

The Zero Lower Bound and Economic Determinants of the Volatility Surface in the Interest Cap Markets

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Abstract

We address an important but not yet answered question: What would the economic determinants of the shape of the implied volatility surface be under the zero lower bound condition? To answer the question, we examine time variations of the implied volatility surface of the cap market and investigate economic determinants of volatility curve slopes and curvatures. We provide empirical evidence that it is beneficial to employ unanticipated structural shocks as additional economic determinants. We associate the need for downward jumps in the volatility dynamics (Jarrow et al. (2007)) with two macroeconomic unanticipated shocks. We find that unexpected unemployment shocks play an important role in explaining implied volatility surfaces with short-term maturity, and unanticipated inflation shocks are important in explaining surfaces with longer-term maturity. Our results provide important information for hedging the risk of increases in the reference rates, and are particularly timely, enabling practitioners to prepare future exit strategies.

Keywords: Interest Rate Caps, Economic Determinants, Zero Lower Bound

JEL classification: G10, G12, G13

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1. Introduction

Since the U.S. Federal Funds Rate hit the zero lower bound¹ (hereafter, ZLB) in December of 2008, this unorthodox condition has created major challenges in many fields. Such challenges are mainly discussed from the monetary policy perspective, and little academic attention has been paid to the interest rate options market, despite its important practical implications for hedging and pricing. The main concern is that the price of a zero strike floor should be equal to zero according to the Black model, while current market quotes have positive values during ZLB periods. Under this unconventional condition, when some of the fixed-income security markets have entered the zone of zero nominal rates or even negative rates, studies of financial instruments embedded with option features, such as caps, floors, and swaptions, face Herculean tasks in improving pricing mechanisms and understanding interest rate options markets. Our primary interest is in examining the implied dynamics of interest rate options markets, focusing on the ZLB period. Specifically, we examine time variations of the implied volatility surface of the cap market, and investigate economic determinants of volatility slopes and curvatures.

Volatility smiles or smirks are omnipresent phenomena in the equity and currency options markets, yet they remain as a puzzling phenomena. The primary focus of the equity options literature has been to model the patterns of the volatility surface by relaxing the Black-Scholes assumptions, for instance, by moving from models with deterministic volatility to models with stochastic volatility (Hull and White (1987), Heston (1993), Dumas et al. (1998)), jumps as in the lévy process (Rubinstein (1994), Merton (1976), Bates (1991), Das and Sundaram (1999)), or both. From the modeling perspective, these models, with the help of liquidity effects or market frictions, have achieved a level of success in explaining the behavior of the observed implied volatility.² Interestingly, Bakshi et al. (1997) find that employing stochastic interest rates, volatility, and stochastic volatility with random jumps does not fully explain the implied volatility dynamics. Proposed inferences about portions not explained are due to the existence of liquidity issues, default risks, and leverage effects from asymmetric volatility responses.

Unlike much of the literature on equity options, research on the volatility smile patterns in interest rate options markets is remarkably sparse and does not directly examine the economic determinants. Most studies have focused on at-the-money options, with little attention to the economic determinants of volatility smiles or skews in interest rate options markets and virtually no focus on the ZLB period. Gupta and Subrahmanyam (2005), Fan et al. (2007), and Jarrow et al. (2007) examine smile effects in interest rate

¹For recent discussion on the zero lower bound, refer to Blinder (2012), Hamilton and Wu (2012), Swanson and Williams (2014), and Wu and Xia (2016).

²See Longstaff (1995), Brenner and Eom (1997), Erderington (2002), Mayhew (2002), Pena et al. (1999, 2001), Bollen and Whaley (2004), and Garleanu et al. (2009). Alternative effects including non-normality on a smile by Corrado and Su (1996), transaction costs Peña et al. (1999)), discrete tick size (Dennis and Mayhew (2009)), net buying pressure and limits to arbitrage (Bollen and Whaley (2004), Shiu et al. (2010)), and trader's behavior (Vagnani (2009)) have been suggested.

options, but only from a modeling perspective. Two exceptions are the papers of Peña et al. (1999) and Deuskar et al. (2008). Peña et al. (1999) examine the determinants of the implied volatility function in the Spanish equity index options market. Deuskar et al. (2008) explore potential determinant factors and find evidence that factors outside the yield curve, such as liquidity and default risk measures, are necessary to fully explain the shape of the smile in the cap market. In the same vein, Li and Zhao (2009) address a similar question by considering the implied risk-neutral distribution extracted from cap prices. They find that external factors significantly affect cap prices even after controlling for yield curve factors. In this paper, we address this issue in the US interest rate options markets by characterizing the time-series dynamics of the volatility surface and examining its economic determinants.

In spite of many similarities among options markets, note that the conclusions of equity options markets are not mechanically transferable to interest rate options because the implied dynamics of interest rate derivatives present some distinctive observed features. Two representative instances are the inverse relationship between the underlying interest rates and the humped or decreasing shape of their term structure (see Rebonato (2002) and Brigo and Mercurio (2002) for stylized facts). These distinctions mainly arise from structural differences between the two markets. One of these structural differences is discussed in terms of market participants. In interest rate options markets, institutional investors with access to homogeneous information set trade interest rate options via over-the-counter (OTC) markets, and the effects of the liquidity and credit risk naturally come into the equation. Another difference comes from the option products' perspective. Because caps and swaptions have a wide range of maturities and contract terms, they are said to be relatively opaque, which may create informational asymmetry, in turn leading to big jumps in implied volatility. From a modeling perspective, Jarrow et al. (2007) find that significant negative jumps in interest rates are needed to capture the smile. Nonetheless, the question of an economic explanation for the use of negative jumps has never been answered.³

We revisit the second question raised by Deuskar et al. (2008) under the ZLB condition. To be specific, we ask the question: What would the economic determinants of the shape of the implied volatility surface be once the nominal interest rate reaches the zero lower bound (ZLB)? To answer the question, we directly examine the economic determinants of the smile using potential factors. We base our choice of factors both on the findings of equity options research and on the interest rate derivatives literature. The first line of factors used to explain the shapes of the slope or curvature include controls for the three term structure factors. The candidate determinants are option volatility, bid-ask spread, and credit risk. Lastly, we employ proxies for unexpected economic shocks

³Investors are ignorant about the most attractive available terms and about whom to contact for given terms. Prices determined in OTC markets vary to the extent that those prices are influenced by the role of brokers and dealers trying to clear opaqueness.

extracted from the SVAR (Structural Vector Autoregressive Regression) equation with a system of three endogenous variables including inflation, unemployment, and the federal fund rate. The rationale for unexpected economic shocks comes both from the observation that the Libor, the reference rate for caps, floors and swaptions display almost perfect correlation with the federal fund rate and the economic equilibrium framework in which adjusting the federal fund rate is supposed to be a function of major economic fundamentals. Another rationale is based on the finding of Jarrow et al. (2007) that significant negative jumps in interest rates are required to capture a smile from the modeling perspective. The existence of negative jumps may play a critical role in explaining the shape of the implied volatility, but economic interpretations are far from perfect. Employing proxies of market frictions, such as liquidity shortage, is not a perfect solution, either. Relating to modeling, our distinctive hypothesis is that volatility skews or smirks may reflect investors' anticipation of downward jumps. In this regard, we associate the need for negative jumps with two unexpected shocks extracted from the dynamic stochastic general equilibrium. We will elaborate this point in section 2.2.

In this paper, we contribute to the literature in two major ways. First, to the best of our knowledge, we present the first empirical analysis of the dynamics of interest rate options markets in the ZLB period. Due to the existence of natural positivity restrictions on the underlying rates, we devise new measures to substitute for two traditional measures, risk reversal and the butterfly spread, and we focus on extracting information set hidden the implied volatility of the cap markets. Second, we provide empirical evidence that it is necessary to employ additional factors to better understand implied volatility surfaces by associating the need for downward jumps in the volatility dynamics (Jarrow et al. (2007)) with two macroeconomic unanticipated shocks from time-series of residuals of inflation and unemployment. Our analysis improves the understanding of the interest rate options market under the ZLB condition and provides important information regarding hedging the risk of an increase in the reference rates. Our result is particularly timely, enabling practitioners to prepare future exit strategies.

Our first set of results is related to unexpected economic shocks and the shape of implied volatility curves. We find that unexpected shocks related to the unemployment rate have strong positive impacts on the slopes of implied volatility curves for short-term maturities. By employing eight economic determinants, the three Nelson-Siegel factors, credit spread, bid-ask spread, ATM (at-the-money) volatility, unexpected inflation shocks, and unexpected unemployment shocks, we find that unexpected unemployment shocks play an important role in explaining the implied volatility surface, and short-term maturity and unanticipated inflation shocks are important in explaining the surface of longer-term maturity. In summary, we provide empirical evidence that it is beneficial to employ structural unanticipated shocks as additional economic determinants. To sum, we verify the existence of two more determinants implied from economic conditions during ZLB periods.

The structure of our paper is as follows. Section 2 describes the data set and presents

summary statistics. Section 3 presents empirical results for determining economic factors to explain the volatility smile in the data. We examine and interpret the impact of two macro-economic variables on these patterns. Section 4 concludes with a summary of the main results.

2. Data

In this section, we define our dependent variables as proxies for the slopes and curvatures of the implied volatility surface of the cap market. We also propose explanatory variables to fully capture information hidden in the surface of the cap market. The variables of interest are classified into three categories: implied volatility of the cap market, the term structure of interest rates, and economic determinants.

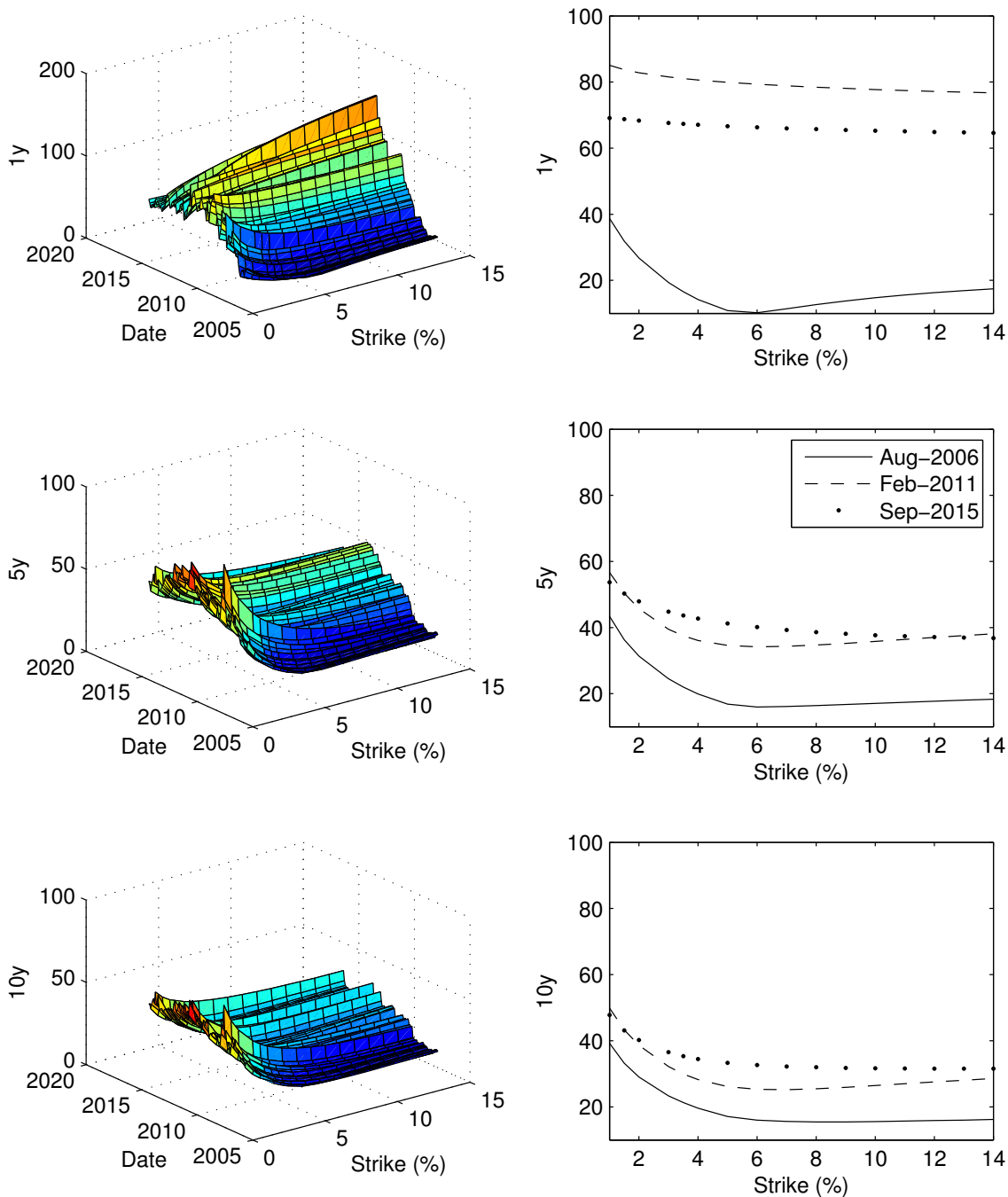
2.1. Slope and curvature proxies

The implied volatilities of the cap market used in this study are drawn from Bloomberg from August 1, 2006 to September, 30 2015. We obtain daily bid, ask, and mid close quotes (2360 daily observations) for 17 maturities (1 year to 30 years) over 15 strikes, which vary from 1% to 14%, including at-the-money. The most difficult task in this study has been to defining the information set contained in the implied volatility surface because the practical measures to proxy for slopes and curvatures do not work under the ZLB condition. The main reason for this is that the ZLB imposes basic positivity restrictions on the underlying process, by its nature.⁴ In practice, the measure of the asymmetry of the implied volatility curve is the risk reversal, the implied volatility difference in the in-the-money and out-of-the-money quotes, and the measure of the curvature is the butterfly spread, the difference between the average of the implied volatilities of two away-from-the-money and the at-the-money quotes.⁵ In the ZLB periods, a problem arises from the fact that there are no reliable quotes on out-of-the-money options because the ATMs are the lowest quotes. These proxies are not suitable for our analysis because of the natural restrictions on implied volatility shapes, which make it hard to construct smile and curvature proxies using traded caps. Incorporating these restrictions into our analysis, we seek to define new measures of slope and curvature with absolute strike levels. Based on the abovementioned arguments, instead of using risk reversal and butterfly spread, we focus on the half sphere of the implied volatility surface as follows.

⁴ $\log(F/K)$ are only defined for positive forward rates F and a positive strike rate K in the Black formula.

⁵The conventional measures are to use at-the-money options, in-the-money options with +0.25 log moneyness ratio (LMR), and out of-the-money options with -0.25 LMR. The +0.25 LMR in-the-money option is also problematic because as the +0.25 LMR point is far less than 1% level, the nearest point to the at-the-money option contains little information about +0.25 LMR point. Furthermore, Deuskar et al. (2008) find that the results from unscaled (absolute) implied volatilities and volatility and maturity adjusted moneyness (in the spirit of Li and Zhao (2006)) are similar.

Figure 1: Implied volatility surfaces with different maturities



Note. The left column illustrates the term structure dynamics of the implied volatilities with maturities of 1, 5, and 10 years with respect to absolute strike price (3M LIBOR rate) from August 1, 2006 to September 1, 2015 for Mid quotes. We obtain all USD cap data from Bloomberg. Bloomberg provides implied volatilities surfaces based on the Black Model (1976) using LIBOR single discount curve. The right column displays implied volatility curves with maturities of 1, 5, and 10 years at three representative days, August 1, 2006, February 1, 2011, and September 1, 2015. We intend to present exemplary shapes of implied volatility curves at three different time points (i.e., before the ZLB, during the ZLB, and after the ZLB.)

Table 1: Definition of slope and curvature measures

| Measures | Definition |
|-----------|---------------------------|
| S_1 | 14% - 1% |
| S_{2-1} | 7% - 1% |
| S_{2-2} | 14% - 7% |
| C_1 | $(1\% + 14\%)/2 - 7.5\%$ |
| C_{2-1} | $(1\% + 7\%)/2 - 4\%$ |
| C_{2-2} | $(7\% + 14\%)/2 - 10.5\%$ |

Note. In the measures' name, S stands for slope and C stands for curvature. The subscript 1 means type 1, which describes the slope or curvature over the whole quotes range (from 1% to 14%). The subscript 2-1 and 2-2 means type 2, which describes the slope of curvature over the near-ATM region (from 1% to 7%) and away-ATM region (from 7% to 14%), divided by the middle point.

The implied volatilities of the caps we use here, and throughout the paper, are those with 1%, 7%, and 14% strikes. The starting point is used to verify whether the implied volatility smiles and smirks exist across different maturities and strikes. The smirks prevailed for the sample period, as in Figure 1. We construct three slopes proxies to investigate information hidden in the volatility surface in detail. The first kind uses the difference between the closest and the farthest values in the implied volatility, which captures overall risk perceptions regarding upward changes in the underlying rates. We also examine whether information hidden in the near-ATM region (1%) is different from that in the away-ATM region (7% and 14%) by constructing two additional slopes as in Table 1. In a similar vein, we construct three curvature proxies by using implied volatilities of the caps with 1%, 4%, 7.5%, 10.5%, and 14% strikes. Our approach is comparable to those measures popular in the equity options literature. For instance, Xing et al. (2010) argue that slope reflects informed investors' demand for out-of-the-money puts in anticipation of bad news.⁶

2.2. List of economic determinants

In traditional option pricing theories, caps are portfolios of options on the interest rate; thus, underlying rates are supposed to be the sole source of uncertainty, and the yield curve should be the major factor. In empirics, however, there exists a puzzle called the unspanned stochastic volatility. Heidari and Wu (2003) initiate the literature on un-

⁶See Yan (2011), who proves that the slope of an out-of-the money put is approximately proportional to the average stock jump size. Toft and Prucyk (1997) relate slope to firm leverage; Dennis and Mayhew (2002) and Bakshi et al. (2003) draw a connection between slope and risk-neutral skewness; Cremers et al. (2008) examine the relationship between slope and credit spread; Bollen and Whaley (2004) show slope to be affected by the net buying pressure from public order flow; and Duan and Wei (2009) find slope to be dependent on the systematic risk proportion in the total risk.

spanned volatility by claiming that the factors explaining the variations in interest rate option prices are not spanned by the factors explaining the variations in the yield curve. In addition, Collin-Dufresne and Goldstein (2002) show that although caps and swaptions are derivatives written on LIBOR and swap rates, their prices appear to be driven by risk factors not encompassed by the factors that explain LIBOR or swap rates. Our research questions are closely associated with the literature on interest rate derivatives that includes the unspanned stochastic volatility puzzle. The rationale for employing other factors, beside the three factors of the term structure, is based on the extant literature on interest rate derivatives.⁷ Our explanatory variables are categorized into three groups: A) term structure's three factors, that is level, slope, and curvature B) liquidity, volatility, and credit risk proxies, and C) unexpected shocks from a SVAR with a system of unemployment, inflation, and federal fund rate equations.

To extract three factors of the yield curve, we rely on the dynamic Nelson-Siegel methodology. According to the parametric description of the yield curve, as suggested by Nelson and Siegel (1987), extended to a dynamic framework by Diebold et al. (2006) and Diebold et al. (2006), a state-space representation of the yield curve factor dynamics, and the time t observed yield curve for specific maturity τ , $y_t(\tau)$ can be written as

$$y_t(\tau) = H \cdot \beta_t + e_t \quad (1)$$

$$\beta_t = \mu + F \cdot \beta_{t-1} + v_t \quad (2)$$

F is the coefficient matrix of the three factors. The observation equation of the model is shown in equation (1), and it provides the cross-sectional relationship that the model imposes between observed yields and the chosen yield curve factor representation, $\beta_t = \{L_t, S_t, C_t\}$. The transition equation is given in equation (2). The parametrization of the factor-loading matrix, H , is given by:

$$H = \left[1, \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}, \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right]$$

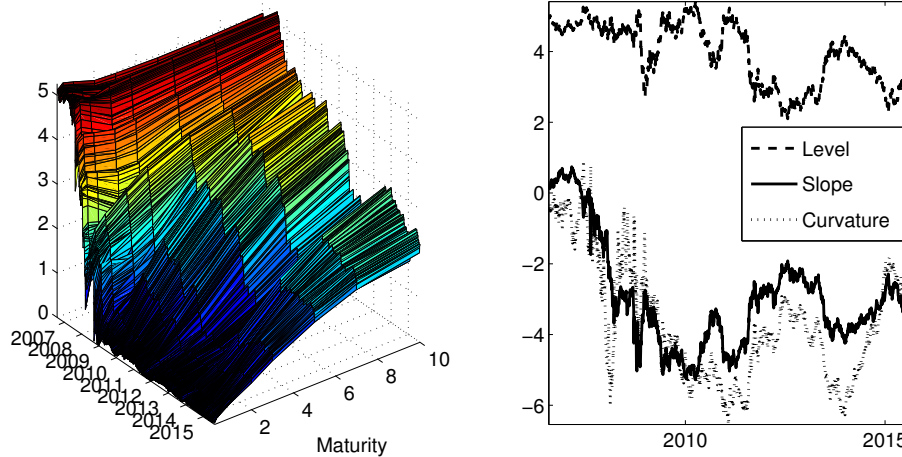
$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) S_t + \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) C_t \quad (3)$$

where λ_t is a time-decay parameter that determines the shape and decay-speed of the slope and curvature loadings. The right panel in Figure 2 captures the time-series of the three factors extracted by equation (3).

The next regressors are ATM volatility (ATMvol), liquidity proxy (BAS), and credit risk (CS) proxies. Contemporaneous studies identify liquidity difference as one of the

⁷Fan et al. (2003) argue that swaptions may be encompassed by bonds and Li and Zhao (2006) show that multifactor DTSMs have serious difficulties in hedging caps and cap straddles. Using an approximate factor model, they find that the three factors from the yield curve explain only about 60% of the variations of the implied volatility surface.

Figure 2: Term structure of Treasury Constant Maturity Rate and three Nelson-Siegel dynamic factors



Note. The left figure illustrates the term structure of the interest rates. To build the term structure of interest rates, we use daily Treasury Constant Maturity Rates ranging from 3-month to 10-year maturities. We obtain all series from the H.15 Selected Interest Rates released by the Board of Governors of the Federal Reserve System (US). The H.15 release contains daily interest rates for selected U.S. Treasury and private money market and capital market instruments. Data spans from June 1 in 2006 to September 30 in 2015. The right figure displays the term structure’s three factors of level, slope, and curvature under the dynamic Nelson-Siegel framework.

important non-default risk components. Plain-vanilla contracts such as caps and floors in the US OTC interest rate derivatives market are extremely competitive. In this regard, in imperfect and incomplete tradable markets, liquidity problems will inevitably impact option prices by increasing the spread between the ask and bid prices. For instance, Peña et al. (2001) suggest that a serious candidate to explain the pronounced pattern of volatility estimates across exercise prices would relate to liquidity costs. The role of liquidity measures in explaining the implied volatility surface is also supported by the literature on the importance of liquidity in explaining credit spreads.⁸ As a measure of liquidity, we use the relative bid-ask spread on ATM caps with corresponding strikes. Another source of risk in the cap market is a probability of counterparty default, intertwined with a credit risk element from the underlying LIBOR rate. From the over the counter market, a counterparty’s default is a risk taken by option buyers, and, if the overall risk of default is higher, it will result in a riskier market and higher option prices. In this analysis, we use the credit spread (CS), which is an aggregation of the probability of default estimated for

⁸For instance, Driessen (2005) follows the Duffie and Singleton (1999) reduced-form approach, and documents that liquidity premia account for a fraction of spreads of 20%, with Perraudin and Taylor (2004) providing a higher estimate (up to 30 bps). In addition, Chen et al. (2007) provide bond-specific liquidity measures for about 4,000 corporate bonds and build a robust connection between liquidity and credit spread.

a large number of corporations across typically very liquid equity markets. The intuition for examining these independent variables is also discussed in Deuskar et al. (2008).

Table 2 reports descriptive statistics for the variables of interest. Panel A reports summary statistics of implied volatilities with maturities of 1, 5, and 10 years and absolute strikes of 1%, 7%, and 14% for ask, bid, and mid quotes, respectively. Panel B reports summary statistics of exploratory economic determinants. Macroeconomic data are obtained from the Federal Reserve Economic Database of the St. Louis Fed, at a monthly frequency.⁹ The first intriguing observation is that time-series dynamics between the implied volatility of short-term maturity (1 year) and longer-term maturities (5 and 10 years) are distinctive. While decreasing shapes are observed for long-term maturities, monotonic increasing patterns are observed for short-term maturities on average. An additional intriguing observation is that the values of implied volatility monotonically decrease from short-term maturity to long-term maturities if strikes are fixed.

The last regressors are two unexpected macroeconomic shocks based on the DSGE (Dynamic Stochastic General Equilibrium) and its econometric counterparts. The role of systematic monetary policy is critical for understanding the way that macroeconomic shocks affect interest rates and the real economy simultaneously. The Taylor principle has been known to be a feature of these empirical responses: if inflationary expectations rise above the inflation target, the federal fund rate increases by more than the inflation gap. In addition, the fund rate rises in response to the output gap (i.e., real GDP above its potential level.) The choice of macroeconomic shocks is theoretically comparable to how the term spread is intertwined with unanticipated total factor productivity (TFP) shocks. Macroeconomic shocks induce changes in output and inflation gaps, and systematic monetary policy adjusts the funds rate accordingly. Long-term interest rates move in anticipation of these systematic policy responses.

The value of the cap markets is heavily dependent on perceptions of what the Federal Reserve will do with the federal fund rate, because the underlying rates, such as Libor or swap, show almost perfect correlation with the federal fund rate. To be specific, those perceptions are formed by the unexpected part of information flow from economic fundamentals, such as inflation and unemployment rate or output gap. Under the ZLB condition, monetary policy loses the ability to stimulate the economy. The monetary authority's ability to stimulate does hold, however, by paving a path for future interest rates, which can change expected inflation to announce "forward guidance" for short-term reference rates. In this regard, every new economic report about CPI and unemployment rates would bring the Fed a step closer to raising rates. These are the risk factors which cause the Fed to give a sudden indication that surprises the market, especially the interest rate options markets. For instance, Fed Chairwoman Janet Yellen signalled to the market that the central bank was in no hurry to raise interest rates in 2016 March, attributing

⁹Quarterly frequencies are converted to monthly frequencies using the dynamic Chow-Lin method proposed by Silva and Cardoso (2001) if necessary.

Table 2: Descriptive statistics

| Panel A | | | | | | | | | |
|------------|----------|----------|----------|--------|-------------|-------------|--------|-------|-------|
| 1% | Ask | | | Bid | | | Mid | | |
| | 1yr | 5yr | 10yr | 1yr | 5yr | 10yr | 1yr | 5yr | 10yr |
| Mean | 73.01 | 58.46 | 49.87 | 71.5 | 57.47 | 48.46 | 72.26 | 57.97 | 49.16 |
| Median | 73.28 | 60.03 | 50.1 | 71.37 | 58.8 | 48.7 | 72.89 | 59.5 | 49.54 |
| Stdev | 19.51 | 8.48 | 5.49 | 19.42 | 8.72 | 5.85 | 19.44 | 8.59 | 5.65 |
| Kurtosis | -0.07 | 0.34 | 0.57 | -0.06 | 0.15 | 0.28 | -0.07 | 0.24 | 0.43 |
| Skewness | -0.43 | -0.11 | 0.13 | -0.47 | -0.13 | 0.13 | -0.46 | -0.12 | 0.13 |
| Maximum | 112.65 | 85.35 | 68.45 | 109.8 | 83.7 | 66.8 | 111.23 | 84.53 | 67.63 |
| Minimum | 26.1 | 38.85 | 36.6 | 24.3 | 38.05 | 34.7 | 25.2 | 38.95 | 35.65 |
| 7% | | | | | | | | | |
| Mean | 71.21 | 34.89 | 26.08 | 69.57 | 34.09 | 25.73 | 70.39 | 34.49 | 25.9 |
| Median | 76.76 | 36.33 | 26.45 | 73.16 | 35.5 | 26.25 | 75.29 | 35.91 | 26.41 |
| Stdev | 34.98 | 11.19 | 6.38 | 34.52 | 11.22 | 6.81 | 34.72 | 11.2 | 6.59 |
| Kurtosis | -0.97 | -0.77 | -0.6 | -0.92 | -0.78 | -0.64 | -0.94 | -0.78 | -0.62 |
| Skewness | -0.35 | -0.21 | -0.22 | -0.31 | -0.17 | -0.15 | -0.34 | -0.19 | -0.19 |
| Maximum | 136 | 55.45 | 40.74 | 136 | 55.15 | 41.67 | 136 | 55.15 | 41.21 |
| Minimum | 8 | 14.05 | 13.6 | 7.25 | 13.25 | 12.75 | 7.63 | 13.65 | 13.18 |
| 14% | | | | | | | | | |
| Mean | 75.02 | 34.96 | 26.13 | 73.26 | 34.08 | 25.96 | 74.14 | 34.52 | 26.04 |
| Median | 75.57 | 38.18 | 27.53 | 73.38 | 37.2 | 27.13 | 74.43 | 37.71 | 27.38 |
| Stdev | 38.98 | 11.52 | 6.85 | 38.3 | 11.3 | 7.22 | 38.6 | 11.39 | 7.03 |
| Kurtosis | -1.01 | -1.04 | -0.92 | -0.92 | -1.02 | -0.94 | -0.97 | -1.04 | -0.94 |
| Skewness | -0.04 | -0.24 | -0.35 | 0.01 | -0.18 | -0.28 | -0.02 | -0.22 | -0.32 |
| Maximum | 156.15 | 57 | 39.9 | 156.15 | 54.8 | 40.33 | 156.15 | 55.2 | 39.9 |
| Minimum | 11.8 | 15.9 | 12.05 | 11.2 | 15.9 | 11.5 | 11.5 | 16 | 11.78 |
| Panel B | | | | | | | | | |
| | L_{TS} | S_{TS} | C_{TS} | CS | BAS_{1yr} | Vol_{1yr} | CPI | UN | FFR |
| Mean | 3.91 | -2.94 | -3.71 | 1.18 | 1.11 | 64.80 | 1.87 | 7.17 | 1.09 |
| Median | 4.07 | -3.07 | -3.99 | 1.00 | 0.45 | 68.59 | 1.87 | 7.30 | 0.15 |
| Stdev | 0.88 | 1.50 | 1.64 | 0.57 | 2.17 | 27.82 | 1.50 | 1.83 | 1.84 |
| Kurtosis | -1.18 | 0.35 | -0.59 | 6.33 | 15.21 | -0.37 | 0.08 | -1.42 | 0.84 |
| Skewness | -0.38 | 0.96 | 0.49 | 2.47 | 3.60 | -0.51 | -0.05 | -0.04 | 1.60 |
| Maximum | 5.31 | 0.70 | 0.52 | 3.49 | 14.00 | 115.90 | -1.96 | 10.00 | 5.26 |
| Minimum | 2.16 | -5.18 | -6.47 | 0.54 | -0.41 | 6.80 | 5.50 | 4.40 | 0.07 |

Note. This table reports the indicated summary statistics of the selected variables at daily frequencies from August 1, 2006 to September 1, 2015. Panel A reports summary statistics of implied volatilities with maturities of 1, 5, and 10 years and absolute strikes of 1%, 7%, and 14% for ask, bid, and mid quotes, respectively. Implied volatilities are extracted by the Black Model (1976) using the LIBOR single discount curve and the reference rate for caps is the 3-month LIBOR. We obtain all quotes from Bloomberg. Panel B reports summary statistics of explanatory candidates for economic determinants.

this stance to slower global growth.

In this regard, we suggest a hypothesis for interest rates: the positive unexpected shock related to unemployment and inflation will raise the slope of implied volatility curve. Based on the abovementioned arguments, we employ a SVAR involving three equations. In the first equation of the corresponding SVAR, inflation is the dependent variable, and the regressors are lagged values of all three variables. In the second equation, the unemployment rate is the dependent variable and the regressors are lags of all three variables plus the current value of the inflation rate. The interest rate is the dependent variable in the third equation, and the regressors are lags of all three variables, and the current value of the inflation rate plus the current value of the unemployment rate. Identification is achieved by imposing short-run restrictions, computed with a Cholesky decomposition of the reduced-form residuals' covariance matrix. The SVAR(1)-process is defined as:

$$\mathbf{y}_t = A_1 \mathbf{y}_{t-1} + \mathbf{u}_t \quad , \quad (4)$$

where the \mathbf{y} vector consists of a set of three endogenous variables $\mathbf{y}_t = (\text{inflation}_t, \text{unemployment}_t, \text{federal fund rate}_t)$ and A_1 is a (3×3) coefficient matrix. \mathbf{u}_t is a three-dimensional process with $E(\mathbf{u}_t) = \mathbf{0}$ and time invariant positive definite covariance matrix $E(\mathbf{u}_t \mathbf{u}_t^\top) = \Sigma_{\mathbf{u}}$. After estimating equation (4), we obtain the time-series of unexpected shocks, with \hat{u}_{1t} and \hat{u}_{2t} capturing unexpected news from inflation and unemployment, respectively. One can think of \hat{u}_{1t} and \hat{u}_{2t} as yield shocks analogous to Ang and Piazzesi (2003)'s vector of latent financial variables.

3. Empirical Results

This section reports empirical results. Our main question is the identification of economic determinants to explain the implied volatility surface in the cap market under the ZLB condition. To seek the information hidden in the hemispherical surface of the cap market, we run contemporaneous regression specifications as follows:

$$y_t = \beta X_t + \epsilon_t \quad (5)$$

where y_t are the dependent measures defined in Table 1, and ϵ_t is assumed to follow $N(0, V)$. X_t contains a group of explanatory variables and our model specifications are dependent on variables in the group. Model (1) is defined if X_t includes only the three term structure factors; Model (2) is defined as Model (1) with three additional variables, credit risk (CS), option volatility (ATMvol), and bid-ask spread (BAS); and Model (3) is defined as Model (2) with two unexpected shocks, unemployment rate and inflation.

3.1. Regression Result

The main estimation results based on equation (5) are shown in Tables 3 and 4. The panels in Table 3 contain the regression results with slope measures as dependent variables and the panels in Table 4 display the results with curvature measures as regressands. The

first notable finding in Table 3 is that unexpected shock related to unemployment exhibits strong positive statistical significance in Model (3) for three slope measures with 1 year maturity. Given the occurrence of positive shocks from unemployment, and equivalently lower than expected unemployment rate, the Fed tends to signal the market with hawkish views on the federal fund rate. In turn, hawkish expressions drive market participants to anticipate an increase in the reference rate, thus increasing the hedging demand for out-of-the-money caps leading to an increase of the slope measures. For an anecdotal event, we cite an article from the Wall Street Journal stating, “In 2016 March, Fed Chairwoman Janet Yellen signalled that the central bank was in no hurry to raise interest rates, citing slower global growth.” The next intriguing result, relevant to the first finding, is that unexpected shock related to inflation exhibits strong negative statistical significance in all specifications for the first two slope measures, with 5- and 10-year maturities, shown in Panel A and B. The unexpected shock related to unemployment exhibits persistent positive statistical significance in the slope measures for all maturities in all models. It is possible that the demand for out-of-the-money caps is higher when interest rates are expected to go up regardless of their maturities in the away-ATM region. Considering the historical inverse relationship between rates of unemployment and corresponding rates of inflation, the results are consistent with the findings of unexpected unemployment shocks.

Combining two of the abovementioned findings yields our main finding, that unexpected unemployment shocks play an important role in explaining the implied volatility surface with short-term maturity and unanticipated inflation shocks are important in explaining the surface of longer-term maturity. In this regard, we associate our findings with Jarrow et al. (2007)’s finding that significant negative jumps in interest rates are needed to capture the smile. Simply put, negative jumps contain information about unexpected macroeconomic shocks. In addition to the main finding, our results verify the usefulness of economic determinants such as credit spread, liquidity cost, and option volatility, suggested by Deuskar et al. (2008), and the three term structure factors prove to be useful in capturing the implied volatility slopes, as shown in all panels of Model (1). One puzzling result is that term structure slope factors lose statistical power in the third slope measure case, as shown in Panel C. Our inference is that the loss of statistical power may be related to the news shock of economics literature. In a standard DSGE model, several papers find that an unanticipated TFP shock decreases the term spread, while the term spread increases following the unanticipated (news) shock. Kurmann and Otrok (2013) find that 50 percent or more of all unpredictable movements in the slope over a ten-year forecast horizon are due to news shocks about future total factor productivity. In this regard, our proposed unexpected shocks may collide with unanticipated TFP shocks, leading to the loss of statistical power in the term spread.

The panels in Table 4 exhibit the results with curvature measures as regressands. Interpretation the curvature measures is nevertheless easy, particularly under the ZLB condition. Considering the fact that the definition of curvature is usually given by the change of the slope, our interpretation for the curvature factor can be associated with

Table 3: Regression results of slopes of implied volatility curve

| Panel A: S_1 | | | | | | | | | |
|--------------------|---------------------------|---------------------------|---------------------------|-------------------------|-------------------------|--------------------------|-------------------------|-------------------------|-------------------------|
| Maturity | 1y | | | 5y | | | 10y | | |
| | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| L_{TS} | -21.4095*** (-9.2277) | -23.4252*** (-9.8881) | -28.9110*** (-10.3831) | -4.3122*** (-5.3724) | -6.0578*** (-3.1968) | -6.4546*** (-2.8467) | -2.0306*** (-3.9639) | -1.5678 (-1.0153) | -3.1759** (-2.1021) |
| S_{TS} | 9.1797*** (3.8669) | 0.3254 (-0.1147) | -8.0317** (-2.1786) | 1.2009 (1.4623) | -1.3612* (-1.8186) | -1.7186** (-2.0268) | -0.4797 (-0.9151) | -1.5425** (-2.5664) | -2.3009*** (-3.6556) |
| C_{TS} | -13.3443*** (-6.1465) | -11.1361*** (-5.0277) | -7.6725*** (-3.1019) | -1.5531** (-2.0679) | 0.1169 (0.1351) | 0.1249 (0.1516) | 0.0619 (0.1291) | 1.1832** (2.4695) | 1.2430** (2.5879) |
| CS | | 14.9930*** (-4.1865) | -18.8471*** (-5.0877) | | -8.7056*** (-7.6001) | -10.1321*** (-8.9498) | | -4.9905*** (-6.5539) | -6.5193*** (-9.5419) |
| BAS | | -1.8162** (-2.0794) | -2.4519*** (-2.8458) | | 0.5019 (0.4940) | -0.1408 (-0.1425) | | 1.2597 (1.2002) | 0.1305 (0.1436) |
| ATM vol | | -0.3490*** (-2.4945) | -0.7204*** (-4.1649) | | -0.1006 (-0.8258) | -0.1131 (-0.7781) | | 0.1034 (0.6491) | -0.0613 (-0.3972) |
| Shock1 | | | -1.1528 (-0.7854) | | | -1.6125*** (-3.4599) | | | -1.6697*** (-6.1956) |
| Shock2 | | | 43.6537*** (3.0604) | | | 1.2328 (0.2721) | | | 0.9599 (0.3962) |
| Adj R ² | 53.43 | 63.37 | 66.52 | 22.05 | 49.43 | 55.55 | 11.5 | 36.28 | 55.09 |
| DW | 0.6603 | 0.7304 | 0.9118 | 0.5109 | 0.7227 | 0.8634 | 0.5327 | 0.7263 | 1.1742 |
| Panel B: S_{2-1} | | | | | | | | | |
| L_{TS} | -18.2516*** (-10.0803) | -19.5177*** (-10.7682) | -23.9096*** (-11.3000) | -4.6214*** (-6.9005) | -5.9768*** (-3.7595) | -5.4181*** (-2.8127) | -1.8257*** (-4.1158) | -0.9165 (-0.6806) | -1.0566 (-0.7751) |
| S_{TS} | 6.3433*** (3.4241) | -0.5774 (-0.2659) | -7.2224** (-2.5780) | 0.1585 (0.2314) | -1.9338*** (-3.0795) | -1.9179*** (-2.6623) | -1.1642** (-2.5651) | -1.9174*** (-3.6581) | -1.9673*** (-3.4643) |
| C_{TS} | -10.8374*** (-6.3966) | -8.5524*** (-5.0467) | -5.9224*** (-3.1508) | -0.5912 (-0.9433) | 0.8535 (1.1761) | 0.7712 (1.1019) | 0.8222* (1.9809) | 1.7925*** (4.2899) | 1.4485*** (3.3426) |
| CS | | -13.2355*** (-4.8305) | -16.5196*** (-5.8683) | | -7.2478*** (-7.5419) | -8.3000*** (-8.6293) | | -4.1946*** (-6.3167) | -5.2560*** (-8.5264) |
| BAS | | -1.5010** (-2.2462) | -2.0345*** (-3.1073) | | 0.6883 (0.8075) | 0.3311 (0.3943) | | 1.1902 (1.3003) | 0.5413 (0.6598) |
| ATM vol | | -0.2339** (-2.1839) | -0.5310*** (-4.0403) | | -0.0691 (-0.6758) | -0.0236 (-0.1913) | | 0.149 (1.0721) | 0.1308 (0.9392) |
| Shock1 | | | -1.2289 (-1.1018) | | | -1.3598*** (-3.4343) | | | -1.4350*** (-5.9018) |
| Shock2 | | | 33.8836*** (3.1260) | | | -1.8299 (-0.4754) | | | -2.8566 (-1.3069) |
| Adj R ² | 57.8 | 68.1 | 71.23 | 29.86 | 54 | 58.54 | 15.27 | 38.13 | 53.33 |
| DW | 0.6585 | 0.7184 | 0.9162 | 0.5318 | 0.7761 | 0.881 | 0.5839 | 0.8213 | 1.1165 |
| Panel C: S_{2-2} | | | | | | | | | |
| L_{TS} | -3.1579*** (-5.8004) | -3.9075*** (-6.7089) | -5.0014*** (-7.1764) | 0.3092 (1.4614) | -0.0810 (-0.1413) | -1.0366 (-1.5004) | -0.2049 (-1.2361) | -0.6513 (-1.1341) | -2.1193*** (-3.5719) |
| S_{TS} | 2.8364*** (5.0918) | 0.9028 (1.2939) | -0.8093 (-0.8770) | 1.0424*** (4.8150) | 0.5726** (2.528) | 0.1993 (0.7714) | 0.6846*** (4.0368) | 0.3749* (1.6772) | -0.3336 (-1.3495) |
| C_{TS} | -2.5068*** (-4.9206) | -2.5837*** (-4.7445) | -1.7501*** (-2.8268) | -0.9620*** (-4.8590) | -0.7366*** (-2.8143) | -0.6463 (-2.5746) | -0.7603*** (-4.9026) | -0.6093*** (-3.4193) | -0.2055 (-1.0895) |
| CS | | -1.7574** (-1.9960) | -2.3275** (-2.5103) | | -1.4577*** (-4.2058) | -1.8324 (-5.3120) | | -0.7958*** (-2.8103) | -1.2633*** (-4.7081) |
| BAS | | -0.3151 (-1.4675) | -0.4174* (-1.9356) | | -0.1864 (-0.6063) | -0.4720 (-1.5672) | | 0.0695 (0.1780) | -0.4107 (-1.1502) |
| ATM vol | | -0.1152*** (-3.3499) | -0.1893*** (-4.3731) | | -0.0315 (-0.8552) | -0.0895** (-2.0203) | | -0.0456 (-0.7688) | -0.1921*** (-3.1690) |
| Shock1 | | | 0.0761 (0.2072) | | | -0.2527* (-1.7795) | | | -0.2347** (-2.2173) |
| Shock2 | | | 9.7701*** (2.7366) | | | 3.0626** (2.2186) | | | 3.8164*** (4.0113) |
| Adj R ² | 36.08 | 44.81 | 47.71 | 18.42 | 30.26 | 37.85 | 17.43 | 21.46 | 38.28 |
| DW | 0.6252 | 0.7608 | 0.8896 | 0.6423 | 0.7094 | 0.858 | 0.5921 | 0.6225 | 0.9469 |

Note. This table reports the regression results of three different models with slopes of implied volatility curves as independent variables. Three slopes are defined in Table 1 for mid quotes. Candidates for economic determinants are the three Nelson-Siegel factors, credit spread (CS), bid-ask spread (BAS), ATM volatility (ATM vol), unexpected economic shocks of inflation (shock1), and unexpected shocks of unemployment rate (shock2). We omit the constant coefficients for brevity.

Table 4: Regression results of curvatures of implied volatility curve

| Panel D: C_1 | | | | | | | | | |
|--------------------|-------------------------|------------------------|--------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Maturity | 1y | | | 5y | | | 10y | | |
| | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| L_{TS} | 7.8667*** (11.0595) | 8.1940*** (11.8527) | 9.9539*** (12.4588) | 2.4595*** (8.1766) | 2.9838*** (4.0386) | 2.3093** (2.5444) | 0.8600*** (3.9652) | 0.217 (0.3170) | -0.2978 (-0.4145) |
| S_{TS} | -2.0452*** (-2.8102) | 0.6397 (0.7724) | 3.2777*** (3.0986) | 0.3267 (1.0614) | 1.1911*** (4.0815) | 1.0310*** (3.0374) | 0.8513*** (3.8359) | 1.1067*** (4.1541) | 0.8493*** (2.8379) |
| C_{TS} | 4.4187*** (6.6387) | 3.2364*** (5.0071) | 2.2594*** (3.1835) | -0.0827 (-0.2940) | -0.7207** (-2.1370) | -0.6418* (-1.9464) | -0.7125*** (-3.5107) | -1.1426*** (-5.3801) | -0.8063*** (-3.5308) |
| CS | | 5.9371*** (6.6810) | 7.3597*** (6.9240) | | 3.0648*** (6.8625) | 3.4491*** (7.6111) | | 1.8012*** (5.3364) | 2.1549*** (6.6332) |
| BAS | | 0.6256** (2.4548) | 0.8526*** (3.4487) | | -0.4256 (-1.0743) | -0.3596 (-0.9088) | | -0.5743 (-1.2344) | -0.4358 (-1.0077) |
| ATM vol | | 0.0701* (1.7156) | 0.1892*** (3.8111) | | 0.0219 (0.4610) | -0.0249 (-0.4282) | | -0.0941 (-1.3321) | -0.1433* (-1.9530) |
| Shock1 | | | 0.6569 (1.5599) | | | 0.5912*** (3.1689) | | | 0.6368*** (4.9695) |
| Shock2 | | | -13.0037*** (-3.1773) | | | 2.1818 (1.2030) | | | 2.9777** (2.5850) |
| Adj R ² | 62.58 | 73.33 | 76.43 | 37.45 | 56.08 | 59.31 | 19.75 | 36.67 | 48.65 |
| DW | 0.6483 | 0.7017 | 0.922 | 0.5817 | 0.8313 | 0.8995 | 0.6389 | 0.8614 | 1.0182 |
| Panel E: C_{2-1} | | | | | | | | | |
| L_{TS} | 5.2558*** (11.0043) | 5.3114*** (11.7801) | 6.4689*** (12.5252) | 1.4569*** (6.8306) | 0.1633*** (2.9591) | 0.9329 (1.3140) | -0.0792 (-0.5035) | -0.7629 (-1.4631) | -1.6045*** (-2.8112) |
| S_{TS} | -1.4092*** (-2.8837) | 0.2711 (0.5019) | 1.9835*** (2.9006) | 0.5238** (2.4002) | 1.0111*** (4.5541) | 0.7869*** (2.9636) | 0.7766*** (4.8234) | 0.8631*** (4.2528) | 0.4516* (1.8992) |
| C_{TS} | 3.0071*** (6.7285) | 2.0013*** (4.7475) | 1.4304*** (3.1178) | -0.4995** (-2.5028) | -0.9120*** (-3.5543) | -0.8400*** (-3.2332) | -0.7963*** (-5.4076) | -1.0742*** (-6.6398) | -0.7260*** (-4.0009) |
| CS | | 4.4349*** (6.5067) | 5.4703*** (7.9612) | | 1.7938*** (5.2793) | 1.8908*** (5.3338) | | 1.0164*** (3.9528) | 1.0590*** (4.1028) |
| BAS | | 0.4344** (2.6131) | 0.5959*** (3.7288) | | -0.3321 (-1.1020) | -0.3923 (-1.2676) | | -0.2552 (-0.7201) | -0.3463 (-1.0081) |
| ATM vol | | 0.0252 (0.9479) | 0.1036*** (3.2279) | | 0.0033 (0.0920) | -0.0445 (-0.9768) | | -0.0860 (-1.5987) | -0.1688*** (-2.8939) |
| Shock1 | | | 0.5856** (2.1516) | | | 0.2616* (1.7924) | | | 0.2969*** (2.9165) |
| Shock2 | | | -8.0187*** (-3.0308) | | | 2.3526* (1.6584) | | | 3.1716*** (3.4654) |
| Adj R ² | 62.58 | 74.84 | 78.16 | 31.37 | 44.52 | 45.66 | 20.09 | 30.48 | 38.68 |
| DW | 0.5969 | 0.6371 | 0.8489 | 0.6917 | 0.9276 | 0.9238 | 0.8083 | 1.0048 | 1.0632 |
| Panel F: C_{2-2} | | | | | | | | | |
| L_{TS} | 0.2897*** (5.7939) | 0.3470*** (6.5565) | 0.4560*** (7.2460) | 0.0159 (0.6171) | 0.0533 (0.7738) | 0.1069 (1.3163) | 0.0780*** (3.7088) | 0.102 (1.4346) | 0.2656*** (3.7796) |
| S_{TS} | -0.3005*** (-5.8743) | -0.1192* (-1.8795) | 0.0465 (0.5576) | -0.1457*** (-5.5325) | -0.0860*** (-3.1633) | -0.0594* (-1.9571) | -0.0948*** (-4.4085) | -0.0570** (-2.0602) | 0.0216 (0.7388) |
| C_{TS} | 0.2410*** (5.1517) | 0.2246*** (4.5395) | 0.1572*** (2.8122) | 0.1107*** (4.5961) | 0.0733** (2.3333) | 0.0690** (2.3412) | 0.0870*** (4.4278) | 0.0596*** (2.7003) | 0.022 (0.9830) |
| CS | | 0.2290*** (2.8620) | 0.3075*** (3.6731) | | 0.1976*** (4.7505) | 0.2540*** (6.2673) | | 0.1298*** (3.7030) | 0.2016*** (6.3435) |
| BAS | | 0.0329* (1.6862) | 0.0458** (2.3508) | | 0.0031 (0.08426) | 0.0337 (0.9530) | | -0.0217 (-0.4488) | 0.0435 (1.0292) |
| ATM vol | | 2.9309*** (2.9309) | 0.0165*** (4.2311) | | 0.0025 (0.5741) | 0.0055 (1.0510) | | 0.0017 (0.2299) | 0.0181*** (2.5214) |
| Shock1 | | | 0.026 (0.7822) | | | 0.0562*** (3.3653) | | | 0.0535*** (4.2698) |
| Shock2 | | | -2.6581*** (-2.6581) | | | -0.1707 (-1.0525) | | | -0.3630*** (-3.2215) |
| Adj R ² | 38.98 | 48.43 | 51.74 | 20.85 | 34.13 | 43.73 | 23.72 | 30.79 | 50.2 |
| DW | 0.6893 | 0.8048 | 0.9764 | 0.6632 | 0.7253 | 0.895 | 0.6244 | 0.6487 | 1.0984 |
| # Obs | 109 | 109 | 109 | 109 | 109 | 109 | 109 | 109 | 109 |
| # var.s | 4 | 7 | 9 | 4 | 7 | 9 | 4 | 7 | 9 |

Note. This table reports the regression results of three different models with curvatures of implied volatility curves as independent variables. Three curvatures are defined in Table 1 for mid quotes. Candidates for economic determinants are the three Nelson-Siegel factors, credit spread (CS), bid-ask spread (BAS), ATM volatility (ATM vol), unexpected economic shocks of inflation (shock1), and unexpected shocks of unemployment rate (shock2). We omit the constant coefficients for brevity.

market perceptions about the speed of adjustment of the federal fund rate. When the positive coefficient for unexpected shocks to curvature measures is observed, it indicates that unexpected shocks are expected to be higher the more hump-shaped, convex, or concave the current implied volatility surface expected to be, and vice versa. Unlike the results in Table 3, the curvature of the smile is negatively related to the unexpected unemployment shocks, with strong statistical significances in all Model (3) scenarios for a 1-year maturity. Along with the signs, the magnitudes of the unexpected unemployment shocks become smaller as the absolute strike moves from near-ATM to away-ATM. It appears that the volatility smiles have more curvature when the term structure is relatively flat, especially during the ZLB period. Relating to this finding, unexpected shock related to inflation exhibits strong positive statistical significance in all specifications with all curvature measures. This is consistent with the findings of the slope measures, given the implication of the Philips curve. It should be noted that the liquidity proxies seem to be losing statistical power, both for slope and curvature measures, with longer term maturities.

3.2. Time-varying effects of unanticipated shocks

We further investigate the time-varying effects of two unexpected shocks. When a Taylor-Rule policy for exiting the ZLB is implemented, the monetary authority modifies the Taylor Rule with a time-varying inflation target after the exit date.¹⁰ We add time-varying unemployment into the Taylor Rule with a time-varying inflation target based on the fact that FED chairwoman, Janet Yellen supports placing reducing unemployment and boosting growth on an equal footing with fighting inflation in the Fed's policy. Based on findings from the monetary economics literature, we examine the time-varying effects of two unexpected shocks on the interest rate options markets. A generic state space model with time-varying coefficients with a pure random walk is given as follows:

$$y_t = \beta_t X_t + \epsilon_t$$

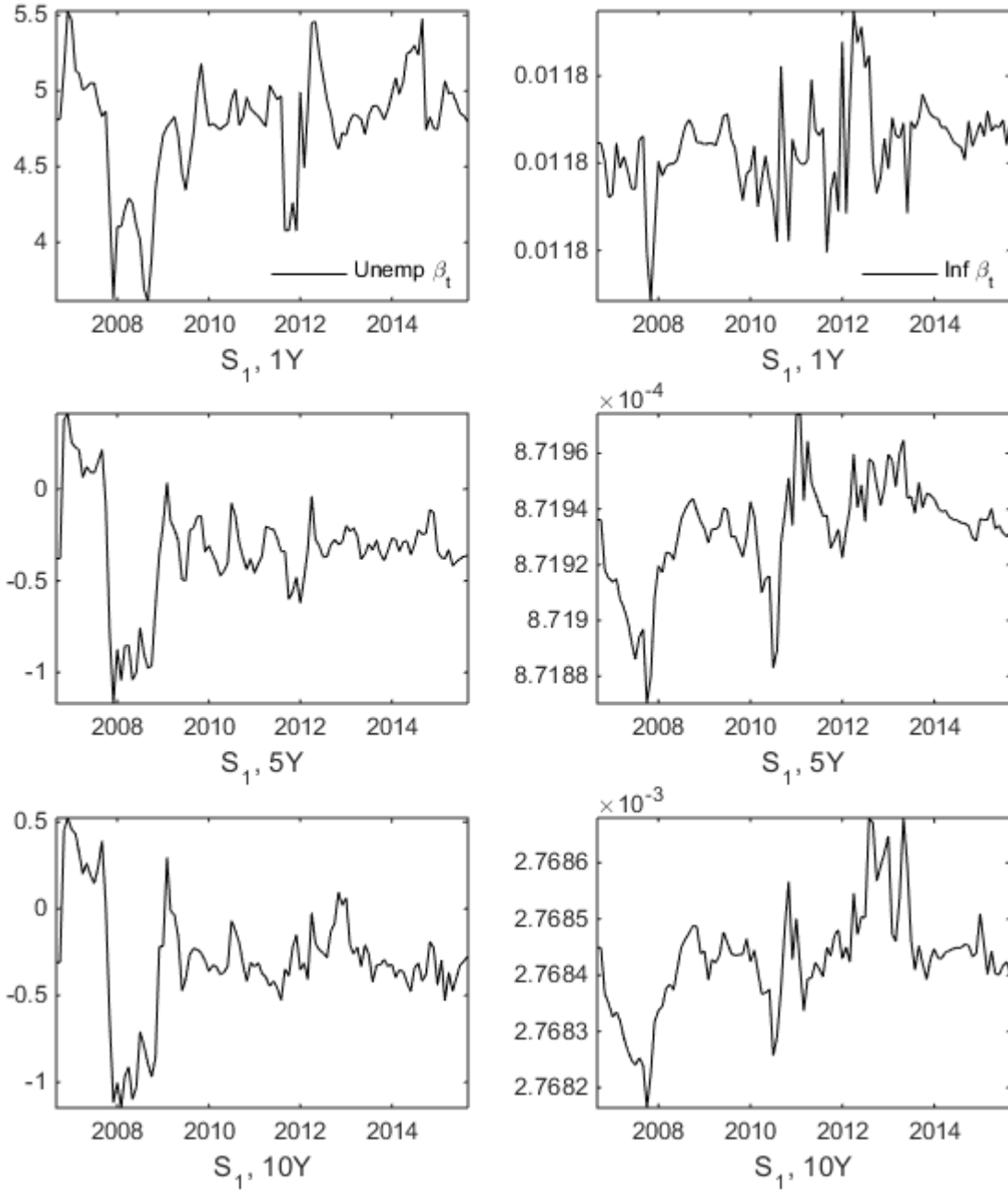
$$\beta_t = \beta_{t-1} + \nu_t$$

where ϵ_t and ν_t are assumed to follow $N(0,R)$ and $N(0,Q)$, respectively, and the model is estimated by Gaussian maximum likelihood. All variables are same the as defined in equation (3).

Figures 3 and 4 depict the Kalman filter inferences on the regression coefficients. These show estimates of how the implied volatility surface has changed in response to two macroeconomic shocks, with changing importance between the two shocks. Figure 3 exhibits time-varying coefficients of two shocks to the first slope measure (S_1) with

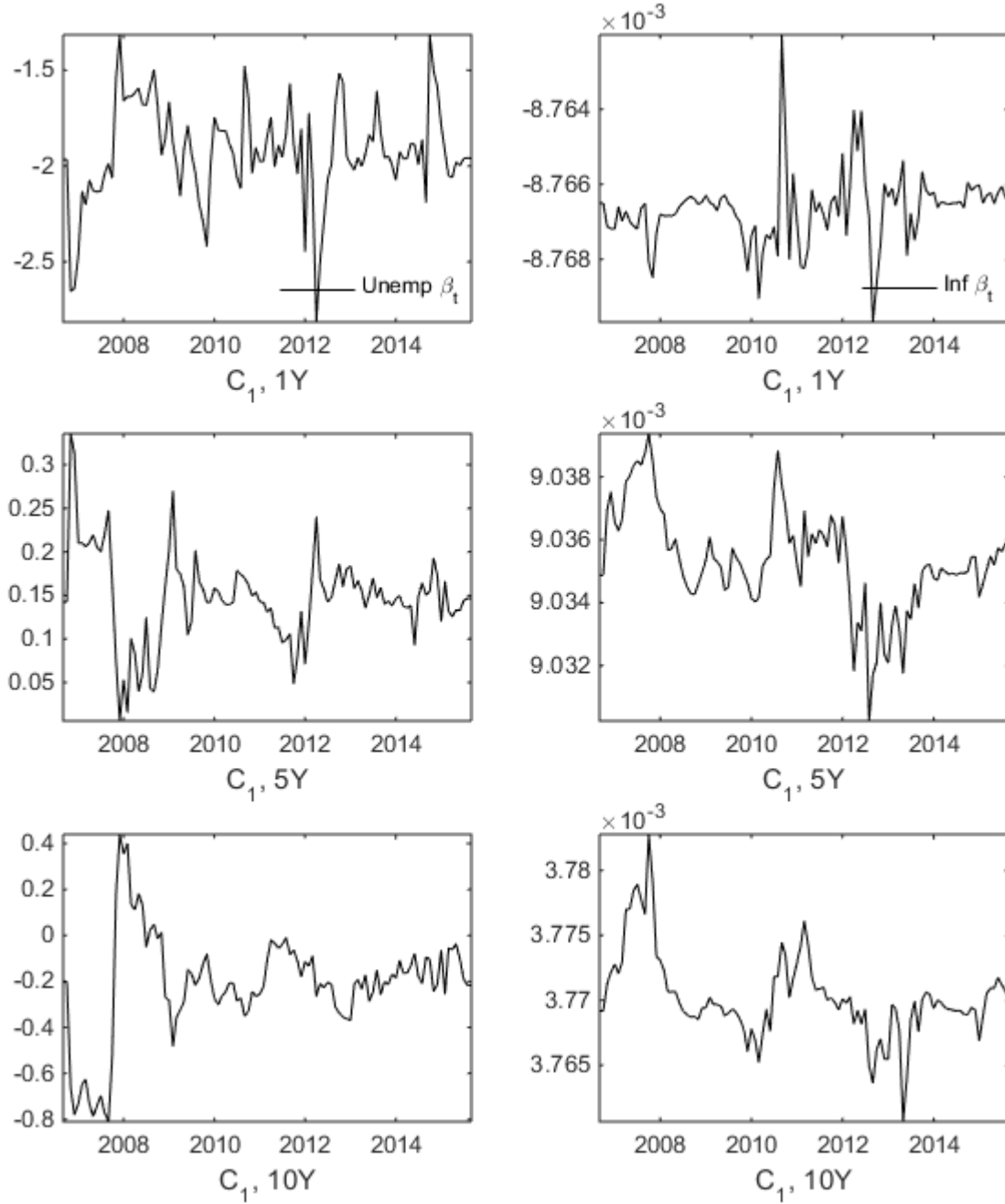
¹⁰Ireland (2007) argues that US inflation can be explained by a New Keynesian model with a Taylor Rule only if the inflation target is allowed to vary over time. Additionally, Kozicki and Tinsley (2005), Rudebusch and Wu (2008), and Grkaynak et al. (2005) provide evidence of a time-varying short-run inflation target for the US.

Figure 3: Time-varying coefficients of two shocks to slope measures



Note. This figure depicts time-varying coefficients of two shocks to the first slope measure (S_1) with different maturities. The first column contains the results for the unemployment shock and the second column contains the results for the inflation shock.

Figure 4: Time-varying coefficients of two shocks to curvature measures



Note. This figure depicts time-varying coefficients of two shocks to the first curvature measure (C_1) with different maturities. The first column contains the results for the unemployment shock and the second column contains the results for the inflation shock.

different maturities. The first column contains the results of the unemployment shocks, and the second column contains the results of the inflation shocks. The first notable finding is that the magnitudes of time-varying coefficients of the two shocks to slope measures with 1-year maturities display larger values than those with longer maturities, and the magnitudes with longer maturities are negative, having hovered near zero after 2008. In addition, the relative importance of the effects of the two shocks seems to be significant with unexpected unemployment shocks being much larger. This result is consistent with the result of Table 3 because unexpected unemployment shocks exhibit strong positive statistical significance in all Model (3) scenarios with 1-year maturities, while they show no statistical power for longer maturities. Interestingly, all magnitudes of time-varying coefficients in Figure 3 display a sudden drop or a decreasing pattern during the financial crisis, meaning that their contribution to explaining the implied volatility surface decreases significantly during the period. Note that the time series patterns of time varying coefficients for both shocks with longer maturities seem to be cast in a similar mold. We attribute these results to the existence of homogeneous information in implied volatilities with longer maturities.¹¹

Figure 4 exhibits time-varying coefficients of two shocks to the first curvature measure (C_1) with different maturities. The first column contains the results of the unemployment shocks, and the second column contains the results for the inflation shocks. Unlike the results of slope measures, the results of curvature measures with 1-year maturities show no discernable patterns, yielding noisy dynamics. The finding that magnitudes of time-varying coefficients of unemployment shock to slope measures with 1-year maturities display larger values than those with longer maturities is similar to Figure 3. In addition, the relative importance between the effects of two shocks seems to be significant with unexpected unemployment shocks being much larger, too. This result is also consistent with the result of Table 4 because unexpected unemployment shocks exhibit strong negative statistical significance in all Model (3) scenarios with 1-year maturities. However, all magnitudes of time-varying coefficients in Figure 4, except the result with 5-year maturities, display a sudden surge or an increasing pattern during the financial crisis, meaning that their contribution to explaining the implied volatility surface increases significantly during the period.

4. Conclusions

This paper answers a question: What would the economic determinants of the shape of the implied volatility surface be under the zero lower bound condition? We consider eight economic determinants in our analysis: three Nelson-Siegel factors, credit spread,

¹¹We have verified that this finding holds for other longer term maturities. Additional results are available upon request. The signs of time-varying coefficients of unexpected inflation shock show no match to the result of Table 3, which give rise to the possible misspecification of the transition equation

bid-ask spread, ATM volatility, unexpected inflation shocks, and unexpected unemployment shocks. We find that unexpected unemployment shocks play an important role in explaining the implied volatility surface with short-term maturity, and unanticipated inflation shocks are important in explaining the surface of longer-term maturity. An intriguing finding is that all magnitudes of time-varying coefficients in the time-varying regression for the slope measures display a sudden drop or a decreasing pattern during the financial crisis, meaning that their contribution to explaining the implied volatility surface decreases significantly during the period. We also find that the explanatory contribution in the time-varying regression for the curvature measures increase significantly during the period. Our results provide important information regarding hedging the risk of an increase in the reference rates and are particularly timely enabling practitioners to prepare future exit strategies. Relating to these findings, we verify the benefits of the presence of two additional unanticipated macroeconomic shocks in explaining the dynamics of the implied volatility of the cap markets.

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