Forward Premium Puzzle and Heterogeneous Beliefs

Abstract

We propose a two-country model with heterogeneous beliefs to understand the forward premium puzzle. The domestic and foreign investors disagree on the expected growth rate of the domestic money supply. Facing a shock to domestic money supply, the disagreement between the domestic and foreign investors shifts the relative wealth of foreign investor, which moves the foreign exchange rate and interest rate differential between foreign and domestic countries in opposite directions. Calibrated to the U.S. and U.K. data, our model reproduces the rejection of the uncovered interest rate parity. Using a monthly index of heterogeneous beliefs based on the Consensus Forecast in 15 major economies, the empirical evidence confirms two implications of our model: 1) a positive relation between heterogeneous beliefs and changes in exchange rate; 2) a negative relation between heterogeneous beliefs and carry trade returns.

JEL Classification: D53, G10, G12

Keywords: Heterogeneous beliefs; Uncovered interest rate parity; Carry trade; Exchange rate volatility

1 Introduction

The failure of uncovered interest rate parity (UIP) has attracted academic attention for a few decades. The UIP predicts that the expected change of the exchange rate (i.e., the price of domestic currency in terms of foreign currency) should be equal to the nominal interest rate differential between foreign and domestic countries. If the uncovered interest parity holds, a regression of the realized exchange rate changes on the interest rate differentials should have a unity slope coefficient and a zero constant. Contrary to the theoretical prediction, the empirical results find a negative relation between the changes in exchange rate and the corresponding interest rate differential, with the slope coefficient less than negative one (e.g., Hansen and Hodrick, 1980; Fama, 1984; Backus, Foresi, and Telmer, 2001). This is called the "forward premium puzzle." The empirical failure of UIP implies that investors can employ currency carry trade strategy to earn a positive average excess return by shorting the currency with low interest rate and investing the funds in the currency with high interest rate.

To provide an explanation of the forward premium puzzle, we construct a heterogeneous belief model building on Lucas (1982) and Croitoru and Lu (2015). In an economy with two groups of investors from two countries, there are one consumption goods and one currency from each country. The monetary policy of foreign country is transparent and thus both domestic and foreign investors have the correct estimates about the growth rate of foreign money supply. However, the monetary policy of domestic country is ambiguous or less transparent, in the sense that the expected growth rate of domestic money supply is unobservable. Domestic and foreign investors may disagree on the expected domestic money growth. In particular, domestic investors are more optimistic while foreign investors are more pessimistic about domestic money growth. They "bet" against each other on domestic monetary policy generating "trading risk" (e.g., David, 2008; Croitoru and Lu, 2015).

The key to produce the failure of UIP is to identify a friction making the changes in exchange rate opposite to the interest rate differential. Our model reveals that the heterogeneity in beliefs between domestic and foreign investors shifts the relative wealth weight of foreign investors, which positively affects foreign exchange rate and negatively affects the interest rate differential. When there is a negative shock to domestic money supply, domestic investors "lose" the bet because they are overly optimistic about the domestic money growth rate and thus their relative wealth decreases. They save less to smooth the consumption. The domestic nominal interest rate then rises, leading to lower nominal interest rate differential between foreign and domestic countries. Meanwhile, everything else being equal, a negative shock to domestic money supply is likely to cause deflation, making domestic goods relatively more competitive. Such an increase in foreign demand for domestic goods leads to more demand for domestic currency and a currency appreciation. Moreover, as the domestic money supply experiencing a negative shock, pessimistic foreign investors "win" the bet because they are correct about the subsided domestic money growth rate and thus their relative wealth increases. They consume more domestic consumption goods in addition to foreign consumption goods through holding more domestic currency. Therefore, the domestic currency further appreciates. Taken together, our model not only generates a negative relation between the change in foreign exchange rate and the interest rate differential, but also produces a higher volatility for the change in foreign exchange rate than the interest rate differential.

In addition to rationalize the well-documented empirical failure of UIP, our model can produce a stochastic volatility of exchange rate with a magnitude comparable to the actual data. In our model, "trading risk", generated by the "bet" between domestic and foreign investors, amplifies the diffusion coefficient of nominal exchange rate caused by domestic monetary risk, making the volatility of nominal exchange rate rate positively related to investors' heterogeneity in beliefs. Consequently, the volatility of exchange rate fluctuates as the dispersion of beliefs varies over time. This is consistent with the empirical findings in Beber, Breedon, and Buraschi (2010), which documents a positive impact of differences in beliefs on the currency return volatility.

We calibrate our two-country model to the U.S. and U.K. data. Following the standard long-horizon predictive regression approach like Fama (1984), we test the UIP directly by regressing the changes in exchange rate on the interest differential. The slope coefficient in our baseline calibration is negative (-1.01) and closely matches the actual data (-1.18). Our model could not only produce the negative correlation between the currency risk premium and the interest rate differential, but also generates higher volatility of foreign exchange rate (6.78%) than the volatility of interest differentials (1.46 bps). The high volatility of the exchange rate in the model is associated with the time-varying degree of heterogeneous beliefs. Using a proxy for the realized heterogeneous beliefs in the U.S. and the U.K., the dynamics of the model generated exchange rate volatility resembles the actual volatility dynamics of the USD/GBP exchange rate.

The calibration uncovers two testable implications of our model. First, the changes in the exchange rate are positively related to the heterogeneity in beliefs among investors about the domestic money growth rate. Second, the carry trade return is negatively related to the heterogeneous beliefs. We test these two hypotheses in 15 major economies by constructing a monthly index of the dispersion of beliefs using the Consensus Forecast published by the Consensus Economics. The Consensus Forecasts contains the estimates of individual panelists for numerous macroeconomic indicators. Our proxy of heterogeneous beliefs is based on the dispersion of the panelists' estimate for the consumer price index (CPI). To test the first hypothesis, the future changes in foreign exchange rates are regressed on the proxy of heterogeneity in beliefs and other control variables. The evidence is overall supportive. The coefficients of dispersion of beliefs are positive in 10 out of the 15 economies, and 8 of them are statistically significant. For the second hypothesis, we regress the 3-month carry trade returns on the dispersion of beliefs. Out of the 15 countries, the regression coefficients for 12 economies have the negative sign, and 9 of them are statistically significant. Our results are robust to controlling for economic uncertainty, as well as using the relative proxy of heterogeneity in beliefs. The relative heterogeneity in beliefs measures the dispersion of beliefs in each country relative to the United States.

Our model is related to several studies that attempt to explain the forward premium puzzle. Verdelhan (2010) provides a habit formation model to produce the failure of UIP. Heyerdahl-Larsen (2014) extends Verdelhan (2010) to a model with consumption home bias and deep habits. In addition to solving for the forward premium puzzle, it can match the dynamics of domestic assets including equity premium, stock return volatility, and upward-sloping real yield curves. Burnside, Han, Hirshleifer and Wang (2011) provide an explanation for the forward premium puzzle assuming investors are overconfident about inflation information. Yu (2013) presents a sentiment-based model which helps explain the forward premium puzzle and the low correlation between consumption growth growth differentials and exchange rate growth. With the same purpose, Osambela (2013) develops a two-country two-good model where domestic and foreign investors have different beliefs about the expected output growth rate in each country. Our paper is most closely related to Osambela (2013). However, there are two differences. First, in our model domestic and foreign investors have different beliefs about the expected domestic money growth, and then we examine the nominal forward premium puzzle. Second, our model generates high exchange rate volatility due to "trading risk", which is comparable to actual data.

Our model is also related to international asset pricing models with heterogeneous beliefs. Li and Muzere (2010) present a model with two countries and two goods in which investors with logarithmic utility have heterogeneous beliefs about expected output growth rate. They find that the exchange rate and stock returns exhibit high volatility. Unlike their paper, we also explain the nominal forward premium puzzle. Dumas, Lewis, and Osambela (2017) propose a sentiment model in which domestic and foreign investors have different beliefs about the information content in public signals. They study several interesting issues such as the co-movement of returns and capital flows, the home equity preference, the dependence of firm returns on home and foreign factors, and the abnormal returns around the listings of foreign firms in the home market. Unlike their paper, our model extends Croitoru and Lu (2015) to an international setup where domestic and foreign investors have heterogeneous beliefs about domestic monetary policy. The research questions addressed are also different from Dumas, Lewis, and Osambela (2017). Moreover, our model is related to two-tree models with heterogeneous beliefs. Buraschi, Trojani, and Vedolin (2014) explain the differential pricing of index and individual equity options. Gallmeyer, Jhang, and Kim (2016) demonstrate that the idiosyncratic cash flow shocks priced via the heterogeneity in beliefs can explain cross-sectional stock returns and cash flows. Han, Lu, and Zhou (2018) show that disagreement among investors in one large firm (i.e., industry leading firm) has a spillover effect on the pricing of other stocks owned by these investors.

The rest of the paper is organized as follows. Section 2 describes the model and solves the dynamic equilibrium. Section 3 calibrates parameters, reproduces several stylized empirical facts, and derives two testable hypotheses. Empirical analysis results are presented in Section 4. Section 5 concludes the paper. The Appendix contains the proof of Propositions.

2 The Model

Our model builds on Lucas (1982) two-country model and Croitoru and Lu (2015) heterogeneous beliefs model. We consider a continuous-time, pure exchange, finite horizon [0,T], and nominal economy with two groups of investors from two countries. We call them domestic country and foreign country respectively, and then domestic investors and foreign investors correspondingly. The uncertainty is generated by a four-dimensional Brownian motion, $\omega = (\omega_\varepsilon, \omega_{\varepsilon^*}, \omega_M, \omega_M^*)^T$. There are one consumption goods and one currency from each country. The domestic consumption goods is assumed to be the numeraire.

2.1 Aggregate Consumption, Money Supply, and the Information Structure

The aggregate consumption goods of domestic country and foreign country are denoted by ε and ε^*, respectively. The money supplies in currency of domestic country and foreign country are represented by M and M^*, respectively. They follow the dynamics given by:

$$\begin{pmatrix} d\ln\varepsilon_t \\ d\ln\varepsilon_t^* \\ d\ln M_t \\ d\ln M_t^* \end{pmatrix} = \begin{pmatrix} \mu_\varepsilon \\ \mu_{\varepsilon^*} \\ \mu_M \\ \mu_{M^*} \end{pmatrix} dt + \begin{pmatrix} \sigma_{\varepsilon\varepsilon} & 0 & 0 & \sigma_{\varepsilon M^*} \\ \sigma_{\varepsilon^*\varepsilon} & \sigma_{\varepsilon^*\varepsilon^*} & 0 & \sigma_{\varepsilon^* M^*} \\ \sigma_{M\varepsilon} & \sigma_{M\varepsilon^*} & \sigma_{MM} & 0 \\ 0 & \sigma_{M^*\varepsilon^*} & 0 & \sigma_{M^*M^*} \end{pmatrix} d\omega_t, \tag{1}$$

where $d\omega = (d\omega_\varepsilon, d\omega_\varepsilon^*, d\omega_M, d\omega_M^*)^T$ and the four Brownian motions $\omega_\varepsilon, \omega_\varepsilon^*, \omega_M$, and ω_M^* are independent. We can allow these four Brownian motions to be dependent but this does not generate new insights.

All investors can observe the processes of domestic and foreign aggregate consumption, ε and ε^*, and money supplies, M and M^*. There is no uncertainty associated with domestic and foreign consumption risks in the sense that all investors can observe the realizations of ω_ε and ω_ε^*. Moreover, the monetary policy of foreign country is transparent and thus all investors have the correct estimate about the growth rate of foreign money supply, μ_M^*. However, the monetary policy of domestic country is ambiguous, in the sense that the expected growth rate of its money supply, μ_M, is unobservable. All investors need to estimate it by observing the realizations of domestic money supply, M. Thus, domestic and foreign investors may have different estimates about μ_M. We denote their estimates by μ_M^D and μ_M^F, respectively.[1] The dynamics of domestic money supply from the perspective of each investor ($i = D, F$) follows the dynamics given by:

$$d \ln M_t = \mu_M^i dt + \sigma_{M\varepsilon} d\omega_{\varepsilon,t} + \sigma_{M\varepsilon^*} d\omega_{\varepsilon^*,t} + \sigma_{MM} d\omega_{M,t}^i, \tag{2}$$

where investor i's perceived innovation process of domestic monetary risk $d\omega_{M,t}^i = \frac{1}{\sigma_{MM}}(\frac{dM_t}{M_t} - \mu_M^i dt - \sigma_{M\varepsilon} d\omega_{\varepsilon,t} - \sigma_{M\varepsilon^*} d\omega_{\varepsilon^*,t})$ and then we have:

$$d\omega_{M,t}^F = d\omega_{M,t}^D + \bar{\mu}_M dt, \tag{3}$$

where $\bar{\mu}_M = \frac{\mu_M^D - \mu_M^F}{\sigma_{MM}}$ denotes the difference in estimated growth rate of domestic money supply between domestic and foreign investors, scaled by the standard deviation of domestic money supply.

2.2 Investment Opportunities

There are four uncertainties, so we need five securities to complete the market: one real riskless bond, B, with zero-supply and paying-off the real interest rate r; two zero-supply, nominally riskless bonds, B_M and B_M^*, in its own currency paying-off nominal interest rate R and R^* (but they are risky in real items); and two stocks S and S^* representing the claims on domestic and foreign aggregate consumption ε and ε^* respectively, and with a supply of one share. The prices of the riskless bond, two nominally riskless bonds, and two stocks follow the dynamics:

$$dB_t = B_t r_t dt, \tag{4}$$

$$dB_{M,t} = B_{M,t}[(\mu_{q,t} + R_t)dt + \sigma_{q,t} d\omega_t] = B_{M,t}[(\mu_{q,t}^i + R_t)dt + \sigma_{q,t} d\omega_t^i],$$

$$dB_{M^*,t} = B_{M^*,t}[(\mu_{q^*,t} + R_t^*)dt + \sigma_{q^*,t} d\omega_t] = B_{M^*,t}[(\mu_{q^*,t}^i + R_t^*)dt + \sigma_{q^*,t} d\omega_t^i],$$

$$dS_t + \varepsilon_t dt = S_t(\mu_{S,t} dt + \sigma_{S,t} d\omega_t) = S_t(\mu_{S,t}^i dt + \sigma_{S,t} d\omega_t^i),$$

$$dS_t^* + p_t^* \varepsilon_t^* dt = S_t^*(\mu_{S^*,t} dt + \sigma_{S^*,t} d\omega_t) = S_t^*(\mu_{S^*,t}^i dt + \sigma_{S^*,t} d\omega_t^i),$$

where p^* is the equilibrium price of foreign consumption goods relative to the price of domestic consumption goods (which is the numeraire), $d\omega^i = (d\omega_\varepsilon, d\omega_\varepsilon^*, d\omega_M^i, d\omega_M^*)^T$ is innovation process

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[1] We can assume investors update their estimates about the expected domestic money growth rate using Bayesian learning, but this doesn't generate any new insight.

perceived by investor i, and the volatility matrix is given by

$$\Omega = \begin{pmatrix} \sigma_q, & \sigma_q^*, & \sigma_S, & \sigma_S^* \end{pmatrix}^T = \begin{pmatrix} \sigma_{q\varepsilon} & \sigma_{q\varepsilon^*} & \sigma_{qM} & \sigma_{qM^*} \\ \sigma_{q^*\varepsilon} & \sigma_{q^*\varepsilon^*} & \sigma_{q^*M} & \sigma_{q^*M^*} \\ \sigma_{S\varepsilon} & \sigma_{S\varepsilon^*} & \sigma_{SM} & \sigma_{SM^*} \\ \sigma_{S^*\varepsilon} & \sigma_{S^*\varepsilon^*} & \sigma_{S^*M} & \sigma_{S^*M^*} \end{pmatrix}. \tag{5}$$

Therefore, we have the following relationship

$$\begin{pmatrix} \mu_q^D - \mu_q^F, & \mu_{q^*}^D - \mu_{q^*}^F, & \mu_S^D - \mu_S^F, & \mu_{S^*}^D - \mu_{S^*}^F \end{pmatrix} = \begin{pmatrix} \sigma_q M, & \sigma_{q^*M}, & \sigma_S M, & \sigma_{S^*M} \end{pmatrix} \bar{\mu}_M. \tag{6}$$

Given a complete market, investor i faces the unique market prices of risk, θ_ε, θ_{ε^*}, θ_M^i and θ_M^*. They are associated with the uncertainties of domestic and foreign consumption goods and money supplies, respectively, and solve for the following equation ($i = D, F$):

$$\Omega \begin{pmatrix} \theta_\varepsilon, & \theta_{\varepsilon^*}, & \theta_M^i, & \theta_M^* \end{pmatrix}^T = \begin{pmatrix} \mu_q^i - r, & \mu_{q^*}^i - r, & \mu_S^i - r, & \mu_{S^*}^i - r \end{pmatrix}^T. \tag{7}$$

Therefore,

$$\theta_M^D - \theta_M^F = \bar{\mu}_M. \tag{8}$$

Under the no arbitrage condition and assuming domestic consumption goods as the numeraire, investor i's perceived state price density in equilibrium is given by (e.g., Buraschi, Trojani, and Vedolin, 2014)

$$d\xi_t^i = -\xi_t^i (r_t dt + \theta_{\varepsilon,t} d\omega_{\varepsilon,t} + \theta_{\varepsilon^*,t} d\omega_{\varepsilon^*,t} + \theta_{M,t}^i d\omega_{M,t}^i + \theta_{M^*,t} d\omega_{M^*,t}), \tag{9}$$

where r_t is the domestic real interest rate.

2.3 Investors' Optimization

We assume that all investors consume both domestic and foreign consumption goods. Foreign investors are assumed to hold both foreign and domestic currencies, while domestic investors only hold domestic currency. Investor i ($i = D, F$) chooses his holdings of domestic and foreign consumption and currencies and maximizes his cumulative lifetime expected utility

$$\max_{c^i, c^{*i}, m^i, m^{*i}} E_t^i \left[\int_0^T e^{-\rho t} u^i \left(c_t^i, c_t^{*i}, q_t m_t^i, \tfrac{q_t^* m_t^{*i}}{p_t^*} \right) dt \right] \tag{10}$$
$$\text{s.t. } E_t^i \left[\int_0^T \xi_t^i \left(c_t^i + p_t^* c_t^{*i} + q_t m_t^i + q_t^* m_t^{*i} \right) dt \right] \leq x_0^i,$$

where

$$\begin{aligned} u^D \left(c^D, c^{*D}, qm^D, \tfrac{q^* m^{*D}}{p^*} \right) &= \alpha \left[\beta^D \log \left(c^D \right) + \left(1 - \beta^D \right) \log \left(c^{*D} \right) \right] + (1 - \alpha) \log \left(qm^D \right), \\ u^F \left(c^F, c^{*F}, qm^F, \tfrac{q^* m^{*F}}{p^*} \right) &= \alpha \left[\left(1 - \beta^F \right) \log \left(c^F \right) + \beta^F \log \left(c^{*F} \right) \right] \\ &\quad + (1 - \alpha) \left[(1 - \gamma) \log \left(qm^F \right) + \gamma \log \left(\tfrac{q^* m^{*F}}{p^*} \right) \right], \end{aligned}$$

where q and q^* denote the equilibrium prices of domestic currency and foreign currency relative to the price of domestic consumption (numeraire).

Following Basak and Cuoco (1998) and Basak (2000), we can construct a "world" representative agent with the following utility function:

$$\begin{aligned} & U(c_t, c_t^*, q_t m_t, \tfrac{q_t^* m_t^*}{p_t^*}; \lambda_t) \\ &= \max_{\substack{c^D + c^F = c, c^{*D} + c^{*F} = c^* \\ m^D + m^F = m, m^{*D} + m^{*F} = m^*}} u^D \left(c_t^D, c_t^{*D}, q_t m_t^D, \tfrac{q_t^* m_t^{*D}}{p_t^*} \right) + \lambda_t u^F \left(c_t^F, c_t^{*F}, q_t m_t^F, \tfrac{q_t^* m_t^{*F}}{p_t^*} \right), \end{aligned} \tag{11}$$

where λ denotes the relative wealth weight of foreign investors, and it is positive and can be stochastic.

2.4 The Equilibrium

We derive the competitive equilibrium following the standard martingale techniques (e.g., Cox and Huang, 1989; Karatzas, Lehoczky, and Shreve, 1990).

Definition 1 *An equilibrium is a price system* $(r, R, R^*, S, S^*, B_M, B_{M^*})$ *and admissible policies* $\left(c^i, c^{*i}, m^i, m^{*i}, \pi^i \right)$ $(i = D, F)$ *such that: (i) investors choose their optimal policies given their beliefs; and (ii) markets for consumption, money and securities clear, i.e.,*

$$\begin{gathered} c_t^D + c_t^F = \varepsilon_t, \ c_t^{*D} + c_t^{*F} = \varepsilon_t^*, \\ m_t^D + m_t^F = M_t, \ m_t^{*D} + m_t^{*F} = M_t^*, \\ \pi_{M,t}^D + \pi_{M,t}^F = 0, \ \pi_{M^*,t}^D + \pi_{M^*,t}^F = 0, \\ \pi_{S,t}^D + \pi_{S,t}^F = 1, \ \pi_{S^*,t}^D + \pi_{S^*,t}^F = 1, \\ X_t^D + X_t^F = S_t + p_t^* S_t^* + q_t M_t + q_t^* M_t^*. \end{gathered} \tag{12}$$

In the following proposition, we present the domestic and foreign nominal interest rates and the market prices of domestic and foreign consumption risk and monetary risk.

Proposition 1 *The domestic and foreign nominal interest rates are*

$$R_t = \tfrac{1+(1-\gamma)\lambda_t}{G_t^D+(1-\gamma)G_t^F\lambda_t}, \tag{13}$$

$$R_t^* = \tfrac{1}{H_t}, \tag{14}$$

where λ is the relative wealth weight of foreign investors and follows the dynamics

$$d\lambda_t = -\lambda_t \bar{\mu}_M d\omega_{M,t}^D, \tag{15}$$

and $G_t^i = \frac{\exp\left\{\left(\frac{1}{2}\sigma_M^2-\mu_M^i\right)(T-t)\right\}-1}{\frac{1}{2}\sigma_M^2-\mu_M^i}$ $(i = D, F)$*, and* $H_t = \frac{\exp\left\{\left(\frac{1}{2}\sigma_{M^*}^2-\mu_{M^*}\right)(T-t)\right\}-1}{\frac{1}{2}\sigma_{M^*}^2-\mu_{M^*}}$.

The market prices of of domestic and foreign consumption and monetary risk perceived by two countries' investors are

$$\begin{gathered}\theta_{\varepsilon,t} = \sigma_{\varepsilon\varepsilon},\ \theta_{\varepsilon^*,t} = 0, \\ \theta_{M^*,t} = \sigma_{\varepsilon M^*}, \\ \theta_{M,t}^D = \tfrac{\left(1-\beta^F\right)\lambda_t\bar{\mu}_M}{\beta^D+(1-\beta^F)\lambda_t},\ \theta_{M,t}^F = -\tfrac{\beta^D\bar{\mu}_M}{\beta^D+(1-\beta^F)\lambda_t}.\end{gathered} \tag{16}$$

Equation (13) shows that investors' disagreement about the expected domestic money growth rate, $\bar{\mu}_M$, affects the domestic nominal interest rate in two ways. First, the investors' disagreement shifts the relative wealth weight of foreign investors, λ. Second, the disagreement affects μ_M^i and then G_t^i. In the presence of the investors' disagreement, a negative shock to domestic money supply moves the domestic nominal interest rate higher. Facing a negative shock to domestic money supply, optimistic domestic investors "lose" the bet because they overestimate the growth rate of domestic money supply and thus their relative wealth decreases (Equation (15)). Therefore, domestic investors save less, pushing domestic nominal interest rate up.

Equation (14) shows that foreign nominal interest rate is nonstochastic. This is intuitive: first, there is no uncertainty associated with foreign money supply; second, the foreign currency is only held by foreign investors. Admittedly, we could instead assume that both domestic and foreign investors hold the foreign currency. Then, the foreign nominal interest rate, which depends on investors' disagreement and relative wealth, would become stochastic. This might help our model match the actual data better including the first and second moment of the foreign nominal interest rate. However, this alternative assumption is not able to generate any new insight in explaining the nominal forward premium puzzle.

Equation (16) describes the individual market prices of domestic and foreign consumption risk and monetary risk, respectively. The expected growth rate of domestic money supply is unobservable and thus domestic and foreign investors may have different perceptions about domestic monetary shock. Therefore, they have different perceived market price of domestic monetary risk. Foreign investors are relatively pessimistic and transfer domestic monetary risk to domestic investors in order to smooth their consumption. Domestic investors require a higher premium for risk compensation and hence a higher perceived market price of monetary risk. The individual market price of domestic monetary risk is affected by investors' disagreement through two channels. First, disagreement directly affects individual market price of domestic monetary risk through risk-sharing effect. Second, the disagreement shifts wealth allocation between domestic and foreign investors, λ, and then affects individual market price of domestic monetary risk. This is consistent with the analysis of Croitoru and Lu (2015).

In the following, we present the forward interest rates for zero-coupon domestic and foreign bonds.

Remark 1 *The time t forward interest rate for $(n-m)$-year zero-coupon domestic bond starting in m years $(n > m > 0)$ is given by*

$$f_{t,m,n} = \tfrac{(1+R_{t,n})^n}{(1+R_{t,m})^m} - 1, \tag{17}$$

where the time t zero-coupon yield with maturity $\tau(\tau = m, n)$ for domestic bond is

$$R_{t,\tau} = \tfrac{1}{\tau}\ln(\tfrac{G_t^D+(1-\gamma)G_t^F\lambda_t]}{G_{t+\tau}^D\exp\left\{(\frac{1}{2}\sigma_M^2-\mu_M^D)\tau\right\}+(1-\gamma)G_{t+\tau}^F\exp\left\{(\frac{1}{2}\sigma_M^2-\mu_M^F)\tau\right\}\lambda_t}). \tag{18}$$

The time t forward interest rate for $(n-m)$-year zero-coupon foreign bond starting in m years $(n > m > 0)$ is given by

$$f^*_{t,m,n} = \tfrac{(1+R^*_{t,n})^n}{(1+R^*_{t,m})^m} - 1, \tag{19}$$

where the time t zero-coupon yield with maturity $\tau(\tau = m, n)$ for foreign bond is

$$R^*_{t,\tau} = \mu_M^F - \tfrac{1}{2}\sigma_M^2 - \tfrac{1}{\tau}\ln(\tfrac{H_{t+\tau}}{H_t}). \tag{20}$$

Similar to the analysis for domestic nominal interest rate, Equation (17) shows that the forward interest rate for domestic bond is affected by the disagreement about domestic monetary policy. However, due to its complicated expression, it is difficult to analyze the relation forward interest rate and disagreement. We leave this issue in the section of numerical analysis. Moreover, it is not surprising that the foreign foreign interest rate is unaffected by the disagreement with the same intuition as that for foreign nominal interest rate.

The next proposition presents the nominal exchange rate and its dynamics.

Proposition 2 *The nominal foreign exchange rate is*

$$e_t^n = \frac{G_t^D + (1-\gamma)\lambda_t G_t^F}{\gamma H_t \lambda_t} \frac{M_t^*}{M_t}. \tag{21}$$

The drift of nominal foreign exchange rate perceived by domestic investors and its diffusion coefficients are

$$\begin{aligned} \mu_{e^n,t}^D &= \mu_{M^*} - \tfrac{1}{2}\sigma_{M^*}^2 - \mu_M^D + \tfrac{1}{2}\sigma_M^2 - \sigma_{M\varepsilon^*}\sigma_{M^*\varepsilon^*} \\ &- \frac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^D\right)(T-t)\right\} + (1-\gamma)\lambda_t \exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^F\right)(T-t)\right\}}{G_t^D + (1-\gamma)\lambda_t G_t^F} \\ &+ \frac{\exp\left\{\left(\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*}\right)(T-t)\right\}}{H_t} + \frac{G_t^D(\bar{\mu}_M - \sigma_{MM})\bar{\mu}_M}{G_t^D + (1-\gamma)\lambda_t G_t^F}, \\ \sigma_{e^n\varepsilon,t} &= -\sigma_{M\varepsilon}, \ \ \sigma_{e^n\varepsilon^*,t} = \sigma_{M^*\varepsilon^*} - \sigma_{M\varepsilon^*}, \\ \sigma_{e^nM,t} &= -\sigma_{MM} + \frac{G_t^D\bar{\mu}_M}{G_t^D + (1-\gamma)\lambda_t G_t^F}, \ \ \sigma_{e^nM^*,t} = \sigma_{M^*M^*}. \end{aligned} \tag{22}$$

The nominal foreign exchange rate is the ratio of the relative price of domestic currency, q, to the relative price of foreign currency, q^*. Equation (21) shows that nominal foreign exchange rate is positively related to foreign money supply and negatively proportional to domestic money supply. Moreover, nominal foreign exchange rate is driven by investors' disagreement about the expected domestic money growth rate. The intuition is similar to the effect of disagreement on individual market price of domestic monetary risk. When there is a negative shock to domestic money supply, the decrease in domestic money supply causes deflation in the domestic country, making domestic goods relatively more competitive. This boosts the foreign demand for domestic goods, leading to domestic currency appreciation. Consequently, the nominal exchange rate (denoted by units of foreign currency per domestic currency) rises. More importantly, pessimistic foreign investors "win" the bet because they have more accurate estimate about the subdued expected growth rate of domestic money supply and thus their relative wealth increases. They consume more domestic consumption goods through holding more domestic currencies. Hence, domestic currency further appreciates, leading to additional increases in the nominal exchange rate.

Equation (22) shows that the drift and diffusion coefficients of nominal exchange rate are related to the investors' disagreement. The drift includes eight items. The first five are standard terms reflecting investors' expectation of domestic and foreign money supply, investors' precaution about domestic and foreign monetary risks, and the covariance between domestic and foreign monetary risks (e.g., Basak and Gallmeyer, 1999). The seventh item vanishes as investors' investment horizon goes to infinity if assuming $\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*} < 0$. Otherwise, the nominal exchange rate is explosive. The sixth and eighth items are new, capturing the effect of investors' disagreement on nominal exchange rate. We examine such effect closely in the model calibration section. The diffusion coefficients of nominal exchange rate includes four components. The item caused by domestic monetary risk, $\sigma_{e^n M,t}$, is positively related to investors' disagreement about the expected domestic money growth rate, $\bar{\mu}_M$. It is intuitive: the "trading risk" generated from investors' disagreement affects investors' consumption, the stochastic discount factor, the relative prices of domestic and foreign currencies, and eventually the exchange rate.

In the next proposition, we present the expected return of domestic and foreign stocks perceived by domestic investors and their diffusion coefficients.

Proposition 3 *The expected return of domestic stock perceived by domestic investors and its diffusion coefficients are*

$$\begin{gathered}
\mu_{S,t}^D = \mu_\varepsilon + \tfrac{1}{2}\sigma_\varepsilon^2, \qquad (23)\\
\sigma_{S\varepsilon,t} = \sigma_{\varepsilon\varepsilon},\ \ \sigma_{S\varepsilon^*,t} = 0,\\
\sigma_{SM,t} = 0,\ \ \sigma_{SM^*,t} = \sigma_{\varepsilon M^*}.
\end{gathered}$$

The expected return of foreign stock perceived by domestic investors and its diffusion coefficients are

$$\begin{gathered}
\mu_{S^*,t}^D = \mu_\varepsilon^* + \tfrac{1}{2}\sigma_{\varepsilon^*}^2 + \tfrac{\left(1-\beta^D-\beta^F\right)\left(1-\beta^F\right)\lambda_t^2\bar{\mu}_M^2}{[(1-\beta^D)+\beta^F\lambda_t][\beta^D+(1-\beta^F)\lambda_t]^2}, \qquad (24)\\
\sigma_{S^*\varepsilon,t} = \sigma_{\varepsilon^*\varepsilon},\ \ \sigma_{S^*\varepsilon^*,t} = 0,\\
\sigma_{S^*M,t} = \tfrac{\left(1-\beta^D-\beta^F\right)\lambda_t\bar{\mu}_M}{[\beta^D+(1-\beta^F)\lambda_t]^2},\ \ \sigma_{S^*M^*,t} = \sigma_{\varepsilon^*M^*}.
\end{gathered}$$

In our model, investors are myopic with logarithmic utility and domestic consumption goods is assumed to be the numeraire. Therefore, the price of domestic stock and its expected return and diffusion coefficients are unaffected by investors' disagreement about domestic monetary risk (e.g., Basak, 2000) (Equation (23)). However, the relative price of foreign consumption is affected by investors' disagreement about the expected growth rate of domestic money supply because pessimistic (foreign) investors transfer monetary risk to optimistic (domestic) investors. Therefore, the price of foreign stock and its expected return and diffusion coefficients are affected by investors' disagreement, $\bar{\mu}_M$ (Equation (24)). This is new in the literature: the disagreement about nominal variable of domestic country (i.e., monetary policy) affects the price and its dynamics of real variable (i.e., stock) of foreign country. Equation (24) also shows that the diffusion coefficient of expected return of foreign stock caused by domestic monetary risk, σ_{S^*M}, is positively related to investors' disagreement, $\bar{\mu}_M$ and then the total volatility of stock return increases with the disagreement.

3 Numerical Analysis and Results

To test the validity of our model, we investigate if our model could replicate some well-documented empirical findings in the literature. The first subsection calibrates the model to U.S. and U.K data. We examine how heterogeneous beliefs are related to the volatility of the exchange rate. The next subsection investigates the ability of the model to reproduce the rejection of uncovered interest rate parity, as well as the joint relations between heterogeneous beliefs, changes in exchange rate, and interest rate differentials.

3.1 Calibration

We calibrate the model parameters to match a set of unconditional moments on real consumption growth, the money supply, the nominal interest rates, and the nominal foreign exchange rate. Following Osambela (2013) and Heyerdahl-Larsen (2014), we consider the United States as the domestic country and the United Kingdom as the foreign country. Our sample contains quarterly observations from 1955 until 2016 obtained from Federal Reserve Economic Data (FRED) database.

The calibrated parameters of aggregate consumption and money supplies are directly matched to unconditional mean and variance of real consumption growth and money supply growth in the United States and the United Kingdom. Time discount factor ρ is set to 0.1, consistent with previous studies (e.g., Dumas, Kurshev, and Uppal, 2009; Dumas, Lewis and Osambela, 2017). Given the life expectancy in the United States and the United Kingdom sits at about 78, the investment horizon is set at 60 years. The initial relative wealth weight of foreign investors, λ, is normalized to 1. To calibrate other parameters in our model, we use the method of simulated moments. Mathematically, let Λ^d be the moments of the data (e.g. mean or variance of real consumption growth and the money supply). Denote Λ^s as the same moments calculated on the simulated data from the model with parameters θ. The Λ^s is averaged across simulations. Then, we obtain the calibrated parameters:

$$\hat{\theta}_{MSM} = \arg\min \hat{m}(x, \theta)' W \hat{m}(x, \theta), \tag{25}$$

where $\hat{m}(x, \theta) = \Lambda^d - \Lambda^s$ and W is a positive-definite weighting matrix. We choose the weighting matrix as a diagonal matrix with an inverse of empirical variances of the moments. Our moment conditions target the mean and variance of real consumption growth and money supply, as well as the mean of foreign exchange rate, domestic and foreign nominal interest rate. Table 1 presents the chosen parameters.

[INSERT TABLE 1 HERE]

Using the calibrated parameters in Table 1, we simulate the model across 10,000 paths of 160 quarterly observations. Table 2 reports the unconditional moments of the data, as well as the average moments obtained from our simulation. The first half of Table 2 reports the moments that we target to match with the actual data. The second half corresponds to other moments that we employ to evaluate the ability of our model to replicate some stylized facts concerning the currency market. Overall, our model could produce comparable moments as the actual data. For example, the second moments of the model-generated interest rate and interest rate differential match the actual data closely. The model implied volatility of nominal interest rate is 0.84 bps and 0.96 bps in the U.S. and the U.K. respectively. The corresponding moments of the actual data are 0.79 bps and 0.92 bps. The volatility of interest rate differential is 1.46 bps in the model, compared to 1.07 bps in the data. The volatility of the foreign exchange rate in the model is 6.78%, which is slightly higher than the 3.98% observed in the data. Noticeably, our model could also produce a higher volatility of foreign exchange rate (6.78%) than the volatility of interest differentials (1.46 bps).[2]

Our model could also produce the correlation of money supply, interest rate, and consumption growth similar to the actual data. The correlation between domestic and foreign interest rate is

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[2] Heyerdahl-Larsen (2014) pointed out that "To reproduce the failure of the UIP, the currency risk premium must...exhibit a higher volatility than the expected depreciation of the exchange rate."

quite high both in the data (0.87) and our model (0.98). The correlation between domestic and foreign consumption is about 0.24 in our model and the data. Our model produces a slightly lower correlation between domestic and foreign money supply (0.12), compared to the actual data (0.26). However, our model has difficulties in characterizing the moments of stock returns. The model generates very low levels of the mean and standard deviation of the stock returns. Understandably, the main purpose of our parsimonious model is to identify the effect of investors' disagreement on the dynamics of the exchange rate and interest rates. Consequently, the assumption of the stock markets in the model might be overly simplified.

[INSERT TABLE 2 HERE]

3.2 Exchange Rate Volatility

Figure 1 (a) plots the volatility of the nominal exchange rate in the baseline calibration against the dispersion of beliefs, measured by the scaled difference in estimates of domestic money growth rate between domestic and foreign investors ($\bar{\mu}_M = (\mu_M^D - \mu_M^F)/\sigma_{MM}$). Consistent with Beber et al. (2010), our model implies a positive relation between investors' disagreement and volatility of the exchange rate. As the dispersion of beliefs fluctuates, the volatility of the exchange rate also varies over time. This implies that the well-documented stochastic volatility in exchange rates could be generated endogenously by the dispersion of beliefs in our model (e.g., Poon and Granger, 2003). Equation (22) shows that "trading risk" generated from the "bet" between domestic and foreign investors amplifies the diffusion coefficient of nominal exchange rate caused by domestic monetary risk, $\sigma_{e^n M,t}$, making the volatility of nominal exchange rate positively related to investors' dispersion in beliefs.

Using the actual proxy for heterogeneous beliefs in the U.S. and the U.K.[3], we compute the model generated exchange rate volatility. Figure 1(b) compares the model generated exchange rate volatility and the actual volatility of the USD/GBP exchange rate. The correlation between the two series is 0.37, which is close to 0.39 reported in Heyerdahl-Larsen (2014). The model generated exchange rate volatility follows the actual data closely: a moderate downward trend starting from

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[3] The proxy for heterogeneous beliefs is based on the Consensus Forecast published by the Consensus Economics. See the next section for more details.

the early 1990s and a high volatility period in the late 2000s. Noticeably, our model captures the spike in volatility during the 2007-2009 financial crisis, albeit overshooting actual data.

[INSERT FIGURE 1 HERE]

3.3 The Uncovered Interest Rate Parity

Following Osambela (2013), this section verifies if our model could replicate the rejection of uncovered interest rate parity (UIP). We run the UIP regression on three samples of data. First, using the calibrated parameters as in Table 1, we simulate 10,000 paths of nominal interest rates of the domestic and foreign markets as well as the nominal foreign exchange rate. We denote this sample as the "baseline calibration." Next, we simulate another 10,000 paths of nominal interest rates and exchange rates without heterogeneous beliefs between the domestic and foreign investors. In other words, we manually set the model parameters $\mu_M^F = \mu_M^D = 2.15\%$ while keeping other parameters unchanged. We refer to this as the "no disagreement" sample. Lastly, we collect the actual quarterly nominal interest rate and exchange rate in the U.S. and the U.K. between 1955 and 2016. For each of the three samples, we run time-series UIP regressions for each path i to derive the estimated slope β_i as well as the estimated intercept α_i. Specifically, the UIP regression is given as

$$e_{t+1} - e_t = \alpha + \beta(R_t^F - R_t^D) + \epsilon_t, \tag{26}$$

where the e_t is the natural log of exchange rate at time t, R_t^D and R_t^F are the domestic and foreign interest rate at time t, respectively.[4] If the UIP holds, then $\alpha = 0, \beta = 1$. In other words, currencies with high interest rate should depreciate, while the low interest rate currencies would appreciate.

Panel A of Table 3 reports the mean of slopes ($\bar{\beta}$), the intercepts ($\bar{\alpha}$), the absolute value of t-statistics (in parentheses), and the adjusted R-squared ($\bar{R}^2$). The baseline calibration suggests that our model can reproduce the rejection of UIP as observed in the empirical data. The coefficient β in our calibrated model (-1.01) is negative and closely resembles the one in the real data (-1.18). The calibrated model reveals that, in contrast to the UIP, the currency with high interest rate appreciates while the low interest rate currency depreciates. On the contrary, the "no disagreement" sample

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[4] Note that the exchange rate e_t is defined as the price of the domestic currency divided by the price of foreign currency. An increase in e_t indicates a depreciation of the foreign currency or an appreciation of the domestic currency.

estimation exhibits a positive β. Without heterogeneous beliefs, our baseline model behaves more similarly to what the standard UIP predicts. The calibration also shows that our model could produce very low levels of R^2, which is another notable feature of the empirical failure of UIP. (Burnside, 2018)

[INSERT TABLE 3 HERE]

Recent empirical studies find that the forecasting horizon is important in determining the exchange rate predictability (Boudoukh, Richardson, and Whitelaw, 2016; Engle, 2016). In particular, Engel (2016) shows that the estimated coefficient β in the UIP regression is negative for a short forecasting horizon but becomes positive as the forecasting horizon increases. In other words, when the interest rate increases, the currency appreciates in the short run, but depreciates in the longer run. To reproduce the role of forecasting horizon in the UIP regression, we run the following regression:

$$e_{t+j+1} - e_t = \alpha + \beta(R_t^F - R_t^D) + \epsilon_t, \tag{27}$$

where j is the forecasting horizon. If $j = 0$, then it becomes the traditional UIP regression in (26). Panel B of Table 3 reports the model estimation for different horizons (j) and dispersion of beliefs. The baseline calibration shows the coefficient β starts from -0.89 and increases above 0.03 as the forecasting horizon grows. In other words, the pattern of exchange rate predictability in our model could change as the forecasting horizon extends.[5] To further pin down the joint effect of the heterogeneous beliefs and the forecasting horizon, we manually change the level of dispersion of beliefs and test its impact on the UIP coefficient. In addition to the "no disagreement' sample, we simulate a "low disagreement" and a "high disagreement" sample. The "Low disagreement" ("High disagreement") sample comes from 10,000 simulations by setting the $\mu_M^D - \mu_M^F = 0.12\%$ ($\mu_M^D - \mu_M^F = 0.4\%$) while keeping other parameters unchanged. Without heterogeneous beliefs, the β fluctuates around 0.38 and exhibits no upward trend as forecasting horizon increases. In the "low

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[5] Although the UIP regression coefficient β flipped sign in the baseline calibration as in Engel (2016), we acknowledge that the increase in β is slower than what the actual data shows as the forecasting horizon j extends. In the baseline calibration, the sign becomes positive only after 24 quarters. Engel (2016) shows that the sign flipping could occur within two years.

disagreement” sample, however, the β grows from -0.58 to 0.14. Meanwhile, the β rises even faster for the “high disagreement” sample, from -5.44 to -1.59 as j grows. In summary, Panel B of Table 3 implies the heterogeneous beliefs could facilitate the changes in exchange rate predictability as the forecasting horizon grows.

3.4 Carry Trade Returns, Changes in Exchange Rate and Heterogeneous Beliefs

The rejection of UIP implies that there is a high average payoff to the carry trade. The carry trade is defined as a domestic investor who borrows at the domestic risk-free rate (R^D), exchanges the proceeds into foreign currency, invest it at the foreign risk-free rate (R^F), and eventually converts all investment back into home currency. The carry trade returns from investing in the currency market are given as:

$$CTR_{t+1} = R^F - R^D - (e_{t+1} - e_t). \tag{28}$$

With heterogeneous investors in our models, the dispersion of beliefs enters the pricing kernel and thus naturally affects the expected currency returns. To illustrate the model implied relation between heterogeneous beliefs and carry trade returns, we conduct a set of simulations using model parameters as those in Table 1, while manually adjust the magnitude of heterogeneous beliefs. For each value of $\bar{\mu}_M$, we simulate 10,000 paths and compute the average carry trade return. Figure 2 plots the model implied relation between heterogeneous beliefs ($\bar{\mu}_M$) and average carry trade return. Consistent with previous studies such as Beber et al. (2010) and Yu (2013), the carry trade returns are negatively associated with the heterogeneity in beliefs.

[INSERT FIGURE 2 HERE]

Beber et al. (2010) decompose carry returns into “returns generated by the interest rate differentials and the returns generated by exchange-rate appreciation/depreciation.” They find that the carry trade return is negatively related to the dispersion of beliefs, while one of its component, the interest rate differential, is positively related to the dispersion of beliefs. Therefore, it is particularly essential to check the underlying model implied relations between the heterogeneity in beliefs in our model and the interest rate differentials, the changes in exchange rate, or the carry trade returns.

Following Beber et al. (2010) and Yu (2013), we decompose the carry trade return into two parts - the interest rate differential ($R^F - R^D$) and the changes in the exchange rate ($e_{t+1} - e_t$), and investigate our model predictions individually. Plot A in Figure 3 presents the positive relation between heterogeneous beliefs and exchange rate movement, and plot B illustrates its negative relation with the interest rate differential. Beber et al. (2010) find that whenever market participants disagree about their future expectations, the adverse spot currency movements makes carry trade unprofitable. This agrees with our model prediction shown in plot A of Figure 3. Meanwhile, the negative relation between investors' disagreement and interest rate differential appears to be moderate. This is similar to Yu (2013), which documents a weak association between the dispersion in domestic-foreign sentiment and the interest rate differential.

[INSERT FIGURE 3 HERE]

To summarize, our model with heterogeneous beliefs has the following two predictions regarding the effects of dispersion in beliefs on the exchange rate and carry trade returns.

Hypothesis 1: Changes in the exchange rate is positively related to the dispersion in beliefs among investors about the expected growth rate of money supply.

Hypothesis 2: Carry trade return is negatively related to the dispersion in beliefs among investors about the expected growth rate of money supply.

3.5 The Underlying Mechanism

The carry return decomposition in Figure 3 reveals the intuition behind the rejection of uncovered interest rate parity in our model. Similar to existing studies such as Yu (2013), our model reproduces the failure of UIP in the following manner. The difference in interest rate between domestic and foreign market is a function of some state variables (e.g., heterogeneous beliefs in our model or the domestic-foreign sentiment differentials in previous studies), and the expected change in exchange rates and the expected return on carry trade are also the functions of the same state variables. If the coefficients on the interest rate differentials and on the expected change in exchange rates are opposite, then the model can produce a negative coefficient in the UIP regression.

Nevertheless, our model portrays a unique mechanism on how the interest rate differential and the changes in exchange relate to the state variables. In our model, the heterogeneity in beliefs among domestic and foreign investors affects foreign exchange rate positively and interest rate differential negatively through the relative wealth weight of foreign investors. Let's take the U.S. (as the domestic country) and U.K. (as the foreign country) as an example. First, when the U.S. money supply experiences a negative shock, the optimistic U.S. investors overestimate the expected domestic money growth rate while the pessimistic beliefs of the U.K. investors are closer to the actual value. The U.S. investors "lose" the bet on the expected domestic money growth rate, misallocate their assets, and thus their relative wealth decreases. They save less to smooth the consumption and therefore the U.S. nominal interest rate (R_t^D) increases. Meanwhile, the U.K. nominal interest rate remains constant because the pound sterling is only held by British investors and there is no uncertainty associated with the U.K. monetary policy (Equation (13)). Consequently, the nominal interest rate differential between the U.K. and the U.S., $R_t^F - R_t^D$, decreases. Second, a negative shock to the U.S. money supply is likely to cause deflation, making American goods relatively more competitive, which boosts the foreign demand. In this way, more demand for American consumption goods leads to the appreciation of the U.S. dollar. Moreover, the heterogeneity in beliefs about expected domestic money growth rate between domestic and foreign investors could induce additional currency appreciation through the relative wealth channel. As the U.K. investors have lower estimates about the growth rate of domestic money supply, they "win" the bet on the growth rate of money supply. They allocate their assets appropriately, leading to an increase in their wealth relative to the U.S. investors. They consume more U.S. consumption goods in addition to their own country consumption goods by holding more U.S. dollar. Therefore, the U.S. dollar appreciates further. Taken together, the heterogeneity in beliefs among domestic and foreign investors influences the relative wealth weight of foreign investors, which positively affects foreign exchange rate and negatively affects the interest rate differential.

Notably, the larger the heterogeneity in beliefs, the stronger the impact of the relative wealth weight of foreign investors on the changes in exchange rate and the interest rate differentials. To see it, note that the change in relative wealth weight of foreign investors, $d\lambda$, is positively related to the heterogeneity in beliefs, $\bar{\mu}_M$, when there is a negative shock to domestic money supply, $d\omega_M < 0$ (Equation (15)). The convexity of in Figure 3(a) confirms this analysis although it is weaker in Figure 3(b).

To reproduce the rejection of the UIP, the expected changes in exchange rate must not only be negatively correlated with the interest rate differential, it must also be more volatile than the exchange rate (Heyerdahl-Larsen, 2014). As discussed above, the dispersion of beliefs could induce added currency movement through the relative wealth channel. The moments of our baseline calibration in Table 2 confirm this intuition. It shows that foreign exchange rate exhibits more volatility than interest rate differential, implying that the coefficient of Equation (26), β, is less than -1.

4 Empirical Tests

This section tests two predictions of our model on the relation between heterogeneous beliefs, exchange rate return, and carry trade returns. First, we describe the proxy for heterogeneous beliefs about money growth rate in the international markets. Then, we report the results of regression-based tests for the two model predictions.

4.1 Proxy for Heterogeneous Beliefs

As the information of heterogeneity in beliefs are not readily available, we construct a monthly index of dispersion of beliefs using the Consensus Forecast published by the Consensus Economics. The Consensus Forecasts ask the world's leading forecasters for their predictions for numerous macroeconomics variables from over 85 nations in Industrialized Countries, Eastern Europe, Asia Pacific, and Latin America. The Consensus Forecasts show the estimates of individual panelists for each economic indicator, along with the mean and the standard deviation. The Consensus Forecasts cover all of the G-7 countries (i.e., Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States) every month since October 1989. The information for the Netherlands, Norway, South Africa, Spain, Sweden, and Switzerland are also available for shorter periods. In December 2002, it also added individual and consensus forecasts for the Eurozone. Patton and Timmermann (2010) and Dovern, Fritsche, and Slacalek (2012) provide detailed explanations about the Consensus Forecasts data.

Because Consensus Forecasts publications do not include the estimates of the growth in money supply, we use the estimate for consumer price index (CPI) as a proxy, similar to Croitoru and

Lu (2015). Following Diether, Malloy, and Scherbina (2002), dispersion in beliefs is defined as the standard deviation of forecasts divided by the absolute value of the mean estimates. We then scale the standard deviation to account for the differences in forecasting horizon.[6] To mitigate the influence of possibly spurious outliers, dispersion in beliefs is winsorized at the 1% and 99%. All dispersion indices in the 15 economies are plotted in Figure 4. Most of the countries experienced a surge in the dispersion of beliefs during the crisis in the late 2000s. This is consistent with the finding of David and Farhat (2018), which documented an elevated level of aggregate dispersion of beliefs from 2006 to 2016. On average, the level of dispersion of beliefs is the highest in Japan. To the contrary, both Norway and Denmark have stable and low levels of dispersion of beliefs. The flat areas in the plot (most noticeable for Japan) are due to winsorization.

[INSERT FIGURE 4 HERE]

4.2 Exchange Rate Changes and Heterogeneous Beliefs

Our empirical tests follow the standard long-horizon predictive regressions (e.g., Fama and French, 1988; Hodrick, 1992; Beber et al., 2010; and Yu, 2013). Specifically, the future changes in foreign exchange rates are regressed on the proxy of heterogeneity in beliefs and other control variables:

$$e_{t+3} - e_t = \alpha + \beta(R^F_{t,t+3} - R^D_{t,t+3}) + \gamma\phi_t + \epsilon_t, \tag{29}$$

where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity, and ϕ_t is the dispersion of beliefs.[7] Similar to Beber et al. (2010), note that the interest rate differential is controlled for on the right-hand side. All observations are at monthly frequency. To adjust for autocorrelated residuals due to overlapping observations, we use Newey-West (1987) standard errors to correct for the autocorrelation and potential heteroscedasticity.

Table 4 reports the results from the monthly overlapping predictive regression of 3-month changes in exchange rates on the dispersion of beliefs. The first two columns present the estimated coefficients of interest rate differential ($R^F - R^D$) and the corresponding t-statistics. Consistent with

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[6] For example, the standard deviation for a t-month forecasting horizon (σ) will be re-scaled as $\sigma\sqrt{\frac{12}{t}}$

[7] In the empirical analysis, we treat the U.S. as the "foreign" country, and each of the 15 major economies as the "domestic" country. This is because in our model, money supply is only nontransparent for the domestic country.

the previous literature, the coefficients of interest rate differential are largely negative, confirming the presence of the forward premium puzzle in our sample.

[INSERT TABLE 4 HERE]

The next two columns report the estimated coefficients of dispersion of beliefs (ϕ) and the corresponding t-statistics. Our model predicts that future changes in foreign exchange rates are positively related to the dispersion of beliefs (Hypothesis 1). The evidence is overall supportive. The coefficient of dispersion of beliefs are positive in 10 out of the 15 economies, and 8 of them are statistically significant.

Although uncertainty is a prerequisite for the existence of heterogeneous beliefs, these two are conceptually different. To single out the effect of the dispersion in beliefs, Table 5 re-estimates the equation (4) after controlling for the uncertainty of the economy, proxied by the VIX index. The results in Table 5 are largely comparable to those in Table 4. After controlling for the volatility, the coefficient of dispersion of beliefs are still positive in 10 out of the 15 countries, and 9 of them are statistically significant.

The evidence in Tables 4 and 5 are based on the absolute level of dispersion of beliefs in each country. An intuitive alternative is measuring the dispersion of beliefs in each country relative to the United States. Thus, the second set of tests based on relative dispersion of beliefs are now presented as additional evidence. The relative dispersion of beliefs (δ_i) in country i are defined as $\delta_i = \log(\phi_i) - \log(\phi_{us})$, where ϕ_i and ϕ_{us} are the dispersion of beliefs in country i and United States respectively. Table 6 presents the estimation results. Similar to the previous estimations, the estimated coefficients of heterogeneous beliefs in most of the countries exhibit have correct signs and 9 out of 15 are statistically significant.

[INSERT TABLE 5 AND 6 HERE]

4.3 Carry Trade Returns and Heterogeneous Beliefs

Next, we consider the model prediction that the carry trade returns are negatively related to the dispersion of beliefs. In particular, we regress the 3-month carry trade returns on the dispersion of beliefs. Table 7 provides evidence supporting this prediction. The regression coefficients for most of the countries have the negative sign, and more than half of them are statistically significant. Table 8 added the volatility (VIX) as a control. Table 9 uses the relative dispersion of beliefs instead of the absolute dispersion of beliefs in each country. All of the results are largely illustrating a negative relation between the carry trade returns and dispersion of beliefs.

[INSERT TABLE 7, 8 AND 9 HERE]

A stylized fact of the carry trade strategy is the so-called "up the stairs, down the elevator" description of the carry trade returns (Boudoukh, Richardson, and Whitelaw, 2016). The carry-trade returns are periodically hit by the possibility of an abrupt adversarial movement in the currency market. In our model, the dispersion in beliefs enters the pricing kernel, which determines how much compensation is required for the potential risk of a sudden loss in value. Consistent with Beber et al. (2016), our empirical results in section 4.2 and section 4.3 confirm that the dispersion of beliefs has some predictive power for the considerable and sudden loss in value that a carry trade strategy suffers from.

Taken together, the empirical evidence is largely supporting our two model predictions. Our results are robust to an alternative measure of dispersion in beliefs as well as controlling for uncertainty in the economy.

5 Conclusion

Using a two-country model coupled with heterogeneous beliefs, this paper provides a disagreement-based interpretation of the forward premium puzzle. In our model, the domestic and foreign investors over- or under-estimate the growth of the domestic money growth rate. In response to a shock to the domestic money supply, the disagreement between the domestic and foreign investors moves the foreign exchange rate and interest rate differential in opposite directions. Consequently, our model could reproduce the rejection of the uncovered interest rate parity. Furthermore, the time-varying disagreement between investors generates a "trading risk,' which leads to fluctuations in exchange rate volatility. In this way, our model could also produce a stochastic volatility of exchange rate with a comparable magnitude to the actual data.

Our model delivers two testable hypotheses: first, a positive relationship between heterogeneous beliefs and changes in exchange rate; second, a negative relation between carry trade return and heterogeneous beliefs. Using a monthly index of heterogeneous beliefs based on the Consensus Forecast in 15 major economies, we confirm the two implications from our model. The empirical results are still valid after controlling for economic uncertainty, as well as using the relative proxy of heterogeneity in beliefs.

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APPENDIX:

Proof of Proposition 1:
Each investor maximizes his problem to solve optimal consumption goods and currency holdings. We insert them into the utility of "world" representative agent given by:

$$\begin{aligned}
&U(\varepsilon, \varepsilon^*, qM, q^*M^*/p^*; \lambda) \\
&= A\left(\lambda\right) + \alpha\left[\beta^D + \left(1-\beta^F\right)\lambda\right]\log\left(\varepsilon\right) + \alpha\left[\left(1-\beta^D\right) + \beta^F\lambda\right]\log\left(\varepsilon^*\right) \\
&+ \left(1-\alpha\right)\left[1 + \left(1-\gamma\right)\lambda\right]\log\left(qM\right) + \left(1-\alpha\right)\gamma\lambda\left[\log\left(\tfrac{q^*}{p^*}M^*\right)\right],
\end{aligned}$$

where

$$\begin{aligned}
A\left(\lambda\right) = {}& \alpha\left[\beta^D\log\left(\tfrac{\beta^D}{\beta^D+(1-\beta^F)\lambda}\right) + \left(1-\beta^D\right)\log\left(\tfrac{1-\beta^D}{(1-\beta^D)+\beta^F\lambda}\right)\right] \\
&+ \alpha\lambda\left[\left(1-\beta^F\right)\log\left(\tfrac{\left(1-\beta^F\right)\lambda}{\beta^D+(1-\beta^F)\lambda_t}\right) + \beta^F\log\left(\tfrac{\beta^F\lambda}{(1-\beta^D)+\beta^F\lambda}\right)\right] \\
&+ \left(1-\alpha\right)\left[\log\left(\tfrac{1}{1+(1-\gamma)\lambda}\right) + \left(1-\gamma\right)\lambda\log\left(\tfrac{(1-\gamma)\lambda}{1+(1-\gamma)\lambda}\right)\right].
\end{aligned}$$

We derive the investor-specific state price density from the first-order conditions of investor's optimization problem using domestic consumption goods as a numeraire (e.g., Buraschi, Trojani, and Vedolin, 2014). Applying Ito's lemma to investor-specific state price density, we can calculate the market price of risk. The nominal interest rates can be calculated as

$$\begin{aligned}
R_t &= \tfrac{U_{M,t}}{U_{\varepsilon,t}} = \tfrac{\partial U_t}{\partial\left(qM\right)_t}\tfrac{1}{\frac{\partial U_t}{\partial\varepsilon_t}} = \tfrac{1-\alpha}{\alpha}\tfrac{1+(1-\gamma)\lambda_t}{\beta^D+(1-\beta^F)\lambda_t}\tfrac{\varepsilon_t}{q_tM_t}, \\
R_t^* &= \tfrac{U_{M^*,t}}{U_{\varepsilon^*,t}} = \tfrac{\partial U_t}{\partial\left(\frac{q_t^*}{p_t^*}M_t^*\right)}\tfrac{1}{\frac{\partial U_t}{\partial\varepsilon_t^*}} = \tfrac{1-\alpha}{\alpha}\tfrac{\gamma\lambda_t}{(1-\beta^D)+\beta^F\lambda_t}\tfrac{p_t^*\varepsilon_t^*}{q_t^*M_t^*}.
\end{aligned}$$

In the following, we derive the prices of foreign consumption goods, domestic currency, and foreign currency relative to the price of domestic consumption goods. First, the price of foreign consumption goods relative to the price of domestic consumption goods is

$$p_t^* = \tfrac{U_{\varepsilon^*,t}}{U_{\varepsilon,t}} = \tfrac{\left(1-\beta^D\right)+\beta^F\lambda_t}{\beta^D+(1-\beta^F)\lambda_t}\tfrac{\varepsilon_t}{\varepsilon_t^*},$$

Next, we calculate the price of domestic currency relative to the price of domestic consumption goods.

$$\begin{aligned}
M_s &= M_t\exp\left\{\mu_M^D\left(s-t\right) + \sigma_{M\varepsilon}\left(\omega_{\varepsilon,s} - \omega_{\varepsilon,t}\right) + \sigma_{M\varepsilon^*}\left(\omega_{\varepsilon^*,s} - \omega_{\varepsilon^*,t}\right) + \sigma_{MM}\left(\omega_{M,s}^D - \omega_{M,t}^D\right)\right\}, \\
\lambda_s &= \lambda_t\exp\left\{-\tfrac{1}{2}\bar{\mu}_M^2\left(s-t\right) - \bar{\mu}_M\left(\omega_{M,s}^D - \omega_{M,t}^D\right)\right\}.
\end{aligned}$$

We have

$$\begin{aligned}
E_t^D\left[\tfrac{1}{M_s}\right] &= \tfrac{1}{M_t}E_t^D\left[\exp\left\{-\mu_M^D\left(s-t\right) - \sigma_{M\varepsilon}\left(\omega_{\varepsilon,s} - \omega_{\varepsilon,t}\right) - \sigma_{M\varepsilon^*}\left(\omega_{\varepsilon^*,s} - \omega_{\varepsilon^*,t}\right) - \sigma_{MM}\left(\omega_{M,s}^D - \omega_{M,t}^D\right)\right\}\right] \\
&= \tfrac{1}{M_t}\exp\left\{\left(\tfrac{1}{2}\sigma_M^2 - \mu_M^D\right)\left(s-t\right)\right\},
\end{aligned}$$

where $\sigma_M^2 = \sigma_{M\varepsilon}^2 + \sigma_{M\varepsilon^*}^2 + \sigma_{MM}^2$. Thus, $\int_t^T E_t^D\left[\frac{1}{M_s}\right]ds = \frac{1}{M_t}\frac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^D\right)(T-t)\right\}-1}{\frac{1}{2}\sigma_M^2 - \mu_M^D}$.

Therefore, we have

$$\begin{aligned}
E_t^D\left[\tfrac{\lambda_s}{M_s}\right] &= \tfrac{\lambda_t}{M_t} E_t^D\left[\exp\left\{\begin{array}{c} -\frac{1}{2}\bar{\mu}_M^2\,(s-t) - \mu_M^D\,(s-t) - \sigma_{M\varepsilon}\,(\omega_{\varepsilon,s} - \omega_{\varepsilon,t}) \\ -\sigma_{M\varepsilon^*}\,(\omega_{\varepsilon^*,s} - \omega_{\varepsilon^*,t}) - (\bar{\mu}_M + \sigma_{MM})\left(\omega_{M,s}^D - \omega_{M,t}^D\right) \end{array}\right\}\right] \\
&= \tfrac{\lambda_t}{M_t}\exp\left\{\left(\tfrac{1}{2}\sigma_M^2 - \mu_M^F\right)(s-t)\right\}. \\
\int_t^T E_t^D\left[\tfrac{\lambda_s}{M_s}\right] ds &= \tfrac{\lambda_t}{M_t}\tfrac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^F\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_M^2 - \mu_M^F}.
\end{aligned}$$

Therefore, the price of domestic currency relative to the price of domestic consumption goods is

$$\begin{aligned}
q_t &= \tfrac{1}{\xi_t^D} E_t^D\left[\int_t^T \xi_s^D R_s q_s ds\right] = \tfrac{(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]} E_t^D\left[\int_t^T \tfrac{1+(1-\gamma)\lambda_s}{M_s} ds\right] \\
&= \tfrac{(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\left[\int_t^T E_t^D\left[\tfrac{1}{M_s}\right] ds + (1-\gamma)\int_t^T E_t^D\left[\tfrac{\lambda_s}{M_s}\right] ds\right] \\
&= \tfrac{(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\left[\tfrac{1}{M_t}\tfrac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^D\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_M^2 - \mu_M^D} + (1-\gamma)\,\tfrac{\lambda_t}{M_t}\tfrac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^F\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_M^2 - \mu_M^F}\right] \\
&= \tfrac{(1-\alpha)\left[G_t^D + (1-\gamma)\lambda_t G_t^F\right]}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\tfrac{\varepsilon_t}{M_t},
\end{aligned}$$

where $G_t^i = \frac{\exp\left\{\left(\frac{1}{2}\sigma_M^2 - \mu_M^i\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_M^2 - \mu_M^i}$ $(i = D, F)$.

Finally, we calculate the price of foreign currency relative to the price of domestic consumption goods.

$$M_s^* = M_t^* \exp\left\{\mu_{M^*}\,(s-t) + \sigma_{M^*\varepsilon^*}\,(\omega_{\varepsilon^*,s} - \omega_{\varepsilon^*,t}) + \sigma_{M^*M^*}\,(\omega_{M^*,s} - \omega_{M^*,t})\right\},$$

We have $\int_t^T E_t^D\left[\frac{1}{M_s^*}\right] ds = \frac{1}{M_t^*}\frac{\exp\left\{\left(\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*}\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*}}$, where $\sigma_{M^*}^2 = \sigma_{M^*\varepsilon^*}^2 + \sigma_{M^*M^*}^2$.

$$\begin{aligned}
q_t^* &= \tfrac{1}{\xi_t^D} E_t^D\left[\int_t^T \xi_s^D R_s^* q_s^* ds\right] = \tfrac{(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]} E_t^D\left[\int_t^T \tfrac{\gamma\lambda_s}{M_s^*} ds\right] \\
&= \tfrac{\gamma(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\int_t^T E_t^D\left[\tfrac{\lambda_s}{M_s^*}\right] ds = \tfrac{\gamma(1-\alpha)\varepsilon_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\int_t^T E_t^D\left[\tfrac{1}{M_s^*}\right] E_t^D\left[\lambda_s\right] ds \\
&= \tfrac{(1-\alpha)\gamma H_t\lambda_t}{\alpha[\beta^D + (1-\beta^F)\lambda_t]}\tfrac{\varepsilon_t}{M_t^*},
\end{aligned}$$

where $H_t = \frac{\exp\left\{\left(\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*}\right)(T-t)\right\} - 1}{\frac{1}{2}\sigma_{M^*}^2 - \mu_{M^*}}$.

Proof of Proposition 2:

The nominal foreign exchange rate is given by

$$e_t = \tfrac{q_t}{q_t^*} = \tfrac{G_t^D + (1-\gamma)\lambda_t G_t^F}{\gamma H_t\lambda_t}\tfrac{M_t^*}{M_t}.$$

Applying Ito's lemma to nominal foreign exchange rate, e_t, we can derive its drift and diffusion coefficients.

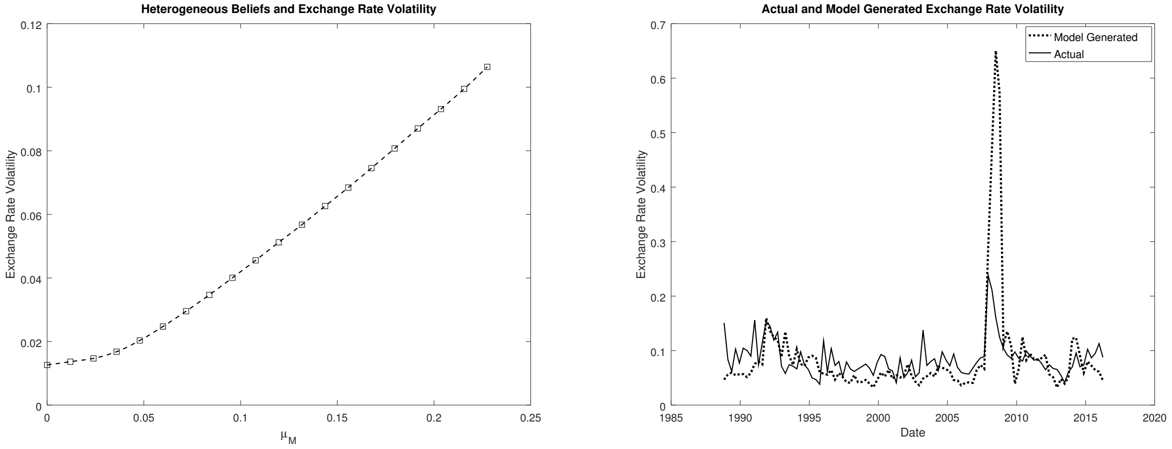
Proof of Proposition 3:

The prices of domestic and foreign stocks are given by

$$S_t = \tfrac{1}{\xi_t^D} E_t^D \left[\textstyle\int_t^T \xi_s^D \varepsilon_s ds \right] = (T-t)\,\varepsilon_t,$$

$$S_t^* = \tfrac{1}{\xi_t^D} E_t^D \left[\textstyle\int_t^T \xi_s^D p_s^* \varepsilon_s^* ds \right] = \tfrac{\left(1-\beta^D\right)+\beta^F \lambda_t}{\beta^D+(1-\beta^F)\lambda_t} S_t = \tfrac{\left(1-\beta^D\right)+\beta^F \lambda_t}{\beta^D+(1-\beta^F)\lambda_t} (T-t)\,\varepsilon_t = (T-t)\,p_t^* \varepsilon_t^*.$$

Applying Ito's lemma to the prices of domestic and foreign stocks, respectively, we can calculate their expected stock returns and diffusion coefficients.



(a) (b)

Figure 1: Exchange Rate Volatility. This figure examines the volatility of the exchange rate generated by our model. To illustrate the model implied relation between heterogeneous beliefs and exchange rate volatility, we conduct a set of simulations using model parameters as those in Table 1, while manually adjust the level of heterogeneous beliefs. For each value of $\bar{\mu}_M = (\mu_M^D - \mu_M^F)/\sigma_{MM}$, we simulate 10,000 paths and compute the average exchange rate volatility. Plot (a) shows the volatility of the nominal exchange rate in the baseline calibration against the dispersion of beliefs, measured by differences in beliefs between domestic and foreign investors ($\bar{\mu}_M$). Using the actual proxy for heterogeneous beliefs in the U.S. and the U.K., we compute the model generated exchange rate volatility. The proxy for heterogeneous beliefs is a monthly index of dispersion of beliefs using the Consensus Forecast published by the Consensus Economics. Figure 1(b) compares the model generated exchange rate volatility and the actual monthly volatility of the USD/GBP exchange rate. The volatility of the USD/GBP exchange rate is computed using daily currency returns in each month.

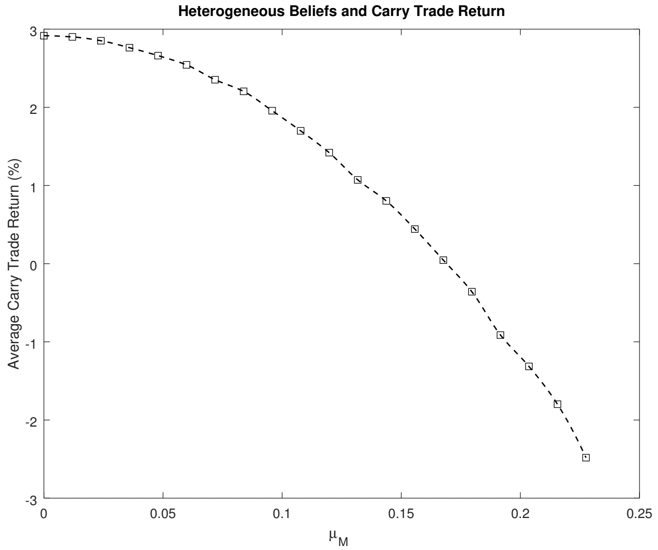
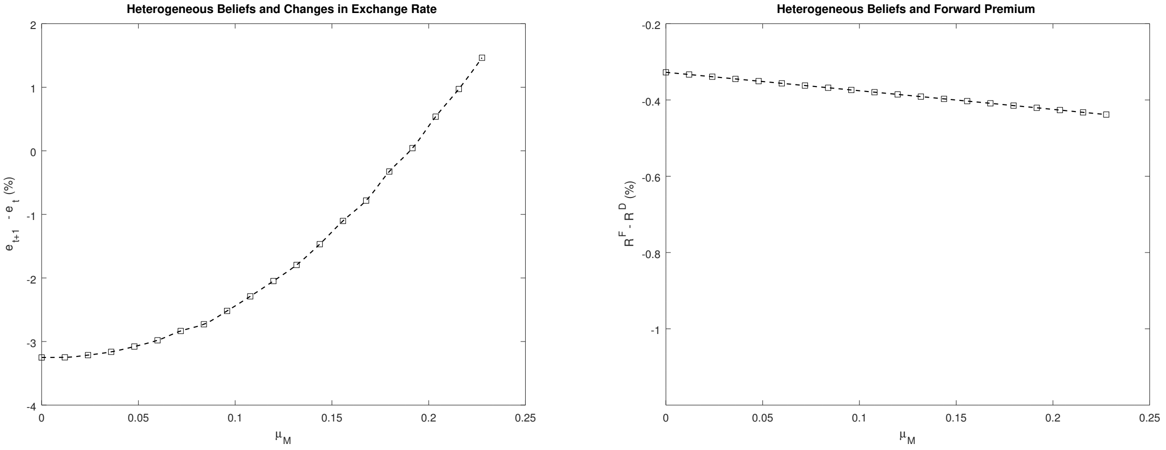


Figure 2: Heterogeneous Beliefs and Carry Trade Returns. This figure plots the model implied relation between heterogeneous beliefs and average carry trade return. We conduct a set of simulations using model parameters as those in Table 1, while manually adjust the heterogeneous beliefs. For each value of $\bar{\mu}_M$, we simulate 10,000 paths and compute the average carry trade return as in equation (23).



(a) Changes in Exchange Rate

(b) Forward Premium

Figure 3: Carry Trade Return Decomposition. This figure decomposes carry returns into returns generated by the interest rate differentials and the returns generated by exchange-rate appreciation/depreciation. We conduct a set of simulations using model parameters as those in Table 1, while manually adjust the level of heterogeneous beliefs. For each value of $\bar{\mu}_M$, we simulate 10,000 paths and compute the average returns of exchange rate ($e_{t+1} - e_t$) and the interest rate differentials between foreign and domestic interest rate ($R_t^F - R_t^D$), where e_t is the natural log of exchange rate at time t, R_t^F and R_t^D are the domestic and foreign interest rate at time t, respectively.

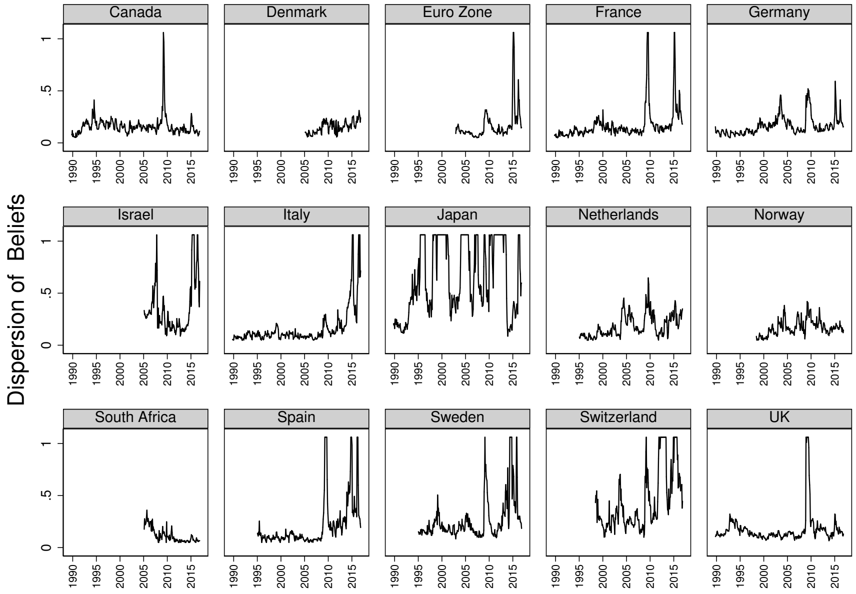


Figure 4: Heterogeneous Beliefs in 15 Economies. This figure plots the monthly index of dispersion of beliefs in 15 major economies. The monthly index of dispersion of beliefs is based on the Consensus Forecast published by the Consensus Economics. The Consensus Forecasts come from the world's leading forecasters' predictions for the consumer price index (CPI). The dispersion in beliefs index is defined as the standard deviation of forecasts divided by the absolute value of the mean estimates. We then scale the standard deviation to account for the differences in forecasting horizon. The Consensus Forecasts cover all of the G-7 countries (the United States, Japan, Germany, France, the United Kingdom, Italy, and Canada) every month since October 1989. The information for the Netherlands, Norway, Spain, Sweden, Switzerland, and South Africa are also available for shorter periods. In December 2002, it also added individual and consensus forecasts for the Eurozone. All series are winsorized at 1%.

Table 1: Model Parameters: baseline calibration

Parameters	Symbols	Values
Aggregate consumptiongoods and money supplies		
Expected growth rate of domestic consumption goods	μ_{ε}	0.805%
Expected growth rate of foreign consumption goods	μ_{ε^*}	0.644%
Expected growth rate of domestic money supplies	μ_M	2.15%
Expected growth rate of foreign money supplies	μ_{M^*}	1.65%
Volatility of domestic consumption goods related to ω_{ε}	$\sigma_{\varepsilon\varepsilon}$	0.0134
Volatility of domestic consumption goods related to ω_{M^*}	$\sigma_{\varepsilon M^*}$	0.0000
Volatility of foreign consumption goods related to ω_{ε}	$\sigma_{\varepsilon^*\varepsilon}$	0.0051
Volatility of foreign consumption goods related to ω_{ε^*}	$\sigma_{\varepsilon^*\varepsilon^*}$	0.0210
Volatility of foreign consumption goods related to ω_{M^*}	$\sigma_{\varepsilon^* M^*}$	0.0000
Volatility of domestic money supplies related to ω_{ε}	$\sigma_{M\varepsilon}$	0.0032
Volatility of domestic money supplies related to ω_{ε^*}	$\sigma_{M\varepsilon^*}$	0.0003
Volatility of domestic money supplies related to ω_M	σ_{MM}	0.0167
Volatility of foreign money supplies related to ω_{ε^*}	$\sigma_{M^*\varepsilon^*}$	0.0037
Volatility of foreign money supplies related to ω_{M^*}	$\sigma_{M^*M^*}$	0.0284
Investors′ preferences and beliefs		
Domestic money growth estimated by domestic investors	μ_M^D	2.25%
Domestic money growth estimated by foreign investors	μ_M^F	2.05%
Difference in beliefs about domestic money growth	$\bar{\mu}_M$	0.1194
Initial relative weight of foreign investors	λ	1.0000
Investment horizon	T	60 Years
Time discount factor	ρ	0.1000
Weight of consumption	α	0.0274
Weight of domestic consumption assigned by domestic investors	β^D	0.3205
Weight of foreign consumption assigned by foreign investors	β^F	0.5288
Weight of foreign money assigned by foreign investors	γ	0.9500

[*] This table presents the calibrated parameters for our baseline model. To calibrate the parameters our model, we use the method of simulated moments. Our moment conditions target the mean and variance of real consumption growth and money supply, as well as the mean of foreign exchange rate, domestic and foreign interest rate.

Table 2: Moments on Key Variables

	Actual	Model
Target moment conditions		
mean of domestic consumption growth rate	0.805%	0.81%
mean of foreign consumption growth rate	0.644%	0.64%
mean of domestic money supply growth rate	1.654%	1.66%
mean of foreign money supply growth rate	2.155%	2.15%
volatility of domestic consumption growth rate	0.680%	0.67%
volatility of foreign consumption growth rate	1.059%	1.08%
volatility of domestic money supply growth rate	0.853%	0.85%
volatility of foreign money supply growth rate	1.504%	1.43%
mean of domestic interest rate	4.50%	3.91%
mean of interest rate differential	2.10%	1.08%
mean of the foreign exchange rate	0.495	0.56
Other moment conditions		
volatility of the foreign exchange rate	3.978%	6.78%
volatility of domestic nominal interest rate ($\times 10^{-4}$)	0.786	0.84
volatility of foreign nominal interest rate ($\times 10^{-4}$)	0.916	0.96
volatility of interest rate differential ($\times 10^{-4}$)	1.065	1.46
correlation of domestic and foreign money supply	0.258	0.12
correlation of domestic and foreign interest rate	0.872	0.98
correlation of domestic and foreign consumption	0.236	0.24
mean of domestic excess return	0.066	0.02
mean of foreign excess return	0.048	0.02
volatility of domestic stock return	0.155	0.01
volatility of foreign stock return	0.198	0.02

[*] This table reports the unconditional moments of our baseline calibration. Using the calibrated parameters in Table 1, we simulate the model across 10,000 paths of 160 quarterly observations each. We report the unconditional moments of the actual data, as well as the average moments obtained from our simulation. The domestic country is the United States, and the foreign country is United Kingdom. Our sample of actual data contains quarterly observations from 1955 until 2016 in the United States and United Kingdom obtained from Federal Reserve Economic Data (FRED) database. The interest rate is the 3-month inter-bank rate. The money supply is the M2 money supply in the US, and M4 money supply in the UK. The stock return is the S&P 500 total return index in the US and the FTSE 100 index return in the UK. The foreign exchange rate is measured by the average USD/GBP rate in each quarter.

Table 3: Uncovered Interest Rate Parity

Panel A: UIP Regression

	Baseline Calibration	No Disagreement	Actual Data
$\bar{\alpha}$	0.002	0.002	-0.037
	(0.99)	(0.88)	(2.37)
$\bar{\beta}$	-1.012	0.231	-1.177
	(1.51)	(3.31)	(2.02)
$\bar{R^2}$	0.006	0.006	0.014

Panel B: Forecasting Horizon and UIP

j	No disagreement	Low disagreement	Baseline Calibration	High Disagreement
1	0.353	-0.578	-0.896	-5.443
4	0.362	-0.489	-0.798	-4.485
8	0.389	-0.320	-0.645	-4.413
12	0.390	-0.163	-0.457	-3.364
16	0.381	-0.063	-0.296	-3.017
20	0.408	0.022	-0.145	-2.484
24	0.399	0.140	0.031	-1.592

[*] This table reports the estimation results of the UIP regression. Using the calibrated parameters as in Table 1, we simulate 10,000 paths of nominal interest rates of the domestic and foreign markets as well as the nominal foreign exchange rate. This sample is referred as the "baseline calibration". The "no disagreement" sample comes from the another 10,000 simulated paths of nominal interest rates and exchange rates without heterogeneous beliefs between the domestic and foreign investors (i.e. $\mu_M^F = \mu_M^D = 2.15\%$ while keeping other parameters unchanged.) The actual data sample contains the quarterly nominal interest rate and exchange rate in the U.S. and the U.K. between 1955 and 2016 from the Federal Reserve Economic Data (FRED) database. The "Low disagreement" ("High disagreement") sample comes from 10,000 simulations by setting the $\mu_M^D - \mu_M^F = 0.12\%$ ($\mu_M^F - \mu_M^D = 0.4\%$) while keeping other parameters unchanged. The UIP regression is given as $e_{t+j+1} - e_t = \alpha + \beta(R_t^F - R_t^D) + \epsilon_t$ where the e_t is the natural log of exchange rate at time t, R_t^D and R_t^F are the domestic and foreign interest rate at time t, respectively. j is the forecasting horizon. We run the regression for each simulated path and the actual data. $\bar{\alpha}$, $\bar{\beta}$, $\bar{R^2}$ are the average intercept, slope and adjusted R-squared across each path. Panel A sets $j = 0$ for all estimations. Panel B reports the estimation results with different levels of j.

Table 4: Exchange rate changes and Heterogeneous Beliefs

	$R^F - R^D$	t-stats	ϕ_t	t-stats	R^2
Canada	-0.315	(-0.16)	0.089***	(3.45)	0.036
Denmark	-1.438***	(-2.90)	0.118	(1.27)	0.060
Euro Zone	-1.102*	(-2.13)	-0.015	(-0.51)	0.028
France	-1.413	(-0.75)	-0.003	(-0.14)	0.002
Germany	-2.444	(-1.18)	0.058*	(1.72)	0.011
Israel	-1.653**	(-2.41)	-0.001	(-0.06)	0.065
Italy	-4.987***	(-3.44)	0.058***	(2.85)	0.048
Japan	-2.946	(-1.38)	-0.009	(-0.80)	0.012
Netherlands	-7.461**	(-2.29)	0.001	(0.03)	0.021
Norway	1.061	(0.34)	0.207***	(3.34)	0.049
South Africa	-1.609**	(-2.54)	0.300**	(2.36)	0.055
Spain	-2.060	(-0.70)	0.025*	(1.67)	0.011
Sweden	-6.692*	(-2.23)	-0.014	(-0.71)	0.021
Switzerland	-8.353**	(-2.23)	0.024*	(1.69)	0.025
United Kingdom	-1.327	(-0.67)	0.052**	(2.43)	0.020

[*] This table reports the estimation results of the standard long-horizon predictive regressions. Specifically, the future changes in foreign exchange rates are regressed on the proxy of heterogeneity in beliefs and other control variables: $e_{t+3} - e_t = \alpha + \beta(R^F_{t,t+3} - R^D_{t,t+3}) + \gamma\phi_t + \epsilon_t$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity, and ϕ_t is the dispersion of beliefs. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate and interest rate is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Table 5: Exchange rate changes, Heterogeneous Beliefs, and VIX

	$R^F - R^D$	t-stats	ϕ_t	t-stats	VIX	t-stats	R^2
Canada	-0.566	(-0.29)	0.076**	(2.90)	0.066**	(2.18)	0.050
Denmark	-3.460***	(-5.63)	0.104**	(2.22)	0.339***	(4.94)	0.020
Euro Zone	-2.337***	(-3.97)	-0.018	(-0.65)	0.228***	(3.91)	0.011
France	-1.470	(-0.79)	-0.006	(-0.28)	0.045	(1.06)	0.005
Germany	-2.350	(-1.14)	0.051*	(1.71)	0.029	(0.67)	0.012
Israel	-1.678**	(-2.43)	-0.002	(-0.13)	-0.023	(-0.47)	0.066
Italy	-4.744**	(-3.22)	0.055**	(2.63)	0.034	(0.78)	0.050
Japan	-3.354*	(-1.77)	-0.009	(-0.80)	0.080*	(1.74)	0.021
Netherlands	-5.664*	(-1.69)	0.011	(0.31)	0.055	(1.26)	0.025
Norway	6.994	(0.93)	0.221***	(3.65)	0.148**	(2.51)	0.082
South Africa	-4.885	(-0.65)	0.234*	(1.84)	0.248**	(2.61)	0.101
Spain	-1.376	(-0.47)	0.023*	(1.79)	0.059	(1.42)	0.018
Sweden	-6.261*	(-2.08)	-0.012	(-0.61)	0.049	(1.05)	0.024
Switzerland	-7.538*	(-2.01)	0.020*	(1.68)	0.038	(0.89)	0.026
U.K.	-1.266	(-0.64)	0.0569*	(2.56)	-0.0293	(-0.70)	0.022

[*] This table reports the estimation results of the standard long-horizon predictive regressions. Specifically, the future changes in foreign exchange rates are regressed on the proxy of heterogeneity in beliefs and other control variables: $e_{t+3} - e_t = \alpha + \beta(R^F_{t,t+3} - R^D_{t,t+3}) + \gamma\phi_t + \text{VIX} + \epsilon_t$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity, and ϕ_t is the dispersion of beliefs. VIX is the CBOE Volatility Index. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate and interest rate is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Table 6: Exchange rate changes, Relative Heterogeneous Beliefs, and VIX

	$R^F - R^D$	t-stats	δ_t	t-stats	VIX	t-stats	R^2
Canada	0.383	(0.19)	0.046	(0.01)	-0.018***	(-2.73)	0.025
Denmark	-3.347***	(-5.52)	0.064*	(1.71)	-0.067***	(-4.94)	0.019
Euro Zone	-2.433***	(-4.19)	0.020*	(2.38)	-0.045***	(-3.87)	0.013
France	0.570	(0.29)	0.024**	(3.00)	-0.009	(-0.98)	0.032
Germany	-2.511	(-1.17)	-0.039	(-0.45)	-0.010	(-1.09)	0.007
Israel	-2.403**	(-3.29)	0.013*	(1.75)	0.035**	(2.22)	0.082
Italy	-4.588**	(-3.24)	0.024***	(4.19)	-0.005	(-0.50)	0.080
Japan	-3.867	(-1.54)	0.049	(0.07)	-0.006	(-0.46)	0.019
Netherlands	-5.816*	(-1.80)	0.065	(0.92)	-0.010	(-1.00)	0.028
Norway	6.228*	(1.69)	0.022**	(2.08)	-0.030*	(-2.46)	0.045
South Africa	-3.644	(-0.46)	-0.019	(-1.33)	-0.081***	(-2.95)	0.090
Spain	-1.520	(-0.52)	0.014**	(2.65)	-0.014	(-1.49)	0.036
Sweden	-4.817	(-1.56)	0.013**	(1.97)	-0.004	(-0.39)	0.037
Switzerland	-5.762*	(-1.70)	0.012*	(1.87)	0.017	(1.29)	0.033
U.K.	1.545	(0.76)	0.014	(0.20)	-0.087	(-0.10)	0.002

[*] This table reports the estimation results of the standard long-horizon predictive regressions. Specifically, the future changes in foreign exchange rates are regressed on the proxy of relative heterogeneity in beliefs and other control variables: $e_{t+3} - e_t = \alpha + \beta(R^F_{t,t+3} - R^D_{t,t+3}) + \gamma\delta_t + \text{VIX} + \epsilon_t$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity, and δ_t is the relative dispersion of beliefs. The relative dispersion of beliefs (δ_i) in country i are defined as $\delta_i = \log(\phi_i) - \log(\phi_{\text{us}})$, where ϕ_i and ϕ_{us} are the dispersion of beliefs in country i and United States respectively. VIX is the CBOE Volatility Index. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate, interest rate and VIX is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Table 7: Carry Trade Returns and Heterogeneous Beliefs

	ϕ_t	t-stats	R^2
Canada	-0.0859***	(-3.39)	0.034
Denmark	-0.0462	(-0.51)	0.002
Euro Zone	0.0161	(0.54)	0.002
France	0.00753	(0.33)	0.010
Germany	-0.0553*	(-1.75)	0.006
Israel	-0.0311*	(-1.76)	0.022
Italy	-0.0459**	(-2.26)	0.015
Japan	-0.0174*	(-1.70)	0.008
Netherlands	-0.0258	(-0.76)	0.002
Norway	-0.207***	(-3.35)	0.049
South Africa	-0.125*	(-1.81)	0.009
Spain	-0.0242*	(-1.63)	0.009
Sweden	-0.0164	(-0.82)	0.003
Switzerland	0.00929	(0.69)	0.002
United Kingdom	-0.0523**	(-2.44)	0.018

[*] This table reports results of regressing 3-month carry trade returns on heterogeneous beliefs(ϕ_t). The carry trade returns from investing in the currency market are given as: $CTR_{t+3} = R^F_{t,t+3} - R^D_{t,t+3} - (e_{t+3} - e_t)$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate and interest rate is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Table 8: Carry Trade Returns, Heterogeneous Beliefs, and VIX

	ϕ_t	t-stats	VIX	t(VIX)	R^2
Canada	-0.072***	(-2.80)	-0.064*	(-2.14)	0.048
Denmark	-0.014	(-0.15)	-0.089	(-1.58)	0.020
Euro Zone	0.018	(0.62)	-0.109*	(-2.13)	0.029
France	0.010	(0.46)	-0.041	(-0.97)	0.003
Germany	-0.049*	(-1.76)	-0.027	(-0.62)	0.008
Israel	-0.030*	(-1.70)	0.014	(0.30)	0.023
Italy	-0.041**	(-2.03)	-0.058	(-1.32)	0.021
Japan	-0.018	(-1.66)	-0.071	(-1.54)	0.015
Netherlands	-0.032	(-0.97)	-0.081	(-1.91)	0.016
Norway	-0.216***	(-3.55)	-0.095	(-1.91)	0.070
South Africa	-0.182*	(-1.68)	-0.290***	(-3.72)	0.100
Spain	-0.022*	(-1.75)	-0.062	(-1.49)	0.017
Sweden	0.014	(0.70)	-0.054	(-1.14)	0.007
Switzerland	0.006	(0.46)	-0.047	(-1.09)	0.007
U.K.	-0.057**	(-2.58)	0.029	(0.71)	0.020

[*] This table reports results of regressing 3-month carry trade returns on heterogeneous beliefs(ϕ_t), and VIX. The carry trade returns from investing in the currency market are given as: $CTR_{t+3} = R^F_{t,t+3} - R^D_{t,t+3} - (e_{t+3} - e_t)$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity. VIX is the CBOE Volatility Index. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate, interest rate, and VIX is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Table 9: Carry Trade Returns, Relative Heterogeneous Beliefs, and VIX

	δ_t	t-stats	VIX	t(VIX)	R^2
Canada	-0.035**	(-2.08)	-0.084**	(-2.79)	0.024
Denmark	-0.003	(-0.31)	-0.084	(-1.45)	0.020
Euro Zone	-0.018**	(-2.01)	-0.075	(-1.43)	0.050
France	-0.025**	(-3.31)	-0.026	(-0.64)	0.035
Germany	0.022	(0.03)	-0.041	(-0.96)	0.003
Israel	0.005	(0.86)	0.306	(0.05)	0.007
Italy	-0.021***	(-3.84)	-0.022	(-0.49)	0.051
Japan	-0.008*	(-1.69)	-0.056	(-1.22)	0.014
Netherlands	-0.007	(-1.05)	-0.058	(-1.27)	0.017
Norway	-0.022*	(-2.09)	-0.136*	(-2.51)	0.035
South Africa	0.014	(1.21)	-0.317***	(-3.68)	0.092
Spain	-0.013***	(-2.61)	-0.042	(-1.00)	0.034
Sweden	-0.017**	(-2.51)	-0.021	(-0.44)	0.029
Switzerland	-0.011*	(-1.74)	-0.022	(-0.48)	0.020
U.K.	-0.002	(-0.25)	0.304	(0.07)	0.001

[*] This table reports results of regressing 3-month carry trade returns on relative heterogeneous beliefs(δ_t), and VIX. The carry trade returns from investing in the currency market are given as: $CTR_{t+3} = R^F_{t,t+3} - R^D_{t,t+3} - (e_{t+3} - e_t)$, where e_t represents the logarithm of the spot exchange rate at time t against the U.S. dollar. $R^D_{t,t+3}$ ($R^F_{t,t+3}$) denotes the domestic (U.S.) interbank rate at time t with a 3-month maturity. The relative dispersion of beliefs (δ_i) in country i are defined as $\delta_i = \log(\phi_i) - \log(\phi_{\text{us}})$, where ϕ_i and ϕ_{us} are the dispersion of beliefs in country i and United States respectively. VIX is the CBOE Volatility Index. The sample period is October 1989 to December 2016. All observations are at monthly frequency. The information on exchange rate, interest rate, and VIX is from the Federal Reserve Economic Data (FRED) database. The proxy of dispersion of beliefs is from the Consensus Forecast published by the Consensus Economics. Standard errors are adjusted as in Newey-West (1987). ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.