Large Swings in Currencies Driven by Fundamentals

Phornchanok Cumperayot
Chulalongkorn University

Casper G. de Vries*
Erasmus University Rotterdam and Tinbergen Institute

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Abstract

Exchange rate returns exhibit distributions with fat tails, i.e. high probability of extreme currency movements. We provide evidence that the apparent non-normality is driven by the tail behavior of macroeconomic fundamentals. Economic and probabilistic arguments are offered for this relationship. The empirical results show that the exchange rate returns and economic fundamentals are asymptotically dependent: when the fundamentals such as money supply, interest rate and price level increase dramatically, large declines in currency prices occur around one third of the times. Their joint occurrence indicates that large swings in currency prices are partly associated with heavy-tailed macroeconomic fundamentals.

Keywords: Exchange rates, fundamentals, fat-tailed distributions.
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1 Introduction

Large swings in currency prices have been thoroughly investigated and led to the hypothesis that exchange rate return distributions exhibit non-normal fat tails, see e.g., Westerfield (1977), Boothe and Glassman (1987), Akgiray, Booth and Seifert (1988), Koedijk, Schafgans and de Vries (1990), Koedijk, Stork and de Vries (1992), Koedijk and Kool (1994) and Susmel (2001).[1] The fat tail nature of the FX returns indicates that the frequent occurrence of extreme market movements is excessive relative to the conventional normal distribution. As the potential catastrophic consequences of these extreme events are relevant for risk management and financial stability, an intriguing open question is: What causes these large swings in currency prices? In this paper, by using Extreme Value Theory (EVT) we give theoretical arguments and empirical tests for the hypothesis that the fat tails are caused by the economic fundamentals that drive the exchange rate.

*Corresponding author's address: Tinbergen Institute, Erasmus University Rotterdam, P.O. Box 1738, Rotterdam 3000 DR, The Netherlands. E-mail address: cdevries@few.eur.nl. E-mail address of Phornchanok Cumperayot: phornchanok.c@chula.ac.th. We like to thank Eric van Wincoop for perceptive comments and helpful suggestions on an earlier draft.

[1] Moreover, it is well documented that at higher frequencies exchange rate returns exhibit volatility clustering, see Diebold (1988). This adds to the fat tail nature of the returns.

Standard exchange rate models, such as the monetary-approach based model, would suggest that this fat tail feature either stems from the nature of the fundamentals' distributions, or is caused by the error term. Using a reduced form of the monetary-approach model, Lux and Sornette (2002) show that the fat tails of the exchange rate returns are driven by rational expectations bubbles, proposed by Blanchard and Watson (1982). However, the numerical prediction resulting from their exogenous bubbles is not consistent with the usual empirical findings.[2] Lux and Marchesi (1999) and Aoki (1999) use the market microstructure exchange rate models to explain the fat-tailed exchange rate returns by generating endogenous bubbles through the trading process. In this paper, we argue that the stylized fact of power-law tails of the exchange rate returns can be due to the heavy tail behavior of economic fundamentals from the monetary-approach exchange rate models.

We first show that within a standard monetary macroeconomic model with Brainard type multiplicative uncertainty, the implied distributions of macroeconomic variables like the inflation rate and money stock can exhibit the heavy tail feature, even if the noise distribution itself has no tails at all such as the uniform distribution. Then, by applying Feller's (1971, VIII.8) convolution theorem in the log-linear exchange rate model we discuss how the heavy tails are carried over to the exchange rate returns. For empirical investigation, our data set consists of monthly observations from 30 countries over the period of 1974-2007.[3] We demonstrate that the fat tail nature is not exclusively for the exchange rate returns, but also for economic fundamentals. Moreover, the exchange rate returns and economic fundamentals are asymptotically dependent, i.e. when the fundamentals take on extreme values the probability of their joint occurrence with large swings in currency prices is positive even in limit.[4]

The failure of standard models which relate exchange rate to macroeconomic variables becomes well known since the seminal work of Meese and Rogoff (1983), as this type of models has had little success compared to naive no change forecasts. However, Engel and West (2005) show that an exchange rate manifests near-random walk behavior if the fundamentals are I(1) and if the factor for discounting expected future fundamentals is close to unity. Therefore, according to Engel, Mark and West (2007) the criterion of outperforming a random walk in forecasting is too strong for the exchange rate models. Engel and West (2005) and Engel, Mark and West (2007) also provide various evidence to support the hypothesis that the exchange rate models are not as bad as we think. Moreover, in their scapegoat theory Bacchetta and Wincoop (2004) exploit parameter instability to explain why the exchange rate models have found so little explanatory power of macroeconomic variables.

Neely and Sarno (2002) discusses developments in exchange rate economics and show that there are always signs indicating the link between macroeconomic variables and the exchange rate even in the works that reject the exchange rate models.[5] In addition, a number of works, e.g. Mark and Sul (2001) and Groen (2005), show that the predictive power of the exchange rate models can be increased by using panel studies and predicting exchange rates at long horizons. Engel, Mark and West (2007) also explain that short-run movements in exchange rates are primarily determined by changes in expectations of future fundamentals. The sample distribution of ex post realizations of economic variables is commonly perceived as a good approximation of the distribution used when making forecasts. Hence, we take these as sufficient basis for investigating the tail relationship between the exchange rate returns and the economic fundamentals from the

[2] The model used in Lux and Sornette (2002) suggests fatter tails than the usual findings as it predicts the tail index below 1, while the empirical estimates for the exchange rate returns are between 2-4.

[3] See Appendix A for a list of countries and data sources.

[4] Asymptotic dependence is the strongest form of tail depepndence. For a bivariate normal distribution, two variables are asymptotically dependent when their correlation coeffficient is equal to one.

[5] This includes the work by Meese and Rogoff (1983) which find some evidence of predictability at long horizons.

monetary-approach exchange rate models.[6]

The rejection of the exchange rate models often comes from the fact that these models cannot correctly predict the exchang rate movement consistently and persistently. However, the academic consensus that macroeconomic variables have little explanatory power for exchange rates is in contradiction to market practices when analyzing the currency price movement (Bacchetta and Wincoop, 2004). In this paper, by using EVT we express the relation between variables in terms of tail index similarity and probabilities. Our measures thus cover cases in which the monetary-approach exchange rate models may hold true, but evidence have been weaken by complications such as parameter uncertainty (Bacchetta and Wincoop, 2004), volatile and inconsistent expectations (Neely and Sarno, 2002) or even nonlinearity (Taylor and Peel, 2000). To our knowledge, this is the first investigation of the linkage between the tails of the fundamentals' distribution and the distribution of exchange rate returns.

To measure the tail fatness, we estimate tail indices α using the Hill estimator[7]. To illustrate the extreme linkage between the economic fundamentals and the exchange rate returns, we calculate the conditional expectation measure proposed in Huang (1992) and used in Hartmann, Straetmans and de Vries (2004, 2010) and de Vries (2005). Further, we analyze the robustness of our results by using an alternative measure of asymptotic dependence in Poon, Rockinger and Tawn (2004), and by testing whether the tail association is preserved under the log-linear exchange rate models. The next section gives a brief account of what we need from extreme value theory (EVT) and makes the argument for the transmission of fat tails from the fundamentals to the exchange rate returns. We combine theoretical economic and probabilistic arguments for the fat tail phenomenon. Section 3 and 4 discuss the estimation methods and empirical results, respectively. Conclusions are presented in Section 5.

2 Theory

Within a standard monetary macroeconomic model, we first show that multiplicative supply-side Brainard type noise with a bounded support induces fat tails on the distribution of the macroeconomic aggregates, even in the setup the noise itself does not have fat tails. Subsequently, we provide a short review of the probabilistic properties of fat-tailed distributed random variables and their scaling properties.

2.1 Tail Events and Macroeconomic Fundamentals

One may wonder why macroeconomic fundamentals have distributions with heavy tails. An early statistically oriented explanation for inflation rates was given by Engle (1982). Engle's ARCH model has random variables follow a martingale process with autoregressive behavior in the second moment causing clusters of high and low volatility. Then even if the innovations are thin-tailed normally distributed, the unconditional distribution ends up having fat tails like the Pareto distribution, see de Haan, Resnick, Rootzen and de Vries (1989). Cumperayot (2002), however, shows that macroeconomic variables significantly exhibit fat tails even after filtering out the ARMA-GARCH components. Thus, a fundamental based explanation for the apparent non-normality is needed.

Here we develop an economic based explanation of how the distribution of a macroeconomic variable like the money stock or rate of inflation can exhibit the fat tail feature. The idea is not

[6] To provide empirical evidence, in this paper we focus on the tail that represents extreme depreciations of the domestic currency relative to the foreign curreny which is relevant for risk management and financial stability.

[7] The Hill (1975) estimator is the most efficient estimator in mean squared error sense for the heavy-tailed distributed variables.

to present a fully fledged theory, as this would be outside the scope of the paper, but to present a coherent argument for two of the macroeconomic variables involved. The next subsection then shows that the heavy tail feature is carried over to the exchange rate.

To this end consider the following stylized monetary macroeconomic model, as presented in, e.g., Walsh (2003, p.440). The aggregate supply curve reads

$$Y_t = A_t(\Pi_t - E_{t-1}[\Pi_t]) + \varepsilon_t, \tag{1}$$

where Y_t is the logarithmic level of output, Π_t is an inflation rate and $E_{t-1}[\Pi_t]$ is the time $t-1$ expected inflation for time t, and ε_t is a noise term. In the short run, deviations from the long-run output level are possible due to expectational errors. The elasticity of output with respect to inflation expectations' errors is A_t. Thus (1) is in a crude way the Lucas type supply curve. Aggregate demand depends on real interest rates, i.e. the nominal interest rate minus expected inflation $I_t - E_t[\Pi_{t+1}]$:

$$Y_t = -b(I_t - E_t[\Pi_{t+1}]) + \eta_t. \tag{2}$$

The reduced-form money market equation is based on the quantity equation

$$M_t = P_{t-1} + \Pi_t + Y_t - gI_t + \nu_t, \tag{3}$$

where M_t and P_{t-1} stand for the logarithms of the quantity of money and price level, respectively. The three disturbances $(\varepsilon_t, \eta_t, \nu_t)$ are assumed to have mean zero i.i.d. noise with thin (exponential decline) or bounded tails (in case of bounded support).

Frequently model estimates and new data lead to parameter revisions, see Sack (2000). We capture the model uncertainty via the Brainard (1967) effect and assume that the coefficient for the short-run Phillips effect A_t is an i.i.d. random variable. Suppose A_t has a beta distribution

$$P\{A \leq x\} = x^{\alpha},\ \ \alpha > 2. \tag{4}$$

The support of this distribution is $[0, 1]$. Note that this distribution is clearly not fat tailed. The fact that zero is in the support reflects the possibility that the short-run supply curve may be vertical, i.e. coincides with the long-run curve.

Suppose the goal of monetary policy is to stabilize the level of inflation around a target π^*. This reflects, e.g., the European Central Bank's single price stability objective, since the ECB does not have real income stabilization or employment as its prime objectives. Specifically, assume that the objective resembling the ECB's main task reads

$$\min_{I_t} E_{t-1}[(\Pi_t - \pi^*)^2]. \tag{5}$$

Based on information available at time $t-1$, the central bank determines the policy interest rate I_t in order to minimize its expected loss function of price instability. The policy interest rate is then set to ensure that the expected value of inflation equals the target level, i.e. $E_t[\Pi_{t+1}] = E_{t-1}[\Pi_t] = \pi^*$. Substituting out Y_t from the first two equations (1) and (2) gives

$$\Pi_t = \frac{(b + A_t)\pi^* - bI_t + \eta_t - \varepsilon_t}{A_t}.$$

Since by assumption $\alpha > 2$ in (4), the $E[1/A_t]$ is bounded (see (6) below). Thus we can take expectations conditional on time $t-1$ information

$$E_{t-1}[\Pi_t] = b(\pi^* - I_t)E_{t-1}[1/A_t] + \pi^*,$$

and equate $E_{t-1}[\Pi_t] = \pi^*$. Hence, given its objective function (5), it is optimal for the central bank to set

$$I_t = \pi^*.$$

This implies for the money equation (3) that

$$M_t = P_{t-1} + \Pi_t + Y_t - g\pi^* + \nu_t.$$

Use the first two equations (1) and (2) to substitute out Y_t and Π_t, to get

$$M_t = P_{t-1} + (1-g)\pi^* + (1 + \frac{1}{A_t})\eta_t - \frac{\varepsilon_t}{A_t} + \nu_t.$$

Solving for the two other endogenous variables we find

$$\Pi_t = \pi^* + \frac{\eta_t - \varepsilon_t}{A_t}$$

and

$$Y_t = \eta_t.$$

Now Π_t and M_t are heavy-tailed distributed since $(\eta_t - \varepsilon_t)/A_t$ is heavy-tailed distributed. This follows from the fact that the random Phillips effect coefficient appears in the denominator. Given the beta distribution assumption (4) regarding A_t, the distribution of the inverse is

$$P\{\frac{1}{A} \leq x\} = 1 - P\{A \leq \frac{1}{x}\} = 1 - \frac{1}{x^\alpha}, \tag{6}$$

with support $x\epsilon[1,\infty)$. Thus the inverse of A has a heavy-tailed Pareto distribution (conditional on the distribution of η_t and ε_t) and has moments k only up to α. This can be easily seen from

$$E[(1/A)^k] = \int_1^\infty x^{k-\alpha-1}dx = \frac{1}{k-\alpha}x^{k-\alpha}|_1^\infty.$$

The power decline of the Pareto density makes that not all moments exist. This is the defining characteristic of heavy-tailed distributions.

As a result, the unconditional distributions of Π_t and M_t are also heavy-tailed. To see this, let $Q = \eta - \varepsilon$ and consider the distribution of Q/A. Suppose the distribution of Q does not have heavy tails, in the sense that all moments are bounded; in particular $E_Q[Q^\alpha] < \infty$. Using the conditioning argument of Breiman and (6) then shows that

$$\begin{aligned} P\{\frac{Q}{A} &> x\} = E_Q[P\{\frac{q}{A} > x|Q=q\}] \\ &= E_Q[P\{\frac{1}{A} > \frac{x}{q}|Q=q\}] \\ &= E_Q[\left(\frac{Q}{x}\right)^\alpha] \\ &= E_Q[Q^\alpha]x^{-\alpha}. \end{aligned} \tag{7}$$

Therefore, due to the random Phillips curve coefficient, the unconditional distributions of Π_t and M_t are also heavy-tailed.[8] Next, we show how the heavy tail feature can be carried over to the exchange rate distribution.

[8] The same argument is applied to the money growth rate ΔM_t which contains $(\eta_t - \varepsilon_t)/A_t$ and $(\eta_{t-1} - \varepsilon_{t-1})/A_{t-1}$.

2.2 Regular Variation and Tail Additivity

The monetary-approach exchange rate model is linear in the macroeconomic fundamental variables (see Neely and Sarno, 2002, and Sarno and Taylor, 2002). Suppose that the distributions of the macroeconomic variables exhibit heavy tails. We first show that if the macroeconomic variables are i.i.d., then the exchange rate also has a distribution with heavy tails. Subsequently, we argue that this result still follows if the macroeconomic variables are dependent. From an economic point of view the independence case is, in a way, the hardest case to treat. Since if, say, the macroeconomic fundamentals are driven by a common component that is heavy-tailed distributed, then it is almost immediate that this property is transferred to the distribution of the exchange rate.[9]

We adopt the following general notion of heavy tails. A distribution function $F(x)$ is said to exhibit heavy tails if its tails vary regularly at infinity. The upper tail varies regularly at infinity with tail index α if [10]

$$\lim_{t\to\infty}\frac{1-F(tx)}{1-F(t)}=x^{-\alpha},\ x>0 \text{ and } \alpha>0. \tag{8}$$

Regular variation implies that the distribution changes at a power rate. This contrasts with, e.g., the normal distribution that has tail probabilities that decline at an exponential rate. The number of bounded moments of $F(.)$ is finite and equals the integer value of α, i.e. the α-moment.[11] One checks that the Student-t distribution satisfies (8) by using L'Hôpital's rule, for the Pareto distribution this is trivial.

Random variables with regularly varying distributions satisfy an important additivity property. Suppose a distribution has heavy tails, so that

$$P\{X>x\}=1-F(x)\sim Ax^{-\alpha},\ as\ x\to\infty. \tag{9}$$

According to Feller's Convolution Theorem (1971, VIII.8), if X_1 and X_2 are i.i.d. with c.d.f. $F(x)$ which has regularly varying tails as in (9), then

$$P\{X_1+X_2>s\}\sim 2As^{-\alpha},\ as\ s\to\infty. \tag{10}$$

If X and Y are i.i.d. and if X has a tail index of α and Y has a lighter tail (e.g. has a hyperbolic tail with a higher power than α or even has an exponential type tail), then analogous to the proof of (7) one shows that

$$P\{X+Y>s\}\sim As^{-\alpha}. \tag{11}$$

In this case the convolution is dominated by the heavier tail.

Some intuition for the Feller theorem is as follows. Let X be i.i.d. Pareto distributed with scale $A=1$. Then for large s

$$1-P\{X_1\le s, X_2\le s\}=1-\left(1-s^{-\alpha}\right)^2\approx 2s^{-\alpha}$$

since the second term $s^{-2\alpha}$ is of smaller order. Thus only the (univariate) probability mass along the axes counts. The mass above the line $X_1+X_2=s$ is also determined by how much probability mass is aligned along the axes above this line, i.e. $2s^{-\alpha}$. The probability mass above the line away from the axes is of smaller order.

[9] For theoretical details on extremal analysis, the reader is referred to, e.g., Longin (1996) and Embrechts, Kluppelberg and Mikosch (1999).

[10] For the lower tail, $\lim_{t\to\infty} F(-tx)/F(-t)=x^{-\alpha}$, $x>0$ and $\alpha>0$.

[11] For instance, the Pareto distribution satisfies the Power law and has a number of bounded moments equal to an integer of α. The Student-t distribution has moments equal to its degree of freedom. The thin-tailed normal distribution has all moments bounded.

The convolution result (10) and (11) are very powerful. To give an illustrative example, let's consider the quasi-reduced-form specification of the exchange rate models in the logarithmic form

$$e = \varphi_0 + \varphi_1 m + \varphi_2 y + \varphi_3 i + \varphi_4 p^e + u, \tag{12}$$

where e, m, y, i and p^e denote changes in the exchange rate, money supply, real income, interest rate and expected price level, respectively, and u is an error term. The exchange rate is quoted as a price of foreign currency in terms of domestic currency, while other variables are the domestic variables relative to the corresponding foreign variables. Monetary neutrality holds if $\varphi_1 = 1$; $\varphi_2 < 0$ in theory, while the sign of φ_3 depends on the version of the model, see Frankel (1979).

In the context of the monetary model (12), the convolution theorem predicts that if the distributions of the changes in the fundamental variables exhibit heavy tails, the exchange rate return distribution should have a heavy tail as well. In particular, (11) constrains the tail shapes of the fundamentals and the exchange rate in the following way

$$\alpha_e = \min(\alpha_m, \alpha_y, \alpha_i, \alpha_{p^e}, \alpha_u), \tag{13}$$

where α_x represents a tail index of a random variable x and where the variables adhere to (12). In particular assume for example that the macroeconomic variables m, y, i and p^e in (12) all have Pareto type tails with identical tail index α as in (9) and unit scale, so that $A = 1$. Moreover assume that the noise u follows a distribution with bounded support. Then

$$\begin{aligned} P\{e > t\} &= P\{\varphi_0 + \varphi_1 m + \varphi_2 y + \varphi_3 i + \varphi_4 p^e + u > t\} \\ &\sim (\varphi_1^\alpha + \varphi_2^\alpha + \varphi_3^\alpha + \varphi_4^\alpha)\, t^{-\alpha}. \end{aligned} \tag{14}$$

We find that the tail shape of the exchange rate returns e is governed by the tail shape of the fundamentals' changes.

The convolution result (10) assumes that the macroeconomic variables from (12) are independent random variables, which is often not the case due to endogeneity. Consider therefore the multivariate extension of (8). Suppose that the vector $\mathbf{x}$ of fundamental variables is multivariate regularly varying in the sense that

$$\lim_{t\to\infty} \frac{1 - F(t\mathbf{x})}{1 - F(t\mathbf{1})} = W(\mathbf{x}), \ \mathbf{x} > 0,$$

where $W(.)$ is a function such that $W(\lambda\mathbf{x}) = \lambda^{-\alpha} W(\mathbf{x})$, $\alpha > 0$, $\lambda > 0$ and $\mathbf{1}$ is the unit vector. Suppose the marginal distributions are as in (9) so that the scales are of the same order, and all the marginal distributions have the same tail index α. Then for any non-zero weight vector $\mathbf{w}$, $P\{\mathbf{w}^T\mathbf{x} > s\} \sim Cs^{-\alpha}$, as $s \to \infty$. Here the scale constant C depends on the type of dependence and can no longer be determined as in (14), i.e. it requires specific knowledge of the copula. Nevertheless, the weighted sum of macroeconomic variables that determines the distribution of the exchange rate still has a Pareto like upper tail with the tail index α. Moreover, it is still the case that if the marginal distributions have different tail indices, the fundamental with the heaviest tail should have a tail index equal to the tail index of the exchange rate returns.

In addition, we like to note that the atemporal convolution result still holds when the economic variables are stationary time series. This is so since the convolution is a 'cross-section' like aggregation at a specific point in time. As the exchange rate and macroeconomic variables display bouts of quiescence and turbulence, changes in the economic variables are often captured by ARMA-GARCH type of models. From the convolution result (10), one can show that when time series are not i.i.d. but serially dependent, the occurrence of extremes may affect the distribution of order statistics, but not the tail index α. That is the exchange rate return distribution still has hyperbolic tails.

The convolution theorem can nevertheless also be used to study the aggregation of time series over time. Suppose for example that m follows the following $MA(1)$ process

$$m_t = \varepsilon_t + \gamma\varepsilon_{t-1}, \text{ and } \gamma > 0,$$

and where the innovations ε are i.i.d. with distribution function as in (9). Then, by Feller's Convolution Theorem

$$P\{m > x\} \sim A\,(1+\gamma^{\alpha})\,x^{-\alpha}, \; as\ x \to \infty.$$

Furthermore, $P\{m_t + m_{t-1} > x\} \sim A\,[1+(1+\gamma)^{\alpha}+\gamma^{\alpha}]\,x^{-\alpha}$, *as* $x \to \infty$. Note that the convolution results show that the scales of the random variables change due to the moving average process, but not the tail index α.

More complicated time series models can also be handled. For instance, Engle's (1982) original contribution modeled the inflation rate by the ARCH process. De Haan et al. (1989) showed that the tail of the stationary distribution of the ARCH process is regularly varying. Basrak et al. (2002) discuss the convolution of GARCH processes.

2.3 Asymptotic Dependence

If the tail fatness of the exchange rate returns is a consequence of heavy-tailed macroeconomic variables. Extreme exchange rate movements should be associated with extreme changes in macroeconomic variables. To investigate the linkage between extreme movements of the two variables we test whether the variables are asymptotically dependent. Asymptotic dependence is the strongest form of four types of the dependence structure, which are independence, perfect dependence, asymptotic independence and asymptotic dependence (see Poon, Rockinger and Tawn, 2004). If we have two variables Y and X and their extreme value statistics are defined when Y and X are beyond the EVT thresholds θ_Y and θ_X, respectively. The variables are asymptotically dependent if the conditional probability $P\{Y > \theta_Y \mid X > \theta_X\}$ remains positive in limit, i.e. when both variables move deeper into the tails and approach infinity.[12]

To examine the connection in the tails of economic fundamentals and exchange rate returns, we exploit the conditional expectation measure $E\{\kappa \mid \kappa \geqslant 1\}$ proposed in Huang (1992) and used in Hartmann, Straetmans and de Vries (2004, 2010) and de Vries (2005), i.e.

$$E\{\kappa \mid \kappa \geqslant 1\} = \frac{P\{X > \theta_X\} + P\{Y > \theta_Y\}}{1 - P\{X \le \theta_X, Y \le \theta_Y\}}, \tag{15}$$

where κ denotes the number of variables in the tail area. This measure simply states the expected number of variables being in the tail area, given that one variable is in the tail. The idea behind this measure is that if two variables are not independent, having some information about one variable, say X, implies that one has also information about the other variable, Y (see Hartmann et al., 2010).

Rather than defining the thresholds θ_Y and θ_X, in this paper we evaluate $E\{\kappa \mid \kappa \geqslant 1\}$ as Y and X approach infinity. Hartmann et al. (2010) show that in the limit $1 \le E\{\kappa \mid \kappa \geqslant 1\} \le 2$. The expected number is equal to 1 when two variables are asymptotically independent and equal to 2 when the variables are perfectly asymptotically dependent. For asymptotically dependent variables, $1 < E\{\kappa \mid \kappa \geqslant 1\} < 2$. To develop some intuition for this measure let Y be an exchange rate return and X be an economic fundamental which determines Y, namely $Y = \delta X + \xi$ when

[12] Without loss of generality, one can also consider minima, as results for one of the two can be immediately transferred.

$0 < \delta \leq 1$. By applying the tail additivity theorem, we analyze two cases: how the coefficient δ and the conditional expectation measure $E\{\kappa \mid \kappa \geqslant 1\}$ are related. For the case in which thin-tailed economic fundamentals result in the exponential tails of the exchange rate returns, Proposition 1 in Hartmann et al. (2010) applies (see page 245). If the variables Y and X follow a bivariate normal distribution with positive correlation, they are asymptotically independent with $E\{\kappa \mid \kappa \geqslant 1\} = 1$ in limit.

Case 1 *Suppose that X and ξ are i.i.d. random variables with regularly varying tails, i.e. as $x \longrightarrow \infty$*

$$P\{X > x\} = P\{\xi > x\} = x^{-\alpha}.$$

Then $\lim_{x\longrightarrow\infty} E\{\kappa \mid \kappa \geqslant 1\} = \frac{\delta^{-\alpha}+2}{\delta^{-\alpha}+1}$. The conditional expectation measure $E\{\kappa \mid \kappa \geqslant 1\}$ is positively related to the coefficient δ. If $\delta = 1$, the conditionally expected number equals 1.5 in the limit.

Proof. By definition

$$\lim_{x\longrightarrow\infty} E\{\kappa \mid \kappa \geqslant 1\} = \lim_{x\longrightarrow\infty} \frac{P\{X > x\} + P\{Y > x\}}{1 - P\{X \leq x, Y \leq x\}} = \lim_{x\longrightarrow\infty} \frac{P\{X > x\} + P\{\delta X + \xi > x\}}{1 - P\{X \leq x, \delta X + \xi \leq x\}}.$$

For the numerator, using Feller's convolution theorem we get

$$P\{X > x\} + P\{\delta X + \xi > x\} = x^{-\alpha} + (\delta^{-\alpha} + 1)x^{-\alpha} = (\delta^{-\alpha} + 2)x^{-\alpha}.$$

For the denominator, by assuming excessive volatile exchange rate we get

$$1 - P\{X \leq x, \delta X + \xi \leq x\} = (\delta^{-\alpha} + 1)x^{-\alpha}.$$

Thus, $\lim_{x\longrightarrow\infty} E\{\kappa \mid \kappa \geqslant 1\} = \frac{\delta^{-\alpha}+2}{\delta^{-\alpha}+1}$.

Case 2 *Suppose that X and ξ are i.i.d. random variables. X has a tail index of α as in the previous case but ξ has a lighter tail. Then $\lim_{x\longrightarrow\infty} E\{\kappa \mid \kappa \geqslant 1\} = \frac{\delta^{-\alpha}+1}{\delta^{-\alpha}}$. The conditional expectation measure $E\{\kappa \mid \kappa \geqslant 1\}$ is positively related to the coefficient δ. If $\delta = 1$, the conditionally expected number equals 2 in the limit. Note that the heavier tail dominates and the variables become complete asymptotically dependent when the correlation is equal to 1.*

■

Proof. By the same definition, using Feller's convolution theorem for the numerator we get

$$P\{X > x\} + P\{\delta X + \xi > x\} = x^{-\alpha} + \delta^{-\alpha}x^{-\alpha} = (\delta^{-\alpha} + 1)x^{-\alpha}.$$

For the denominator, by assuming excessive volatile exchange rate we get

$$1 - P\{X \leq x, \delta X + \xi \leq x\} = \delta^{-\alpha}x^{-\alpha}.$$

Thus, $\lim_{x\longrightarrow\infty} E\{\kappa \mid \kappa \geqslant 1\} = \frac{\delta^{-\alpha}+1}{\delta^{-\alpha}}$. ■

These two cases demonstrate the hypothetical asymptotic dependence between exchange rate returns and economic fundamentals when the variables are linearly associated as suggested in the monetary models of exchange rates. However, since the concept of asymptotic dependence simply expresses the relation between variables in terms of limiting conditional probability or the related probability measure, the probability measures allow broader but more precise investigation beyond a simple log-linear model. These include cases in which the monetary-approach exchange rate models may hold true, but the models have been complicated by the factors such as parameter uncertainty (Bacchetta and Wincoop, 2004), volatile and inconsistent expectations (Neely and Sarno, 2002) or even nonlinearity (Taylor and Peel, 2000). Based on EVT, we directly measure the probability of the joint occurrence of variables at a particular quantile, or vice versa.[13]

3 Estimation

In this section, we explain the semi-parametric tail estimation and the extreme linkage measure used to examine whether large swings in currency prices are associated with the extreme behavior of macroeconomic fundamentals.

3.1 Tail Estimators

To estimate the tail index α from (8), the semi-parametric estimators like the log-moment based Hill (1975) estimator and the Dekkers-Einmahl-de Haan (1989) (DEdH) estimator are natural candidates. Starting from the presumption that the distributions have fat tails, the Hill estimator is the more efficient estimator in mean squared error sense. Define the ascending order statistics from the sample of size n, as $X_{(1)} \leqslant X_{(2)} \leqslant ... \leqslant X_{(n)}$. The Hill (1975) estimator reads

$$\widehat{\gamma}(m) = \frac{1}{m}\sum_{i=1}^{m}[\log X_{(n+1-i)}/X_{(n-m)}], \tag{16}$$

where γ denotes the inverse tail index $1/\alpha$ and $X_{(n-m)}$ is a suitable threshold. Thus, there are m observations above the threshold. For $m(n) \to \infty$, while $m(n)/n \to 0$, $\widehat{1/\alpha}$ is asymptotically normally distributed with zero mean and variance $1/\alpha^2$.

The use of extreme value theory for economics and finance resides in the estimation of extreme probability-quantile $(\widehat{p}_q, q)$ combinations, where $\widehat{p}_q = 1 - F(q)$ and q is at the border or outside the sample. For example, the banking industry uses this to provide stress test estimates. De Haan, Jansen, Koedijk and de Vries (1994) developed the following probability estimator:

$$\widehat{p}_q = \frac{m}{n}(\frac{X_{(n-m)}}{q})^{\widehat{\alpha}}. \tag{17}$$

For the reverse problem of estimating the quantile at a certain low probability level, one simply inverts (17). One can show that the statistical properties of $\widehat{p}_q$ are driven by the statistical properties of the tail index estimator, since this statistic appears in (17) as a power. Thus $\widehat{p}_q$ is also asymptotically normally distributed. Our results from the tail index estimation can then be useful for risk management and stress testing at macroeconomic level.

An essential step in the computation of any tail index estimator is the selection of the number of upper order statistics m, in other words the selection of the threshold, $X_{(n-m)}$ in (16). The statistical properties of the Hill estimator crucially depend on the selection of the threshold. Too few observations enlarge the variance of the estimator, while too many observations reduce the variance at the expense of biasedness (by including observations from the central range, the first order approximation (9) becomes marred by second order terms). There are several automated

[13] Although we assess the extreme linkage in the limit, the measure provides a good approximation of tail relation at high but finite extreme levels, see, e.g., Balkema and De Haan (1974) and Hartmann et al. (2010). Our study is thus relevant for FX risk management and economic stability in general.

procedures available to deal with this trade-off problem, but these bootstrap based procedures only work in large samples such as are available in the high frequency domain.[14]

Our study relies on rather coarse macroeconomic data, ranging between 200 to 407 monthly observations from the period 1974-2007, and we are mainly interested in tail index values. As suggested by Loretan and Phillips (1994) and Embrechts et al. (1997), in this paper an appropriate threshold is thus selected from a plateau on which the Hill estimate appears relatively stable. By plotting the series of $\widehat{\gamma}(m)$ against different values of m, the appropriate m is chosen from the one whose value of $\widehat{\gamma}(m)$ first stabilizes. We, then, verify the eye-balling result by reporting the estimates for two typical tail sizes (5 and 2.5 percent of the overall sample size), as suggested by Lux and Sornette (2002). We also use the plots of the Dekkers-Einmahl-de Haan (1989) (DEdH) estimates to confirm the fat tail property of the data.[15]

3.2 Measure of Extreme Linkage

To study the dependence structure, the influence of the marginal distributions of Y and X, i.e. F_Y and F_X, is conventionally eliminated by transforming the raw data to a common marginal distribution.[16] To extract the dependence between exchange rate returns and economic fundamentals, we follow Hartmann, Straetmans and de Vries (2005) by transforming the variables Y and X to unit Pareto marginals $\widetilde{Y}$ and $\widetilde{X}$:

$$\widetilde{Z} = \frac{1}{1 - F_Z(Z)},$$

where $F_Z(Z)$ is the marginal cumulative distribution function for Z. The variables $\widetilde{Y}$ and $\widetilde{X}$ have the same distribution function, and they possess the same dependence structure as Y and X. Thus, the transformation allows us to focus on differences in distributions that are purely due to dependence of extremes.

After the transformation, we can rewrite the conditional expectation measure (15) as

$$E\{\kappa \mid \kappa \geqslant 1\} = \frac{P\left\{\widetilde{X} > s\right\} + P\left\{\widetilde{Y} > s\right\}}{1 - P\left\{\widetilde{X} \leq s, \widetilde{Y} \leq s\right\}} = \frac{P\left\{\widetilde{X} > s\right\} + P\left\{\widetilde{Y} > s\right\}}{P\left\{\max\left[\widetilde{X}, \widetilde{Y}\right] > s\right\}}, \tag{18}$$

where all the probabilities in the expectation measure now have the same Pareto tail. Hartmann et al. (2010) then substitute the probability estimator (17) for an extreme probability-quantile $(\widehat{p}_s, s)$ in equation (18). By using a common tail cutoff point, i.e. $X_{(n-m)}$ in (17), the estimator of extreme linkage becomes a simple count measure:

$$\widehat{E}\{\kappa \mid \kappa \geqslant 1\} = \frac{m_{\widetilde{X}} + m_{\widetilde{Y}}}{m_{\max}}, \tag{19}$$

where $m_{\widetilde{X}}$ ($m_{\widetilde{Y}}$) is the number of order statistics above the tail cutoff point for the $\widetilde{X}$ ($\widetilde{Y}$) series and $m_{\max}$ is the corresponding number for the max $\left[\widetilde{X}, \widetilde{Y}\right]$ sequence. Note that since the marginal distributions are unknown, Hartmann et. al (2005) replace them with their empirical counterparts:

[14] It can be shown that the Hill estimator's rate of convergence is best under the mean squared error criterion, see Hall and Welsh (1984).

[15] To save space, the results are available upon request. For other tail estimation methods, see, e.g., Beirlant, Goegebeur, Segers and Teugels (2004).

[16] Hartmann, Straetmans and de Vries (2005) transform the data to unit Pareto marginals, while Poon et al. (2004) apply unit Frechet marginals. Beirlant et al. (2004) discuss other choices of marginal distribution transformation and also state that the precise choice of transformation is not so important.

$$\widetilde{Z} = \frac{n+1}{n+1-R_Z},$$

where $R_Z = rank\left(Z_{(l)}, l = 1, ..., n\right)$.[17]

Using simulated data, Hartmann et al. (2010) show the behavior of the linkage estimator $\widehat{E}$ in (19) for different tail cutoff points ranging from high to low. The difference between asymptotically independent and dependent data is that in the former case the plot first lingers in the neighborhood of 1 (due to asymptotic independence) before rising towards 2, while in case of asymptotic dependence not far from the origin there emerges a stable plateau at a level above 1. To measure the extreme linkage, the conditionally expected value $\widehat{E}$ is computed using a tail cutoff point where the plot of $\widehat{E}$ is stable. Similar to the Hill estimation, we verify the eye-balling result by reporting the estimates using the tail cutoff points at the largest 2.5, 5 and 10 percent of sample thresholds. Moreover, for robustness check we estimate $\widehat{E}$ using the raw data and the unit Frechet transformed data.[18] Also we show the consistence between our extreme linkage measure and the asymptotic dependence tests proposed by Poon, Rockinger and Tawn (2004).[19]

To deal with the small number of observations resulting from low frequency macroeconomic variables, we pool the data across countries. Observations used to estimate and test the extreme linkage between exchange rate returns and economic fundamentals, then, range from 9611 to 10996. The panel analysis of the tail event is partly justified by the similarity of tail indices across countries as demonstrated latter in Figure 1. Moreover, following from the exchange rate model in equation (12) we examine the contemporaneous relation between the exchange rate returns and economic fundamentals in the tails.

4 Empirical Results

This section is devoted to the empirical investigation of the extreme behavior of exchange rate returns and macroeconomic variables and their extreme linkages. The first subsection presents the estimated tail indices of the exchange rate returns and changes in the fundamentals. The second subsection investigates the extreme linkages between these variables. The last subsection illustrates the robustness of our empirical results.

4.1 Tails of Economic Variables

From the Hill estimates of the inverse tail index $\widehat{\gamma} = 1/\widehat{\alpha}$ for different tail thresholds, i.e. different $X_{(n-m)}$ in (17), we select the number of observations m in the tail at a threshold where there first exists a plateau on which the Hill estimate appears relatively stable. Figure 1 illustrates the estimated tail index $\widehat{\alpha}$ on the $y-$axis for exchange rate returns and economic variables from 30 countries.[20] The left panel shows for each country the estimated tail index (denoted by a bar) and its 95\% confidence interval (a straight line). By using different shades of grey, the right panel compares for each country the estimated tail indices using the 5\% tail observations, the 2.5\% tail observations and m observations from an eye-balling approach, respectively.

[17] For more details, see Hartmann et. al (2010).

[18] For unit Frechet marginal distributions, the variables Y and X are transformed to $\widetilde{Y}$ and $\widetilde{X}$ where $\widetilde{Y} = -1/\log F_Y(Y)$ and $\widetilde{X} = -1/\log F_X(X)$. To save space, the results are available upon request.

[19] In Appendix B, we briefly describe the asymptotic dependence tests in Poon, Rockinger and Tawn (2004). For other measures of extreme dependence, readers are referred to Beirlant et al. (2004).

[20] The variables are monthly absolute changes in the exchange rate, money supply, real income, interest rate and price level relative to the corresponding foreign levels. Due to the small number of observations in each country we assume symmetric distributions. Our estimates are thus dominated by the heaviest tail.

On the left panel, most of the point estimates of the tail index are below 4 which imply that the fourth and higher moments do not exist. The upper bounds of the 95\% confidence interval are moreover in single digit for majority of the cases. For financial variables like exchange rate and interest rate, the tail indices are between 1 and 4. For the monetary variables, e.g. money supply and inflation, the tail indices linger between 2 and 4, while the real variable like output tends to have less fat tails among all.[21] Additionally, the right panel shows that our findings are robust for different tail thresholds, as we observe clusters of tail indices for each country while using different tail thresholds.[22]

Further, Table 1 provides descriptive statistics for the estimated tail indices of absolute changes in the exchange rate (e), money supply (m), real income (y), interest rate (i) and price level (p), using the two tail sizes, i.e. 5 and 2.5 percent of the overall sample size. The results appear to be similar. From the averages, only the mean and variance exist for the exchange rate returns. The fourth and higher moments are infinite for money growth and inflation, while for the interest rate it is still debatable whether variance is bounded. Evidence indicates that not only are the exchange rate returns heavy-tailed distributed, but the fundamental variables also exhibit heavy tails.

Nevertheless, even if the distribution of fundamentals exhibits fat tails just like the distribution of exchange rate returns, how do we know that the latter feature is induced by the former? Next we examine whether large swings in exchange rates are associated with the heavy-tailed macroeconomic fundamentals.

Table1: Tail index estimates of FX returns and fundamentals

	5 percent tail				
	e	m	y	i	p
Mean	2.2964	3.5027	5.2466	1.9793	3.5142
Median	2.0768	3.0117	4.1980	2.0448	3.1635
Max	6.0547	10.4102	14.1220	3.8133	7.2348
Min	0.4357	1.1189	1.4823	0.4551	1.6057
Std. Dev.	1.2708	1.7957	2.9821	0.8557	1.4840
	2.5 percent tail				
	e	m	y	i	p
Mean	2.6087	3.9615	6.74574	2.3682	3.8851
Median	2.2442	3.9258	5.1587	2.1779	3.4690
Max	7.4797	11.4010	21.1251	5.6337	7.6681
Min	0.5488	0.9026	1.4639	0.4694	1.6168
Std. Dev.	1.5379	2.1621	4.4774	1.2471	1.6614

4.2 Extreme Linkages

Feller's (1971, VIII.8) convolution theorem helps introduce the invariance principle regarding the tail index under convolution. From the convolution theorem, if the exchange rate is an additive function of macroeconomic fundamentals, the tails of the exchange rate return distribution should be governed by the heaviest tails of the fundamentals' rates of change. The theorem provides a simple but elegant way to examine the validity of the log-linear monetary models in the tail area. Therefore, as preliminary evidence Figure 2 shows the plots between tail indices of the exchange

[21] The plots of the DEdH (1989) estimator suggest the same. The plots are available upon request.

[22] To save space, we do not display the plots of tail indices and their 95\% confidence intervals using 5\% and 2.5\% of total observations. To conclude, the point estimates are rather similar, while the 95\% band tends to be larger when smaller number of observations in the tail used in computing the Hill estimates.

rate returns and the minimum tail indices of the fundamentals from four different exchange rate models nested in equation (12).

By constraining some coefficients in equation (12), for each country we compare the tail index of the exchange rate returns α_e with the minimum tail index from 1) the traditional fundamental model, i.e. $\min(\alpha_m, \alpha_y)$, 2) the flexible-price monetary model with interest rate, i.e. $\min(\alpha_m, \alpha_y, \alpha_i)$, 3) the flexible-price monetary model with inflation, i.e. $\min(\alpha_m, \alpha_y, \alpha_{p^e})$, and 4) the sluggish-price monetary model, i.e. $\min(\alpha_m, \alpha_y, \alpha_i, \alpha_{p^e})$.[23] From an equality imposed in (13), we should ideally observe a 45 degree line (the solid line). However, from the point estimates using the 5 percent tail observations the plots lie along the 45 degree line.[24] The correlation coefficients between tail indices on the left and right sides of (12) are 0.4227**, 0.0459, 0.3867** and 0.0471, respectively.[25][26]

Feller's (1971, VIII.8) convolution theorem provides a guideline for the relationship in the tails between the exchange rate returns and economic fundamentals, especially in the first and third models when the interest rate is excluded. The tail index α determines the tail shape of fat-tailed distributions: a smaller α implies a slower rate at which the probability density function approaches infinity. The higher probability mass in the tail area implies the fatter tail and more extreme behavior of the random variable. Besides, the integer value of α is the highest bounded moment of the distribution. However, to determine whether the similarity of the tail behaviors is not just a coincidence is more difficult to establish. Even if there is a similarity between tail indices of the exchange rate returns and economic fundamentals, it would constitute no sufficient proof of the existing relation.

Next, we estimate the extreme linkage between the exchange rate returns and economic fundamentals using the conditional expectation measure. After the unit Pareto transformation, we compute $\widehat{E}$ as graphically shown in Figure 3. While all the plots travel from 1 to 2, in Figure 3 we illustrate the behavior of the linkage estimator for the 20\% largest sample thresholds. For the series of real income $(e, y, -)$, the plot indicates a case of asymptotic independence, as it does not leave the neighborhood of 1 at first, and latter on slowly rises towards 2 in the end. For the negative relation between exchange rate returns and interest rate changes $(e, i, -)$, the linkage estimates first lingers around 1 before rising towards 2 which is also a case of asymptotic independence. For the rest, we observe a dramatic rise at first, then a stable plateau after the first hundred to two hundred top observations (roughly 1-2 percent of the sample size).

To measure the extreme linkage between exchange rate returns and economic fundamentals, Table 2 presents the estimates of the linkage measure using the 2.5\%, 5\% and 10\% of top observations. In Table 2, we see that the estimates do not vary much even using different tail cutoff points. As in Figure 3, we find that the positive exchange rate returns are asymptotically independent with negative changes in the real income and the interest rate. However, for the fundamentals such as increases in money supply, interest rate and price level the estimates of extreme linkage are around 1.3, 1.2 and 1.38, respectively. It implies that the exchange rate returns and these economic fundamentals are expected to be jointly in the tails approximately 30\%, 20\% and 38\% of the times, respectively.[27]

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[23] Note that for the expected price level p^e the current price level (p) is used as a proxy.

[24] To save space, we do not report the plots that use 2.5 and 10 percent tail observations from which the plots look rather similar, while the plots using an eye-balling approach provide even stronger evidence.

[25] Note that throughout the paper an asterisk indicates significance at the 10\% level while two and three asterisks show significance at the 5\% and 1\% levels, respectively.

[26] If we take out Brazil, the correlation coefficients jump to 0.5095***, 0.2493, 0.4084** and 0.2632, respectively. The returns on Brazilian real tend to have thin tails, indicated by both Hill and DEdH estimates, which are contradictory to their currency crisis experiences.

[27] The estimation results are very similar to the case of unit Frechet transformation. The estimates using transformed data tend to behave slightly different from the ones using the raw data. Results are available upon request.

Table 2: The conditional expectation measure

	2.5%	5%	10%
$(e, m, +)$	1.3499	1.3164	1.2688
$(e, y, -)$	1.0126	1.0207	1.0411
$(e, i, +)$	1.2281	1.1977	1.1703
$(e, i, -)$	1.0717	1.0875	1.0970
$(e, p, +)$	1.3898	1.3985	1.3618

The empirical results show that the exchange rate returns and economic fundamentals are asymptotically dependent. When changes in fundamentals, such as money supply, interest rate and price level, are extremely large, the probability of an extreme domestic currency depreciation is positive even in limit, i.e. when both variables move very deep into the tail area. Noteworthily, asymptotic dependence is the strongest form of tail dependence.[28] Their joint occurrences in the tails between exchange rate returns and economic fundamentals thus indicates that large swings in currency prices are associated with the heavy tail behavior of macroeconomic fundamentals. The fat tail feature of exchange rate returns are caused by these macroeconomic fundamentals.

4.3 Robustness Analysis

In this subsection, we analyze the robustness of our findings in two directions: 1) use other measures of asymptotic dependence, and 2) test the relation between tails of the exchange rate returns and the composite fundamentals constructed from the exchange rate models in equation (12).

4.3.1 Other Measures of Asymptotic Dependence

We reexamine our findings by using the asymptotic dependence tests proposed by Poon, Rockinger and Tawn (2004). To identify the type of extremal dependence structure, Poon et al. (2004) examine the limiting conditional probability by using a pair of distribution-free dependence structure measures $(\chi, \overline{\chi})$, that can be non-parametrically estimated and statistically tested. The parameter χ measures the limiting probability that extreme values of the variables $\widetilde{Y}$ and $\widetilde{X}$ occur simultaneously. If $\chi = 0$, $\widetilde{Y}$ and $\widetilde{X}$ are asymptotically independent, i.e. the extreme values of the variables in limit occur independently. If $\chi > 0$, $\widetilde{Y}$ and $\widetilde{X}$ are asymptotically dependent, while if $\chi = 1$, the variables are perfectly dependent.

For variables that are asymptotically independent, Poon et. al. (2004) measure their extreme dependence by using the second measure of extremal dependence $\overline{\chi}$, which measures the rate at which $P\left\{\widetilde{Y} > s \mid \widetilde{X} > s\right\}$ approaches 0,

$$\overline{\chi} = \lim_{s\to\infty} \frac{2\log P\left\{\widetilde{X} > s\right\}}{\log P\left\{\widetilde{Y} > s, \widetilde{X} > s\right\}} - 1.^{29}$$

Note that $-1 \leq \overline{\chi} \leq 1$. If $\widetilde{Y}$ and $\widetilde{X}$ are asymptotically dependent, $P\left\{\widetilde{Y} > s, \widetilde{X} > s\right\} = P\left\{\widetilde{X} > s\right\}$ and $\overline{\chi} = 1$. If $\widetilde{Y}$ and $\widetilde{X}$ are independent, $P\left\{\widetilde{Y} > s, \widetilde{X} > s\right\} = \left[P\left\{\widetilde{X} > s\right\}\right]^2$ and

[28] Further, this type of tail dependence between exchange rate returns and economic variables is not very common. By using a pooled sample of 46 countries in the period 1974-2008, Cumperayot and Kouwenberg (2012) show that out of 18 current and lagged economic variables only two variables (the real interest rate and the real interest rate differential) that are asymptotically dependent with the exchange rate returns.

[29] See also Ledford and Tawn (1996) and Coles, Heffernan and Tawn (1999).

thus, $\overline{\chi} = 0$. Positive and negative values of correspond to positive and negative extreme association, respectively.

Thus, we test for asymptotic dependence by computing $\overline{\chi}$. If the estimated is significantly less than 1, the variables are said to be asymptotically independent with $\chi = 0$. However, if the null hypothesis that $\overline{\chi} = 1$ cannot be rejected, we can then estimate χ, i.e. the limit probability that the extreme values of the variables $\widetilde{Y}$ and $\widetilde{X}$, and thus the raw variables Y and X, occur simultaneously. That is the extreme values of the variables are asymptotically associated. Table 3 shows the estimated $\overline{\chi}$, standard deviation and $z - stat$ for the null hypothesis that $\overline{\chi} = 1$. Since the estimation of $(\chi, \overline{\chi})$ involves the computation of the Hill estimator[30], the selection of the threshold is very crucial. In Table 3, we report the results using 2.5\%, \5% and \10% tail observations together with using an eye-balling approach.

Table 3: The estimated $\overline{\chi}$

	2.5%	5%	10%	Eye-balling
$(e, m, +)$	0.8397*	1.1712	1.1040	1.0961
std.	0.1115	0.0931	0.0638	0.0968
$z - stat$	-1.4374	1.8396	1.6307	0.9928
$(e, y, -)$	-0.0618***	-0.1175***	-0.0706***	-0.0606***
std.	0.0588	0.0391	0.0291	0.0622
$z - stat$	-18.0732	-28.5982	-36.7903	-17.0474
$(e, i, +)$	1.1181	1.1388	0.7477***	1.1070
std.	0.1294	0.0924	0.0534	0.1357
$z - stat$	0.9124	1.5028	-4.7274	0.7885
$(e, i, -)$	0.6460***	0.4937***	0.2857***	0.5922***
std.	0.1005	0.0645	0.0393	0.1026
$z - stat$	-3.5208	-7.8482	-18.1894	-3.9756
$(e, p, +)$	0.7594**	0.9731	1.2213	0.8865
std.	0.1061	0.0841	0.0670	0.0983
$z - stat$	-2.2680	-0.3197	3.3048	-1.1541

From Table 3, regardless of the tail cutoff points we can reject the null hypothesis of $\overline{\chi} = 1$ at the 1\% significant level for $(e, y, -)$ and $(e, i, -)$, while for the fundamentals like money supply, interest rate (positive) and price level we cannot reject the null hypothesis for most cases. Therefore, our conclusion remains that the positive exchange rate returns are asymptotically dependent with increases in money supply, interest rate and price level, whereas evidence shows the asymptotic independence between positive exchange rate returns, and negative changes in the real income and the interest rate. For the cases which the null hypothesis of $\overline{\chi} = 1$ cannot be rejected, we then estimate χ under the assumption that $\overline{\chi} = 1$. Table 4 illustrates the estimated χ.

[30] See the estimation method in Appendix B.

Table 4: The estimated χ

	2.5%	5%	10%	Eye-balling
$(e, m, +)$		0.4130***	0.4027***	0.4331***
std.		0.0173	0.0116	0.0196
$z - stat$		23.9299	34.7696	22.1390
$(e, i, +)$	0.2995***	0.2803***		0.3025***
std.	0.0181	0.0118		0.0193
$z - stat$	16.5793	23.7532		15.7017
$(e, p, +)$		0.5446***	0.4556***	0.5556
std.		0.0226	0.0130	0.0284
$z - stat$		24.0616	34.9610	19.5126

The estimated χ, which is the limiting probability that extreme values of the exchange rate returns and economic fundamentals occur simultaneously, is significantly different from zero for all cases. However, this measure predicts higher probabilities of extreme coexistence than the conditional expectation measure. Based on the estimated χ, the large positive exchange rate returns, and extremely increases in money supply, interest rate and price level are expected to jointly occur roughly 40\%, 30\% and 50\% of the times, respectively.

4.3.2 Composite Fundamentals

We have so far investigated the bilateral associations between exchange rate returns and individual economic fundamentals in equation (12). Evidence indicates that there are tail relations between exchange rate returns and economic fundamentals such as money supply, interest rate and price level. Based on four exchange rate models in equation (12), we now construct the series of composite fundamentals. Then, we test asymptotic dependence between the exchange rate returns and 1) the composite fundamentals and 2) the error terms. If the log-linear monetary model of exchange rates holds and Feller's (1971, VIII.8) convolution theorem for heavy-tailed random variables applies, the fat tail feature and thus the tail association should be preserved under addition. We should then observe the asymptotic dependence between the exchange rate returns and the composite fundamentals. Moreover, we would like to compare between the composite fundamentals and the error terms: which one is more associated with the exchange rate returns?

Table 5 reports the conditionally expected value $\widehat{E}$ between the exchange rate returns and the composite fundamentals on the left columns and between the exchange rate returns and the error terms on the right columns.[31] On the first column, TF, FPI, FPP and SP indicate the composite fundamentals and the error terms constructed from the traditional fundamental model, the flexible-price monetary model with interest rate, the flexible-price monetary model with inflation, and the sluggish-price monetary model, respectively. As before, we report the estimates of the linkage measure using the 2.5\%, 5\% and 10\% of top observations. The estimated coefficients of these four exchange rate models are shown in Appendix C.

[31] All series have been transformed to unit Pareto marginals.

Table 5: The conditional expectation measure

	Composite fundamentals			Error terms		
	2.5%	5%	10%	2.5%	5%	10%
TF	1.3333	1.2709	1.2254	1.5849	1.5324	1.4809
FPI	1.3351	1.2596	1.2128	1.5455	1.5167	1.4688
FPP	1.3564	1.3302	1.2908	1.3144	1.3116	1.3127
SP	1.3444	1.3122	1.2703	1.2923	1.3004	1.3133

As in previous cases, the estimates are rather stable regardless of the tail thresholds used in computation. For the composite fundamentals, the estimates range between 1.2 to 1.35 which coincide with the estimates between the exchange rate returns and individual economic fundamentals. The estimates slightly increase from TF and FPI to FPP and SP, when price is incorporated into the model. For TF and FPI, the estimates of extreme linkage between the exchange rate returns and the error terms are higher than those from the composite fundamental. However, for FPP and SP the extreme linkages between the exchange rate returns and the composite fundamentals are as high as those estimates from the error terms. The behavior of these extreme linkage measures are graphically depicted in Figure 4. None of the plots lingers around one, as would be the case of asymptotic independence.

To this point, we conclude that the exchange rate returns and economic fundamentals are asymptotically dependent. The probability that the extreme values of the variables occur simultaneously is non-zero even in limit. The composite fundamentals, constructed from the exchange rate models in equation (12), provide similar estimates as the estimates from individual fundamentals in the models. Evidence helps strengthen our theoretical argument: given the log-linear exchange rate model heavy-tailed economic fundamentals are a reason for the fat tail feature of exchange rate returns, as according to the convolution theorem the heavy tail feature is preserved under addition. The asymptotic association between the exchange rate returns and economic fundamentals is not higher than the exchange rate returns' association with the error terms. However, the measures of extreme linkage become similar when the inflation is taken into account (see the case of FPP and SP).

5 Conclusion

Exchange rate returns exhibit distributions with fat tails. The fat tail nature of the FX returns indicates that the frequent occurrence of extreme market movements is excessive relative to the conventional normal distribution. A number of works try to examine causes of the tail fatness of exchange rate returns, a topic that is highly relevant for risk management and financial stability. However, little attention has been put to the fact that the fat tail nature is not exclusively for the exchange rate returns, but also for macroeconomic fundamentals. Further, if the macroeconomic fundamentals are heavy-tailed distributed, then it is likely that this property can be transferred to the distribution of the exchange rate returns. In other words, the fat tails of the FX returns are driven by the economic fundamentals.

In this paper, by introducing the random Phillips curve coefficient into a standard monetary macroeconomic model (Walsh, 2003) we show that the unconditional distributions of macroeconomic variables such as inflation and money supply are heavy-tailed. To illustrate the tail relationship between exchange rates and economic fundamentals we exploit the log-linearity of the monetary model of exchange rates. Quite a bit is known about the sum of random variables that exhibit heavy tails. Feller's (1971, VIII.8) convolution theorem for heavy-tailed random variables holds that the fat tail feature is preserved under addition. Thus if the fundamentals' distributions exhibit fat tails and the log-linear monetary model applies, the fat tail feature is transferred to the log exchange rate.

For empirical investigation, we use a panel data set consisting of monthly observations from 30 countries over the period of 1974-2007. First, based on the Hill estimator we demonstrate that both exchange rate returns and economic fundamentals from the monetary-approach exchange rate models are heavy tailed. From the point estimates, the fourth and higher moments do not exist in most cases, except for the real income. Second, we show the similarity of tail indices between left and right sides of the exchange rate models, as constrained by the convolution theorem. Then, to investigate the tail dependence between these variables we apply the conditional expectation measure proposed in Huang (1992) and an alternative measure of asymptotic dependence in Poon, Rockinger and Tawn (2004).

From the pooled data, we find that the exchange rate returns and contemporaneous economic fundamentals are asymptotically dependent, i.e. when the fundamentals take on extremely large values the probability of their joint occurrence with large swings in currency prices is positive even in limit. The positive exchange rate returns are asymptotically dependent with increases in money supply, interest rate and price level, whereas evidence shows the asymptotic independence between positive exchange rate returns, and negative changes in the real income and the interest rate. The composite fundamentals, constructed from the log-linear exchange rate models, provide similar estimates of extreme linkage as the estimates from individual fundamentals. Evidence helps strengthen our theoretical argument that the heavy tail feature, and thus the tail relationship, are preserved under addition.

Asymptotic dependence is the strongest form of tail dependence. The joint occurrences in the tails between exchange rate returns and economic fundamentals therefore indicates that large swings in currency prices are associated with the heavy tail behavior of macroeconomic fundamentals. The fat tail feature of exchange rate returns are caused by these macroeconomic fundamentals. Nevertheless, we also find that factors outside the monetary exchange rate models, namely the error terms, also cause the tail fatness of the exchange rate returns. The measures of extreme linkage between the exchange rate returns and economic fundamentals is not higher than the exchange rate returns' association with the error terms. However, they are equivalent in the models where inflation is taken into account.

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6 Appendix A: Data Source

The data are monthly observations on the exchange rate, money supply (M2), production index, interest rate and consumer price index. For most countries, variables are relative to the US and the data ranges from February 1974 to December 2007. For the European countries, variables are relative to Germany and the data ends in December 1998. The data source is the IMF International Financial Statistics (IFS). For most countries, the US dollar exchange rates from IFS are coded AE. The monetary aggregate is a sum of IFS codes 34A and B. Industrial or manufacturing production index, code 66 or 66Y, is used as a proxy for real income. For the interest rate, we use the money market rate or deposit rate (code 60B or 60L). The consumer price index (code 64) is used for the price level.

A list of 30 countries used in our study: Argentina, Austria, Bolivia, Brazil, Canada, Chile, Columbia, Denmark, Ecuador, Finland, France, India, Indonesia, Israel, Japan, Jordan, Korea, Malaysia, Mexico, Netherlands, Norway, Pakistan, Peru, Philippines, Spain, South Africa, Sweden, UK, Venezuela and Zimbabwe.

7 Appendix B: Estimation of Extremal Dependence

To estimate the pair of bivariate extremal dependence measures $\overline{\chi}$ and χ, recall that after applying the unit Pareto transformation to the original pair of variables Y and X, the transformed variables $\widetilde{Y}$ and $\widetilde{X}$ have the same distribution function and possess the same dependence structure as the original pair. Under the condition of regular variation, the joint cumulative distribution function of the transformed variables $\widetilde{Y}$ and $\widetilde{X}$ can be written as

$$P\left\{\widetilde{X} > s, \widetilde{Y} > s\right\} \sim L(s)s^{-1/\xi}; s \to \infty,$$

where $\xi \in (0,1]$ and $L(s)$ is a slowly varying function.

Following from Poon et al. (2004), the parameter ξ can be estimated as a tail index of the univariate variable Z, with $Z = \min(\widetilde{X}, \widetilde{Y})$. Given the estimate $\widehat{\xi}$ and the tail threshold $Z_{(n-m)}$, with $Z_{(n-m)}$ is the m-th largest observation from a sample of size n, the estimator for $\overline{\chi}$ is

$$\begin{aligned} \widehat{\overline{\chi}} &= 2\widehat{\xi} - 1, \\ \text{with } Var\left(\widehat{\overline{\chi}}\right) &= \frac{\left(\widehat{\overline{\chi}} + 1\right)^2}{m}. \end{aligned}$$

Using the fact that the Hill estimator follows a normal distribution asymptotically, we then test the null hypothesis $\overline{\chi} = 1$. In case that the null hypothesis cannot be rejected, i.e. the variables are asymptotically dependent, we then proceed to estimate χ.

The maximum likelihood estimator of χ and its variance are

$$\begin{aligned} \widehat{\chi} &= \frac{m}{n} Z_{(n-m)}, \\ \text{with } Var\left(\widehat{\chi}\right) &= \frac{\left[Z_{(n-m)}\right]^2 m(n-m)}{n^3}. \end{aligned}$$

Appendix C: Dynamic OLS Regression

To construct the composite fundamental series, instead of using given parameters like in Flood and Rose (1995, 1999), from equation (12) we apply the Stock and Watson (1993) dynamic OLS estimation by adding the one period leads and lags of the first differences of the regressors in the exchange rate model. For the traditional fundamentals (TF) $\varphi_3 = \varphi_4 = 0$, for the flexible-price monetary model with interest rate (FPI) $\varphi_4 = 0$, for the flexible-price monetary model with inflation (FPP) $\varphi_3 = 0$ and no coefficient equals zero in the sluggish-price model (SP). According to equation (12), the dynamic OLS equation for the traditional fundamentals (TF) can be written as

$$e_t = a_0 + a_1 m_t + a_2 y_t + a_5 \Delta m_{t+1} + a_6 \Delta y_{t+1} + a_9 \Delta m_{t-1} + a_{10} \Delta y_{t-1}.$$

In Table A1, the columns m, y, i, r and p contain the estimated coefficients of the variables. Types of the estimated models are described in the second column.

Table A1: Parameters of the monetary-based exchange rate models

		m	y	i	p^e
Austria	TF	0.5105***	-0.0110*		
	FPI	0.5229***	-0.0120*	-0.0018***	
	FPP	-0.0168	-4.10E-05		1.0581***
	SP	-0.0277**	-0.0008	0.0007*	1.0764***