Modeling the Joint Dynamics of Risk Neutral Stock Index and Bond Yield Volatilities

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Abstract

This study investigates the joint evolution of risk neutral stock index and bond yield volatility using the Chicago Board Option Exchange S&P500 volatility index (VIX) and the Bank of America Merrill Lynch Treasury Option Volatility Estimate Index (MOVE). I use bivariate regime switching models to investigate risk neutral volatility linkages due to common information and information spillover, as well as volatility clustering and asymmetry. Common information about economic and financial conditions appear to drive VIX and MOVE fluctuations between "risk-on" and "risk-off" regimes. Two-regime specifications also reveal novel associations between information spillover and stock-bond linkage. Ignoring regime shifts leads to spurious extreme persistence and incomplete inferences about asymmetric volatility.

Key Words: option-implied volatility, asymmetric correlation, common information, information spillover, regime-switch

JEL Codes: G11, G12, C32

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1. Introduction

Volatility has been recognized as one of the most important determinants of asset value for stocks and bonds, the two most important asset classes. Expectations of future market volatility and their linkages have important implications for asset pricing, portfolio management and hedging effectiveness. If volatility is directly related to the rate of information flow (Ross, 1989), volatility expectations may persist over time due to the gradual incorporation of information (Anderson and Bollerslev, 1997) or incomplete information of traders and subsequent revisions in beliefs after a structural break (Timmermann, 2001). Investors can also react differently to positive and negative information of the same magnitude. Therefore, asymmetric volatility is documented for both stocks (Black, 1976; Campbell and Hentschel, 1992; Daouk and Ng, 2011) and interest rates (Chan, Karolyi, Longstaff and Sanders, 1992; Jarrow, Li and Zhao, 2007). Furthermore, the linkages between the stock and bond markets reflect common information (Ederington and Lee, 1993) and cross-market information spillover effects (Fleming, Kirby and Ostdiek, 1998).

Typically, researchers estimate volatility from the time series of historical price changes.[1] However, such volatility estimates are *ex post* measures and reflect only part of the impact of information arrival on perceptions of volatility. Information does not only cause asset price changes but also induces revisions of investor beliefs about the future volatility of asset prices and macroeconomic variables (Stulz, 1986). Although not directly observable, implied volatility

[1] Two approaches for measuring volatility are parametric estimation and more direct nonparametric measures. Among parametric methods, the ARCH (Engle, 1982) class of models formulates volatility as a function of past returns and other directly observable variables while stochastic volatility in discrete-time models incorporates past returns as well as latent state variables. In contrast, nonparametric volatility measurements are generally data driven and model free, including ARCH filters and smoothers designed to measure volatility over infinitesimally short horizons, and realized volatility measures for (non-trivial) fixed-length time intervals.

estimates are derived from prices of options or other derivatives to represent investor beliefs about the underlying asset price volatility (Patell and Wolfson, 1979). Recently, implied volatility has gained more popularity in the literature and among practitioners. In contrast to *ex post* physical volatility measures, implied volatility is the *ex ante* risk neutral expectation of future volatility and reflects both immediate and longer term effects of information flow. Another problem of volatility is its high persistence, which is a sign of structural change in variance and can be better characterized empirically by regime switching models.[2]

It is widely recognized that there is substantial time-variation and regime-dependence in the relation between stock and bond returns (Fama and French, 1989; Connolly et al., 2005, 2007). Multivariate regime-switching models have become increasingly popular in investigating asset allocation between stock and bonds (Guidolin and Timmermann, 2005, 2006, 2007; Baele, Bekaert and Inghelbercht, 2010; Yang, Zhou and Wang, 2010; Chan, Treepongkaruna, Brooks and Gray, 2012). Also, regime switching models have been used to study asymmetric correlations across asset returns and to draw implications for asset allocation (Ang and Bekaert, 2002; Ang and Chen, 2002). All these studies have demonstrated that the regime switching model is better than the single-state model in capturing the joint return distribution of stock and bond markets. However, very few studies have explored the regime shifts for joint distribution of risk neutral stock and bond volatilities. To fill this gap in the existing literature, I develop bivariate regime switching models to examine joint evolution of risk neutral volatilities and their asymmetric correlations due to common information and information spillover, as well as volatility clustering and asymmetry. The paper makes several contributions to the literature.

First, with daily implied volatility indices for the S&P500 stock index and US Treasury bond yields from 1990 to 2010, I find that bivariate two-state regime switching models fit the

[2] See Lamoureux and Lastrapes (1990), Cai (1994); Hamilton and Susmel (1994).

data much better than a single-regime model, suggesting substantial regime dependence in the relation between risk neutral stock and bond volatilities. These models are particularly appealing for implied volatilities because news about business cycles and financial conditions can simultaneously alter investor expectations in both stock and bond markets as indicated in Timmerman (2001). In particular, the two regimes in my model can be characterized as “risk-on” and “risk-off” regimes. During risk-on markets, bother stock and bond risk neutral volatilities are higher. Moreover, these ex ante volatilities are more volatile and have stronger correlation. By contrast, the risk-off regime is associated with lower volatility expectations, lower volatility of volatilities and weaker cross-market linkages between ex ante volatilities. I also report strong evidence that macroeconomic and financial variables commonly used in the literature, such as the short-term interest rate, Treasury bond term structure spread, and corporate bond default spread, predict the transition probability of regime switches. Thus, common information about economic and financial conditions, especially the default spread, causes regime shifts in the joint evolution of volatility expectations of stock and bond markets.

Second, allowing for regime shifts reveals important new associations between stock-bond market links and the degree of information spillover. Without regime shifts, the stock volatility expectation and its change predicts its interest rate volatility counterparts, implying that information alters the expectations of stock investors and then spreads to the bond market through trading. However, parameters estimated for the regime switching models reveal bi-directional information spillover between stock and bond markets in the risk-on regime, an important source of correlation that is 77% larger relative to the normal regime. This implies a much lower benefit from “flight to quality” because linkages between ex ante volatilities strengthen across stock and bond markets during a crisis. These results extend the volatility linkage literature[3] by empirically distinguishing between information spillover and common information effects and revealing an association between the extent of information spillover and the degree of stock-bond linkage.

Third, I document additional new evidence on volatility clustering and asymmetry. Volatility expectation forms clusters in each regime, suggesting gradual incorporation of information. Moreover, low volatility expectation persists for 4.37 days while high volatility expectation persists for 15.15 days. Ignoring regime shifts leads to the spurious appearance of extreme persistence. Also, a very significant and robust negative relation between innovations in stock returns and expected stock volatility exists and is consistent with the asymmetric volatility literature using implied volatilities (for example, Dennis, Mayhew and Stivers, 2006). A notable new finding is that the asymmetric volatility effect is much larger in the risk-on regime with higher volatilities. This suggests that non-diversifiable stock market volatility as an asset class[4] should be very appealing for stock portfolio diversification, especially in bad times. Moreover, the relation between bond yield implied volatility and the level of the long-term interest rate is regime-dependent, negative in the risk-on regime but positive in the normal regime. This adds to the literature on interest rate volatility that typically examines volatility of the short-term interest rate and finds mixed relationships (Trolle and Schwartz, 2009). It may also shed some new light on volatility of interest rate derivatives (Li and Zhao, 2006).

Finally, this study features two very prominent volatility indicators, the Chicago Board Option Exchange's S&P500 volatility index (VIX) and Bank of America Merrill Lynch's Treasury Option Volatility Estimate Index (MOVE). VIX is widely covered by the financial

[3] Fleming et al (1998) find volatility linkages between stock, bond and money markets became stronger following the 1987 stock market crash

[4] Stock market volatility is now traded in the US and Europe. In particular, VIX futures and options saw a dramatic increase in volume in the past few years.

media, and is even included on the ticker of the CNBC financial news cable television network. Investors view the VIX index as reflecting both fear and the demand for portfolio insurance (Whaley, 2000; 2008) while academics find VIX an increasingly useful and interesting measure of the market's expected future stock index volatility.[5] MOVE is a widely-followed measure of government bond yield volatility.[6] MOVE is also included by the IMF in a statistical appendix of Global Financial Stability Reports together with VIX. However, MOVE is seldom studied in the literature, either by itself or in relation to VIX.[7] My study fills this gap.

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 discusses the regime-switching models and develops testable hypotheses. Section 4 presents the empirical results. Finally, Section 5 offers concluding remarks.

2. Data

The VIX index is the square root of market price average for selected out-of-the-money call and put options written on the S&P 500 index at two of the nearest maturities.[8] The squared VIX approximates the model-free implied variance of Britten-Jones and Neuberger (2000) and the risk-neutral expected value of return variance of Carr and Wu (2009) over a 30-day horizon. Similarly, the MOVE index is a weighted average of the normalized implied yield volatility for 1-month Treasury options on the two-year (20% weight), five-year (20% weight), 10-year (40% weight), and 30-year (20% weight) U.S. Treasury bonds. The options underlying the MOVE Index have expiration dates of approximately one month; thus, the MOVE index measures the

[5] Strictly speaking, the VIX adds higher order cumulants beyond risk neutral ex ante volatility when there are jumps in the underlying returns (Carr and Lee, 2009; Martin, 2011). In this case, the VIX should be interpreted more broadly as uncertainty or risk.
[6] For example, a recent story in the Wall Street Journal (Blumberg, 2010) attributes s a rise in MOVE from 75 basis points in August 2010 to 109 basis points in December 2010 to concerns about the fiscal health of euro-zone nations.
[7] To the best of my knowledge, only Markellos and Psychoyios (2011) include comparison of VIX and MOVE, but the focus is on interest rate volatility indices they construct.
[8] See Carr and Wu (2006) and Chicago Board Options Exchange (2009) for detailed construction of VIX index.

implied volatility of long-term yields over a relatively short horizon. However, the exact methodology used to normalize the interest rate volatility and derive it from options is not disclosed. Also note that VIX is quoted in percentages while MOVE is expressed in basis points.

Daily observations of VIX and MOVE are downloaded from Bloomberg. The sample begins on January 2, 1990 and ends on December 31 2010 for a total of 5232 observations. Figure 1 plots daily values of VIX and MOVE during the sample period. As shown in the figure, VIX and MOVE spike sharply and frequently during times of high uncertainty, such as the first Gulf War in 1990 - 1991, the Asian financial crisis in 1997 - 1998, the terrorist attacks of September 2001, and the peak of the financial crisis in the fall of 2008. Most notably, VIX and MOVE soared to record levels following the collapse of Lehman Brothers in September 2008.

[Figure 1 here]

To show the relationship between implied volatilities and their underlying assets, I collect S&P 500 closing prices and 10-year Treasury yields from St. Louis Fed's FRED. We can see from Panel A of Figure 2 that VIX usually jumps upward as the S&P500 dropped. In Panel B, the spikes in MOVE occur when 10-year Treasury yield moves either upward or downward.

[Figure 2 here]

Panel A of Table 1 reports summary statistics. During the sample period, the average VIX and MOVE are 20.383 percent (0.2038) and 102.867 basis points (0.0103) respectively, suggesting that stocks are more volatile than Treasury bonds. The standard deviation of VIX is 8.253 percent, greater than that of MOVE (24.838 basis points). Skewness is positive for both VIX and MOVE. All excess kurtosis are greater than 3, meaning that volatilities and their changes have sharp peaks and fat tails. In particular, VIX has more excess kurtosis than MOVE , suggesting more spikes in the stock index volatility.[9]

[Table 1 here]

As shown in Panel B of Table 1, all four variables exhibit significant daily serial correlation at the 1% level. In particular, VIX and MOVE are quite persistent while volatility changes are not. Furthermore, squared terms are strongly serial correlated across all five variables under consideration, suggesting the existence of ARCH effects.

Given that VIX and MOVE are quite persistent, it is important to address concerns about stationarity. Therefore, I estimate augmented Dickey-Fuller unit root tests for all four variables and select optimal lag lengths based on the Bayesian Information Criterion (BIC). Table 2 reports the ADF t-test statistics in regressions with an intercept and the ADF ρ-test statistics in regressions with an intercept and a time trend. The statistics for VIX and MOVE are smaller than the critical values at the 1% significant level. Therefore, I can reject the null hypothesis that volatilities follow unit root processes. In other words, VIX and MOVE can be considered stationary series.

[Table 2 here]

The next question is how persistent VIX and MOVE are. A widely used measure is the sum of the autoregressive coefficients (SAC) or equivalently cumulative impulse function (CIR), which is 1/(1-SAC). I estimate autoregression models with the optimal lag lengths identified in Table 2. The sums of the autoregressive coefficients, 0.989 for VIX and 0.986 for MOVE respectively, suggest extremely high persistence of both stock and bond volatility expectations.

[9] Note the skewness and kurtosis are standardized and measure-free. Therefore, skewness and kurtosis of VIX are directly comparable to MOVE counterparts although VIX and MOVE are in terms of percentages and basis points respectively.

More intuitively, the CIRs are 92.42 for VIX and 72.82 for MOVE, meaning that stock and bond volatility expectations will persist for 92.42 days and 72.82 days respectively.

Another preliminary test examines how implied volatility transmits between stock and bond markets. Therefore, bivariate Granger-causality tests are applied to the pair of VIX and MOVE. For simplicity, the same lag length, k, is set for *y* and *x* in a regression as follows:

$$y_t = a + \sum_{i=1}^{k} b_i y_{t-i} + \sum_{j=1}^{k} c_j x_{t-j} + \varepsilon_t \ .$$

The results of this Granger Causality are represented in Table 3. It is clear that VIX Granger-causes MOVE. In other words, stock volatility expectation and its changes can be used to predict bond volatility counterparts, implying that information alters expectation of stock investors and thus spreads to bond market probably through cross-market trading. However, the opposite is not true.

[Table 3 here]

Next, I collect and construct the following economic indicators, which are commonly used in the literature and available at daily frequency from St. Louis Fed's FRED. While other macroeconomic variables have been used in the literature, the data typically exist only at monthly or even lower frequency.

First, the 3-month Treasury yield is used as a proxy for short-term interest rate. The short rate is closely attuned to discount rate and has significant predictive power for future asset returns with a long history of empirical support (Fama and Schwert, 1977). Second, the slope of the yield curve has also been shown to be a good predictor of both consumption and output growth (Harvey, 1988). The slope is typically measured by the term spread (*TERM*), which is the difference between the 10-year and 2-month Treasury yields. Most recessions are preceded by a sharp decline in the term spread and frequently by an inverted yield curve (Ferson and Harvey, 1991). Third, the default spread (*DEF*) is measured as the difference between Moody BAA and AAA bond yields. This spread is an indicator of perceived business conditions in the general economy. If business conditions are poor, then there is an increase in the likelihood of default in lower quality corporate bonds, which leads to increase in default spread. A number of studies on stock and bond markets (Chen, Roll, and Ross, 1986; Fama and French, 1989; Ferson and Harvey, 1991) identify the default and term spreads as business cycle risk factors that drive asset prices. In particular, the term spread is a proxy for longer term business conditions while the default spread measures short term business conditions (Fama and French, 1989).

[Table 4 here]

Table 4 shows the correlation matrix of implied volatilities, stock index return, bond yield[10] and instruments. Note that all correlations are significant at 5% level at least, except for stock return correlations with bond yield and instruments. Several correlations are also notable as follows. First, VIX has a high positive correlation of 0.603 with MOVE, suggesting a strong linkage between stock and bond implied volatilities. Second, VIX and MOVE are negatively correlated with stock return and bond yield, implying negative return-volatility relationships. Next, the correlations between short rate and implied volatilities are negative while there are positive correlations between volatilities and two business cycle factors, term and default spreads. Finally, among the three instruments, the default spread may have the most significant predictive power because its correlations with VIX and MOVE are the strongest.

3. Methodology and Hypotheses

[10] I use 10-year Treasury bond yield because the MOVE index measures implied yield volatility and 10 year Treasury bonds account for the biggest weight.

Following much of the literature mentioned in the introduction, I focus on the two-state regime-switching model, which has an appealing interpretation. Moreover, testable hypotheses are developed for both common information and information spillover effects, as well as volatility clustering and asymmetry.

The general form of a basic bivariate regime-switching model can be written as follows:

$$
\text{(1)} \qquad \begin{aligned}
&\mathbf{IV_t} = \boldsymbol{\mu}_{\mathbf{it}} + \boldsymbol{\varepsilon}_{\mathbf{it}},\\
&\boldsymbol{\mu}_{\mathbf{it}} = E(\mathbf{r_t} | s_t = i, \mathbf{F_{t-1}}),\\
&\boldsymbol{\varepsilon}_{\mathbf{it}} \,|\, \mathbf{F_{t-1}} \sim (\mathbf{0}, \boldsymbol{\Omega}_{\mathbf{it}}),
\end{aligned}
$$

where $IV_{\mathbf{t}} = (VIX_t\,, MOVE_t\,)'$ is a 2×1 vector of stock and bond implied volatilities at time t. s_t is the unobserved regime at time t, which takes value 1 or 2. $\boldsymbol{\mu}_{\mathbf{it}} = (\mu_{it}^s,\, \mu_{it}^b)'$ is a 2×1 vector of means given regime *i* conditioned on the past information set $\mathbf{F_{t-1}}$. $\boldsymbol{\varepsilon}_{\mathbf{it}} = (\varepsilon_{it}^s,\, \varepsilon_{it}^b)'$ is a 2×1 vector of innovations given regime *i*. $\boldsymbol{\Omega}_{\mathbf{it}}$ is the conditional variance-covariance matrix of IV_t given regime i since

$$
\text{(2)} \qquad \mathrm{var}(\mathbf{r}_t | s_t = i, \mathbf{F_{t-1}}) = \mathrm{var}(\boldsymbol{\varepsilon}_{\mathbf{it}} | \mathbf{F_{t-1}}) = \boldsymbol{\Omega}_{\mathbf{it},} \quad i \in \{1, 2\}.
$$

Specifically, after the preliminary search on the VAR lag length with AIC and BIC, I consider

the following VAR(1) [11] with contemporaneous stock return and interest rate specification for the conditional means

$$
\text{(3)} \qquad \boldsymbol{\mu}_{\mathbf{it}} = \boldsymbol{\mu}_{\mathbf{i}} + \boldsymbol{\lambda}_{\mathbf{i}} VIX_{t-1} + \gamma_i MOVE_{t-1} + \theta_i \mathbf{r}_t, \quad i \in \{1, 2\},
$$

where $\boldsymbol{\mu}_{\mathbf{i}} = (\mu_i^s,\, \mu_i^b)'$ is a 2×1 vector of the constant means given regime *i*. $\boldsymbol{\lambda}_{\mathbf{i}} = (\lambda_i^s,\, \lambda_i^b)'$ is a 2×1 vector of regression coefficients on the first lagged implied stock volatility VIX_{t-1} given regime *i*.

[11] Further inclusion of more lags in VAR does not make the regime switching model perform better.

$\gamma_{\mathbf{i}} = (\gamma_i^s, \gamma_i^b)'$ is a 2×1 vector of regression coefficients on the first lagged implied bond volatility $MOVE_{t-1}$ given regime *i*. $\theta_i = \begin{pmatrix} \theta_i^s & 0 \\ 0 & \theta_i^b \end{pmatrix}$ is a 2×2 matrix of regression coefficients on the vector of $\mathbf{r}_t = (r_t^s, r_t^b)'$, where r_t^s is log stock return and r_t^b is the 10-year Treasury bond yield. For simplicity, I only consider the contemporaneous relations between the stock return (interest rate) and VIX (MOVE). The purpose is not to investigate economic source of asymmetric volatility, such as leverage effect (Black, 1976) or risk premium arguments (Campbell and Hentschel, 1992), but to draw implications for portfolio diversification.

Assuming that $\boldsymbol{\varepsilon}_{\mathbf{it}}$ follows an i.i.d. bivariate normal distribution, the conditional distribution of $\mathbf{IV_t}$ follows a mixture of two i.i.d. bivariate normal distributions, which can approximate a very broad set of density families:

(4) $$\mathbf{IV_t} \mid \mathbf{F_{t-1}} \sim \begin{cases} IIN(\boldsymbol{\mu}_{\mathbf{1t}}, \boldsymbol{\Omega}_{\mathbf{1t}}),\ w.p. \quad p_{1t}, \\ IIN(\boldsymbol{\mu}_{\mathbf{2t}}, \boldsymbol{\Omega}_{\mathbf{2t}}),\ w.p. \quad p_{2t}. \end{cases}$$

Moreover, I assume that the variances and correlation are constant within each regime because further inclusion of time-varying conditional variances and correlation in each regime is statistically insignificant (Guidolin and Timmermann, 2005). Therefore, the conditional variance-covariance matrices can be specified as follows:

(5) $$\boldsymbol{\Omega}_{\mathbf{it}} = \boldsymbol{\Omega}_{\mathbf{i}} = \mathbf{D_i R_i D_i}, \quad \mathbf{D_i} = \begin{pmatrix} \sqrt{\sigma_i^s} & 0 \\ 0 & \sqrt{\sigma_i^b} \end{pmatrix}, \quad \mathbf{R_i} = \begin{pmatrix} 1 & 0 \\ \rho_i & 1 \end{pmatrix}, \quad i \in \{1, 2\},$$

where σ_i^s and σ_i^b are the constant conditional variances of stock and bond volatility expectations given regime *i*. ρ_i is the constant conditional correlation between stock and bond volatility expectations given regime *i*. Note that regimes can be characterized by volatility expectation linkages suggested by ρ_i.

A natural point to introduce regimes is to parameterize S_t as a first-order Markov chain, meaning that the probability of regime shift depends only on the most recent regime as follows:

$$\begin{aligned}&\Pr(S_t = j \mid S_{t-1} = i, S_{t-2} = k, \cdots) = \Pr(S_t = j \mid S_{t-1} = i) = p_{ij}, \quad i \in \{1, 2\}, \\ &0 \le p_{ij} \le 1, \sum_{j=1}^{2} p_{ij} = 1 \textit{ for all } i\end{aligned} \tag{6}$$

p_{ii} measures the persistence of regime i with duration of.

$$D_i = \frac{1}{1 - p_{ii}} \text{ days} \tag{7a}$$

It can also be shown that unconditional probability of being in regime i is

$$\Pr(S_t = i) = \frac{1 - p_{jj}}{2 - p_{ii} - p_{jj}} \textit{ for } i \neq j. \tag{7b}$$

The parameters of this regime switch model with time-invariate transition probabilities are $(\mu_i^j, \lambda_i^j, \gamma_i^j, \theta_i^j, \sigma_i^j, \rho_i, p_{ii})$, where $i \in \{1,2\}$ and $j \in \{s, b\}$.

Testable hypotheses are developed as follows. First, we can perform a likelihood-ratio test (LR test hereafter) against the benchmark of the single-regime model. The null hypothesis is

(H1) $\mu_1^j = \mu_2^j, \lambda_1^j = \lambda_2^j, \gamma_1^j = \gamma_2^j, \theta_1^j = \theta_2^j, h_1^j = h_2^j, \ j \in \{s, b\}$, and $\rho_1 = \rho_2$.

If the null hypothesis above is strongly rejected, the two-state model fits the data much better than the single-regime model. In other words, regimes are really present in the distribution of stock and bond implied volatilities.

Second, if volatility expectation within a regime is clustering caused by gradual incorporation of information, VIX and MOVE should exhibit significantly positive serial correlation as formulated in the following hypothesis.

(H2a) $\lambda_i^s > 0$ and $\gamma_i^b > 0$.

Moreover, if investors revise their beliefs following a structural change, each regime should persist less than the single-regime duration measured by the cumulative impulse response (CIR) in Section 2, and the persistence of each regime measured by duration D_i should be different. Such persistence effect of regime shifts can be formulated as follows.

(H2b) $$D_i < CIR \text{ and } D_1 \neq D_2 .$$

Third, for well documented stock volatility asymmetry, I expect significantly negative coefficient(s) between stock return and VIX. The economic significance is that non-diversifiable stock market volatility could be an asset class, which offers very good diversification opportunity for stock portfolio. Moreover, if asymmetric volatility effect is stronger in the bad time, hedging is more effective. Assuming that regime 1 is the regime with higher volatility and thus lower return, I test the following hypothesis for stock volatility asymmetry.

(H3a) $$\theta_1^s < \theta_2^s < 0 .$$

For bond volatility asymmetry, it is unclear what I can predict based on the literature. Therefore, I test the following null hypothesis and let data speak the signs of coefficients:

(H3b) $$\theta_i^b = 0 .$$

If the null hypothesis is rejected and the coefficient is significantly negative, bond yield volatility is a good hedge against long term interest rate.

More importantly, information can alter investor perceptions of one market's risk, which in turn affects expected volatility in other markets if investors rebalance their portfolios across markets. For example, portfolio managers often shift funds from stocks into bonds when they expect stock market volatility to increase. But the effectiveness of flight to quality is weakened if expected volatilities across markets become increasingly correlated. Therefore, I test the following hypothesis without information spillover effect.

(H4) $$\lambda_i^b = 0 \text{ and } \gamma_i^s = 0$$

Furthermore, allowing time varying transition probabilities can reveal more aspects of the joint distribution of implied volatilities (Gray, 1996). Conditional on $\mathbf{F_{t-1}}$, the time-varying transition probabilities can be written as

(8) $$\begin{aligned} &\Pr(S_t = j \mid s_{t-1} = i,\ \mathbf{F_{t-1}}) = p_{ij,t},\ i, j \in \{1,\ 2\}, \\ &0 \le p_{ij,t} \le 1,\ \sum_{j=1}^{2} p_{ij,t} = 1 \text{ for all } i. \end{aligned}$$

Motivated by a common information effect (Ederington and Lee, 1993), I specify the transition probabilities to be a function of the lagged instruments, which proxy for common information about economic prospects:

(9) $$p_{ii,t} = p(S_t = i \mid S_{t-1} = i, \mathbf{F_{t-1}}) = \Phi(a_i + b_i Instrument_{t-1}), \quad i \in \{1, 2\},$$

where $Instrument_{t-1} \in \{SHORT_{t-1}, TERM_{t-1}, DEF_{t-1}\}$ is one of the macroeconomic variables commonly used in the literature. a_i and b_i are unknown parameters and Φ is the cumulative normal distribution function, which ensures that $0 < p_{ii,t} < 1$. This specification makes transition probabilities monotonic in the instrument, thus facilitating the interpretations of the parameters. By choosing one particular instrument each time, we can tell how predictive power varies from across instruments. [12]

The parameters of the time-varying regime switch model are $(\mu_i^j, \lambda_i^j, \gamma_i^j, \theta_i^j, \sigma_i^j, \rho_i, a_i, b_i)$, where $i \in \{1,2\}$ and $j \in \{s, b\}$. Note that b_i measures the dependence of the probability of staying in regime i on a particular instrument. We can perform a LR test against the time-invariate regime switch model with the following null hypothesis

(H5) $$b_i = 0 .$$

[12] I have also tried to use all instruments all at once. Unfortunately, the estimation is hard to converge.

If the null hypothesis is strongly rejected, there is substantial time-variation in transition probability and the instrument(s) can predict regime switch in stock and bond implied volatilities. In other words, information implied by macroeconomic variables can simultaneously alter investor expectations in both stock and bond markets and drive anticipated volatilities to fluctuate between regimes of high versus low volatility, supporting a common information effect.

To further determine whether the two-state model does a better job at accounting for the characteristics of stock and bond implied volatilities, a variety of diagnostic tests are conducted on the standardized residuals from the regime switch models.

4. Results

The estimation results are reported in Tables 5 and 6. From the estimated log-likelihood values and the resulting likelihood ratio statistics in Table 5, regime-switching models perform far better than the single-regime model, suggesting the existence of regimes and thus supporting the first hypothesis. Thus, we focus attention on the regime-switching models. In Table 5, I first examine the parameters for the variance equation of the regime switch model with constant transition probability. The estimate for the correlation between expectations of stock and bond volatilities under regime one (17.5%) is about twice as large as that for the cross-market correlation under regime two (9.9%). Arguably, regime one is a risk-on regime because of stronger linkages while regime two is a normal regime. The variations of both stock and bond volatility expectations are larger in the risk-on regime than in the normal regime. The movement of MOVE under regime one is about seven times as large as under regime two. The contrast is much more striking for the changes of VIX in different regimes.

[Table 5 here]

Besides stronger linkages and bigger variation of volatility expectations, the risk-on regime is also characterized by higher volatility expectations. This can be seen from the parameters in the mean equation in Table 5. The magnitudes of the intercepts suggest that expectation in both stock and bond volatilities under regime one is about three times as large as under regime two.

Confirming the volatility expectation clustering hypothesis within each regime (H2a), both VIX and MOVE are quite persistent no matter in which regime they stay since the estimates for λ_i^s and γ_i^b are close to unity and highly significant. An obvious explanation is gradual incorporation of information. Moreover, the parameters of the time-invariate transition probabilities support the persistent effect of regime shifts (H2b). The probability that high volatility expectation will be followed by another day of high expected volatility is p_{11}= 77.1%, which indicates that episodes will typically persist on average for 1/(1-0.771)=4.37 days. The probability that low volatility expectation will be followed by low perceived volatility is p_{22}= 93.8%, so that this regime will persist for 1/(1-0.934)=15.15 days.[13] Recall the preliminary analysis in Section 2 that stock and bond volatility expectations will persist for 92.42 days and 72.82 days respectively if regime shifts are not taken into consideration. Put in another way, ignoring regime shifts in volatility expectations can lead to the spurious appearance of extreme persistency.

Moreover, the estimates of θ_i^s are significantly negative with $\hat{\theta}_1^s = -1.145 < \hat{\theta}_2^s = -0.771$, confirming the hypothesis (H3a) that asymmetric stock volatility effect is not only present in each regime but also stronger in the risk-on regime. This finding carries an important implication

[13] On average, the unconditional probability that volatility is in the risk-on regime is (1-0.934)/(2-0.771-0.934) = 22.4%, which implies less than a quarter of time for market turmoil and risk-on to occur.

that non-diversifiable stock market volatility as an asset class is a good hedge against stock portfolio, especially in bad times. It suggests that VIX futures, options and other related products are useful developments for stock market risk management. For the second part of the hypothesis on bond volatility asymmetry (H3b), the null hypothesis is rejected. The relation between long term bond yield and ex ante interest rate volatility is regime-dependent: negative in the risk-on regime but positive in the normal regime. This evidence is new to the literature on interest rate volatility because researchers typically examine volatility of the short-term interest rate and find mixed relationships (Trolle and Schwartz, 2009). The negative relation in the risk-on regime also suggests some potential for interest rate volatility instruments to hedge against interest rate risk.

The null hypothesis of no information spillover effect (H4) is rejected for the risk-on regime. As indicated by estimates of λ_i^b, higher VIX can predict higher MOVE next period in regime one but not in regime two. Similarly, estimates for γ_i^s suggest that higher MOVE has predictive power for higher VIX in regime one only. This evidence indicates that the information spillover effect is bi-directional and regime-specific, which is not found in the Granger-causality tests of Table 3. Arguably, such bi-directional volatility transmission is an important source of correlation that is about 77% (= (0.175-0.099)/0.099) larger in the risk-on regime relative to the normal regime. Suppose that portfolio managers realize the presence of regimes. When they expect stock market volatility to increase in the bad time, they also expect increased bond volatility because linkages between volatility expectations strengthen across stock and bond markets. Therefore, a flight to quality by shifting funds from stocks into bonds becomes less effective than it does in the normal state.

However, constant transition probability cannot directly test for a common information effect. To address this issue, I turn to the more general model with time-varying transition probability to see whether common information about economic and financial conditions can drive anticipated volatilities to fluctuate between risk-on and normal regimes.

[Table 6 here]

As shown in Table 6, the estimated log-likelihood values and the resulting likelihood ratio statistics suggest that the time-varying transition probability model fits significantly better than the constant transition probability model, underscoring the importance of a more flexible modeling of transition probability. The results in the mean and variance equations are qualitatively consistent with and quantitatively similar to those in the time-invariate regime switch model. Most notably, all three instruments are helpful in predicting the probabilities of a regime switch. First, if the short rate is used as instrument, both b_1 and b_2 are positive, but only b_1 is significant. This implies that as the short rate increases, the probability of staying in the risk-on regime increases, which may characterize periods of high interest rates, asset price volatilities, and stock-bond co-movement. Second, the slope of the yield curve provides additional information for regime classification. With decreasing term spread that can predict recessions, the probability of staying in the risk-on regime increases, with higher expected volatilities and cross-market linkages. Note that such predictive power is not significant in the normal regime. Third, as suggested by the log-likelihood values and the resulting likelihood ratio statistics, the default spread has the greatest influence on the transition probability among three instruments. The reason may be that the default spread better captures information about short term business conditions. Both b_1 and b_2 are significantly negative. If stock and bond are perceived as less volatile and correlated in the current regime, improving business conditions indicated by lower default spread confirms that markets are in good shape and offer diversification opportunities. In contrast, under the risk-on regime with higher expected volatilities and cross-market linkages, a relatively better business situation will not drive a regime switch in the short run. In terms of magnitudes, the effect is bigger in the normal regime than in the risk-on regime.

Also, a variety of diagnostic tests on the standardized residuals (available on request) suggest that the evidence of misspecification is far weaker for the two-state models.

5. Concluding Remarks

Using daily implied volatility indices for S&P500 returns and US Treasury bond yields from 1990 to 2010, I examine the joint evolution of expected stock and bond volatility. While there are many studies of the Chicago Board Option Exchange S&P500 volatility index (VIX), this study also features the bond market's equivalent to VIX,[14] the Bank of America Merrill Lynch Treasury Option Volatility Estimate Index (MOVE). The bivariate two-state regime switching models developed in the paper fit the data much better than a single-regime model and also allow for *ex ante* volatility linkages due to common information and information spillover, as well as volatility clustering and asymmetry. There is strong evidence that common information about economic and financial conditions plays a significant role in driving the dynamics of regime switches between normal and risk-on regimes, where the risk-on regime is characterized by higher expected volatility and stronger cross-market linkages between ex ante volatilities. Allowing for regimes also reveals novel associations between stronger information spillover and increased linkages in the risk-on regime, suggesting lower effectiveness of flight to quality portfolio rebalancing strategies. Ignoring regime shifts in volatility expectations can lead to spurious extreme persistence and incomplete inferences about asymmetric volatility.

[14] Harley Bassman of Bank of America Merrill Lynch described the MOVE index as the bond market's equivalent to VIX. This argument has been widely cited among practitioners.

Future research can more thoroughly examine the time series properties of MOVE, which is still little explored relative to VIX, especially if the exact methodology used to normalize the interest rate volatility and to derive it from options is disclosed. More implications can be discussed for portfolio diversification and risk management, especially on an out-of-sample basis. Another dimension for future research is to investigate stock and bond ex ante volatilities across countries and relate associations to international macroeconomic and financial conditions. Finally, derivatives exchanges have recently expanded the range of implied volatility indexes to oil, gold, exchange rates, and other stock indexes, suggesting further room for studies of the joint evolution of implied volatility across asset classes.

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Table 1: Preliminary Data Analysis

The table reports preliminary analysis result for stock and bond implied volatilities from Bloomberg. VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is Bank of America Merrill Lynch Treasury Option Volatility Estimate Index. VIX is expressed in percentage and MOVE is quoted in basis points. ΔVIX and ΔMOVE are the first differences of VIX and MOVE indices respectively. "Nobs" is the numbers of daily observations for the series after deleting unmatched data. "Lag x" denotes autocorrelation at x period lag. LB Q(10) is the Ljung-Box's Q (10) statistics with *, **, and *** denoting significance at 10%, 5%, and 1%, respectively.

Panel A: Summary Statistics

Period	Stat	Nobs	mean	Std	Min	Max	Skew	Excess Kurt
From 1/2/1990	VIX	5232	20.383	8.253	9.310	80.860	2.023	7.285
To 12/31/2010	MOVE	5232	102.867	24.838	51.200	264.600	1.231	3.737

Panel B: Serial Correlation

Period	Stat	Lag1	Lag5	Lag10	LB Q(10)	LB Q(10) for squared term
From 1/2/1990	VIX	0.983	0.941	0.910	46419.461***	43777.029***
to 12/31/2010	MOVE	0.985	0.933	0.886	45474.504***	44265.317***

Table 2: ADF Test Results for Unit Roots

The table reports the results of augmented Dickey-Fuller test for unit roots. VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is Bank of America Merrill Lynch Treasury Option Volatility Estimate Index. "lags" means the optimal lag length used in regression to test for the presence of a unit root and it is chosen based on Bayesian information criterion (BIC). "Intercept" means that the regression equation used to test for the presence of a unit root includes an intercept term, where the ADF t-test statistics is reported. "Intercept & Trend" means that the regression equation used to test for the presence of a unit root includes an intercept and a time trend term, where the ADF ρ-test statistics is reported. *, **, and *** denote significance at 10%, 5%, and 1%, respectively.

Variable	lags	Intercept	Intercept & Trend
VIX	10	-4.555***	-43.103***
MOVE	4	-5.473***	-60.877***

Table 3: Pair-wise Granger Causality tests

The table reports the results of pair-wise granger causality tests VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is Bank of America Merrill Lynch Treasury Option Volatility Estimate Index. “lags” means the lag length used in regression to test for the presence of Granger causality. For simplicity, a same lag length, *k*, is set for *y* and *x* in a regression as follows

$$y_t = a + \sum_{i=1}^{k} b_i y_{t-i} + \sum_{j=1}^{k} c_j x_{t-j} + \varepsilon_t$$

When *k*=1, chi-squared t statistics is reported. When *k*>1, F-test statistics is reported. *, **, and *** denote significance at 10%, 5%, and 1%, respectively.

lags	MOVE →VIX	VIX → MOVE
k=1	1.035	9.153***
k =2	0.239	7.173***
k =3	0.297	4.489***
k =4	0.428	3.691***
k =5	0.318	2.888**
k =6	1.380	2.423**
k =7	1.082	2.272**
k =8	1.175	2.115**
k =9	1.015	2.005**
k =10	0.914	1.909**

Table 4: Correlation Matrix

The table reports correlation matrix for VIX, MOVE, stock return and bond yield as well as various instruments. VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is Bank of America Merrill Lynch Treasury Option Volatility Estimate Index. Stock return, denoted by r_s, is the log difference of S&P500 closing prices while r_b denotes the 10-year US Treasury bond yield. *SHORT* is the 3-month US Treasury bill yield. *TERM* is the difference between the US 10-year and 2-year Treasury bond yields. *DEF* is the difference between Moody BAA and AAA bond yields. *, **, and *** denote significance at 10%, 5%, and 1%, respectively.

	VIX	MOVE	r_s	r_b	SHORT	TERM	DEF
VIX	1.000						
MOVE	0.603***	1.000					
r_s	-0.122***	-0.034**	1.000				
r_b	-0.314***	-0.146***	0.010	1.000			
SHORT	-0.278***	-0.349***	0.009	0.810***	1.000		
TERM	0.177***	0.400***	-0.009	-0.356***	-0.810***	1.000	
DEF	0.676***	0.542***	-0.018	0.448***	-0.500***	0.387***	1.000

Table 5: Estimation Results of the Time-Invariate Regime-Switching Model

The table estimates two nested models for stock and bond implied volatility using VIX and MOVE daily data from the beginning of 1990 through the end of 2010. Two-regime model is unrestricted:

$$\begin{pmatrix} VIX_t \\ MOVE_t \end{pmatrix} = \begin{pmatrix} \mu_i^s \\ \mu_i^b \end{pmatrix} + \begin{pmatrix} \lambda_i^s \\ \lambda_i^b \end{pmatrix} VIX_{t-1} + \begin{pmatrix} \gamma_i^s \\ \gamma_i^b \end{pmatrix} MOVE_{t-1} + \begin{pmatrix} \theta_i^s & 0 \\ 0 & \theta_i^b \end{pmatrix} \begin{pmatrix} r_s \\ r_b \end{pmatrix} + \begin{pmatrix} \varepsilon_{it}^s \\ \varepsilon_{it}^b \end{pmatrix}, \text{ where } \begin{pmatrix} \varepsilon_{it}^s \\ \varepsilon_{it}^b \end{pmatrix} \sim \text{IIN}(\mathbf{0}, \mathbf{\Omega}_{\mathbf{it}}),$$

$$\mathbf{\Omega_{it}} = \mathbf{D_{it} R_{it} D_{it}}, \quad \mathbf{D_{it}} = \begin{pmatrix} \sqrt{\sigma_i^s} & 0 \\ 0 & \sqrt{\sigma_i^b} \end{pmatrix}, \quad \mathbf{R_{it}} = \begin{pmatrix} 1 & 0 \\ \rho_i & 1 \end{pmatrix}, \quad i \in \{1, 2\},$$

and transition probabilities $p_{ii} = \Pr(S_t = i \mid S_{t-1} = i, S_{t-2} = k, \cdots) = \Pr(S_t = i \mid S_{t-1} = i), \quad i \in \{1, 2\}$, where VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is the Merrill Option Volatility Estimate Index. S_t is the unobserved regime at time t. Single-regime model is restricted with the following constraints: $\mu_1^j = \mu_2^j$, $\lambda_1^j = \lambda_2^j$, $\gamma_1^j = \gamma_2^j$, $\theta_1^j = \theta_2^j$, $\sigma_1^j = \sigma_2^j$, $j \in \{s, b\}$, and $\rho_1 = \rho_2$. The parameter estimates are the QMLE. The t-statistics are reported in parentheses. The log-likelihood values are also reported. The likelihood ratio test is a test of the regime switch model against the single-regime model. *, **, and *** denote significance at 10%, 5%, and 1% respectively.

	Single-regime model	Two-regime model	
		Regime 1	Regime 2
μ_i^s	0.269***	0.563***	0.305***
t-stat.	(4.830)	(11.868)	(31.389)
μ_i^b	1.365***	5.301***	0.931***
t-stat.	(3.826)	(24.286)	(19.823)
λ_i^s	0.985***	0.975***	0.984***
t-stat.	(489.174)	(801.311)	(2147.739)
λ_i^b	0.040***	0.060***	0.004
t-stat.	(4.054)	(9.285)	(1.636)
γ_i^s	0.001	0.001***	-0.000
t-stat.	(0.848)	(4.405)	(-1.642)
γ_i^b	0.977***	0.952***	0.985***
t-stat.	(320.712)	(610.023)	(2212.378)
θ_i^s	-0.977***	-1.145***	-0.771***
t-stat.	(85.471)	(-73.759)	(-89.013)
θ_i^b	0.026	-0.139***	0.049***
t-stat.	(0.639)	(-3.112)	(5.905)
σ_i^s	0.878***	2.505***	0.328***
t-stat.	(48.074)	(44.258)	(41.392)
σ_i^b	17.985***	53.478***	7.421***
t-stat.	(51.450)	(32.513)	(42.395)
ρ_i	0.170***	0.175***	0.099***
t-stat.	(12.256)	(6.381)	(5.328)
p_{ii}		0.771***	0.934***
t-stat.		(53.127)	(14.135)
Likelihood	-21986.758	-10682.399	
LR statistics		22608.718***	

Table 6: Estimation Results of Time-Varying Regime-Switching Models for Level Analysis

The table estimates three time-varying regime switch models for stock and bond implied volatility using VIX and MOVE daily data from the beginning of 1990 through the end of 2010.

$$\begin{pmatrix} VIX_t \\ MOVE_t \end{pmatrix} = \begin{pmatrix} \mu_i^s \\ \mu_i^b \end{pmatrix} + \begin{pmatrix} \lambda_i^s \\ \lambda_i^b \end{pmatrix} VIX_{t-1} + \begin{pmatrix} \gamma_i^s \\ \gamma_i^b \end{pmatrix} MOVE_{t-1} + \begin{pmatrix} \theta_i^s & 0 \\ 0 & \theta_i^b \end{pmatrix}\begin{pmatrix} r_s \\ r_b \end{pmatrix} + \begin{pmatrix} \varepsilon_{it}^s \\ \varepsilon_{it}^b \end{pmatrix}, \text{ where } \begin{pmatrix} \varepsilon_{it}^s \\ \varepsilon_{it}^b \end{pmatrix} \sim \text{IIN}(\mathbf{0}, \mathbf{\Omega}_{\text{it}}),$$

$$\mathbf{\Omega_{it}} = \mathbf{D_{it}}\mathbf{R_{it}}\mathbf{D_{it}}, \quad \mathbf{D_{it}} = \begin{pmatrix} \sqrt{\sigma_i^s} & 0 \\ 0 & \sqrt{\sigma_i^b} \end{pmatrix}, \quad \mathbf{R_{it}} = \begin{pmatrix} 1 & 0 \\ \rho_i & 1 \end{pmatrix}, \quad i \in \{1, 2\},$$

and transition probabilities $p_{ii,t} = p(s_t = i \mid s_{t-1} = i, \mathbf{F_{t-1}}) = \Phi(a_i + b_i Instrument_{t-1}), \quad i \in \{1, 2\},$

where VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is the Merrill Option Volatility Estimate Index. S_t is the unobserved regime at time t. $\mathbf{F_{t-1}}$ is the past information set. Φ is the cumulative normal distribution function. The parameter estimates are the QMLE. The t-statistics are reported in parentheses. The log-likelihood values are also reported. The likelihood ratio test is a test of the time-varying model against time invariate model in Table 4. *, **, and *** denote significance at 10%, 5%, and 1% respectively.

	Instrument=Short rate		Instrument=TERM spread		Instrument=DEF spread	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
μ_i^s	0.586***	0.310***	0.573***	0.306***	0.587***	0.319***
t-stat.	(12.194)	(31.828)	(11.971)	(31.470)	(12.091)	(32.928)
μ_i^b	5.253***	0.942***	5.290***	0.934***	5.511***	0.916**
t-stat.	(23.759)	(20.078)	(24.032)	(19.887)	(24.641)	(19.543)
λ_i^s	0.976***	0.984***	0.976***	0.984***	0.975***	0.983***
t-stat.	(796.258)	(2136.764)	(796.532)	(2142.420)	(788.263)	(2104.758)
λ_i^b	0.061***	0.004	0.061***	0.004*	0.062***	0.003
t-stat.	(9.3446)	(1.598)	(9.360)	(1.669)	(9.391)	(1.101)
γ_i^s	0.001***	-0.001*	0.001***	-0.000	0.001***	-0.000**
t-stat.	(3.743)	(-1.702)	(4.039)	(-1.643)	(4.039)	(2.040)
γ_i^b	0.952***	0.985***	0.952***	0.985***	0.951***	0.985***
t-stat.	(602.884)	(2207.599)	(605.274)	(2208.059)	(596.737)	(2194.469)
θ_i^s	-1.142***	-0.774***	-1.144***	-0.772***	-1.140***	-0.777***
t-stat.	(-72.962)	(-88.567)	(-73.262)	(-88.994)	(-72.005)	(-86.451)
θ_i^b	-0.125***	0.050***	-0.132***	0.049***	-0.147***	0.055***
t-stat.	(-2.756)	(5.974)	(-2.914)	(5.858)	(-3.185)	(6.658)
σ_i^s	2.531***	0.330***	2.521***	0.329***	2.563***	0.326***
t-stat.	(43.866)	(41.395)	(43.956)	(41.415)	(43.244)	(41.050)
σ_i^b	54.072***	7.411***	53.846***	7.428***	54.418***	7.390***
t-stat.	(32.053)	(42.432)	(32.239)	(42.432)	(31.931)	(42.390)
ρ_i	0.175***	0.100***	0.175***	0.100***	0.175***	0.097***
t-stat.	(6.322)	(5.336)	(6.341)	(5.346)	(6.291)	(5.136)
a_i	-1.011***	1.490***	-0.591***	1.516***	-0.219***	1.946***
t-stat.	(-20.586)	(40.961)	(-12.185)	(41.711)	(-4.311)	(53.142)
b_i	0.091***	0.001	-0.117***	-0.011	-0.370***	-0.538***
t-stat.	(7.101)	(0.176)	(-3.723)	(-0.442)	(-8.659)	(-14.655)
Likelihood	-10675.509		-10680.268		-10652.880	
LR	13.78***		4.262***		59.038***	

Figure 1: Historical VIX and MOVE

The plot shows daily movement of VIX and MOVE from the beginning of January 1990 through the end of December 2010. VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is the Bank of America Merrill Lynch Treasury Option Volatility Estimate Index. VIX is in terms of percentage while MOVE is in terms of basis points.

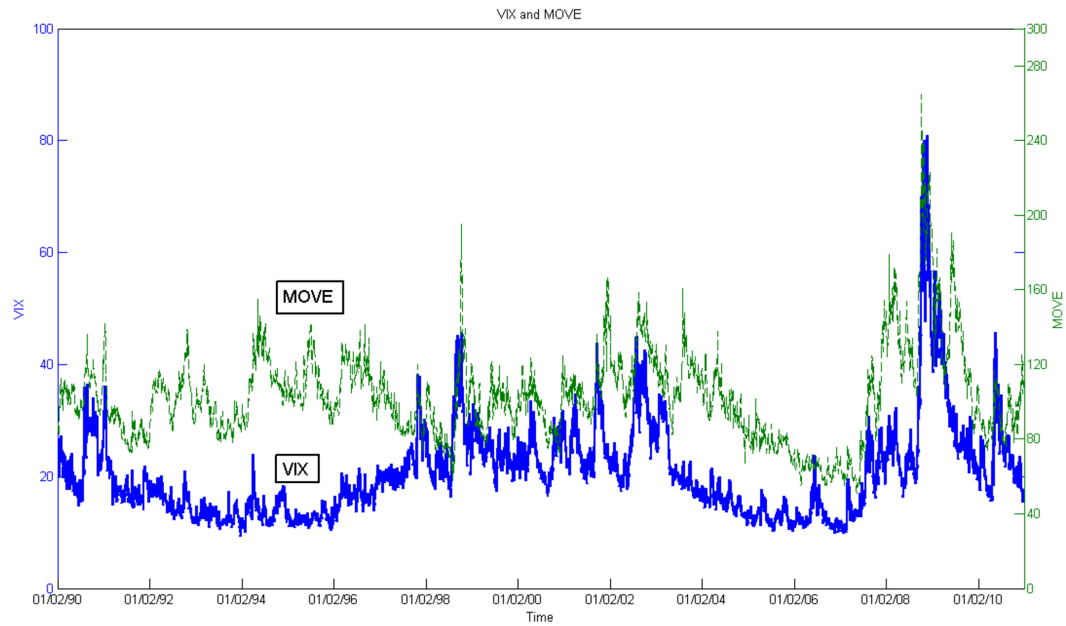
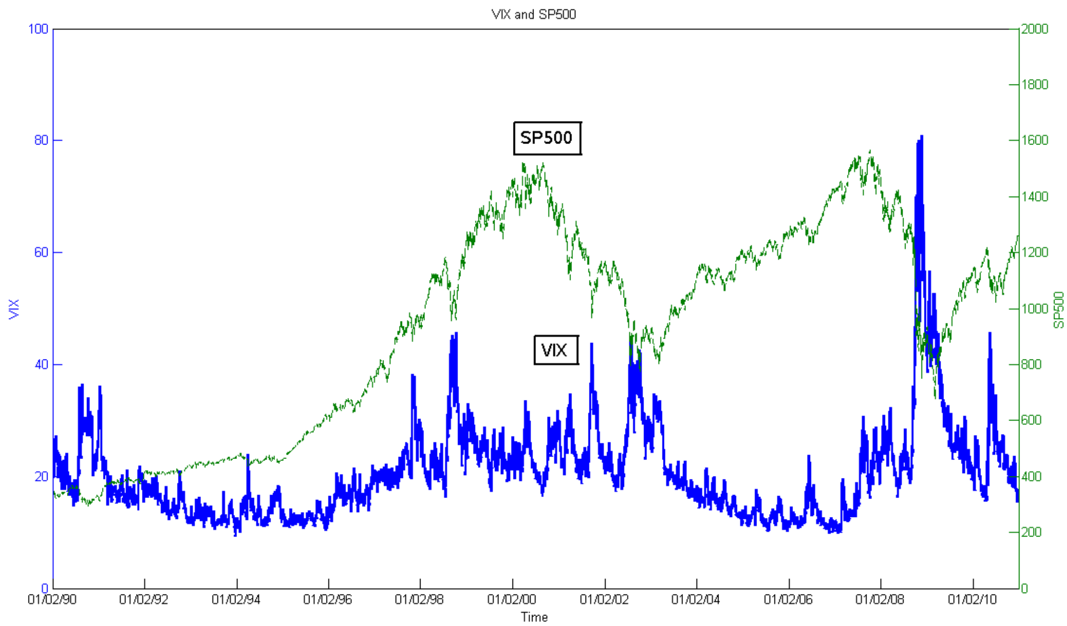


Figure 2: Volatilities and their underlying assets

The plots show daily movement of the VIX and S&P 500 closing price in Panel A and daily movement of MOVE and 10-year Treasury bond yield in Panel B from the beginning of January 1990 through the end of December 2010. VIX is the Chicago Board Option Exchange S&P500 volatility index. MOVE is the Merrill Option Volatility Estimate Index in which 10-year U.S Treasury bonds accounts for 40%.

Panel A



Panel B