Long Run Risks in FX Markets: Are They There?*

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ABSTRACT

We uncover a tight relation between long run consumption risks (LRRs), currency risk premia and global currency risk factors. Countries that suffer a bad relative LRR shock experience a decline in their currency risk premium: their currencies appreciate initially before subsequently depreciating to deliver lower expected returns. Furthermore the High-Minus-Low (HML) carry trade sorted on interest rate differentials and the HML dollar beta portfolio sorted on conditional dollar exposures are highly correlated with global and US LRRs respectively. Finally US LRRs are a unique source of global risk driving the global exchange rate factor structure, a novel insight that has received surprisingly little emphasis thus far. An international LRR model where US and global LRRs constitute two distinct sources of global risk quantitatively accounts for these empirical findings.

Keywords: International Finance, Asset Pricing, Long Run Risks, Currency Risk Premia

JEL Codes: E0, F3, F4, G1

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1 Introduction

Over the last two decades one of the major paradigms for thinking about exchange rate determination has been the long run risk (LRR) framework. In a multi-country, multi-good framework with i) Epstein and Zin (1989) (EZ) preferences, ii) frictionless markets, iii) consumption home bias and iv) highly correlated growth prospects, a tight connection exists between relative expected future consumption growths (LRRs hereafter), currency excess returns, currency risk premia and global currency risk factors. In such models, a relative LRR shock to country i vis-*á*-vis the US lowers country i's currency risk premium, inducing: i) an immediate appreciation of currency i against the dollar, followed by ii) a subsequent long-run depreciation moving forward (Colacito et al, 2018).[1]

This recursive interpretation of currency dynamics has proven very successful as a unified framework for thinking about canonical international finance puzzles. Firstly, it can reproduce i) a sizeable equity premium in each country and ii) smooth bilateral real exchange rates that are negatively correlated with relative consumption growths, simultaneously overcoming the equity premium, FX volatility and Backus-Smith puzzles (Colacito and Croce, 2011, 2013). Secondly traditional global currency risk factors are rationalised as compensation for bearing LRRs. For example the profitability of the High-Minus-Low (HML) currency carry trade sorted on interest rate differentials is tied to risk compensation for global LRRs (Colacito et al, 2018).

Yet despite the theoretical success of this framework, a large chasm still exists between theory and empirics. We simply do not know if the key predictions of the international LRR framework for currency dynamics are consistent with exchange rate data. Do the currencies of countries that suffer bad relative LRR shocks actually i) appreciate on impact before ii) subsequently depreciating? Is the profitability of the HML carry trade truly linked to an empirically constructed global LRR factor?

The answers to these questions are not well understood because there is a dearth of empirical literature studying the interaction between exchange rates and LRRs. The reason is simple and well studied: LRRs are empirically difficult to identify using aggregate consumption data (Schorfheide,

[1] The key mechanism driving these currency dynamics is a recursive risk-sharing scheme for LRR shocks. For further details, we direct the reader to Colacito and Croce (2013) and Colacito et al (2018).

Song, and Yaron, 2016). The statistical power to detect any long run persistence in consumption growth is small and made even more challenging by traditional mismeasurement issues associated with the collection of aggregate consumption data (Slesnick, 1998).

To bridge this divide between the theory and empirics, we use the ICAPM-VAR framework of Campbell et al (2017) to estimate stock-market cash flow news. Using stock market cash flow news as a proxy for LRRs is appropriate given the high degree of portfolio home bias in international portfolios (French and Poterba, 1992), implying that local stock market cash flow news are intimately connected to the equilibrium path of future local consumption streams. Furthermore unlike other identification approaches considered in previous work, this approach is highly tractable because it only requires the use of asset market data, enabling us to estimate stock market cash flow news for a large panel of countries. This allows us to empirically characterise the joint dynamics between relative LRRs, currency excess returns, currency risk premia and global currency risk factors using a large panel dataset of bilateral exchange rates and currency portfolios.

With this identification strategy in hand we uncover four main results regarding these empirical joint dynamics that are broadly consistent with the international LRR models. Firstly currency excess returns and relative LRRs are negatively correlated: the currencies of countries that suffer bad relative LRR shocks vis-*á*-vis the US appreciate against the dollar on impact. Secondly currency risk premia and relative LRRs are positively correlated: over the long run such currencies depreciate against the dollar, delivering lower expected currency excess returns moving forward.

Thirdly, we find that profitable currency strategies such as the high minus low (HML) carry trade sorted on interest rate differentials and the HML dollar beta portfolio sorted on time varying dollar exposures (ROW) are tightly constructed with our empirically constructed global and US LRR factors respectively. These results empirically validate the intuition from international LRR models that the profitability of the HML carry trade constitutes compensation for bearing global LRRs because it goes short (long) the currencies of countries with high (low) exposures to global LRRs (Colacito et al, 2018).

Furthermore we find a tight connection between the HML dollar beta portfolio and our empirically constructed US LRR factor. These results are very revealing when considered in the light of recent literature. In an important recent contribution, Verdelhan (2018) shows that there are two currency portfolios at the heart of all systematic variation in bilateral exchange rates: i) the traditional HML carry trade and ii) the HML dollar beta portfolio. Thus our results linking the former to global LRRs and the latter to US LRRs have important theory implications: they suggest that these two global currency risk factors are simply proxies for US and global LRRs respectively. Hence they suggest that an international LRR model where two LRR factors - US and global- constitute two distinct sources of global risk in the world economy can explain systematic variation in bilateral exchange rates.

To formalise this argument, we conclude the paper by calibrating an international LRR model with these two sources of global risk. The model is closely related to Colacito et al (2018): there are N countries, N goods and financial and goods markets are internationally complete. Each country is endowed with a representative investor with EZ preferences and consumes a home biased index of all consumption goods. The point of departure in our model is that there are two sources of global risk that countries are heterogeneously exposed to: US and global LRRs.

The model qualitatively and quantitatively matches all our empirical findings. We interpret this as evidence supporting our argument that US LRRs are a distinct source of global risk pricing currency markets. Thus US growth prospects are an important economic driver behind the global exchange rate factor structure, a novel insight that hasn't received much emphasis thus far in the international macro-finance literature.

Related Literature: The first strand of literature that our paper is related to is the international asset pricing literature using recursive preferences. This literature applies an international extension of Bansal and Yaron (2004) to resolve classic international puzzles such as the FX volatility puzzle (Colacito and Croce, 2011; Bansal and Shaliastovich, 2013), Backus-Smith puzzle (Colacito and Croce, 2013) and the carry trade puzzle (Colacito et al, 2018). Crucial to these resolutions are highly correlated long run risks (LRRs).

The novelty of our work is to take these models to the data using technology from the intertemporal asset pricing literature that explores an ICAPM decomposition of the stochastic discount factor (SDF), using Epstein-Zin (EZ) preferences. The origins of this literature begin with the seminal contribution of Campbell (1993) who combines a first order log-linearization of the intertemporal budget constraint of an EZ representative investor around the deterministic steady state with the EZ euler equation to derive a discrete-time version of Merton's ICAPM. Campbell and Vuolteenaho (2004) use this framework to decompose the SDF into a cash flow and a discount rate component and use a highly tractable VAR framework to estimate both of these components. Campbell et al (2017) and Bansal, Kiku, Shaliastovich and Yaron (2017) extend these approaches to account for stochastic volatility.

In this paper we extend these methods to an international context, using the ICAPM-VAR framework to estimate LRRs as the cash flow news component of the SDF decomposition. This method is highly tractable because it only requires the use of asset market data alone since cash flow news are not measured directly but are backed out of discount rate news using the Campbell-Shiller decomposition. With this identification strategy in hand, we uncover novel evidence in favour of the LRR view of exchange rate determination and currency dynamics, a valuable contribution that can help address the asset pricing dark matter criticisms that are often levelled at this literature (Chen, Dou and Kogan, 2019).

Finally our paper is related to a growing literature tying global sources of risk to US specific state variables. Most related to our work is Boehm and Kroner (2020) who show that US growth prospects or US LRRs are important drivers of global sources of risk in equity markets. Jiang (2022) ties the HML dollar beta portfolio and global currency risk to the US fiscal condition. Finally Brusa et al (2020) and Mueller et al (2017) tie global equity and currency risk to US monetary policy.

2 Theory Framework

Here we present a general theoretical framework where two LRR factors- US and global- drive global sources of risk in the world economy. We use the framework to formalise testable predictions of the international LRR framework that we take to the data. They are formalised via propositions 1-4. The framework is remarkably general and encapsulates a broad class of multi-country, multi-good models that feature i) EZ preferences and ii) internationally frictionless financial markets. These include models that feature international trade (Colacito and Croce, 2013; Colacito et al, 2018) and those that abstract from it (Colacito and Croce, 2011; Andrews et al, 2021).

2.1 Overview

Environment: There are $N+1$ countries indexed by $i \in \{0, 1, 2, ..., N\}$. Country 0 is the base country: without loss of generality we set the US as the base country. Each country is home to a representative investor with Epstein and Zin (1991) (EZ) recursive preferences who solves the following intertemporal consumption and savings problem:

$$\max_{\{C_t^i, W_{t+1}^i\}_{t=0}^{\infty}} U_t^i = [(1-\delta)(C_t^i)^{1-\frac{1}{\psi}} + \delta(E_t {U_{t+1}^i}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$s.t.\ W_{t+1}^i = (W_t^i - C_t^i)(R_{m,t+1}^i) \tag{1}$$

δ: *Time Preference*

ψ: *Intertemporal Elasticity of Substitution (IES)*

γ: *Relative Risk Aversion*

Wealth: Country i's wealth portfolio W_t^i is defined as the present discounted value of a perpetual claim to country i's consumption stream: $\{C_t^i\}_{t=0}^{\infty}$.[2] It is priced by the local log SDF m_{t+1}^i through the standard euler equation:

$$\mathbb{E}_t[e^{m_{t+1}^i + r_{m,t+1}^i}] = 1 \tag{2}$$

SDF: As shown by Epstein and Zin (1992), m_{t+1}^i takes the form:

$$m_{t+1}^i = \theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1}^i - (1-\theta) r_{m,t+1}^i \tag{3}$$

Here $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $r_{m,t+1}^i$ is the log return on country i's aggregate wealth portfolio. The Campbell and Shiller (1988) approximation implies that:

$$r_{m,t+1}^i \approx \kappa_0 + \kappa_1 wc_{i,t+1} - wc_{i,t} + \Delta c_{t+1}^i \tag{4}$$

[2] We remain agnostic about the role of international trade: C_t^i can be interpreted as either a local consumption good (no trade) or a local consumption basket (trade). Specifying the nature of this consumption is not essential for our purposes.

κ_0, κ_1 are constant coefficients of log-linearisation. Thus m^i_{t+1} can be rewritten in terms of consumption growths Δc^i_t and the wealth-consumption ratio wc^i_t:

$$m^i_{t+1} = \underbrace{(\theta - 1 - \frac{\theta}{\psi})}_{-\gamma} \Delta c^i_{t+1} + (\theta - 1)\kappa_1 wc_{i,t+1} \tag{5}$$

2.2 Exchange Rates

Risk Sharing Condition: Financial markets are internationally complete. Therefore the real exchange rate $\mathcal{E}_{i,t}$, defined as country i consumption units per US consumption units, adjusts to enforce the equality of marginal utility growths between US and country i (Backus, Foresi and Telmer, 2001):

$$\Delta \mathcal{E}_{i,t+1} = m^{US}_{t+1} - m^i_{t+1} \ , \ \forall i \in \{1, 2, ..., N\} \tag{6}$$

$\mathcal{E}_{i,t}$: country i consumption units per US consumption units

m^i_t: Country i's real SDF in local consumption units

EZ: Substituting (5) into (6) implies:

$$\Delta \mathcal{E}_{i,t+1} \approx \gamma \underbrace{(\Delta c^i_{t+1} - \Delta c^{US}_{t+1})}_{\mathcal{C}_{t+1}} + \kappa_1 (1 - \theta) \underbrace{(wc^i_{t+1} - wc^{US}_{t+1})}_{\mathcal{W}_{t+1}} \tag{7}$$

κ_1: Log-linearization coefficient.[3]

γ: Coefficient of relative risk aversion

θ: Uncertainty resolution parameter.[4]

LRR: To link exchange rates to LRRs, we now substitute the wealth component $\mathcal{W}_{t+1}$ out of (7) in terms of long-run consumption news. To do this, we substitute the Campbell-Shiller (1988)

[3] κ_1 is associated with Campbell and Shiller (1988) approximation

[4] $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ where γ is risk aversion and ψ is intertemporal elasticity of substitution (IES)

approximation (4) into the Euler equation (2). This yields a recursive equation in wc_t^i:

$$wc_t^i = ln\delta + \kappa_0 + (1 - \frac{1}{\psi})\Delta c_{t+1}^i + \kappa_1 wc_{t+1}^i \tag{8}$$

Recursively solving this equation forward and imposing the standard transversality condition: $\lim_{s\to\infty} \kappa_1^s wc_{t+s}^i = 0$ implies the following expression for $\mathcal{W}_{t+1}$:

$$\mathcal{W}_{t+1} = (1 - \frac{1}{\psi})(E_{CF,t+1}^{US} - E_{CF,t+1}^i) \tag{9}$$

$E_{CF,t+1}^i$ represents country level future consumption growth expectations:

$$E_{CF,t+1}^i = \mathbb{E}_{t+1} \sum_{s=1}^{\infty} \rho^s \Delta c_{t+1+s}^i, \;\; i \in \{1, 2, 3, ..., N\} \tag{10}$$

Finally substituting (9) back into (7) ties exchange rate movements to short run and long run consumption risks:

$$\Delta\mathcal{E}_{i,t+1} \approx \gamma(\underbrace{\Delta c_{t+1}^i - \Delta c_{t+1}^{US}}_{\mathcal{C}_{t+1}}) + \kappa_1(\gamma - \frac{1}{\psi})\underbrace{(E_{CF,t+1}^i - E_{CF,t+1}^{US})}_{\mathcal{L}_{t+1}} \tag{11}$$

Discussions: (11) is the central equation that captures the LRR view of exchange rate determination. It states that in an EZ world, there are two drivers of bilateral exchange rates: relative consumption growths $\mathcal{C}_{t+1}$ and relative news about future consumption growths $\mathcal{L}_{t+1}$. The second component is the LRR component that is the focus of international LRR models. (11) implies that if country i's EZ agent exhibits a preference for early resolution of uncertainty, a bad relative LRR shock for country i ($\mathcal{L}_{t+1} \uparrow$) is associated with a real dollar depreciation against currency i.

Models: Standard international LRR models generally impose an affine structure on $E_{CF,t+1}^i$. For example Colacito and Croce (2011) assumes a two country extension of Bansal and Yaron

(2004):

$$\Delta c^i_{t+1} = \mu + x^i_t + \xi^c_{t+1}$$
$$x^i_t = \rho_x x^i_{t-1} + \xi^x_t \tag{12}$$

This implies that $E^i_{CF,t}$ is linear in a single state variable x_t:

$$E^i_{CF,t} = \frac{x^i_t}{1-\rho_x\delta} \tag{13}$$

Thus bilateral exchange rates in Colacito and Croce (2011) follow a simple two factor structure:

$$\Delta\mathcal{E}_{i,t+1} = \gamma(\Delta c^i_{t+1} - \Delta c^{US}_{t+1}) + \frac{\kappa_1}{1-\rho_x\delta}(\gamma - \frac{1}{\psi})(x^i_{t+1} - x^{US}_{t+1}) \tag{14}$$

2.3 Currency Excess Returns

Overview: A key implication of (11) is that in an EZ world, investing in foreign currency is tantamount to placing a contrarian bet on that country's future growth prospects. If the given foreign country suffers a bad relative LRR shock vis-*á*-vis the US, its currency appreciates against the dollar in real terms, all else being equal. Thus the log excess return rx^i_{t+1} associated with investing in foreign currency i should be decreasing in country i's relative LRR shock. To formalise this, we introduce the following lemma about currency excess returns:

Lemma 2.1. *(Currency Excess Returns). Let rx^i_{t+1} denote the realized log excess real return of going long currency i from the perspective of a USD investor. Then* (11) *implies that rx^i_{t+1} takes the following form up to a Jensen's inequality term J_t:*

$$rx^i_{t+1} - \frac{1}{2}J_t = \gamma(N^{US}_{C,t+1} - N^i_{C,t+1}) + \kappa_1(\gamma - \frac{1}{\psi})\underbrace{(N^{US}_{CF,t+1} - N^i_{CF,t+1})}_{LRR^i_t} \tag{15}$$

J_t takes the form:

$$J_t = var_t m^{US}_{t+1} - var_t m^i_{t+1}$$

Proof is contained in the online appendix. $N^i_{C,t+1}$ and $N^i_{CF,t+1}$ denotes a contemporaneous consumption growth shock and a LRR shock for country i realized at time $t+1$:

$$N^i_{C,t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t)\Delta c^i_{t+1}, \;\; i \in \{1, 2, 3, ..., N\}$$
$$N^i_{CF,t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{s=1}^{\infty} \rho^s \Delta c^i_{t+1+s}, \;\; i \in \{1, 2, 3, ..., N\}$$

(15) implies the following testable prediction about the link between currency excess returns and LRRs that emerges from the international LRR framework:

Proposition 1. *(Currency Excess Returns). For an EZ investor with a preference for early resolution of uncertainty ($\gamma > \frac{1}{\psi}$), the log excess return associated with going long in currency i against the dollar is* ***decreasing*** *in country i's relative LRR shock vis-á-vis the US (LRR^i_t).*

2.4 Currency Risk Premia

Overview: Now we move onto currency risk premia. A key implication from (15) is that the log currency risk premium associated with investing in currency i is proportional to the Jensen's inequality term J_t. In an EZ environment with LRRs this term is proportional to the relative variance of LRR shocks. We formalise this insight below:

Lemma 2.2. *(Currency Risk Premia) Let crp^i_t denote the ex-ante log currency risk premium a USD investor demands for going long currency i. Then (15) implies that crp^i_t takes the form:*

$$\begin{aligned} crp^i_t =& \mathbb{E}_t rx^i_{t+1} \\ &\propto J_t \\ &\approx \gamma^2 \mathcal{V}_{C,t} + \kappa_1^2(\gamma - \frac{1}{\psi})^2 \mathcal{V}_{CF,t} \end{aligned} \tag{16}$$

$\mathcal{V}_{C,t}$ and $\mathcal{V}_{CF,t}$ denote the relative variances of contemporaneous consumption shocks and LRR shocks respectively:

$$\mathcal{V}_{C,t} = var_t N^{US}_{C,t+1} - var_t N^{i}_{C,t+1}$$
$$\mathcal{V}_{CF,t} = var_t N^{US}_{CF,t+1} - var_t N^{i}_{CF,t+1} \tag{17}$$

Discussion: Lemma 2.2 implies that in an EZ world where agent's exhibit a preference for early resolution ($\gamma > \frac{1}{\psi}$), crp^i_t is *increasing* in $\mathcal{V}_{CF,t}$. In other words, the currency risk premium that an EZ investor demands for going long currency i against the dollar is ***increasing*** *in the variance differential between US LRR shocks and country i's LRR shocks.* Thus *the currencies of countries with **less** volatile future consumption profiles are **more** risky* to an EZ investor. This implies that the link between LRRs and currency risk premia is governed by how the variance of LRR shocks is affected by its level.

Currency Risk Premia and LRR Models: In the open economy LRR models of Colacito and Croce (2013) and Colacito et al (2018), the variance of country i's relative LRR shock ($var_t N^i_{CF,t+1}$) is decreasing in its level ($N^i_{CF,t+1}$). This model feature stems from the recursive risk sharing scheme for LRR shocks that emerges in equilibrium due to international trade.[5] Thus Lemma 2.2 implies that in the LRR models the currency risk premium associated with going long currency i declines in response to a bad LRR shock to country i. This prediction is summarized below:

Proposition 2. *(Currency Risk Premia). From a USD investor's perspective, crp^i_t is **increasing** in country i's relative LRRs vis-á-vis the US. This implies the following currency dynamics in response to a bad relative LRR shock to country i that lowers crp^i_t:*

- ***Short Run**: Currency i **appreciates** against the dollar on impact*
- ***Long Run**: Currency i subsequently **depreciates** against the dollar over the long run to deliver lower future returns for a USD investor that goes long currency i.*

[5]For further details on this equilibrium risk-sharing scheme, we direct the reader to Colacito and Croce (2013)

2.5 Global Currency Risk Factors

Overview: Having characterised the international LRR model's key testable predictions for bilateral currency excess returns and currency risk premia we now move towards characterising its key testable predictions for global currency risk factors. Recent work by Verdelhan (2018) suggests that there is a two factor structure driving the global exchange rate factor structure. These two factors are : i) the High-Minus-Low (HML) carry trade sorted on interest rate differentials and ii) the HML dollar beta portfolio sorted on time varying dollar exposures. In this section we show that in our framework these currency portfolios are compensation for bearing global and US LRRs respectively.

Global Shocks: To derive our predictions for these two global currency risk factors, we impose the following global factor structure for LRRs:

$$N^i_{CF,t+1} = \alpha^i_2 + \beta^i_{CF} N^G_{CF,t+1} + \beta^i_{US} N^{US}_{CF,t+1} + \epsilon^i_{2,t+1} \tag{18}$$

Shocks are gaussian: $\epsilon^i_{2,t+1} \sim i.i.d\ N(0,\, \tau^2)$. Country level LRRs loads on two sources of global risk: global LRRs: $N^G_{CF,t+1}$ and US LRRs: $N^{US}_{CF,t+1}$. β^i_{CF} and β^i_{US} capture country level loadings on these two global risk factors.

2.5.1 HML Carry Trade

Carry Factor: By construction, the HML carry trade sorts currencies into portfolios on the basis of interest rate differentials. Thus HML carry trade returns correspond to average exchange rate changes in the high vs low interest rate currency portfolios. Thus (18) implies that unexpected carry trade returns $\widetilde{HML}_{t+1} = HML_{t+1} - \mathbb{E}_t HML_{t+1}$ follow:

$$\begin{aligned}\widetilde{HML}_{t+1} =& \underbrace{\kappa_1(\frac{1}{\psi} - \gamma)}_{<0}[N^G_{CF,t+1}(\overline{\beta}^H_{CF} - \overline{\beta}^L_{CF}) + N^{US}_{CF,t+1}\underbrace{(\overline{\beta}^H_{US} - \overline{\beta}^K_{US})}_{\approx 0}] \\ \approx& \underbrace{\kappa_1(\frac{1}{\psi} - \gamma)}_{<0}\underbrace{(\overline{\beta}^H_{CF} - \overline{\beta}^L_{CF})}_{<0} N^G_{CF,t+1}\end{aligned} \tag{19}$$

$\overline{\beta}^H_{CF}, \overline{\beta}^H_{US}$ and $\overline{\beta}^L_{CF}, \overline{\beta}^L_{US}$ captures the average exposure of currencies in the high interest rate (low interest rate) portfolios to the two global LRR factors respectively. In the second line, we impose the standard restriction in international LRR models that on average high interest rate countries only differ in their exposures to the global LRR factor relative to low interest rate countries (Colacito et al, 2018). If high interest rate currencies are assumed to load less on global LRRs ($\overline{\beta}^H_{CF} - \overline{\beta}^L_{CF} < 0$), then we recover the prediction from Colacito et al (2018) that the HML carry trade loads positively on global LRRs: $N^G_{CF,t+1}$ and hence its profitability is tied to global LRR compensation. This is summarised in the proposition below:

Proposition 3. *(HML Carry Trade Returns). If global LRR betas are monotonically declining in interest rates ($\overline{\beta}^H_{CF} - \overline{\beta}^L_{CF} < 0$), the HML carry trade loads positively on global LRRs: it's returns* ***increase*** *in response to a positive global long-run shock: $N^{US}_{CF,t}$*

This proposition is the key prediction for the HML carry trade that emerges from the international LRR framework that we will take to the data. The intution behind this result is that the HML carry trade goes long the currencies of countries that are adversely exposed to the global LRR factor against the currencies of countries that have low exposures to this factor. To formalise this, note that (19) implies the following lemma about HML carry trade betas:

Lemma 2.3. *(HML Betas). Currency i's loading on the HML factor is: $\beta^i_{HML} = \frac{cov_t(rx^i_{t+1}, \widetilde{HML}_{t+1})}{var_t(HML_{t+1})}$ is:*

$$\beta^i_{HML} = \beta^i_{CF} \underbrace{(\overline{\beta}^H_{CF} - \overline{\beta}^F_{CF})}_{<0} \underbrace{\frac{\kappa_1^2 \tau^2 (\gamma - \frac{1}{\psi})^2}{(\overline{\beta}^H_{CF} - \overline{\beta}^F_{CF})^2 \tau^2}}_{>0} \tag{20}$$

Thus carry trade betas are decreasing in the global LRR beta β^i_{CF} and the HML carry trade longs (shorts) currencies of countries with low (high) exposure to global LRRs.

2.5.2 Dollar Beta Portfolio

Finally we characterise testable implications for the dollar beta portfolio, a strategy that sorts currencies into portfolios on the basis of their time varying exposures to the dollar factor. By construction, the dollar factor $Dollar_{t+1}$ shorts the dollar against the ROW and takes the following form in our framework:

Lemma 2.4. *(Dollar Excess Returns). Lemma 2.2 implies that the* $\widetilde{Dollar}_{t+1} = Dollar_{t+1} - \frac{1}{2}J_t^{US}$ *take the following form:*

$$\begin{aligned}
\widetilde{Dollar}_{t+1} =& \gamma\kappa_1(\gamma - \frac{1}{\psi})[\underbrace{(\beta_{CF}^{US} - \overline{\beta}_{CF})}_{\approx 0} N_{CF,t+1}^{G} + (1-\overline{\beta}_{US})N_{CF,t+1}^{US}] \\
=& \underbrace{\kappa_1(\gamma - \frac{1}{\psi})}_{>0}\underbrace{(1-\overline{\beta}_{US})}_{>0} N_{CF,t+1}^{US}
\end{aligned} \tag{21}$$

The Jensen's term takes the form:

$$J_t^{US} = var_t m_{t+1}^{US} - \frac{1}{N}\sum_{i=1}^{N} var_t m_{t+1}^{i}$$

$\overline{\beta_{CF}}, \overline{\beta_{US}}$ captures the average non-US world's exposure to the two LRR factors. The second line imposes the simplifying restriction that the US has average exposure to global LRRs. If in addition, the ROW exposure to US LRRs is lower than the US ($1 - \overline{\beta}_{US} > 0$), then lemma 2.4 indicates that the profitability of the dollar factor represents compensation for bearing US LRRs. Moving to dollar betas, note that lemma 2.4 implies the following lemma about dollar betas:

Lemma 2.5. *(US LRRs and Dollar Betas). Currency i's loading on the dollar factor:* $\beta_{Dollar}^{i} = \frac{cov_t(rx_{t+1}^{i}, \widetilde{Dollar}_{t+1})}{var_t(\widetilde{Dollar}_{t+1})}$ *is:*

$$\beta_{Dollar}^{i} = (1-\beta_{US}^{i})\underbrace{(1-\overline{\beta}_{US})}_{>0}\underbrace{\frac{\kappa_1^2\tau^2(\gamma-\frac{1}{\psi})^2}{(1-\overline{\beta}_{US})^2\tau^2}}_{>0} \tag{22}$$

Lemma 2.6 suggests that dollar betas are declining in country specific exposures to US LRRs: β_{US}^{i}. Thus when the dollar appreciates against the ROW, it does more so against the currencies of countries that have low US LRR exposures. Thus the HML dollar beta portfolio that longs (shorts) low (high) dollar beta currencies positively loads on the US LRR factor:

Lemma 2.6. *(US LRRs and HML Dollar Beta Portfolio). Lemma 2.6 implies that returns on the HML portfolio that goes long (short) a portfolio of high (low) dollar beta portfolios: $Dollar_t^{Global}$ takes the form:*

$$Dollar_t^{Global} = \underbrace{(\overline{\beta}_{US}^{L} - \overline{\beta}_{US}^{H})(1-\overline{\beta}_{US})}_{>0}\underbrace{\frac{\kappa_1^2\tau^2(\gamma-\frac{1}{\psi})^2}{(1-\overline{\beta}_{US})^2\tau^2}}_{>0} N_{CF,t}^{US} \tag{23}$$

Here $\overline{\beta}_{US}^{H}(\overline{\beta}_{US}^{L})$ denotes the average US LRR exposures in the high (low) dollar beta portfolios respectively. These results give rise to the following testable proposition below:

Proposition 4. *(US LRRs, the Dollar and HML Dollar Beta Portfolio). Both the dollar carry trade and the HML dollar beta portfolio load on US LRRs: their returns **increase** in response to a positive US LRR shock: $N_{CF,t}^{US}\uparrow$*

3 News Identification

3.1 Framework

Overview: Taking propositions 1-4 to the data requires an identification of country level consumption news $N_{CF,t+1}^{i}$. We identify these terms using the ICAPM-VAR framework championed by Campbell and Vuolteenaho (2004) and Campbell et al (2017). This methodology estimates stock market cash flow news using a VAR approach, which we use as our proxy for country level LRRs. We discuss the specific details in this section.

VAR System: Following Campbell et al (2017), we model country i's aggregate wealth returns $r_{m,t+1}^{i}$ as being jointly determined as part of a heteroskedastic first order VAR system. In specific terms, the state system z_t^i is driven by the following process:

$$z_{t+1}^i = \mu_i + \Gamma(z_t^i - \mu_i) + \sigma_t^i \xi_{t+1}^i \tag{24}$$

$$\xi_{t+1}^i \sim i.i.d\ N(0,\, I) \tag{25}$$

Discount Rate News: Under this structural assumption, discount rate news on country i's aggregate wealth portfolio $N_{DR,t+1}^i = (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{s=1}^{\infty} \rho^s r_{m,t+1+s}^i$ is affine in the state vector z_t^i:

$$N_{DR,t+1}^i = (e_1' \lambda)\sigma_t^i \xi_{t+1}^i \tag{26}$$

Here $\lambda = \kappa\Gamma(I - \kappa\Gamma)^{-1}$. I is an $N \times N$ identity matrix and Γ is an $N \times N$ matrix of parameters associated with the VAR system. e_1 is a vector that include one as its first element and zero for all other elements: $e_1 = [1, 0, 0, ..., 0]^T$. In other words e_1 picks out $r_{m,t+1}^i$ from the state vector z_{t+1}^i. Finally κ is a log-linearization parameter that captures the average dividend yield or the average consumption-wealth ratio. We follow Campbell et al (2017) and set $\kappa = 0.95^{\frac{1}{12}} = 0.995$.

Cash Flow News: With this estimate of discount rate news, we can then back out cash flow news on country i's wealth portfolio $N_{CF,t+1}^i = (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{s=0}^{\infty} \rho^s \Delta c_{t+1+s}^i$ as the residual from the wealth return decomposition derived by Campbell and Vuolteenaho (2004) tying wealth return shock $\tilde{r}_{m,t+1}^i = (\mathbb{E}_{t+1} - \mathbb{E}_t) r_{m,t+1}^i$ to cash flow and discount rate news on the aggregate wealth portfolio:

$$\tilde{r}_{m,t+1}^i = N_{CF,t+1}^i - N_{DR,t+1}^i \tag{27}$$

Combining (26) and (27) then implies that $N_{CF,t+1}^i$ is the following affine function of the state vector:

$$N_{CF,t+1}^i = (e_1' + e_1'\lambda)\sigma_t^i \xi_{t+1}^i \tag{28}$$

Proxy: We estimate the VAR system at the country level, giving us an estimate $\hat{N}^i_{CF,t+1}$ for each country i. This is our empirical proxy for country i's LRR.

3.2 State System

State Vector: To estimate (28), we need to take a stand on the specification of the state vector z^i_t. Here we follow Campbell et al (2017): the first two elements of z^i_t are country level equity returns $r^i_{m,t+1}$ and its volatility σ^i_t. We specify the remaining $n-2$ elements here. In total we assume that z^i_{t+1} is a four dimensional vector with the following elements:

$$z^i_t = \begin{bmatrix} r^i_{m,t} & \sigma^i_t & DY^i_t & TS^i_t \end{bmatrix}^T \tag{29}$$

State Variables

$r^i_{m,t+1}$: *Country i's aggregate market Return*

DY^i_{t+1}: *Country i's aggregate dividend yield*

TS^i_{t+1}: *Country i's Term Spread*

σ^i_{t+1}: ***Conditional** volatility of country i's aggregate market return*

Conditional Market Volatility: The second state variable: *conditional* market volatility σ^i_t needs to be estimated. To achieve this we follow the approach of Campbell et al (2017). First we construct realized quarterly market variance $RVAR_{t+1}$ from daily market return data by cumulating squared market returns for all days i within quarter $t+1$:

$$RVAR_{t+1} = \sum_{i \in t+1}^{N} r_i^2$$

Define r_i as the log daily return on the MSCI total return index where day i is inside quarter $t+1$. We then run the following predictive regression:

$$RVAR^i_{t+1} = \alpha + \phi_1 RVAR^i_t + \phi_2 r^i_{m,t} + \phi_3 TS^i_t + \phi_4 DY^i_t \tag{30}$$

To control for the heteroskedasticity of innovations to our state variables, this regression is estimated as a Weighted Least Squares (WLS) regression where each observation is weighted by the inverse of the realized variance $RVAR_t^{-1}$. Using the above estimates of $\hat{\alpha}$ and $\hat{\phi}_1 - \hat{\phi}_4$, I estimate σ_t as:

$$\hat{\sigma}^i_{t+1} = \hat{\alpha} + \hat{\phi}_1 RVAR^i_t + \hat{\phi}_2 r^i_{m,t} + \hat{\phi}_3 TS^i_t + \hat{\phi}_4 DY^i_t \tag{31}$$

3.3 Data

Table 1: State Variable Sample Information

State Variables	Source	Sample Period
$\hat{r}^i_{m,t+1}$	MSCI Global	Jan 1973 - Dec 2019
$\hat{\sigma}^i_t$	MSCI Global	Jan 1973 - Dec 2019
TS^i_{t+1}	Global Financial Data	Jan 1973 - Dec 2019
DY^i_{t+1}	MSCI Global	Jan 1973 - Dec 2019

Coverage: We focus our analysis on developed countries. This includes 11 countries: Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom and the United States.

Data Appendix: In the interests of space, we relegate a more detailed discussion of our data to the online appendix.

3.4 Discussion

Other Identification Schemes: The identification strategy outlined in section 3 is highly tractable: only asset market data is required to generate our country level cash flow news estimates $\hat{N}^i_{CF,t+1}$. This is what makes our approach highly tractable relative to other identification schemes used in prior literature to identify long run risks. Other identification schemes are not tractable enough to enable us to construct country level measures for consumption news.

For example Bansal, Kiku, Shaliastovich and Yaron (2017) (BKSY) follow a large prior literature in defining the aggregate wealth portfolio as a value weighted portfolio of stock market and human capital wealth adopted in prior literature (Jagannathan and Wang, 1996; Campbell, 1996;

Lustig and Van Nieuwerburgh, 2007). Such an approach is only feasible for the US context: labor income data required to compute human capital returns are not readily available across a wide enough panel of countries for the BKSY framework to be useful for our purposes. Similar feasibility concerns apply to other identification schemes for long run risks considered in the literature (Schorfheide and Yaron, 2018; Liu and Matthies, 2021).

Consumption News vs Cash Flow News: Our approach implicitly assumes a mapping between stock market cash flows news and long run consumption news. We believe this approach is reasonable: a large literature exists documenting persistent home bias in global equity portfolios (Poterba, 1992; Couerdacier and Rey, 2013). Thus stock market cash flows broadly captures the equilibrium path of future local consumption streams available to each country. To support this contention, we show in the online appendix that our cash flow news estimates $\hat{N}^i_{CF,t+1}$ are highly correlated with other long run risk proxies such as the GDP growth forecasts. Thus a mapping exists between our cash flow news measure and country level LRRs.

4 Empirical Long Run Risks

LRR Risk Sharing: Here we describe how our empirical LRR proxy $N^i_{CF,t+1}$ behaves in the data. In particular, we focus on the cross-country correlation in LRRs. This is an important quantity in international LRR models: high ***positive*** cross-country correlations in LRRs are a crucial ingredient in the LRR resolutions to the classic international finance puzzles (Colacito and Croce, 2011, 2013). To verify this assumption in our data, we start by plotting correlation of our identified country level LRRs ($N^i_{CF,t}$) with US LRRs ($N^{US}_{CF,t}$) for 6 major developed countries. This is depicted in Figure 1. Plot 1 reveals that LRRs are indeed highly correlated across countries, with the correlations ranging from 0.58 to 0.82 for these countries. These plots are suggestive evidence in favour of the view that there is a high degree of LRR risk sharing across countries, a critical assumption in workhorse international LRR models.

To formalise the degree of LRR risk sharing across countries, we construct a formal measure that is motivated by the canonical international risk sharing index of Brandt, Cochrane and Santa

Clara (2007). We present this LRR risk sharing index below:

$$LRR^i_j = 1 - \frac{var[(N^i_{CF,t+1}) - (N^j_{CF,t+1})]}{var[N^i_{CF,t+1}] + var[N^j_{CF,t+1}]} \tag{32}$$

This index captures the ratio of LRRs across countries i and j to the total amounts of LRRs to share across the two countries. The numerator measures how much LRR is not shared: how different the variance of LRR is across countries. The denominator measures how much LRR there is to share: the total variance of consumption news across the two countries. Hence $LRR^i_j = 1$ implies that the LRR risk sharing across countries i and j is perfect. Conversely $LRR^i_j = 0$ suggests that there is no LRR risk sharing across the two countries.

Panels A, B and C of figure 2 reports the level of LRR risk sharing at the bilateral country level for the full sample, pre-2007 sample and post-2007 sample respectively. The results reinforce the fact that a high degree of LRR risk sharing occurs between countries. The degree of risk sharing is noticeably smaller whenever Japan is involved, however for all other bilateral pairs the recorded values of the risk sharing index are relatively close to 1.

Figure 1: *Long Run Risks Correlations with US*

Description: This figure plots US consumption news $N_{CF,t}^{US}$ (blue) against foreign consumption news $N_{CF,t}^{US}$ (red). The correlations are reported in the bottom right portion of each panel.

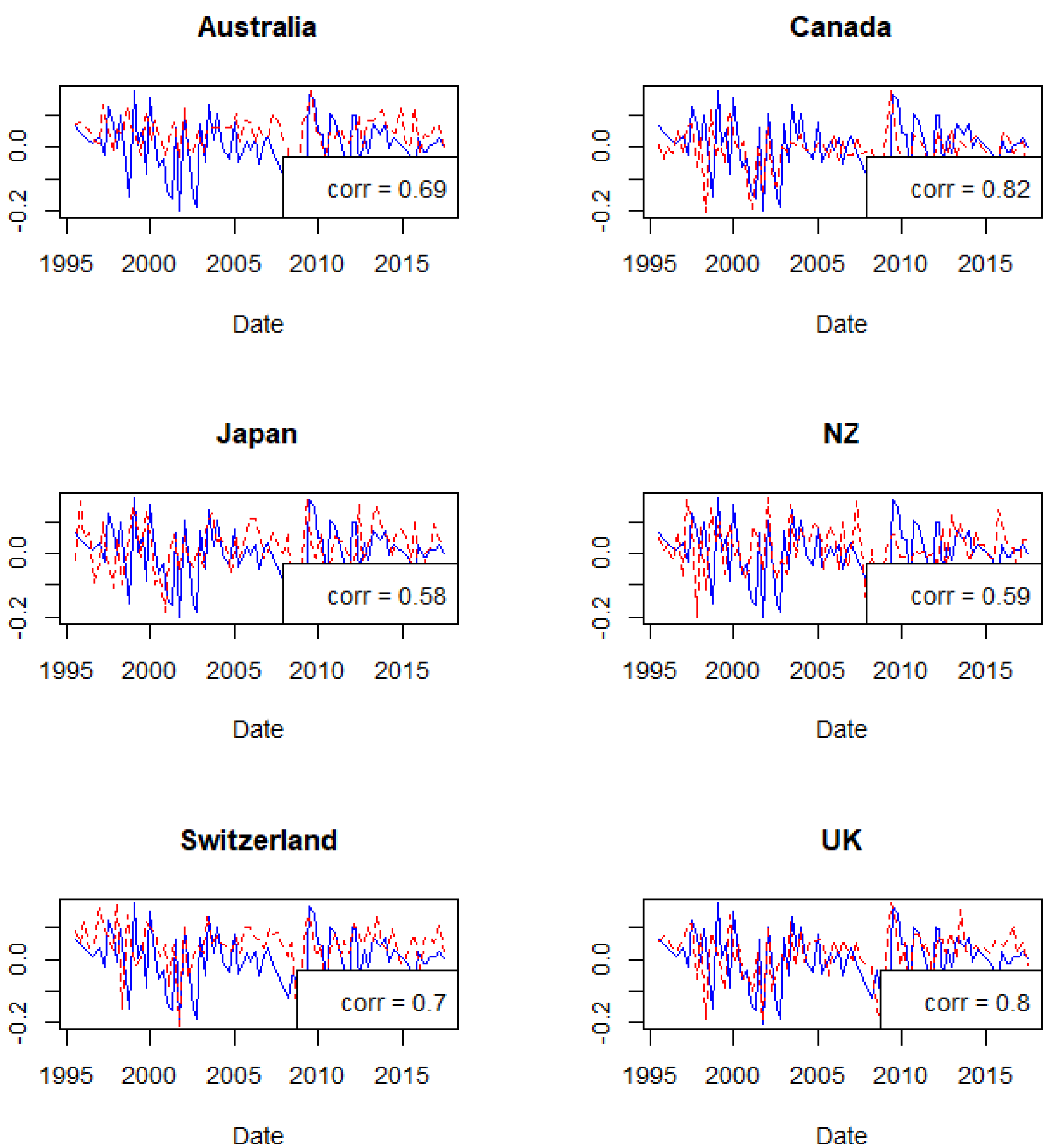


Table 2: *LRR Risk Sharing*

Description: This table reports the computations for LRR risk sharing index LRR_j^i defined by (32). Panel A, B and C reports the results for the full, pre-2007 and post-2007 samples respectively.

Panel A: Full Sample

	AUS	CAN	DEN	JAP	NZL	NOR	SWE	SWI	UK	USA
AUS	1									
CAN	0.592	1								
DEN	0.666	0.622	1							
JAP	0.600	0.391	0.551	1						
NZL	0.651	0.496	0.426	0.375	1					
NOR	0.628	0.695	0.800	0.541	0.448	1				
SWE	0.615	0.717	0.758	0.547	0.445	0.794	1			
SWI	0.561	0.678	0.664	0.426	0.447	0.690	0.770	1		
UK	0.700	0.779	0.632	0.497	0.600	0.733	0.744	0.792	1	
USA	0.668	0.810	0.690	0.570	0.518	0.687	0.775	0.700	0.776	1

Panel B: Pre-2007 Sample

	AUS	CAN	DEN	JAP	NZL	NOR	SWE	SWI	UK	USA
AUS	1									
CAN	0.596	1								
DEN	0.498	0.693	1							
JAP	0.445	0.449	0.387	1						
NZL	0.771	0.537	0.417	0.414	1					
NOR	0.405	0.721	0.735	0.376	0.476	1				
SWE	0.461	0.730	0.701	0.545	0.411	0.764	1			
SWI	0.464	0.719	0.671	0.343	0.392	0.742	0.768	1		
UK	0.668	0.793	0.619	0.447	0.622	0.722	0.705	0.790	1	
USA	0.638	0.868	0.664	0.575	0.549	0.674	0.780	0.738	0.780	1

Panel C: Post-2007 Sample

	AUS	CAN	DEN	JAP	NZL	NOR	SWE	SWI	UK	USA
AUS	1									
CAN	0.600	1								
DEN	0.765	0.572	1							
JAP	0.721	0.326	0.683	1						
NZL	0.557	0.453	0.424	0.336	1					
NOR	0.786	0.676	0.848	0.690	0.414	1				
SWE	0.818	0.694	0.829	0.549	0.516	0.834	1			
SWI	0.657	0.614	0.659	0.524	0.540	0.630	0.776	1		
UK	0.738	0.759	0.660	0.548	0.608	0.753	0.812	0.807	1	
USA	0.697	0.727	0.717	0.563	0.477	0.702	0.767	0.641	0.775	1

5 LRRs and Currency Dynamics

5.1 LRRs and Currency Excess Returns

Specifications: Having described the empirical properties of our identified country level LRRs, we now move to our main empirical investigation: the joint dynamics between ***relative*** LRRs and currency dynamics. We start with currency excess returns: to take proposition 1 to the data, we consider panel specifications of the following form:

$$rx_t^i = \alpha + \beta_1(\hat{N}_{CF,t}^{US} - \hat{N}_{CF,t}^i) + \beta_2(\Delta c_t^{US} - \Delta c_t^i) + \epsilon_t^i \tag{33}$$

$$rx_{q,t}^i = \alpha + \beta_1(\hat{N}_{CF,t}^{US} - \hat{N}_{CF,t}^i) + \beta_2(\Delta c_t^{US} - \Delta c_t^i) + \epsilon_t^i \tag{34}$$

rx_t^i: ***Nominal*** excess return associated with going long currency i against the dollar

$rx_{q,t}^i$: ***Real*** excess return associated with going long currency i against the dollar

Construction: We follow Lustig and Verdelhan (2011)'s approach to constructing currency excess returns. Denote by $s_{i,t}$ and $f_{i,t}$ the log spot and forward exchange rates defined as units of USD per foreign currency i. Thus the log currency excess return $rx_{i,t}$ on buying foreign currency i in the forward market and then selling it in the spot market after one quarter is:

$$rx_t^i = s_{i,t} - f_{i,t-1} \tag{35}$$

$q_{i,t}$ is the ***real*** exchange rate: US consumption units per country i's consumption unit. This is constructed by adjusting the nominal spot rate $s_{i,t}$ by the realized GDP deflator. Similarly define the real forward exchange rate $f_{i,t}^q$ as the nominal forward rate $f_{i,t}$ adjusted by the realized 3-month inflation differential measured by the GDP deflator. Then the real currency excess return $rx_{q,t}^i$ is:

$$rx_{q,t}^i = q_{i,t} - f_{i,t-1}^q \tag{36}$$

Hypothesis: The central coefficient of interest from the panel specification (33) and (34) is β_1. Proposition 1 implies that a bad relative LRR shock to country i ($\hat{N}^{US}_{CF,t} - \hat{N}^{i}_{CF,t} > 0$) *increases* excess log returns for a US investor going long currency i. Thus the international LRR model places the following restrictions on β_1:

Table 3: Testable Implications for proposition 1

Specification	Testable Implication
Nominal	$\beta_1 > 0$
Real	$\beta_1 > 0$

Discussion: The results for the baseline specification in (33) and (34) are presented in table 4. In line with international LRR models $\beta_1 > 0$: bad relative LRR shocks to country i ($\hat{N}^{US}_{CF,t} - \hat{N}^{i}_{CF,t} > 0$) are associated with an ***appreciation*** of currency i against the dollar and consequently an ***increase*** in currency excess returns associated with going long currency i against the dollar. These results are robust across sub-samples and hold equally well for nominal currency excess returns as well as real currency excess returns.

A comment about magnitudes is in order. Whilst ***qualitatively*** the signs are consistent with the international LRR models, ***quantitatively*** the magnitudes are rather modest: a 1% bad relative LRR shock to country i vis-*á*-vis the US results in a 10 basis point increase in average currency excess returns on impact. Furthermore the R^2 values are relatively low, especially for the univariate regressions reported in panel A. This indicates that whilst relative LRRs are important drivers of currency excess returns, their contribution to the overall variance is relatively small.

This finding is consistent with Verdelhan (2018) who finds that the share of bilateral exchange rate volatility emanating from local shocks is small. Thus we should not be surprised that empirically local LRR shocks have a relatively small contribution to overall FX volatility. Further this finding is not at odds with the international LRR models: these models impose high cross-country correlation in LRRs. Thus relative differences in SDFs are weakly responsive to local LRR shocks, resulting in a low share of FX volatility coming from these shocks (Colacito and Croce, 2011, 2013).

Table 4: *Currency Excess Returns and LRRs*

Description: This table reports estimation results for specifications captured by equations (33), (34). Panel A reports the univariate regressions. Panels B and C report the multivariate results where various FX and risk controls are added. Country fixed effects are added to each regression and standard errors are clustered at the country level. Sample period is 1980Q1-2017Q1.

	Panel (a): Univariate Regressions					
	Nominal: rx_t^i			Real: $rx_{q,t}^i$		
	Full	Pre-2007	Post-2007	Full	Pre-2007	Post-2007
$N_{CF,t}^{US} - N_{CF,t}^i$	0.101***	0.095***	0.126*	0.092***	0.083**	0.124*
	(0.031)	(0.035)	(0.073)	(0.032)	(0.034)	(0.073)
$\Delta c_t^{US} - c_t^i$	0.009	0.026	-0.013	0.010	0.030	-0.032
	(0.021)	(0.028)	(0.037)	(0.023)	(0.029)	(0.036)
Country FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Observations	1,115	696	419	1,115	696	419
Adjusted R^2	0.005	0.001	0.004	0.008	0.004	0.004
	Panel (b): FX Controls					
	Nominal: rx_t^i			Real: $rx_{q,t}^i$		
	Full	Pre-2007	Post-2007	Full	Pre-2007	Post-2007
$N_{CF,t}^{US} - N_{CF,t}^i$	0.090**	0.089**	0.121**	0.098**	0.099**	0.121**
	(0.039)	(0.043)	(0.060)	(0.039)	(0.044)	(0.061)
$\Delta c_t^{US} - \Delta c_t^i$	0.006	0.045**	0.088	0.008	0.044**	0.085
	(0.024)	(0.018)	(0.081)	(0.022)	(0.018)	(0.082)
$q_{i,t}$	0.0001	-0.0004***	-0.0002	0.0001	-0.0004**	-0.0002
	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)
$fd_{i,t}$	0.011**	-0.003	0.037***	0.011***	-0.005	0.039***
	(0.004)	(0.006)	(0.008)	(0.004)	(0.006)	(0.008)
$basis_t$	0.068***	0.036***	0.119***	0.070***	0.039***	0.122***
	(0.010)	(0.011)	(0.019)	(0.010)	(0.012)	(0.020)
US Surplus-Debt Ratio$_t$	-0.012***	-0.010	-0.017***	-0.012***	-0.008	-0.019***
	(0.003)	(0.006)	(0.004)	(0.003)	(0.007)	(0.004)
Country FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Observations	996	624	372	996	624	372
Adjusted R^2	0.115	0.101	0.231	0.120	0.102	0.233
Note:			*p<0.1; **p<0.05; ***p<0.01			

Controls: Since our identification strategy indirectly estimates stock market cash flow news as the residual from the wealth return decomposition captured by (27), a common concern is that our LRR proxy variable ($\hat{N}^i_{CF,t}$) may also capture discount rate news if the VAR is misspecified (Chen and Zhao, 2009). Thus any positive result may simply be capturing the link between relative discount rate news movements and currency excess returns, a fact that is already known (Chiang and Mo, 2022).

To alleviate these concerns, we augment the baseline specification with currency risk premium proxies that are correlated with country level discount rate news. Following empirical international finance literature, the set of controls we consider are:

$$X^W_t = \begin{bmatrix} s_t & q_{i,t} & fd_{i,t} & basis_t & \text{US Surplus-Debt Ratio}_t \end{bmatrix}^T \tag{37}$$

$s_{i,t}$: Level of the nominal exchange rate: Foreign Currency i/USD

$q_{i,t}$: Level of real exchange rate: FCU/USD

$fd_{i,t}$: Forward discount against currency i.

$basis_t$: US Treasury Basis

US Surplus-Debt Ratio$_t$: US Surplus-Debt Ratio

Multivariate: Panels B and C of table 4 present multivariate extensions of the baseline specifications: (33) and (34) with the controls described above. They clearly indicate that the strong ***negative*** relationship between relative LRRs and currency excess returns are not spanned by discount rate news proxied by traditional global currency risk factors. Thus the results in table 4 are genuinely capturing a tight ***negative*** link between relative LRRs and currency excess returns, as predicted by proposition 1.[6]

5.2 Currency Risk Premia

Specifications: We now investigate the link between ***relative*** LRRs and currency risk premia, or ***expected*** currency excess returns. To take proposition 2 to the data, we consider predictive

[6]Online appendix shows robustness of these results using other controls considered by Verdelhan (2018).

regressions of the form:

$$rx^i_{t+j,t+k} = \alpha + \beta_{j,k}(\hat{N}^{US}_{CF,t} - \hat{N}^i_{CF,t}) + \epsilon_{i,t} \tag{38}$$

Notation: $rx^i_{t+j,t+k}$ is the *nominal* currency excess return for an investor that longs currency i against the dollar during periods $t+j$ and $t+k$. As before, the frequency of the periods is *quarterly*.

Hypothesis: The central coefficient of interest is $\beta_{j,k}$. Proposition 2 implies that a bad relative LRR shock to country i ($N^{US}_{CF,t} - N^i_{CF,t} > 0$) *decreases* the log currency risk premium that a USD investor demands for going long currency i. To deliver these lower expected future returns, currency i appreciates on impact before depreciating moving forward. Thus international LRRs models impose the following restrictions on these coefficients:

Table 5: Testable Implications for proposition 2

Specification	Testable Implication
Short Run	$\beta_{j,k} > 0$ for $j, k = 0$
Long Run	$\beta_{j,k} < 0$ for $\forall j, k > 0$

Overview: The results for the baseline predictive regressions outlined in (38) are captured in table 15. Panel A reports the univariate case and panels B and C report the multivariate case where the same FX controls and risk controls as before are added.

Discussion: Table 15 broadly supports the LRR view of currency risk premia. Upon receipt of a bad relative LRR shock vis-á-vis the US ($N^{US}_{CF,t} - N^i_{CF,t} > 0$), the currency of country i depreciates over the long run, as evidenced by the negative coefficients for $\beta_{j,k}$. This predictability largely dies out 4 years after the shock, as evidenced by the reversal of the coefficient sign from negative to positive when $rx^i_{t+12,t+16}$ is the dependent variable.

Table 6: *LRRs and Currency Risk Premia*

Description: This table reports estimation results for the predictive regressions outlined by (38). Panel A reports the univariate regressions. Panels B and C report the multivariate results where various FX and risk controls are added. Country fixed effects are added and standard errors are clustered at the country level. Sample period is from 1980Q1-2017Q1.

	Panel (a): Univariate						
	rx^i_t	$rx^i_{t,t+1}$	$rx^i_{t,t+4}$	$rx^i_{t+4,t+8}$	$rx^i_{t+8,t+12}$	$rx^i_{t+12,t+16}$	$rx^i_{t+16,t+20}$
$N^{US}_{CF,t} - N^i_{CF,t}$	0.091***	-0.050**	-0.053	-0.192***	-0.217***	0.130**	-0.003
	(0.032)	(0.020)	(0.041)	(0.073)	(0.056)	(0.064)	(0.042)
$\Delta c^{US}_t - \Delta c^i_t$	-0.011	-0.007	0.014	0.105	0.118**	0.096*	0.023
	(0.024)	(0.023)	(0.094)	(0.070)	(0.053)	(0.054)	(0.043)
Country FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	1,068	1,059	1,032	996	962	926	891
Adjusted R^2	0.005	0.005	0.009	0.012	0.018	0.002	0.011
	Panel (b): FX Controls						
	rx^i_t	$rx^i_{t,t+1}$	$rx^i_{t,t+4}$	$rx^i_{t+4,t+8}$	$rx^i_{t+8,t+12}$	$rx^i_{t+12,t+16}$	$rx^i_{t+16,t+20}$
$N^{US}_{CF,t} - N^i_{CF,t}$	0.090**	-0.041**	-0.012	-0.181**	-0.201***	0.199**	0.058
	(0.039)	(0.020)	(0.035)	(0.086)	(0.064)	(0.080)	(0.057)
$\Delta c^{US}_t - \Delta c^i_t$	0.006	0.002	0.076	0.117**	0.162***	0.116**	0.081*
	(0.024)	(0.015)	(0.048)	(0.053)	(0.039)	(0.054)	(0.047)
$q_{i,t}$	0.0001	0.001***	0.005***	0.004***	0.001***	0.002**	0.001*
	(0.0001)	(0.0001)	(0.0004)	(0.0003)	(0.0004)	(0.001)	(0.0003)
$fd_{i,t}$	0.011**	0.011***	0.010	0.007	0.0002	0.010*	0.003
	(0.004)	(0.003)	(0.007)	(0.007)	(0.005)	(0.006)	(0.008)
$basis_t$	0.068***	0.066***	0.055***	-0.049***	-0.045***	-0.019**	0.047***
	(0.010)	(0.012)	(0.016)	(0.013)	(0.014)	(0.008)	(0.015)
US Surplus-Debt Ratio$_t$	-0.012***	-0.025***	-0.098***	-0.062***	-0.001	0.017*	0.051***
	(0.003)	(0.003)	(0.010)	(0.008)	(0.007)	(0.010)	(0.007)
Country FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	996	987	960	924	890	854	819
Adjusted R^2	0.121	0.141	0.320	0.173	0.068	0.105	0.126
Note:	*p<0.1; **p<0.05; ***p<0.01						

5.3 Global Currency Risk Factors

5.3.1 HML Carry Trade

Overview: Having concluded our review of bilateral currency dynamics, we now shift our analysis towards global currency risk factors. We start our analysis by looking at the HML carry trade sorted on interest rate differentials. Recall from proposition 3 that in a LRR world, HML carry trade loads positively on global LRRs: it can be viewed as a strategy that longs currencies of countries that are ***less*** exposed to global long run risks and shorts currencies of countries that are ***more*** exposed to global long run risks. In other words HML carry trade profitability represents risk compensation for bearing global LRRs.

Proxy: To take this prediction to the data, we must construct an empirical proxy for the global LRR factor $N^G_{CF,t+1}$. Our empirical proxy is an equally weighted average of our country level LRR measure:

$$\hat{N}^G_{CF,t+1} = \frac{1}{N}\sum_{i=1}^{N} \hat{N}^i_{CF,t+1} \tag{39}$$

Specification: To link our global LRR factor to the HML carry trade, we follow Lustig and Verdelhan (2011) and construct 6 interest rate sorted currency portfolios. We then run the following specification at the portfolio level:

$$HML^i_{t+1} = \alpha + \beta_i \hat{N}^G_{CF,t+1} + \epsilon_{i,t+1} \;,\; \forall i \in \{1, 2, ..., 6\} \tag{40}$$

Results: The estimation results associated with this specification are presented in table 7. These regressions confirm that interest rate sorted portfolios are monotonically increasing in their exposures to the global long run risk factor $N^G_{CF,t+1}$. Furthermore the HML carry trade return $(HML^6_t - HML^1_t)$ is increasing in $N^G_{CF,t+1}$. Thus HML carry trade profitability can be interpreted as risk compensation for bearing global LRRs, in line with the international LRR models.

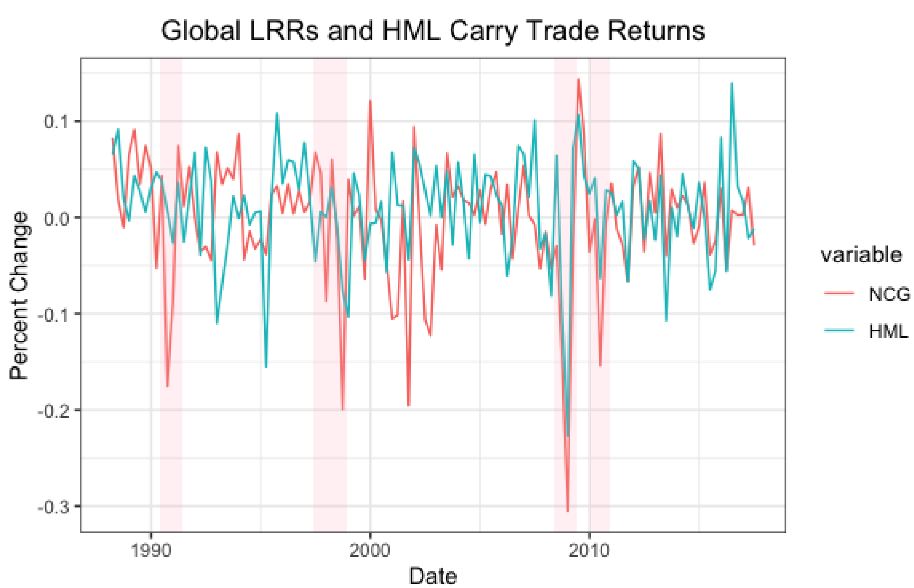
Table 7: *Interest Rate Sorted Portfolios and Global Long Run Risks*

Description: This table reports estimation results for specifications captured by equation (40). Standard errors for these regressions are heteroskedasticity-robust. fd_t is the average forward discount against the USD. $Basis_t$ is the US treasury premium from Du, Im and Schreger (2017). Sample period is from 1988Q1-2017Q2.

	Dependent variable: Interest Rate Sorted Currency Portfolios						
	HML_t^1	HML_t^2	HML_t^3	HML_t^4	HML_t^5	HML_t^6	$HML_t^6 - HML_t^1$
$N^G_{CF,t+1}$	-0.119**	-0.016	-0.008	-0.028	0.068	0.107*	0.226***
	(0.048)	(0.044)	(0.040)	(0.050)	(0.052)	(0.06)	(0.064)
US Govt Surplus-Debt Ratio	-0.009*	-0.004	-0.003	-0.005	-0.012**	-0.002	0.007
	(0.005)	(0.005)	(0.004)	(0.006)	(0.006)	(0.007)	(0.007)
	(0.0003)	(0.0004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
fd_t	0.002	0.001	0.007	-0.001	0.001	0.008	0.006
	(0.007)	(0.006)	(0.006)	(0.007)	(0.008)	(0.010)	(0.009)
$basis_t$	0.037***	0.068***	0.073***	0.063***	0.065***	0.071***	0.034*
	(0.014)	(0.012)	(0.011)	(0.014)	(0.015)	(0.019)	(0.018)
Observations	118	118	118	118	118	118	118
Adjusted R^2	0.119	0.223	0.269	0.162	0.213	0.120	0.116
Note:							*p<0.1; **p<0.05; ***p<0.01

Figure 2: *Global LRRs and HML Carry Trade Returns*

Description: This figure plots the global LRR factor $N^G_{CF,t+1}$ (orange) against HML carry trade returns (green). Pink bands correspond to the following global downturns: 1990Q4-1991Q4 (1990's global recession), 1997Q2-1998Q4 (Asian Financial Crisis), 2008Q2-2009Q2 (Global Financial Crisis), 2010Q1-2010Q4 (European Debt Crisis).



Graph: To complement these results, I plot the global LRR factor against HML carry trade returns in figure 2. The pink bands highlight well established global downturns. In each case, the global long run risk factor tracks the HML carry trade returns: both crash during periods of global economic downturns. This supports the LRR interpretation of the HML carry trade as a strategy that loads on global LRRs. Since global downturns often coincide with deteriorating global growth expectations, HML carry trade returns crash during these periods.

5.3.2 Dollar Beta Portfolio

Overview: Now we draw our attention to the dollar carry trade and the corresponding HML dollar beta portfolio. Recall from proposition 4 that both the dollar carry trade and the HML dollar beta portfolio load positively on the US LRR factor: it performs well when US receives a good LRR shock relative to the ROW. Consequently the HML dollar beta portfolio also loads positively on US LRRs: it longs (shorts) currencies that are weakly (strongly) exposed to US LRRs.

Proxy: Testing these predictions requires an empirical proxy for US relative LRRs vis-*á*-vis the ROW: $LRR^{US}_{t+1} = N^{US}_{CF,t+1} - \overline{N}^{ROW}_{CF,t+1}$. To construct such a proxy, we define $\overline{N}^{ROW}_{CF,t+1}$ as an equal weighted average of country specific LRRs excluding the US:

$$\overline{N}^{ROW}_{CF,t+1} = \frac{1}{N} \sum_{i \neq US} N^{i}_{CF,t+1} \tag{41}$$

Specifications: To take proposition 4 to the data, we project the dollar carry trade $Dollar_t$ and the HML dollar beta portfolio return $Dollar^{Global}_t$ onto LRR^{US}_t:

$$DOLLAR_{t+1} = \alpha + \beta_1 LRR^{US}_{t+1} + \epsilon_{t+1} \tag{42}$$

$$DOLLAR^{Global}_{t+1} = \alpha + \beta_1 LRR^{US}_{t+1} + \epsilon_{t+1} \tag{43}$$

Hypothesis: proposition 4 implies that $\beta_1, \beta_2 > 0$.

Construction: To construct the dollar carry trade, we follow Lustig, Roussanov and Verdelhan

(2014) and Verdelhan (2018) and define it as the equally weighted average dollar appreciation against the ROW:

$$Dollar_t = \sum_{i=1}^{N} \Delta s_{i,t} \tag{44}$$

As before $s_{i,t}$ is the nominal spot exchange rate of currency i per dollar. To estimate time varying dollar betas, we follow Verdelhan (2018) and extract them from rolling regressions using a 60 month rolling window. The factor model takes the form:

$$\Delta s_{i,\tau} = \alpha + \beta^i_{Dollar,t} Dollar_\tau + \beta^i_{HML,t} HML_\tau + \epsilon^i_\tau, \text{ for } \tau = t-60, ..., t-1 \tag{45}$$

Currencies are then sorted into six portfolios at time t based on dollar betas $\beta^i_{Dollar,t}$. Portfolio 1 contains currencies with lowest dollar betas ($\beta^i_{Dollar,t}$), while portfolio 6 contains currencies with the highest dollar betas ($\beta^i_{Dollar,t}$). Going long portfolio 6 and short portfolio 1 is what we refer to as the HML dollar beta portfolio.

Results: Table 8 reports the results for the dollar carry trade regressions. We present the univariate regression as well as a multivariate version that controls for known drivers of the dollar exchange rate such as the US surplus-debt ratio (Jiang, 2021), the average forward discount against the USD fd_t and the US treasury basis $basis_t$. In both cases, an increase in our empirical US LRR factor (LRR^{US}_t) is associated with high excess returns on the dollar carry trade. However the contribution of US relative LRRs to the overall variance of dollar carry trade return seems quantitatively small especially when compared against the explanatory power of the US treasury basis.

Moving on to the dollar beta portfolio returns, the results are even stronger: table 9 documents a tight connection between LRR^{US}_t and the HML dollar beta portfolio. Consistent with proposition 4, the results suggest that the dollar beta portfolios are increasing in their exposure to the US LRR factor and that the HML dollar beta portfolio returns are positively correlated with this factor. Since this HML portfolio isolates the global risk information contained in the dollar carry trade, these results suggest that US LRRs are an important source of global currency risk.

Table 8: *Dollar Factor and US Relative LRRs*

Description: This table reports estimation results associated with equation (42). As before FX risk controls are added to the regressions. Standard errors are heteroskedasticity-robust. Sample period is 1988Q-2017Q2.

	Dependent variable: Dollar Factor	
	$Dollar_t$	$Dollar_t$
LRR_t^{US}	0.158*	0.134*
	(0.083)	(0.072)
US Surplus-Debt Ratio		-0.010
		(0.007)
$basis_t$		0.086***
		(0.015)
fp_t		0.004
		(0.008)
q_t		0.0002
		(0.0004)
Constant	-0.001	-0.017
	(0.004)	(0.047)
Observations	118	118
Adjusted R^2	0.022	0.272
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 9: *Dollar Beta Portfolios and US LRRs*

Description: This table reports estimation results associated with estimating equation (43). As before FX risk controls are added to the regressions. Standard errors are heteroskedasticity-robust. Sample period is 1988Q1-2017Q2.

	Dependent variable: Dollar Beta Portfolios: rp_t^i						
	rp1	rp2	rp3	rp4	rp5	rp6	$Dollar_t^{Global}$
LRR_t^{US}	-0.032	0.026	0.009	0.203***	0.198**	0.211**	0.243**
	(0.031)	(0.053)	(0.078)	(0.072)	(0.088)	(0.095)	(0.098)
US Surplus-Debt $Ratio_t$	-0.0001	-0.008**	-0.010*	-0.006	-0.002	-0.007	-0.007
	(0.002)	(0.004)	(0.005)	(0.006)	(0.007)	(0.009)	(0.009)
fp_t	-0.0003	0.010**	0.006	0.001	-0.004	0.005	0.005
	(0.002)	(0.004)	(0.006)	(0.009)	(0.010)	(0.012)	(0.012)
$basis_t$	0.006	0.038***	0.051***	0.069***	0.086***	0.109***	0.103***
	(0.004)	(0.010)	(0.013)	(0.018)	(0.024)	(0.026)	(0.027)
Constant	0.001	0.004	0.002	0.010	0.018	0.020	0.019
	(0.004)	(0.007)	(0.009)	(0.009)	(0.012)	(0.014)	(0.014)
Observations	115	115	115	115	115	115	115
Adjusted R^2	-0.017	0.141	0.138	0.224	0.216	0.266	0.247
Note:				*p<0.1; **p<0.05; ***p<0.01			

Table 10: *US LRRs and Currency Excess Returns across Numeraires*

Description: This table reports estimation results for the regression below:

$$rx^i_{j,t} = \alpha + \beta LRR^{US}_t + \epsilon^i_{j,t}$$

The regressions rotate the numeraire currency j. The univariate regressions are reported in panel A and the multivariate regressions are reported in B. Sample period is 1988Q1-2017Q2

	Panel (a): Univariate Regressions							
	USD	AUD	CAD	DKK	JPY	NZD	CHF	GBP
LRR^{US}_t	0.101***	0.146**	0.189***	0.264***	0.375***	0.105*	0.221***	0.175***
	(0.035)	(0.062)	(0.051)	(0.072)	(0.065)	(0.062)	(0.063)	(0.060)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,115	1,115	1,115	1,115	1,115	1,115	1,115	1,115
Adjusted R^2	0.001	0.004	0.005	0.006	0.029	0.007	0.004	0.001

	Panel (b): With Controls							
	USD	AUD	CAD	DKK	JPY	NZD	CHF	GBP
LRR^{US}_t	0.137***	0.257***	0.269***	0.339***	0.376***	0.210***	0.275***	0.259***
	(0.034)	(0.059)	(0.049)	(0.071)	(0.065)	(0.059)	(0.062)	(0.058)
HML_t	0.068**	0.381***	0.235***	0.170***	−0.211***	0.363***	0.003	0.158***
	(0.033)	(0.056)	(0.048)	(0.061)	(0.055)	(0.057)	(0.059)	(0.056)
$(i^i_t - i^{US}_t)HML_t$	−0.342***	−0.464***	−0.487***	−0.375***	−0.272***	−0.417***	−0.619***	−0.714***
	(0.050)	(0.086)	(0.072)	(0.102)	(0.094)	(0.087)	(0.091)	(0.085)
$Dollar^{Global}_t$	-0.001**	-0.003***	-0.002***	-0.002**	-0.003***	-0.003***	-0.001*	-0.002**
	(0.0004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,115	1,115	1,115	1,115	1,115	1,115	1,115	1,115
Adjusted R^2	0.067	0.132	0.131	0.052	0.062	0.115	0.062	0.121

Note: *p<0.1; **p<0.05; ***p<0.01

US LRRs and Global FX Factor Structure: To build on these results suggesting a link between US LRRs and global currency risk, we investigate the general link between our US LRR factor and the global exchange rate factor structure. In specific terms we regress currency excess returns against our US LRR factor LRR_t^{US} and rotate the numeraire currency used to compute these excess returns. In specific terms, we run specifications of the form:

$$rx_{j,t}^i = \alpha + \beta LRR_t^{US} + \epsilon_{j,t}^i$$

$rx_{j,t}^i$ denotes the currency excess returns for an investor in country j who goes long currency i. We rotate the numeraire currency j and investigate the link between US relative LRRs and average currency excess returns denominated in any numeraire currency j. We present the results from these regressions in table 10.

Discussions: These results complement the prior result: they suggest a tight connection between US relative LRRs and the global factor structure in currency excess returns. A good US relative LRR shock ($LRR_t^{US} \uparrow$) increases average currency excess returns not only for a USD investor, but investors from the other G9 developed countries. These results are not spanned by other known global currency risk factors uncovered by Verdelhan (2018), as shown in panel B.

5.4 US LRRs vs Global LRRs

Overview: One obvious concern with the results shown in the previous section is that the link between US LRRs and the global FX factor structure is spanned by the global LRR factor. It might simply be the case that US LRRs simply load the most on this global LRR factor, explaining the strong connection between US LRRs and currency excess returns shown in the previous section.

Specification: To rule this possibility out, we define $\tilde{N}^{G}_{CF,t+1}$ as the global LRR factor orthogonalised w.r.t US LRR factor $N^{US}_{CF,t+1}$. We run the same currency-numeraire level regressions as before but using $\hat{N}^{G}_{CF,t+1}$ as the main independent variable instead:

$$rx^{i}_{j,t} = \alpha + \beta \tilde{N}^{G}_{CF,t} + \epsilon^{i}_{j,t}$$

Discussion: We present the results from this regression in table 11. These results suggest that the relationship between this orthogonalised global LRR factor and the global exchange rate factor structure is weaker than the corresponding relationship with US LRRs exhibited in table 10. Whilst there is significance in the univariate regressions, they largely wash away once the currency factor model of Verdelhan (2018) is accounted for. This is documented in panel B of table 11.

Big Picture: The results in table 10 and 11 suggest that US LRRs are a distinct source of global risk that is separate from the global LRR factor emphasised by Colacito et al (2018). Thus we interpret our evidence as supporting an international LRR model where two LRR factors-US and global- drive systematic variation in all bilateral exchange rates. We formalise this argument in the next section through a formal simulation exercise.

Table 11: *Global LRRs and Currency Excess Returns across Numeraires*

Descriptions: This table reports estimation results from the following regressions:

$$rx^i_{j,t} = \alpha + \beta NC^G_{CF,t} + \epsilon^i_{j,t}$$

Numeraire currency j is rotated across the regressions. Panel A reports the univariate regressions and Panel B reports the multivariate regressions.

	Panel (a): Univariate Regressions							
	USD	AUD	CAD	DKK	JPY	NZD	CHF	GBP
$NC^G_{CF,t}$	0.038	0.287***	0.194***	0.137***	-0.040	0.195***	0.008	0.127***
	(0.025)	(0.043)	(0.036)	(0.047)	(0.045)	(0.044)	(0.045)	(0.043)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,115	1,115	1,115	1,115	1,115	1,115	1,115	1,115
Adjusted R^2	-0.007	0.034	0.020	-0.001	-0.010	0.011	-0.009	-0.0001

	Panel B: With Controls							
	USD	AUD	CAD	DKK	JPY	NZD	CHF	GBP
$NC^G_{CF,t}$	-0.038	0.092*	0.044	-0.006	0.040	-0.007	−0.099**	-0.043
	(0.027)	(0.047)	(0.039)	(0.058)	(0.055)	(0.048)	(0.049)	(0.046)
HML_t	0.073**	0.322***	0.199***	0.158**	−0.266***	0.351***	0.020	0.148**
	(0.036)	(0.061)	(0.052)	(0.070)	(0.062)	(0.062)	(0.064)	(0.061)
$(i^i_t - i^{US}_t)HML_t$	-0.345***	-0.427***	-0.460***	-0.317***	-0.196**	-0.404***	-0.630***	-0.718***
	(0.051)	(0.087)	(0.073)	(0.106)	(0.099)	(0.088)	(0.092)	(0.086)
$Dollar^{Global}_t$	-0.0004	-0.002***	-0.001**	-0.001*	-0.003***	-0.002***	-0.001	-0.001
	(0.0004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,115	1,115	1,115	1,115	1,115	1,115	1,115	1,115
Adjusted R^2	0.052	0.116	0.102	0.024	0.023	0.097	0.045	0.099

Note: *p<0.1; **p<0.05; ***p<0.01

6 Calibration Exercise

Overview: Our empirical evidence presented thus far *qualitatively* supports the predictions of an international LRR model where there are two sources of global risk: US LRRs and global LRRs respectively. To show that such a model can also *quantitatively* match our evidence, we calibrate such a model in this section. The model is closely related to Colacito et al (2018), with the key departure being that the endowments of non-US countries load on both US and global long-run shocks. Conversely the only source of global risk the US is exposed to is the global long-run shock.

6.1 International LRR Model

Overview: There are $N+1$ countries indexed by $i \in \{0, 1, 2, ..., N\}$. Country 0 is the model analogue to the United States (US) and the remaining N countries compose the non-US world. Each country is an endowment economy that is home to a unique tradable good and is populated by a representative agent with Epstein and Zin (1989) and Weil (1991) preferences.

Processes: Key model dynamics are described below:

$$x^i_{t+1} = \mu + x^i_t - \tau(x^i_t - \frac{1}{N}\sum_{j=0}^{N+1} x^j_t) + \xi^i_{t+1} + z^i_t \qquad \{\text{Endowments}\}$$

$$z^i_{t+1} = \begin{cases} \rho_x z^i_t + \epsilon^i_{x,t+1} + \beta^i_{CF,t}\epsilon^G_{x,t+1} + \beta^i_{US,t}\epsilon^{US}_{x,t+1} & \text{if } i \neq US \\ \rho_x z^i_t + \beta^{US}_{CF,t}\epsilon^G_{x,t+1} + \epsilon^{US}_{x,t+1} & \text{if } i = US \end{cases} \qquad \{\text{Persistent Component}\}$$

Parameters

μ: *Mean Endowment Growth Rate*

τ: *Degree of Cointegration*[7]

β^i_{US}: *Country i's LRR Exposure to US LRRs*

β^i_{CF}: *Country i's LRR Exposure to Global LRRs*

[7] Colacito, Croce and Liu (2019) show that cointegration is required to ensure a well-defined ergodic distribution of the relative supply of the two goods.

Global shock exposures follow a slow-moving AR(1):

$$\beta^i_{CF,t} = \alpha + \tau\beta^i_{CF,t-1} + \xi_{CF,t} \tag{46}$$

$$\beta^i_{US,t} = \alpha + \tau\beta^i_{US,t-1} + \xi_{US,t} \tag{47}$$

Shocks: All shocks are IID and gaussian: contemporaneous shocks are uncorrelated across countries but long-run shocks are correlated: $cor(\xi^i_{t+1}, \xi^j_{t+1}) = \rho$ for $i \neq j$.

6.1.1 Consumption Markets

Consumption Preferences: Consumption streams for each country are defined over a general CES aggregator of the $N+1$ goods:

$$C^i_t = [\sum_{i=1}^{N+1} \alpha_{i,j}^{\frac{1}{\phi}} (C^i_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{48}$$

$C^i_{j,t}$: *country i's consumption of good j*

$\alpha_{i,j}$: *Country i's preference for good j*

ϕ: *Elasticity of Substitution across goods*

Consumption Home Bias: I assume that $\alpha_{i,i} = \alpha \in (\frac{1}{2}, 1)$, $\forall i$. Since $\alpha < 1$, agents value both goods so there will be international trade in equilibrium. However since $\alpha > \frac{1}{2}$, international risk sharing will be limited by a natural desire for a *home biased consumption basket*. Preference for all other foreign goods are symmetric: $\alpha_{i,j} = \frac{1-\alpha}{N}$, $\forall j \neq i$

Goods Prices: All consumption goods are internationally tradable at prices $\{p_{i,t}\}^N_{i=0}$ which are denominated in units of the *global numeraire*. I fix the *consumption basket of the US (country* 0*)* as the *global numeraire*. This means that goods prices $\{p_{i,t}\}^N_{i=0}$ are denominated in units of the US consumption basket.

Price Levels: Denote by Q_t^i the relative price of country i's consumption in units of the global numeraire. By construction:

$$Q_t^i = \begin{cases} \mathcal{E}_{i,t} = [\sum_{j=1}^{N+1} (\frac{1-\alpha}{N})^{\frac{1}{\phi}} (p_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} & \text{if } i \neq 0 \\ 1 & \text{if } i = 0 \end{cases} \tag{49}$$

Proof of these results are contained in theory appendix 9.7.1. Note that since country 0 (US)'s consumption basket is the global numeraire, Q_t^i is the real dollar exchange rate $\mathcal{E}_t$ denoted by country i's consumption units per country 0 (US) consumption units.

6.1.2 Preferences

Each country is populated by a representative investor that has Epstein and Zin (1989) and Weil (1989) recursive preferences. These preferences are defined over the local consumption basket C_t^i defined in 6.1.1. Thus, the lifetime utility of investor i satisfies:

$$U_t^i = [(1-\delta)(C_t^i)^{1-\frac{1}{\psi}} + \delta(E_t {U_{t+1}^i}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}, i \in \{0, 1, 2, ..., N\}$$

Parameters

δ: *Time Preference*

ψ: *Intertemporal Elasticity of Substitution (IES)*

γ: *Relative Risk Aversion*

C_t^i: *Consumption for country i at time t*

6.1.3 Financial Markets

Financial markets are dynamically complete: dollar appreciation rate $\Delta\mathcal{E}_t$ is pinned down by the equality of marginal utility growths (Backus, Foresi and Telmer, 2001):

$$\Delta\mathcal{E}_{i,t} = m_t^0 - m_t^i \tag{50}$$

6.1.4 Investor's Problem

Overview: Since markets are dynamically complete, the intertemporal budget constraint (IBC) can be written in static form:

$$\max_{\{\{C^i_{j,t}\}_{j=0}^{N+1}, W^i_{t+1}\}_{t=0}^{\infty}} U_0^i \tag{51}$$

$$s.t.\ \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i C_t^i \leq \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i W_t^i \tag{52}$$

$$Q_t^i C_t^i = \sum_{j=0}^{N} p_{j,t} C^i_{j,t} \tag{53}$$

$$C_t^i = [\sum_{i=1}^{N+1} \alpha_{i,j}^{\frac{1}{\phi}} (C^i_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{54}$$

$$\alpha_{i,i} = \alpha \in (\frac{1}{2}, 1),\ \alpha_{i,j} = \frac{1-\alpha}{N},\ \forall i \neq j \tag{55}$$

Λ_t is the world state price density that prices country i's wealth portfolio in units of the global numeraire.

6.1.5 Market Clearing Conditions and Equilibrium

Goods markets clears:

$$X_t^i = \sum_{j=0}^{N} C^i_{j,t}\ ,\ \forall i \tag{56}$$

Equilibrium: Equilibrium is defined as a set of prices: $\{p_{j,t}\}_{j=0}^{N+1}$, quantities: $\{C^i_{j,t}\}$ and wealth processes: $\{W^j_{t+1}\}_{j=0}^{N+1}$ such that: i) each household maximises utility (51) s.t (52) - (55), ii) goods markets clear according to (56).

6.2 Solution Method

Pareto Weight: The equilibrium system of equations is presented in table 13.[8] I follow Colacito et al (2018) and Anderson (2005) and recast the equilibrium in terms of the pareto weight

[8] I relegate the proof of this equilibrium system to theory appendix 9.7.2.

distribution $\{S_{i,t}\}_{i=1}^{N}$. $S_{i,t}$ denotes country i's ***relative*** *pareto weight* vis-*á*-vis country 0 (US). The equilibrium system implies that $\{S_{i,t}\}_{i=1}^{N}$ is a key variable that determines equilibrium consumption allocations (A1-A4), relative prices (A7-A10) and consequently asset prices (A14).

Solution Method: We numerically approximate the model to third order using dynare ++. The approximation point is the symmetric steady state where global consumption and wealth are equally shared. At this steady state, $S_{i,t} = \overline{\mathcal{S}} = 1\ \forall i$, $wc_t^i = pd_t^i = \overline{\mathcal{P}} = \frac{\delta}{1-\delta}$, $R_{m,t+1}^i = R_{t+1}^i = \overline{\mathcal{R}} = \frac{1}{\delta}$,$C_{i,t}^i = \alpha$, $C_{j,t}^i = 1-\alpha$, $C_t^i = \overline{\mathcal{C}} = 1$, $p_{i,t} = \mathcal{E}_t = 1$ and $M_t^i = \overline{\mathcal{M}} = e^{\delta}$. Taking at least a third order approximation is necessary to guarantee time varying currency risk premia in the model.

Baseline Calibration: I set $N = 6$, a small number to make the simulation tractable. During the burn-in period, country 6 is initialised to have the highest exposures and country 1 the lowest exposures. Country specific exposures are equally spaced between 0.1 and 0.6: $\beta_{CF}^i, \beta_{US}^i \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6\}$. All other parameters follows a symmetric calibration:[9]

Table 12: *Baseline Calibration*

Panel A: Preference Parameters		
Parameter	Description	Value
γ	Relative Risk Aversion	7.5
ψ	Intertemporal Elasticity of Substitution	2
α	Home Bias Parameter	0.96
δ	Discount Factor	0.99
ϕ	Elasticity of Substitution across Goods	0.2
Panel B: Endowment Parameters		
Parameter	Description	Value
μ	Mean Endowment Growth Rate	0.005
β	Cointegration Parameter	0.05
ρ_x	LRR Persistence	0.98
ρ	LRR correlations	0.98
τ	Exposure persistence	0.99

[9] Detailed discussion of calibration choices are contained in the online appendix.

Table 13: Equilibrium System

Exogenous Processes	
$(A1): x^i_{t+1} = \mu + x^i_t - \tau(x^i_t - \frac{1}{N}\sum_{j=0}^{N+1} x^j_t) + \xi^i_{t+1} + z^i_t$	{Endowments}
(A2): $z^i_{t+1} = \begin{cases} \rho_x z^i_t + \epsilon^i_{x,t+1} + \beta^i_{CF,t}\epsilon^G_{x,t+1} + \beta^i_{US,t}\epsilon^{US}_{x,t+1} & \text{if } i \neq US \\ \rho_x z^i_t + \beta^{US}_{CF,t}\epsilon^G_{x,t+1} + \epsilon^{US}_{x,t+1} & \text{if } i = US \end{cases}$	{Persistent Component}
$(A3): \beta^i_{CF,t} = \alpha + \tau\beta^i_{CF,t-1} + \xi_{CF,t}$	
(A4): $\beta^i_{US,t} = \alpha + \tau\beta^i_{US,t-1} + \xi_{US,t}$	
Consumption FOCs	
$(A4):\ C^i_{i,t} = X^i_t[1 + \frac{1-\alpha}{\alpha(N-1)}\sum_{j\neq i}\frac{S_{j,t}}{S_{0,t}}]^{-1}$	
$(A5):\ C^i_{j,t} = \frac{1-\alpha}{\alpha}\frac{1}{N-1}\frac{S_{j,t}}{S_{i,t}}C^i_{i,t}$	
Net Exports (vis-*á*-vis the US)	
$(A6):\ NX^i_t = X^i_t - C^i_{0,t}$	
Consumption Aggregators	
$(A7): C^i_t = [\sum_{j=1}^{N+1}\alpha_{i,j}^{\frac{1}{\phi}}(C^i_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}}$	
Relative Prices	
$(A8):\ p_{i,t} = (\alpha_{0,i}\frac{C^0_t}{C^0_{i,t}})^{\frac{1}{\phi}}$	
Price Levels	
$(A9):\ Q^i_t = \begin{cases} \mathcal{E}_{i,t} = [\sum_{j=1}^{N+1}(\frac{1-\alpha}{N})^{\frac{1}{\phi}}(p_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} & \text{if } i \neq 0 \\ 1 & \text{if } i = 0 \end{cases}$	
State Variable	
$(A10):\ S_{i,t} = S_{i,t-1}(\frac{M^i_t}{M^0_t})^{\phi}(\frac{C^i_t/C^i_{t-1}}{C^0_t/C^0_{t-1}})$	
Global Consumption Shares	
$(A11):\ SWC^i_t = \frac{Q^i_t C^i_t}{\sum_{j=0}^{N+1} p_{j,t}X_{j,t}}$	
Wealth-Consumption Ratios	
$(A12):\ wc^i_t = [\mathbb{E}_t e^{\theta[ln\delta + (1-\frac{1}{\psi})\Delta c^i_{t+1} + log(1+wc^i_{t+1})]}]^{\frac{1}{\theta}}$	
Wealth Returns	
$(A13):\ R^i_{m,t+1} = \frac{(1+wc^i_{t+1})e^{\Delta c^i_{t+1}}}{wc^i_t}$	
Price-Dividend Ratios	
$(A14):\ pd^i_t = \mathbb{E}_t e^{\theta ln\delta - \frac{\theta}{\psi}\Delta c^i_{t+1} + (\theta-1)log(R^i_{m,t+1}) + log(1+pd^i_{t+1}) + \Delta x^i_{t+1} + \Delta p^i_{t+1}}$	
Equity Returns	
$(A15):\ R^i_{t+1} = \frac{(1+pd^i_{t+1})e^{\Delta x^i_{t+1}}}{pd^i_t}$	
SDFs	
$(A16):\ M^i_{t+1} = e^{\theta ln\delta - \frac{\theta}{\psi}\Delta c^i_{t+1} + (\theta-1)log(R^i_{m,t+1})}$	
Exchange Rate	
$(A17):\ \Delta\mathcal{E}_{i,t+1} = log(\frac{M^0_{t+1}}{M^i_{t+1}})$	

6.3 Simulation Results

Simulated Regressions: To evaluate the quantitative performance of the model, we compare empirical regression results against their theory counterparts using simulated data from the model. The model is a quarterly calibration where the average results over 1,000 simulations of 100 quarters each is used to estimate the model regressions. The results are depicted in table 14.

Table 14: *Model vs Simulated Regressions*

Description: This table compares regression results from the data against the simulation from the calibrated LRR model. For the data regressions, sample period is 1988Q1-2017Q2. For the model regressions, simulations are over 100 quarters for 1,000 simulations.

	Coefficient	*Data*	*Model*
Panel (a): Bilateral Currency Excess Returns			
$\mathrm{rx}^q_{i,t} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	0.092	0.582
$\mathrm{rx}^q_{i,t,t+4} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	-0.058	-0.155
		(0.030)	
$\mathrm{rx}^q_{i,t,t+8} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	-0.233	-0.238
		(0.073)	
$\mathrm{rx}^q_{i,t,t+12} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	-0.430	-0.331
		(0.072)	
$\mathrm{rx}^q_{i,t,t+16} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	-0.287	-0.388
		(0.121)	
$\mathrm{rx}^q_{i,t,t+20} = \alpha + \beta(N^{US}_{CF,t} - N^i_{CF,t}) + \epsilon$	β	-0.258	-0.431
		(0.152)	
Panel (a): HML Carry Trade			
$\mathrm{HML}_t = \alpha + \beta N^G_{CF,t} + \epsilon$	β	0.226	0.594
		(0.064)	
Panel (a): HML Dollar Beta Portfolio			
$\mathrm{Dollar}^{Global}_t = \alpha + \beta N^{US}_{CF,t} + \epsilon$	β	0.243	0.624
		(0.098)	
Note:	*p<0.1; **p<0.05; ***p<0.01		

Discussion: Consistent with our early analysis, the simulation results show that the model qualitatively matches the empirical regression results: when country i suffers a bad LRR shock, currency i appreciates against the dollar before subsequently depreciating moving forward. They also show that the HML carry trade sorted on interest rate differentials and the HML dollar beta portfolio load on global and US LRRs respectively, consistent with our empirical evidence. Quantitatively these model responses are by and large close to the empirical estimates, suggesting that an international LRR model with two global factors- US and global LRRs respectively, can largely account for systematic variation in all bilateral exchange rates.

7 Conclusion

In conclusion, this paper investigates the joint dynamics between long run consumption risks (LRRs), currency excess returns and global currency risk factors. Using a novel identification strategy to identify country level LRRs, we uncover four main results. Firstly, currency excess returns and relative LRRs are negatively correlated: the currencies of countries that suffer bad relative long run shocks vis-a-vis the US appreciate against the dollar on average. Secondly, currency risk premia and relative LRRs are positively correlated: over the long run such currencies depreciate against the dollar, resulting in lower expected currency returns moving forward. Thirdly the High-Minus-Low (HML) and dollar carry trades are highly correlated with appropriately constructed global and US LRR factors respectively. Finally US relative long run shocks vis-a-vis the ROW drive traditional global currency risk factors.

Taken together, these four facts support an international LRR model where two LRR factors- US and global- drive the global factor structure in bilateral exchange rates. To formalise this argument, we calibrate an international LRR model where US and global LRRs drive common sources of risk in the world economy. The model qualitatively and quantitatively matches all our empirical findings. We interpret this as evidence that US LRRs are a distinct source of global risk pricing currency markets. Thus US growth prospects are an important economic driver behind the global exchange rate factor structure, a novel insight unique to this paper that should inform future work in international macro-finance.

8 References

Bansal, R., Yaron, A. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59(4), 1481-1509.

Bansal, R; Kiku, D; Yaron, A. 2010. Long run risks, the Macroeconomy and Asset Prices. *American Economic Review* 100(2): 542-546

Bansal, R; Kiku, D; Shaliastovich, I; Yaron, A. 2014. Volatility, the Macroeconomy and Asset Prices. *The Journal of Finance* 69(6): 2471-2511

Bansal, R; Shaliastovich, I. 2013. A long run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies* 26: 1-33

Bekaert, G., Harvey, C. R. 1997. Emerging equity market volatility. *Journal of Financial economics*, 43(1), 29-77.

Bekaert, G., Harvey, C. R. 2017. Emerging equity markets in a globalizing world. Available at SSRN 2344817.

Bekaert, G., Harvey, C. R., Lumsdaine, R. L. 2002. Dating the integration of world equity markets. *Journal of Financial Economics* 65(2): 203-247.

Bekaert, G., Wang, X. 2010. Inflation risk and the inflation risk premium. *Economic Policy*, 25(64), 755-806.

Boehm, C., Kroner, N. 2020. The US, economic news, and the global financial cycle. *Working Paper.*

Brandt, M. W., Cochrane, J. H., Santa-Clara, P. 2006. International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics* 53(4): 671-698.

Brusa, F., Savor, P., Wilson, M. (2020). One central bank to rule them all. *Review of Finance*, 24(2), 263-304.

Burnside, C., Eichenbaum, M., Rebelo, M. 2011. Do peso problems explain the returns to the carry trade?. The Review of Financial Studies, 24(3), 853-891.

Campbell, J. 1993. Intertemporal asset pricing without consumption data. *American Economic Review* 83: 487—512

Campbell, J., Shiller, R. (1988a). The dividend-price ratio and expectations of future dividends and discount factors. The Review of Financial Studies, 1(3), 195-228.

Campbell, J; Giglio, S; Polk, C; Turley, R. 2018. An intertemporal CAPM with stochastic volatility. *Journal of Financial Economics* 128(2): 207—233

Campbell, J; Vuolteenaho, T. 2004. Bad beta, good beta. *American Economic Review* 94: 1249-1275

Chiang, I., Ethan, H., Mo, X. N. 2022. A'Bad Beta, Good Beta'Anatomy of Currency Risk Premiums and Trading Strategies. Available at SSRN 3393265.

Chen, H., Dou, W. W., Kogan, L. 2019. Measuring "Dark Matter" in Asset Pricing Models. [National Bureau of Economic Research].

Chen, L., Zhao, X. 2009. Return Decomposition. *Review of Financial Studies* 22:5213—5249.

Colacito, R; Croce, M. 2011. Risks for the long run and the real exchange rate. *Journal of Political Economy* 119(1): 153-181

Colacito, R; Croce, M. 2013. International Asset Pricing with Recursive Preferences. *The Journal of Finance* 68(6): 2651-2686

Colacito, R; Croce, M; Gavazzoni, F; Ready, R. 2018. Currency risk factors in a recursive multicountry setting. *The Journal of Finance* 73(6): 2719-2756

Engle, R. F., Kroner, K. F. 1995. Multivariate simultaneous generalized ARCH. *Econometric theory*: 122-150.

Engel, C. 2014. Exchange rates and interest parity. *Handbook of international economics*, 4, 453-522.

Fama, E. F. 1984. Forward and spot exchange rates. *Journal of monetary economics*, 14(3), 319-338.

Hansen, L., Hodrick, R. (1980). Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *Journal of political economy*, 88(5), 829-853.

Jiang, Z. 2021. US Fiscal Cycle and the Dollar. *Journal of Monetary Economics, Forthcoming*

Lustig, H., Roussanov, N., Verdelhan, A. 2011. Common risk factors in currency markets. *The Review of Financial Studies* 24(11): 3731-3777.

Lustig, H., Roussanov, N., Verdelhan, A. 2014. Countercyclical currency risk premia. *Journal of Financial Economics* 111(3): 527-553.

Mueller, P., Tahbaz-Salehi, A., Vedolin, A. 2017. Exchange rates and monetary policy uncertainty. *The Journal of Finance*, 72(3), 1213-1252.

Restoy, F., Weil, P. 2011. Approximate equilibrium asset prices. *Review of Finance* 15(1): 1-28.

Schorfheide, F; Song, D; Yaron, A. 2018. Identifying long-run risks: A Bayesian mixed-frequency approach. *Econometrica* 86(2): 617-654.

Slesnick, D. 1998. Are our data relevant to the theory? The case of aggregate consumption. *Journal of Business and Economic Statistics* 16: 52–61.

Stambaugh, R. 1999. Predictive regressions. *Journal of Financial Economics* 54(3): 375-421.

Verdelhan, A. 2018. The Share of Systematic Variation in Bilateral Exchange Rates. *The Journal of Finance* 123 (1): 375-418

9 Online Appendix

9.1 Data

9.1.1 State Variables

Market Return Data: We obtain equity index returns from the MSCI Indexes available via Thomson Datastream. We collect total return indices in order to capture the effect of reinvested dividends on equity returns. Index data is denominated in the respective domestic countries and contains the same 41 countries in the dividend yield sample along with the United States. Following Cenedese et al (2016)'s approach we complement MSCI data with individual index returns for Japan, the United States, France and Germany using SP500, FTSE,CAC,DAX and TOPIX data. This is because individual index data for these countries is available over a longer time series than the MSCI data, but as soon as the MSCI series is available we use those data instead.

Dividend Yields: We obtain raw dividend yield series from Thompson Datastream Equity Index which covers the period from January 1965 to December 2019 for the following 41 countries: Australia, Austria, Argentina, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Croatia, China, Colombia, Denmark, Egypt, Finland, France, Germany, Greece, Hungary, Indonesia, Ireland, Israel, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, Norway, Peru, Philipines, Poland, Portugal, Russia, Singapore, Slovenia, South Africa, South Korea, Sweden, Switzerland, Thailand, United Kingdom. In addition, we collect US Dividend Yield Data from Robert Shiller's public website. This gives us a longer time series for the US beginning in January 1923.

Term Spread: We collect interest rate data on Treasury Bills (90 day) and 10 year inflation indexed treasury bonds from global financial data. We define the term spread as the difference between these two rates for each country.

9.1.2 Macro Data

Consumption Data: To use consumption growth differentials $\Delta c_t^{US} - \Delta c_t^i$ as a control in our regressions, we obtain country level data for consumption from the OECD at the quarterly frequency. Following Colacito et al (2018), we use the volume index of private consumption expenditure for each country as the consumption series for each country. The full dataset is an unbalanced panel for each of the developed countries starting from 1961Q1 to 2021Q1. We define consumption growth for each country as log quarterly changes in the volume index for each country.

Price Level: To construct real currency excess returns, we need a measure inflation differentials. We follow standard practice and use the GDP deflator that is publicly available from Oxford Economics via Datastream.

9.1.3 Exchange Rate Data

Exchange Rates: We obtain end-of-month spot and one month forward exchange rate data from the WMR / Reuters and Barclays Bank International (BBI) databases available via Thomson Reuters Datastream. Currencies are indirectly quoted against the USD with our data documenting exchange rates in terms of 1 USD. Following Burnside, Eichenbaum and Rebelo (2011), we use BBI data from December 1983 to November 1996 and then use WMR quotes as soon as they are available from December 1986. We also follow their methodology of converting WMR quotes against the GBP to the USD for dates between January 1976 to November 1983. This extends our time series by 7 years, starting in January 1976 for our sample of 11 developed countries: Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom and the United States.

Portfolios: We use the High-Minus-Low (HML) carry portfolios (HML1-HML6) and the dollar beta sorted portfolios (rp1-rp6) from Adrien Verdelhan's website: http://web.mit.edu/adrienv/www/Data.html. This dataset constructs these portfolios at a monthly frequency. Since we work at a quarterly frequency, we simply aggregate the returns to a quarterly frequency.

9.2 Robustness Checks

Other Proxies: Here we explore the robustness of our results for bilateral currency dynamics using a different set of currency risk premium proxies. In particular we consider the currency factor model posited by Verdelhan (2018) which has been shown to capture a large portion of common variation in bilateral exchange rates. This factor model is contained in vector X_t^{RISK}:

$$X_t^{RISK} = \begin{bmatrix} HML_t & HML_t(i_t^{US} - i_t^i) & Dollar_t^{Global} \end{bmatrix}^T \tag{57}$$

HML_t: Unconditional HML Carry Trade

$HML_t(i_t^{US} - i_t^i)$: Conditional HML Carry Trade

$Dollar_t^{Global}$: HML Dollar Beta Currency portfolio constructed by Verdelhan (2018)

Table 15: *Verdelhan (2018) robustness check.*

Description: This table reports estimation results for the predictive regressions outlined by (38). Panel A reports the univariate regressions. Panels B and C report the multivariate results where various FX and risk controls are added. Country fixed effects are added and standard errors are clustered at the country level.

	Panel (c): Risk Controls						
	rx_t^i	$rx_{t,t+1}^i$	$rx_{t,t+4}^i$	$rx_{t+4,t+8}^i$	$rx_{t+8,t+12}^i$	$rx_{t+12,t+16}^i$	$rx_{t+16,t+20}^i$
$N_{CF,t}^{US} - N_{CF,t}^i$	0.121***	0.008	0.009	−0.211**	−0.247***	0.161**	0.024
	(0.034)	(0.013)	(0.046)	(0.092)	(0.071)	(0.072)	(0.045)
$\Delta c_t^{US} - \Delta c_t^i$	0.032***	0.022**	0.116*	0.139*	0.177***	0.134**	0.117*
	(0.012)	(0.010)	(0.063)	(0.080)	(0.056)	(0.063)	(0.063)
HML_t	0.087	0.032	0.171***	0.026	0.108**	−0.143***	−0.298***
	(0.090)	(0.031)	(0.052)	(0.066)	(0.052)	(0.051)	(0.053)
$(i_t^i - i_t^{US})HML_t$	-0.354***	-0.012	0.564***	0.080	0.042	0.029	-0.429***
	(0.080)	(0.042)	(0.120)	(0.084)	(0.059)	(0.025)	(0.075)
$Dollar_t^{Global}$	-0.001***	-0.009***	-0.011***	-0.002***	0.004***	-0.001***	-0.00005
	(0.0003)	(0.001)	(0.001)	(0.0005)	(0.001)	(0.001)	(0.001)
Country FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	996	987	960	924	890	854	819
Adjusted R^2	0.092	0.437	0.192	0.020	0.043	0.014	0.013
Note:	*p<0.1; **p<0.05; ***p<0.01						

Other Numeraires: In this section, we explore whether the baseline results linking relative LRRs to currency excess returns is specific to the numeraire choice (USD). To investigate this, we rotate the numeraire and re-estimate the specifications in table 4. This involves regressing relative LRRs between the numeraire country 0 and country i and currency excess returns rx^i_{t+1} from the perspective of that numeraire country. These results are presented below:

Table 16: *Other Numeraires*

Description: This table re-estimates equation (33) after rotating the numeraire currency. Results are reported for both real and nominal currency excess returns

	Dependent variable: Nominal Currency Excess Returns						
	AUD	CAD	DKK	JPY	NZD	CHF	GBP
$N^{US}_{CF,t+1} - N^i_{CF,t+1}$	0.003	0.094***	0.032	-0.071**	0.022	0.037	0.138***
	(0.039)	(0.033)	(0.040)	(0.030)	(0.038)	(0.039)	(0.042)
$N^{US}_{C,t+1} - N^i_{C,t+1}$	0.012	0.019	0.003	0.013	0.006	-0.030	0.003
	(0.022)	(0.026)	(0.028)	(0.019)	(0.026)	(0.021)	(0.030)
Observations	1,115	1,115	1,115	1,115	1,115	1,115	1,115
	Dependent variable: Real Currency Excess Returns						
	AUD	CAD	DKK	JPY	NZD	CHF	GBP
$N^{US}_{CF,t+1} - N^i_{CF,t+1}$	-0.007	-0.094***	0.029	0.072**	0.024	0.035	-0.139***
	(0.039)	(0.033)	(0.041)	(0.030)	(0.038)	(0.040)	(0.042)
$N^{US}_{C,t+1} - N^i_{C,t+1}$	0.012	0.018	0.001	0.014	0.009	-0.031	0.001
	(0.022)	(0.026)	(0.028)	(0.019)	(0.026)	(0.021)	(0.030)
Observations	1,101	1,069	834	882	1,026	1,119	1,119
Note:					*p<0.1; **p<0.05; ***p<0.01		

These results are comparatively weaker than the results presented in table 4 using the USD numeraire, suggesting that the link between relative LRRs and currency excess returns is more powerful using the USD numeraire. However the table does suggest that the negative relationship between currency excess returns and relative LRRs between numeraire country 0 and country i predicted by proposition 1 seems to hold equally well for other well known numeraires such as the Canadian Dollar (CAD) and the UK Pound (GBP). Evidence in favour of proposition 1 seems to be weaker for other numeraires however.

Volatility Modelling: The baseline VAR framework outlined in the main text that was used to estimate country specific consumption news $N^i_{CF,t}$ assumed an *affine* structure for stochastic volatility. However this is likely to be a misspecification: volatility news $N^i_{v,t+1}$ is unlikely to be linearly related to the state variable vector z^i_{t+1}. Realized volatility is highly persistent and is highly correlated across the world: volatility spillovers are common, especially across developed markets.[10]

To remedy this shortcoming we propose moving away from the affine structure and adopting a non-linear specification that takes these volatility dynamics more seriously. The natural specification is a GARCH modelling environment: We now assume that the exogenous state vector z^i_{t+1} follows an MGARCH-VAR:

$$\begin{aligned}
&z^i_{t+1} = \mu_i + \Gamma(z^i_t - \mu_i) + \xi^i_{t+1} \\
&\xi^i_{t+1} = H_t^{\frac{1}{2}} \epsilon^i_{t+1} \\
&H_t = C_v + A'\xi^i_t(\xi^i_t)'A + G'H_{t-1}G \\
&\epsilon^i_t, \sim N(0,\, I)
\end{aligned} \tag{58}$$

H_t is an $n \times n$ conditional variance-covariance matrix that governs the conditional covariances of shocks to the state vector ξ^i_{t+1}. H_t drives the conditional mean of z^i_t through gaussian shocks ϵ^i_t. C_v, A and G are all $n \times n$ constant parameter matrices. We relegate full technical discussion of the MGARCH VAR estimation prodcedure in appendix 9.6.2

We estimate the MGARCH VAR system under three sets of assumptions for A and G. First we estimate consumption news $N^i_{CF,t}$ for each country without restricting the parameter matrices A and G. **?** refer to this as the *full BEKK*. We then estimate by restricting the off-diagonal parameters of matrices A and G to be zero, known as the *diagonal BEKK*. Finally, we estimate by restricting all parameters of matrix G to zero, that is, we estimate an *ARCH* model.

Once convergence is achieved the parameter estimates and shocks to the state variable process are then used to estimate $N^i_{CF,t+1}$ in the manner described in appendix 9.6.2. Subsequently we re-estimate the baseline specification captured in table 4 using the MGARCH-VAR framework.

[10]See Bekaert and Harvey (1997); Ng (2000) and Bekaert, Harvey and Lumsdaine 2002

Table 17: *MGARCH-VAR Specification*

Description: This table re-estimates equation (33) after re-estimating consumption news $N^i_{CF,t}$ using the MGARCH-VAR framework outlined in (85). Results are presented for the Full BEKK, Diagonal BEKK and the ARCH cases.

	Dependent variable: rx^i_{t+1}		
	Full BEKK	Diagonal BEKK	ARCH
$N^{US}_{CF,t+1} - N^i_{CF,t+1}$	0.076***	0.095***	0.063***
	(0.029)	(0.036)	(0.026)
Observations	996	996	996
Adjusted R^2	0.020	0.073	0.064

Note:
*p<0.1; **p<0.05; ***p<0.01

These results suggest that the baseline results in favour of the LRR framework are not compromised by the affine volatility assumption. Adopting a more sophisticated approach to modelling volatility news does not remove our baseline results in favour of the LRR framework. It is still the case that good US relative consumption news vis-*á*-vis country i is stil associated with higher log excess returns for a US investor going long in currency i.

9.3 LRR Identification Issues

Overview: One might be concerned that the ICAPM-VAR framework used to identify consumption news may not be appropriately capturing LRRs in consumption. A common critique of this framework is that since the methodology backs out consumption news from discount rate news using the **?** decomposition, the accuracy of the consumption news terms $N^i_{CF,t}$ in measuring LRRs is sensitive to the specification of the state system. Thus misspecification of the state system can result in $N^i_{CF,t}$ also capturing discount rate news that $N^i_{CF,t}$ may be a poor of country i's long run consumption news.

Growth Forecasts: To address this criticism, we establish that $N^i_{CF,t}$ is tightly connected to future GDP growth forecasts as measured by the OECD. These growth forecasts are one quarter ahead forecasts that are available at the country level. We denote these forecasts as $Forecast_t^i$ and regress them against our consumption news terms $N_{CF,t}^i$ and control for macro fundamentals.

These regressions are presented in table 18 and suggest that there is a strong positive relationship between the two: increases in country i's consumption news $N_{CF,t}^i$ are indeed associated with increased growth forecasts for that country. These results are not spanned by macro fundamentals. Hence our empirical proxy for LRRs does indeed track shifts in country level growth expectations.

Table 18: *Consumption News and GDP Growth Forecasts*

Description: This table reports estimation results of GDP Growth forecast Changes $\Delta Forecast_t^i$ against consumption news $N_{CF,t}^i$:

$$\Delta Forecast_t^i = \alpha + \beta N_{CF,t}^i + \epsilon_t \tag{59}$$

The regressions control for know macro fundamentals such as consumption growth Δc_t^i, IP growth ΔIP_t^i and GDP growth GDP_t^i.

	Dependent variable: $\Delta Forecast_t^i$			
	(1)	(2)	(3)	(4)
$N_{CF,t}^i$	0.027**	0.029**	0.026**	0.031***
	(0.011)	(0.012)	(0.011)	(0.012)
Δc_t^i		-0.005		
		(0.003)		
ΔIP_t^i			0.038***	
			(0.010)	
ΔGDP_t^i				0.014
				(0.011)
Time FE	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes
Observations	1,458	1,295	1,452	1,295
Note:		*p<0.1; **p<0.05; ***p<0.01		

9.4 Other Empirical Results

9.4.1 Dollar Risk Premium

Dollar Risk Premium: Here we consider the link between US relative LRRs and the dollar risk premium: the ex-ante risk premium associated with the dollar carry trade. We consider predictive regressions of the form:

$$rx^{Dollar}_{t+j,t+k} = \alpha + \beta_{j,k} LRR^{US}_t + \epsilon_t \tag{60}$$

Hypothesis: The central coefficient of interest is $\beta_{j,k}$. Proposition 4 implies that a good US relative LRR shock ($LRR^{US}_t \uparrow$) *decreases* the risk premium that a USD investor demands for shorting the dollar against a basket of foreign currencies. To deliver these lower expected future returns on the dollar carry trade, the dollar depreciates on impact before appreciating moving forward. Thus international LRRs models impose the following restrictions on these coefficients:

Table 19: *Testable Implications for Dollar Risk Premia*

Specification	Testable Implication
Short Run	$\beta_{j,k} > 0$ for $j, k = 0$
Long Run	$\beta_{j,k} < 0$ for $\forall j, k > 0$

The estimation results from the predictive regressions proposed by (60) are presented in the table 20. Panel A reports the univariate predictive regressions and panel B reports the multivariate specification result.

Discussions: Consistent with the international LRR models, table 20 suggests that a positive US relative LRR shock ($LRR^{US}_t \uparrow$) ***lowers*** the dollar risk premium, ***lowering*** expected future returns on the dollar carry trade. Consistent with earlier results on bilateral currency risk premia, these effects largely mean-revert after three years, as evidenced by the negative coefficients on $rx^{Dollar}_{t,t+4}, rx^{Dollar}_{t+4,t+8}, rx^{Dollar}_{t+8,t+12}$. However note that the magnitudes of the future dollar return response are larger than the bilateral case reported earlier: the cumulative 3 year return response ($rx^{Dollar}_{t,t+12}$) is close to a 100 basis point decline in the univariate case and a 70 basis point decline for the multivariate case.

Table 20: *US LRRs and Dollar Risk Premia*

Description: This table reports predictive regression results for US relative LRRs: LRR^{US}_t for dollar carry trade returns. Panel A reports the univariate results and Panel B reports the multivariate extension. Standard errors are heteroskedasticity-robust. Sample period is 1988Q1-2017Q2

	Panel (a): Univariate						
	rx^{Dollar}_t	$rx^{Dollar}_{t,t+1}$	$rx^{Dollar}_{t,t+4}$	$rx^{Dollar}_{t+4,t+8}$	$rx^{Dollar}_{t+8,t+12}$	$rx^{Dollar}_{t+12,t+16}$	$rx^{Dollar}_{t+16,t+20}$
LRR^{US}_t	0.158*	-0.135*	-0.042	-0.533***	-0.255*	0.378**	0.001
	(0.088)	(0.074)	(0.160)	(0.140)	(0.150)	(0.167)	(0.185)
Constant	-0.001	0.001	0.003	0.008	0.003	-0.001	-0.001
	(0.004)	(0.004)	(0.009)	(0.008)	(0.009)	(0.009)	(0.009)
Observations	118	117	114	110	106	102	98
Adjusted R^2	0.022	0.014	-0.008	0.070	0.009	0.031	-0.010

	Panel (b): FX Controls						
	rx^{Dollar}_t	$rx^{Dollar}_{t,t+1}$	$rx^{Dollar}_{t,t+4}$	$rx^{Dollar}_{t+4,t+8}$	$rx^{Dollar}_{t+8,t+12}$	$rx^{Dollar}_{t+12,t+16}$	$rx^{Dollar}_{t+16,t+20}$
LRR^{US}_t	0.134*	-0.113*	0.062	-0.440***	-0.192	0.472***	0.103
	(0.071)	(0.068)	(0.102)	(0.131)	(0.155)	(0.150)	(0.156)
US Surplus-Debt Ratio$_t$	-0.010	-0.030***	-0.101***	-0.060***	0.0003	0.015	0.037*
	(0.008)	(0.008)	(0.012)	(0.013)	(0.012)	(0.013)	(0.019)
$basis_t$	0.086***	0.049***	0.046	-0.048**	-0.046**	-0.016	0.060**
	(0.018)	(0.013)	(0.030)	(0.024)	(0.023)	(0.025)	(0.026)
fp_t	0.004	0.014*	0.009	-0.001	0.002	0.027*	0.023
	(0.009)	(0.008)	(0.014)	(0.016)	(0.016)	(0.015)	(0.016)
q_t	0.0002	0.002***	0.006***	0.004***	0.001*	0.002**	0.002**
	(0.0003)	(0.0004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Constant	-0.017	-0.218***	-0.806***	-0.537***	-0.170*	-0.198*	-0.133
	(0.045)	(0.049)	(0.075)	(0.095)	(0.093)	(0.111)	(0.106)
$basis_t$	0.068***	0.066***	0.055***	-0.049***	-0.045***	-0.019**	0.047***
	(0.010)	(0.012)	(0.016)	(0.013)	(0.014)	(0.008)	(0.015)
Constant	-0.012***	-0.025***	-0.098***	-0.062***	-0.001	0.017*	0.051***
	(0.003)	(0.003)	(0.010)	(0.008)	(0.007)	(0.010)	(0.007)
Observations	118	117	114	110	106	102	98
Adjusted R^2	0.272	0.216	0.494	0.277	0.039	0.170	0.206
Note:	*p<0.1; **p<0.05; ***p<0.01						

Risk Premium on $Dollar^{Global}_t$: To further illustrate the connection between US relative LRRs and $Dollar^{Global}_t$, we investigate the predictive power of US relative LRRs for $Dollar^{Global}_t$. These results are shown in table 21.

Table 21: *US LRRs and $Dollar_t^{Global}$*

Description: This table reports estimation results associated with predictive regressions of the form::

$$Dollar^{Global}_{t+j,t+k} = \alpha + \beta_{j,k} LRR^{US}_t + \epsilon_t \tag{61}$$

Panels A and B reports the univariate and multivariate regressions respectively.

	Panel (a): Univariate						
	$Dollar_t^{Global}$	$Dollar_{t,t+1}^{Global}$	$Dollar_{t,t+4}^{Global}$	$Dollar_{t+4,t+8}^{Global}$	$Dollar_{t+8,t+12}^{Global}$	$Dollar_{t+12,t+16}^{Global}$	$Dollar_{t+16,t+20}^{Global}$
LRR_t^{US}	0.273**	-0.201*	-0.113	-0.792***	-0.099	0.342*	-0.028
	(0.118)	(0.106)	(0.200)	(0.174)	(0.183)	(0.184)	(0.217)
Constant	0.003	0.005	0.019*	0.021**	0.018	0.011	0.015
	(0.005)	(0.005)	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)
Observations	115	114	111	107	103	99	95
Adjusted R^2	0.048	0.022	-0.007	0.115	-0.008	0.015	-0.011

	Panel (b): FX Controls						
	$Dollar_t^{Global}$	$Dollar_{t,t+1}^{Global}$	$Dollar_{t,t+4}^{Global}$	$Dollar_{t+4,t+8}^{Global}$	$Dollar_{t+8,t+12}^{Global}$	$Dollar_{t+12,t+16}^{Global}$	$Dollar_{t+16,t+20}^{Global}$
LRR_t^{US}	0.247**	-0.176*	-0.007	-0.741***	-0.067	0.436**	0.071
	(0.098)	(0.095)	(0.151)	(0.174)	(0.191)	(0.175)	(0.207)
US Surplus-Debt Ratio_t	-0.009	-0.033***	-0.103***	-0.037**	0.008	0.0004	0.015
	(0.011)	(0.010)	(0.015)	(0.017)	(0.017)	(0.016)	(0.021)
$basis_t$	0.104***	0.055***	0.047	-0.019	-0.037	0.010	0.023
	(0.028)	(0.019)	(0.037)	(0.033)	(0.033)	(0.032)	(0.033)
fp_t	0.006	0.021*	0.017	-0.013	0.008	0.037**	0.001
	(0.013)	(0.011)	(0.019)	(0.020)	(0.020)	(0.018)	(0.021)
q_t	0.0001	0.002***	0.006***	0.002***	0.0004	0.002**	0.002**
	(0.0004)	(0.0005)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Constant	0.002	-0.222***	-0.785***	-0.320***	-0.020	-0.244*	-0.219*
	(0.059)	(0.064)	(0.099)	(0.120)	(0.119)	(0.143)	(0.129)
Observations	115	114	111	107	103	99	95
Adjusted R^2	0.240	0.155	0.328	0.159	-0.026	0.085	0.042
Note:	*p<0.1; **p<0.05; ***p<0.01						

9.5 Proofs

9.5.1 Equilibrium SDF

Since international financial markets are dynamically complete, both the home and foreign SDF will price the wealth portfolios of each country. For simplicity we use the home SDF to price the home wealth portfolio and the foreign SDF to price the foreign wealth portfolio. This implies the following asset pricing restrictions hold:

$$\mathbb{E}_t[e^{m^H_{t+1}+r^H_{m,t+1}}] = 1 \tag{62}$$

$$\mathbb{E}_t[e^{m^F_{t+1}+r^F_{m,t+1}}] = 1 \tag{63}$$

SDF: For recursive utility, the equilibrium SDF for country i follows:

$$m^i_{t+1} = \theta ln\delta - \frac{\theta}{\psi}\Delta c^i_{t+1} + (\theta-1)r^i_{m,t+1} \tag{64}$$

Utilizing the Campbell-Shiller (1989) approximation, $r^H_{m,t+1}$ follows:

$$r^i_{m,t+1} = \kappa_0 + \kappa_1 wc_{i,t+1} - wc_{i,t} + \Delta c^i_{t+1} \tag{65}$$

$wc_{i,t}$ is the log wealth-consumption ratio for country i. Plugging (65) into (65) implies that SDF shocks follow:

$$\tilde{m}^i_{t+1} = \underbrace{(\theta - 1 - \frac{\theta}{\psi})}_{-\gamma}\Delta\tilde{c}^i_{t+1} + (\theta-1)\kappa_1\tilde{wc}_{i,t+1} \tag{66}$$

9.5.2 Exchange Rates

Since international financial markets are dynamically complete, the following perfect international risk sharing condition must hold:

$$\tilde{\mathcal{E}}_t = \tilde{m}^F_{t+1} - \tilde{m}^H_{t+1} \tag{67}$$

(66) implies that this takes the form in the text:

$$\tilde{\mathcal{E}}_{t+1} \approx \gamma(\underbrace{\Delta\tilde{c}^H_{t+1} - \Delta\tilde{c}^F_{t+1}}_{\mathcal{C}_{t+1}}) + \kappa_1(1-\theta)(\underbrace{\tilde{wc}^F_{t+1} - \tilde{wc}^H_{t+1}}_{\mathcal{W}_{t+1}}) \tag{68}$$

9.5.3 Wealth Processes

Home Wealth Process: I start with pricing the home wealth portfolio using the home SDF. Utilizing the Campbell-Shiller (1989) approximation, $r_{q,t+1}$ follows:

$$r^H_{m,t+1} = \kappa_0 + \kappa_1 wc_{H,t+1} - wc_{H,t} + \Delta c^H_{t+1} \tag{69}$$

Plugging the approximation into the Euler Equation for the home wealth portfolio (62) and linearizing around the deterministic steady state yields a recursive equation in $\omega_{H,t}$:

$$wc_{H,t} = ln\delta + \kappa_0 + (1-\frac{1}{\psi})\Delta c^H_{t+1} + \kappa_1 wc_{H,t+1} \tag{A38}$$

Recursively solving this equation forward yields:

$$wc_{H,t} = (\theta ln\delta + \kappa_0)\sum_{j=0}^{s-1}\kappa_1^j + (1-\frac{1}{\psi})\sum_{j=0}^{s-1}\kappa_1^j \mathbb{E}_t \Delta c^H_{t+j+1} \tag{70}$$

Impose the transversality condition that $\omega_{H,t}$ is stationary:

$$\lim_{s\to\infty} \kappa_1^s wc_{H,t+s} = 0$$

This implies that shocks to the home wealth-consumption ratio follows:

$$\tilde{wc}_{H,t} = (1-\frac{1}{\psi})N^H_{c,t+1} \tag{71}$$

$N^H_{c,t+1}$ represents news to future consumption growth expectations:

$$N^H_{c,t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{s=1}^{\infty}\rho^s \Delta c^H_{t+1+s} \tag{72}$$

Pricing the foreign wealth portfolio using the foreign SDF yields a symmetric expression for the home wealth-consumption ratio:

$$\tilde{wc}_{F,t} = (1 - \frac{1}{\psi})N^F_{c,t+1} \tag{73}$$

Substituting (71) into (66) yields the following expressions for the level of the log SDF and shocks to the log SDF:

$$m^i_{t+1} = -\gamma\Delta c^i_{t+1} - \kappa_1(\gamma - \frac{1}{\psi})E^i_{CF,t+1} \tag{74}$$

$$\tilde{m}^i_{t+1} = -\gamma\Delta\tilde{c}^i_{t+1} - \kappa_1(\gamma - \frac{1}{\psi})N^i_{CF,t+1} \tag{75}$$

This implies the exchange rate process given by (11):

$$\mathcal{E}_{i,t+1} \approx \gamma(\underbrace{\Delta c^0_{t+1} - \Delta c^i_{t+1}}_{\mathcal{C}_{t+1}}) + \kappa_1(\gamma - \frac{1}{\psi})\underbrace{(E^0_{CF,t+1} - E^i_{CF,t+1})}_{\mathcal{L}_{t+1}} \tag{76}$$

9.5.4 Proofs (Currency Excess Returns)

From country 0's perspective, the log currency excess return rx^i_{t+1} associated with going long currency i is:

$$\begin{aligned}
rx^i_{t+1} =& i^i_{t+1} - i^0_{t+1} - \Delta\mathcal{E}_{i,t+1} \\
=& log\mathbb{E}_t M^i_{t+1} - log\mathbb{E}_t M^0_{t+1} - [\mathbb{E}_t m^i_{t+1} - \mathbb{E}_t m^0_{t+1}] \\
=& \tilde{m}^0_{t+1} - \tilde{m}^i_{t+1} + [log\mathbb{E}_t M^i_{t+1} - \mathbb{E}_t m^i_{t+1}] - [log\mathbb{E}_t M^0_{t+1} - \mathbb{E}_t m^0_{t+1}]
\end{aligned}$$

Here $\tilde{m}^i_{t+1} = m^i_{t+1} - \mathbb{E}_t m^i_{t+1}$. The third line results from adding and subtracting $m^i_{t+1} - m^0_{t+1}$ to the RHS. Now evaluating $\tilde{m}^i_{t+1}$ using (75) and making use of Jensen's inequality: $\frac{1}{2}var_t m^i_{t+1} = log\mathbb{E}_t M^i_{t+1} - \mathbb{E}_t m^i_{t+1})$ yields the expression in (15):

$$rx^i_{t+1} - \frac{1}{2}J_t = \gamma(N^0_{C,t+1} - N^i_{C,t+1}) + \kappa_1(\gamma - \frac{1}{\psi})(N^0_{CF,t+1} - N^i_{CF,t+1}) \tag{77}$$

J_t takes the form:

$$J_t = var_t m^0_{t+1} - var_t m^i_{t+1}$$

9.6 VAR Framework Derivations

9.6.1 News Terms

To simplify notation, we drop country specific notation i.

Discount Rate News: $N^i_{D,t}$ is defined as:

$$\begin{aligned}
N_{d,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \\
=& e_1' \sum_{j=1}^{\infty} \rho^j \Gamma^j \sigma_t \xi_{t+1} \\
=& e_1' \underbrace{\rho\Gamma(I - \rho\Gamma)^{-1}}_{\lambda} \sigma_t \xi_{t+1} \\
=& e_1' \lambda \sigma_t \xi_{t+1}
\end{aligned} \tag{78}$$

Consumption News: $N^i_{CF,t}$ is then backed out from Campbell-Shiller decomposition:

$$\begin{aligned}
N_{CF,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \\
=& r_{m,t+1} - \mathbb{E}_t r_{m,t+1} + N_{d,t+1} \\
=& e_1' \sigma_t \xi_{t+1} + e_1' \lambda \sigma_t \xi_{t+1} \\
=& (e_1' + e_1' \lambda) \sigma_t \xi_{t+1}
\end{aligned} \tag{79}$$

VOL News: $N_{V,t}^i$ follows:

$$\begin{aligned}
N_{v,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j var_t(m_{t+1} + r_{m,t+1}) \\
=& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j \tau \sigma_{t+j}^2 \\
=& \tau e_2' \sum_{j=1}^{\infty} \rho^j \Gamma^j \sigma_t \xi_{t+1} \\
\propto& e_2' \underbrace{\rho\Gamma(I - \rho\Gamma)^{-1}}_{\lambda} \sigma_t \xi_{t+1} \\
=& e_2' \lambda \sigma_t \xi_{t+1}
\end{aligned} \tag{80}$$

9.6.2 MGARCH VAR Estimation Procedure

The exogenous state vector z_t follows an MGARCH-VAR:

$$\begin{aligned}
z_{t+1} &= \mu + \Gamma(z_t - \mu) + \xi_{t+1} \\
\xi_{t+1} &= H_{t+1}^{\frac{1}{2}} \epsilon_{t+1} \\
H_{t+1} &= C_v + A' \xi_{t+1} (\xi_{t+1})' A + G' H_t G \\
\epsilon_{t+1} &\sim N(0,\, I)
\end{aligned} \tag{81}$$

H_t is an $n \times n$ variance-covariance matrix that governs the conditional covariances of shocks to the state vector ξ_{t+1}^i. H_t drives the conditional mean of z_t^i through gaussian shocks ϵ_t^i. C_v, A and G are all $n \times n$ parameter matrices associated with the MGARCH VAR system.

Estimation of cash flow news $N_{c,t+1}$ and discount rate news $N_{d,t+1}$ in this framework is identical to Campbell et al (2017). $N_{d,t+1}$ measures time $t+1$ shocks to long run discount rate expectations:

$$\begin{aligned}
N_{d,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} \\
=& (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j e_1^T \Gamma^j H_t^{\frac{1}{2}} \epsilon_t \\
=& e_1' \lambda H_t^{\frac{1}{2}} \epsilon_{t+1}
\end{aligned} \tag{82}$$

I again refer to e_1' as an $n \times 1$ vector with 1 tied to the element linked with market returns $r_{m,t+1}$ and 0 for all other elements. The goal of this vector is to extract aggregate market returns $r_{m,t+1}$ from the state vector. The multi-period forecast of future market return shocks from the VAR system is $(\mathbb{E}_{t+1} - \mathbb{E}_t)r_{m,t+j+1} = e_1'\Gamma^{j+1}\xi_t$. This allows me to move from the first to the second and third lines. As before $N_{c,t+1}$ is backed out from the Campbell-Shiller decomposition:

$$\begin{aligned} N_{c,t+1}^i =& (\mathbb{E}_{t+1} - \mathbb{E}_t)r_{m,t+1}^i + N_{d,t+1}^i \\ =& (e_1' + e_1'\lambda)H_t^{\frac{1}{2}}\epsilon_{t+1} \end{aligned} \tag{83}$$

Expectation terms $E_{c,t}$ and $E_{d,t}$ continue to be defined in accordance with (90). However expected volatility $E_{v,t}$ is now different:

$$\begin{aligned} E_{v,t} =& \mathbb{E}_t \sum_{j=1}^{\infty} \rho^j H_{t+j} \\ =& \underbrace{\sum_{j=1}^{\infty} \rho^j [A'^j H_t A^j + G'^j H_t G^j]}_{V_t^*} + \underbrace{\sum_{j=1}^{\infty} \rho^{j-1} [A'^j C_v A^j + G'^j C_v G^j]}_{\overline{V}} \\ =& V_t^* + \overline{V} \end{aligned} \tag{84}$$

$\overline{V}$ represents the permanent component of conditional volatility H_t. V_t^* represents the time varying component of conditional volatility. These components follow:

$$\begin{aligned} \overline{V} &= \rho\Omega C_v \\ V_t^* &= \rho\tilde{\Omega} H_t \end{aligned} \tag{85}$$

where:

$$\begin{aligned} \Omega &= (I \otimes I - A' \otimes A' - G' \otimes G')^{-1} \\ \tilde{\Omega} &= (I \otimes I - \rho(A' \otimes A') - \rho(G' \otimes G^T))^{-1} \end{aligned} \tag{86}$$

$\otimes$ denotes the kronecker product. Note that above solutions for (85) are only valid if and only if the inverse exists. Engle and Kroner (1995) prove formally that the above MGARCH process requires the eigenvalues of $A + G$ to be less than one in modulus.[11]

To map this process to volatility news I follow Bansal, Kiku, Shaliastovich and Yaron (2017) in assuming that $N_{v,t+1}$ and $E_{v,t}$ are the expectations and news to the volatility of cash flow news $N_{c,t+1}$. This implies that $E_{v,t}$ and $N_{v,t+1}$ take the following form:

$$\begin{aligned} N_{v,t+1} &= (e_1' + e_1'\lambda)(\rho\Omega(H_t - H_{t-1}))(e_1' + e_1'\lambda) \\ E_{v,t+1} &= (e_1' + e_1'\lambda)H_t(e_1' + e_1'\lambda) \end{aligned} \tag{87}$$

To see this, denote the variance of cash flow news by $V_{c,t+1}$. Then we have:

$$\begin{aligned} N_{v,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^j V_{c,t+1+j} \\ =& (\mathbb{E}_{t+1} - \mathbb{E}_t)(e_1^T + e_1^T\lambda)\sum_{j=1}^{\infty}\rho^j V^*_{t+j-1}(e_1' + e_1'\lambda) \\ =& (e_1' + e_1'\lambda)((\mathbb{E}_{t+1} - \mathbb{E}_t)V^*_{t+1})(e_1' + e_1'\lambda) \\ =& (e_1' + e_1'\lambda)(\rho\Omega(H_t - H_{t-1}))(e_1' + e_1'\lambda) \end{aligned} \tag{88}$$

Taking expectations of $V_{c,t+1}$ yields the expression for $E_{v,t+1}$ in (87). Thus news terms with the MGARCH VAR follow:

$$\begin{aligned} N_{c,t+1} &= (e_1' + e_1'\lambda)H_t^{\frac{1}{2}}\epsilon_{t+1} \\ N_{d,t+1} &= e_1'\lambda H_t^{\frac{1}{2}}\epsilon_{t+1} \\ N_{v,t+1} &= (e_1' + e_1'\lambda)(\rho\Omega(H_t - H_{t-1}))(e_1' + e_1'\lambda) \end{aligned} \tag{89}$$

[11] Refer to Engle and Kroner (1995) for discussion of regularity conditions under which this condition is satisfied

Expectation terms are the same as baseline case except for $E^i_{v,t}$:

$$
\begin{aligned}
E_{c,t} &= (e_1' + e_1'\lambda)\Gamma z^i_t \\
E_{d,t} &= e_1'\lambda\Gamma z^i_t \\
E_{v,t} &= (e_1' + e_1'\lambda)H_t(e_1' + e_1'\lambda)
\end{aligned} \tag{90}
$$

9.7 Model Proofs

9.7.1 Price Level

Overview: The price level P^i_t for country 0 is the solution to the following cost minimization problem:

$$\min_{\{\{C^0_{j,t}\}_{j=0}^{N+1}\}} \sum_{j=0}^{N+1} p_{j,t}C^i_{j,t} \tag{91}$$

subject to the consumption aggregator:

$$C^0_t = [\sum_{j=1}^{N+1} \alpha_{i,j}^{\frac{1}{\phi}}(C^0_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{92}$$

FOCs with respect to $C^i_{i,t}$ and $C^i_{j,t}$ imply:

$$p_{0,t} = \lambda_t(\alpha\frac{C^0_t}{C^0_{0,t}})^{\frac{1}{\phi}} \tag{93}$$

$$p_{i,t} = \lambda_t(\alpha_{0,i}\frac{C^0_t}{C^0_{i,t}})^{\frac{1}{\phi}} \tag{94}$$

Finally simple algebra can confirm that the home price level P^H_t takes the form:

$$\lambda_t = P^0_t = [\sum_{j=1}^{N+1}(\frac{1-\alpha}{N})^{\frac{1}{\phi}}(p_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{95}$$

Going through symmetric steps for the foreign country yields similar expression for foreign price levels. Thus Q^i_t: the relative price of country i's consumption in units of the global numeraire follows:

$$Q_t^i = \begin{cases} \mathcal{E}_{i,t} = [\sum_{j=1}^{N+1} (\frac{1-\alpha}{N})^{\frac{1}{\phi}} (p_{j,t})^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} & \text{if } i \neq 0 \\ 1 & \text{if } i = 0 \end{cases} \tag{96}$$

This is the expresssion in the main text.

9.7.2 Consumption FOCs

Overview: Since markets are dynamically complete internationally, I can rewrite the IBC in a static form for the country i's rep investor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i C_t^i \leq \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i W_t^i \tag{97}$$

Notice that $Q_t^0 = 1$ since country 0 (US)'s consumption basket is the global numeraire. Λ_t is the world state price density that prices all assets in the world economy. Hence the problem for country i's rep investor can be rewritten as a time zero problem:

$$\max_{\{\{C_{j,t}^i\}_{j=0}^N, W_{t+1}^i\}_{t=0}^\infty} U_0^i \tag{98}$$

$$s.t.\ \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i C_t^i \leq \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} Q_t^i W_t^i \tag{99}$$

$$Q_t^i C_t^i = \sum_{j=0}^{N} p_{j,t} C_{j,t}^i \tag{100}$$

$$C_t^i = [\sum_{i=1}^{N+1} \alpha_{i,j}^{\frac{1}{\phi}} (C_{j,t}^i)^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}} \tag{101}$$

$$\alpha_{i,i} = \alpha \in (\frac{1}{2}, 1),\ \alpha_{i,j} = \frac{1-\alpha}{N},\ \forall i \neq j \tag{102}$$

First order conditions for consumption allocations: $C^i_{j,t}, \forall i,j \in \{0,1,2,...,N\}$ are as follows:

$$[C^i_{i,t}]: \ [\prod_{k=0}^{t-1} V^i_{2,k}]V^i_{1,t}(\alpha \frac{C^i_t}{C^i_{i,t}})^{\frac{1}{\phi}} = \mu^H \frac{\Lambda^t}{\Lambda_0} p^i_t \tag{103}$$

$$[C^i_{j,t}]: \ [\prod_{k=0}^{t-1} V^i_{2,k}]V^i_{1,t}[\frac{1-\alpha}{N} \frac{C^i_t}{C^i_{j,t}}]^{\frac{1}{\phi}} = \mu^i \frac{\Lambda^t}{\Lambda_0} p^j_t, \ \forall i \neq j \tag{104}$$

$$[C^j_{i,t}]: \ [\prod_{k=0}^{t-1} V^j_{2,k}]V^j_{1,t}[\frac{1-\alpha}{N} \frac{C^j_t}{C^j_{i,t}}]^{\frac{1}{\phi}} = \mu^j \frac{\Lambda^t}{\Lambda_0} p^i_t, \ \forall i \neq j \tag{105}$$

Here $V^i_{1,t} = \frac{\partial U^i_t}{\partial C^i_t}$ and $V^i_{2,t} = \frac{\partial U^i_t}{\partial U^i_{t+1}}$. Combining (103) with (105) yields:

$$p^i_t = [\prod_{k=0}^{t-1} V^i_{2,k}]V^i_{1,t}[\frac{\alpha C^i_t}{C^i_{i,t}}]^{\frac{1}{\phi}} \frac{1}{\mu^i \frac{\Lambda_t}{\Lambda_0}} = [\prod_{k=0}^{t-1} V^j_{2,k}]V^j_{1,t}[\frac{(1-\alpha)C^j_t}{C^j_{i,t}}]^{\frac{1}{\phi}} \frac{1}{\mu^j \frac{\Lambda_t}{\Lambda_0}} \tag{106}$$

$\frac{\Lambda_t}{\Lambda_0}$ can be pinned down by combining (103) and (104). Multiply both sides of (103) by $C^i_{i,t}$ and both sides of (104) by $C^i_{j,t}, \forall i \neq j$ and adding the resulting products yield:

$$\mu^i \frac{\Lambda^t}{\Lambda_0}[\sum_{j=0}^{N} p^j_t C^i_{j,t}] = [\prod_{k=0}^{t-1} V^i_{2,k}]V^i_{1,t}(C^i_t)^{\frac{1}{\phi}} \underbrace{[\alpha^{\frac{1}{\phi}}(C^i_{i,t})^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}}(C^i_{j,t})^{\frac{\phi-1}{\phi}}]}_{(C^i_t)^{\frac{\phi-1}{\phi}}}$$

Note by construction $\sum_{j=0}^{N} p^j_t C^i_{j,t} = Q^i_t C^i_t$. Also note that since Country 0's consumption basket is the global numeraire: $\sum_{j=0}^{N} p^j_t C^0_{j,t} = C^0_t$. This fact pins down $\frac{\Lambda_t}{\Lambda_0}$:

$$\frac{\Lambda^t}{\Lambda_0} = \frac{[\prod_{j=0}^{t-1} V^0_{2,j}]V^0_{1,t}}{\mu^0} \tag{107}$$

As in Colacito et al (2018), I write FOCs in terms of country 0 (US)'s pseudo-pareto weight vis-*á*-vi country i: $S_{i,t}$. I define $S_{i,t}$ as:

$$S_{i,t} = [\frac{(\prod_{k=0}^{t-1} V^0_{2,k})V^0_{1,t}}{(\prod_{k=0}^{t-1} V^i_{2,k})V^i_{1,t}} \frac{\mu^i}{\mu^0}]^{\phi}[\frac{C^0_t/C^0_{t-1}}{C^i_t/C^i_{t-1}}] \tag{108}$$

Recursively solving backwards yields the following law of motion for S_t:

$$S_{i,t} = S_{i,t-1}(\frac{M_t^0}{M_t^i})^{\phi}[\frac{C_t^0/C_{t-1}^0}{C_t^i/C_{t-1}^i}] \tag{109}$$

Combine (108) with (103), (104) and (105). This yields:

$$S_{i,t}\frac{\alpha}{(1-\alpha)/N}\frac{C_{i,t}^0}{C_{i,t}^i} = 1 \tag{110}$$

$$\frac{S_{j,t}}{S_{i,t}}\frac{(1-\alpha)/N}{\alpha}\frac{C_{F,t}^F}{C_{F,t}^H} = 1 \tag{111}$$

Combining (110) and (111) with the consumption market clearing conditions yields the presentation of the first order conditions described in the text:

$$C_{i,t}^i = X_t^i[1 + \frac{1-\alpha}{\alpha(N-1)}\sum_{j\neq i}\frac{S_{j,t}}{S_{0,t}}]^{-1},\ \forall i \tag{112}$$

$$C_{j,t}^i = \frac{1-\alpha}{\alpha}\frac{1}{N-1}\frac{S_{j,t}}{S_{i,t}}C_{i,t}^i,\ \forall i \neq j \tag{113}$$

9.8 Other Equilibrium Equations

Aggregate Consumption: Plug the consumption FOCs into the consumption aggregators ((101)) yields (A7) in the equilibrium system:

$$C_t^i = [\sum_{j=1}^{N+1}\alpha_{i,j}^{\frac{1}{\phi}}(C_{j,t}^i)^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}},\ \forall i \tag{114}$$

Net Exports: By construction country 0 (US)'s net exports to country i $NX_t^i = C_{0,t}^0 - \sum_{j\neq i} C_{j,t}^0$. The consumption FOCS ((103)- (105)) imply the result in lemma **??** the main text:

$$NX_t^i = A_2 X_t^0 \frac{\sum_{j\neq i} S_{j,t}}{S_{i,t} + \sum_{j\neq i} S_{j,t}} \tag{115}$$

Relative Prices: To characterise relative prices p_t^i, combine (106) and (107) yields the following expressions:

$$p_{i,t} = (\alpha_{0,i}\frac{C_t^0}{C_{i,t}^0})^{\frac{1}{\phi}}, \forall i \tag{116}$$

9.9 Calibration Choices

Consumption Home Bias: I follow Colacito et al (2018) and set $\alpha_{i,i} = \alpha > \frac{1}{2}$ and $\alpha_{i,j} = \frac{1-\alpha}{N}$. Thus each agent i's preferences over foreign goods are symmetric. My chosen value of α is 0.96: this is in line with standard calibration choices for home bias used in the open economy macro literature (Lewis, 2011).

Elasticity of Substitution: I choose a low elasticity of substitution across goods ϕ of 0.2. This choice is motivated by empirical evidence documenting a low elasticity of substitution across consumption goods (Couerdacier and Rey, 2013).

IES: I choose a high IES value of $\psi = 2$. This choice is motivated by standard calibration choices made in the international asset pricing literature using recursive preferences (Colacito and Croce, 2013; Colacito et al, 2018).

Cointegration: I calibrate the cointegration parameter β to 0.05. This is larger than standard calibrations in the recursive utility literature, where β is set to a smaller number.[12] I motivate this choice due to the match the mild persistence of currency risk premia suggested by my empirical evidence.

Other Parameters: I set mean endowment growth $\mu = \mu_H = \mu_F = 0.005$. Since this is a quarterly calibration, this corresponds to an annualized mean growth of 2%, as commonly assumed in conventional calibrations.

[12]In Colacito and Croce (2013), $\beta = 0.005$. These calibration choices are also adopted by Colacito et al, 2018 and Colacito et al (2021)