d,t}
\end{aligned} \tag{B.47}$$

After some terms canceling out,

$$\sum_{i=1}^{h_t}(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})[(\frac{\lambda_t^2}{\gamma_i}-\frac{\delta-r_t}{\gamma_i}+\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t}+\frac{Y_t}{r_t+\rho_t\mu_y\lambda_t-\mu_y})dt$$
$$+\pi_{i,t}^*\sigma_{s,t}^d dW_{dt}+\pi_{i,t}^*\sigma_{s,t}^y dW_{y,t}+\frac{Y_t\sigma_y dW_{y,t}}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}]$$
$$+\sum_{i=h_t+1}^{N}(r_t+\frac{\delta-r_t}{\gamma_i})[(-\frac{\delta-r_t}{\gamma_i})(X_{i,t}+\frac{Y_t}{r_t-\mu_y})dt+\frac{Y_t\sigma_y dW_{y,t}}{r_t-\mu_y}]$$
$$=N\cdot Y_t\mu_y dt+N\cdot Y_t\sigma_y dW_{y,t}+D_t\mu_d dt+D_t\sigma_d dW_{d,t} \tag{B.48}$$

Deterministic terms of the above equation is

$$\sum_{i=1}^{h_t}(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(\frac{\lambda_t^2}{\gamma_i}-\frac{\delta-r_t}{\gamma_i}+\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y})$$
$$+\sum_{i=h_t+1}^{N}(r_t+\frac{\delta-r_t}{\gamma_i})(-\frac{\delta-r_t}{\gamma_i})(X_{i,t}+\frac{Y_t}{r_t-\mu_y})=N\cdot Y_t\mu_y+D_t\mu_d \tag{B.49}$$

This provides the equation for the risk-free rate. For expositional convenience, we can re-write the equation using the optimal consumption.

$$\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y})}(\frac{\lambda_t^2}{\gamma_i}-\frac{\delta-r_t}{\gamma_i}+\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y})$$
$$+\sum_{i=h_t+1}^{N}\frac{C_{i,t}^*}{(X_{i,t}+\frac{Y_t}{r_t-\mu_y})}(-\frac{\delta-r_t}{\gamma_i})(X_{i,t}+\frac{Y_t}{r_t-\mu_y})=N\cdot Y_t\mu_y+D_t\mu_d \tag{B.50}$$

Solving the above equation for r_t gives

$$r_t=\delta+(\mu_d D_t+\mu_y NY_t)(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1}-(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1}\frac{\lambda_t^2}{2}\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i}(1+\frac{1}{\gamma_i}) \tag{B.51}$$

3. Stock volatility

Diffusion terms of (B.48) is

$$\sum_{i=1}^{h_t}(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(\pi_{i,t}^*\sigma_{s,t}^d dW_{dt}+\pi_{i,t}^*\sigma_{s,t}^y dW_{y,t}+\frac{Y_t\sigma_y dW_{y,t}}{r_t+\rho_t\sigma_y\lambda_t-\mu_y})$$
$$+\sum_{i=h_t+1}^{N}(r_t+\frac{\delta-r_t}{\gamma_i})(\frac{Y_t\sigma_y dW_{y,t}}{r_t-\mu_y})=D_t\sigma_d dW_{d,t}+N\cdot Y_t\sigma_y dW_{y,t} \tag{B.52}$$

This gives the following two equations for the stock volatility.

$$\sum_{i=1}^{h_t}(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})\pi_{i,t}^*\sigma_{s,t}^d = D_t\sigma_d \tag{B.53}$$

$$\begin{aligned}&\sum_{i=1}^{h_t}(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(\pi_{i,t}^*\sigma_{s,t}^y + \frac{Y_t\sigma_y}{r_t + \rho_t\sigma_y\lambda_t - \mu_y})\\ &+ \sum_{i=h_t+1}^{N}(r_t + \frac{\delta - r_t}{\gamma_i})(\frac{Y_t\sigma_y}{r_t - \mu_y}) = N \cdot Y_t\sigma_y\end{aligned} \tag{B.54}$$

For expositional convenience, we can re-write (B.53)

$$\sum_{i=1}^{h_t}(\frac{C_{i,t}^*}{X_{i,t} + g(\boldsymbol{\theta}_t)Y_t})[\frac{\lambda_t}{\gamma_i}(X_{i,t} + g(\boldsymbol{\theta}_t)Y_t) - \rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)]\frac{\sigma_{s,t}^d}{\sigma_{s,t}} = D_t\sigma_d \tag{B.55}$$

By solving the above equation for λ_t

$$\lambda_t = (\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i})^{-1}[D_t\sigma_d\frac{\sigma_{s,t}}{\sigma_{s,t}^d} + \sum_{i=1}^{h_t}(\frac{C_{i,t}^*}{X_{i,t} + g(\boldsymbol{\theta}_t)Y_t})\rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)] \tag{B.56}$$

From (B.45),

$$\frac{\sigma_{s,t}\sum_{i=1}^{N}X_{i,t} + h_t\rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)}{\sum_{i=1}^{h_t}\frac{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}{\gamma_i}} = (\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i})^{-1}[D_t\sigma_d\frac{\sigma_{s,t}}{\sigma_{s,t}^d} + \sum_{i=1}^{h_t}(\frac{C_{i,t}^*}{X_{i,t} + g(\boldsymbol{\theta}_t)Y_t})\rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)] \tag{B.57}$$

Then, we obtain the expression for $\sigma_{s,t}$

$$\begin{aligned}&\sigma_{s,t}\\ &= [(\sum_{i=1}^{h_t}\frac{X_{i,t} + g(\boldsymbol{\theta}_t)Y_t}{\gamma_i})^{-1}(\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i})^{-1}(D_t\sigma_d\frac{\sigma_{s,t}}{\sigma_{s,t}^d} + \sum_{i=1}^{h_t}\frac{C_{i,t}^*\rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)}{X_{i,t} + g(\boldsymbol{\theta}_t)Y_t})\\ &- h_t\rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)](\sum_{i=1}^{N}X_{i,t})^{-1}\end{aligned} \tag{B.58}$$

(B.54) also can be re-written as

$$\sigma_{s,t} = \frac{\sum_{i=1}^{h_t}(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})[\frac{\lambda_t}{\gamma_i}(X_{i,t} + g(\boldsymbol{\theta}_t)Y_t) - \rho_t\sigma_yY_tg(\boldsymbol{\theta}_t)]\sigma_{s,t}^y}{\sigma_yY_t(N - \sum_{i=1}^{h_t}(r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})g(\boldsymbol{\theta}_t) - \sum_{i=h_t+1}^{N}(r_t + \frac{\delta - r_t}{\gamma_i})\frac{1}{r_t - \mu_y})} \tag{B.59}$$

To summarize, the following four equations constitute the set of equations to determine the four asset parameters $(\lambda_t, r_t, \sigma_{s,t}^d, \sigma_{s,t}^y)$.

$$
\begin{aligned}
&1. \sum_{i=1}^{h_t}\Big(\frac{\lambda_t}{\gamma_i\sigma_{s,t}}\big(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\big)-\frac{1}{\sigma_{s,t}}\frac{\rho_t\sigma_yY_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)=\sum_{i=1}^{N}X_{i,t}\\
&2. \sum_{i=1}^{h_t}\Big(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big(\frac{\lambda_t^2}{\gamma_i}-\frac{\delta-r_t}{\gamma_i}+\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)\\
&\quad+\sum_{i=h_t+1}^{N}\Big(r_t+\frac{\delta-r_t}{\gamma_i}\Big)\Big(-\frac{\delta-r_t}{\gamma_i}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t-\mu_y}\Big)=N\cdot Y_t\mu_y+D_t\mu_d\\
&3. \sum_{i=1}^{h_t}\Big(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big[\frac{\lambda_tX_{i,t}}{\gamma_i\sigma_{s,t}}+\frac{\lambda_t-\rho_t\sigma_y\gamma_i}{\gamma_i\sigma_{s,t}}\Big(\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)\Big]\sigma_{s,t}^d=D_t\sigma_d\\
&4. \sum_{i=1}^{h_t}\Big(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big(\Big[\frac{\lambda_tX_{i,t}}{\gamma_i\sigma_{s,t}}+\frac{\lambda_t-\rho_t\sigma_y\gamma_i}{\gamma_i\sigma_{s,t}}\Big(\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)\Big]\sigma_{s,t}^y\\
&\quad+\frac{Y_t\sigma_y}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)+\sum_{i=h_t+1}^{N}\Big(r_t+\frac{\delta-r_t}{\gamma_i}\Big)\Big(\frac{Y_t\sigma_y}{r_t-\mu_y}\Big)=N\cdot Y_t\sigma_y
\end{aligned}
\tag{B.60}
$$

4. Stock price

Consumption clearing condition is

$$
\begin{aligned}
&\sum_{i=1}^{h_t}\Big(r_t+\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)\\
&+\sum_{i=h_t+1}^{N}\Big(r_t+\frac{\delta-r_t}{\gamma_i}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t-\mu_y}\Big)=N\cdot Y_t+D_t
\end{aligned}
\tag{B.61}
$$

By taking r_t from summation and considering $\sum_{i=1}^{N}X_{i,t}=S_t$. We can obtain the following equation.

$$
\begin{aligned}
&r_tS_t=NY_t+D_t-r_t\frac{Y_th_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}-r_t\frac{Y_t(N-h_t)}{r_t-\mu_y}\\
&-\sum_{i=1}^{h_t}\Big(\frac{\delta-r_t}{\gamma_i}-\frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t+\rho_t\sigma_y\lambda_t-\mu_y}\Big)-\sum_{i=h_t+1}^{N}\Big(\frac{\delta-r_t}{\gamma_i}\Big)\Big(X_{i,t}+\frac{Y_t}{r_t-\mu_y}\Big)
\end{aligned}
\tag{B.62}
$$

By solving for S_t and rearranging term, S_t can be expressed as

$$\begin{aligned}S_t = &\frac{D_t}{r_t} + Y_t[\frac{N}{r_t} - \frac{h_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y} - \frac{N-h_t}{r_t - \mu_y}] \\ &- \frac{1}{r_t}\sum_{i=1}^{h_t}(\frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{r_t}\sum_{i=h_t+1}^{N}(\frac{\delta - r_t}{\gamma_i})(X_{i,t} + \frac{Y_t}{r_t - \mu_y})\end{aligned} \tag{B.63}$$

By factoring terms with respect to Y_t and substituting the equilibrium risk-free rate for r_t,

$$\begin{aligned}S_t = &\frac{D_t}{r_t} + Y_t[\frac{N}{r_t} - \frac{h_t + \frac{1}{r_t}\sum_{i=1}^{h_t}(\frac{\delta-r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})}{r_t + \rho_t\sigma_y\lambda_t - \mu_y} - \frac{N - h_t + \frac{1}{r_t}\sum_{i=h_t+1}^{N}(\frac{\delta-r_t}{\gamma_i})}{r_t - \mu_y}] \\ &+ \frac{1}{r_t}[(\mu_d D_t + \mu_y N Y_t)(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1}\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i}(1 + \frac{1}{\gamma_i})]\sum_{i=1}^{N}\frac{X_{i,t}}{\gamma_i} \\ &- \frac{\lambda_t^2}{2r_t}\sum_{i=1}^{h_t}(\frac{\gamma_i - 1}{\gamma_i^2})X_{i,t}\end{aligned} \tag{B.64}$$

B.4 Proof of the equation (25)

Consider a representative agent economy where there is no labor income risk. The Euler equation in continuous time is

$$0 = \Lambda_t D_t dt + E_t[d(\Lambda_t S_t)] \tag{B.65}$$

where Λ_t is the state price density and S_t is the stock price. By applying the Itô's product and dividing terms by $\Lambda_t S_t$,

$$0 = \frac{\Lambda_t D_t}{\Lambda_t S_t}dt + E_t[\frac{S_t d\Lambda_t + \Lambda_t dS_t + dS_t d\Lambda_t}{\Lambda_t S_t}] = E_t[dR_t] - r_t dt + E_t[\frac{dS_t d\Lambda_t}{S_t \Lambda_t}] \tag{B.66}$$

where $dR_t \equiv \frac{dS_t + D_t dt}{S_t}$. By rearranging terms,

$$E_t[dR_t] - r_t dt = -E_t[dR_t \frac{d\Lambda_t}{\Lambda_t}] \tag{B.67}$$

In the meantime, the state price density is defined as

$$\Lambda_t \equiv e^{-\delta t}u'(C_t) \tag{B.68}$$

Also, its dynamics is

$$\frac{d\Lambda_t}{\Lambda_t} = -\delta dt + \frac{C_t u''(C_t)}{u'(C_t)}\frac{dC_t}{C_t} + \frac{1}{2}\frac{u'''(C_t)}{u'(C_t)}dC_t dC_t \tag{B.69}$$

Therefore, (B.67) is

$$E_t[dR_t] - r_t dt = -E_t[dR_t \frac{C_t u''(C_t)}{u'(C_t)} \frac{dC_t}{C_t}] = \gamma_t E_t[dR_t \frac{dC_t}{C_t}] = \gamma_t Cov_t[dR_t, \frac{dC_t}{C_t}] \tag{B.70}$$

where $\gamma_t \equiv -\frac{C_t u''(C_t)}{u'(C_t)}$. (B.70) can be re-written as

$$E_t[dR_t^e] = \gamma_t Cov_t[dR_t, \frac{dC_t}{C_t}] = \gamma_t Cov_t[dR_t^e, \frac{dC_t}{C_t}] \tag{B.71}$$

where $E_t[dR_t^e] = E_t[dR_t] - r_t dt$

B.5 Proof of Proposition 4

Consider conditional covariance between stock returns and aggregate consumption growth. The aggregate consumption can be decomposed into the consumption of stockholders and that of non-stockholder.

$$\begin{aligned}
&Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*}) \\
&= \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*}) + \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} Cov_t(dR_t^e, \frac{d\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=h_t^*+1}^N C_{i,t}^*})
\end{aligned} \tag{B.72}$$

We only need to consider the diffusion terms of dynamics

$$\begin{aligned}
=&\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t})(\pi_{i,t}^*\sigma_{s,t}^d(\sigma_{s,t}^d + \rho\sigma_{s,t}^y) + \pi_{i,t}^*\sigma_{s,t}^y(\rho\sigma_{s,t}^d + \sigma_{s,t}^y) + Y_t\sigma_y g(\boldsymbol{\theta}_t)(\rho\sigma_{s,t}^d + \sigma_{s,t}^y))dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \\
&+ \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=h_t^*+1}^N \frac{C_{i,t}^*}{X_{i,t}+\frac{Y_t}{r_t-\mu_y}}(\frac{\sigma_y Y_t(\rho\sigma_{s,t}^d+\sigma_{s,t}^y)}{r_t-\mu_y})dt}{\sum_{i=h_t^*+1}^N C_{i,t}^*}
\end{aligned} \tag{B.73}$$

By substituting $\pi_{i,t}$ into the equation

$$\begin{aligned}
=&\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t})(\frac{\sigma_{s,t}\lambda_t}{\gamma_i}(X_{i,t} + g(\boldsymbol{\theta}_t)Y_t) - \sigma_{s,t}\rho_t\sigma_y Y_t g(\boldsymbol{\theta}_t) + Y_t\sigma_y g(\boldsymbol{\theta}_t)(\rho\sigma_{s,t}^d + \sigma_{s,t}^y))dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \\
&+ \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=h_t^*+1}^N \frac{C_{i,t}^*}{X_{i,t}+\frac{Y_t}{r_t-\mu_y}}(\frac{\sigma_y Y_t(\rho\sigma_{s,t}^d+\sigma_{s,t}^y)}{r_t-\mu_y})dt}{\sum_{i=h_t^*+1}^N C_{i,t}^*}
\end{aligned} \tag{B.74}$$

After some terms canceling out,

$$=\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\lambda_t\sigma_{s,t}\sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{\gamma_i})dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} + \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=h_t^*+1}^N \frac{C_{i,t}^*}{X_{i,t}+\frac{Y_t}{r_t-\mu_y}}(\frac{\sigma_y Y_t(\rho\sigma_{s,t}^d+\sigma_{s,t}^y)}{r_t-\mu_y})dt}{\sum_{i=h_t^*+1}^N C_{i,t}^*} \tag{B.75}$$

Solving (B.75) for $\lambda_t \sigma_d dt$ yields

$$\lambda_t \sigma_{s,t} dt = E_t[dR_t^e] =$$

$$\frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}) - \frac{\sum_{i=h_t^*+1}^{N} \frac{C_{i,t}^* \sigma_y(\rho\sigma_{s,t}^d + \sigma_{s,t}^y)}{X_{i,t}(r_t - \mu_y)/Y_t + 1}}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \tag{B.76}$$

Also, from (B.75)

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*}) = \frac{\lambda_t \sigma_{s,t} \sum_{i=1}^{h_t^*} (\frac{C_{i,t}^*}{\gamma_i})}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \tag{B.77}$$

Therefore,

$$\lambda_t \sigma_{s,t} dt = E_t[dR_t^e] = \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*}) \tag{B.78}$$

Appendix C. Conditional CCAPM

In this section, we provide a novel equation for the conditional consumption-based asset pricing model. First, when there are K number of individual stocks. Then, the HJB equation for the stockholder i is

$$\begin{aligned} 0 = \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} & \frac{C_{i,t}^{1-\gamma_i}}{1-\gamma_i} - \delta V + [\sum_{k=1}^{K} \pi_{i,k,t}(\mu_{k,t} - r_t) + r_t X_{i,t} + Y_t - C_{i,t}]V_x \\ & + \frac{1}{2} V_{xx} (\sum_{k=1}^{K} \pi_{i,k,t}^2 + 2\sum_{k\neq l} \pi_{i,k,t}\pi_{i,l,t}\sigma_{k,l,t}) + \mu_y Y_t V_y + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy} + Y_t V_{xy} \sum_{k=1}^{K} \pi_{i,k,t}\sigma_{k,y,t} \\ & \forall i = 1, 2, ..., h_t \end{aligned} \tag{C.1}$$

where $\sigma_{k,l,t}$ is the covariance between stock returns k and l and $\sigma_{k,y,t}$ is the covariance between stock returns k and the labor income growth.

Similar to solving the HJB under one stock, its optimal stock holding for the stock k is

$$\pi_{i,k,t}^* = \frac{\mu_{k,t}^e}{\gamma_i \sigma_{k,t}^2}(X_{i,t} + g(\boldsymbol{\theta}_t)Y_t) - \frac{\sum_{k\neq l} \pi_{i,l,t}\sigma_{k,l,t}}{\sigma_{k,t}^2} - \frac{g(\boldsymbol{\theta}_t)Y_t\sigma_{k,y,t}}{\sigma_{k,t}^2} \tag{C.2}$$

It shows that investors not only care about the intertemporal hedging motive arising from the labor income risk, but also care about the hedging among the stocks.

Second, the covariance of stockholders' consumption growth with a stock returns k is

$$Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}})$$

$$= \frac{\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{X_{i,t}+g(\boldsymbol{\theta}_t)Y_t}(\sum_{k\neq l} \pi_{i,l,t}\sigma_{k,l,t} + \pi_{i,k,t}\sigma^2_{k,t} + g(\boldsymbol{\theta}_t)Y_t\sigma_{k,y,t})}{\sum_{i=1}^{h_t^*} C^*_{i,t}} \tag{C.3}$$

By substituting (C.2) for $\pi_{i,k,t}$ to obtain,

$$Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}}) = \mu^e_{k,t}\frac{\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{\gamma_i}}{\sum_{i=1}^{h_t^*} C^*_{i,t}} \tag{C.4}$$

Finally, the equilibrium excess returns of stock k is

$$E_t[dR^e_{k,t}] = \mu^e_{k,t} = \frac{\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{\gamma_i}} Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}}) \tag{C.5}$$

Using the **Proposition 4** in (26), it can re-written as

$$E_t[dR^e_{k,t}] = \frac{Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}})}{Cov_t(dR^e_{t}, \frac{d\sum_{i=1}^{h_t^*} C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}})} E_t[dR^e_{m,t}] \tag{C.6}$$

where $dR^e_{m,t}$ denotes the market excess returns.

If there is no labor income, the ratio of covariances in (C.6) becomes to the standard CAPM beta ($= \frac{Cov_t(dR^e_{k,t}, dR^e_{m,t})}{\sigma^2_{m,t}}$).

Appendix D. Data

In this section, we describe the data sources and variables used in this paper. Throughout the paper, we use monthly frequency. All data are converted to the real dollars using September 2010 dollars. The sample number is 238 observations from April 1996 through December 2015. April 1996 is chosen by the data availability of CE (Consumer Expenditure).

Excess equity returns and instruments

Excess monthly returns to the aggregate stock market is measured by log real per capita growth of the CRSP value-weighted NYSE/Amex/Nasdaq index minus 1-month Treasury bill. Equity data and 1-month Treasury bill data are obtained from the Center for Research in Security Prices (CRSP). All instruments for excess equity returns $X_{r^e,t-1}$ are obtained from Amit Goyal website[54]. They are measured as follows. 1. Log dividend-price ratio (D/P_{t-1}) : log of a 12-month moving sum of dividends paid on the S&P500 index minus the log of stock prices. 2. Stock variance $(SVAR_{t-1})$: monthly sum of squared daily returns on the S&P 500 index. 3. Book-to-market ratio (BM_{t-1}) : book-to-market value ratio for the DJIA. 4. Net equity expansion $(NTIS_{t-1})$: ratio of a 12-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks. 5. Long-term yield (LTY_{t-1}) : Long-term government bond yield.

Consumption data

Monthly consumption data are collected from two data sets. One is from sample household data CE (Consumption Expenditure) where we can 'likely' identify stockholders. The other is from the total U.S. consumption (NIPA; National Income and Product Accounts) measured by the Bureau of Economic Analysis. We calculate separate quarterly consumption growth rate at a monthly frequency for stockholders, aggregate sample households from CE, and aggregate total U.S. households from NIPA. For NIPA, we calculate log real per capita change in nondurable and services. The NIPA total U.S. consumption data from 1960 to 2016 is used to obtain target consumption growth moments in Table 4. These are also used to construct the surplus consumption measure in Section 5.

CE survey is conducted for the Bureau of Labor Statistics by the U.S. Census Bureau as a monthly basis. A selected family is interviewed every 3 months over four times. After the last interview (fourth), the sample family is dropped from the survey and a new sample family is introduced. Therefore, the composition of interviewed households in a month is different from the next month, and thus, we can calculate the quarterly consumption growth at a monthly frequency. Finance asset holding information is collected in the last interview.[55]

We construct consumption based on the Interview Survey part of the CE. As a definition of consumption, we use items in CE which match the definitions of nondurables and services in NIPA. We exclude housing expenses (but not costs of household operations), medical care costs, and education costs due to its substantial durable components, following Malloy et al. (2009) and Attanasio and Weber (1995). We also mainly follow Malloy et al. (2009) for the sample choice. We drop household-quarters in which a household reports negative consumption. Extreme outliers having consumption growth $(C_{i,t+1}/C_{i,t})$ more than 5.0 and less than 0.2 are dropped since these could be reporting or coding errors. Moreover, nonurban households and households residing in student housing are dropped due to incomplete income responses. To identify the stockholders, we refer to the question of "As of today, what is the total value of all directly-held stocks, bonds, and mutual funds?". After constructing the aggregate consumption growth and aggregate stockholders' consumption growth, we regress them on the monthly dummies to control for seasonality and use the residual series for analysis.

[54] http://www.hec.unil.ch/agoyal/

[55] For a more detailed information, see https://www.bls.gov/opub/hom/cex/data.htm

Dividend and labor income data

Both dividend and labor income data from 1960 to 2015 are obtained from U.S. Bureau of Economic Analysis (BEA Account Code: A2218C1 and A4102C1, respectively). By calculating log change in real per capita dividend and labor, the first and second moments of dividend and labor income growth corrected for Jensen's inequality are used to choose the parameter values in Table 1.

Appendix E. Description of Probit regression

In order to proxy for the time-invariant relative risk aversion coefficient for each household in CE data, we consider a Probit regression model to predict the probability that household owns stock based on the SCF (Survey of Consumer Finances) data. The Survey of Consumer Finances (SCF) is a cross-sectional survey of U.S. families conducted by the Federal Reserve Board every three years. The survey data cover a wide variety of information on families balance sheets, pensions, income, and demographic characteristics. Unlike CE data, the SCF directly asks households whether respondents have any stock (Variable name:hstocks) or mutual funds excluding MMMFs (hnmmf). However, since the survey is conducted on a triennial basis, the data cannot be used for conditional asset pricing test. Using the SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, and 2013, we run a Probit regression of whether a household owns stock or mutual fund on a set of observable characteristics that are known to affect the stock investment and exist in the CE data. They are age of household, age squared, an indicator for race not being white/Caucasian, the number of kids, an indicator for at least 12 but less than 16 years of education for head of household (*highschool*), an indicator for 16 or more years of education (*college*), the log of real total household income before taxes, the log of real dollar amount in checking and savings account (set to zero if checking and savings = 0), and indicator for checking and savings account = 0, an indicator for dividend income, and year dummies. They are chosen by Malloy, Moskowitz, and Vissing-Jorgensen (2009) and we additionally include the number kids which has a strong explanatory power. All dollar values are in 2013 dollars. The Probit model estimation is reported in Table X.

The estimates of the coefficients from the Probit model in the SCF data are applied to the CE data to obtain the probability of being a stockholder for each household. In calculating the probability of being stockholders in CE data, we use the 1992 dummy coefficient for the years 1991-1993, the 1995 dummy coefficient for the years for 1994-1996, the 1998 dummy coefficient for the years for 1997-1999, the 2001 dummy coefficient for the years for 2000-2002, the 2004 dummy coefficient for the years for 2003-2005, the 2007 dummy coefficient for the years for 2006-2008, the 2010 dummy coefficient for the years for 2009-2011, and the 2013 dummy coefficient from 2012 onward. We then define the relative risk aversion of each household is 0.5 divided by the estimated probability of being a stockholder. The value 0.5 is chosen to make the household whose probability of being stockholder is average cross households after 1.5% and 98.5% winsorization have the relative risk aversion 5.

Appendix F. Cross-sectional variation in Consumption

Our model finds the consumption level is positively associated with risk aversion. To understand the underlying mechanism which drives the association, we can study the following.

The optimal consumption of stockholders in **Proposition 1.** in Section 2 is

$$C_{i,t} = (r_t + \frac{\delta - r_t}{\gamma_i} - \frac{1-\gamma_i}{\gamma_i^2}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \tag{F.1}$$

Given the same level of $X_{i,t}$ for all stockholders, the partial derivative of the optimal consumption with respect to risk aversion is

$$\frac{\partial C_{i,t}}{\partial \gamma_i} = (\frac{r_t - \delta}{\gamma_i^2} - \frac{\gamma_i - 2}{\gamma_i^3}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \tag{F.2}$$

Since the equilibrium risk-free rate is following,

$$r_t = \delta + (\mu_d D_t + \mu_y N \cdot Y_t)(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i})^{-1}\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i}(1+\frac{1}{\gamma_i}), \tag{F.3}$$

we can rewrite the partial derivative as

$$\frac{\partial C_{i,t}}{\partial \gamma_i} = (\frac{\frac{\mu_d D_t + \mu_y N \cdot Y_t}{\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i}} - \frac{\lambda_t^2}{2}\frac{\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i}(1+\frac{1}{\gamma_i})}{\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i}}}{\gamma_i^2} - \frac{\gamma_i - 2}{\gamma_i^3}\frac{\lambda_t^2}{2})(X_{i,t} + \frac{Y_t}{r_t + \rho_t\sigma_y\lambda_t - \mu_y}) \tag{F.4}$$

Therefore, $\frac{\partial C_{i,t}}{\partial \gamma_i} > 0$ is equivalent to

$$\frac{\mu_d D_t + \mu_y N \cdot Y_t}{\sum_{i=1}^{N} C_{i,t}^*}(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}\frac{1}{\gamma_i})^{-1} - \frac{\lambda_t^2}{2}[\frac{\sum_{i=1}^{h_t}\frac{C_{i,t}^*}{\gamma_i}(1+\frac{1}{\gamma_i})}{\sum_{i=1}^{N}\frac{C_{i,t}^*}{\gamma_i}} + \frac{\gamma_i - 2}{\gamma_i}] > 0 \tag{F.5}$$

The first term is the expected aggregate consumption growth multiplied by the consumption-weighted harmonic mean of aggregate risk aversion. If the expected aggregate consumption growth is high enough to offset the second term, the more risk-averse the investors are, the more they consume. This is because the investors with relatively high risk aversion have relatively low EIS (Elasticity of Intertemporal Substitution) and the consumption smoothing motive is very strong. Therefore, when the consumption is expected to grow fast, investors with high risk aversion (low EIS) consume a lot for the consumption smoothing.

The second term consists of the Sharpe ratio and other term. When the economic uncertainty and volatility are high, the Sharpe ratio is high. This is the moment the more risk-averse investors really care about. Since the more risk-averse investors are more sensitive to the consumption risk, they consume less than the other investors if the second term is higher than the first term.

Please note that only the second term depends on the market participation level. In the case where investors are faced with labor income risk, the market risk-sharing is not perfect. However, as more investors decide to be a stockholder, the risk is more effectively shared out and thus the Sharpe ratio is decreasing. By contrast, without labor income risk, the market is complete and the Sharpe ratio is simply dividend volatility multiplied by the consumption-weighted harmonic mean of stockholders' risk aversion. Although the full market participation is attained in this case, we can consider the hypothetical Sharpe ratio depending on the market participation to compare the Share ratio in the one under the incomplete market.

Figure 6 plots the Sharpe ratio when each stockholder from $i = 1$ to $i = 30$ is the cut-off stockholder at time 0 ($t = 0$) as in Figure 3. The dashed line represents the result for the incomplete market case. Since the risk-sharing is perfect, the variation in the Sharpe ratio only comes from the stockholders' average risk aversion. As investors whose risk aversion is higher than the existing stockholders enter the market, the average risk aversion level increases, thus increasing the Sharpe ratio. Since the market is perfect, the full market participation is attained at $i = N = 30$. Contrary to this, for the case where the market is incomplete, the Sharpe ratio is decreasing as risk-sharing is more effective. Due to this mechanism, the second term in (F.5) is sufficiently low under the incomplete market, resulting in the positive association between the consumption level and risk aversion.

Furthermore, note that the general level of Sharpe ratio is higher without labor income than the one with labor income. This is because with labor income, the perfect correlation between consumption growth and stock returns is no longer the case because labor income is only *partially* correlated with stock returns. Therefore, the consumption risk is lower than otherwise without labor income, resulting in the low level of the Sharpe ratio. This in turn leads to the positive relation between the consumption level and risk aversion.

Appendix G. Stock Volatility

Since the equilibrium stock price is affected by labor income shock as well as dividend shock in our model, the stock volatility is affected by both dividend volatility and labor income volatility. Therefore, it is of particular interest to examine the role of labor income volatility on the stock volatility. In the simulation of the current paper, the dividend volatility 9% and the labor income volatility 3% are used based on the U.S. data. We examine how the stock volatility varies depending on different dividend volatility and labor income volatility. In Panel A of Figure 7, stock volatility is plotted as a function of dividend volatility σ_d and labor income volatility σ_y. The range of σ_d is from 5.5% to 9% and the range of σ_y is from 1.5% to 5%. The figure shows given higher dividend volatility than labor income volatility, the stock volatility varies more sensitively with dividend volatility than labor income volatility.

To decompose the stock volatility, for each value of dividend volatility, we compute the average of stock volatility levels which differ due to different labor income volatility levels. Then, in Panel B, the stock volatility is decomposed into dividend volatility, labor volatility, and other remaining part. It shows the dividend volatility accounts for 30% to 47% stock volatility while the labor income volatility accounts for only 16% on average (The range is 12% to 20%.). Also, as the dividend volatility level increases, it accounts for higher proportion of the stock volatility. This implies that one unit increase in dividend volatility lead to an increase in stock volatility less than one unit. In addition, this figure shows that labor income volatility does not fully explain the excess volatility. As discussed in Section 3, the remaining excess volatility (white area) is generated by heterogeneity.