Default Probabilities of Privately Held Firms

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ABSTRACT

We estimate term structures of default probabilities for private firms using data consisting of 1,759 default events from 29,894 firms between 1999 and 2014. Each firm's default likelihood is characterized by a forward intensity model employing macro risk factors and firm-specific attributes. As private firms do not have traded stock prices, we devise a methodology to obtain a public-firm equivalent distance-to-default by projection which references the distance-to-defaults of public firms with comparable attributes. The fitted model provides accurate multi-period forecasts of defaults, leading to both economically and statistically significant benefits over benchmark models. The reported interest rates charged to private firms are reflective of the estimated default term structure.

Keywords: Default probability; Term structure; Privately held firm; Interest charge

JEL Classification: E43, E47, G33

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1. Introduction

The appropriate assessment of credit risk is not only of interest to academics, but even more important for commercial lenders who must decide both whether to lend and how much of a credit spread to charge for a given loan application. Although the academic literature has been rife with studies of credit risk assessment ever since the early works of Altman (1968), most of the related works, whether structural or non-structural in nature, focus on publicly-traded firms (see Beaver (1966), Bharath & Shumway (2008), Campbell, Hilscher & Szilagyi (2008), Chava & Jarrow (2004), Hillegeist, Keating & Cram (2004), Ohlson (1980), Duffie, Saita & Wang (2007), Duan, Sun & Wang (2012), and many others).

In contrast, defaults of privately held firms mainly remain in the realm of commercial interest, and the research findings are kept proprietary. Academic research on the subject of private firm defaults is skimpy. Other than Altman (2013)'s work, there are only a few studies, mostly from the practitioners' perspective, that examine credit risk of private firms. For instance, Cangemi, Servigny & Friedman (2003) of Standard and Poor's examined the default risk of French private firms based on maximum expected utility (MEU) approach. Falkenstein, Boral & Carty (2000) of Moody's proposed a non-structural approach to assess credit risk of private firms in the U.S. market. This relative paucity of academic attention is partly due to the lack of publicly available data on privately held firms. Even if financial statement data on privately held firms were widely available, there is no market data, such as stock prices, to offer an important dimension of timely information on these firms. As recent advancements in credit risk model typically requires some form of market information, the absence of market data thus poses an additional obstacle to studying defaults of private firms.

In this study, we devise a way to utilize timely market information. Specifically, we estimate a powerful market information measure, known as distance-to-default (DTD), for private firms by referring to the universe of public firms for similar characteristics. Our approach can thus help assess whether using a modified version of the credit risk model that requires market data to predict defaults of private firms actually adds any value.

In addition, we adopt the newly developed doubly stochastic Poisson forward-intensity default modelling technique of Duan et al. (2012) to estimate the term structure of default probabilities for privately held firms. By directly modelling forward intensities, one can directly relate future defaults in any particular time period to the current information set characterized by some market-wide common risk factors and firm-specific attributes.

Using forward as opposed to spot intensities, one in effect bypasses the challenging task of modelling very high dimensional time series of covariates arising from firm-specific attributes due to the sheer number of firms in the data sample.

We investigate both financial and non-financial private firms. Needless to say, financial firms are of great importance. Despite their relevance, the literature on corporate default/bankruptcy typically ignore financial firms, in part because financial firms are highly leveraged making them somewhat distinct from non-financial firms. Technically speaking, reliable DTDs for financial firms is more difficult to obtain. Duan et al. (2012), however, demonstrated that using properly estimated DTDs in corporate default predictions can yield a universal model (i.e., financial and non-financial firms share the same default prediction model) that performs equally well for the subsamples of financial and non-financial firms in terms of the accuracy ratio.[1]

In this paper, we evaluate the credit risk of Korean private firms, both financial and non-financial, based on aforementioned approach. There are three broad reasons why we focus on Korea. First, Korean regulations on default disclosures allow us to assemble a comprehensive dataset on all default events by all corporations, both public and private, and all individuals who have checking accounts.[2] Whenever there is a bounced check issued by any entity within the Korean banking system, Korea Financial Telecommunications and Clearing Institute (KFTC), the official check clearing house in Korea, discloses the detailed identity of the check issuers, including the names, addresses, the date of default, and the first 7 digits of identification codes. This unique feature of Korean market allows us to assemble a dataset that is not only large but also comprehensive by containing the population of all defaults triggered by bounced checks for all businesses so that it can be free from potential selection bias. This is a significant advantage over existing commercial databases in the U.S. in terms of the quality and range of default information.

Second, firm-specific attributes for our sample of private firms are more reliable, as all the firms are externally audited. Specifically, Korean auditing regulations require all corporations whose total assets exceed KRW 10 billion (roughly USD 10 million) to hire an external auditor to audit their financial statements. We are unaware of similar regulations in other markets which require mandatory external auditing even for private firms. For example, there is little known about financial information of large commodity

[1] For further details on estimating DTDs for financial firms, please refer to Duan & Wang (2012).

[2] Unlike in U.S. where anyone with a valid address can open up a checking account and issue personal checks, such payment mechanism is rarely used by common households in Korea. Rather, checking accounts are mostly used by businesses, both corporations and sole proprietorships.

trading companies, as most of them are private, except for Glencore, even though they account for the majority of commodity trading around the world. In fact, one of Moody's reports on private firm defaults (Moody's RiskCalc 3.1 Korea Report) documents that accuracy ratio for audited firms in Korea is higher than the corresponding number for U.S. private firms. The higher ratio may be attributed to higher quality of information provided by the auditing process. This feature should clearly enhance the accuracy of the default prediction model.

Finally, not only is financial information of Korean private firms externally audited, but also it contains detailed information on the amount and interest rates charged for short-term and long-term loans, as well as repayment schedule and collateral information by each loan facility providing institution. We are also unaware of availability of such detailed information for private firms in other markets, including U.S. The availability of forward-looking information on interest charges conditional on maturity allows us to test whether default probabilities are appropriately reflected in the term structure of private borrowers.

Our data consists of 1,759 default events from a sample of 29,894 Korean private firms between 1999 and 2014. Due to the unique features of our sample, our tests are likely to be reliable and provide meaningful guidance with regards to lending decision to commercial lenders whose customers are in most cases private firms and individuals.

From lenders' perspective, an appropriate assessment of both financial and non-financial private firms' credit risks remains a fundamental task. This practical demand for the appropriate assessment of private firms' credit risk partly explains the degree of interest that commercial credit rating agencies have had in this issue relative to academia. Related to our study are Kocagil & Reyngold (2003) and Hood & Zhang (2007) of Moody's who employ binary probit models to estimate firm-level default probabilities for privately held Korean non-financial companies using information conveyed by financial statements. In contrast to the existing literature focusing only on non-financial firms, our study additionally investigates financial firms, and employs a more advanced econometric model to produce term structure of default probabilities.[3] In addition, we have incorporated an innovative implementation feature that factors in public-firm equivalent DTDs for privately held firms.

[3]Integrating financial and non-financial firms in a unified sample does not reduce predictive power of our analysis. In fact, independent estimation for financial firms and non-financial firms, respectively, does not improve accuracy ratios for either of them across various forecasting horizons in our sample. Our unified approach allows us to take advantage of a broader set of default events which yields more accurate inferences for both financials and non-financials.

The risk premia that a private firm is required to pay on its debts of different maturities are obviously an important matter. With the default term structure in place, one can begin to answer this related question of interest. There is a large literature on pricing credit risk, and Duffie & Singleton (1999), Driessen (2005), Pan & Singleton (2008), Jarrow, Lando & Yu (2005) and Azizpour, Giesecke & Kim (2011) are some examples. In the context of our paper, a pricing model will be normative in nature, simply because there are hardly any traded credit instruments for checking the performance of a pricing model. However, we can study whether the interest rates charged to private firms are reflective of their default likelihoods to ascertain the usefulness of the default term structure model. Based on the reported interest rates in a fiscal year, we are able to come up with an interest rate of a private firm and a maturity proxy for that firm-year, and show that interest rates are indeed positively related to their corresponding default probabilities. Moreover, we show that the conclusion is robust to factoring in various control variables.

We further investigate the economic magnitude of default predictability implied by our proposed methodology over various benchmark approaches. Referring to Stein & Jordão (2003) and Stein (2005), we find that the adopting the forward intensity model leads to substantial industry-wide economic benefit ranging from \$94.15 million to \$902.22 million per year over alternative models under a reasonable set of assumptions on banks' lending practices to Korean SMEs. The amount of increased profitability confirms the contribution of our approach to robust credit risk management for both private firms and their creditors.

The remainder of the paper is organized as follows. Section 2 explains how we develop our model of credit risk and term structure estimation for private firms. Section 3 provides a detailed description of the data sources, sample construction process, and definitions of key variables. Section 4 outlines our empirical results. Section 5 makes our concluding remarks.

2. Modeling framework

In this section, we specify the modeling framework for the estimation of the term structure of physical default probabilities for privately held firms in Korea. Our goal is two-fold. First, we estimate the term structure of physical default probabilities for privately held firms. Second, we use them to test whether the observed interest rates charged to the

Korean private firms properly reflect their credit risks.[4]

Our default term structure model follows that of Duan et al. (2012) by adopting *forward* intensities, which extend *spot* intensities of Duffie et al. (2007) as follows. The i-th private firm's default is assumed to be signaled by a jump in a doubly-stochastic Poisson process, N_t^i, which is governed by a non-negative spot *default* intensity, λ_t^i. Let τ_D^i be the i-th firm's default time, which is the first time that N_t^i reaches 1. Thus, $N_t^i - \int_0^t \lambda_s^i ds$ is a martingale relative to $\mathbb{F}$ and P, and we are only interested in this process up to the stopping time τ_D^i. The default intensity process λ_t^i is also the conditional default rate in the sense that $P\left(\tau_D^i \leq t + \Delta | \mathcal{F}_t\right) \approx \lambda_t^i \Delta$ for sufficiently small $\Delta > 0$, prior to its default.

In addition to default events, we factor in exits for reasons other than defaults/bankruptcies to avoid censoring bias. An example of other form of exits is merger/acquisition. We also assume that the other exit for the i-th firm in a group is governed by a separate doubly-stochastic Poisson process M_t^i. We assume that there is a non-negative spot *other exit* intensity process ϕ_t^i so that $M_t^i - \int_0^t \phi_s^i ds$ is also a martingale relative to $\mathbb{F}$ and P.[5] If we denote the i-th firm's *combined exit* time by τ_C^i, then by design the condition $\tau_D^i \geq \tau_C^i$ holds, and the instantaneous combined exit intensity is $\lambda_t^i + \phi_t^i$ at time t. It subsequently follows that the time-t conditional survival probability over the period $[t, t+\tau]$ can be expressed as

$$s_t^i(\tau) = E_t\left[\exp\left(-\int_t^{t+\tau}\left(\lambda_s^i + \phi_s^i\right)ds\right)\right], \tag{1}$$

and the default probability over $[t, t+\tau]$ is given by

$$p_t^i(\tau) = E_t\left[\int_t^{t+\tau}\exp\left(-\int_t^{s}\left(\lambda_u^i + \phi_u^i\right)du\right)\lambda_s^i ds\right]. \tag{2}$$

The Duan et al. (2012) approach that we adopt begins to deviate from spot intensity model by introducing a *forward intensity* version of the above model as a new tool for default prediction over a range of horizons. We first denote by $f_t^i(\tau)$ the forward *default*

[4]The uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, P)$, where P is the physical (statistical) probability measure. The information flow is represented by a right-continuous and complete filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions stated in Protter (2004). Expectation conditional on $\mathcal{F}_t$ is denoted by $E_t(\cdot)$.

[5]Note that λ_t^i and ϕ_t^i need not be two independent processes, but they must be adapted to the filtration $\mathbb{F}$. In fact, they are likely to be dependent when both are defined as functions of some common stochastic covariates. Although intensity processes can be dependent, N_t^i and M_t^i are assumed to be independent once being conditioned on λ_t^i and ϕ_t^i.

intensity specific to the i-th firm, having not defaulted until time t, as

$$f_t^i(\tau) = s_t^i(\tau) \cdot \lim_{\Delta \downarrow 0} \frac{P(t+\tau < \tau_D^i \leq t+\tau+\Delta | \mathcal{F}_t)}{\Delta}, \tag{3}$$

where the survival probability $s_t^i(\tau)$ is given by (1) above. Similarly, we define the forward *combined exit* intensity as

$$g_t^i(\tau) = s_t^i(\tau) \cdot \lim_{\Delta \downarrow 0} \frac{P(t+\tau < \tau_C^i \leq t+\tau+\Delta | \mathcal{F}_t)}{\Delta}. \tag{4}$$

Notice that *spot* intensity is a special example of *forward* intensity in that $f_t^i(0) = \lambda_t^i$ and $g_t^i(0) = \lambda_t^i + \phi_t^i$. Equivalently, we can also express (1) and (2) as

$$s_t^i(\tau) = \exp\left(-\int_0^\tau g_t^i(s)ds\right), \tag{5}$$

$$p_t^i(\tau) = \int_0^\tau \exp\left(-\int_0^s g_t^i(u)du\right) f_t^i(s)ds. \tag{6}$$

Although spot intensity has served as the main tool for modeling defaults in the literature, Duan et al. (2012) have shown the superiority of forward-intensity approach in application. To put it simply, the forward-intensity approach allows users to bypass the task of modelling the very high-dimensional stochastic covariates, for which a suitable model is hard to come by and its estimation inevitably challenging. As the name suggests, the forward-intensity model explicitly absorbs into a set of forward intensity functions the effects arising from the evolution of future spot intensities. The forward intensities corresponding to different forward starting times are functions of variables (i.e., stochastic covariates) observable at the time of making predictions. In short, predictions for various future horizons can be made without having to know the dynamics of the stochastic covariates.

In this paper, we further follow Duan et al. (2012) by specifying the following family of forward intensity functions:

$$f_t^i(\tau) = \exp\left(\alpha_0(\tau) + \sum_{j=1}^k \alpha_j(\tau) x_t^i(j)\right) \tag{7}$$

$$g_t^i(\tau) = f_t^i(\tau) + \exp\left(\beta_0(\tau) + \sum_{j=1}^k \beta_j(\tau) x_t^i(j)\right), \tag{8}$$

where $X_t^i = (x_t^i(1), x_t^i(2), \cdots, x_t^i(k))$ is the set of the stochastic covariates (common risk factors and firm specific attributes) that affect the forward intensities for the i-th firm. Please note that the forward-intensity functions are specific to the forward starting time through τ-specific coefficients. To implement the model empirically, we use a discrete-time version of the model by setting the basic time interval to one month. Thus, we in effect have a discrete-time model on a monthly basis. In the empirical section, we will describe the stochastic covariates being used.

3. Data and sample

This section describes the default and accounting data, the explanatory covariate data, their sources, and the sample construction of our dataset. In addition, we explain how the public-firm equivalent DTDs are estimated, how the interest rate proxies are derived from reported interest charges, and how the approximate maturities are determined

3.1. Default and accounting data sources

Our initial default dataset is created from the Korea Financial Telecommunications and Clearings Institute (KFTC) website. The KFTC keeps track of all suspensions of checking accounts triggered by bounced checks for all accounts in the Korean banking system, and it publicly discloses this information electronically. The dataset is updated every day and covers all default events by all corporations, both public and private, as well as all individuals.[6] As our default dataset is literally comprehensive, it is free from any potential selection issues and thus may be considered superior to the existing commercial databases available in the U.S. that offer limited coverage based on information provided by the participating banks.[7]

The data items available from this list are the first six or seven digits of the issuer identification codes, similar to Tax Identification Number (TIN) or Social Security Number (SSN) in the US, the name and address of the account holder, and the exact date of the suspension. This unique dataset provides us with a precise measure of default that does not rely on any proxies of financial distress: the eschewal of such proxies is one of the key advantages of this paper. One drawback is that the KFTC website publicly discloses

[6] Personal checks issued by individual households that we typically observe in the US are virtually non-existent in Korea. Entities that issue checks are typically corporations or individual entrepreneurs, allowing the KFTC to track and disclose all suspended accounts within the Korean banking system.

[7] One such example is Moody's Credit Research Database (CRD). The description in Falkenstein et al. (2000) provides a detailed account of this dataset.

default events only for the most recent two years in an effort to protect privacy.

To extend the dataset beyond the limited time frame mentioned above, we resort to two major business daily newspapers in Korea, Maeil Business Newspaper and the Korea Economic Daily, which have been (and still are) reporting the same default information provided by the KFTC since even before the KFTC started distributing this information on its website. To ensure consistency between information provided by the two business dailies and KFTC-released default data, we randomly selected 30 days during the most recent two years, and verified that for the selected days, the data provided by both sources essentially contain the same set of default information. We also examined the consistency between the two business dailies beyond the most recent two years by randomly selecting one day from every month, and we found that they are almost perfectly consistent after 2000.

Our accounting data are drawn from TS2000, compiled by the Korea Listed Companies Association (KLCA): TS2000 is comparable to the Compustat provided by Standard and Poor's. One advantage of TS2000 over Compustat is that TS2000 provides extensive coverage of private firms whose total assets exceed a certain threshold.[8] Since the financial statements are audited by external auditors, we can be reasonably comfortable that the data are accurate and credible even for private firms, making this dataset superior to those provided in typical commercial databases in terms of quality.[9] The data for private firms have been made available on an annual basis since 1999 and covers roughly 100 data items for some 30,000 unique private (closely-held) firms.

3.2. Sample construction

After we assemble our initial default dataset and extract accounting information for private firms, we merge these two datasets. Our matching is mainly performed through identification codes and addresses whenever identification codes are available. When identification codes are unavailable, we compare company names, CEO names, and addresses, and designate a match when at least two of the three variables match.

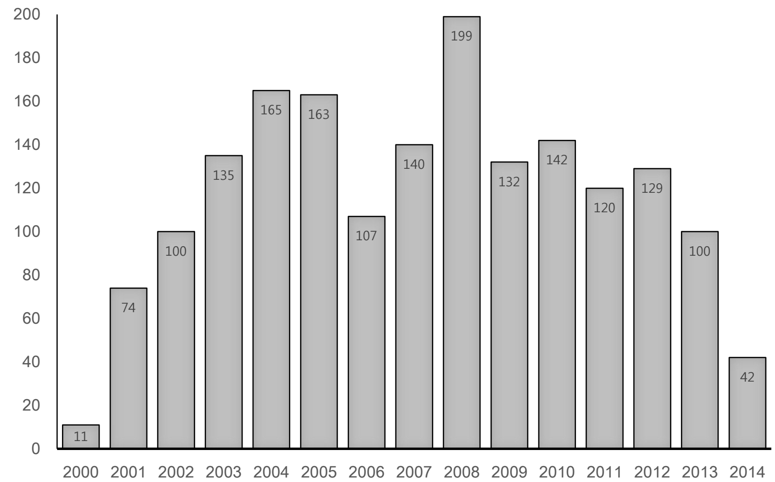
As our default dataset is mostly reliable after 2000 and accounting information for private firms is mostly available from December 1999, we naturally start our sample period from then. More precisely, our final default sample starts in 2000 and ends in June 2014,

[8]Korean auditing regulations require that all corporations whose total assets are greater than KRW 10 billion (roughly USD 10 million) hire an external auditor (accounting firm) to audit their financial statements every fiscal year. This information is compiled by the Korea Listed Companies Association.

[9]For example, only 28% of the financial statements used in Falkenstein et al. (2000) are audited.

Figure 1: Annual default numbers of privately held firms

This figure shows the annual number of default events in our final sample of private firms in Korea. Private firms are those whose assets are in excess of KRW 10 billion (roughly USD 10 million). We observe 1,759 default events from 29,894 unique private firms in our dataset.



while our accounting data ranges from December 1999 to June 2011.[10] Figure 1 shows annual default numbers of private firms for each year during our sample period. There are a total of 1759 default events by the corresponding number of unique private firms during these 14.5 years. The numbers reported in Figure 1 are comparable to those reported in Falkenstein et al. (2000) who use Moody's Credit Research Database (CRD).[11]

3.3. Covariates

To characterize the forward intensity functions specified in Section 2, we employ both (1) macro risk factors and (2) firm-specific attributes based on the financial statements. We selected the covariates from a high-dimensional set of variables based on literature review so that they best fit our data set. The selected covariates are used to infer the likelihood of observing defaults for private firms.

(1) Common variables: The following two macro risk factors are motivated by Duffie et al. (2007) and Duan et al. (2012).

[10]That is, we stop to estimate the model in June 2011 and exploit the default sample between July 2011 and June 2014 for out-of-sample analysis. In our subsequent main analysis, we use the previous year's accounting information to map with the current year's default event. Because private firms' accounting information has been available since 1999, we do not include private firm defaults that occurred during 1999 in our final sample.

[11]In Falkenstein et al. (2000)'s sample, there are a total of 24,718 unique firms with 1,621 default events over the 11-year period from 1989 to 1999.

- CP: The yield on 91-day commercial paper.
- KOSPI: The trailing one-year return on the Korea Composite Stock Price Index.

(2) Firm-specific variables: We have explored a set of candidate variables that are known to represent firm characteristics by the prior literature and research findings, such as Duan et al. (2012), Kocagil & Reyngold (2003), and Hood & Zhang (2007), among others. The last variable (maturity mismatch) is motivated by Adrian & Brunnermeier (2016).

- GP/CA: The ratio of gross profit over current asset as a measure of profitability.
- Debt/EBITDA: The firm's interest bearing debt over earnings before interest, taxes, depreciation and amortization as a measure of debt coverage. The amount of interest bearing debt is calculated as the sum of short term borrowing, long term borrowing, current portion of the long term debt and bond.
- DTD: The estimated firm-level distance-to-default as a measure of volatility-adjusted leverage. See Section 3.4 for details of its computation.
- CASH/CA: The ratio of cash and short term investments over current asset as a measure of liquidity.[12]
- Size: The amount of total assets as a measure of size.[13]
- MM: Current liability minus cash then divided by total liabilities as a measure of maturity mismatch.

For the common macro variables, we collect historical month end data whereas for the firm-specific attributes, we employ audited financial statements. These macro variables are obtained from the Risk Management Institute (RMI) at the National University of Singapore. The firm-specific variables start from the period end of the statement but are lagged by three month to ensure that default predictions are made on the available information at the time of prediction. Table 1 reports the monthly summary statistics of the selected variables (Panel A), and the correlation coefficients among the selected firm-specific attributes (Panel B) to check for excessive multicollinearity and potential over-fitting.

[12]Although it is standard to divide CASH by total asset (TA), we take this liquidity measure as it provides a better fit to the data.

[13]Although it is common to take the natural logarithm of total assets as a proxy for firm size, we find that the range of this log-transformed variable is too restrictive to allow for sufficient variation in the universe of private firms.

Table 1: Descriptive Statistics of Covariates

This table reports the summary statistics of the variables at monthly frequency for the period between November 1999 and June 2011. CP is the yields on 91-day commercial paper (in percent), KOSPI is the trailing one-year return on the Korea Composite Stock Price Index, DTD is distance-to-default, GP/CA is gross profit over current asset, Debt/EBITDA is interest bearing debt over earnings before interest, taxes, depreciation and amortization, CASH/CA is the cash over current asset, Size is the amount of total assets, and MM is the maturity mismatch measure defined as current liability minus cash then divided by total liabilities.

Panel A: Summary Statistics						
	Obs.	Mean	Stdev	Min	Median	Max
CP	140	4.7082	1.3458	2.6200	4.6250	7.8500
KOSPI	140	0.1362	0.3024	-0.5092	0.1744	1.2056
GP/CA	1807764	0.6783	1.2873	-0.5438	0.3642	9.3395
Debt/EBITDA	1807764	0.0653	38.8428	-251.03	1.6989	173.33
DTD	1822381	1.4996	1.4227	-1.8748	1.4581	5.6005
CASH/CA	1807771	0.1407	0.1900	0.0000	0.0660	0.9300
Size	1807773	0.4977	1.2248	0.0178	0.1700	9.9759
MM	1807771	0.5141	0.6025	-3.4428	0.6639	0.9995

Panel B: Correlation Matrix						
	GP/CA	Debt/EBITDA	DTD	CASH/CA	Size	MM
GP/CA	1.0000	0.0709	0.0031	0.2242	-0.0192	0.0028
Debt/EBITDA	0.0709	1.0000	0.0515	0.0048	-0.0081	0.0035
DTD	0.0031	0.0515	1.0000	0.1822	-0.0393	-0.3720
CASH/CA	0.2242	0.0048	0.1822	1.0000	0.0190	-0.4980
Size	-0.0192	-0.0081	-0.0393	0.0190	1.0000	-0.0525
MM	0.0028	0.0035	-0.3720	-0.4980	-0.0525	1.0000

Our approach differs in several ways from that of Duan et al. (2012). First, we exclude several variables that are available to listed firms, such as the ratio of a firm's market equity value to the average market equity value of the market index portfolio (SIZE) and the market-to-book asset ratio (M/B). Also, we add or modify certain input variables that are used in Duan et al. (2012) so that the variable selection is better suited for the private firms in our dataset. Furthermore, we consider only the value of each variable, rather than its trend, because of the annual frequency of the financial statement data for private firms.[14]

3.4. Public-firm equivalent distance-to-default

One of the key variables that we use in the subsequent analysis is firm-level DTDs estimated at different points of time. Firms that exhibit large DTD estimates are expected to be more resilient and less likely to default. This measure, originally developed by Merton (1974), needs firm's asset value and volatility. Modern techniques exist for the estimation of these unknown quantities, but these techniques require knowing firm's equity market capitalization. Obviously, privately held firms by definition do not have traded stocks for

[14]Duan et al. (2012) consider the trend, which is computed as the current value of the variable less the one-year average of the measure, to address a momentum effect.

one to assess their equity market capitalizations. For this, we devise a way to estimate DTDs for private firms indirectly by projecting onto the universe of public firms.

We first obtain monthly DTD estimates for public firms in Korea.[15] Then, we regress these monthly DTD estimates on monthly macro variables and on firm characteristics that have been identified in the previous literature as determinants of default probabilities. We assign a 3 month-lag after the previous fiscal year-end to allow time for information dissemination. Specifically, for December fiscal-year-end firms, the accounting information for that fiscal year is matched with calendar months starting from March of the next year up to February of the following year. As shown in Table 2, we run 12 separate regressions and obtain 12 different sets of coefficients based on the number of months since the most recent fiscal year-end to reflect the age of the information in the reported annual financial statements. For example, DTD in March 2002 is run against financial data ending at December 2001, denoted as Model 0 in Table 2. DTD for the same firm in February 2003 is run against financial data ending at December 2001, and is denoted as Model 11 in Table 2.[16]

Once we obtain these coefficient estimates from the 12 models, we then locate financial information for private firms. Similar to public firms, we also allow a 3 month-lag for possible information dissemination. That is, for December fiscal year-end firms, financial information is applied to March of the next year until February of the following year. For example, March DTD of a December fiscal-year-end private firm is obtained by multiplying coefficients estimates from model 0 to financial numbers from previous December. Similarly, May DTD of the same firm is obtained by multiplying corresponding coefficients from model 2 to the same financial information. In essence, we try to incorporate staleness of information at both stages of the estimation procedure. We run these regressions separately for financial firms and non-financial firms.[17] Then, we generate the public-firm equivalent DTD estimate for private firms based on the fitted regression model as an input covariate of the forward intensity model.

Our approach based on the public-firm equivalent DTD has several merits: (i) We can avoid the *over-fitting* problem by adopting the proposed two-step estimation approach to penalize overly complex models. In this context, we directly show in Section 4.3 (see Figure

[15]Monthly DTD estimates for all public firms in Korea and many other economies are calculated and provided on a regular basis by the Risk Management Institute of the National University of Singapore. The DTD data are freely retrievable at its web site.

[16]If the public firms' fiscal year-end is June, then DTD in September (until next August) is matched with the fiscal information ending in previous June. In this case, September DTD would be a part of model 0 and March DTD would be a part of model 6.

[17]The DTD estimates for public firms are winsorized at the first and 99th percentiles.

4) that the alternative one-step approach with too many covariates in the forward intensity model results in the excessive degree of freedom, leading to poor predictive performance in general. (ii) Our proposed DTD-based approach provides a *universal* way to analyze both financial and non-financial firms, as the public-firm's DTD estimation methodology proposed by Duan & Wang (2012) can deal with financial firms in a comparable manner with non-financial firms despite their uniqueness in capital structure.[18] Note that we can freely choose the determinants of both financial and non-financial DTDs to optimize the predictive performance. (iii) We can successfully accommodate a so-called *age-of-information* issue, which comes from the annual updates of the financial statements for the private firms in our dataset, by adopting the proposed DTD-based estimation on a monthly basis. For example, a financial statement variable updated eleven months ago cannot carry the same quality of information as one updated one month ago, even if their values remain the same. Our regression specification to estimate the public-firm equivalent DTD certainly mitigates this problem, as we update the information from the obsolete firm-specific attributes based on the DTD information that is updated more frequently. (iv) Most importantly, we can utilize the timely stock market information by introducing the public-firm equivalent DTD in that we make a projection to the universe of public firms through this intermediate variable.

3.5. Interest rate, maturity and collateral-to-debt ratio

In our subsequent analysis, we employ three other firm-level variables: the interest rate, the proxy maturity of debt, and the collateral-to-debt ratio for private firms. As the information on these variables is generally unavailable in electronic format, we resort to the footnotes in audited financial statements in text format from DART (Data Analysis, Retrieval and Transfer system) of the Financial Supervisory Service based on an adaptive keyword search method.

The interest rate is defined as the weighted average of interest rates on outstanding interest bearing bank loans for each firm-year.[19] One of the footnotes contains detailed

[18]It is noteworthy that the related literature has devoted little attention to financial firms, as traditional Moody's KMV method tends to neglect a substantial part of a financial firm's debts, producing unreliable DTD estimates for financial firms.

[19]Alternatively, one may consider *implied* interest rate defined as the realized interest expense for a given fiscal period scaled by the outstanding interest bearing debt as of the previous fiscal year end (i.e., short-term borrowing, current portion of long-term debt, bonds, and long-term borrowing). Similar approach is commonly used in the accounting literature to back out the overall cost of debt capital even for publicly traded firm (e.g., Pittman & Foretin (2004)). We also resort to this measure in inferring private firm's DTD from those of public firms (Table 2). But this measure is obviously backward looking and as such a crude (and usually overestimating) proxy for the true interest rate that the firm is facing

Table 2: Coefficient Estimates for Public Firms' Distance to Default (DTD)

These tables present the OLS coefficient estimates where the dependent variable is monthly distance to default estimates between January 1993 to June 2011 for publicly traded non-financial firms (Panel A) and financial firms (Panel B) in Korea based on the Merton (1974) model, respectively. Firm characteristics are as of the most recent fiscal year end. We estimate 12 separate regressions based on the number of months since the most recent fiscal year end. The t-statistics are shown in parentheses. (*** significant at 1% level, ** significant at 5% level, * significant at 10% level)

Panel A: Non-financial Firms

	Number of Months Since the Most Recent Fiscal Year End											
	0	1	2	3	4	5	6	7	8	9	10	11
Constant	8.4676***	7.7772***	7.4379***	7.7605***	7.7879***	8.1532***	8.3263***	8.5390***	8.0548***	9.0883***	8.9534***	8.5968***
	(82.6054)	(80.2545)	(86.6063)	(77.5243)	(73.9752)	(74.2349)	(75.3441)	(71.1906)	(69.0867)	(79.5610)	(75.6902)	(81.3681)
Net Income / Total Assets	-0.2791***	-0.1886**	-0.1891**	-0.1690*	-0.2227***	-0.4238***	-0.5030***	-0.6445***	-0.6753***	-0.5255***	-0.6174***	-0.6208***
	(-3.0917)	(-2.2101)	(-2.2198)	(-1.9289)	(-2.5802)	(-4.7176)	(-5.3429)	(-6.4743)	(-7.0598)	(-5.4486)	(-6.0826)	(-6.2229)
Book Equity / Total Liabilities	0.0730***	0.0853***	0.0847***	0.0544***	0.0546**	0.0570***	0.0765***	0.0847***	0.0878***	0.0930***	0.0934***	0.0917***
	(16.2675)	(20.1161)	(20.0254)	(12.6315)	(12.8695)	(13.2622)	(17.4002)	(17.8558)	(19.2924)	(20.2604)	(19.4527)	(19.4669)
Total Liabilities / Total Assets	-2.2152***	-2.2032***	-2.1834***	-2.3516***	-2.4162***	-2.4400***	-2.3738***	-2.2434***	-2.2435***	-2.1950***	-2.1905***	-2.2456***
	(-32.2017)	(-34.0087)	(-33.7322)	(-35.3957)	(-36.8763)	(-35.9316)	(-33.9100)	(-30.5209)	(-31.7385)	(-30.8482)	(-29.4213)	(-30.7151)
Sales / Total Assets	-0.0981***	-0.0455*	-0.0493**	0.0823***	0.1484***	0.1445***	0.0634***	0.0798***	0.0800***	0.0594**	0.0713***	0.0773***
	(-3.9532)	(-1.9368)	(-2.1060)	(3.4261)	(6.2765)	(6.0459)	(2.6198)	(3.1023)	(3.2406)	(2.3967)	(2.7544)	(3.0427)
Interest Expense / Operating Income	-1.7822***	-1.5533***	-1.6010***	-1.4539***	-1.3713***	-1.3591***	-1.3293***	-1.3179***	-1.2846***	-1.3288***	-1.3565***	-1.3496***
	(-37.5598)	(-34.6389)	(-35.7619)	(-31.6677)	(-30.3403)	(-29.4802)	(-27.9930)	(-26.0919)	(-26.4882)	(-27.1842)	(-26.5096)	(-26.8678)
FX rate (KRW/USD)	-0.0039***	-0.0035***	-0.0031***	-0.0036***	-0.0038***	-0.0041***	-0.0042***	-0.0045***	-0.0040***	-0.0049***	-0.0048***	-0.0044***
	(-47.7691)	(-45.4542)	(-48.0562)	(-44.9827)	(-43.8308)	(-45.6228)	(-46.6369)	(-45.6838)	(-42.9650)	(-53.6405)	(-51.0819)	(-54.4868)
Obs.	16433	16477	16480	16647	16636	16536	17472	16155	16161	16360	16320	16309
$R2$	0.3958	0.3969	0.4078	0.3642	0.3624	0.3532	0.3342	0.3196	0.3246	0.3573	0.3349	0.3509

Panel B: Financial Firms

	Number of Months Since the Most Recent Fiscal Year End											
	0	1	2	3	4	5	6	7	8	9	10	11
Constant	6.1005***	6.3672***	6.7796***	6.2762***	5.7483***	5.6288***	6.3279***	6.1042***	6.2145***	6.6867***	5.8653***	5.8280***
	(15.7211)	(15.8924)	(17.8773)	(14.7542)	(12.7348)	(13.0175)	(15.1598)	(14.5267)	(15.6846)	(17.3411)	(15.0980)	(16.6014)
Book Equity / Total Liabilities	0.0985***	0.1069***	0.1101***	0.1139***	0.0817***	0.0880***	0.1002***	0.0878***	0.0860***	0.0877***	0.0892***	0.0909***
	(8.1625)	(8.9196)	(9.3687)	(9.1609)	(6.5096)	(7.1188)	(8.0507)	(6.7847)	(6.6862)	(6.6671)	(7.1930)	(7.3815)
Total Liabilities / Total Assets	-2.2672***	-2.3613***	-2.2594***	-1.6756***	-1.9475***	-1.9346***	-2.5028***	-2.2270***	-2.2145***	-2.1669***	-2.3846***	-2.4110***
	(-9.1694)	(-9.6095)	(-9.3875)	(-6.5908)	(-7.5698)	(-7.7847)	(-9.8513)	(-8.7661)	(-8.7588)	(-8.3814)	(-9.5521)	(-9.7286)
Sales / Total Assets	0.8654***	0.8512***	0.8564***	1.0154***	0.9291***	0.8908***	1.0326***	0.8593***	0.9780***	1.0629***	1.0512***	0.9631***
	(5.2346)	(5.1694)	(5.2965)	(5.9091)	(5.3360)	(5.3251)	(6.1742)	(5.2526)	(6.0086)	(6.4095)	(6.5369)	(6.0377)
FX rate (KRW/USD)	-0.0032***	-0.0034***	-0.0038***	-0.0039***	-0.0034***	-0.0032***	-0.0035***	-0.0034***	-0.0035***	-0.0039***	-0.0030***	-0.0029***
	(-10.4066)	(-10.6793)	(-12.8748)	(-11.4873)	(-9.0228)	(-9.0640)	(-10.2485)	(-9.9899)	(-11.0399)	(-12.7723)	(-9.7451)	(-10.7250)
Obs.	974	974	975	985	953	951	1015	979	979	981	982	982
R^2	0.3413	0.3620	0.3877	0.3253	0.2715	0.2876	0.3466	0.3002	0.3135	0.3295	0.3219	0.3347

information on the amount and the interest rate of short-term and long-term loans provided by each loan facility providing institution. We calculate the weighted average of interest rates with the outstanding balances as weights. If an interest rate is expressed as a floating rate such as LIBOR (London Interbank Offered Rate), certificate of deposit, or the spread of them, we refer to the rates in the corresponding period from Bloomberg and Economics Statistics System (ECOS) of the Bank of Korea. After calculating the interest rates of short-term and long-term loans respectively, we finally obtain the interest rate for each firm-year as the weighted average of them with the amounts of long-term and short-term borrowings in the accounting data drawn from TS2000 as weights.

We extract maturity information from the repayment schedule section in the footnotes of audited statements. For long-term loans, amount of loans to be retired for each year, up to 4 years, and for all remaining years aggregated from year 5 is available. We employ this information to construct a value-weighted maturity variable assuming that short-term loans' maturity is 6 months, and long-term loans to be retired after 5 years has a maturity of 6 years, which is determined by exponentially decreasing the weights for each year by half.[20]

Finally, we obtain the collateral information from the collateral section of the footnotes in the audited statements, where the maximum credit amount is available for each collateral asset. Although most of the collaterals are on loans, it is sometimes hard to tell if the collateral is on loans or other types of debts such as corporate bonds. For this reason, we define the collateral-to-debt ratio as the sum of maximum credit amounts of the collaterals scaled by the sum of long-term borrowing, short-term borrowing, current portion of long-term debt and corporate bonds.

4. Empirical analysis

This section presents an empirical analysis of the model calibration, the parameter estimates, the forecasting accuracy of the fitted model, and how the interest charges are related to the estimated default term structures.

going forward.

[20]Specifically, note that $\sum_{n=0}^{\infty}(5+n)\left(\frac{1}{2}\right)^{(n+1)} = 6$. We have also considered different assumptions on the weighting schemes, but the results were not very sensitive.

4.1. Calibrating the forward intensity model

Calibration of the forward intensity model can be performed by maximizing a so-called overlapped pseudo-likelihood function. Statistical inference can utilize the model's large sample properties, even though the objective function does not satisfy the standard assumptions on likelihood functions. We fit the model to our dataset of monthly frequency.

The model's implementation is based on the assumption that firms' default activities are conditionally independent given the common factors and firm-specific attributes, which are not affected by any firm's default or other exit. Suppose that there are N firms in our dataset, and our sample period is $[0, T]$, which is discretized into $T/\Delta t$ periods. Under this assumption, we can decompose the pseudo-likelihood function into horizon-specific pseudo-likelihood functions as in Duan et al. (2012). Naturally, the forecasting horizon τ must be smaller than T to the extent that there are enough observations to determine the forward-intensity function of horizon τ.

These horizon-specific pseudo-likelihood functions can be separately maximized using numerical optimization methods, because the original pseudo-likelihood function to be maximized is conveniently the product of the horizon-specific pseudo-likelihood functions. This decomposability allows the entire calibration procedure to be separated into completely unrelated sub-modules. Owing to the large sample size of our dataset, this property certainly increases the computational efficiency.[21]

4.2. Parameter estimates

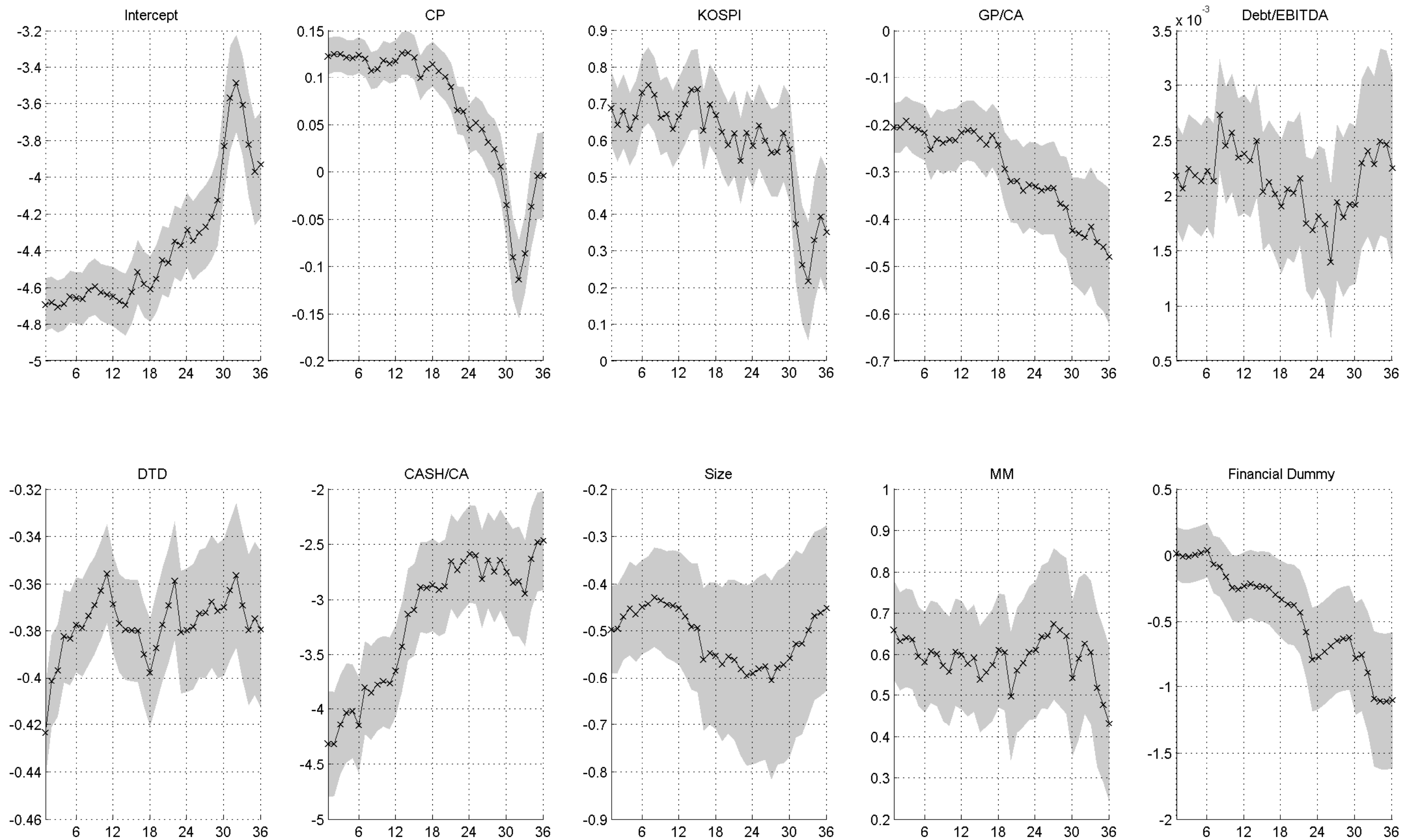
We next discuss the statistical implication of the selected covariates in the forward-intensity model. Figure 2 reports the maximum pseudo-likelihood estimates for $\alpha(\tau)$ in equation (7) with different prediction horizons ranging from 1 month to 36 months.[22] The fitted forward default intensities tend to increase with the yields on 91-day commercial paper for prediction horizons shorter than 2.5 years, whereas the coefficients become negative and lose their significance for longer horizons. This observation is consistent with the fact that higher interest rates force firms to carry heavier burden to cover interest expenses; however, such an effect seems to fade in the long run. Admittedly, this

[21]Refer to A for details of the maximum pseudo-likelihood estimation. The numerical experiments in our analysis were performed based on code written in MATLAB. We are grateful to Tao Wang for providing the sample codes to implement the pseudo-likelihood estimation of the forward intensity model. Details are available upon request.

[22]The forward other-exit intensity model should be estimated as well, but we do not report the result here. The maximum pseudo-likelihood estimates for $\beta(\tau)$ in (8) are available upon request.

Figure 2: Maximum pseudo-likelihood estimates for forward default intensity

This figure shows the maximum pseudo-likelihood estimates of $\alpha(\tau)$ for 1-36 months horizons, along with one-standard-deviation error bands. KOSPI is the trailing one-year retun on the Korea Composite Stock Price Index, CP is the yields on 91 day commercial papers, DTD is distance-to-default, GP/CA is gross profit over current asset, EBITDA/IE is earnings before interest, taxes, depreciation and amortization over interest expense, CASH/CA is the cash over current asset, TA is total assets adjusted by GDP deflator, and MM is the maturity mismatch measure defined as current liability minus cash then divided by total liabilities, and Financial Dummy is a dummy variable which takes 1 if it is a financial firm and 0 otherwise.



phenomenon runs counter to the results obtained by Duffie et al. (2007) in that lower short-term interest rates were used as a policy instrument to boost the economy during recessions. For Korean private firms, we find that the former effect outweighs the latter, offsetting each other for longer prediction horizons, along with business cycles.[23]

Controlling for other covariates, the forward default intensities are estimated to increase in the trailing one-year return of the KOSPI for all prediction horizons considered. While this observation is certainly counterintuitive the perspective of univariate reasoning, Duffie et al. (2007) and Duan et al. (2012) also report the same result for the effect of the one-year S&P500 index return on the default intensities of the US public firms. This relationship could be explained by the fact that the KOSPI return is a lagging business indicator because of its trailing nature in relation to business cycles.

It turns out that a private firm's profitability signaled by the GP/CA ratio plays a significant role in the prediction of defaults. This measure was originally proposed by Hood & Zhang (2007) for predicting private company defaults in Korea. Holding other covariates fixed, the estimated forward default intensities in our analysis are decreasing with the ratio of the gross profit over the current asset for almost all prediction horizons.

Similarly, a firm's debt coverage measured by the Debt/EBITDA ratio is estimated to significantly increase the forward default intensities across different prediction horizons. The inclusion of this covariate is also motivated by Hood & Zhang (2007). The positive sign of the coefficients is consistent with a simple univariate reasoning.

We also confirm that the DTD measure, which can be interpreted as a volatility-adjusted measure of leverage, is one of the most crucial attributes in distinguishing distressed firms from others. Although we use a proxy for private firms' DTDs because we are unable to observe their stock prices, the result shows that a smaller value of a firm's DTD foreshadows a higher default likelihood with a strong statistical significance. To the best of our knowledge, this is the first study that proposes a way to use public-firm equivalent DTDs to gauge the default probabilities of privately held firms. Our finding of its statistical significance in default prediction is consistent with those public-firm studies as reported in Bharath & Shumway (2008), Duffie et al. (2007), Duan et al. (2012), and many others.

We find a significantly negative relationship between the fitted forward default intensities and the CASH/CA ratio after controlling for other covariates. This result is

[23]In the analysis performed by Duan et al. (2012) on the U.S. public firms, the forward default intensities are estimated again to decrease with the three-month Treasury bill rate when the prediction horizon is shorter than one year but to increase for longer horizons.

consistent with a univariate reasoning, because this attribute is assumed to represent the degree of a firm's liquidity to meet its financial obligations in the near term. Note that Duan et al. (2012) reports a similar estimation result with the CASH/TA ratio, which is found to be less indicative in our dataset.

The estimated forward default intensity is, ceteris paribus, significantly decreasing with the firm's size measured by its inflation-adjusted value of total assets (normalized by the Korean GDP deflator) for all horizons. Similar results have been reported in the prior research such as Kocagil & Reyngold (2003), Hood & Zhang (2007), Duffie et al. (2007), and Duan et al. (2012), among others.

A firm's maturity mismatch profile is measured by the current liability minus the cash then divided by the total liabilities. It reflects the tendency of a business to mismatch its balance sheet in the sense that liabilities exceed assets in the short run and that medium- and long-term assets dominate the corresponding obligations. Our estimation results report that the estimated coefficients for this attribute are significantly positive in the forward default intensity model for all prediction horizons. In particular, the maturity mismatch profile makes a strong contribution to the characterization of short-term default likelihood.

Our forward default intensity model contains a financial dummy variable that takes a value of 1 if the firm is a financial private firm, and 0 otherwise. The estimated coefficients are found to be negative but statistically significant in the long run, implying that a financial firm is exposed to a smaller default risk than an otherwise identical non-financial firm.

4.3. Forecasting accuracy analysis

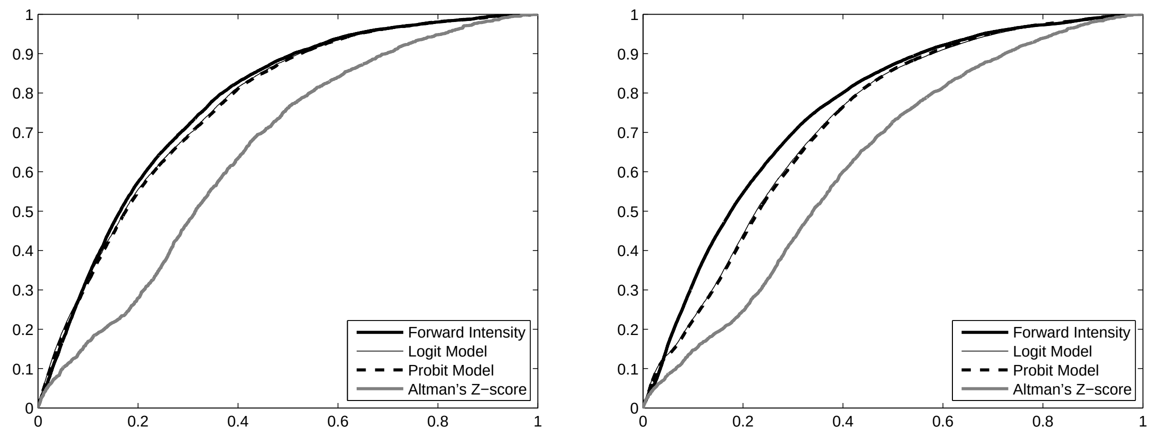
This section presents our testing results after performing a prediction accuracy analysis based on the cumulative accuracy profile of the fitted model. The cumulative accuracy profile, along with the accuracy ratio as its summary statistic, is in practice the most popular validation technique to evaluate the prediction power of any default risk ranking system.

For completeness, we briefly review the concept of the cumulative accuracy profile.[24] First, we compute the cumulative default probabilities implied by our fitted forward intensity model at a conditioning time point and rank each of the private firms in our dataset from the riskiest to safest according to the estimated cumulative default proba-

[24] A detailed explanation of the cumulative accuracy profile can be found in Vassalou & Xing (2004).

Figure 3: Out-of-sample Cumulative Accuracy Profiles

This figure shows the out-of-sample cumulative accuracy profiles based on all private firms in our dataset from Dec 1999 to Jun 2011 for different modeling approaches for one-year (left panel) and three-year (right panel) prediction ahead. The fitted logit and probit models share the same risk factors with the forward intensity model.



bilities. Then, for a given fraction x of the total number of private firms ordered by their respective risk scores (i.e., default probabilities), we generate a curve by calculating the percentage of the defaulters whose risk score is equal to or smaller than the maximum score of each fraction x, ranging from 0 to 1. At the same time, we construct the same type of curve with a hypothetically perfect rating model, which generates a curve that increases linearly and then holds constant at one if the fraction x is equal to or larger than the proportion of firms that default over the risk horizon. Finally, we consider a random model without any prediction power, which generates a linear curve from 0 to 1 with a slope of 45°. The accuracy ratio is defined as the ratio of the area between the curve of the model being tested and that of the random model over the area between the curve of the perfect model and that of the random model. The better the prediction power of a tested model, the larger the value of its accuracy ratio with the ideal value being one.

We conduct an out-of-sample analysis in the time dimension using a moving-window approach. Specifically, we re-estimate the model at each month-end from January 2004 with all the data available up to that time and compute the out-of-sample accuracy ratio over different future periods. That is, the left-hand side of window moves from January 2004 to June 2011 and we use the default data from July 2011 and thereafter for the out-of-sample validation. By doing so, we fully utilize the historical default data up to June 2014 for out-of-sample analysis with three years of prediction horizon.

Figure 3 plots the out-of-sample cumulative accuracy profiles of the fitted forward

Table 3: Out-of-sample Accuracy Ratios

This table reports the out-sample accuracy ratios based on all private firms in our dataset from December 1999 to June 2014 for different modeling approaches for the prediction horizons of 1 year, 2 years and 3 years, respectively. The fitted logit and probit models share the same risk factors with the forward intensity model. The 95 percent confidence intervals are calculated based on DeLong, DeLong & Clarke-Pearson (1988). The Z-statistics are calculated by dividing the difference between two *correlated* accuracy ratios of forward intensity model and each of benchmark models ($AR_{\text{forward intensity}} - AR_{\text{benchmark}}$) by the standard error of the difference following the method of DeLong et al. (1988), where p-values represent the probability that the two accuracy ratios are equal given the same sample data.

		Forward Intensity	Logit Model	Probit Model	Altman's Z-score
1 year	Accuracy Ratio	0.5569	0.5435	0.5396	0.3051
	(95% C.I.)	(0.5501, 0.5638)	(0.5364, 0.5506)	(0.5326, 0.5466)	(0.2970, 0.3132)
	Z-statistics	–	19.677	24.487	57.147
	(p-value)		(<0.0001)	(<0.0001)	(<0.0001)
2 years	Accuracy Ratio	0.5465	0.5139	0.5128	0.2877
	(95% C.I.)	(0.5409, 0.5520)	(0.5083, 0.5195)	(0.5072, 0.5184)	(0.2814, 0.2939)
	Z-statistics	–	22.782	23.256	75.774
	(p-value)		(<0.0001)	(<0.0001)	(<0.0001)
3 years	Accuracy Ratio	0.5359	0.4698	0.4665	0.2638
	(95% C.I.)	(0.5310, 0.5407)	(0.4649, 0.4748)	(0.4616, 0.4713)	(0.2563, 0.2693)
	Z-statistics	–	41.502	43.399	89.700
	(p-value)		(<0.0001)	(<0.0001)	(<0.0001)

intensity model and other alternative models for one-year (left panel) and three-year (right panel) prediction horizons for the full sample, where the fitted logit and probit models share the same risk factors with the forward intensity model.[25] Note that the forward-intensity model differs from Altman (2013)'s Z-score model both in the statistical method and the set of explanatory variables. In comparison to the binary response models, the forward-intensity model only differ in the econometric method not the explanatory variables.

For the one-year ahead prediction, we can see that the fitted forward intensity model with an accuracy ratio of 0.5569 outperforms the alternatives models: the re-estimated Altman (2013)'s Z-score model for private firms has an accuracy ratio of 0.3051, and the two binary response models (logit and probit regressions) with the same set of explanatory variables as in the forward-intensity model exhibit accuracy ratios of 0.5435 and 0.5396, respectively. Applying the formal testing methodology proposed by DeLong et al. (1988), we find that the differences in the accuracy ratios implied by the fitted forward intensity model and each of benchmark models are statistically significant at the standard confidence level.[26]

[25] When we estimate the binary response models, we deal with other exits as non-default cases.

[26] When the two accuracy ratios are estimated based on tests performed on the same data, statistical analysis on their differences should take into account the *positively correlated* nature of the samples. In

Furthermore, the fitted forward intensity model still maintains its superiority over the alternative models for longer horizons. For three-year ahead prediction, the fitted forward intensity model achieves an accuracy ratio of 0.5359, while the prediction accuracy ratios for the binary response models (logit model: 0.4698, probit model: 0.4665) significantly deteriorate with the same explanatory variables. Table 3 summarizes the out-of-sample accuracy ratios of the fitted forward intensity model and the alternative models for different prediction horizons. The result shows that the prediction power of the fitted forward intensity model does not deteriorate for longer horizons relative to other modeling approaches.

We evaluate the effectiveness of using the DTD-based approach by comparing the out-of-sample accuracy ratios between the fitted forward intensity model using the DTD-based approach (With DTD) and the alternative forward intensity model (Without DTD) by incorporating all variables that we use in the first and second stages.[27]

Figure 4 shows the one-standard-deviation error bands (boxes) and the 95 percent confidence intervals (whiskers) of the out-of-sample accuracy ratios based on the two fitted forward intensity models for one-year (left panel) and three-year (right panel) prediction ahead. As shown, the public-firm equivalent DTD variables are significantly helpful to predict private firms' default for short-term forecasting horizon. In other words, the alternative one-step estimation cannot outperform our proposed two-step approach, even if the deviation becomes less pronounced as we increase the forecasting horizon.[28] Figure 5 shows the time-series behavior of the estimated median default probabilities in our sample with risk horizons ranging from 1 month to 36 months. We can observe the economic vulnerability caused by the global financial crisis of 2008-2009.

Figure 6 illustrates the contribution of each firm-specific attribute to the out-of-sample prediction power for the fitted forward intensity model. Specifically, we compare two out-of-sample accuracy ratios of the original forward intensity model (full model) and

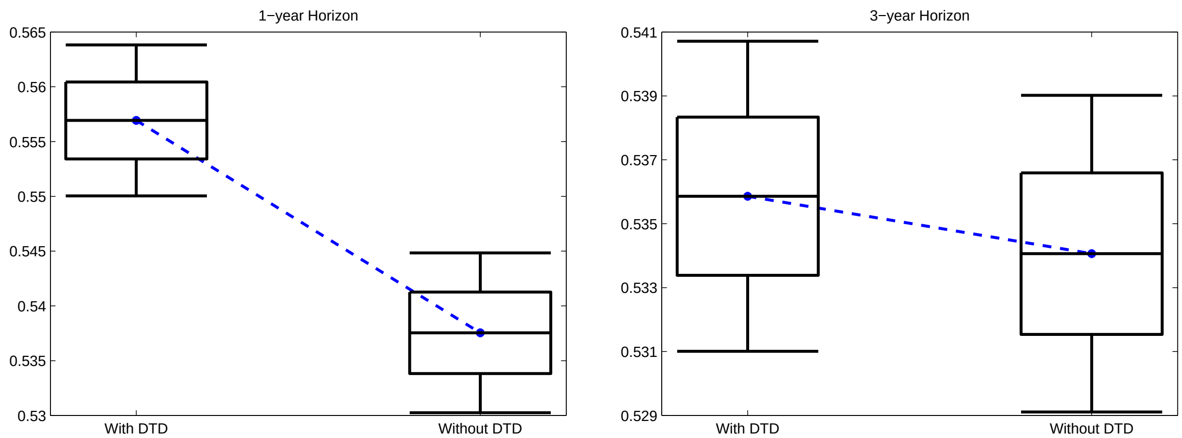
Table 3, the Z-statistics are calculated by dividing the difference between two *correlated* accuracy ratios by the standard error of the difference following the method of DeLong et al. (1988), where p-values represent the probability that the two accuracy ratios are equal given the same sample data after taking possible correlation into account.

[27]Note that there exists a discrepancy in the determinants of DTD between financial and non-financial firms. We incorporate a union set of these determinants in the one-step estimation approach, where the 'Interest Expense / Operating Income' variable, which is available for non-financial firms only, is replaced by the 'Non-operating Expense / Operating Income' for financial firms. In this sense, the comparison test takes a conservative perspective by penalizing the two-step approach, as the alternative one-step approach utilizes a larger firm-specific information set.

[28]According to DeLong et al. (1988), the standardized Z-statistics of the difference in the accuracy ratios are 11.6370 (p-value < 0.0001) for 1-year horizon and 1.5771 (p-value $= 0.1148$) for 3-year prediction ahead, respectively.

Figure 4: The effectiveness of using the DTD-based approach

This figure compares the out-of-sample accuracy ratios between the fitted forward intensity model using the DTD-based approach (With DTD) and the alternative forward intensity model (Without DTD) by incorporating all variables that we use in the first and second stages. The box plots indicate the one-standard-deviation error bands (boxes) and the 95 percent confidence intervals (whiskers) of the out-of-sample accuracy ratios based on the fitted forward intensity models for one-year (left panel) and three-year (right panel) prediction ahead.



a benchmark model, both evaluated at their respective maximum likelihood estimators, where the alternative specification does not include each of the selected firm-specific attribute. Then, we obtain the contribution ratio by dividing the difference between the two accuracy ratios by the accuracy ratio of the full model. It is remarkable that none of other firm-specific attributes contribute to the out-of-sample forecasting power of the forward intensity model above and beyond the public-firm equivalent DTD across different prediction horizons.

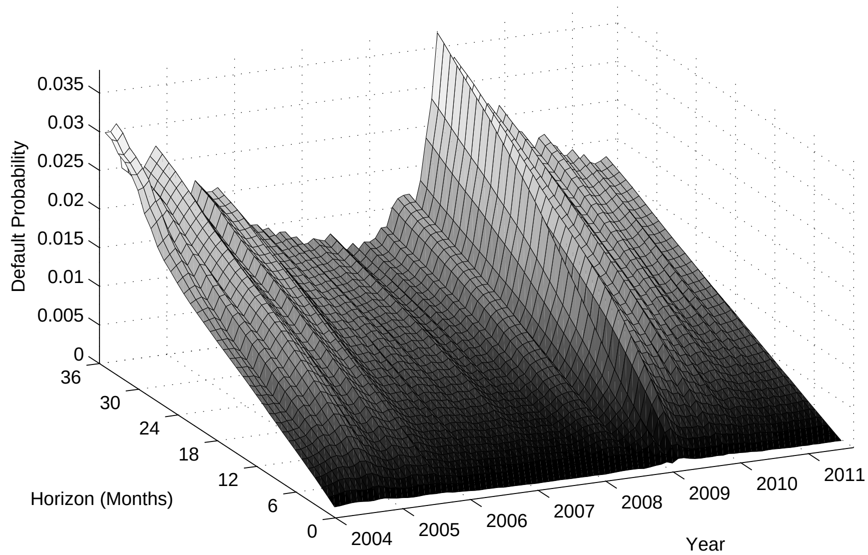
Overall, considering the lack of available data for private firms, the prediction power of the forward intensity model is impressive, not to mention its ability to perform dynamic estimation over multiple future periods.

4.4. Relationship between interest charge and default risk

Having estimated the term structure of default probabilities for our sample of private firms based on the forward intensity model, we investigate whether the reported interest rates of our sample firms actually reflect the credit risk captured by the estimated default term structure. Specifically, we regress the risk premium, defined as the differencial between the interest rate on outstanding debts for each firm-year and the risk-free rate, on the fitted default probability, controlling for other potential factors.

Figure 5: Fitted term structures of median default probabilities

This figure shows the time-series behavior of the estimated median default probabilities in our sample with risk horizons ranging from 1 month to 36 months for conditioning times varying monthly between January 2004 and June 2011.

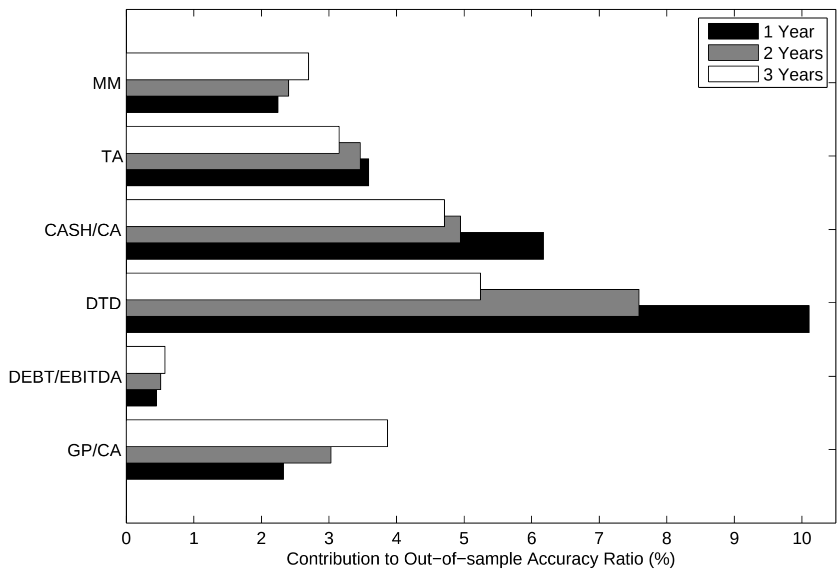


For this analysis, interest rate is defined as the weighted average of interest rates. The risk-free interest rate is estimated for each maturity τ using the standard cubic spline interpolation from the yields of 91-day certificate of deposit and Korean Governments Bonds for multiple maturities of 1, 3 and 5 years, obtained from the Economics Statistics System (ECOS) of the Bank of Korea. Term spread is defined as the yield differential between the 5-year Korean Government Bonds and the 91-day certificate of deposit. We define credit spread as the difference between the three-year yield on the corporate bond issued by AA- rated Korean firms from ECOS and the government bond yield of the same maturity. Default probability is calculated from the fitted forward intensities by applying equation (6) for a particular maturity τ. We consider its annualized value, $\frac{1}{\tau}p_t^i(\tau)$, making it directly compatible with the annualized interest rate. To consider other factors that may be relevant to the value-weighted interest rate across firms and over time, we include collateral-to-debt ratio, term spread, credit spread, industry dummies, and several firm characteristics that significantly affect interest rates.[29] In the following analysis we exclude

[29]As the selected set of firm-specific control variables, we employ the ratio of cash over total assets, the ratio of net income over total assets, the ratio of retained earnings over total assets, current assets less current liabilities then divided by total assets, the ratio of sales over total assets, earnings before interest, taxes, depreciation and amortization divided by total assets, the ratio of gross profit over current assets, total asset sizes, and the maturity mismatch ratio defined as current liability minus cash then divided by total liabilities. We exclude the estimated firm-level DTD from this analysis because of multicollinearity

Figure 6: Contribution of Firm-specific Attributes to Out-of-sample Accuracy Ratios

This figure shows the contribution ratio of each firm-specific attribute to the out-of-sample prediction power of the fitted forward intensity model. Contribution ratio is measured as the ratio of the difference between two accuracy ratios of the original forward intensity model (full model) and an alternative specification that does not include the selected firm-specific attribute, both evaluated at their respective maximum likelihood estimators, over the accuracy ratio of the full model.



all firm-years where the value-weighted interest rate is smaller than the risk-free interest rate of the same maturity.[30] Furthermore, all variables (except for dummy variables) are winsorized at the first and 99th percentiles to ensure that the results are not unduly influenced by outliers.

Table 4 provides summary statistics for all the variables after the winsorization (Panel A) and the results of regressing risk premium on the explanatory variables (Panel B). The numbers in Panel A of Table 4 indicate that our sample private firms pay roughly 2.37% per annum on their debt instruments over the risk-free rate. Annualized default probabilities are somewhere around 1.07% on average. The average collateral-to-debt ratio is roughly 1.12, while the average estimated maturity is 1.78 years. Average term spread is approximately 0.97% point, while the credit spread is around 1.1% point on

with the fitted default probability and other selected firm-specific attributes. We have also considered other variables, including the regional dummies and the year dummies, but regional dummies do not show any impact on interest rates while year dummies have confounding effects with the macro level interest rates.

[30]These are likely to reflect government subsidized loans to small and medium size firms. The funds are originally provided by government budget, while intermediaries bear all credit risk. Interest rates for these loans are pre-determined by government policy.

average. The negative minimum term spread suggests that there is sometimes a reversal in yield curve in the Korean debt market. Panel B of Table 4 summarizes the regression results where the risk premium, which is the difference between the value-weighted interest rate and the risk-free rate, is the dependent variable and the independent variables are the fitted default probability, collateral-to-debt ratio, estimated time to maturity, term spread, credit spread, industry dummy variables, and the selected firm-specific attributes. To check robustness, we also run similar regressions with firm fixed effects (after excluding industry dummies and firm-specific attributes) to eliminate time-invariant unobserved heterogeneity across private firms.

The results clearly indicate that the fitted default probability is a strong predictor of the risk premium to be charged across different regression specifications. The estimates are not only statistically significant, but also economically substantial. For example, the fitted full models (both 4 and 5) suggest that a one standard deviation increase in default probability leads to 0.24 to 0.31% point increase in risk premium for our sample private firms after controlling for other variables. These numbers suggest that our forward intensity model is a useful tool in determining appropriate interest rates charged to private firms in Korea, and that creditors indeed factor in default probabilities in setting interest charges.

The shape of interest-rate term structure inferred from the relationship between the interest rate and the maturity variable seems non-standard. In a standard context, both default-free interest rates and the yields of high-rated corporate bonds tend to go up as the time to maturity increases, a pattern commonly known as the normal yield curve. In contrast, our results based on the universe of Korean private firms show a highly nonlinear shape of term structure. The estimated term structure shows a significantly downward sloping term structure for shorter maturities, but the slope becomes positive for longer than 3.58 to 5.18 years of remaining time to maturity. Our seemingly abnormal finding suggests that there exists significant roll-over risk premium in the commercial lending market for private firms, as a firm is exposed by short-term debt to roll-over risk of not being able to settle its maturing debt. If a firm is subject to a substantial roll-over risk for its shorter value-weighted maturity, the lender may charge a higher interest rate at the time of additional loan origination.[31] However, such an effect tends to disappear as

[31] This observation is consistent with the finding of Gopalan, Song & Yerramilli (2014) in that bond market investors require premia for taking roll-over risk arising from a firm's debt maturity structure, and the effects are even stronger among firms with a speculative grade rating. Moreover, poorly-rated firms are usually eligible only for short-term high-yield loans, whereas their highly-rated counterparts are more likely to be granted longer-term borrowings and allowed to pay lower interest charges.

the firm's quality becomes sufficiently high for long-term borrowings.[32]

As shown, the collateral-to-debt ratio is negatively associated with the risk premium across different model specifications. The rationale behind this relationship is that commercial lenders tend to lower interest charges when the debt is secured by collaterals to guarantee a higher recovery rate at default, though the estimated coefficient loses its significance when firm fixed effects are included in the specification. Ceteris paribus, we find a significantly positive relationship between risk premium and term spread. This result is consistent with univariate reasoning, as the term spread measures excess premium that investors require to commit to holding a long-term bond instead of a series of otherwise identical shorter-term bonds. As expected, risk premium significantly increases with market-wide credit risk premium. An increase in credit spread implies that private firms with higher default probabilities are willing to borrow with a higher interest rate, reflecting a higher market-wide credit risk premium. Overall, the results suggest that default probabilities obtained through the forward intensity model is a significant statistic that explains the observed risk premium charged to private firms in excess of the risk-free rate.

4.5. Economic benefit of the increased accuracy

We further investigate the economic benefit of adopting our proposed forward intensity approach over the selected benchmarks.[33] Motivated by Stein & Jordão (2003) and Stein (2005), we suppose that banks set a level of lending cut-off ($= x$) in order to minimize the cost function (C_S) given by

$$C_S(x) = p(D)\left[c(FN)[1 - CAP(x)] - b(TP)CAP(x)\right] + [1 - p(D)]\left[c(FP)x - b(TN)(1 - x)\right], \quad (9)$$

where $p(D)$ is unconditional probability distribution of defaulters in the population of our sample, $b(\cdot)$ is the benefit of a correct prediction (True Positive or True Negative), $c(\cdot)$ is the cost of a specific type of error (False Positive or False Negative), and $CAP(x)$ is the value of cumulative accuracy profile associated with the respective risk score of $x \in (0, 1)$. The first order condition by differentiating $C_S(x)$ with respect to x gives the slope of a line with marginal cost equal to zero, or equivalently, the *iso-performance* line with slope S given by

$$S = \frac{1 - p(D)}{p(D)} \cdot \frac{c(FP) + b(TN)}{c(FN) + b(TP)} \quad (10)$$

[32] Similarly, Diamond (1991) also reports a non-linear relationship between debt maturity structure for borrowers and their future credit ratings. Specifically, borrowers with high and low credit ratings are more likely to issue short-term debt as compared to firms in the middle of the credit quality spectrum.

[33] We thank an anonymous referee for raising the issue and pointing out the direction of the analysis.

Table 4: Regression of the interest rates charged to private firms on default probabilities

Panel A reports summary statistics for the sample used in the regressions and Panel B reports the results of interest rate regressions. The risk premium is the difference between the interest rate and the risk-free rate. The interest rate is the weighted average of interest rates on outstanding debts for each firm-year. Risk-free rate is estimated for each maturity using the cubic spline interpolation from the yields of 91-day certificate of deposit and Korean Governments Bonds for multiple maturities of 1, 3 and 5 years. Default probability is the annualized value of equation (6) obtained from the fitted forward intensities. Maturity is the value-weighted average of the associated maturities for each debt class, where the weights are the relative proportions of each debt class of the estimated maturity for that firm year. Maturity^2 is the quadratic term of the Maturity. Collateral-to-debt ratio is the sum of maximum credit amounts of the collaterals scaled by the sum of long-term borrowing, short-term borrowing, current portion of long-term debt and corporate bonds. Term spread is the yield differential between the 5-year Korean Government Bonds and the 91-day certificate of deposit. Credit spread is the difference between the three-year yield on the corporate bond issued by AA- rated Korean firms and the government bond yield of the same maturity. Firm characteristics are the ratio of cash over total assets, the ratio of net income over total assets, the ratio of retained earnings over total assets, current assets less current liabilities then divided by total assets, the ratio of sales over total assets, earnings before interest, taxes, depreciation and amortization divided by total assets, the ratio of gross profit over current assets, total assets adjusted by GDP inflator, and current liability minus cash then divided by total liabilities. Firm fixed effects denote whether firm fixed effects are included in the specification. A constant is included in each specification but not reported in the table. The t-statistics are presented in parentheses and are computed using heteroscedasticity-robust standard errors, clustered by both firm and year. (*** significant at 1% level, ** significant at 5% level, * significant at 10% level)

Panel A: Summary Statistics

	Mean	Std. Dev	Min	Median	Max
Risk premium	0.0237	0.0161	0.0006	0.0206	0.0743
Default probability	0.0107	0.0088	0.0001	0.0087	0.0459
Maturity	1.7835	1.2253	0.5063	1.4324	5.7000
Collateral-to-debt ratio	1.1197	0.9480	0.0153	0.9577	6.7928
Term spread	0.0097	0.0077	-0.0012	0.0078	0.0204
Credit spread	0.0109	0.0073	0.0037	0.0098	0.0245

Panel B: Regressions

	Dependent variable: Risk premium				
	Model 1	Model 2	Model 3	Model 4	Model 5
Default probability	0.9722***		0.9548***	0.3530***	0.2725***
	(6.6763)		(6.6104)	(9.2051)	(5.4258)
Maturity		-0.0063***	-0.0057***	-0.0064***	-0.0051***
		(-6.3904)	(-5.2321)	(-5.9007)	(-6.6772)
Maturity^2		0.0008***	0.0008***	0.0008***	0.0005***
		(6.0820)	(4.2467)	(8.0002)	(6.8202)
Collateral-to-debt ratio		-0.0007***	-0.0004***	-0.0006***	-0.0001
		(-3.4804)	(-2.7330)	(-3.2849)	(-0.9051)
Term spread				0.4732***	0.5633***
				(5.1991)	(4.0258)
Credit spread				0.4958***	0.5345**
				(4.2237)	(3.3313)
Firm characteristics	Yes	Yes	Yes	Yes	No
Industry dummy	Yes	Yes	Yes	Yes	No
Firm fixed effects	No	No	No	No	Yes
Number of obs.	28,717	28,717	28,717	28,717	28,717
Adjusted R^2	0.3127	0.1999	0.3261	0.4511	0.7282

Table 5: Assumptions for baseline underwriting costs and profits

This table provides the assumed costs and profits for underwriting to a typical client of a bank as a baseline case scenario. The first row shows the assumed baseline probability of default in the population. The second and third rows indicate the fees and revenue the bank will generate by making a loan, respectively. The fourth and fifth rows represent the costs associated with a default. The sixth row is the median value of the yields of 1-year Korean Governments Bond in the sample period. The last row shows the average loan amount to small and medium business companies per bank as of December 2015. All costs and profits are quoted as percentages of a dollar loaned.

Variable	Value	Source
p(D)	5.88%	(# of default observation)/(# of firms in the sample)
Interest spread (per annum)	2.06%	Median of (interest rate − risk-free rate) in the sample
Underwriting fees (up front)	0.50%	Refer to Table 1 of Stein (2005)
Workout fees (on default)	2.00%	Refer to Table 1 of Stein (2005)
Loss given default	35.0%	A report by Korea Institute of Finance (2007)
Risk-free rate (per annum)	4.15%	Median of the yields of 1-year Korean Governments Bond
b(TN)	2.56%	Interest spread + Underwriting fees
c(FN)	35.03%	(Workout fees + Loss given default) / (1 + Risk-free rate)
b(TP) = c(FP)	0.00%	Refer to Appendix A of Stein (2005)
Annual new loan origination to SMEs	$200.0B	Estimated by the Financial Services Commission (2013-14)

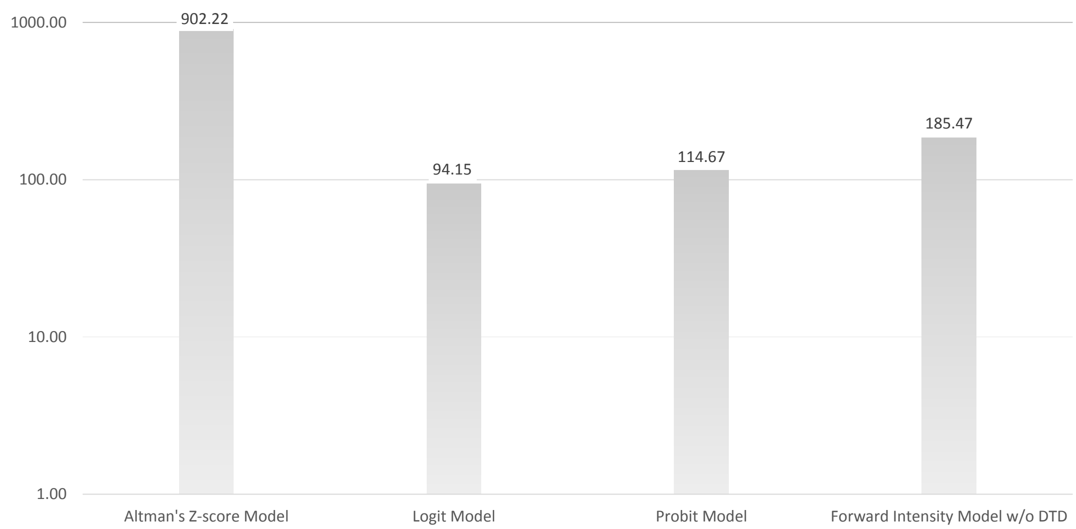
forms a tangent to the CAP curve at the optimal lending cut-off x^*.

Guided by the baseline case scenario in Appendix A of Stein (2005), we assume the baseline underwriting costs and profits as reported in Table 5 to calculate the slope of iso-performance line (10). We further assume for tractability that the lending banks grant loans that mature in one year, making one payment at the maturity date and that defaulting firms default at maturity without paying accrued interest.

Figure 7 shows the estimated economic benefit of using the forward intensity model over alternative models. As shown, the economic benefit of adopting the forward intensity model with the public-firm equivalent DTD approach is significant, as it would generate additional profits on the order of about $902.22 million per year over the Altman's Z-score model, an easy-to-calculate assessment of a private firm's financial distress as a benchmark. Notice that the economic benefit is about 67.95% of the total expected annual profit of lending based on the Altman's Z-score model. The economic benefit of adopting the proposed approach over the Logit model amounts to $94.15 million per year (which is about 4.41% of the Logit-based expected annual profit), $114.67 million per year over the Probit model (about 5.42% of the Probit-based expected annual profit). The economic benefit of adopting the forward intensity model with the public-firm equivalent DTD approach amounts to $185.47 million per year over the alternative forward intensity model without DTD by incorporating all variables that we use in the first and second stages; this extra benefit is approximately 9.07% of the total expected annual profit from adopting the forward intensity model without the projected DTD variable. This empirical

Figure 7: Economic benefit of adopting forward intensity model over alternatives

This figure illustrates the estimated annual economic benefit of adopting the forward intensity model over alternative models based on the assumptions in Table 5. The numbers are quoted in million US dollars.



finding implies that utilizing forward intensity to model credit behaviors can meaningfully contribute to credit risk management for privately held firms in Korea as well as for their creditors.

5. Conclusion

This paper proposes a methodology for estimating default term structure for private firms. To the best of our knowledge, this is the first study to investigate the dynamic behavior of default risk for private firms over different future horizons. Our analysis is feasible due to the unique Korean regulatory environment which requires even private firms to file their audited financial statements once they exceed a certain size threshold. From the commercial lenders' perspective, the proposed framework can be readily applied in practice to help make credit decisions related to privately held firms.

We adopt a forward-intensity model to characterize multiperiod default likelihoods using two macro risk factors, six firm-specific attributes, and one dummy variable to distinguish financial from non-financial firms. The forward-intensity model is calibrated via maximizing an overlapped pseudo-likelihood. Our out-of-sample test results indicate that the prediction power of the fitted forward-intensity model is superior to other alternative models considered, especially for longer prediction horizons.

This study provides a better understanding of the lending practice in granting private firm loans, which constitutes a vital segment of the economy. With the default term structure in place, we are able to examine whether the interest rates charged to private firms are positively related to their default risks. The results are consistent with the notion that default risk is priced in credit contracts and gets manifested in higher interest rates. We confirm that the economic benefit of adopting our proposed approach over the use of alternatives is substantial. Our findings suggest that mandating public disclosure of certain financial information, even for private firms, may facilitate better information flow between lenders and creditors which could ultimately lead to a more efficient pricing of credit instruments. An interesting direction for future research would be to explicitly incorporate possible frailty – unobservable explanatory variables that may be correlated across firms – and how this may affect the computation and accuracy of private firm default estimation.

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A. Overlapped pseudo-likelihood function

This appendix provides an outline of our model calibration scheme based on forward intensities. Sample period is from 0 to T over monthly time grid. Using the notations in Section 2, the pseudo-likelihood function for prediction horizon τ measured in months with each equal to Δt (= 1 month) can be expressed under the doubly stochastic assumption (also known as conditional independence assumption) as

$$\mathcal{L}_\tau\left(\alpha, \beta; \tau_C, \tau_D, X\right) = \prod_{i=1}^{N} \prod_{k=0}^{\lceil T/\Delta t \rceil - 1} \mathcal{L}^i_{\tau, k\Delta t}(\alpha, \beta; \tau^i_C, \tau^i_D, X^i), \tag{11}$$

where $\alpha = \{\alpha(0), \ldots, \alpha(\tau-1)\}$, $\beta = \{\beta(0), \ldots, \beta(\tau-1)\}$, $\tau_C = \left\{\tau_C^1, \ldots, \tau_C^N\right\}$, $\tau_D = \left\{\tau_D^1, \ldots, \tau_D^N\right\}$, $X = \left\{X^1, \ldots, X^N\right\}$, and

$$\begin{aligned}
\mathcal{L}_{\tau,t}^i(\alpha, \beta; \tau_C^i, \tau_D^i, X^i) &= 1_{\left\{t_0^i \le t,\ \tau_C^i > t+\tau\right\}} \mathcal{L}_t\left(\tau_C^i; \tau_C^i > t+\tau\right) \\
&+ 1_{\left\{t_0^i \le t < \tau_C^i \le t+\tau,\ \tau_D^i = \tau_C^i\right\}} \mathcal{L}_t\left(\tau_C^i; \tau_C^i \le t+\tau,\ \tau_D^i = \tau_C^i\right) \\
&+ 1_{\left\{t_0^i \le t < \tau_C^i \le t+\tau,\ \tau_D^i \neq \tau_C^i\right\}} \mathcal{L}_t\left(\tau_C^i; \tau_C^i \le t+\tau,\ \tau_D^i \neq \tau_C^i\right) \\
&+ 1_{\left\{t_0^i > t\right\}} + 1_{\left\{\tau_C^i \le t\right\}}, \qquad (12)
\end{aligned}$$

where t_0^i is the first month that it appeared in the data set, and the parameters α and β characterize the forward default intensity $f_t^i(\tau)$ and forward other exit intensity $g_t^i(\tau) - f_t^i(\tau)$, respectively. The first term on the right-hand side of (12) refers to the likelihood of surviving both forms of exit. The second term represents the likelihood that the i-th firm defaults at a particular time point. The third term is the likelihood that the firm exits due to other reasons. If the firm does not appear in the sample in month t as shown in the last two terms, we set the pseudo-likelihood to 1, which is transformed to 0 in the logarithm.

The pseudo-likelihood function in (11) can be expressed as the product of separate terms involving α and β. Thus, we can maximize its two components separately to obtain the maximum pseudo-likelihood estimates.[34] In addition, the pseudo-likelihood function for α or β can be further decomposed into separate terms involving $\alpha(\tau)$ or $\beta(\tau)$ across different τ's. Hence, we can obtain their maximum pseudo-likelihood estimates without having to perform estimation sequentially from shorter to longer prediction horizons. The horizon-specific pseudo-likelihood functions are given by

$$\mathcal{L}_s^\alpha(\alpha; \tau_C, \tau_D, X) = \prod_{i=1}^{N} \prod_{k=0}^{\lceil (T-s)/\Delta t \rceil - 1} \mathcal{L}_{s,k\Delta t}^{\alpha,i}(\alpha(s); \tau_C^i, \tau_D^i, X^i) \qquad (13)$$

$$\mathcal{L}_s^\beta(\beta; \tau_C, \tau_D, X) = \prod_{i=1}^{N} \prod_{k=0}^{\lceil (T-s)/\Delta t \rceil - 1} \mathcal{L}_{s,k\Delta t}^{\beta,i}(\beta(s); \tau_C^i, \tau_D^i, X^i) \qquad (14)$$

for $s = 0, \Delta t, 2\Delta t, \cdots, \tau - \Delta t$ as intended prediction horizons measured in months. This leads to the following expressions in the form of

[34] Refer to Proposition 2 of Duffie et al. (2007) for similar arguments.

$$
\begin{aligned}
\mathcal{L}^{\alpha,i}_{s,t}(\alpha(s);\tau^i_C,\tau^i_D,X^i) &= 1_{\left\{t^i_0\le t,\ \tau^i_C>t+\Delta t+s\right\}}\exp\left[-f^i_t(s)\Delta t\right] \\
&+ 1_{\left\{t^i_0\le t,\ \tau^i_D=\tau^i_C=t+\Delta t+s\right\}}\left(1-\exp\left[-f^i_t(s)\Delta t\right]\right) \\
&+ 1_{\left\{t^i_0\le t,\ \tau^i_D\ne\tau^i_C=t+\Delta t+s\right\}}\exp\left[-f^i_t(s)\Delta t\right] \\
&+ 1_{\left\{t^i_0>t\right\}}+1_{\left\{\tau^i_C<t+\Delta t+s\right\}}, \qquad (15) \\
\mathcal{L}^{\beta,i}_{s,t}(\beta(s);\tau^i_C,\tau^i_D,X^i) &= 1_{\left\{t^i_0\le t,\ \tau^i_C>t+\Delta t+s\right\}}\exp\left[-h^i_{k\Delta t}(s)\Delta t\right] \\
&+ 1_{\left\{t^i_0\le t,\ \tau^i_D=\tau^i_C=t+\Delta t+s\right\}} \\
&+ 1_{\left\{t^i_0\le t,\ \tau^i_D\ne\tau^i_C=t+\Delta t+s\right\}}\left(1-\exp\left[-h^i_{k\Delta t}(s)\Delta t\right]\right) \\
&+ 1_{\left\{t^i_0>t\right\}}+1_{\left\{\tau^i_C<t+\Delta t+s\right\}}, \qquad (16)
\end{aligned}
$$

where $h^i_t(\tau)=g^i_t(\tau)-f^i_t(\tau)$ and $s=0,\Delta t,2\Delta t,\cdots,\tau-\Delta t$.

Note that the pseudo-likelihood function is constructed with observations from overlapped periods when the prediction horizon τ is longer than Δt. Because of the overlapping nature of the pseudo-likelihood function, the associated inference violates the standard assumption. In this regard, Duan et al. (2012) characterize and derive the large sample properties of the estimator based on maximizing the pseudo-likelihood function; see Appendix A therein. Under mild regularity conditions, they prove its asymptotic consistency by showing that the difference between the maximum pseudo-likelihood estimator and the true data-generating parameter converges weakly to a vector whos distribution is joint normal with mean zero.[35]

[35] A more detailed proof of consistency is available upon request.