

# Pre-emptive sovereign debt restructuring and holdout litigation

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## Abstract

We offer an analytical framework for studying “pre-emptive” debt exchanges. Countries can tailor a sovereign bankruptcy framework by choosing provisions (or ‘haircuts’) ex ante, but must contend with the market discipline of holdout litigation ex post. Secondary markets play a role in shaping the holdout costs facing the sovereign, and our results suggest that it is optimal to prioritise the rights of holdout creditors during litigation so that they are always paid in full. We clarify how macroeconomic and legal factors influence the choice of ex ante haircut. Our model contributes to the debate on sovereign debt restructuring by formalizing Bolton and Skeel’s (2004) notion of a “Designer SDRM”.

*Keywords:* Sovereign debt restructuring, holdout creditors, bankruptcy procedures, absolute priority

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## 1. Introduction

The messy legal disputes surrounding Argentina’s recent default and the European debt crisis of 2012 have rekindled interest in frameworks for sovereign bankruptcy. Policymakers have shifted away from concerns that sovereign debtors might default opportunistically towards advocating “pre-emptive” debt restructuring prior to default (IMF, 2013). The holdout problem has also been rejuvenated. The high-profile court rulings against Argentina in New York have cast a spotlight on the legal sophistication of holdout creditors and the implications of their techniques for the enforcement of sovereign debt contracts.

Debates on sovereign debt restructuring typically emphasise a trade-off between ex ante and ex post efficiency. Proponents of bankruptcy procedures (e.g. Krueger, 2002; IMF, 2002) highlight the ex post inefficiency posed by costly default, and emphasise the importance of mitigating these via institutional or market-based remedies. Critics of such proposals (e.g. Dooley, 2000; Shleifer, 2003) counter that sovereign debt is feasible and affordable only because of the threat of costly crises. Bankruptcy mechanisms that lower ex post costs of debt crises or shield the debtor from the threat of holdout litigation may do more harm than good by increasing moral hazard problems ex ante.

Statutory approaches to balancing ex ante incentives and ex post crisis costs have tended to highlight a “double trigger” mechanism for a viable sovereign bankruptcy regime (Gai et al., 2004; Brookings-CIEPR, 2013).<sup>1</sup> A sovereign first makes a request for assistance, in a manner akin to filing for bankruptcy protection under Chapter 11 of the US bankruptcy code, and then a bankruptcy court (or the IMF) rules on the case. Gai et al. (2004) show formally that ex ante and ex

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<sup>1</sup>Other contributions that examine how policy interventions in sovereign lending influence both ex ante and ex post efficiency include Ghosal and Miller (2003) and Bolton and Jeanne (2007).

post efficiency can be balanced in a welfare maximising way, provided that the bankruptcy court is reasonably able to distinguish “ability to pay” and “willingness to pay” problems, as well as judging if ability-to-pay crises are the result of genuine bad luck on the part of the sovereign.

In an important contribution to the debate, [Bolton and Skeel \(2004\)](#) question the feasibility of statutory approaches to sovereign debt restructuring. Drawing on US bankruptcy history, they suggest that tailored approaches may be more politically palatable. Allowing sovereigns to have the choice of opting out of the provisions of a bankruptcy framework depending on their economic circumstances would increase their willingness to adopt a sovereign bankruptcy framework. [Bolton and Skeel \(2004\)](#) also emphasise the importance of preventing the dilution of the claims of existing creditors to ensure ex ante efficient levels of borrowing. The enforcement of absolute priority emerges as a key element of a robust sovereign bankruptcy framework.

The changing nature of holdout litigation has also altered the balance between ex ante and ex post efficiency. Ad hoc debt exchanges in the past were largely smooth because a sovereign could unilaterally bypass holdout creditors through take-it-or-leave-it offers backed by the agreement of a simple majority of bondholders. But the New York District Court rulings on Argentina have made it easier for holdout creditors to litigate other creditors and, hence, indirectly harass the sovereign ([Brookings-CIEPR, 2013](#)). By lowering the costs of litigation, such developments have made participation in debt exchanges harder to coordinate.

In this paper, we present a model in which countries can tailor a sovereign bankruptcy framework to fit their own circumstances, but must contend with the market discipline of holdout litigation. Our model thus formalizes elements of the [Bolton and Skeel](#) proposal, and contributes to the debate on whether sovereign debt restructuring mechanisms should be mandatory or permit opt-out options. In

our optimal contracting framework, a sovereign issues bonds to fund a risky project and chooses the bankruptcy provisions and exemptions *ex ante* to maximize domestic welfare. By claiming exemptions, for example, the sovereign ensures that domestic welfare does not fall below a certain level when output from the risky project is low and the sovereign files for bankruptcy. This, however, induces the sovereign to offer a higher haircut to creditors during the restructuring. In what follows, we adopt the convention that the sovereign chooses the haircut *ex ante*.

The *ex ante* choice of the haircut has two important incentive effects, *ex post*. First, it influences the sovereign's willingness to repay. All other things being equal, a larger haircut means that the sovereign can more easily restructure its debts. This, in turn, increases the sovereign's incentives to default and file for bankruptcy. The second effect is on the creditors' incentives to holdout during the restructuring. In our model, creditors are *ex post* heterogeneous and have different costs of pursuing litigation against the sovereign. Litigants who reject the sovereign's haircut on their claims petition a bankruptcy judge that they be repaid in full. The judge's ruling reflects the extent to which creditors' rights are favored in the restructuring mechanism. Successful litigants are paid in full, while unsuccessful litigants walk away with nothing. Thus, an increase in the haircut means that more creditors will holdout and pursue litigation.

The haircut also influences creditors' ability to trade their bonds prior to the restructuring phase. If the *ex ante* haircut is high, then creditors with high litigation costs have a strong incentive to sell their bonds to more litigious creditors, *i.e.*, those with low litigation costs. This, in turn, exacerbates holdouts and litigation. On the other hand, if the *ex ante* haircut is low, the incentives for high-litigation cost creditors to sell their claims to more litigious creditors are eliminated. This is because the gains from trade are lower than the cost of participating in the secondary market for the high-litigation cost creditors.

The sovereign's optimal ex ante haircut is influenced by the funding conditions in the primary market and factors that influence output. An increase in the world interest rate, which is a measure of the opportunity cost of lending to the sovereign, leads to an increase in the promised repayment to creditors. But, with a higher repayment, the sovereign finds it more favorable to default and file for bankruptcy so as to reduce the amount it must pay to creditors. The sovereign responds by choosing a higher haircut, which leads to more creditors holding out and litigating during the restructuring phase. This second effect counters the first effect and reduces the sovereign's incentives to default.

Our model predicts that a decrease in expected output, or an increase in its riskiness also leads to an increase in the optimal haircut. In both cases, the probability that the sovereign defaults and files for bankruptcy is increased. An increase in the haircut leads to more holdouts and litigation, which reduces the sovereign's incentives to default. Thus, by opting for a high haircut ex ante, the sovereign optimally trades off the deterioration in the output distribution versus the increase in the repayment incentives.

The legal environment also influences the choice of haircut. If the bankruptcy mechanism's ability to mitigate holdout costs is increased, this increases the incentives for the sovereign to default and file for bankruptcy. In sum, however, the expected holdout costs increases, which reduces domestic welfare. The sovereign can mitigate this effect by choosing a higher haircut, which reduces the sovereign's incentives to file for bankruptcy. Improvements to the rights of holdout creditors' during litigation lowers the ex ante haircut. There are two effects at play. First, an improvement in the rights of holdout creditors implies that it is more likely that the sovereign must repay holdout creditors. This reduces the sovereign's incentives to file for bankruptcy. At the same time, creditors' incentives to holdout and litigate increase. To counter this, the sovereign lower the ex ante haircut, thereby offering more to creditors during the restructuring so as to offset their

incentives to holdout. Our model also shows that it is optimal to prioritise the rights of holdout creditors during litigation so that they are always paid in full. This is consistent with the notion of strict adherence to absolute priority during debt restructuring that [Bolton and Skeel](#) also emphasise in their proposal.

Our work is related to the literature on creditor litigation in sovereign debt, notably [Haldane et al. \(2005\)](#), [Engelen and Lambsdorff \(2009\)](#), [Pitchford and Wright \(2012\)](#), [Bai and Zhang \(2012\)](#), and [Schumacher et al. \(2015\)](#). But these papers focus on the ex post renegotiation stage rather than the ex ante implications for sovereign borrowing and the design of the bankruptcy framework considered here. Recent work has also highlighted the importance of the creditor base in sovereign debt enforcement. [Broner et al. \(2010\)](#) develop a model in which, once it becomes apparent that default is looming and penalties are insufficient, foreign creditors can sell debt securities on the secondary market to domestic residents. Since domestic residents expect the government to enforce domestic debts, they purchase the securities at face value. Trading in the secondary market thus allows foreign creditors to circumvent default. The secondary market channel through which debt is enforced in our model is very different, however.

Our paper is also related to recent work on pre-emptive sovereign debt restructuring. While many papers simply assume that restructurings are preceded by a default, [Asonuma and Trebesch \(2016\)](#) assume that this is not the case. They develop a quantitative sovereign debt model that emphasises the Nash bargaining game between the debtor and its creditors, and which incorporates the possibility of both pre-emptive and post-default restructuring. [Asonuma and Trebesch](#) show that pre-emptive restructuring is optimal if the sovereign expects a high probability of default and faces high output costs as a result. In their model, pre-emptive deals also lead to lower haircuts, quicker renegotiations, and lower crisis costs. Empirical studies of pre-emptive restructuring include [Diaz-Cassou et al. \(2008\)](#), [Panizza et al. \(2009\)](#), and [Erce \(2013\)](#).

## 2. Model

A small open economy extends over two periods,  $t = 0, 1$ . There is a single consumption good and a representative agent – the sovereign – is risk neutral and cares only about consuming in period 1.

At  $t = 0$ , the sovereign can borrow funds from the international capital market to finance a risky project that yields  $\theta$  units of output at  $t = 1$ . We suppose that the sovereign issues a unit of debt under international law – an infinitely divisible one period bond that pays  $R > 1$  on maturity at  $t = 1$ . The realized ex post return on the project is a random variable,  $\theta \in [\underline{\theta}, \bar{\theta}]$ , drawn from the cumulative distribution function  $G(\theta)$ . There is a large pool of foreign creditors who are ex ante identical and risk neutral. These creditors are not capital constrained and are able to borrow and lend as much as needed at the constant risk-free world interest rate,  $\bar{r}$ .

In the presence of absolute sovereign immunity in international law, it is difficult (if not impossible) for foreign creditors to “attach” a sovereign’s property in the event of a default. Even though trading and credit relationships may be impaired in ways that diminish the sovereign’s output as a result of default, lenders receive nothing. Denoting the output cost of default by  $\delta$ , the sovereign is only willing to repay creditors in full provided

$$\theta - R > (1 - \delta)\theta. \tag{1}$$

To avoid the messy costs associated with default, we allow the sovereign to tailor bankruptcy procedures ex ante. Specifically, the contract with foreign creditors at  $t = 0$  stipulates a haircut,  $h \in [0, 1]$ , that creditors will be offered at  $t = 1$  in the event that the sovereign files for bankruptcy in international courts to seek a restructuring. Filing for bankruptcy has two effects. First, it helps mitigate

the output cost of default to  $\sigma\delta \leq \delta$ , where  $\sigma \in [0, 1]$  captures the ability of the bankruptcy mechanism to shield the sovereign from output costs. But as equation (1) makes clear, this also has the effect of increasing the incentive to default. Second, the recourse to a bankruptcy procedure ensures that creditors receive some payment in states of the world where they would otherwise have received nothing.

If the sovereign files for bankruptcy at  $t = 1$ , it offers creditors a payment of  $R(1 - h)$ . Each creditor must decide whether to accept the haircut or litigate in the courts to recover the full payment. Litigation is costly and private litigation costs,  $\ell_i \in [0, 1]$ , are drawn i.i.d from the unit interval. It is also risky since the full claim can only be recovered with success probability  $\phi$ . Litigation fails with probability  $1 - \phi$ , in which case a holdout creditor receives nothing. Following [Schumacher et al. \(2015\)](#),  $\phi$  can be interpreted as a measure of the strength of creditor rights. Holdouts are costly for the economy and reduce output by the fraction  $f(\mu)$ , where  $\mu$  is the mass of litigious creditors. We assume that  $f' > 0$ ,  $f'' > 0$ , and litigation costs are uniformly distributed in what follows.

Finally, creditors have access to a secondary market at  $t = 1$ . Upon discovering their litigation cost, bondholders may opt to sell their claims to those with low litigation costs. The secondary market for bonds is perfectly competitive with market-clearing price,  $P$ .

Table 1 illustrates the time-line of events. We solve the model by backward induction. The pure strategy sub-game perfect Nash equilibrium consists of a bankruptcy threshold,  $\theta^*$ , litigation threshold,  $\ell^*$ , mass of holdouts,  $\mu^*$ , secondary market clearing price,  $P^*$ , repayment,  $R^*$ , and haircut,  $h^*$ , such that at  $t = 1$

- conditional on the sovereign filing for bankruptcy, individual creditors litigate whenever  $\ell_i < \ell^*$ , given the repayment,  $R^*$  and haircut  $h^*$ ;
- the secondary market clears so that aggregate demand and supply of bonds



Ex ante ( $t = 0$ )	Ex post ( $t = 1$ )
1. Sovereign issues bonds with repayment $R$ and haircut $h$	1. Output, $\theta$ , realised 2. Sovereign repays or defaults 3. If sovereign repays: - Creditors consume $R$ - Sovereign consumes $\theta - R$ 4. If sovereign defaults: - Creditors trade bonds at price $P$ - Litigious creditors reject haircut $h$ - Litigations succeed with probability $\phi$ - Non-holdouts consume $R(1 - h)$ - Successful litigants consume $R$ - Sovereign consumes rest minus costs

Table 1: Timeline of events

yield the equilibrium price  $P^*$ ;

- creditors optimally choose to either demand or supply bonds, given their litigation costs,  $\{\ell_i\}_{i=0}^1$ , the price,  $P^*$ , the repayment,  $R^*$ , and the haircut  $h^*$ ;
- the sovereign files for bankruptcy if  $\theta < \theta^*$ , given the mass of holdout creditors,  $\mu^*$ , repayment,  $R^*$ , and haircut  $h^*$ ;

And, at  $t = 0$ ,

- the sovereign optimally chooses  $R^*$ , and  $h^*$  given the bankruptcy threshold,  $\theta^*$ , the equilibrium secondary market price  $P^*$ , and mass of holdouts,  $\mu^*$ .

### 2.1. Filing for bankruptcy and secondary markets

If the sovereign files for bankruptcy and offers creditors a haircut,  $h$ , then an individual creditor,  $i$ , who accepts the offer receives  $R(1 - h)$ . But if the creditor rejects the offer and litigates with success probability  $\phi$ , the expected payoff after

incurring litigation costs is  $R(\phi - \ell_i)$ . So creditor  $i$  holds out whenever

$$R(\phi - \ell_i) > R(1 - h), \quad (2)$$

i.e., whenever  $\ell_i < \ell^* \equiv h - (1 - \phi)$ .

At the same time, however, creditors can buy or sell their bond holdings once they learn their litigation costs. In particular, those with high litigation costs would like to sell their claims to those with low litigation costs. The assumption that creditors are ex ante identical implies that, prior to trading, they all hold an equal amount of bonds. A creditor with litigation cost  $\ell_i$  will buy additional bonds if  $R(\phi - \ell_i) - P > 0$ , i.e., if  $\ell_i < \phi - P/R$ . But if  $\ell_i > \phi - P/R$ , the creditor attempts to sell the bonds. Since aggregate demand must equal aggregate supply to clear the secondary market, it follows that  $\phi - P^*/R = 1 - [\phi - P^*/R]$ . And accordingly,  $P^* = R[\phi - 1/2]$ .

At the market clearing price,  $P^*$ , it must also be the case that a creditor with litigation cost  $\ell^*$ , i.e., is indifferent between holding out and accepting the sovereign's offer, should not wish to purchase bonds on the secondary market. By contradiction, if this creditor did demand bonds then it implies there is another creditor with litigation cost  $\ell^* + \varepsilon$ , where  $\varepsilon \ll 1$ , who also demands bonds but who would strictly accept the restructuring offer and never hold out. The maximum price such a creditor would willingly pay is  $R(1 - h)$ . But at this price there is no gain from trade for other creditors with higher litigation costs. If there is an infinitesimally small cost that all creditors face in order to participate in the market, then all creditors with costs above  $\ell^*$  would not enter and there would be no trade. So we require that

$$R(\phi - \ell^*) - P^* < 0, \quad (3)$$

which, on rearranging, gives

$$h > \hat{h} \equiv \min \left\{ \frac{3}{2} - \phi, 1 \right\}. \quad (4)$$

So for bonds to be traded on the secondary market, the haircut set by the sovereign must be sufficiently high so that less litigious creditors are encouraged to sell their claims to more litigious creditors. All the bonds are thus held by litigious creditors who holdout during the restructuring.

Turning to the sovereign's decision to file for bankruptcy, there are two cases to consider: (i) when bonds are traded, i.e.  $h > \hat{h}$ ; and (ii) when bonds are not traded, i.e.  $h \leq \hat{h}$ . We consider each in turn.

If creditors trade bonds after the sovereign files for bankruptcy, then  $\mu^* = 1$  and all creditors holdout and litigate successfully with probability  $\phi$ . In this case, the willingness to pay constraint is

$$\theta - R > (1 - \sigma\delta - f(1))\theta - R\phi, \quad (5)$$

which can be rearranged to obtain the bankruptcy threshold

$$\theta > \theta^* \equiv \frac{R(1 - \phi)}{\sigma\delta + f(1)}. \quad (6)$$

In the second case, the sovereign still incurs output costs  $\sigma\delta\theta$  while it files for bankruptcy. But the output cost from holdouts is now  $f(\ell^*)\theta$ . Successful holdout creditors receive  $R$  with probability  $\phi$  and creditors who accept the sovereign's offer get  $R(1 - h)$ . Accordingly, the willingness to pay constraint is

$$\theta - R > (1 - \sigma\delta - f(\ell^*))\theta - R\theta\ell^* - R(1 - h)(1 - \ell^*), \quad (7)$$

which gives an alternate bankruptcy threshold,

$$\theta > \theta^{**} \equiv \frac{R(h - \ell^*(h))^2}{\sigma\delta + f(\ell^*(h))}. \quad (8)$$

Proposition 1 summarizes.

**Proposition 1.** *The mass of litigants is given by*

$$\mu^* = \begin{cases} 1 & \text{if } h > \hat{h} \\ \ell^* \equiv h - (1 - \phi) & \text{if } h \in [1 - \phi, \hat{h}] \\ 0 & \text{otherwise} \end{cases}.$$

*The secondary market bond price is*

$$P^* = \begin{cases} R(\phi - \frac{1}{2}) & \text{if } h > \hat{h} \\ \text{Indeterminate} & \text{otherwise} \end{cases}.$$

*And the resulting sovereign bankruptcy threshold is*

$$\begin{cases} \theta^* & \text{if } h > \hat{h} \\ \theta^{**} & \text{otherwise} \end{cases}.$$

Figures 1 and 2 illustrate these results.<sup>2</sup> In Figure 1, we plot the mass of litigious creditors as a function of the haircut. When haircuts are small,  $h \leq 1 - \phi$ , no creditors has an incentive to holdout. In the intermediate range,  $1 - \phi < h \leq \hat{h}$ , creditors with low litigation costs opt to hold out, whereas others do not. Since

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<sup>2</sup>In producing all figures, we set  $\bar{r} = 0$ ,  $R = 1.5$ ,  $\phi = 0.6$ ,  $\epsilon = 1.1$ ,  $\delta = 1$  and  $\sigma = 0$ . Output is distributed according to an exponential distribution with rate parameter  $\lambda = 1/7$ .

there is no trade in this range, high-litigation cost creditors cannot sell their bonds to low litigation cost creditors. And when  $h > \hat{h}$ , bonds are traded in the secondary market and are all held by litigious creditors.

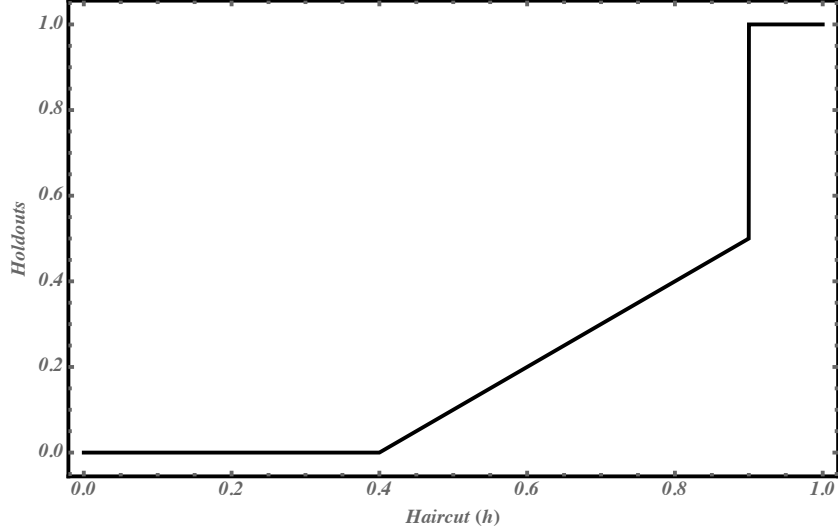


Figure 1: The fraction of litigious bond holders as a function of the haircut

Figure 2 plots the bankruptcy threshold as a function of the haircut. A marginal increase in the haircut has an ambiguous impact on the bankruptcy threshold. Specifically,

$$\frac{\partial \theta^{**}}{\partial h} \propto (\sigma\delta + f(\ell^*))(1 - 2\ell^*) - (h - (\ell^*)^2)f'(\ell^*), \quad (9)$$

and the sign of the partial derivative depends on the semi-elasticity of holdout costs, with respect to the mass of holdouts, and the net benefit to the sovereign from repaying. If

$$\frac{f'(\ell^*)}{\sigma\delta + f(\ell^*)} < \frac{1 - 2\ell^*}{h - \ell^*(h)^2}, \quad (10)$$

then the relative increase in the cost of holdouts is less than the relative increase in the benefit from repaying. So the sovereign prefers to default and  $\frac{\partial \theta^{**}}{\partial h} > 0$ . Reversing the inequality implies that the relative increase in the cost of holdouts exceeds the relative increase in the benefits of repaying. In this case,  $\frac{\partial \theta^{**}}{\partial h} < 0$  and the sovereign prefers to repay.

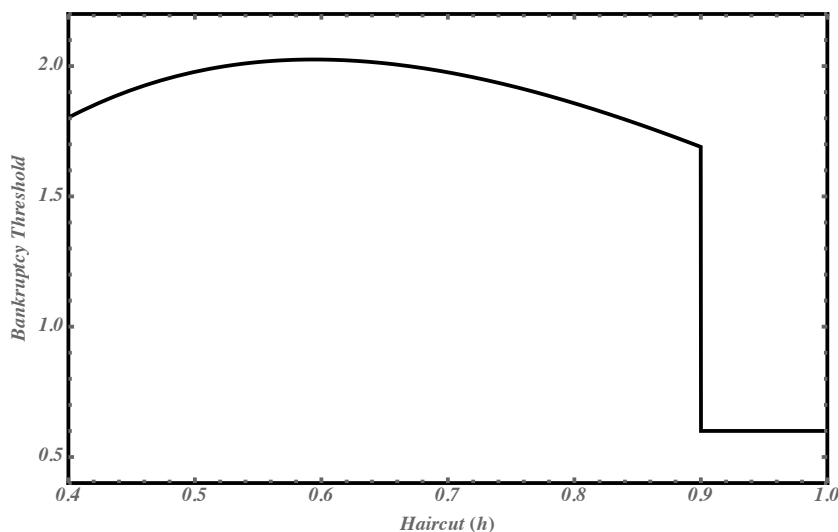


Figure 2: The sovereign's bankruptcy threshold as a function of the haircut.

As shown in Figure 2, when the haircut is small, an increase in  $h$  increases the incentives to default since the relative holdout cost is low. But as the haircut continues to increase, the rise in holdout costs is such that, for any further increase in the haircut, the sovereign's default incentives decrease. Once the critical level  $\hat{h}$  is reached, there is a discontinuous shift reflecting the reallocation of all bonds to litigious creditors via the secondary market. The bankruptcy threshold is constant for  $h > \hat{h}$ .

Finally, note that an increase in the creditor-friendliness of the bankruptcy regime,  $\phi$ , decreases the bankruptcy threshold. Also, an increase in repayments,  $R$ , heightens the incentives to file for bankruptcy because the benefits of reducing the debt burden via restructuring become more significant.

## 2.2. Optimal haircut and repayment

In our model, a sovereign would never choose a haircut above the threshold  $\hat{h}$ . At  $t = 0$ , the sovereign's ex ante expected welfare is expected output net of expected holdout costs and expected repayment to creditors. In the case of  $h < \hat{h}$ ,

expected welfare is

$$\begin{aligned}
 EW &= E[\theta] - (\sigma\delta + f(\ell^*(h))) \int_{\underline{\theta}}^{\theta^{**}(h)} \theta dG(\theta) \\
 &\quad - R\left\{1 - G(\theta^{**}(h))(h - \ell^*(h)^2)\right\}, \tag{11}
 \end{aligned}$$

and, when  $h > \hat{h}$ , expected welfare at  $t = 0$  is

$$\widehat{EW} = E[\theta] - (\sigma\delta + f(1)) \int_{\underline{\theta}}^{\theta^*} \theta dG(\theta) - R\left\{1 + G(\theta^*)(1 - h)\right\}. \tag{12}$$

where  $E[\theta]$  is the expected output. In [Appendix A](#), we demonstrate that  $EW > \widehat{EW}$  provided that the bankruptcy regime is sufficiently creditor-friendly, in the sense that the probability of successful holdout litigation is sufficiently large. [Figure 3](#) illustrates the ex ante expected welfare of the sovereign as a function of the haircut – the optimal haircut (the grey dotted line) lies below  $\hat{h}$ .

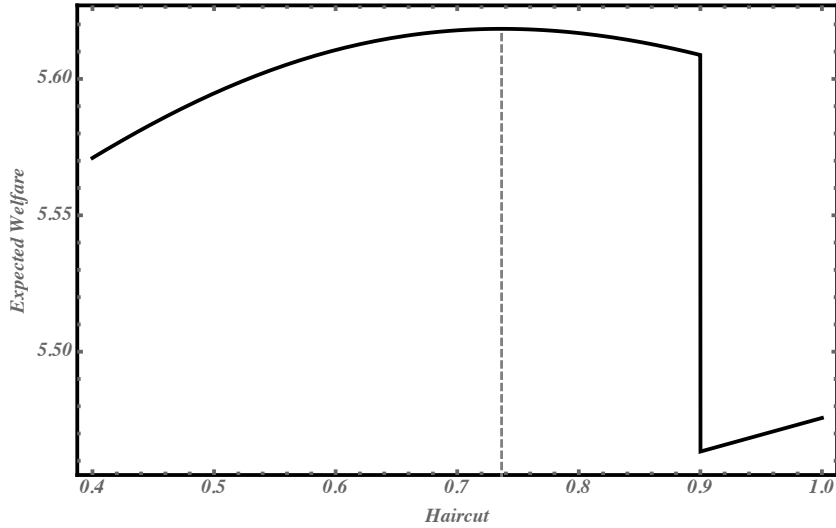


Figure 3: Expected domestic welfare as a function of the haircut

In choosing the optimal haircut to maximize expected welfare, the sovereign

takes the required repayment as given. So, we have that

$$h^*(R) = \max_h EW(R, h) \quad (13)$$

such that

$$h \leq \hat{h}$$

The required repayment is determined by the creditors, via their break-even condition, namely

$$R \left\{ 1 - G(\theta^{**}(h)) \left( h - \frac{\ell^*(h)^2}{2} \right) \right\} = 1 + \bar{r} \quad (14)$$

In general, the optimal haircut and repayment implied by equations (13) and (14) are not amenable to analytical solutions. We therefore impose some assumptions on the function  $f(\mu)$  and the distribution of output to make further headway. Specifically, we suppose that  $f(\mu) = e^{\epsilon(\mu-1)}$  so that the semi-elasticity of holdout costs is a constant, i.e  $f'/f = \epsilon$ , and that shocks to output are uniformly distributed over  $[\underline{\theta}, \bar{\theta}]$ . For sharper results, we further assume that the bankruptcy mechanism fully shields the sovereign from output losses, namely  $\sigma = 0$ .<sup>3</sup>

For a given repayment,  $R$ , the sovereign's choice for the haircut,  $h^* \equiv h^*(R)$ , that maximizes domestic welfare is given by  $EW_h(h^*, R) = 0$ , where

$$EW_h(h, R) = R(1 - 2\ell^*(h)) - \frac{\epsilon}{2} f(\ell^*(h))(\theta^{**}(h) + \underline{\theta}). \quad (15)$$

At the optimum,  $h^*$ , the bankruptcy threshold is a decreasing function in the haircut, i.e  $\frac{\partial \theta^{**}}{\partial h} \Big|_{h=h^*} < 0$ . Proposition 2 provides the comparative statics for the optimal haircut.

**Proposition 2.** *The optimal haircut is increasing in the repayment,  $R$ . A leftward shift in the output distribution, i.e a decrease in  $\underline{\theta}$ , raises the optimal haircut, as*

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<sup>3</sup>The results that follow continue to hold in numerical exercises for different output distributions and positive values of  $\sigma$ .



*does increased riskiness (in the sense of a mean-preserving shift in the output distribution). Greater creditor-friendliness of the bankruptcy regime lowers the optimal haircut, while an increased elasticity of holdout costs raises the optimal haircut.*

*Proof.* See [Appendix A](#). □

As repayments increase, they induce two opposing effects on the sovereign's choice of haircut. First, an increase in  $R$  heightens the sovereign's incentive to file for bankruptcy. Second, because of the heightened likelihood of default, the holdout costs incurred by the sovereign are also greater. The first effect induces the sovereign to opt for a higher haircut, while the second effect acts in the opposite direction. From the first-order condition, the first effect always dominates.

Our model predicts that countries with weak and risky macroeconomic fundamentals will seek higher haircuts ex ante. A leftward shift in the output distribution lowers expected output. Moreover, with a decrease in  $\underline{\theta}$ , the range of  $\theta$  over which the sovereign would default and incur the holdout costs is larger. At the same time, by increasing the haircut, the sovereign's default point,  $\theta^{**}$ , is reduced, thus lowering the incentives for the sovereign to default. Thus, by opting for a high haircut ex ante, the sovereign optimally trades off the deterioration in the output distribution versus the increase in the repayment incentives.

Increasing the variance of the output distribution, while leaving the mean unchanged also has the effect of enlarging the range of  $\theta$  over which the sovereign defaults and incurs holdout costs. The greater risk of suffering an output cost again induces a higher haircut.

An improvement in the creditor-friendliness of the bankruptcy regime induces two effects. First, it becomes more likely that the sovereign must repay holdout

creditors. This reduces the sovereign's incentives to file for bankruptcy. And second, the creditors' incentives to holdout and litigate increase. To counter this, the sovereign lower the ex ante haircut, thereby offering more to creditors during the restructuring so as to offset their incentives to holdout.

Finally, an increase in elasticity of holdout costs implies that holdout costs are more sensitive to a change in the mass of holdouts. At the same time, since  $f(\ell) \leq 1$ , and  $f(1) = 1$ , an increase in the elasticity must be reflected by a decrease in the holdout costs. This, in turn, increases the sovereign's incentives to default and seek bankruptcy. If  $\epsilon$  is large enough, then the default incentives outweigh the potential holdout costs incurred, and so the risk of default increases. This, in turn, induces the sovereign seeks a higher haircut.

Turning to the creditors' break-even condition, for any given haircut,  $h$ , the repayment,  $R^* \equiv R^*(h)$ , is given by the solution to

$$V(h, R^*) \equiv R^* \left[ 1 - \left( \frac{\theta^{**}(R^*, h) - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \right) \left( h - \frac{\ell^*(h)^2}{2} \right) \right] = 1 + \bar{r}. \quad (16)$$

Proposition 3 summarizes the properties of  $R^*$ .

**Proposition 3.** *The repayment,  $R^*$ , is decreasing in the haircut. A decrease in the expected output, (either a fall in  $\underline{\theta}$  or  $\bar{\theta}$ ) and an increase in the mean-preserving spread of the output distribution increase repayment. Greater creditor-friendliness of the bankruptcy regime decreases repayment, while an increase in the elasticity of holdout costs increases repayments. Finally, higher world interest rates also lead to greater repayment.*

*Proof.* See [Appendix B](#). □

The intuition behind the results of Proposition 3 is easily understood. Any change to the output distribution that increases the risk of default (a fall in  $\underline{\theta}$  or  $\bar{\theta}$ )

leads to an increase in repayment demanded by creditors faced with greater default risk. Increased riskiness of the output distribution, as reflected by an increase in the mean-preserving spread, has the same effect. As the creditor friendliness of the bankruptcy regime increases, the sovereign's incentives to default are reduced. Moreover, if the sovereign did file for bankruptcy, more creditors would holdout and claim the full repayment with greater probability. Thus, the value of creditors' claims against the sovereign increase and, thus, the required repayment decreases. On the other hand, an increase in the elasticity of holdout costs leads to a decrease in holdout costs, for a given mass of holdout creditors. This, in turn, increases the sovereign's incentives to default and file for bankruptcy. Thus, creditors seek a higher repayment in order to break even in expectation. Higher world interest rates increase the outside option for creditors who must, therefore, be compensated for sovereign lending with higher repayments.

We close the discussion of our model with Proposition 4, which describes the joint equilibrium. The comparative statics directly follow from our earlier analysis of the schedules  $h^*(R)$  and  $R^*(h)$  in Propositions 2 and 3, respectively.

**Proposition 4.** *The optimal haircut and repayment, i.e.  $h^{**}$  and  $R^{**}$ , respectively, jointly solves*

$$\begin{aligned} EW_h(h^{**}, R^{**}) &= 0 \\ V(h^{**}, R^{**}) &= 1 + \bar{r}. \end{aligned}$$

*An increase in the world interest rate,  $\bar{r}$ , leads to an increase in both the haircut and repayment. An increase in the maximum output,  $\bar{\theta}$ , leads to a decrease in both the haircut and repayment. An increase in the minimum output,  $\underline{\theta}$ , has an ambiguous effect on repayment but leads to a decrease in the haircut. A mean-preserving shift in the output distribution leads to increase in  $h^{**}$ , but has an ambiguous effect on  $R^{**}$ . An improvement in the creditor-friendliness of the bankruptcy regime*

leads to a decrease in the haircut, but has an ambiguous effect on repayment. Finally, an increase in the elasticity of holdout costs leads to an increase in  $h^{**}$  but has an ambiguous effect on  $R^{**}$ .

### 3. Implications for sovereign debt restructuring

Early proposals for sovereign bankruptcy frameworks (Krueger, 2002; IMF, 2002) typically assumed a ‘one-size-fits-all’ approach based on a uniform and mandatory set of provisions. In an important contribution to the debate, Bolton and Skeel (2004) propose that sovereigns be permitted to opt out of some aspects of the bankruptcy procedure ex ante. They argue that a tailored approach to sovereign bankruptcy is easier to sell politically than more ambitious statutory approaches. Debtor nations that typically express strong reservations about sovereign debt restructuring regimes may be more willing to participate if they are able to obtain exemptions to provisions in accordance with local conditions.<sup>4</sup> But there is a risk that countries may adopt overly harsh provisions ex ante, for fear of losing their access to credit.

Our model helps shed light on this debate, and on the disciplining role played by holdout litigation in determining ex ante exemptions (or haircuts). Our findings suggest that tailored frameworks will vary across countries according to their default risk – countries with weak or risky macroeconomic fundamentals will be more likely to seek out softer provisions. And the more sensitive are the output costs of holdout litigation to the mass of holdout creditors in the sovereign bond market, the softer will be the provisions sought. In our optimal contracting framework, contractual flexibility leads to an efficient outcome. As such, we are unable

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<sup>4</sup>Bolton and Skeel (2004) base their argument on US bankruptcy history – to pass the first US bankruptcy law at the Federal level in the 19th century, each state could determine what collateral a debtor could exempt from its creditors if it filed for bankruptcy. State lawmakers could thus adjust their exemptions to local norms and had discretion over how much collateral a debtor could retain. Bankruptcy law was Federal, but tailored on a state-by-state basis.

to address the normative issue of whether sovereigns should be precluded from adopting overly harsh provisions that make restructuring more, rather than less, painful.<sup>5</sup>

The analysis also clarifies the extent to which a sovereign debt restructuring should favour creditors. To the extent that an international body seeks to maximise global welfare, i.e. the sum of domestic welfare and repayment to creditor expected payoffs, it can choose the degree of creditor protection,  $\phi$ , to maximise

$$GW(R, h; \phi) = E[\theta] - f(\ell^*) \int_{\theta}^{\theta^{**}} \theta dG(\theta) - RG(\theta^{**}) \frac{(\ell^*)^2}{2} \quad (17)$$

where  $GW$  is the sum of expected output, loss to the sovereign from holdouts, and the cost to creditors from litigation. Numerical exercises with both uniform and exponential distributions for  $\theta$ , constant semi-elasticity  $\epsilon$ , and  $\sigma \geq 0$ , confirm that  $\phi^* = 1$  is the optimum choice for the international body.

The situation  $\phi^* = 1$  is consistent with the notion of absolute priority, namely the prioritisation by the bankruptcy regime of the payments of bonds issued first before those issued later. [Bolton and Skeel \(2004\)](#) emphasise the importance of enforcing absolute priority as the basis of any bankruptcy procedure and our model confirms this perspective. The lack of adherence to absolute priority is damaging for global welfare in our model since, by diluting the claims of exiting creditors, it curbs market discipline and results in higher borrowing costs for sovereigns. Reinforcing creditor priorities within the bankruptcy mechanism, thus, helps balance the trade-off between ex post crisis costs and ex ante incentives.

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<sup>5</sup>Myopic policymakers who quickly want to access international capital markets, but who are unlikely to be around to face the costs of future defaults, are likely to be more willing to advocate quick and creditor-friendly restructuring. Extending the model to capture such political distortion is one way to explore this issue. See [Acharya and Rajan \(2013\)](#) for a model in this vein.

## 4. Conclusion

Institutional solutions to sovereign debt restructuring entailing uniform and mandatory bankruptcy provisions are often criticised because there are too many political obstacles to implementation. As a result, contractual and market solutions have tended to be favoured by many policymakers and commentators.

In this paper, we have sought to examine a middle ground in which sovereigns are given the choice of opting out of a statutory bankruptcy framework by crafting their own provisions. We present a model that sheds light on the role played by holdout creditors, macroeconomic fundamentals, and the legal environment on the choice of exemptions (or haircut) sought *ex ante*. Our model formalizes some of the insights of the [Bolton and Skeel \(2004\)](#) proposal advocating “Designer SDRMs” as a possible transitory step on the road to a statutory bankruptcy regime.

The lack of sufficient support for a statutory mechanism has now led the IMF to encourage countries to restructure their debts pre-emptively (i.e. launched prior to default), as a way of strengthening the existing contractual framework ([IMF, 2013](#)). Our paper thus contributes to a broader policy debate on how countries might be prevented from defaulting too late and too little. Extending the model to allow for government myopia would allow analysis of whether sovereigns should be precluded from adopting overly harsh provisions.

Finally, the legal environment in our model is rudimentary. Enriching the analysis of sovereign debt enforcement to allow for judicial micro-foundations and a deeper understanding of sovereign immunity (in the spirit of [Miller and Thomas, 2007](#) and [Weidemaier and Gulati, 2015](#)) is an important area for future work.

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## Appendix A. Proof for Proposition 2

We first establish that  $h \leq \hat{h}$ . It follows that for  $EW > \widehat{EW}$ , we require that

$$R(1-h)G(\theta^*) \left\{ \frac{G(\theta^{**}(h))}{G(\theta^*)} \left( \frac{h - \ell^*(h)^2}{1-h} \right) + 1 \right\} \\ + (\sigma\delta + f(\ell^*(h)))\mathbb{E}[\theta|\theta < \theta^{**}(h)] \left\{ \frac{(\sigma\delta + f(1))\mathbb{E}[\theta|\theta < \theta^*]}{(\sigma\delta + f(\ell^*(h)))\mathbb{E}[\theta|\theta < \theta^{**}(h)]} - 1 \right\} > 0.$$

The first line in the equation above is always positive. As for the second line, this is also positive as long as  $(\sigma\delta + f(1))\mathbb{E}[\theta|\theta < \theta^*] > (\sigma\delta + f(\ell^*(h)))\mathbb{E}[\theta|\theta < \theta^{**}(h)]$ . Since  $(\sigma\delta + f(\ell^*(h)))\mathbb{E}[\theta|\theta < \theta^{**}(h)] < (\sigma\delta + f(\ell^*(h)))\mu_\theta$ , a more strict sufficient condition would require that  $(\sigma\delta + f(1))\mathbb{E}[\theta|\theta < \theta^*] > (\sigma\delta + f(\ell^*(h)))\mu_\theta$ , where the right-hand side is increasing in  $h$ . If we therefore evaluate the right-hand side at  $h = \hat{h}$ , we get the sufficient condition  $(\sigma\delta + f(1))\mathbb{E}[\theta|\theta < \theta^*] > (\sigma\delta + f(1/2))\mu_\theta$ , which can be recast as a lower-bound condition on  $\phi > \tilde{\phi}$ , where  $\tilde{\phi}$  is given by  $(\sigma\delta + f(1))\mathbb{E}[\theta|\theta < \theta^*(\tilde{\phi})] - (\sigma\delta + f(1/2))\mu_\theta = 0$ .

Next, we show that at the optimal haircut,  $h^*$ , the bankruptcy threshold is decreasing. In particular,

$$\begin{aligned} \left. \frac{\partial \theta^{**}}{\partial h} \right|_{h=\tilde{h}} &\propto f(\ell^*(\tilde{h}))(1 - 2\ell^*(\tilde{h})) - (\tilde{h} - \ell^*(\tilde{h})^2)f'(\ell^*(\tilde{h})) \\ &= f'(\ell^*(\tilde{h})) \left\{ \frac{f(\ell^*(\tilde{h}))}{2R} [\theta^{**}(\tilde{h}) + \underline{\theta}] - (\tilde{h} - \ell^*(\tilde{h})^2) \right\} \\ &= f(\ell^*(\tilde{h})) \frac{f'(\ell^*(\tilde{h}))}{2R} \left\{ \underline{\theta} - \frac{R(\tilde{h} - \ell^*(\tilde{h})^2)}{f(\ell^*(\tilde{h}))} \right\} \\ &= -f(\ell^*(\tilde{h})) \frac{f'(\ell^*(\tilde{h}))}{2R} \left\{ \theta^{**}(\tilde{h}) - \underline{\theta} \right\} < 0. \end{aligned}$$

To ensure that the solution  $h^*$  is a maximum, we require that domestic welfare

is concave in the haircut. This requires that  $EW_{hh}(h^*) < 0$ . We find that

$$EW_{hh}(\tilde{h}) = -2R - \frac{1}{2}f''(\ell^*(\tilde{h}))(\theta^{**}(\tilde{h}) + \underline{\theta}) + \frac{\epsilon}{4}f'(\ell^*(\tilde{h}))(\theta^{**}(\tilde{h}) - \underline{\theta}).$$

A sufficient condition for  $h^*$  to be a maximum is that

$$f''(\ell^*(\tilde{h})) > \frac{\epsilon}{2}f'(\ell^*(\tilde{h})),$$

which is always true as long as the semi-elasticity,  $\epsilon$ , is greater than 1/2.

For the comparative statics of  $h^*$ , we first derive the effects of a change in parameters on the first-order condition. In particular,

$$\begin{aligned} EW_{hR}(\tilde{h}) &= \frac{1}{2}f'(\ell^*(\tilde{h}))\frac{(\theta^{**}(\tilde{h}) + \underline{\theta})}{R} - \frac{1}{2}f'(\ell^*(\tilde{h}))\frac{(\theta^{**}(\tilde{h}))}{R} = \frac{1}{2}f'(\ell^*(\tilde{h}))\frac{\underline{\theta}}{R} > 0 \\ EW_{h\underline{\theta}} &= -\frac{1}{2}f'(\ell^*(h)) < 0 \\ EW_{h\Delta} &= \frac{1}{2}f'(\ell^*(h)) > 0, \end{aligned}$$

where  $\Delta$  is a symmetric increase (decrease) in the upper (lower) bound of the output distribution. The signs of the comparative statics follow from the direct application of the implicit function theorem.

For the legal parameters,  $\phi$  and  $\epsilon$ , we have that

$$\begin{aligned} EW_{h\phi} &= -2R - \frac{1}{2}f''(\ell^*(h))(\theta^{**}(h) + \underline{\theta}) + \frac{1}{2}f'(\ell^*(h))\text{abs}\left(\frac{\partial\theta^{**}(h)}{\partial\phi}\right) \\ EW_{h\epsilon} &= -\frac{1+\epsilon^2}{2}f(\ell^*(h))(\theta^{**}(h) + \underline{\theta}) - \frac{\epsilon}{2}f(\ell^*(h))\underbrace{\frac{\partial\theta^{**}}{\partial\epsilon}}_{>0} < 0. \end{aligned}$$

where

$$\frac{\partial\theta^{**}}{\partial\phi} = -R\frac{2\ell^*(h) + \epsilon(h - \ell^*(h)^2)}{f(\ell^*(h))^2} < 0.$$

Using the fact that  $f(\ell^*(h)) = e^{\epsilon(\ell^*(h)-1)}$ , we can re-write  $EW_{h\phi}$  as

$$-2R - \frac{\epsilon^2}{2} f(\ell^*(h)) [\theta^{**}(h) + \underline{\theta}] + \frac{\epsilon}{2} R [2\ell^*(h) + \epsilon(h - \ell^*(h)^2)].$$

Grouping the terms involving  $R$ , a sufficient condition for  $EW_{h\phi} < 0$  is therefore

$$\frac{\epsilon}{2} [2\ell^*(h) + \epsilon(h - \ell^*(h)^2)] < 2,$$

which is always satisfied as long as

$$\frac{\epsilon}{2} \left( 1 + \epsilon \frac{5}{4} \right) < 2.$$

## Appendix B. Proof of Proposition 3

From the creditor's break-even condition we obtain that

$$\frac{\partial V}{\partial R} \propto \bar{\theta} - (1 - \xi)\underline{\theta} - 2\xi\theta^{**}(h),$$

where  $\xi = h - \ell^*(h)^2/2 < 1$ . Since  $\partial V/\partial R$  is decreasing in  $\xi$ , a sufficient condition for the partial derivative to be positive, i.e.,  $\partial V/\partial R > 0$ , is that  $\theta^{**}(h) < \bar{\theta}/2$ . Since the bankruptcy threshold is increasing in  $R$ , we can evaluate it at  $R = \bar{\theta}$ , which yields the sufficient condition

$$f(0) > 2 \left( \frac{5}{4} - \phi \right).$$

Next, to determine how a change in the sovereign's ex ante haircut choice influences the repayment, we find that

$$\frac{1}{R} \frac{\partial V}{\partial h} = \frac{\epsilon}{2} (\theta^{**}(\tilde{h}) - \underline{\theta}) \left( \tilde{h} - \frac{\ell^*(\tilde{h})^2}{2} \right) - (\theta^{**}(\tilde{h}) - \underline{\theta}) (1 - \ell^*(\tilde{h})).$$

If the derivative is negative, this implies that following an increase in the haircut, the effect of increased litigation is smaller than the reduction in the overall payment by the sovereign, which in turn, reduces the value of the creditor's claims. If, however, the derivative is positive, then an increase in the haircut leads to greater litigation, which enhances the value of the claims. In what follows, we focus on the latter case. Therefore, a sufficient condition for  $\partial V/\partial h > 0$  is  $\epsilon > 5/2 - 2\phi$ .

For the remaining comparative statics, note that

$$\begin{aligned} \frac{\partial V}{\partial r} &< 0 \\ \frac{\partial V}{\partial \theta} &> 0 \\ \frac{\partial V}{\partial \underline{\theta}} &> 0 \\ \frac{\partial V}{\partial \Delta} &< 0 \\ \frac{\partial V}{\partial \phi} &> 0 \\ \frac{\partial V}{\partial \epsilon} &< 0 \end{aligned}$$

The signs of the comparative static exercises follow directly from the implicit function theorem.

The results in Proposition 4 follow directly from applying the results of Propositions 2 and 3 to determine how the curves  $h^*(R)$  and  $R^*(h)$  shift following changes in the underlying parameters.