Asset pricing implications of your mutual fund manager's constraints

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June 22, 2017

Abstract

By the end of 2015, U.S. mutual funds managed $15 trillion in assets. These funds control about 25% of the equity and 40% of the commercial paper market. As a result, regulations impacting these funds have asset pricing implications. In this paper, we analyze the liquidity management constraint imposed on these funds by the Investment Company Act of 1940. Due to the Act, some funds do not trade illiquid stocks. The non-tradability of these stocks leads to sub-optimal risk sharing. In a competitive equilibrium, we show that this constraint generates the "betting against beta" phenomenon. Moreover, because of the constraint, alpha is non-zero in general. That is, adding factors to eliminate alpha is a futile exercise. Lastly, we empirically corroborate the theory.

JEL classification: G01; G11; G12; G14; G15

Key words: Non-tradability; Distress Risk; Idiosyncratic Volatility; Anomaly; betting against beta

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1 Introduction

By the end of 2015, U.S. mutual funds managed around \$15 trillion in assets.[1] This number, worldwide, is approximately \$37 trillion. These funds are heavily regulated. In the U.S., the Investment Company Act of 1940 (hereafter referred to as the "Act") regulates mutual funds as well as exchange-traded funds, closed-end funds and unit investment trusts. The main goal of the Act, enforced by the SEC, is to "protect investors". In that regard, the Act imposes various constraints. For example, section 18(f) introduces leverage constraints; rule 0-1 and section 15 impose board governance constraints; section 8(b) imposes portfolio constraints. Rule 22c-1 requires the mutual fund shares to be redeemable on a daily basis. Additionally, the Act introduces annual reporting requirements, privacy rules, anti-money laundering rules, and specific book-keeping rules. The Act also prohibits mutual funds from investing more than 15% of their net assets in illiquid securities.[2]

The performance of mutual fund managers relative to a benchmark also generates investment constraints. Due to agency conflicts, fund managers are often subject to both explicit and implicit tracking-error constraints, which further limit investments in stocks that are not part of the benchmark.[3] Due to the sheer size of the mutual fund industry, the illiquidity constraints have economy-wide implications. In this paper, we study the asset pricing implications of these "non-tradability" constraints.

In order to conform with the Act, some mutual funds do not trade some stocks. For example, a fund family's board of directors prohibits mutual-fund managers from investing in certain stocks by adding restrictions to their charter. We develop a model that explicitly considers the non-tradability of stocks deemed illiquid by the Act. We consider heterogeneous investors with mean-variance preferences. The investors can be different both in wealth and risk aversion. They belong to two categories: constrained and unconstrained in terms of their ability to invest

[1] `http://www.icifactbook.org/ch1/16_fb_ch1`

[2] The definition of illiquid securities is, understandably, not concrete. Loosely speaking, according to the Act, securities for which market quotations are not readily available are illiquid. Restricted securities and other investments that generally cannot be sold within seven days at approximately the price at which they are carried by the fund are also illiquid.

[3] Cao, Han, and Wang (2017) provides an excellent summary of the asset pricing literature dealing with investment constraints.

in stocks. The investors form portfolios to maximize wealth by investing in stocks, which also belong to two categories: over-the-counter (OTC) traded and exchange traded. OTC stocks are illiquid while exchange stocks are liquid. Due to the non-tradability constraint, constrained investors cannot invest in the OTC stocks. In our model, constrained investors resemble mutual funds. The modeling setup mimics the classic CAPM framework with one exception – there is a non-tradability constraint. Importantly, several anomalies arise naturally. Specifically, we show that:

1. *The bet against beta strategy is due to the non-tradability constraint.*

2. *The idiosyncratic risk anomaly arises naturally due to the non-tradability constraint.*

3. *Factor mining to eliminate alpha is a futile exercise.*

To understand the bet against beta strategy, consider the impact of the non-tradability constraint. Since constrained investors do not invest in OTC stocks, there are fewer investors trading OTC stocks. As a result, the risk sharing opportunities in the market change. The inefficient risk sharing is systematic: it affects both constrained and unconstrained investors. In this setting, our model demonstrates that alpha is decreasing in beta and illustrates the "bet against beta" strategy popularized by Frazzini and Pedersen (2014).

The idiosyncratic risk anomaly arises naturally. In our setup, correlation between stocks is governed by a $K \geq 1$ factor model. The beta turns out to be positively related to the idiosyncratic risk. Thus, stocks with high idiosyncratic risk have high beta and hence negative α. In this manner, the relationship between the idiosyncratic risk anomaly and the bet against beta strategy is mathematical.

Lastly, we show that even with an arbitrary K factor model, the alpha is non-zero. Harvey et al. (2015) documents 316 factors corresponding to hundreds of anomalies. Fama and French (2016) boil the factors down to five. We demonstrate that given any number of factors, one can always create a long-short portfolio with positive alpha. That is, due to the non-tradability constraint, factor mining is futile.

We empirically test the model by providing an alternate explanation for the distress risk anomaly. We study distress risk for several reasons. Fama and French (1993) suggest that the size and value factors are related to the distress risk. In addition, Fama and French (2008) show that the anomalies related to price momentum, earnings momentum, credit risk, dispersion, idiosyncratic volatility, asset growth, and capital investments are concentrated in the worst-rated stocks (i.e. distressed stocks). Finally, the main empirical implication of the model is to analyze the viability of the beta against beta strategy. To that end, we use distress risk as the sorting mechanism of β, as in Frazzini and Pedersen (2014), to highlight the robustness of our results.[4]

To test our model, we form portfolios sorted by the KMV distress measure as in Crosbie and Bohn (2003). For robustness, we also form portfolios using the hazard model in Campbell, Hilscher, and Szilagyi (2008), the Z-score, and the O-score. Our results remain qualitatively the same. In the spirit of our model, following Campbell et al. (2008), we include both OTC and exchange (i.e. AMEX and NYSE) traded stocks. Consistent with the model, we find that α is inversely related to the beta of the distress risk sorted portfolios. With regard to OTC stocks sorted by distress, we find that as the distress decile increases, the loading on the market factor increases. The loading on the SML and the HML factor does not follow a consistent pattern. That being said, the increase in loading of the market factor is sufficiently large to generate the betting against beta phenomenon. Finally, we show that the distress risk anomaly is muted in exchange traded stocks.

The next section describes the related literature. Section 3 provides a simple framework that highlights the impact of non-tradability constraints in equilibrium. Section 4 describes a general model and its asset pricing implications. Section 5 empirically tests the distress risk anomaly and Section 6 concludes.

2 Related Literature

To be clear, we are not the first to explain the betting against beta phenomenon. Fama and French (2004) lucidly write that "there is a positive relation between the expected return and

[4] In fact, any portfolio sorting mechanism that generates a positive relationship between the sorting measure and β is viable.

beta but it is too flat."[5] Theoretically, our model is closest to Merton (1987). Using incomplete information, Merton (1987) explains the flat beta. In his model, investors are unaware of the return generating process of some stocks. As a result, they do not trade those stocks. The lack of information creates an aggregate non-tradability constraint. Our model differs in two ways. First, Merton (1987) considers a one-factor model, while we consider an arbitrary K factor model. Second, due to lack of information, the shadow cost of non-tradability in Merton (1987) is strictly positive. Consequently, Merton (1987) cannot explain the idiosyncratic risk anomaly while it is natural in our model.

By appealing to the leverage constraint stemming from the Act, Frazzini and Pedersen (2014) offer an alternative explanation for the flat beta. They claim that since mutual funds are leverage constrained, the demand for high beta stock increases, which in turn, causes negative α. Instead of leverage, we use the non-tradability constraint stemming from the Act to explain the flat beta. Distinct from them, we show that even with the K factor model, the alpha is non-zero and factor mining is futile.

To summarize, several studies use various constraints to theoretically explain departures from CAPM. Like Brennan (1993), Alankar et al. (2013) use tracking error constraints arising due to the principal-agent issues in asset management to explain CAPM deviations. Hindy (1995) and Cuoco (1997) use portfolio constraints; Garleanu and Pedersen (2011) use margin constraints; Hong and Sraer (2016) use short sale constraints and heterogeneous beliefs; Bali, Brown, Murray, and Tang (2014) use investor preferences for lotteries to explain flat beta. Empirically, Drechsler and Drechsler (2014) use the cheap-minus-expensive-to-short factor and Stambaugh et al. (2014) use the investor sentiment factor to explain CAPM deviations. Adding to this literature, we use the non-tradability constraint.

This article is also related to the idiosyncratic risk anomaly literature. Ang, Hodrick, Xing, and Zhang (2006) show that domestic securities with high idiosyncratic risk under-perform securities with low idiosyncratic risk, while Ang, Hodrick, Xing, and Zhang (2009) show the same phenomenon internationally. Baker, Bradley, and Wurgler (2011) claim:

[5] Starting from Blume (1970), Jensen et al. (1972), Blume and Friend (1973) and Stambaugh (1982), the Sharpe-Lintner version of CAPM has been firmly rejected.

> "... among the many candidates for the greatest anomaly in finance, a particularly compelling one is the long-term success of low-volatility and low-beta stock portfolios."

In our model, beta turns out to be positively related to idiosyncratic risk in the presence of non-tradability constraints. This relationship arises naturally.

Lastly, the paper is related to the distress risk anomaly. The relationship between distress risk and expected return is subject to intense debate in finance. On one hand, starting from Merton (1974), increases in distress risk correspond to increases in beta. As a result, expected return is supposed to increase with distress risk. Vassalou and Xing (2004) find positive relationship between distress risk and average returns. However, Dichev (1998), Campbell, Hilscher, and Szilagyi (2008), and Friewald, Wagner, and Zechner (2014) document that high distress risk securities deliver abnormally low returns. In contrast to this literature, we explain the anomaly using the non-tradability constraint. Specifically, we show that the anomaly is pronounced in OTC stocks but significantly muted among exchange traded stocks. In our setting, OTC stocks are deemed illiquid and therefore non-tradable under Act.

George and Hwang (2010) theoretically explain the distress risk puzzle using heterogeneous financial distress costs. They claim that firms with high costs of financial distress choose low leverage to avoid distress. Then, in their model, in a recession, low leverage firms suffer more than high leverage firms. In this manner, high leverage firms are actually less risky than low leverage firms. Garlappi, Shu, and Yan (2008) and Garlappi and Yan (2011) explain the distress risk anomaly using a shareholder recovery model. In Garlappi and Yan (2011), debt of securities close to default gets renegotiated. As a result, the beta of distressed securities is not as high as it seems based on the Merton (1974) model.

This article contributes to the literature in two ways. First, we study the liquidity constraints of the Act and show that these constraints generate the viability of the betting against the beta strategy. We also theoretically show that the idiosyncratic risk anomaly is related to betting against beta. Taken together, we show that the α is always non-zero due to the constraint. Therefore, the practice of adding factors to explain "alpha" is futile. Second, we empirically corroborate the model by offering a new explanation of the distress risk anomaly. Our results show that the distress anomaly is primarily driven by OTC stocks. Removing OTC stocks from a portfolio significantly reduces the size of the distress risk anomaly. Lastly, we illustrate the relation between the distress risk anomaly and the bet against the beta strategy.

3 A simple framework showing the effect of non-tradability

What is the impact of the "non-tradability"? The following framework conveys the basic intuition of the paper. Consider a representative agent who chooses a portfolio of N stocks. The mean of the random return, $\tilde{R}_n$, is $\bar{R}_n$. The covariance of return between the n^{th} and the m^{th} stock, $\text{Cov}\left(\tilde{R}_n, \tilde{R}_m\right)$, is σ_{nm}. Additionally, there is no risk-free asset and the variance-covariance matrix of returns is non-singular. The representative agent has mean-variance preferences with the risk-aversion coefficient equal to one. This framework mimics the classic CAPM framework with one exception — there is a "non-tradability" friction.

In our framework, non-tradability means that some investors cannot trade some stocks. Mathematically, a positive Lagrange multiplier is associated with the investor's non-tradability constraint. Upon aggregation, these Lagrange multipliers, λ, act like a "trading cost" paid by the representative agent to trade these stocks. In this spirit, suppose explicitly that the cost to trade the n^{th} stock equals λ_n. To make the basic point, suppose that the cost is either zero or positive. That is, if there are no trading restrictions on the n^{th} stock, the trading cost, $\lambda_n = 0$. On the other hand, if there are restrictions, $\lambda_n > 0$.

The representative agent chooses portfolio $\{\pi_n\}$ to maximize the following Lagrangian:[6]

$$L = \sum_{n=1}^{N} \pi_n \bar{R}_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \pi_n \pi_m \sigma_{nm} - \sum_{n=1}^{N} \lambda_n \pi_n.$$

The first component of the Lagrangian is the expected return; the second component is the portfolio variance (scaled by risk aversion); the last component concerns trading costs. Upon inspection, the portfolio optimization problem is convex; the optimal portfolio, $\{\pi_n^*\}$, is unique.

[6] We do not include the full investment constraint: $\sum_{n=1}^{N} \pi_n = 1$. This constraint simply scales the portfolios; it does not change the main implications. In the next section depicting the general model, we include this constraint.

The optimality condition is

$$\frac{\partial L}{\partial \pi_n} = 0 \quad \Longrightarrow \quad \sum_{n=1}^{N} \pi_n^* \, \sigma_n^2 \, + \sum_{m=1;m \neq n}^{N} \pi_m^* \sigma_{nm} \, = \, \bar{R}_n \, - \, \lambda_n.$$

The left hand side depicts the marginal risk which includes two terms. The first term reflects the variance and the second term reflects the covariance. The right hand side depicts the marginal benefit: the difference between the reward, $\bar{R}_n$ and the trading cost λ_n.

Using the aggregation property of the covariance operator, the left hand side of the optimality condition can be re-written as

$$\sum_{n=1} \pi_n^* \, \sigma_n^2 + \sum_{m=1;m \neq n}^{N} \pi_m^* \sigma_{nm} \, = \sum_{m=1}^{N} \pi_m^* \sigma_{nm} \, = \sum_{m=1}^{N} \pi_m^* \, \mathrm{Cov}\Big(\tilde{R}_n, \tilde{R}_m\Big) \, = \, \mathrm{Cov}\Big(\tilde{R}_n, \sum_{m=1}^{N} \pi_m^* \, \tilde{R}_m\Big).$$

In equilibrium, due to market clearing, the representative agent's portfolio has to be the market portfolio. As a result, the optimality condition is

$$\mathrm{Cov}\Big(\tilde{R}_n, \tilde{R}_{mkt}\Big) \, = \, \bar{R}_n \, - \, \lambda_n \quad \text{where} \quad \tilde{R}_{mkt} \equiv \sum_{n=1}^{N} \pi_n^* \, \tilde{R}_n. \tag{1}$$

Multiplying equation (1) by π_n^* and adding across all N stocks, we get

$$\mathrm{Var}\Big(\tilde{R}_{mkt}\Big) \, = \, \bar{R}_{mkt} \, - \, \lambda_{mkt}; \quad \text{where} \quad \bar{R}_{mkt} \equiv \sum_{n=1}^{N} \pi_n^* \, \bar{R}_n; \quad \text{and} \quad \lambda_{mkt} \equiv \sum_{n=1}^{N} \pi_n^* \, \lambda_n. \tag{2}$$

The parameter λ_{mkt} represents the value-weighted trading cost. Upon inspection, $\lambda_{mkt} > 0$. Equation (2) is standard. In the case without trading costs, $\lambda_{mkt} = 0$ and the expected market return equals the market variance (since the risk aversion coefficient equals one). In the case with trading costs, the market variance is lower than the expected market return.

Using the definition, $\beta_n \equiv \mathrm{Cov}\left[\tilde{R}_n, \tilde{R}_{mkt}\right] / \mathrm{Var}\left[\tilde{R}_{mkt}\right]$, slight algebraic manipulation of equations (1) and (2) yields:

$$\bar{R}_n \, = \, \beta_n \, \bar{R}_{mkt} \, + \, \alpha_n \quad \text{with} \quad \alpha_n \, \equiv \, \lambda_n \, - \, \beta_n \, \lambda_{mkt}. \tag{3}$$

Equation (3) is the basis of the paper. The first component of the expected return, $\beta_n \bar{R}_{mkt}$, is the security market line of CAPM. The second component, consistent with the literature, is the residual: α_n. In the absence of the non-tradability constraints, $\lambda_n = 0$ and hence $\alpha_n = 0$. This is the standard CAPM result. In the presence of the non-tradability constraints, on the other hand, $\lambda_n \geq 0$ and hence $\alpha_n \neq 0$. With equation (3), we illustrate the following points:

1. **Bet against beta:** Upon inspection, clearly we have that α is decreasing in β:

$$\frac{d\,\alpha_n}{d\,\beta_n} = -\,\lambda_{mkt} < 0.$$

2. **Factor Mining Futility:** The covariance matrix of returns is completely general. This indicates that an arbitrary K factor model does not change the main result. Therefore, adding factors in the hope to eliminate α is a futile exercise. Also, note that the general covariance structure leads to a one-factor model. That is, the K factors fold into one factor. The folding property, while trivial, is not well appreciated.

Next, we show a general one-period model endogenizing the shadow cost λ_n. The model also clarifies that our story is neither related to risk-aversion nor related to heterogeneous wealth.

4 The Model

4.1 Factor model

We assume a one-period model with N stocks. The stocks fall into two categories. The first category, labeled "O", involves firms traded on an Over-The-Counter (OTC) exchange. The second category, labeled "E", involves firms traded on a "normal" exchange like NYSE. For the sake of concreteness, the E category stocks abide by SEC regulations while O category stocks do not. This assumption resembles reality. Regulations concerning O category stocks are considerably more lax. Each firm, denoted by $n = 1, 2, \ldots N$, has a market value of V_n and a market share of $\pi_n^* \equiv V_n \,/\, \sum_{n=1}^{N} V_n$. Since these stocks are held in positive net supply, these stocks comprise the "primary" assets.

The stock returns follow a linear K factor structure as in Ross (1976).[7] That is, the return is composed of the expected return and the sum of two separate sources of random returns: factor return and idiosyncratic return:

$$\tilde{R}_n = \bar{R}_n + \sum_{k=1}^{K} b_{nk}\,\tilde{F}_k + \tilde{\epsilon}_n. \tag{4}$$

For $n, m \in \{1, 2, \ldots, N\}$; and $k, l \in \{1, 2, \ldots, K\}$, we assume

$$\begin{array}{llll} \mathbb{E}\left[\tilde{F}_k\right] = 0; & \mathbb{E}\left[\tilde{\epsilon}_n\right] = 0; & \mathbb{E}\left[\tilde{\epsilon}_n \,|\, \tilde{F}_k\right] = 0; & \mathbb{E}\left[\tilde{F}_k^2\right] = 1; \\ \mathbb{E}\left[\tilde{\epsilon}_n^2\right] = s_n^2; & \mathrm{Cov}\left[\tilde{\epsilon}_n, \tilde{\epsilon}_{m\neq n}\right] = 0; & \mathrm{Cov}\left[\tilde{F}_k, \tilde{F}_{l\neq k}\right] = 0; & \mathrm{Cov}\left[\tilde{\epsilon}_n, \tilde{F}_k\right] = 0. \end{array}$$

The number of factors, K, is known. A zero-mean assumption for the factor and idiosyncratic risk is without loss of generality. The mean of idiosyncratic risk conditional on each of the K factors is zero. This assumption implies that the returns follow a "strict" factor structure. In order to resolve rotational indeterminacy, we assume that the factors have unit variance.[8] Additionally, factors and idiosyncratic risk are uncorrelated with each other.

The variance of the firm return is

$$\begin{aligned} \mathrm{Var}\big(\tilde{R}_n\big) &= \mathrm{Var}\Big(\bar{R}_n + \sum_{k=1}^{K} b_{nk}\,\tilde{F}_k + \tilde{\epsilon}_n\Big) = \mathrm{Var}\Big(\sum_{k=1}^{K} b_{nk}\,\tilde{F}_k\Big) + \mathrm{Var}\big(\tilde{\epsilon}_n\big) \\ &= \sum_{k=1}^{K} b_{nk}^2\,\mathrm{Var}\big(\tilde{F}_k\big) + s_n^2 = \sum_{k=1}^{K} b_{nk}^2 + s_n^2. \end{aligned}$$

[7]Grinblatt and Titman (1983), Chamberlain and Rothschild (1983), Connor (1984) and Huberman (1989) study extensions of the factor model. The handbook chapter of Connor and Korajczyk (1995) provides an excellent summary of the factor models in asset pricing.

[8]This choice to resolve rotational indeterminacy is not unique. Chamberlain and Rothschild (1983) use a eigenvector decomposition approach. Connor and Korajczyk (1995) gives an excellent summary of econometric analysis related to these two approaches.

The market portfolio, as expected, also has a factor structure:

$$
\begin{aligned}
\tilde{R}_{mkt} &= \sum_{n=1}^{N} \pi_n^* \tilde{R}_n = \sum_{n=1}^{N} \pi_n^* \left(\bar{R}_n + \sum_{k=1}^{K} b_{nk} \tilde{F}_k + \tilde{\epsilon}_n \right) \\
&= \bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n \quad \text{with} \quad b_k \equiv \sum_{n=1}^{N} \pi_n^* b_{nk}. \qquad (5)
\end{aligned}
$$

The parameter, b_k, is the economy wide exposure to factor $\tilde{F}_k$. Throughout the paper, we assume that b_k is not affected by individual stock attributes. Two remarks are in order related to the factor structure.

Remark 1 *The market variance increases with idiosyncratic variance, s_n^2. To see this, the market variance is*

$$
\sigma_{mkt}^2 \equiv Var\left(\bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n \right) = \sum_{k=1}^{K} b_k^2 + \sum_{n=1}^{N} (\pi_n^*)^2 s_n^2.
$$

Standard differentiation shows that

$$
\frac{\partial \sigma_{mkt}^2}{\partial s_n^2} = (\pi_n^*)^2 > 0.
$$

Remark 2 *The covariance of stock n with the market increases with the idiosyncratic variance, s_n^2. Furthermore, the market beta, β_n, also increases with the idiosyncratic variance, s_n^2, as long as $\beta_n < (\pi_n^*)^{-1}$. To see this, standard calculation shows that the covariance is*

$$
\sigma_{n,mkt} \equiv Cov\left[\tilde{R}_n, \tilde{R}_{mkt} \right] = Cov\left(\tilde{R}_n \, , \, \bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n \right) = \sum_{k=1}^{K} b_{nk} b_k + \pi_n^* s_n^2. \qquad (6)
$$

Using the definition, $\beta_n \equiv \sigma_{n,mkt} / \sigma_{mkt}^2$, standard differentiation shows that

$$
\frac{\partial \sigma_{n,mkt}}{\partial s_n^2} = (\pi_n^*) > 0; \quad \text{and} \quad \frac{\partial \beta_n}{\partial s_n^2} = \frac{\pi_n^*}{\sigma_{mkt}^2} \left(1 - \beta_n \pi_n^* \right) > 0 \quad \text{if} \quad \beta_n < (\pi_n^*)^{-1}.
$$

Empirically, the condition $\beta_n < (\pi_n^*)^{-1}$ holds in a well-diversified market. Remark 2 shows that the idiosyncratic risk is related to stock beta and in turn, idiosyncratic risk has asset pricing implications.

4.1.1 Return dynamics of Secondary Assets

In addition to the N primary assets, $K+1$ derivative assets are also traded. Following Cox et al. (1985), these derivative assets, although held in zero net supply, "complete" the market. More importantly, these derivative assets are exchange traded — all agents can trade them without incurring any cost. The return dynamics of the derivative assets are

$$\tilde{R}_{N+k} = \bar{R}_{N+k} + \tilde{F}_k \qquad \text{for} \quad k = 1, 2, \ldots, K. \tag{7}$$

Finally, the $N+K+1$ asset is a risk-free bond offering a sure return, R_f:

$$\tilde{R}_{N+K+1} = R_f. \tag{8}$$

To summarize, the return dynamics of both the primary and the secondary assets follow a factor structure. The O-category stocks are less regulated relative to the E-category stocks. As a result, some investors, as shown below, cannot trade the O-category stocks. Other than that, markets are perfect. There are no taxes. All borrowing and lending activities are frictionless. All agents understand the market structure perfectly; there is no incomplete information. Lastly, all agents have homogeneous expectations; there is no disagreement. Next, we describe the agents and their portfolio choice.

4.2 Investor Preferences and Portfolio Choice

There are J investors who fall into two categories: Constrained "C" and Unconstrained "U". Constrained investors cannot trade O-category firms, while Unconstrained investors can. Each investor, j, has mean-variance preferences with wealth $W(j)$ and risk-aversion coefficient $\delta(j)$. The preference of investor j is represented as:

$$U(j) = \mathbb{E}\left[\tilde{R}(j)\, W(j)\right] - \frac{\delta(j)}{2\, W(j)} \operatorname{Var}\left[\tilde{R}(j)\, W(j)\right],$$

where $\tilde{R}(j)$ is the random future gross return.

In order to maximize utility, each investor chooses a fraction $\{x_n(j)\}$ to invest in each of the N primary assets, a fraction $\{y_k(j)\}$ to invest in each of the K derivative assets and a fraction $\{z(j)\}$ to invest in the risk-free bond. Note that a constrained investor cannot trade OTC traded stocks: $x_n(j) = 0$ for $j \in$ C and $n \in O$. The portfolio return, $\tilde{R}(j)$, is

$$\tilde{R}(j) = \sum_{n=1}^{N} x_n(j)\, \tilde{R}_n + \sum_{k=1}^{K} y_k(j)\, \tilde{R}_{N+k} + z(j)\, R_f.$$

Using the budget constraint,

$$\sum_{n=1}^{N} x_n(j) + \sum_{k=1}^{K} y_k(j) + z(j) = 1;$$

slight algebra shows that the portfolio return of investor j also follows a factor structure:

$$\tilde{R}(j) = \bar{R}(j) + \sum_{k=1}^{K} b_k(j)\, \tilde{F}_k + \sum_{n=1}^{N} x_n(j)\, \tilde{\epsilon}_n, \quad \text{with} \quad b_k(j) \equiv \sum_{n=1}^{N} x_n(j)\, b_{nk} + y_k(j); \tag{9}$$

$$\bar{R}(j) \equiv R_f + \sum_{n=1}^{N} x_n(j)\, R_n^e + \sum_{k=1}^{K} b_k(j)\, \big(\bar{R}_{N+k} - R_f\big); \tag{10}$$

and

$$R_n^e \equiv \bar{R}_n - R_f - \sum_{k=1}^{K} b_{nk}\, \big(\bar{R}_{N+k} - R_f\big). \tag{11}$$

The term $b_k(j)$ shows the factor k exposure of investor j. The term R_n^e is stock n's excess return after accounting for the factor exposure. The return variance, in turn, is

$$\mathrm{Var}\Big(\tilde{R}(j)\Big) = \sum_{k=1}^{K} b_k^2(j) + \sum_{n=1}^{N} x_n^2(j)\, s_n^2. \tag{12}$$

Upon inspection of equations (10) and (12), it is easier to work with $b_k(j)$ as opposed to $y_k(j)$. Also, equation (9) shows that there is a one to one mapping between $b_k(j)$ and $y_k(j)$.

Then, the investor j chooses $\{x_n(j)\}$ and $\{b_k(j)\}$ to

$$\begin{aligned} \text{Maximize} \quad & \bar{R}(j) - \frac{\delta(j)}{2} \mathrm{Var}\Big(\tilde{R}(j)\Big), \\ \text{subject to} \quad & x_n(j) = 0 \quad \text{if} \quad \{j \in C; n \in O\}. \end{aligned} \tag{13}$$

The last equation reflects the non-tradability constraint faced by the constrained investor. In the next subsection, we describe the equilibrium and its implications.

4.3 Equilibrium and its implications

Definition 1 *A competitive equilibrium is defined as a strategy profile of all investors where*

1. *Each investor j chooses portfolio $\{x_n(j)\}$ consisting of primary assets and factor k exposure $\{b_k(j)\}$ to maximize her utility described in equation* (13)*;*

2. *The primary asset market clears:*

$$\sum_{j=1}^{J} x_n(j)\, W(j) = V_n \qquad \textit{for} \quad n = 1, 2, \ldots, N;$$

3. *The secondary asset market and the bond market clear:*

$$\sum_{j=1}^{J} y_k(j)\, W(j) = 0 \quad \textit{for} \quad k = 1, 2, \ldots, K \quad \textit{and} \quad \sum_{j=1}^{J} z(j)\, W(j) = 0.$$

With the equilibrium definition and the linear factor structure, investor j's optimization is relatively easy to analyze. The Lagrangian of investor j is

$$L(j) = \bar{R}(j) - \frac{\delta(j)}{2} \mathrm{Var}\Big(\tilde{R}(j)\Big) - \sum_{n=1}^{N} \lambda_n(j)\, x_n(j)\, \mathcal{I}\big(j \in C; n \in O\big),$$

where $\mathcal{I}(\cdot)$ is the indicator function. The first two components of the Lagrangian reflect the risk-return tradeoff. The last component reflects the non-tradability constraint. The indicator function, $\mathcal{I}$, shows that the constrained investors cannot invest in O-category stocks. The term

$\lambda_n(j) > 0$ is the Lagrange multiplier associated with the non-tradability constraint for investor j. The first order condition of all investors with respect to the factor exposure, $b_k(j)$, is

$$\frac{\partial L(j)}{\partial b_k(j)} = 0 \quad \Longrightarrow \quad b_k(j) = \frac{\bar{R}_{N+k} - R_f}{\delta(j)}. \tag{14}$$

Equation (14) is standard. The numerator is the excess return from taking one more unit of the factor k exposure. The denominator is the marginal risk scaled by risk aversion.[9] The first order condition of all investors with respect to primary assets, $x_n(j)$, is

$$\frac{\partial L(j)}{\partial x_n(j)} = 0 \quad \Longrightarrow \quad x_n(j) = \frac{\bar{R}^e_n - \lambda_n(j)\,\mathcal{I}\big(j \in C;\, n \in O\big)}{\delta(j)\, s^2_n}; \tag{15}$$

or

$$x_n(j) = \begin{cases} \dfrac{\bar{R}^e_n}{\delta(j)\, s^2_n} & n \in E \quad \text{or} \quad j \in U; n \in O \quad (16) \\ 0 & j \in C; n \in O \quad (17) \end{cases}$$

Equation (15) is also standard. There are three different cases to consider. First, consider the E-category stocks in equation (16). Since all investors can invest in them, the proportion, $x_n(j)$, increases with the excess return and decreases with the risk aversion coefficient and idiosyncratic volatility. Second, consider unconstrained investors trading O-category stocks. The optimal proportion, $x_n(j)$, remains the same. Third, consider constrained investors trading O-category stocks. Due to the non-tradability constraint, the optimal proportion, $x_n(j)$, is zero (equation (17)) or

$$\lambda_n(j) = \begin{cases} 0 & n \in E \quad \text{or} \quad j \in U; n \in O \quad (18) \\ R^e_n & j \in C; n \in O \quad (19) \end{cases}$$

Equation (19) plays a critical role going forward. For O-category stocks, the shadow cost of non-tradability for the constrained investor j, $\lambda_n(j)$, is of the same order as the excess return. The shadow cost of E-category stocks is zero trivially. In order to get further insights, we analyze market clearing next.

[9] Recall that the volatility of the factor equals 1 by assumption.

4.4 Aggregation, Market clearing and Asset pricing implications

Going forward, we need three market-wide terms. Let $W \equiv \sum_{j=1}^{J} W(j)$ be the market wealth; let

$$\frac{1}{W} \sum_{j=1}^{J} \frac{W(j)}{\delta(j)} \equiv \frac{1}{\delta} \tag{20}$$

be the risk tolerance of the representative agent; and let

$$\frac{q_c}{\delta} \equiv \sum_{j \in C} \frac{1}{W} \frac{W(j)}{\delta(j)}, \tag{21}$$

be the fraction of the representative agent's risk tolerance contributed by the constrained investors. The term q_c plays a crucial role in our set up. If q_c is low, the fraction of wealth owned by constrained investors is relatively low and in turn, the asset pricing implications of the non-tradability constraint are low. Also note that $q_c = 0$ in the classic CAPM case. With slight abuse, we interpret q_c to be the proportion of constrained investors. With the three terms, we answer some asset pricing questions:

4.4.1 What is the equilibrium return of factor k

Multiplying equation (14) by $W(j)$, adding over all investors, dividing by the market wealth and using equation (20), we get

$$b_k \equiv \sum_{j=1}^{J} \frac{b_k(j)\, W(j)}{W} = \frac{\bar{R}_{N+k} - R_f}{\delta} \tag{22}$$

Additional results about b_k can be better understood by analyzing the relationship between $y_k(j)$ and $b_k(j)$ in equation (9). Multiplying equation (9) by $W(j)$, adding over all investors, dividing by the market wealth, we have

$$\frac{1}{W} \sum_{j=1}^{J} y_k(j)\, W(j) = \sum_{n=1}^{N} b_{nk} \left(\sum_{j=1}^{J} \frac{x_n(j)\, W(j)}{W} \right) - \sum_{j=1}^{J} \frac{b_k(j)\, W(j)}{W}.$$

The left hand side is the derivative asset market clearing condition. Since the derivative assets are held in zero net supply; noting that the term in the parenthesis is the market share; using equation (22), the above expression simplifies to

$$\sum_{n=1}^{N} b_{nk}\, \pi_n^* \;=\; b_k \;=\; \frac{\bar{R}_{N+k} \;-\; R_f}{\delta}. \tag{23}$$

Equation (23) is expected. Since the variance of the factor is normalized to unity, the expected return of the aggregate factor k exposure, b_k, equals the excess factor return scaled by the representative agent's risk tolerance, δ.

4.4.2 What is the cost of non-tradability in equilibrium?

The shadow cost of non-tradability depends on the Lagrange multiplier of investor j. Then, the aggregate shadow cost is the sum of the Lagrange multipliers of all investors. Then, upon inspection of equation (18), the shadow cost of non-tradability of an E-category stock is zero. Multiplying equation (19) by $W(j)/\delta(j)$, adding over all constrained investors and dividing by market wealth, we have

$$R_n^e \sum_{j \in C} \frac{W(j)}{\delta(j)\, W} \;=\; \sum_{j \in C} \frac{\lambda_n(j)\, W(j)}{\delta(j)\, W} \;\equiv\; \frac{\lambda_n}{\delta}.$$

The parameter λ_n represents the shadow cost of non-tradability of the n^{th} OTC stock. Using equation (21), the shadow cost becomes

$$R_n^e\, q_c \;=\; \lambda_n. \tag{24}$$

Equation (24) shows that the shadow cost increases linearly with the proportion of constrained investors, q_c, and the excess return, R_n^e. Unfortunately, the excess return is endogenous. Further insight of the shadow cost comes from the market clearing of O-category stocks.

Multiplying equation (16) by $W(j)$, adding over all unconstrained investors and dividing by market wealth, we have

$$\sum_{j \notin C} \frac{x_n(j)\, W(j)}{W} \;=\; \frac{R_n^e}{s_n^2} \sum_{j \notin C} \frac{W(j)}{\delta(j)\, W}.$$

Noting that the left hand side is the market share of stock n, using equations (21) and (24), we have

$$\lambda_n \;=\; \pi_n^* \, s_n^2 \, \delta \frac{q_c}{1 \,-\, q_c}. \tag{25}$$

The next proposition summarizes the effect of economic variables on the aggregate shadow cost.

Proposition 1 *The aggregate shadow cost of an OTC traded stock, λ_n, increases with the market size, π_n^*, the idiosyncratic variance, s_n^2, and the proportion of constrained investors, q_c. The aggregate shadow cost of an Exchange traded stock is zero.*

4.4.3 What is the expected excess return in equilibrium?

Multiplying equations (17) and (16) by $W(j)$, adding over all unconstrained investors and dividing by market wealth, the excess return can be written as

$$R_n^e \;=\; \pi_n^* \, s_n^2 \, \delta \,+\, \lambda_n; \quad \text{or} \quad \bar{R}_n \,-\, R_f \;=\; \sum_{k=1}^{K} b_{nk} \left(\bar{R}_{N+k} \,-\, R_f\right) \,+\, \pi_n^* \, s_n^2 \, \delta \,+\, \lambda_n.$$

The second equality stems from the definition of the excess return in equation (11). Using equations (23) and using the covariance with respect to the market return (equation (6)), the excess expected return of stock n becomes

$$\bar{R}_n \,-\, R_f \;=\; \delta \mathrm{Cov}\Big(\tilde{R}_n, \tilde{R}_{mkt}\Big) \,+\, \lambda_n. \tag{26}$$

Multiplying equation (26) by the market share π_n^* and adding over all primary assets, we have

$$\delta \;=\; \frac{\bar{R}_{mkt} \,-\, R_f \,-\, \lambda_{mkt}}{\mathrm{Var}\Big(\tilde{R}_{mkt}\Big)} \tag{27}$$

Substituting equation (27) in equation (26), we get the main equation of the paper

$$\bar{R}_n - R_f = \beta_n \left(\bar{R}_{mkt} - R_f\right) + \underbrace{\lambda_n - \beta_n \lambda_{mkt}}_{\equiv \alpha_n}. \tag{28}$$

Equation (28) leads to several implications:

Implication 1 ***Bet against beta strategy:*** *For all stocks,*

$$\frac{d\,\alpha_n}{d\beta_n} = -\lambda_{mkt} < 0.$$

Then, a long portfolio of low beta stocks financed by a short portfolio of high beta stocks generates α*. Frazzini and Pedersen (2014) empirically corroborate this strategy.*

Corollary 1 ***What does bet against beta mean in a multi-factor model:*** *The beta is a weighted average sum of the factor loadings. Mathematically, since* $\pi_n^* s_n^2 \approx 0$*,*

$$\beta_n \approx \sum_{k=1}^{K} b_{nk}\,\gamma_k \quad \text{with} \quad \gamma_k \equiv \frac{b_k}{\sum_{k=1}^{K} b_k^2}.$$

Corollary 2 ***Folding property of the K factor model:*** *Even though the return generating process is generated by a* $K \geq 1$ *factor model, the equilibrium return only depends on one factor — the market factor. In other words, the* K *factor model folds into a one factor model where the market beta of stock* n*,* β_n*, is given in Remark 2.*

Implication 2 ***Idiosyncratic risk anomaly is related to the bet against beta strategy:*** *For Exchange traded stocks,*

$$\frac{d\,\alpha_n}{ds_n^2} = \underbrace{\frac{d\alpha_n}{d\,\beta_n}}_{<0} \times \underbrace{\frac{d\,\beta_n}{d\,s_n^2}}_{>0} < 0,$$

since $\lambda_n = 0$*. The second inequality stems from Remark 2.*

Implication 3 ***Factor mining is futile:*** *For all stocks,* $\alpha_n \neq 0$ *(except for knife edge cases). This observation is independent of the number of factors* K*. Adding factors in the hope of eliminating* α *is a futile exercise.*

4.5 What are the empirical implications?

There are three main testable hypotheses:

1. **Hypothesis 1:** α_n and β_n are inversely related.

2. **Hypothesis 2:** Adding high beta OTC stocks to a portfolio in the hope to capture the "illiquidity premium" does not work. Adding high beta OTC stocks can actually reduce the α.

3. **Hypothesis 3:** If stock n has more exposure to all of the factors relative to stock m, then $\alpha_n < \alpha_m$. Mathematically,

$$\forall\, k \quad \text{if} \quad b_{nk} > b_{mk} \qquad \Longrightarrow \qquad \alpha_n < \alpha_m.$$

The first hypothesis is a direct test of equation (28), the main equation of the paper. The second hypothesis is more nuanced. Loosely speaking, it is commonly understood that illiquid stocks command an "illiquidity premium". Equation (28) shows that the illiquidity premium explanation is true for low beta stocks. However, high beta, illiquid stocks can suffer from an "illiquidity discount" and hence α can be negative. In the context of non-tradability, since the shadow cost of non-tradability for OTC stocks is positive, $\lambda_n > 0$, one is tempted to include OTC stocks in a portfolio. Positive λ_n implies an "illiquidity premium". This temptation, however, can lead to a worse portfolio because if an OTC stock have a high β, its α can be negative. The third hypothesis directly stems from Corollary 1.

In the next section, we corroborate these hypotheses by studying the distress risk puzzle.

5 Taking the model to the data

The empirical analysis follows the methodology of Campbell et al. (2008). Consistent with their model and methodology, we include both OTC and exchange traded[10] stocks. We include

[10]The exchanges include NYSE, NYSE AMEX, NASDAQ-NMS Stock markets, NASDAQ OMX BX, Midwest exchange (Chicago), NYSE Arca, and the Philadelphia exchange

OTC stocks for two reasons. First, Campbell et al. (2008) includes them and, importantly, Table VIII of their article hints at the relationship between institutional ownership and distress risk. Institutional ownership is a common proxy for constrained investors, which leads to the second reason. Since OTC stocks are illiquid, institutional investors do not trade them. Ang et al. (2013) empirically show that OTC stocks are predominantly traded by retail investors — not institutional investors. As a result, analyzing OTC stocks naturally relates to analyzing the asset pricing implications of the non-tradability constraint. In summary, we follow methodology of Campbell et al. (2008) as it is amenable to our theory.

As stated, we sort portfolios by their distress risk. We measure distress risk as the Distance To Default (DDF) using the Moody's KMV approach. Crosbie and Bohn (2003), Vassalou and Xing (2004), Da et al. (2010) and Campbell et al. (2008) also use the KMV approach to measure distress risk. In the Appendix, as a robustness check, we show results when the distress risk is measured by the hazard model of Campbell et al. (2008). We have also checked our results using traditional distress measures like the Altman Z-score and the Ohlson O-score. Dichev (1998) and Griffin and Lemmon (2002) use these two measures to study the distress risk puzzle. For brevity, we do not report these results as they are qualitatively similar to the results derived using the KMV measure. By sorting portfolios using the distress risk, we are able to also address the distress risk anomaly. However, as stated before, any sorting mechanism will work, as long as we identify the OTC and exchange traded stocks.

We divide the analysis into two parts. In the first part, we corroborate the three hypotheses using the CAPM (one-factor) model. In the second part, we corroborate the hypotheses using the three-factor Fama French model. Lastly, to address the distress specific macroeconomic events of the 1980s highlighted by Chava and Purnanandam (2010), we also report results for two different periods: 1980:1 - 1995:12 and 1996:1 - 2010:12.

5.1 Firm characteristics and Return Data

The sample consists of all domestic stocks[11], available simultaneously on the Center for Research in Security Prices (CRSP) monthly data set and COMPUSTAT. The sample also includes de-listed returns. The analysis period ranges between January 1981 to December 2010.[12]

In order to reduce the look-ahead bias, we assume that the COMPUSTAT data is available three months after the stated date. For example, we assume that the month-end December 2000 COMPUSTAT accounting data is available by March 2001. Then, we match this information with the March 2001 CRSP market data. The three month period is consistent with the literature. For example, Dichev (1998) assumes a six-month lag and Campbell et al. (2008) assumes a two-month lag. Consistent with Campbell et al. (2008), we winsorize the variables in our model at the 5th and 95th percentiles of their pooled distributions across all firm-months. That is, we replace any observation below the 5th percentile with the 5th percentile, and any observation above the 95th percentile with the 95th percentile. We remove all zero-leverage stocks from our sample. Lastly, we also exclude any stocks with either a negative book-to-market ratio or stocks that have a price less than or equal to $1. The price exclusion minimizes the turn-over costs and dampens the effect of bid-ask bounce.

Each month, we sort stocks according to the distress level. Afterward, we form ten portfolios representing ten deciles of distress risk and perform the standard asset pricing tests. We repeat the procedure for the sample with and without the OTC-traded stocks. In this manner, we link the distress risk puzzle to the liquidity risk of the OTC-traded stocks.

5.2 Summary statistics of DDF sorted portfolios

We follow Vassalou and Xing (2004) and Bharath and Shumway (2004)'s approach to calculate the DDF. The model requires four inputs: the annualized volatility of stock returns,

[11]Specifically, we use COMPUSTAT exchange codes for the NYSE, NYSE AMEX, NASDAQ-NMS Stock markets, NASDAQ OMX BX, Midwest exchange (Chicago), NYSE Arca, and the Philadelphia exchange.

[12]We link the datasets using firm level 8 digit CUSIP. COMPUSTAT is set for PROSRC = D (Domestic - US and Canadian companies), CONSOL = C (consolidated financial statements) and DATAFMT = STD (standardized).

σ_E; the face value of debt, F; the risk free rate, R_F; and time to maturity, T. We estimate σ_E using the prior year's daily stock returns from the daily CRSP dataset.[13] Following Vassalou and Xing (2004), we estimate F to be the debt in current liabilities plus one-half of long-term debt from the quarterly COMPUSTAT dataset. We use the one year Treasury Constant Maturity rate as a proxy for the risk-free rate R_f.[14] Using these inputs and the Bharath and Shumway (2004) numerical approach, we calculate the DDF.

The first panel of Table 1 reports the DDF for the full sample period and the two sub-sample periods: 1981:1 - 1995:12 and 1996:1 - 2010:12. Specifically, the table shows DDF for ten value-weighted portfolios sorted by their DDF. By construction, the DDF increases monotonically across the different deciles. The average DDF varies significantly across the two periods. The sample period ending in December 1995 has a higher DDF than the later period. This can be either due to the Great Recession or due or due to the distress related macroeconomic events that took place prior to the 1980s (Chava and Purnanandam (2010)).

The second panel of Table 1 provides the portfolio characteristics of the ten portfolios. The pattern of average returns across different portfolios is surprising. The monthly average return of both the least distressed portfolio and the second most distressed portfolio is 1.13%. The average monthly return for the fifth least distressed portfolio is 1.35%. That is, the average returns do not monotonically increase with distress. The statistics in the third and fourth panel show that distressed firms tend to be smaller, and have a high book-to-market ratio. Specifically, as the DDF increases, the firm size decreases and the book-to-market ratio increases. This result is consistent with the literature. Lastly, the behavior of average monthly returns and portfolio characteristics along the ten deciles is consistent across the two different sample periods.

Tables 2 and 3 show the same results separately for the OTC and exchange-traded stocks respectively. The second panel of both tables is the most interesting. The average monthly return across all portfolio for OTC stocks is lower than the average return of exchange stocks. For example, the average return of the most distressed portfolio (decile 10) of exchange traded stocks is 1.23%; the average return of the OTC counterpart is -0.76%. In fact, OTC stocks

[13] Values calculated with less than 50 daily observations are deleted.

[14] The data for the one year Treasury rate is from the Board of Governors of the Federal Reserve system available at http://research.stlouisfed.org/fred/data/irates/gs1.

suffer from a negative average return across most distress risk portfolios. The third panel of both tables compares sizes of both OTC and exchange stocks. For a given decile, OTC stocks are smaller than exchange stocks. Also, for both OTC and exchange stocks, most distressed firms tend to be the smallest. Lastly, the fourth panel shows that the most distressed firms tend have the highest book-to-market ratios.

5.3 Asset pricing implications from a one-factor CAPM model

Table (4) reports asset pricing implications using the CAPM specification. Each portfolio corresponds to one column of the table. Panel A reports the alphas with t-statistics below in parentheses. It also reports the beta with respect to the market and the adjusted R square. Panel B reports the alphas and betas for OTC stocks; Panel C reports the alphas and betas for exchange stocks.

The market beta does not increase monotonically with distress risk. In fact, the beta behavior is hump-shaped. That is, the betas increase initially with distress risk and then decrease. This behavior is not consistent with the Merton model. Relatedly, the alphas also do not monotonically change with the distress risk. These two observations highlight the reason for the conflicting evidence of the distress risk puzzle.

The alpha and beta pattern in Panels B and C highlights the main point of the empirical analysis. Across all distress risk profiles, the market beta for OTC stocks is higher than the market beta of exchange stocks. More importantly, consistent with the results from the previous two tables, the alphas for OTC stocks are significantly lower than exchange stocks. In fact, the OTC stock alphas are largely negative. Ang et al. (2013) also highlight the negative alpha from investment in OTC stocks. The magnitude of the negative alpha is surprising. Across all distress risk portfolios (except for decile 1 in which the alpha is not statistically significant), the alphas are lower than -10%.

The alphas of most of the exchange stocks are not statistically different than zero. As a result, since the all stock portfolios are a value-weighted combination of both exchange and OTC stocks, their alphas are lower than the exchange stocks. This confirms the second hypothesis

— Adding high beta OTC stocks is hazardous to the portfolio.

Figure 1 which graphically summarizes the behavior of alphas and betas gives a slightly different story. Upon inspection, the inverse relationship between alpha and beta is clear. Frazzini and Pedersen (2014) were probably the first to associate the distress risk puzzle with the bet against the beta strategy. We reaffirm their findings.

To summarize, our evidence shows that the distress risk puzzle is largely driven by OTC stocks. The distress risk anomaly is also related to the bet against beta strategy. Lastly, our evidence suggests that OTC stocks do not provide an "illiquidity premium"; they suffer from negative alpha and command an "illiquidity" discount.

5.4 Asset pricing implications from a three-factor Fama French model

Table 5 reports asset pricing implication using the three-factor specification. Figure 2 shows the factor loadings for the three factors for both OTC and exchange stocks. Figure 3 shows the corresponding three-factor annualized alphas. There is striking variation in factor loadings across the portfolios. For exchange stocks, the market beta increases with distress risk. That is, the least distressed portfolio has a market beta below one; while the most distressed portfolio has a market beta above one. For exchange stocks, least distressed portfolios also have negative loadings on both the value and the size factors. This behavior is expected: distressed portfolios tend to consist of small value stocks. To summarize, the factor loadings increase with distress risk for exchange stocks.

However, for OTC stocks, factor loadings do not increase monotonically with distress risk. Relative to exchange stocks, the size beta for OTC stocks is higher across all distress risk portfolios. In fact, for most of the distress sorted portfolios, both market beta and value beta are higher for OTC stocks relative to exchange Stocks. That being said, for OTC stocks, the value beta does not seem to increase with distress risk. Judging from the coefficient of variation (R^2), the three-factor model explains less variation for OTC stocks relative to exchange stocks. Therefore, for OTC stocks, a three-factor model, may not be sufficient.

Due to the higher factor loadings across distressed portfolios, from the third implication, it is clear that the overall beta increases with distress risk. As a result, the alphas should decrease with distress risk. Figure 3 confirms the inverse relationship between alpha and beta in a multi-factor model. The inverse relationship is more pronounced for OTC stocks. For exchange stocks, the inverse relationship holds for the most distressed portfolios.

To summarize, Figure 3 shows that the distress risk anomaly is significantly muted for exchange stocks. In fact, distress risk anomaly even by absent for exchange stocks. Friewald et al. (2014) gives an excellent summary of the conflicting evidence of the distress risk anomaly. Figure 3 also shows that the distress risk anomaly is really pronounced for OTC stocks. Just like the previous one-factor result, distress risk puzzle is largely driven by OTC stocks. The distress risk anomaly is also related to the bet against beta strategy.

6 Conclusion

This paper makes two main contributions to the asset pricing literature. First, we study the liquidity constraints of the Investment Act of 1940. These constraints generate the viability of the betting against the beta strategy. Relatedly, we show what betting against beta means in a multi-factor model. We also theoretically show that idiosyncratic risk anomaly is related to betting against beta. Taken together, we show that the α is always non-zero due to the constraint. Therefore, the practice of adding factors to explain "alpha" is futile.

Second, we empirically corroborate the model by offering a new explanation of the distress risk anomaly. Our results show that the distress anomaly is primarily driven by OTC stocks. Removing OTC stocks from a portfolio significantly reduces the size of the distress risk anomaly. Lastly, we illustrate the distress risk anomaly is related to the bet against the beta strategy.

Appendix

In this section, we report the asset pricing implications when the distress risk is measured using the hazard model of Campbell et al. (2008). From Campbell et al. (2008), the optimal conditional probability default at time t + 12 given survival until t + 11 is:

$$DP_t = \frac{1}{(1 + exp(A))}$$

where

$$\begin{aligned} A &\equiv -9.16 - 20.26 \times NIMTAAVG + 1.42 \times TLMTA - 7.13 \times EXRETAVG \\ &+ 1.41 \times SIGMA - 0.045 \times RSIZE - 2.13 \times CASHMTA + 0.075 \times MB \\ &- 0.058 \times PRICE. \end{aligned}$$

Explanatory variable NIMTAAVG represents a moving average of the net income to market value of total assets (NIMTA). Specifically, $\text{NIMTAAVG}_{t-1,t-12} = \frac{1-\phi^3}{1-\phi^{12}}(\text{NIMTA}_{t-1,t-3} + \cdots + \phi^9\text{NIMTA}_{t-9,t-12})$, with $\phi = 2^{-\frac{1}{3}}$, implying that the weight is halved each quarter[15]. Variable TLMTA represents the total liabilities to market value of total assets; CASHMTA represents the cash and short-term assets to the market value of total assets, MB represents the market-to-book ratio; and EXRETAVG represents the moving average of the monthly log excess return on each firm's equity relative to the S&P500 index (EXPRET). Again, $\text{EXRETAVG}_{t-1,t-12} = \frac{1-\phi}{1-\phi^{12}}(\text{EXRET}_{t-1} + \cdots + \phi^{11}\text{EXRET}_{t-12})$. SIGMA is a proxy for the standard deviation of each firm's daily stock return over the past three months[16]; RSIZE is the relative size of each firm measured as the log ratio of its market capitalization to that of the S&P500 index, and finally, the variable PRICE is each firm's log price per share, truncated above at 15.

Panel A of Table 6 provides summary statistics of the explanatory variables. Each

[15] We do not force the average NIMTA to be consecutive quarters. Enforcing consecutive quarters caused a substantial reduction in the sample of firms with complete information.

[16] SIGMA is calculated as an annualized 3-month rolling sample standard deviation centered around zero in lieu of the mean: $\text{SIGMA}_{i,t-1,t-3} = \left(252 * \frac{1}{N-1}\sum_{k\in(t-1,t-2,t-3)} r_{i,k}^2\right)^{\frac{1}{2}}$. Values of SIGMA calculated with less than 50 daily observations are deleted.

observation is weighted equally in the panel. There is a clear difference in the mean and the standard deviation of both OTC and exchange traded stocks.[17] For example, the OTC traded stocks tend to smaller in size, to have less net income and to be more volatile.

Panel B of Table 6 provides summary statistics for the ten value-weighted portfolios. Specifically, the statistics span the whole analysis period. Again, there is a clear difference in the default probability of both OTC and exchange traded stocks. The mean portfolio returns decrease with distress risk as expected. For example, the most distressed OTC traded securities earn an average of -4% per month while the least distressed firms earn an average of 1% per month. It also turns out that as the distress risk increases, size decreases. In the same fashion, as the distress risk increases, the book-to-market ratio also increases. In summary, distress risk is related to both the size and the value factor. This result is consistent with findings in Griffin and Lemmon (2002).

Table 7 shows the asset pricing results. Level of distress increases from left to right, with portfolio 10 denoting most distressed stocks. The one factor CAPM beta increases with leverage and the α simultaneously decreases. The betting against beta phenomenon is clearly visible in our results. The regression results show that α is driven by non-tradability. Only the OTC traded stocks exhibit a statistically significant breakdown in the risk return relationship, defined as the presence of statistically significant negative alphas. We note that return distributions of distressed stocks are more positively skewed than S&P500 return distributions.[18] Next, we show the asset pricing results for the three-factor Fama-French model in Table 8. The results are largely consistent with Campbell et al. (2008). As with the CAPM specification, 3-factor betas increase as the leverage increases, while alphas decrease. We again confirm the betting against beta phenomenon. We demonstrate a breakdown in the risk-return trade-off in the presence of non-tradability: only OTC stocks have significantly negative alphas for all leverage levels.

[17]The COMPUSTAT CRSP universe of merged stocks also includes some erroneously classified private companies (exchg code = 0), erroneously classified non-traded stocks (exchg code = 1), a limited number of LBO firms (exchg code = 3), a nominal number of Canadian firms that trade on the Toronto exchange (exchg code = 7), and unlisted evaluated equity (exchg code = 20).

[18]We evaluate excess return defined as the return over the risk free rate of interest from the Ken French website: monthly Fama/French Factors.

References

Alankar, A., P. Blaustein, and M. S. Scholes (2013). The cost of constraints: Risk management, agency theory and asset prices. *Working paper*.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The Journal of Finance 61*(1), 259–299.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2009). High idiosyncratic volatility and low returns: International and further us evidence. *Journal of Financial Economics 91*(1), 1–23.

Ang, A., A. A. Shtauber, and P. C. Tetlock (2013). Asset pricing in the dark: The cross-section of otc stocks. *Review of Financial Studies 26*(12), 2985–3028.

Baker, M., B. Bradley, and J. Wurgler (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal 67*(1), 40–54.

Bali, T. G., S. J. Brown, S. Murray, and Y. Tang (2014). Betting against beta or demand for lottery. *Unpublished working paper. McDonough School of Business, Georgetown*.

Bharath, S. T. and T. Shumway (2004). Forecasting default with the kmv-merton model. *SSRN eLibrary. 2004. Available at: http://ssrn.com/paper=637342*.

Blume, M. E. (1970). Portfolio theory: a step toward its practical application. *The Journal of Business 43*(2), 152–173.

Blume, M. E. and I. Friend (1973). A new look at the capital asset pricing model. *The journal of finance 28*(1), 19–34.

Brennan, M. (1993). Agency and asset pricing.

Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In search of distress risk. *The Journal of Finance 63*(6), 2899–2939.

Cao, J., B. Han, and Q. Wang (2017). Institutional investment constraints and stock prices. *Journal of Financial and Quantitative Analysis 52*(2), 465–489.

Chamberlain, G. and M. Rothschild (1983). Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica: Journal of the Econometric Society*, 1281–1304.

Chava, S. and A. Purnanandam (2010). Is default risk negatively related to stock returns? *Review of Financial Studies 23*(6), 2523–2559.

Connor, G. (1984). A unified beta pricing theory. *Journal of Economic Theory 34*(1), 13–31.

Connor, G. and R. A. Korajczyk (1995). The arbitrage pricing theory and multifactor models of asset returns. *Handbooks in operations research and management science 9*, 87–144.

Cox, J. C., J. E. Ingersoll Jr, and S. A. Ross (1985). An intertemporal general equilibrium model of asset prices. *Econometrica: Journal of the Econometric Society*, 363–384.

Crosbie, P. and J. Bohn (2003). Modeling default risk.

Cuoco, D. (1997). Optimal consumption and equilibrium prices with portfolio constraints and stochastic income. *Journal of Economic Theory 72*(1), 33–73.

Da, Z., P. Gao, et al. (2010). Clientele change, liquidity shock, and the return on financially distressed stocks. *Journal of Financial and Quantitative Analysis 45*(01), 27–48.

Dichev, I. D. (1998). Is the risk of bankruptcy a systematic risk? *The Journal of Finance 53*(3), 1131–1147.

Drechsler, I. and Q. F. Drechsler (2014). The shorting premium and asset pricing anomalies.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2004). The capital asset pricing model: Theory and evidence. *The Journal of Economic Perspectives 18*(3), 25–46.

Fama, E. F. and K. R. French (2008). Dissecting anomalies. *The Journal of Finance 63*(4), 1653–1678.

Fama, E. F. and K. R. French (2016). Dissecting anomalies with a five-factor model. *Review of Financial Studies 29*(1), 69–103.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of Financial Economics 111*(1), 1–25.

Friewald, N., C. Wagner, and J. Zechner (2014). The cross-section of credit risk premia and equity returns. *The Journal of Finance 69*(6), 2419–2469.

Garlappi, L., T. Shu, and H. Yan (2008). Default risk, shareholder advantage, and stock returns. *Review of Financial Studies 21*(6), 2743–2778.

Garlappi, L. and H. Yan (2011). Financial distress and the cross-section of equity returns. *The Journal of Finance 66*(3), 789–822.

Garleanu, N. and L. H. Pedersen (2011). Margin-based asset pricing and deviations from the law of one price. *Review of Financial Studies 24*(6), 1980–2022.

George, T. J. and C.-Y. Hwang (2010). A resolution of the distress risk and leverage puzzles in the cross section of stock returns. *Journal of Financial Economics 96*(1), 56–79.

Griffin, J. M. and M. L. Lemmon (2002). Book-to-market equity, distress risk, and stock returns. *Journal of Finance 57*(5), 2317–2336.

Grinblatt, M. and S. Titman (1983). Factor pricing in a finite economy. *Journal of Financial Economics 12*(4), 497–507.

Harvey, C. R., Y. Liu, and H. Zhu (2015). ... and the cross-section of expected returns. *Review of Financial Studies*, hhv059.

Hindy, A. (1995). Viable prices in financial markets with solvency constraints. *Journal of Mathematical Economics 24*(2), 105–135.

Hong, H. and D. A. Sraer (2016). Speculative betas. *The Journal of Finance*.

Huberman, G. (1989). A simple approach to arbitrage pricing theory. *Theory of Valuation. Frontiers of Modern Financial Theory 1*, 289–297.

Jensen, M. C., F. Black, and M. S. Scholes (1972). The capital asset pricing model: Some empirical tests.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance 29*(2), 449–470.

Merton, R. C. (1987). A simple model of capital market equilibrium with incomplete information. *The journal of finance 42*(3), 483–510.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of economic theory 13*(3), 341–360.

Stambaugh, R. F. (1982). On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis. *Journal of financial economics 10*(3), 237–268.

Stambaugh, R. F., J. Yu, and Y. Yuan (2014). The long of it: Odds that investor sentiment spuriously predicts anomaly returns. *Journal of Financial Economics 114*(3), 613–619.

Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *Journal of Finance 59*(2), 831–868.

Figure 1: This graph shows CAPM annualized alphas (in %) for exchange-traded stocks (diamond-shaped marker), OTC-traded stocks (cross marker), and all stocks (circle marker), as a function of CAPM beta. Data are monthly from 1981:1 to 2010:12. Stocks are assigned to ten deciles according to their KMV distance to default measure.

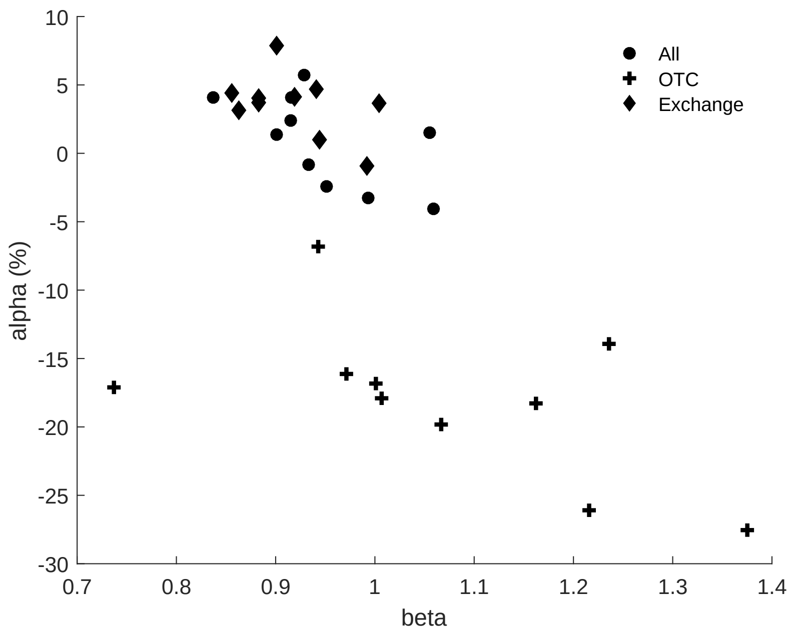

Figure 2: This figures shows factor loadings from Fama-French three-factor model. Dotted line shows the market beta for the ten distress sorted portfolios. Dashed line shows that value beta and solid line shows the size beta. Panel A is for Exchange-traded (non-OTC) stocks. Panel B is for OTC stocks. Data are monthly from 1981:1 to 2010:12. Horizontal axis is for distress bins from 1 (least distressed) to 10 (most distressed), calculated using the KMV-Merton distress measure.

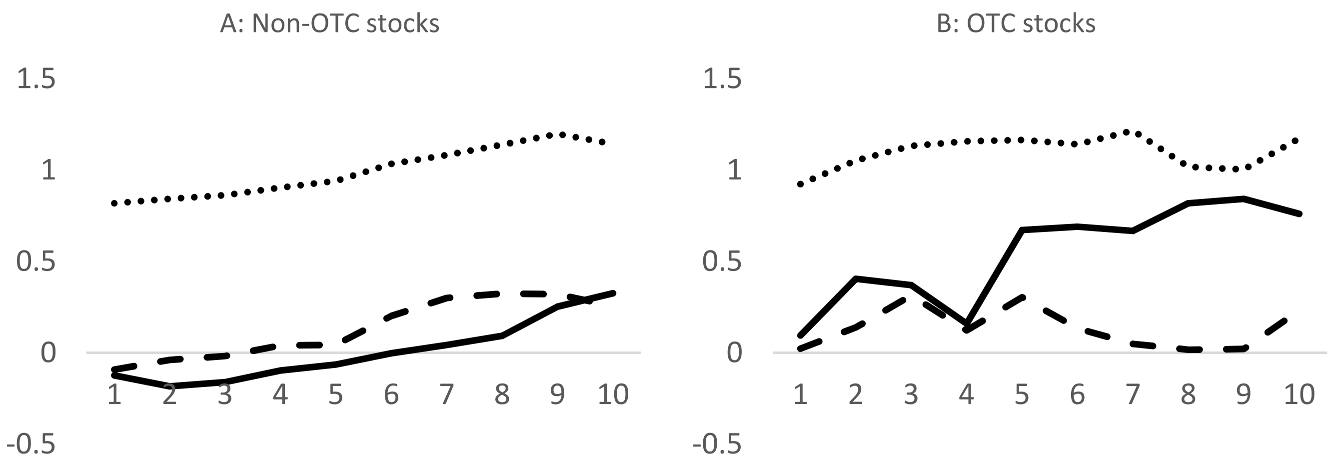

Figure 3: This graph shows Fama-French three-factor annualized alphas (in %) for Exchange-traded stocks (Panel A), and OTC stocks (Panel B). Data are monthly from 1981:1 to 2010:12. Dotted lines depict 99% confidence bands. Horizontal axis is for distress bins from 1 (least distressed) to 10 (most distressed), calculated using the KMV-Merton distress measure.

Table 1: KMV summary statistics: Full sample

The first panel of this table shows the distance to default calculated using the KMV-Merton model for the full sample period 1981:1 - 2010:12 and for two sub-periods. We sort all stocks based on the DDF and divide them into ten value-weighted portfolios. For example, first decile (1) represents portfolio with the lowest distress risk (lowest DDF) and the tenth decile (10) represents portfolio with the highest distress risk (highest DDF). The second panel shows monthly returns; the third panel shows the log of the market value of equity (in millions); the fourth panel shows the book value of equity over the market value of equity across the ten portfolios. The portfolios are formed using both OTC and Exchange traded stocks.

	1	2	3	4	5	6	7	8	9	10
KMV default measure (DDF), basis points										
Sample: Full	0.00	0.00	0.00	0.02	0.10	0.38	1.12	2.87	7.41	26.66
Sample: 1981:1 - 1995:12	0.00	0.00	0.00	0.00	0.01	0.12	0.58	2.28	7.94	34.44
Sample: 1996:1 - 2010:12	0.00	0.00	0.00	0.03	0.15	0.52	1.41	3.18	7.12	22.46
Monthly Portfolio Returns, %										
Sample: Full	1.13	0.97	0.82	1.21	1.35	1.07	0.67	0.66	1.13	0.70
Sample: 1981:1 - 1995:12	1.52	1.15	1.06	1.27	1.17	1.09	0.74	1.03	0.96	1.00
Sample: 1996:1 - 2010:12	0.73	0.78	0.58	1.14	1.52	1.05	0.59	0.29	1.29	0.40
Log Market Value										
Sample: Full	6.76	6.59	6.31	6.06	5.72	5.46	5.18	4.89	4.51	3.96
Sample: 1981:1 - 1995:1	5.41	5.34	5.02	4.70	4.42	4.10	3.83	3.55	3.15	2.59
Sample: 1996:1 - 2010:12	7.48	7.27	7.01	6.80	6.43	6.20	5.92	5.62	5.25	4.69
Book to Market Ratio										
Sample: Full	0.42	0.48	0.53	0.56	0.60	0.64	0.70	0.78	0.88	1.16
Sample: 1981:1 - 1995:1	0.45	0.51	0.56	0.60	0.63	0.69	0.74	0.80	0.89	1.14
Sample: 1996:1 - 2010:12	0.41	0.47	0.51	0.55	0.58	0.62	0.68	0.76	0.88	1.17

Table 2: KMV summary statistics: OTC-Traded

The first panel of this table shows the distance to default calculated using the KMV-Merton model for the full sample period 1981:1 - 2010:12 and for two sub-periods. We sort all stocks based on the DDF and divide them into ten value-weighted portfolios. For example, first decile (1) represents portfolio with the lowest distress risk (lowest DDF) and the tenth decile (10) represents portfolio with the highest distress risk (highest DDF). The second panel shows monthly returns; the third panel shows the log of the market value of equity (in millions); the fourth panel shows the book value of equity over the market value of equity across the ten portfolios. The portfolios are formed using only OTC stocks.

	1	2	3	4	5	6	7	8	9	10
KMV default measure (DDF), basis points										
Sample: Full	0.00	0.00	0.00	0.01	0.05	0.25	0.82	2.56	7.47	32.25
Sample: 1981:1 - 1995:12	0.00	0.00	0.00	0.00	0.02	0.15	0.74	2.90	8.64	38.28
Sample: 1996:1 - 2010:12	0.00	0.00	0.00	0.02	0.10	0.42	0.94	2.06	5.86	26.02
Monthly Portfolio Returns, %										
Sample: Full	0.31	0.60	-0.81	-0.72	-0.28	-0.83	-0.46	-1.17	-1.16	-0.76
Sample: 1981:1 - 1995:12	1.26	0.89	-0.46	0.11	-0.35	-1.03	-1.03	-0.96	-0.47	-0.04
Sample: 1996:1 - 2010:12	-0.64	0.31	-1.14	-1.56	-0.20	-0.64	0.13	-1.38	-1.85	-1.50
Log Market Value										
Sample: Full	3.98	4.07	4.03	3.93	3.79	3.61	3.54	3.35	3.14	2.76
Sample: 1981:1 - 1995:12	3.55	3.51	3.39	3.31	3.14	2.93	2.82	2.64	2.42	1.78
Sample: 1996:1 - 2010:12	5.30	5.25	5.26	5.14	5.00	4.75	4.64	4.39	4.13	3.78
Portfolio Book to Market Ratio										
Sample: Full	0.49	0.51	0.52	0.53	0.56	0.58	0.63	0.71	0.80	1.12
Sample: 1981:1 - 1995:12	0.47	0.48	0.48	0.52	0.54	0.55	0.62	0.69	0.78	1.07
Sample: 1996:1 - 2010:12	0.55	0.59	0.60	0.56	0.60	0.62	0.65	0.73	0.81	1.17

Table 3: KMV summary statistics: Exchange-Traded

The first panel of this table shows the distance to default calculated using the KMV-Merton model for the full sample period 1981:1 - 2010:12 and for two sub-periods. We sort all stocks based on the DDF and divide them into ten value-weighted portfolios. For example, first decile (1) represents portfolio with the lowest distress risk (lowest DDF) and the tenth decile (10) represents portfolio with the highest distress risk (highest DDF). The second panel shows monthly returns; the third panel shows the log of the market value of equity (in millions); the fourth panel shows the book value of equity over the market value of equity across the ten portfolios. The portfolios are formed using only Exchange traded stocks.

	1	2	3	4	5	6	7	8	9	10
KMV default measure, basis points										
Sample: Full	0.00	0.00	0.00	0.02	0.11	0.40	1.18	2.95	7.38	23.46
Sample: 1981:1 - 1995:12	0.00	0.00	0.00	0.00	0.01	0.11	0.51	1.92	7.38	30.16
Sample: 1996:1 - 2010:12	0.00	0.00	0.00	0.03	0.15	0.53	1.47	3.36	7.39	21.10
Monthly Portfolio Returns, %										
Sample: Full	1.18	1.11	0.95	1.14	1.51	1.18	0.96	0.84	1.29	1.23
Sample: 1981:1 - 1995:12	1.51	1.43	1.30	1.29	1.22	1.22	1.14	1.30	1.34	1.43
Sample: 1996:1 - 2010:12	0.86	0.79	0.60	0.99	1.79	1.13	0.79	0.38	1.24	1.03
Log Market Value										
Sample: Full	7.03	6.82	6.56	6.34	6.02	5.80	5.55	5.32	5.02	4.64
Sample: 1981:1 - 1995:12	5.85	5.68	5.39	5.09	4.84	4.54	4.29	4.07	3.75	3.50
Sample: 1996:1 - 2010:12	7.56	7.36	7.10	6.91	6.54	6.34	6.08	5.81	5.49	5.04
Monthly Portfolio Book to Market Ratio										
Sample: Full	0.42	0.48	0.53	0.57	0.60	0.65	0.72	0.80	0.91	1.18
Sample: 1981:1 - 1995:12	0.45	0.51	0.58	0.62	0.66	0.74	0.80	0.86	0.97	1.21
Sample: 1996:1 - 2010:12	0.41	0.46	0.50	0.54	0.58	0.62	0.69	0.77	0.89	1.17

Table 4: Returns on KMV-Merton DDF sorted portfolios – CAPM

We sort all stocks using the KMV-Merton model into ten portfolios based on distance to default cutoffs. In the table, we show the CAPM regression results. Panel A shows CAPM alphas and betas for the full sample period, 1981:1 - 2010:12. For each alpha, we also report its t-statistic (in parentheses). Panels B and C show CAPM alphas and betas for OTC stocks and Exchange stocks.

Panel A: All stocks

Portfolio	1	2	3	4	5	6	7	8	9	10
α (%)	4.07**	1.37	-0.83	4.11	5.71**	2.41	-3.27	-4.08	1.49	-2.43
	(2.09)	(0.65)	(-0.33)	(1.63)	(2.00)	(0.90)	(-1.26)	(-1.53)	(0.41)	(-0.63)
β	0.84	0.90	0.93	0.92	0.93	0.91	0.99	1.06	1.06	0.95
Adj. R-Square	0.62	0.60	0.55	0.55	0.51	0.50	0.56	0.57	0.44	0.34

Panel B: OTC stocks

Portfolio	1	2	3	4	5	6	7	8	9	10
α (%)	-6.80	-13.91***	-18.30***	-16.85***	-16.14***	-17.10***	-27.53***	-26.07***	-17.93***	-19.81**
	(-1.27)	(-2.60)	(-3.54)	(-3.00)	(-2.94)	(-3.13)	(-5.30)	(-4.62)	(-3.22)	(-2.56)
β	0.94	1.24	1.16	1.00	0.97	0.74	1.37	1.22	1.01	1.07
Adj. R-Square	0.22	0.28	0.28	0.19	0.19	0.12	0.31	0.24	0.19	0.13

Panel C: Exchange stocks

Portfolio	1	2	3	4	5	6	7	8	9	10
α (%)	4.40**	3.13	3.71	4.13*	7.89***	4.05	0.98	-0.94	3.65	4.70
	(2.27)	(1.44)	(1.49)	(1.77)	(2.62)	(1.59)	(0.39)	(-0.33)	(1.02)	(1.26)
β	0.86	0.86	0.88	0.92	0.90	0.88	0.94	0.99	1.00	0.94
Adj. R-Square	0.64	0.59	0.52	0.58	0.47	0.51	0.55	0.52	0.43	0.36

Table 5: Returns on KMV-Merton portfolios: Fama-French

We sort all stocks using the KMV-Merton model into ten portfolios based on distance to default cutoffs. In the table, we show the three factor Fama-French (3-F) regression results. Panel A shows 3-F alphas and betas for the full sample period, 1981:1 - 2010:12. For each alpha, we also report its t-statistic below. Panels B and C show 3-F alphas and betas for OTC and Exchange traded securities.

Panel A: All stocks – Fama-French

Portfolio	1	2	3	4	5	6	7	8	9	10
Alpha(%)	6.47***	1.62	-2.10	2.39	3.83	0.25	-5.78**	-6.55**	-1.45	-6.93*
	(3.32)	(0.73)	(-0.84)	(0.93)	(1.32)	(0.09)	(-2.25)	(-2.48)	(-0.39)	(-1.86)
MKT	0.79	0.92	0.96	0.95	0.98	0.94	1.05	1.08	1.09	1.04
HML	-0.30	-0.03	0.17	0.22	0.23	0.27	0.34	0.33	0.38	0.61
SMB	-0.13	-0.19	0.07	0.09	0.04	0.29	0.14	0.36	0.33	0.33
Adj. R-Square	0.65	0.61	0.55	0.56	0.52	0.53	0.58	0.61	0.47	0.39

Panel B: OTC stocks – Fama-French

Portfolio	1	2	3	4	5	6	7	8	9	10
Alpha(%)	-5.99	-14.10***	-18.01***	-17.89***	-18.13***	-18.15***	-29.23***	-25.94***	-19.80***	-20.93***
	(-1.06)	(-2.76)	(-3.25)	(-3.14)	(-3.28)	(-2.93)	(-5.64)	(-4.57)	(-3.55)	(-2.71)
MKT	0.83	1.12	1.04	0.95	0.93	0.69	1.32	1.07	0.99	0.99
HML	-0.13	0.02	-0.04	0.17	0.34	0.18	0.34	0.01	0.33	0.23
SMB	0.54	0.81	0.76	0.59	0.81	0.59	0.93	0.99	0.63	0.88
Adj. R-Square	0.26	0.34	0.34	0.21	0.24	0.15	0.37	0.31	0.22	0.16

Panel C: Exchange-traded stocks – Fama-French

Portfolio	1	2	3	4	5	6	7	8	9	10
Alpha(%)	6.96***	3.38	2.47	3.04	5.74*	2.21	-1.13	-3.07	1.14	-0.41
	(3.52)	(1.58)	(0.94)	(1.30)	(1.91)	(0.84)	(-0.46)	(-1.10)	(0.31)	(-0.12)
MKT	0.80	0.89	0.91	0.94	0.95	0.90	0.98	1.02	1.03	1.05
HML	-0.32	-0.03	0.16	0.14	0.26	0.23	0.27	0.28	0.32	0.65
SMB	-0.13	-0.24	0.04	0.06	0.05	0.26	0.17	0.24	0.29	0.31
Adj. R-Square	0.67	0.61	0.52	0.58	0.48	0.54	0.56	0.54	0.45	0.43

Table 6: Campbell et al. (2008) portfolios summary statistics

Panel A of this table includes the following variables (various adjustments are described in the data description section): net income to market value of total assets (NIMTA), total liabilities to market value of total Assets (TLMTA), cash and short-term assets to the market value of total assets (CASHMTA, CASH for short in table), market-to-book ratio (MB), monthly log excess return on each firm's equity relative to the S&P500 index (EXRET), the standard deviation of each firm's daily stock return over the past three months (SIGMA), the relative size of each rm measured as the log ratio of its market capitalization to that of the S&P 500 index (RSIZE), and finally, each firm's log price per share, truncated above at 15 (PRICE) for the sample period 1981:1 - 2010:12. In Panel B of this table we sort all stocks based on their default measure in basis points and divide them into 10 portfolios based on percentile cutoffs. For each portfolio we includes value weighted observations for the default measure in basis points, monthly returns, annualized returns, log of the market value of equity (in millions), and the book value of equity over the market value of equity. The portfolios in Panel B are presented for all firms in the sample and then subgroups formed based on the exchange on which the stocks trade.

Panel A

	Stats	NIMTA	TLMTA	CASH	MB	EXRET	SIGMA	RSIZE	PRICE
Full Sample	mean	-0.00	0.42	0.09	2.40	-0.00	0.53	-10.65	2.19
	sd	0.05	0.28	0.10	2.04	0.14	0.30	1.98	0.71
OTC Traded	mean	-0.01	0.44	0.10	2.35	-0.02	0.68	-12.11	1.65
	sd	0.08	0.29	0.11	2.26	0.18	0.33	1.56	0.83
Exch. Traded	mean	0.00	0.42	0.09	2.41	-0.00	0.49	-10.31	2.31
	sd	0.05	0.27	0.10	1.99	0.13	0.27	1.91	0.62

Panel B

	1	2	3	4	5	6	7	8	9	10
Monthly Portfolio Default Measure, basis points										
Sample: Full	1.79	2.56	3.16	3.84	4.65	5.69	7.11	9.40	14.18	37.79
Sample: OTC Traded	1.67	2.55	3.18	3.87	4.72	5.76	7.17	9.49	14.36	41.33
Sample: Exchange Traded	1.80	2.56	3.16	3.83	4.64	5.67	7.09	9.37	14.10	34.68
Monthly Portfolio Returns, %										
Sample: Full	1.14	1.05	1.10	1.06	1.16	1.01	1.05	0.86	0.33	-0.13
Sample: OTC Traded	1.19	0.85	0.46	0.39	-0.07	-0.73	-1.33	-2.06	-2.41	-3.54
Sample: Exchange Traded	1.14	1.05	1.13	1.13	1.13	1.13	1.06	1.28	0.86	0.53
Log Market Value										
Sample: Full	5.93	6.32	6.16	5.91	5.63	5.34	4.96	4.50	4.03	3.34
Sample: OTC Traded	4.17	4.47	4.39	4.35	4.20	4.05	3.87	3.58	3.33	2.88
Sample: Exchange Traded	6.13	6.48	6.33	6.10	5.84	5.56	5.21	4.78	4.36	3.75
Monthly Portfolio Book to Market Ratio										
Sample: Full	0.62	0.53	0.57	0.62	0.68	0.73	0.79	0.84	0.91	1.06
Sample: OTC Traded	0.85	0.74	0.72	0.73	0.76	0.79	0.82	0.85	0.87	1.00
Sample: Exchange Traded	0.59	0.52	0.56	0.61	0.66	0.72	0.78	0.84	0.93	1.10

Table 7: Returns on Campbell et al. (2008) portfolios: CAPM

We sort all stocks using the Campbell et al. (2008) 12-month hazard rate model and sort them into 10 portfolios based on percentile cutoffs. In the table below we show results from regressions of value weighted excess returns on a constant and the market excess return. Panel A shows CAPM alphas and betas, as well as corresponding t-stats below (for alphas) over the full sample period, 1981:1 - 2010:12. Panels B and C show CAPM alphas and betas, as well as corresponding t-stats below (for alphas) over subgroups formed based on the exchange on which the stocks trades.

Panel A: All stocks – CAPM

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.12***	6.88***	7.57***	6.52***	7.31***	4.83***	5.35***	2.16	-4.60**	-8.94***
	(6.35)	(7.30)	(8.62)	(6.72)	(7.23)	(3.80)	(3.27)	(1.06)	(-2.00)	(-3.64)
MKT	0.82	0.83	0.84	0.91	0.97	1.05	1.06	1.16	1.23	1.11
Adj. R-Square	0.80	0.88	0.90	0.90	0.90	0.87	0.80	0.76	0.72	0.62

Panel B: OTC stocks – CAPM

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.19***	2.28	-1.84	-3.32	-8.33***	-15.96***	-22.46***	-28.89***	-31.47***	-40.79***
	(3.32)	(0.94)	(-0.67)	(-1.05)	(-2.92)	(-5.47)	(-7.59)	(-9.72)	(-9.68)	(-13.34)
MKT	0.91	1.12	1.05	1.15	1.11	1.21	1.31	1.27	1.18	1.26
Adj. R-Square	0.57	0.66	0.56	0.54	0.55	0.57	0.58	0.54	0.44	0.48

Panel C: Exchange-traded stocks – CAPM

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.20***	6.99***	8.12***	7.55***	7.29***	6.86***	5.60***	7.78***	2.12	-1.51
	(6.11)	(7.03)	(8.05)	(7.60)	(7.39)	(5.54)	(3.90)	(3.97)	(0.95)	(-0.64)
MKT	0.81	0.83	0.81	0.89	0.92	0.99	1.03	1.11	1.16	1.12
Adj. R-Square	0.78	0.87	0.87	0.89	0.90	0.86	0.83	0.76	0.72	0.67

Table 8: Returns on Campbell et al. (2008) portfolios: Fama-French

We sort all stocks using the Campbell et al. (2008) 12-month hazard rate model into 10 portfolios based on percentile cutoffs. In the table below we show results from regressions of value weighted excess returns on a constant and the market excess return. Panel A shows Fama-French 3-factor alphas and betas, as well as corresponding t-stats (for alphas) below over the full sample period, 1981:1 - 2010:12. Panels B and C show Fama-French 3-factor alphas and betas, as well as corresponding t-stats (for alphas) below over subgroups formed based on the exchange on which the stocks trades.

Panel A: All stocks – Fama-French

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.69***	7.00***	7.60***	6.17***	6.72***	3.29***	3.78**	0.31	-6.06***	-10.79***
	(6.95)	(7.98)	(8.91)	(6.26)	(6.45)	(2.63)	(2.23)	(0.16)	(-2.67)	(-4.48)
MKT	0.82	0.86	0.86	0.94	1.00	1.10	1.10	1.20	1.23	1.10
HML	-0.08	-0.01	0.00	0.06	0.10	0.24	0.24	0.28	0.22	0.29
SMB	-0.10	-0.15	-0.10	-0.10	-0.04	0.03	0.09	0.20	0.33	0.49
Adj. R-Square	0.81	0.89	0.91	0.91	0.91	0.88	0.81	0.78	0.74	0.67

Panel B: OTC stocks – Fama-French

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.38***	1.60	-3.94	-4.81	-9.92***	-16.87***	-23.25***	-29.22***	-31.96***	-41.67***
	(3.17)	(0.67)	(-1.44)	(-1.52)	(-3.55)	(-6.29)	(-8.24)	(-10.48)	(-10.81)	(-14.28)
MKT	0.89	1.09	1.06	1.16	1.08	1.13	1.25	1.15	1.05	1.19
HML	-0.04	0.08	0.32	0.23	0.24	0.12	0.11	0.01	0.04	0.17
SMB	0.10	0.34	0.43	0.28	0.55	0.70	0.60	0.79	0.89	0.71
Adj. R-Square	0.57	0.69	0.60	0.56	0.60	0.66	0.63	0.64	0.56	0.54

Panel C: Exchange-traded stocks – Fama-French

	1	2	3	4	5	6	7	8	9	10
Alpha(%)	8.81***	7.16***	8.10***	7.47***	6.86***	5.77***	3.85***	5.77***	-0.04	-3.27
	(6.63)	(7.78)	(8.19)	(7.64)	(6.66)	(4.72)	(2.73)	(2.83)	(-0.02)	(-1.39)
MKT	0.81	0.85	0.84	0.92	0.94	1.03	1.09	1.17	1.21	1.13
HML	-0.09	-0.01	0.02	0.02	0.07	0.17	0.27	0.30	0.33	0.27
SMB	-0.09	-0.17	-0.13	-0.11	-0.03	-0.05	0.05	0.09	0.24	0.33
Adj. R-Square	0.79	0.88	0.88	0.90	0.90	0.88	0.85	0.78	0.75	0.69