Asymmetries of the

Intraday Return-Volatility Relation

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Abstract

This study investigates the asymmetry of the intraday return-volatility relation at different return horizons ranging from 1, 5, 10, 15, up to 60 minutes and compares the empirical results with results for the daily return horizon. Using a sample of S&P 500 and VIX from September 25, 2003 to December 30, 2011 and a Quantile-Regression approach, the results generally confirm the strong negative return-volatility relation over all return horizons. However, this negative relation is asymmetric in three different aspects. First, the effect of positive and negative returns on volatility is different and slightly more pronounced for negative returns. Second, for both positive and negative returns, the effect is conditional on the distribution of volatility changes. The absolute effect is up to five times larger in the extreme tails of the distribution. Third, at the intraday level, there is evidence of both autocorrelation in volatility changes and cross-autocorrelation with returns. This lead-lag relation with returns is also very asymmetric and more pronounced in the tails of the distribution. These effects are, however, not found for the daily return horizon.

JEL Codes: C21, G12, G13

Keywords: Asymmetric return-volatility relation, implied volatility, index options, intraday, quantile regression, VIX

1. Introduction

The relationship between risk and return is a fundamental principle in finance and has extensively been examined in the past four decades (Markowitz and Blay, 2013). Moreover, the relationship between volatility and equity returns has commonly been documented to be asymmetric. Returns and volatility are negatively related and this relationship is more prominent for negative returns (Black, 1976, Christie, 1982, French, et al., 1977, Bekaert and Wu, 2000).

In this paper, we take a new look at the risk and return relationship by examining the intra-daily effects of negative and positive stock index returns over various parts of the conditional volatility index (VIX) distribution. Our approach further allows us to investigate the cases of extreme asymmetric volatility in more depth. As the level of volatility increases, e.g. during financial crises, it is expected that the negative asymmetric return-volatility relation will be significantly more pronounced in the extreme parts of the conditional VIX distribution than what traditional models, e.g., the Ordinary Least Squares (OLS), will predict. Our methodology, Quantile Regression analysis, allows modelling of the return-volatility relation with emphasis on different parts of the conditional volatility distribution, including the extreme tails. By using a combination of the robust Quantile Regression approach and a data set of varying high-frequency returns and VIX, our study is able to monitor the strong contemporaneous negative asymmetric return-volatility relation across the conditional VIX distribution. Well known hypotheses put forward in the literature for this relationship, such as the *leverage effect* and the *volatility feedback effect*, have not been able to completely characterize such a strong contemporaneous relation at stock index level. Additional investigation of the asymmetric relationship between equity returns and volatility is vital as it has important implications for asset pricing models, option pricing and risk management practices.

Overall, we observe that the strength of the asymmetric return-volatility relation increases with return horizon and is strongest for daily returns. We further note that the asymmetry increases monotonically from the median to the tails of the distribution. As a consequence, OLS will underestimate the asymmetry of this relation beyond the median. Moreover, OLS reveals no asymmetry in the relation at higher frequency, e.g., one minute interval, whereas Quantile Regression shows that there is a strong asymmetric relation between return-volatility at the tails of the conditional VIX distribution. At higher frequencies lagged effects also become more pronounced. Finally, across all frequencies, we find that OLS underestimates the stronger relation in the tails of the distribution.

The remainder of the paper is structured as follows. Section 2 briefly reviews the literature on the return-volatility relation. Section 3 discusses the data used in the study and Section 4 presents the methodology applied. Section 5 reports on the results and Section 6 finally summarizes and concludes.

2. The Asymmetric Return-Volatility Relation

2.1. The Leverage and Volatility Feedback Explanations

Two hypotheses explain the existence of the asymmetric return-volatility relation. These are the *leverage effect* and the *feedback effect* hypotheses. The leverage hypothesis, proposed by Black (1976) and Christie (1982), attributes the asymmetric return-volatility relation to the financial leverage of a firm. When a firm's debt increases the firm's debt/equity-ratio and risk level increases and as a consequence the value of its equity declines. As the risk level increases, the volatility of the equity is also expected to increase. In contrast, the feedback hypothesis put forward by French et al. (1987), Campbell and Hentschel (1992) and Bekaert and Wu (2000), attribute the asymmetric return-volatility relation to a volatility feedback effect.[1] The volatility feedback hypothesis states that increases in volatility imply that required future returns will increase and, as a result, current stock prices decline. These financial and economic explanations might be important for the characterization of the asymmetric relation at lower frequencies, such as monthly or quarterly, but not at daily or higher frequencies.[2]

There is an abundance of studies that examine the return-volatility relation. However, the empirical studies on the asymmetric return-implied volatility relation are relatively recent and fewer in number (including Fleming et al., 1995; Whaley, 2000; Low, 2004; Giot, 2005; Dennis et al., 2006; Hibbert et al. 2008; Frijns et al., 2010; Badshah, 2013). The study of Fleming et al. (1995) is the first on the relation between S&P 100 (OEX) returns and VXO (the predecessor of VIX) changes and finds a significant negative contemporaneous asymmetric relation between OEX returns and VXO changes, while other lagged variables are found to be insignificant or marginally significant.[3] Low (2004) is the first to explain the dynamics of the contemporaneous asymmetric relation between OEX returns and changes in VXO using behavioral explanations of loss-aversion (Kahneman and Tversky, 1979), in which losses loom longer than gains. He shows that the return-volatility relation is

[1] Poterba and Summers (1986) characterize the volatility feedback effect through the economic explanation that time-varying risk premia induce volatility feedback because it represents the link between changes in volatility and returns.

[2] Schwert (1990) and Bollerslev et al. (2006) among others, argue that the asymmetry in volatility is too strong to be explained by the leverage effect. Also previous empirical studies show that the volatility feedback hypothesis is not always consistent. Furthermore, some studies find that there is not always a positive correlation between current volatility and expected future returns (e.g., Breen et al., 1989). However, other studies support the hypothesis (e.g., French et al., 1987; Campbell and Hentschel, 1992, among others).

[3] Similarly, Giot (2005) confirms strong negative contemporaneous asymmetric relations in both the SPX and NDX stock market indexes.

asymmetric and nonlinear. This nonlinear characteristic is best described as a downward sloping reclined S-curve. The partition of downside returns has a convex profile and the partition of upside returns has a concave profile. Convexity implies accelerating increases in the VXO and concavity accelerating decreases in the VXO. Hibbert et al. (2008) investigates the negative asymmetric return-volatility relation between the SPX (NASDAQ-100 index, NDX) returns and changes in the new VIX (VXN) at intraday and daily frequencies. They confirm that the asymmetric return-volatility relation is a contemporaneous rather than a lagged phenomenon. This indicates a rejection of both the leverage and the volatility feedback hypotheses. Instead, Hibbert et al. (2008) present behavioral explanations. They argue that as both hypotheses are based on fundamentals, the effect of return on volatility, and vice versa, should involve a longer lag in lower frequencies than in higher frequencies. Earlier, Dennis et al. (2006) confirm that it is a market-wide phenomenon rather than an individual stock-level characteristic. More recently, Badshah (2013), applying Quantile Regression model, examines the asymmetric return-volatility relation at a daily frequency for several stock market indexes. He observes strong negative asymmetric return-volatility relation in the tails of the conditional volatility changes distribution, and finds that OLS underestimates (overestimates) this relation at the positive (negative) conditional volatility changes distribution. In this paper, we explore the intraday asymmetric return-volatility relation at high frequency, such as 1 minute, which has never been documented before.

2.2. Investor Heterogeneity and the Return-Volatility Relation

The new, model-free implied volatility index (MFIV), VIX uses a full range of strike prices and therefore better captures market-wide sentiments (errors in investors' beliefs) of fear and exuberance.[4] In the market, investors have heterogeneous beliefs about stock fundamentals leading to different stock price forecasts. The disagreements in beliefs during bear markets are higher than during bullish markets as a consequence stock-market volatility is higher during bear market.[5] Shefrin (2008) confirms heterogeneity in beliefs through survey results and argues that heterogeneous beliefs play an important role in asset pricing. The survey results show that the distribution of expected returns in the US stock market is multimodal and fat-tailed. Returns are not distributed near the mean but are distributed with fat-tails and with clusters at each extreme. Shefrin (2008) attributes these clusters to extreme beliefs of optimistic investors on the right-end tail (i.e., who expect high returns) and to a cluster of pessimistic investors on the left-end tail (i.e., who expect low returns). This implies that optimists (pessimists) overestimate (underestimate) expected returns and underestimate (overestimate) volatility. Consistent with the survey results, institutional investors (the pessimists) are actively hedging their portfolios by buying out of the money (OTM) puts. Therefore the demand for OTM puts increases and pushes their prices upward beyond the efficient level, resulting in poor contract price judgment. The explanation is consistent with Bollen and Whaley (2004) and Hans (2008) who show that the volatility skew is primarily caused by the demand for OTM puts. Jackwerth and Rubinstein (1996) label the fear of a crash as "crashophobia", and Shiller (2000) further confirms the fear of crash through a survey in which investors assume more than a 10% probability of market crash within the next six months.

Shefrin (2001, 2008) suggests that investors' heterogeneity leads to a multimodal and fat-tailed stock index return distribution. The OLS regression estimates (using the conditional

[4] The MFIV measure is consistent with the volatility skew pattern, which is a documented pattern in stock index implied volatilities (e.g., Badshah, 2008, and others). Whaley (2009) explains the new VIX and suggests that it is a better gauge of market fear as it accounts for OTM puts.

[5] Li (2007) explains the role of heterogeneous beliefs in asset prices and volatilities, and Buraschi and Jiltsov (2006) explain the role of heterogeneous beliefs in option prices.

mean function) how the dependent variable on average responds to changes in a set of covariates. The OLS approach assumes that this is a good description for the entire conditional distribution of the dependent variable. Hence, this traditional approach ignores any deviations of the relation at the tails of the distribution and fails to account for the effect of the observed investors' heterogeneity. However, Quantile Regressions are consistent with the heterogeneity hypothesis and allow for different responses in different parts of the conditional distribution of the dependent variable (Koenker and Basset, 1978; Koenker and Hallock, 2001; Koenker, 2005).

3. Data

Intraday data for the SPX and the VIX are obtained from the Thomson Reuters Tick History database maintained by SIRCA.[6] Data are sampled for the period when both the new VIX index and intraday data are available. The resulting sample period starts September 25, 2003 and ends December 30, 2011 covering a total of 2,160 trading days. Intraday, both the SPX and the VIX are computed at a 15 second frequency. For the empirical analyses, data is sampled at various frequencies, namely 1-, 5-, 10-, 15-, 60-minutes. Besides the intraday samples, as a benchmark data are also collected at a daily frequency. The various data frequencies are used to examine how the asymmetric return-volatility relation is affected by the sampling frequency. From the raw data, we compute percentage continuously compounded returns and first differences in the VIX.

INSERT TABLE 1 HERE

[6] Securities Industry Research Centre of Asia-Pacific.

Table 1 reports summary statistics for the SPX-returns and for first differences of the VIX. The sample size increases from 2,160 observations at a daily frequency to 778,417 observations at a one-minute frequency. As expected, for returns on the SPX (first columns of Table 1), the average return increases as the sampling frequency decreases (going from 1 minute to daily). As expected, the standard deviations of returns increase as the sampling frequency decreases. Furthermore, the distributions of high frequency returns appear considerably more non-normal than the distribution of daily returns. This is confirmed by the Jarque-Bera statistics. At all sampling frequencies there is evidence of autocorrelation. Although statistically significant, the autocorrelation is relatively small in absolute terms. Finally, the ADF tests reject the null hypotheses of unit roots in the return series.

The last columns of Table 1 report summary statistics for the first difference in VIX (ΔVIX). As with the return data, mean changes and standard deviations increase as the sampling frequency decreases. Overall, ΔVIX is positively skewed and this skewness is higher at the daily frequency than at high intraday frequencies. On the other hand, the kurtosis is considerably higher for high-frequency intraday data than for daily data indicating increasingly fatter tails with increasing sampling frequency. This again results in intraday ΔVIX being more non-normally distributed than for daily data, as evidenced by the Jarque-Bera statistics. As with return data, evidence of ΔVIX being autocorrelated is found. However, these autocorrelations are again small in absolute terms. The null hypotheses of the presence of a unit roots in ΔVIX are rejected.

4. Methodology

The Quantile Regression model approach (QRM) is utilized to assess the intraday asymmetric relation between returns on the S&P 500 and ΔVIX. The model is similar to the QRM framework of Badshah (2013). With the exception of Badshah (2013), most studies have employed traditional regression models that focus on the average relation (at the mean of the distribution) between changes in VIX and SPX returns. This traditional approach might lead to a situation where important information about this relation is not correctly modelled if, e.g., the relation is asymmetric or different in the tails of the conditional distribution. Focusing on the tails of this distribution is important as the tails represent extreme changes in VIX, typically observed in crisis periods. The approach of this study allows modelling of the return-volatility relation with focus on different parts of the conditional ΔVIX distribution, including the extreme tails. Furthermore, as quantile regression requires weaker distributional assumptions, it provides a distributionally more robust method of modelling the conditional ΔVIX distribution and is, hence, less sensitive to extreme observations. With implied volatility changes it is often the case that the distributions are skewed and leptokurtic.

Before specifying the robust QRM model for the intraday asymmetric return-volatility relation a standard benchmark mean regression model (MRM) similar to that of Low (2004), Giot (2005), and Hibbert et al. (2008) is specified. For the analysis, ΔVIX_{it} is defined as the percentage changes in VIX of frequency *i* where *i= 1, 5, 10, 15, 60* minutes, and daily. R_{it} is the percentage continuously compounded return of the S&P500 index of frequency *i* where *i= 1, 5, 10, 15, 60* minutes, and daily. For assessing asymmetry we define positive and negative returns as

$$R_{it}^{+} = \begin{cases} R_{it} \ if \ R_{it} > 0 \\ 0 \quad if \ R_{it} < 0 \end{cases} \ and \ \ R_{it}^{-} = \begin{cases} R_{it} \ if \ R_{it} < 0 \\ 0 \quad if \ R_{it} > 0. \end{cases} \tag{1}$$

The benchmark Standard Mean-Regression model (MRM) for the intraday asymmetric return-volatility relation has the following form:

$$\Delta VIX_{it} = \alpha + \sum_{L=1}^{3} \beta_{iL} \Delta VIX_{it-L} + \sum_{L=0}^{3} \gamma_{iL} R^{+}_{it-L} + \sum_{L=0}^{3} \delta_{iL} R^{-}_{it-L} + u_t, \qquad (2)$$

where α is the intercept, β_{iL} are the coefficients for the lagged ΔVIX for return horizon i, where $L = 1 \;\; to\, 3$, The terms γ_{iL} are the coefficients for the positive returns and δ_{iL} are the coefficients for the negative returns on the SPX index for frequency i, in both cases lags run from $L = 0 \;\; to\, 3$. The residuals u_t are assumed to be independently and identically distributed (*i.i.d.*) with zero mean. Consequently, the MRM assumes that the effects of both types of returns are constant across different sizes of ΔVIX_{it}. Hence, this traditional approach might neglect important information across quantiles of the ΔVIX distribution if the effect is not constant. The QRM approach is able to monitor the effect across the ΔVIX distribution.[7]

Similar to the MRM, the *q*th QRM for examining the asymmetric return-volatility relation has the form,

$$\Delta VIX_{it} = \alpha^{(q)} + \sum_{L=1}^{3} \beta^{(q)}_{iL} \Delta VIX_{it-L} + \sum_{L=0}^{3} \gamma^{(q)}_{iL} R^{+}_{it-L} + \sum_{L=0}^{3} \delta^{(q)}_{iL} R^{-}_{it-L} + u_t, \qquad (3)$$

where $\alpha^{(q)}$ is the intercept, and $\beta^{(q)}_{i}$:s are the coefficients for the lagged ΔVIX for return horizon i. The parameters $\gamma^{(q)}_{iL}$ are the coefficients for positive returns and $\delta^{(q)}_{iL}$ are the coefficients for negative returns on the SPX index for frequency i, where the lag L runs from 0 to 3 in both cases. The residuals, u_t, are assumed to be independent and derived from the error distribution $\Phi_q(u_t)$ with the q^{th} quantile equal to zero. The main feature of the quantile-

[7] Meligkotsidou et al. (2009) provides a usefull discussion on the advantages of the QRM over the MRM.

regression framework is that the conditional effects of the changes in the explanatory variables, that are measured by $\beta_{iL}^{(q)}, \gamma_{iL}^{(q)}$, and $\delta_{iL}^{(q)}$, are functions of the quantile parameter *q*, $q \in (0,1)$. We estimate the QRM in (3) using the method proposed by Koenker and Bassett (1978).[8]

By applying the Quantile Regression model to our data set, the following empirical hypotheses can be tested:

Hypothesis I. Contemporaneous negative and positive returns are the sole drivers of changes in the implied volatility.

Hypothesis II. Past returns or past changes in implied volatilities are important determinants of changes in current implied volatility.

Hypothesis III. The return-volatility relation is asymmetric, that is, implied volatility reacts differently to negative and positive returns.

Hypothesis IV. The relation between return and volatility is asymmetric and more pronounced in the extreme tails of the ΔVIX distribution.

Hypothesis V. The asymmetric volatility remains the same across frequencies, i.e. 1, 5, 10, 15, 60 minutes, and daily.

5. Results

This section presents the results for the Quantile Regression analysis. Results are first reported for the highest (1-minute) frequency and subsequently it is shown how data aggregation alters the relation between returns and volatility.

[8] See Koenker (2005) for mathematical details on the quantile models and their estimation techniques.

5.1. The Intraday Asymmetric Relation between SPX Returns and VIX Changes

Table 2 reports the results for the MRM in (2) and the QRM in (3) for the intraday (1 minute) asymmetric relation between VIX changes and SPX returns. The model contains 11 covariates and an intercept.[9] In the context of the QRM, for each of the 12 coefficients, 19 quantile-regression coefficient estimates for each q in the set $q = \{0.05, 0.1, ..., 0.9, 0.95\}$ are obtained. The estimates of the benchmark MRM are reported in the 12th row of Table 2.

INSERT TABLE 2 HERE

The contemporaneous positive and negative return covariates, with their 19 Quantile-Regression estimates, are plotted in Figure 1 as a dashed curve with squares. The VIX responses to positive (negative) returns are plotted in the Panel A (Panel B) of Figure 1.[10] In each plot the *x*-axis shows the quantile parameter (or *q*), and the *y*-axis indicates the covariate effect as a percentage. For each covariate, the estimates can be interpreted as the conditional effect of a percentage-point change of the covariate on volatility changes, holding other covariates constant. For the MRM the constant OLS estimates are shown in both plots as solid, straight lines with circles over the different quantiles. Noticeably, QRM estimates of the contemporaneous effect of positive returns in Panel A are more negative than the corresponding OLS estimates for quantile values lower than 0.35. On the other hand, the QRM estimates are less negative for all quantiles larger than 0.35. Furthermore, the variation in the positive return-volatility relation is considerable over the range of quantiles. The

[9] It is interesting to compare intraday QRM and MRM estimates with the daily QRM and MRM estimates as most of the previous studies document the daily asymmetric return-volatility relation. In order to facilitate these comparisons daily QRM and MRM estimates are presented for the same sample in Table 3. A corresponding graphical representation is given in Figure 2.

[10] The conventional 95% confidence level is used for the quantile-regression estimates.

coefficient for contemporaneous positive returns varies from -1.262 at the lower end of the distribution up to -0.248 at the upper end. The lower panel of Table 2 confirms that the variations in the estimated coefficients are statistically significant. Table 2 also shows that the autocorrelation structure of the ΔVIX is robust over the different quantiles. For cross-autocorrelations with SPX returns, however, the significant negative cross-autocorrelations tend to decrease and even become significantly positive with increasing lag-length in the upper part of the distribution. This result is consistent with the view that negative returns increase future volatility and positive returns decrease future volatility. Here again, the OLS estimates are not representative for describing the cross-autocorrelations between positive returns and volatility.

The plot for the contemporaneous effect of negative returns on ΔVIX in Panel B of Figure 1 shows a mirror image of the results for positive returns. QRM estimates of the contemporaneous effect of negative returns in Panel B are less negative than the corresponding OLS estimates for quantile values lower than 0.65. On the other hand, the QRM estimates are more negative for all quantiles larger than 0.65. Furthermore, the variation the negative return-volatility relation is also considerable over the range of quantiles. The coefficient for contemporaneous negative returns varies from -0.228 at the lower end of the distribution down to -1.218 at the upper end. The lower panel of Table 2 also confirms that the variations in the estimated coefficient are statistically significant. For cross-autocorrelations with negative SPX returns, the significant negative cross-autocorrelations tend to decrease and become significantly positive with increasing lag-length in the lower part of the distribution. Here, again, the OLS estimates are not representative for describing the cross-autocorrelations between negative returns and volatility.

The empirical results presented in Table 2 and Figure 1 support Hypothesis IV that the return-volatility relation asymmetric across different sizes of volatility changes are more pronounced in the tails of the conditional distribution. As a consequence, OLS, which determines the relation at the mean, is unable to capture the intraday asymmetric return-volatility conditional relation at the different parts of the ΔVIX distribution.

INSERT FIGURE 1 HERE

The estimated coefficients of covariates R_t^+ and R_t^- presented in columns 3 and 4 of Table 2, respectively, represent the contemporaneous intraday return-volatility relation. If these coefficients are compared with the coefficients of corresponding lagged covariates it becomes apparent that both contemporaneous and even lagged returns are important for determining changes in the VIX. The coefficients are statistically significant at the 1% level across all quantiles. The empirical results on the significant impact of lagged covariates at this high frequency have not been reported in the literature. On the other hand, when comparing the magnitudes of the coefficients it is apparent that even at the 1-minute frequency the contemporaneous returns seem to be more important determinants of the changes in volatility than the lagged covariates. Thus, these results do not fully support Hypothesis I that contemporaneous returns are the sole source of changes implied volatility. This hypothesis would imply that fundamental explanations for the return-volatility relation, such as the leverage and volatility feedback, cannot explain the intraday dynamic return-volatility relation. However, a significant up to three-minute lagged effect cannot fully be regarded as evidence of leverage and volatility feedback as these explanations relate to a longer-term lagged effect between return and volatility, or vice versa.

It is further evident from the absolute differences in the estimated coefficients of covariates R_t^+ and R_t^- in Columns 3 and 4 of Table 2, respectively, that there are asymmetric effects for all quantile-regression estimates. Wald tests are applied to test whether the difference between the coefficients $\gamma_t^{(q)}$ and $\delta_t^{(q)}$ in (3) is statistically significant. The null hypothesis (i.e., the coefficients for contemporaneous positive and negative returns are equal) for the Wald test is rejected for each of the quantile regressions.[11] These results imply that there exists an asymmetric return-volatility relation. As a consequence, these empirical results support Hypothesis III: The return-volatility relation is asymmetric, that is, implied volatility reacts differently to negative and positive returns.

More specifically, each individual row of Table 2 (i.e., for each specific *q*-value) shows that the impact of negative and positive SPX returns on VIX changes are changing and highly asymmetric. The changing nature of the quantile estimates provides an interesting picture of how changes in volatility depend on the contemporaneous and lagged covariates.[12] The absolute value of R_t^- monotonically increases when moving from a lower quantile to an upper quantile; i.e., the marginal effect of the negative returns is larger in upper quantiles. For example the absolute effect is over 5 times higher for $q=0.95$ than for $q=0.05$. This situation is reversed for positive returns.[13] Thus, these asymmetric responses across the quantiles of the conditional distribution of implied volatility changes confirm Hypothesis IV: The relation between return and volatility is asymmetric and more pronounced in the extreme tails of the ΔVIX distribution.

[11] Wald tests results are not reported here to save space.

[12] The OLS regression estimates are close to the $q = 0.5$ (median)-Quantile Regression estimates.

[13] The equality of the coefficients across quantiles is tested using the Wald test. This test tests the equality of quantile slope coefficients of each variable across quantiles, hence testing the null hypothesis that the coefficients of a particular covariate across quantile are the same. The test results are reported in the last panel of Table 2 that rejects the null hypothesis of the equality of the coefficients (the contemporaneous and lagged negative and positive returns) across quantiles.

INSERT TABLE 3 HERE

As most of the previous studies have focused on the asymmetric return-volatility relation at a daily frequency, it is interesting to compare the high frequency (1 minute interval) results of this study with corresponding results at a daily frequency. Empirical results for the daily return horizon are presented in Table 3. There are three important differences in results for the 1-minute and the daily interval. First, the relation between return and volatility is much more pronounced at the daily level than at the 1-minute level. The absolute values of the coefficients of R_t^+ and R_t^- are higher at the daily level across all quantiles. This is also evident from the OLS regressions. Second, the coefficients of lagged covariates (negative and positive returns and lagged volatility) are mostly significant at the intraday 1-minute level. However, at the daily level autocorrelation in ΔVIX almost completely disappears and the cross-autocorrelation with returns is significant only for *q*-values lower than 0.05. This indicates that the effect of negative return shocks on volatility is persistent whereas the effect of positive return shocks is not at the daily level. Third, in comparison to the 1-minute level the R-squared values are higher for the daily level across all *q*-values.

5.2. Comparisons of the Intraday Asymmetric Return-Volatility Relations across Sampling Frequency

In this section, the robustness of the empirical results on the short-term asymmetric return-volatility relations is investigated over different intraday return horizons. The return horizons are 1, 5, 10, 15, 60 minutes, and daily. The results for the QRM estimates of model (3) are reported in Table 4. The results for all six time-intervals are grouped according to each *q*-value. The estimates of the MRM in (2) are reported in the last four rows of Table 4.

Additionally, the two positive and negative returns covariates with their 19 quantile-regression estimates are graphed in Figure 3 for each intraday time-interval, where $q = \{0.05, 0.1, ..., 0.9, 0.95\}$. The ΔVIX_i, (i= *1, 5, 10, 15, 60,* and daily) responses to positive (negative) returns are plotted in the upper (lower) panel of Figure 3. Generally, the graphs confirm the robustness of the results across the different intraday return horizons. The intraday return-volatility relation is highly asymmetric across the conditional distribution of volatility changes. Furthermore, the response to negative returns (in the lower panel) seems to be a monotonically decreasing function of the return horizon. However, for positive returns (in the upper panel) the relation to the return horizon is less clear. Still the 1-minute interval has the lowest absolute values across all q-values.

To analyze the asymmetry, Figure 4 compares ΔVIX responses to negative and positive returns across the six time-intervals for each q-value separately. These results indicate the asymmetry, interpreted as the vertical distance between the two lines of the graphs of Figure 4, is a decreasing function of the length of the return horizon and, hence, is lowest at the daily return horizon for q-values below the median. On the other hand, for q-values from median and above the asymmetry is an increasing function of the return horizon and is most pronounced at the daily return horizon. This conditional asymmetric behavior of the return-volatility relation has not been documented in the literature before. As a consequence of this finding, Hypothesis V, stating that the asymmetry in the return-volatility relation is robust across different intraday return horizons is not supported by our findings. This could be due to option market investors' changing their position (or rebalance their portfolios) slowly.[14]

[14] Bollen and Whaley (2004) empirically show that higher implied volatility is purely induced by net-buying pressure of put options. As risk-averse investors who always hedge their underlying portfolio by taking positions in index puts, and due to limits to arbitrage, market makers want compensation for the induced risk, hence they increase option prices which ultimately increases implied volatility (as both have a monotonic relationship).

INSERT TABLE 4 HERE

INSERT FIGURE 3 HERE

Comparing the estimated contemporaneous coefficients of covariates R_t^+ and R_t^- in columns 4 and 5 of Table 4 with those of the corresponding lagged covariates, it is evident it is that contemporaneous negative and positive returns are the most important factors among the covariates that determine changes in the volatility index. This pattern is robust over the different *q*-values for each of the time-intervals. The contemporaneous covariates are robustly statistically significant at the 1% level. However, the lagged covariates are also significant, especially at shorter return horizons. Thus, these results do not fully support Hypothesis I that contemporaneous returns are the sole source of changes implied volatility. This hypothesis would imply that fundamental explanations for the return-volatility relation, such as the leverage and volatility feedback, cannot explain the intraday dynamic return-volatility relation. However, the longest lag for a significant autocorrelation or cross-autocorrelation appears at a lag of three hours. This time span is also very short for drawing conclusions regarding fundamentally based explanations such as leverage and volatility feedback.

Table 4 further shows that the absolute values of the coefficients for R_t^+ consistently are higher than the corresponding coefficients for R_t^-. These results validate Hypothesis III. Furthermore, according to each row of Table 4 (i.e., each quantile of the estimates), the absolute value of R_t^- monotonically increases when moving from lower to higher *q*-values, i.e., the marginal effect of negative returns is much larger in the upper quantiles. The situation is reversed for positive returns. Again, these results support Hypothesis IV.

6. Conclusions

This paper examines the intraday asymmetric relation between return and volatility by analyzing the relation at different parts of the conditional distribution of volatility changes. The S&P 500 index and the VIX index are sampled for different frequencies, ranging from 1, 5, 10, 15, 60 minutes, to one day, over the period from September 25, 2003 to December 30, 2011. The results indicate that the relation between return and volatility is not robust across different parts of the distribution of volatility changes. These results are consistent for all the different sampling frequencies. The effects of return shocks are more pronounced in the tails of the conditional distribution of volatility changes. Furthermore, the asymmetry between effects of positive and negative return shocks is varying over different quantiles of the distribution of volatility changes. Finally, at the intraday level, our study finds statistically significant autocorrelation and cross-autocorrelation patterns for the implied volatility changes that are not observed at the daily level.

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Table 1

Descriptive statistics of the high frequency and daily.

The intraday (1, 5, 10, 15, 60 minutes) and daily percentage continuously compounded returns of SPX stock market index and the daily percentage changes of VIX stock market volatility index.

	SPX(1M)	SPX(5M)	SPX(10M)	SPX(15M)	SPX(60M)	SPX(Daily)	ΔVIX(1M)	ΔVIX(5M)	ΔVIX(10M)	ΔVIX(15M)	ΔVIX(60M)	ΔVIX(Daily)
Mean	-0.00001	-0.00010	-0.00004	-0.00026	0.00075	0.00957	-0.00016	-0.00076	-0.00145	-0.00258	-0.00403	0.00174
Median	0.00000	0.00000	0.00150	0.00310	0.00700	0.05415	0.00000	0.00000	0.00000	0.00000	-0.02000	-0.07000
Maximum	1.4545	1.9990	3.3709	3.5723	4.94220	10.9572	2.7200	2.9900	3.8800	5.1900	5.52000	16.5400
Minimum	-1.8048	-2.6425	-2.3560	-2.7019	-4.63230	-9.4695	-2.9000	-2.9100	-3.6300	-4.9200	-5.67000	-17.3600
Std. Dev.	0.05084	0.11570	0.16077	0.19526	0.35879	1.34827	0.06154	0.14062	0.20314	0.26084	0.49045	1.92981
Skewness	-0.04459	-0.00217	0.25177	0.28827	0.68892	-0.30873	0.04917	0.14964	0.02619	0.29662	0.37738	0.57154
Kurtosis	36.6009	27.4520	23.9133	25.6492	30.68201	13.5128	134.4004	44.4036	35.6729	38.6380	23.35870	21.5306
JarqueBera	36618950.0	3893843.7	1426045.6	1145371.8	395618.8	9981.1	560000000	11164740.1	3478668.3	2834687.6	213748.8	31022.1
Prob	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ρ_1	0.047***	0.004*	-0.008**	0.01**	-0.005	-0.121***	0.016***	0.103***	0.134***	0.131***	0.065***	-0.148***
ρ_2	0.004***	-0.015***	0.01***	0.017***	0.007	-0.050***	0.038***	0.07***	0.065***	0.05***	-0.042***	-0.078***
ρ_3	-0.002***	-0.001***	0.017***	0.007***	-0.030***	0.026***	0.022***	0.03***	0.031***	0.01***	-0.023***	-0.030***
ADF	-443.74***	-136.19***	-158.01***	-160.11***	-44.64***	-37.106***	-150.45***	-135.33***	-146.15***	-112.69***	-45.41***	-22.32***
No. Obs	778,417	156,300	78,207	53,551	12,360	2,160	778,417	156,300	78,207	53,551	12,360	2,160

Note. This table reports the descriptive statistics for the daily percentage continuously compounded returns on S&P 500 stock index and for the daily percentage changes in the VIX volatility Index both sampled at different frequencies such as 1 minute, 5 minute, 10 minute, 15 minute, 60 minute and daily. The autocorrelation coefficients ρ, the Jarque-Bera and the Augmented Dickey-Fuller (ADF) (an intercept is included in the test equation) test values are reported.

***, ** and * denote rejection of the null hypothesis at the 1% , 5% and 10% significance levels, respectively.

Table 2

Quantile Regression Results: Response variable intraday (1 minute) ΔVIX1.

q	Intercept	R_t^+	R_t^-	R_{t-1}^+	R_{t-2}^+	R_{t-3}^+	R_{t-1}^-	R_{t-2}^-	R_{t-3}^-	ΔVIX_{t-1}	ΔVIX_{t-2}	ΔVIX_{t-3}	$R^2(\%)$
0.05	-0.014***	-1.262***	-0.228***	-0.563***	-0.249***	-0.189***	-0.022***	0.078***	0.101***	-0.129***	-0.015***	0.004	40.2
	(-81.71)	(-146.18)	(-46.26)	(-67.73)	(-28.38)	(-29.72)	(-3.76)	(11.58)	(14.90)	(-30.36)	(-3.81)	(1.42)	
0.10	-0.010***	-1.080***	-0.278***	-0.435***	-0.170***	-0.123***	-0.074***	0.025***	0.047***	-0.122***	-0.013***	0.003	35.5
	(-113.84)	(-222.23)	(-75.88)	(-81.09)	(-40.25)	(-33.41)	(-21.32)	(6.89)	(10.77)	(-37.34)	(-5.42)	(1.40)	
0.15	-0.007***	-0.967***	-0.322***	-0.367***	-0.135***	-0.096***	-0.105***	-0.0001***	0.026***	-0.121***	-0.013***	0.002	32.1
	(-95.74)	(-234.38)	(-101.10)	(-81.21)	(-41.88)	(-35.32)	(-36.73)	(-0.06)	(9.03)	(-44.42)	(-7.19)	(1.10)	
0.20	-0.006***	-0.885***	-0.356***	-0.325***	-0.116***	-0.081***	-0.125***	-0.0121***	0.011***	-0.122***	-0.015***	0.001	29.2
	(-93.56)	(-232.87)	(-127.76)	(-85.40)	(-39.02)	(-32.41)	(-48.86)	(-4.26)	(4.36)	(-50.89)	(-7.60)	(0.95)	
0.25	-0.004***	-0.820***	-0.388***	-0.297***	-0.101***	-0.068***	-0.139***	-0.025***	0.003	-0.122***	-0.015***	0.002	27.5
	(-80.76)	(-248.54)	(-167.57)	(-92.86)	(-41.21)	(-25.26)	(-64.07)	(-9.80)	(1.42)	(-58.34)	(-9.42)	(1.49)	
Median	-0.0002***	-0.580***	-0.566***	-0.196***	-0.058***	-0.027***	-0.205***	-0.063***	-0.031***	-0.119***	-0.017***	0.004***	21.5
	(-5.58)	(-272.64)	(-291.77)	(-80.23)	(-29.15)	(-15.28)	(-96.01)	(-30.84)	(-17.19)	(-63.17)	(-14.54)	(3.51)	
0.75	0.003***	-0.396***	-0.806***	-0.124***	-0.019***	0.007***	-0.302***	-0.110***	-0.070***	-0.115***	-0.016***	0.005***	27.4
	(49.71)	(-152.27)	(-242.74)	(-43.62)	(-9.34)	(3.22)	(-103.93)	(-40.95)	(-29.55)	(-49.37)	(-12.16)	(4.74)	
0.80	0.005***	-0.364***	-0.872***	-0.108***	-0.010***	0.017***	-0.334***	-0.124***	-0.084***	-0.115***	-0.015***	0.005***	29.4
	(63.45)	(-135.94)	(-239.42)	(-35.52)	(-4.20)	(6.66)	(-98.69)	(-38.50)	(-30.73)	(-49.72)	(-10.86)	(3.91)	
0.85	0.007***	-0.332***	-0.953***	-0.087***	0.005***	0.031***	-0.379***	-0.142***	-0.102***	-0.115***	-0.014***	0.007***	32.4
	(74.81)	(-120.36)	(-230.05)	(-23.97)	(2.10)	(10.40)	(-100.73)	(-39.75)	(-31.60)	(-44.94)	(-9.00)	(4.58)	
0.90	0.009***	-0.291***	-1.064***	-0.052***	0.028***	0.055***	-0.444***	-0.178***	-0.125***	-0.113***	-0.013***	0.009***	35.7
	(88.35)	(-83.27)	(-196.23)	(-10.54)	(7.80)	(14.82)	(-86.89)	(-38.25)	(-33.39)	(-34.72)	(-7.19)	(4.79)	
0.95	0.013***	-0.248***	-1.218***	0.011	0.083***	0.114***	-0.580***	-0.262***	-0.198***	-0.119***	-0.014***	0.013***	40.2
	(74.67)	(-43.48)	(-145.39)	(1.34)	(12.95)	(17.80)	(-76.75)	(-39.02)	(-27.05)	(-24.72)	(-4.79)	(4.32)	
OLS	-0.0003*	-0.705***	-0.674***	-0.278***	-0.100***	-0.043***	-0.317***	-0.098***	-0.045***	-0.160***	-0.021***	0.007	37.6
	(-1.77)	(-79.45)	(-60.27)	(-27.15)	(-12.11)	(-5.39)	(-35.09)	(-11.47)	(-5.43)	(-16.23)	(-2.92)	(1.21)	
Quantile Slope Equality Test Results: Only significant results of asymmetry are reported.													
		0.2-0.4***	0.2-0.4***	0.2-0.4***	0.2-0.4***	0.2-0.4***	0.2-0.4***	0.2-0.4***	0.2-0.4***				
		0.4-0.5***	0.4-0.5***	0.4-0.5***	0.4-0.5***	0.4-0.5***	0.4-0.5***	0.4-0.5***	0.4-0.5***			0.5-0.6*	
		0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***	0.5-0.6***			
		0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8***	0.6-0.8*		0.6-0.8*	

Note. The MRM and QRM specification 6 and 7 respectively are estimated for the asymmetric return-volatility relation between changes in the VIX and SPX return. In the context of QRM, the standard errors are obtained using the bootstrap method; therefore, robust t-statistics (in parentheses) are computed for each of the quantile estimates. The MRM specification 6 is estimated with Nawey-West (Nawey and West, 1987) correction for heteroscedasticity and autocorrelation.

***, **, and * denote rejection of the null hypothesis at the 1% , 5% and 10% significance levels, respectively.

Table 3

Quantile Regression Results: Response variable daily ΔVIXD.

q	Intercept	R_t^+	R_t^-	R_{t-1}^+	R_{t-2}^+	R_{t-3}^+	R_{t-1}^-	R_{t-2}^-	R_{t-3}^-	ΔVIX_{t-1}	ΔVIX_{t-2}	ΔVIX_{t-3}	$R^2(\%)$
0.05	-0.187***	-1.496***	-0.810***	-0.338***	-0.216***	-0.310***	0.387***	0.371***	0.329**	-0.059	-0.068	-0.065	57.7
	(-3.24)	(-10.73)	(-9.75)	(-3.17)	(-2.61)	(-3.37)	(2.74)	(3.60)	(1.91)	(-0.86)	(-1.27)	(-1.27)	
0.10	-0.161***	-1.240***	-0.841***	-0.205**	-0.143*	-0.194***	0.298***	0.363***	0.226**	-0.063	-0.023	-0.024	52.8
	(-2.75)	(-16.20)	(-14.59)	(-2.42)	(-1.83)	(-2.88)	(2.65)	(3.55)	(2.57)	(-1.08)	(-0.40)	(-0.54)	
0.15	-0.117***	-1.156***	-0.971***	-0.188***	-0.175***	-0.208***	0.180**	0.324***	0.206***	-0.075	-0.022	-0.0351	49.7
	(-2.60)	(-15.80)	(-20.65)	(-2.91)	(-2.97)	(-3.06)	(2.14)	(4.25)	(2.95)	(-1.585)	(-0.554)	(-0.84)	
0.20	-0.068*	-1.077***	-0.990***	-0.148**	-0.166***	-0.164***	0.165**	0.265***	0.228***	-0.059	-0.027	-0.028	47.4
	(-1.77)	(-24.47)	(-21.72)	(-2.52)	(-2.84)	(-2.80)	(2.34)	(4.17)	(4.04)	(-1.32)	(-0.80)	(-0.74)	
0.25	-0.056	-1.057***	-1.00***	-0.129***	-0.150***	-0.125**	0.136**	0.224***	0.181***	-0.048	-0.043	-0.023	45.7
	(-1.35)	(-18.85)	(-22.04)	(-2.70)	(-2.68)	(-2.57)	(2.09)	(4.43)	(3.70)	(-1.24)	(-1.43)	(-0.82)	
Median	0.019	-0.822***	-1.268***	-0.104**	0.016	0.016	0.100**	0.111*	0.125***	-0.046	-0.026	0.018	43.9
	(0.57)	(-21.08)	(-31.90)	(-2.22)	(0.38)	(0.38)	(2.00)	(1.78)	(2.48)	(-1.39)	(-0.96)	(0.67)	
0.75	0.115**	-0.635***	-1.538***	-0.025	0.025	0.03	0.007	0.036	0.057	-0.059*	-0.004	0.009	50.8
	(2.57)	(-14.53)	(-27.85)	(-0.47)	(0.46)	(0.51)	(0.12)	(0.51)	(0.96)	(-1.70)	(-0.10)	(0.30)	
0.80	0.143***	-0.617***	-1.602***	0.004	0.092	0.083	0.001	0.036	0.076	-0.046	0.011	0.033	53.2
	(2.77)	(-9.69)	(-26.43)	(0.075)	(1.53)	(1.55)	(0.02)	(0.42)	(1.14)	(-1.39)	(0.26)	(0.98)	
0.85	0.158***	-0.489***	-1.670***	0.050	0.147**	0.098*	0.020	-0.039	0.072	-0.007	0.026	0.038	56.5
	(2.91)	(-8.36)	(-24.03)	(0.77)	(2.40)	(1.80)	(0.32)	(-0.49)	(1.05)	(-0.20)	(0.58)	(1.28)	
0.90	0.274***	-0.454***	-1.935***	0.056	0.148*	0.078	0.061	-0.041	0.042	0.032	0.011	0.036	60.7
	(4.06)	(-7.87)	(-21.27)	(0.72)	(1.79)	(1.22)	(0.68)	(-0.45)	(0.48)	(0.69)	(0.20)	(0.97)	
0.95	0.373***	-0.430***	-2.171***	0.089	0.266***	0.083	0.008	0.043	-0.124	0.061	0.075	-0.024	67.0
	(4.61)	(-5.14)	(-21.37)	(0.84)	(2.59)	(1.07)	(0.06)	(0.34)	(-1.12)	(1.09)	(1.15)	(-0.42)	
OLS	0.040	-0.966***	-1.382***	-0.123**	-0.053	-0.118*	-0.006	0.054	0.135**	-0.104**	-0.036	-0.049	72.8
	(1.08)	(-11.64)	(-22.58)	(-2.06)	(-0.90)	(-1.75)	(-0.06)	(0.46)	(2.01)	(-2.27)	(-0.61)	(-1.15)	
Quantile Slope Equality Test Results: Only significant results of asymmetry are reported.													
		0.2-0.4***	0.2-0.4***		0.2-0.4***	0.2-0.4***		0.2-0.4**	0.2-0.4*				
		0.4-0.5**	0.4-0.5***										
		0.5-0.6***	0.5-0.6***										
		0.6-0.8**	0.6-0.8***										

Note. The MRM and QRM specification 6 and 7 respectively are estimated for the asymmetric return-volatility relation between changes in the VIX and SPX return. In the context of QRM, the standard errors are obtained using the bootstrap method; therefore, robust t-statistics (in parentheses) are computed for each of the quantile estimates. The MRM specification 6 is estimated with Nawey-West (Nawey and West, 1987) correction for heteroscedasticity and autocorrelation.

***, **, and * denote rejection of the null hypothesis at the 1% , 5% and 10% significance levels, respectively.

Table 4

Quantile Regressions Results: response comparision across time intervals (1,5,10,15,60 minutes and day) at each quantile.

Variable	q	Intercept	R_t^+	R_t^-	R_{t-1}^+	R_{t-2}^+	R_{t-3}^+	R_{t-1}^-	R_{t-2}^-	R_{t-3}^-	ΔVIX_{t-1}	ΔVIX_{t-2}	ΔVIX_{t-3}	R^2(%)
ΔVIX1	0.05	-0.014***	-1.262***	-0.228***	-0.563***	-0.249***	-0.189***	-0.022***	0.078***	0.101***	-0.129***	-0.015***	0.004	40.2
ΔVIX5	0.05	-0.023***	-1.353***	-0.455***	-0.522***	-0.296***	-0.222***	0.010	0.116***	0.150***	-0.051***	0.022**	0.014**	49.3
ΔVIX10	0.05	-0.034***	-1.415***	-0.569***	-0.464***	-0.327***	-0.261***	-0.008	0.141***	0.215***	-0.011	-0.003	0.013	51.6
ΔVIX15	0.05	-0.046***	-1.441***	-0.658***	-0.469***	-0.296***	-0.237***	0.048	0.240***	0.235***	0.015	0.002	0.005	51.4
ΔVIX60	0.05	-0.105***	-1.587***	-0.700***	-0.289***	-0.152***	-0.157***	0.108*	0.463***	0.298***	-0.026	0.010	0.000	53.1
ΔVIXD	0.05	-0.187***	-1.496***	-0.810***	-0.338***	-0.216***	-0.310***	0.387***	0.371***	0.329*	-0.059	-0.068	-0.065	57.7
ΔVIX1	0.10	-0.010***	-1.080***	-0.278***	-0.435***	-0.170***	-0.123***	-0.074***	0.025***	0.047***	-0.122***	-0.013***	0.003	35.5
ΔVIX5	0.10	-0.016***	-1.212***	-0.503***	-0.392***	-0.199***	-0.149***	-0.045***	0.054***	0.098***	-0.041***	0.018***	0.020***	45.5
ΔVIX10	0.10	-0.023***	-1.261***	-0.607***	-0.380***	-0.229***	-0.157***	-0.039***	0.067***	0.135***	-0.015*	-0.002	0.013	47.8
ΔVIX15	0.10	-0.030***	-1.319***	-0.664***	-0.390***	-0.208***	-0.131***	-0.021	0.141***	0.167***	0.001	0.014	0.018**	47.8
ΔVIX60	0.10	-0.074***	-1.420***	-0.752***	-0.232***	-0.094***	-0.139***	0.091**	0.223***	0.225***	-0.011	-0.013	-0.003	49.3
ΔVIXD	0.10	-0.161***	-1.240***	-0.841***	-0.205**	-0.143*	-0.194***	0.298***	0.363***	0.226**	-0.063	-0.023	-0.024	52.8
ΔVIX1	0.15	-0.007***	-0.967***	-0.322***	-0.367***	-0.135***	-0.096***	-0.105***	-0.0001	0.026***	-0.121***	-0.013***	0.002	32.1
ΔVIX5	0.15	-0.012***	-1.117***	-0.542***	-0.321***	-0.156***	-0.113***	-0.069***	0.021***	0.067***	-0.042***	0.015***	0.021***	42.9
ΔVIX10	0.15	-0.017***	-1.167***	-0.643***	-0.326***	-0.178***	-0.126***	-0.055***	0.036**	0.090***	-0.015**	0.002	0.009	45.3
ΔVIX15	0.15	-0.022***	-1.234***	-0.695***	-0.333***	-0.169***	-0.110***	-0.056***	0.095***	0.121***	-0.004	0.009	0.011	45.4
ΔVIX60	0.15	-0.058***	-1.350***	-0.815***	-0.195***	-0.072***	-0.070***	0.059*	0.185***	0.184***	-0.010	-0.015	0.001	46.6
ΔVIXD	0.15	-0.117***	-1.156***	-0.971***	-0.188***	-0.175***	-0.208***	0.180**	0.324***	0.206***	-0.075	-0.022	-0.0351	49.7
ΔVIX1	0.20	-0.006***	-0.885***	-0.356***	-0.325***	-0.116***	-0.081***	-0.125***	-0.0121***	0.011***	-0.122***	-0.015***	0.001	29.2
ΔVIX5	0.20	-0.009***	-1.038***	-0.574***	-0.278***	-0.127***	-0.087***	-0.083***	0.006	0.047***	-0.041***	0.013***	0.021***	40.9
ΔVIX10	0.20	-0.013***	-1.105***	-0.665***	-0.281***	-0.138***	-0.088***	-0.074***	0.014*	0.067***	-0.018***	0.006	0.012**	43.5
ΔVIX15	0.20	-0.017***	-1.158***	-0.718***	-0.276***	-0.129***	-0.081***	-0.062***	0.069***	0.096***	-0.003	0.013**	0.015**	43.6
ΔVIX60	0.20	-0.046***	-1.275***	-0.847***	-0.156***	-0.069***	-0.052***	0.022	0.157***	0.147***	-0.008	-0.012	0.002	44.5
ΔVIXD	0.20	-0.068*	-1.077***	-0.990***	-0.148**	-0.166***	-0.164***	0.165**	0.265***	0.228***	-0.059	-0.027	-0.028	47.4
ΔVIX1	0.50	-0.0002***	-0.580***	-0.566***	-0.196***	-0.058***	-0.027***	-0.205***	-0.063***	-0.031***	-0.119***	-0.017***	0.004***	21.5
ΔVIX5	0.50	-0.001***	-0.769***	-0.757***	-0.150***	-0.048***	-0.015***	-0.161***	-0.052***	-0.009**	-0.040***	0.011***	0.022***	35.7
ΔVIX10	0.50	-0.002***	-0.825***	-0.836***	-0.141***	-0.040***	-0.006	-0.155***	-0.023***	-0.005	-0.015***	0.019***	0.017***	39.1
ΔVIX15	0.50	-0.002***	-0.878***	-0.884***	-0.130***	-0.026***	0.000	-0.143***	-0.012	0.015*	0.005	0.020***	0.017***	39.5
ΔVIX60	0.50	-0.012***	-0.960***	-1.035***	-0.036***	0.042***	0.027***	-0.057*	0.051***	0.073***	0.013	0.012	0.013	40.6
ΔVIXD	0.50	0.019	-0.822***	-1.268***	-0.104**	0.016	0.016	0.100**	0.111*	0.125**	-0.046	-0.026	0.018	43.9
ΔVIX1	0.80	0.005***	-0.364***	-0.872***	-0.108***	-0.010***	0.017***	-0.334***	-0.124***	-0.084***	-0.115***	-0.015***	0.005***	29.4
ΔVIX5	0.80	0.008***	-0.580***	-1.047***	-0.065***	0.006	0.033***	-0.296***	-0.122***	-0.064***	-0.035***	0.018***	0.029***	41.2
ΔVIX10	0.80	0.010***	-0.643***	-1.130***	-0.043***	0.033***	0.044***	-0.291***	-0.093***	-0.055***	0.006	0.028***	0.026***	41.1
ΔVIX15	0.80	0.011***	-0.683***	-1.201***	-0.023*	0.058***	0.053***	-0.288***	-0.093***	-0.040***	0.021**	0.030***	0.021***	44.8
ΔVIX60	0.80	0.024***	-0.733***	-1.422***	0.075***	0.126***	0.095***	-0.192***	0.009	0.017	0.031*	0.031*	0.020	47.2
ΔVIXD	0.80	0.143***	-0.617***	-1.602***	0.004	0.092	0.083	0.001	0.036	0.076	-0.046	0.011	0.033	53.2

Table 4

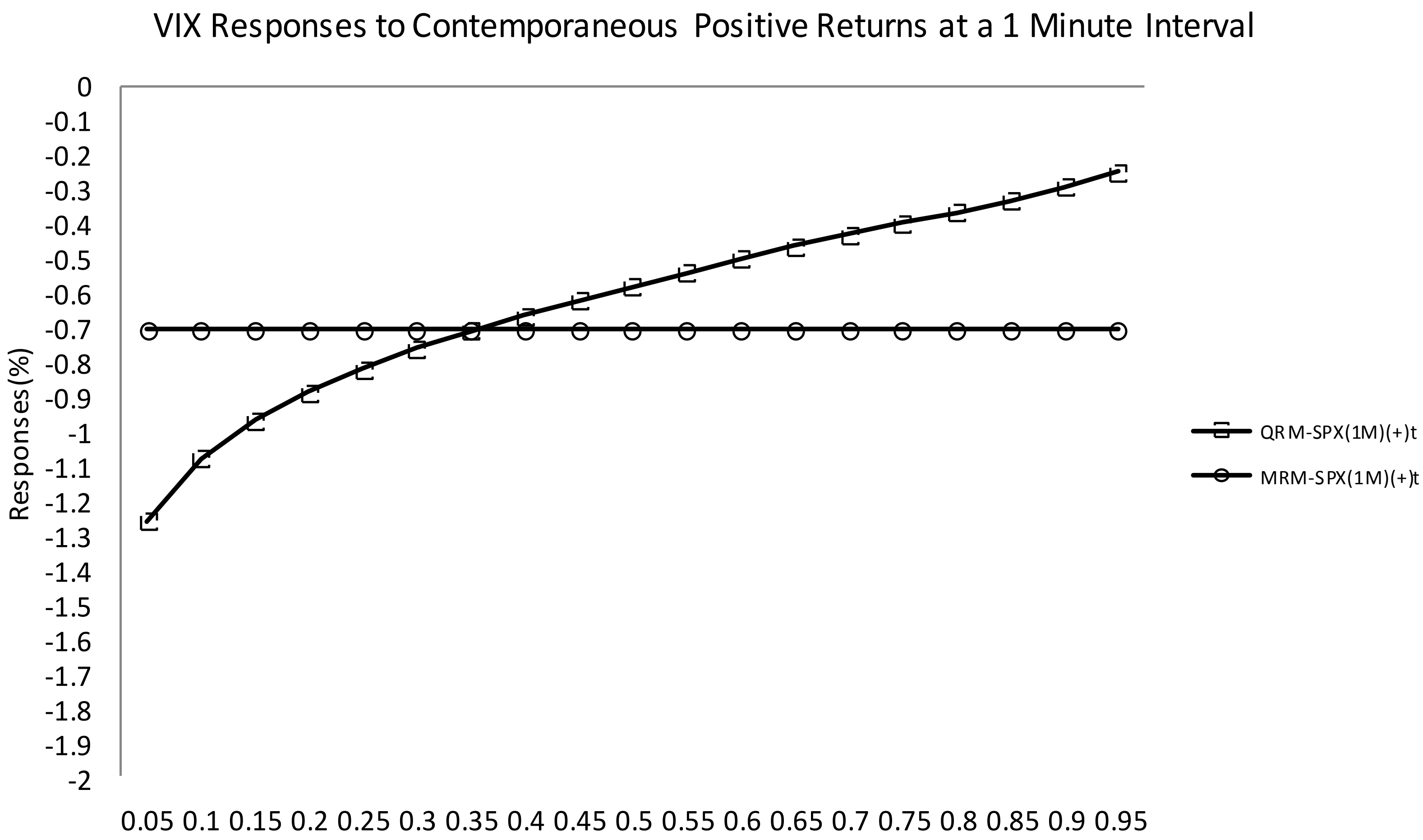
Continued.

Variable	q	Intercept	R_t^+	R_t^-	R_{t-1}^+	R_{t-2}^+	R_{t-3}^+	R_{t-1}^-	R_{t-2}^-	R_{t-3}^-	ΔVIX_{t-1}	ΔVIX_{t-2}	ΔVIX_{t-3}	$R^2(\%)$
ΔVIX1	0.85	0.007***	-0.332***	-0.953***	-0.087***	0.005***	0.031**	-0.379***	-0.142***	-0.102***	-0.115***	-0.014***	0.007***	32.4
ΔVIX5	0.85	0.010***	-0.544***	-1.132***	-0.041***	0.027***	0.049***	-0.351***	-0.151***	-0.079***	-0.034***	0.026***	0.032***	43.3
ΔVIX10	0.85	0.013***	-0.611***	-1.208*	-0.020*	0.056***	0.064***	-0.343***	-0.119***	-0.071***	0.009	0.032***	0.029***	46.1
ΔVIX15	0.85	0.015***	-0.657***	-1.284***	0.004	0.094***	0.079***	-0.328***	-0.107***	-0.070***	0.023***	0.040***	0.020***	46.7
ΔVIX60	0.85	0.034***	-0.715***	-1.504***	0.142***	0.167***	0.133***	-0.206***	-0.033	0.010	0.053***	0.029	0.030*	49.5
ΔVIXD	0.85	0.158***	-0.489***	-1.670***	0.050	0.147**	0.098*	0.020	-0.039	0.072	-0.007	0.026	0.038	56.5
ΔVIX1	0.90	0.009***	-0.291***	-1.064***	-0.052***	0.028***	0.055***	-0.444***	-0.178***	-0.125***	-0.113***	-0.013***	0.009***	35.7
ΔVIX5	0.90	0.013***	-0.499***	-1.227***	-0.006	0.060***	0.082***	-0.421***	-0.201***	-0.115***	-0.032***	0.031***	0.034***	46.1
ΔVIX10	0.90	0.018***	-0.571***	-1.306***	0.012	0.096***	0.103***	-0.404***	-0.173***	-0.096***	0.010	0.030***	0.034***	48.6
ΔVIX15	0.90	0.022***	-0.620***	-1.383***	0.032*	0.146***	0.108***	-0.403***	-0.131***	-0.115***	0.020*	0.045***	0.015	49.2
ΔVIX60	0.90	0.049***	-0.673***	-1.641***	0.175***	0.232***	0.193***	-0.245***	-0.061	-0.017	0.061***	0.039*	0.041*	52.5
ΔVIXD	0.90	0.274***	-0.454***	-1.935***	0.056	0.148*	0.078	0.061	-0.041	0.042	0.032	0.011	0.036	60.7
ΔVIX1	0.95	0.013***	-0.248***	-1.218***	0.011	0.083***	0.114***	-0.580***	-0.262***	-0.198***	-0.119***	-0.014***	0.013***	40.2
ΔVIX5	0.95	0.019***	-0.428***	-1.379***	0.052***	0.123***	0.138***	-0.549***	-0.288***	-0.193***	-0.040***	0.039***	0.036***	49.8
ΔVIX10	0.95	0.027***	-0.512***	-1.455***	0.092***	0.159***	0.166***	-0.563***	-0.244***	-0.164***	0.015	0.033***	0.034***	52.0
ΔVIX15	0.95	0.034***	-0.548***	-1.582***	0.097***	0.235***	0.168***	-0.574***	-0.213***	-0.158***	0.007	0.057***	0.014	52.7
ΔVIX60	0.95	0.060***	-0.536***	-1.843***	0.259***	0.287***	0.234***	-0.487***	-0.153**	-0.148**	0.077**	0.025	0.025	57.0
ΔVIXD	0.95	0.373***	-0.430***	-2.171***	0.089	0.266***	0.083	0.008	0.043	-0.124	0.061	0.075	-0.024	67.0
ΔVIX1	OLS	-0.0003*	-0.705***	-0.674***	-0.278***	-0.100***	-0.043***	-0.317***	-0.098***	-0.045***	-0.160***	-0.021***	0.007	37.6
ΔVIX5	OLS	-0.001	-0.876***	-0.901***	-0.224***	-0.064***	-0.041***	-0.223***	-0.070***	-0.009	-0.046***	0.039***	0.021**	56.5
ΔVIX10	OLS	-0.001	-0.929***	-0.986***	-0.189***	-0.092***	-0.052***	-0.211***	-0.060***	-0.001	0.007	0.003	0.007	60.6
ΔVIX15	OLS	-0.004**	-0.994***	-1.067***	-0.190***	-0.025	-0.025	-0.203***	-0.012	0.022	0.008	0.032	0.002	62.4
ΔVIX60	OLS	-0.027***	-1.004***	-1.176***	-0.048	0.042	0.032	-0.092**	0.013	0.039	0.022	-0.025	0.009	64.2
ΔVIXD	OLS	0.040	-0.966***	-1.382***	-0.123**	-0.053	-0.118*	-0.006	0.054	0.135**	-0.104**	-0.036	-0.049	72.8

Note. The MRM and QRM specification 6 and 7 respectively are estimated for the asymmetric return-volatility relation for each of the volatility index separately. In the context of QRM, the standard errors are obtained using the bootstrap method; therefore, robust t-statistics (in parentheses) are computed for each of the quantile estimates. The MRM specification 6 is estimated with Nawey-West (Nawey and West, 1987) correction for heteroscedasticity and autocorrelation. ***, **, and * denote rejection of the null hypothesis at the 1% , 5% and 10% significance levels, respectively.

Figure 1. Asymmetric Relation between changes in VIX and returns on the S&P500 at a 1 minute frequency

Panel A. Positive Returns



Panel B. Negative Returns

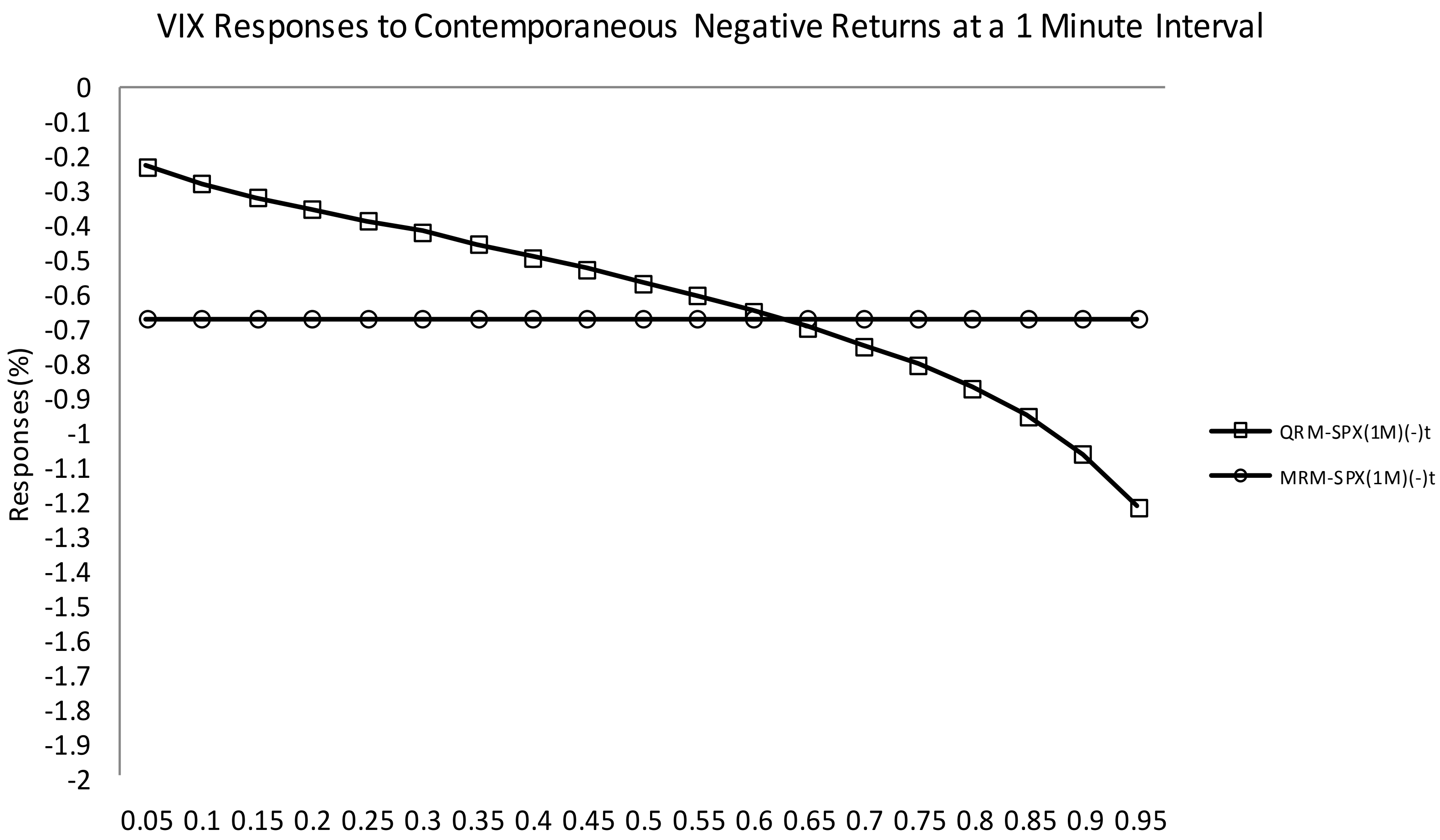
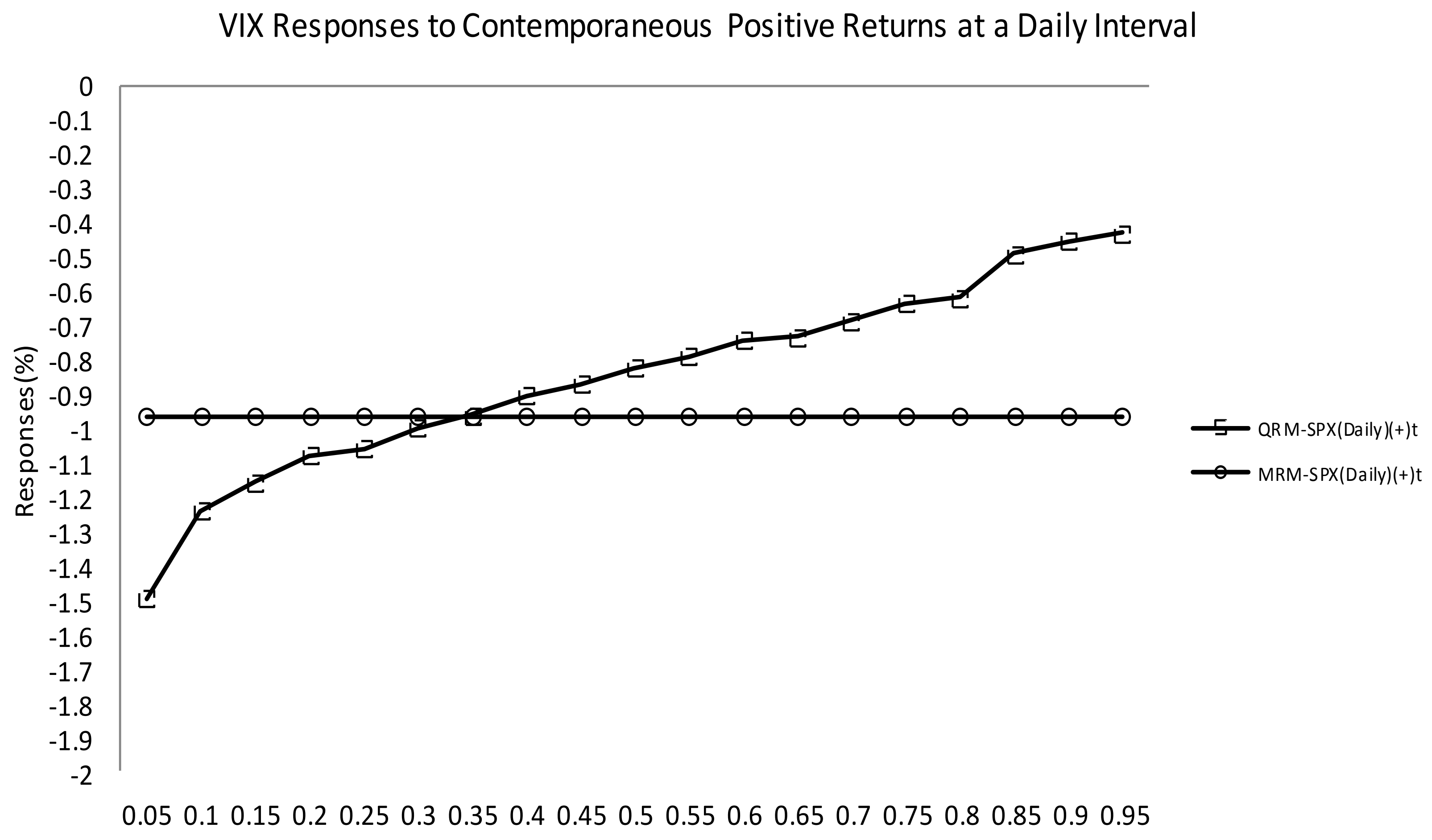
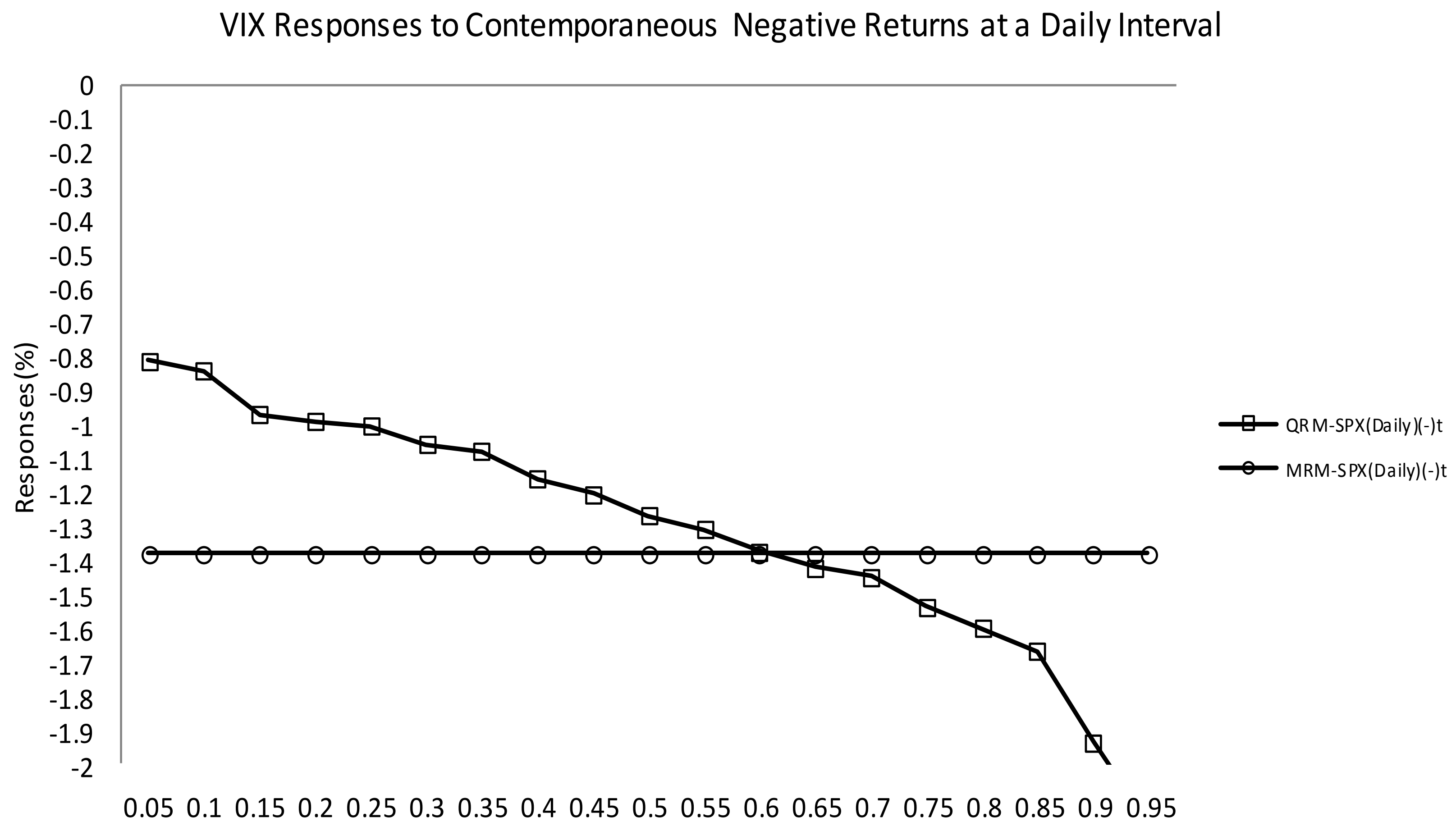
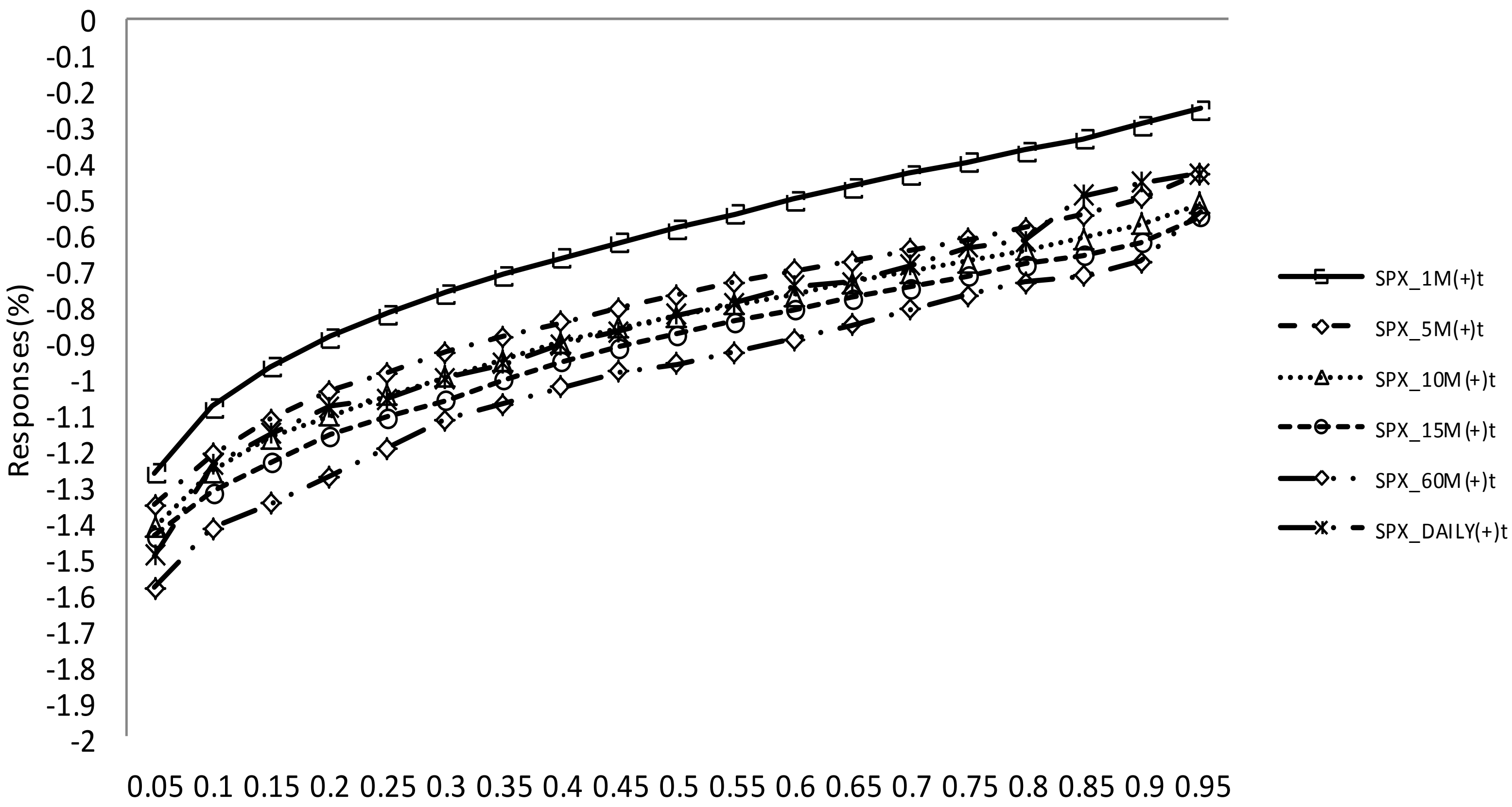


Figure 2: Asymmetric Relation between changes in VIX and returns on the S&P500 at a daily frequency




VIX Responses to Contemporaneous Negative Returns at a Daily Interval
Responses(%)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
-1.7
-1.8
-1.9
-2
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95
QRM-SPX(Daily)(-)t
MRM-SPX(Daily)(-)t

VIX Responses to Contemporaneous Positive Returns (1, 5, 10, 15, 60 Minutes, and Daily)



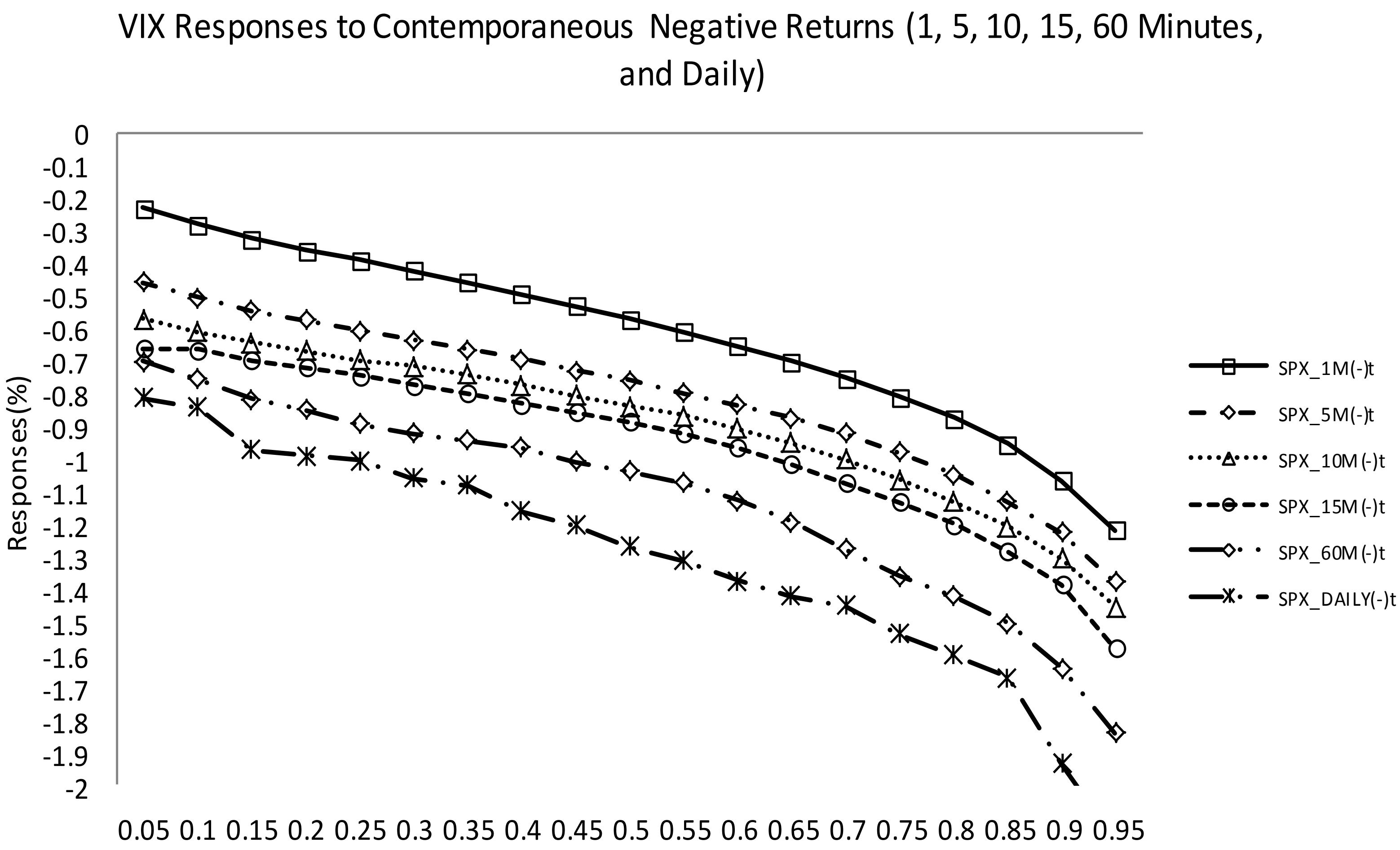
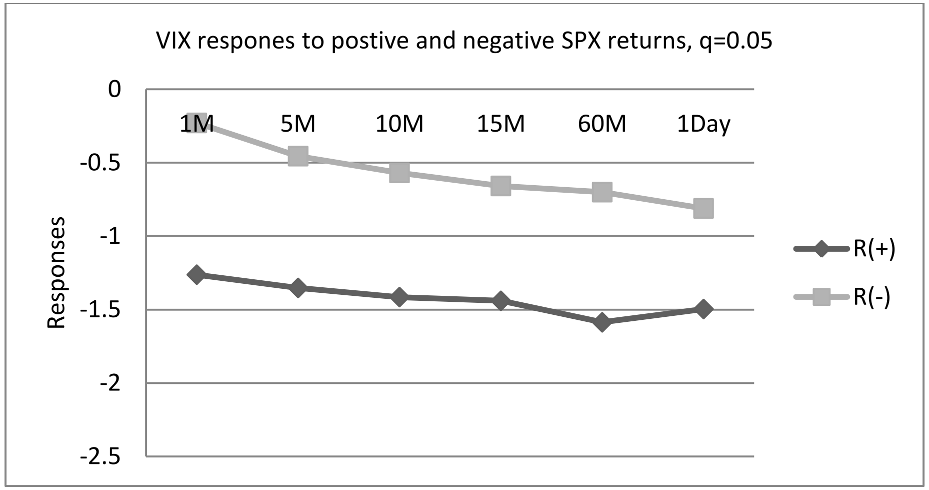
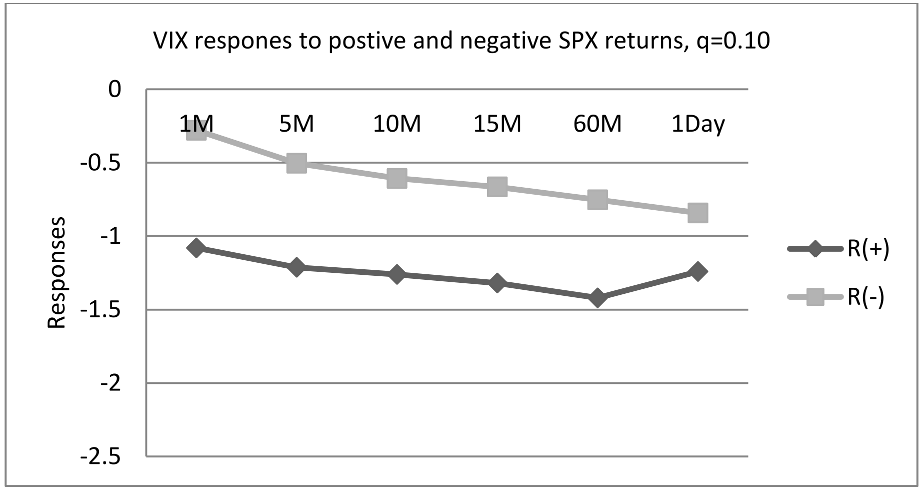
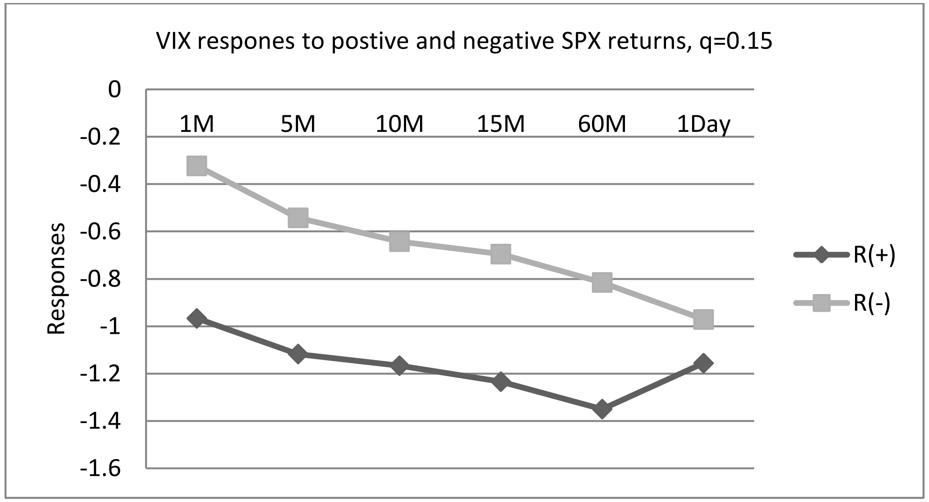
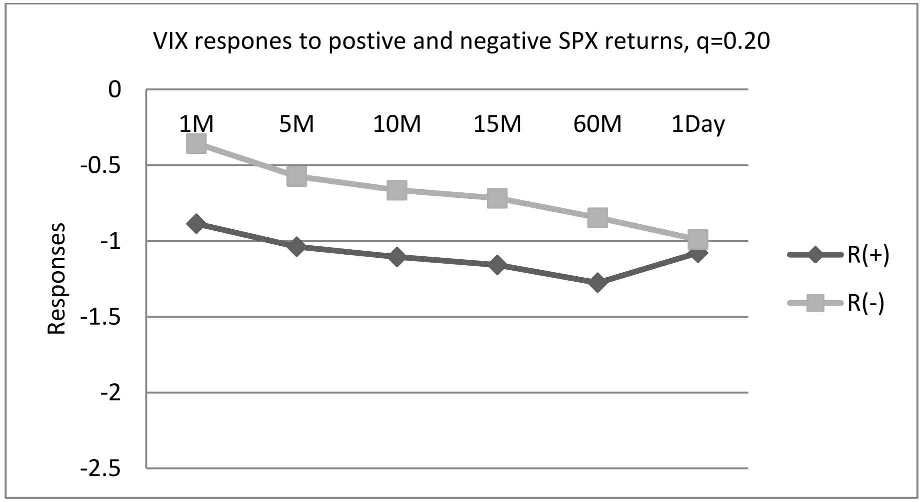


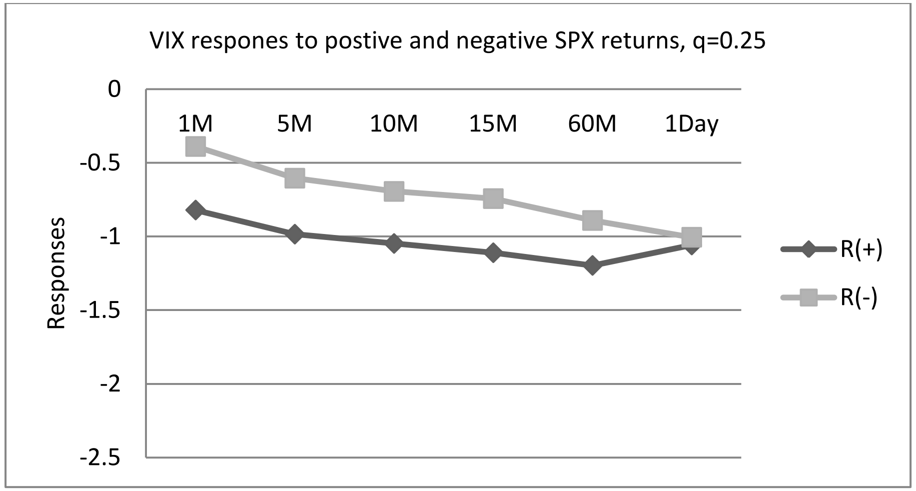
Figure 3: QRM estimates across quantiles: Response variable ΔVIX at 1, 5, 10, 15, 60 minutes and daily frequency.

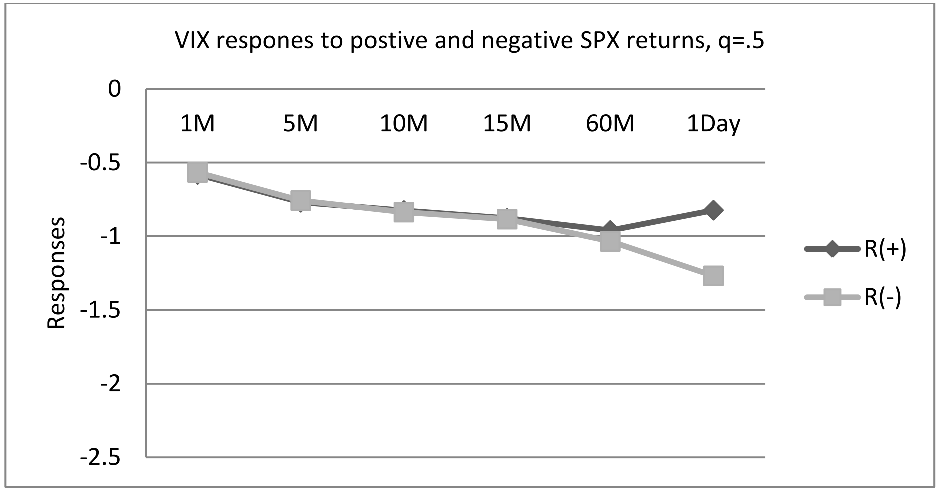

VIX respones to postive and negative SPX returns, q=0.05
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)

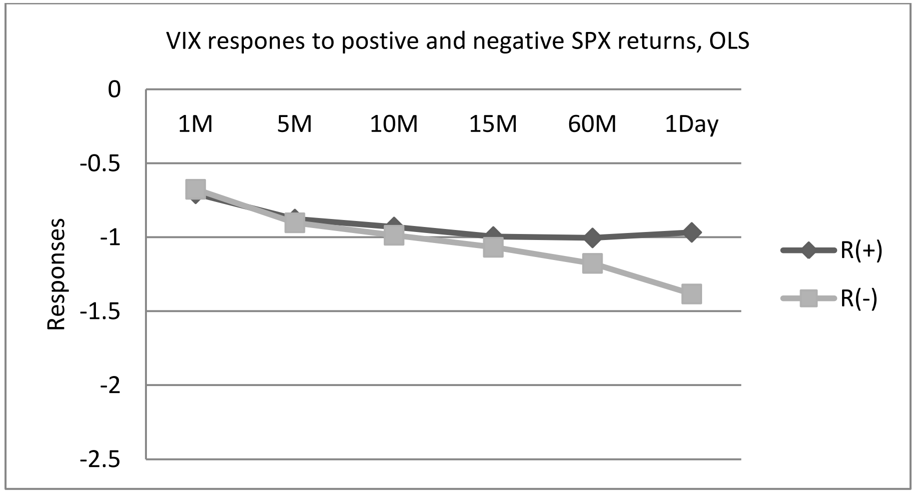

VIX respones to postive and negative SPX returns, q=0.10
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)

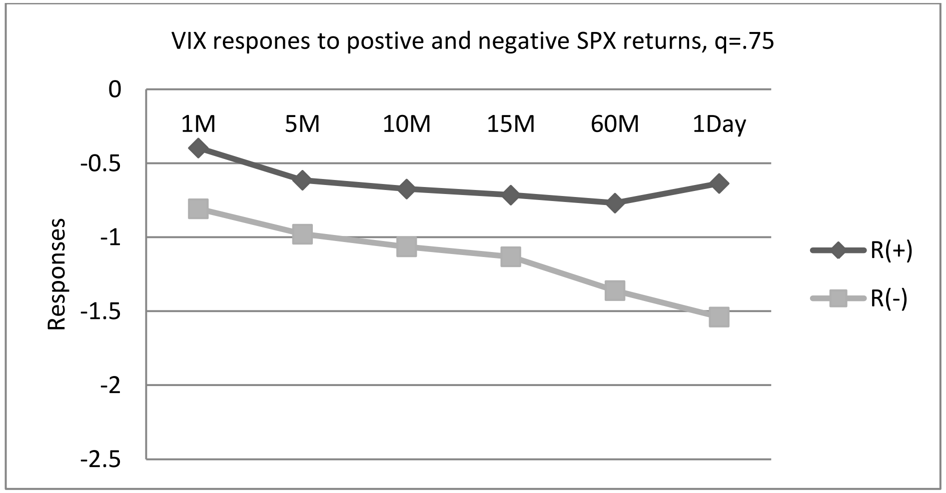

VIX respones to postive and negative SPX returns, q=0.15
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)


VIX respones to postive and negative SPX returns, q=0.20
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)


VIX respones to postive and negative SPX returns, q=0.25
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)


VIX respones to postive and negative SPX returns, q=.5
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)


VIX respones to postive and negative SPX returns, OLS
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)


VIX respones to postive and negative SPX returns, q=.75
0
-0.5
-1
-1.5
-2
-2.5
Responses
1M
5M
10M
15M
60M
1Day
R(+)
R(-)

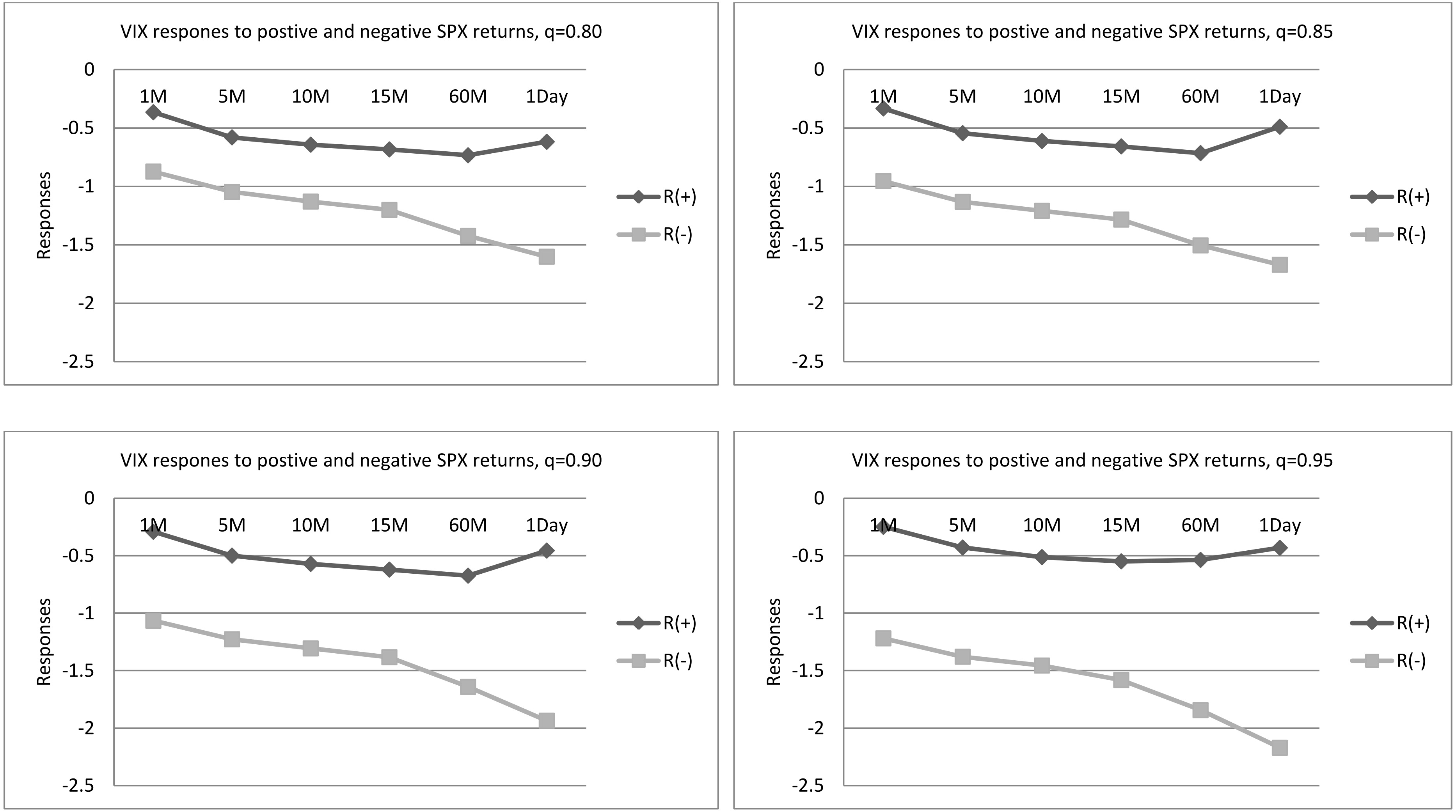


Figure 4: QRM estimates: VIX response comparision across time intervals (1,5,10,15,60 minutes and day) at each quantile