TECHNICAL ANALYSIS WITH UNCERTAIN PREDICTIVE POWER: THE EFFECTS ON PORTFOLIO CHOICE

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Abstract. Deviating from a conventional statistical testing approach, we analyze the economic relevance of technical analysis. Specifically, we assess how uncertainty in predictive power of technical analysis affects investors' portfolio choice. Calibrating our model with CRSP stock index data, we find that, accounting for such uncertainty, investors allocate substantially less to stocks and time the market less aggressively. These effects are stronger under longer investment horizons. Furthermore, the utility loss of ignoring this uncertainty is sizable and increases with horizon at an increasing rate.

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Technical Analysis with Uncertain Predictive Power: The Effects on Portfolio Choice

1 Introduction

When an investor receives a "buy" signal from a technical trading rule, how much should he trust this signal and adjust his allocations to stocks? The literature provides limited guidance in answering this question because it is commonly assumed that investors use an all-or-nothing strategy, namely they allocate 100% of wealth to stocks whenever a buy signal is observed but nothing otherwise. This assumption, however, is unrealistic because it overlooks at least two important issues.

First, investors do not necessarily have strong faith in technical analysis. After all, empirical findings of its usefulness are mixed and inconclusive. Early studies, for example, Fama and Blume (1966) and Jensen and Benington (1970), question the profitability of technical analysis in stock markets. Although later studies increasingly provide evidence defending the usefulness of technical analysis, for example, Brock, Lakonishok, and LeBaron (1992)[1] and Lo, Mamaysky, and Wang (2000)[2], there is still counter evidence such as the works by Allen and Karjalainen (1999), Ready (2002), and Bajgrowicz and Scaillet (2012).

Second and more importantly, investors' allocations to stocks should be optimally chosen in a utility maximization framework: an investor's optimal allocation may depend on, for example, his degree of risk aversion, wealth level, prior belief and current assessment of the predictive power of the trading

[1] Sullivan, Timmermann, and White (1999) show that their results are robust even after adjustment is made for data-snooping. However, the results are weakened in out-of-sample testing.
[2] More recent studies include Han, Yang, and Zhou (2013) and Neely et al. (2014).

rule. When stock returns are possibly predictable, the optimal strategy may also depend on the investment horizon.

Failing to account for these two issues, the *ad hoc* all-or-nothing assumption is unlikely to represent a sensible use of technical analysis in real world. In this paper, we study portfolio choice with technical analysis in a utility maximization framework, with an emphasis on how uncertainty in predictive power of technical analysis affects portfolio choice. We consider an investor who uses a linear prediction model to forecast stock returns with a binary "technical signal" (buy or sell) as the predictive variable. In this context, uncertainty in predictive power is equivalent to an unknown slope parameter. We assume that the investor follows a simple Bayesian approach to use the sample evidence to update prior beliefs about the model parameters[3] and incorporates the precision of the estimators into his expected utility. He then chooses how to allocate his wealth optimally, between a risk-free asset and a risky stock, by maximizing expected utility. This optimization problem is typically called "portfolio choice with parameter uncertainty". Also, the uncertainty in model parameters is commonly called "estimation risk".

We focus on the moving average crossover rule[4] as an example of technical analysis. This rule uses two moving averages[5] to generate technical signals: a "buy" signal is generated when the shorter average is above the longer average and a "sell" signal *vice versa*. The principle of this rule is to follow a market and detect if a new upward trend has begun or that an old trend has ended or reversed.

Our perspective differs from most previous studies in that we view technical analysis as a source of information that is potentially useful to forecast

[3] We also assume that the intercept parameter is unknown.

[4] The moving average crossover rule is one of the most widely used technical trading rules by practitioners and is commonly studied in the literature.

[5] For discretely observed data, an n-period moving average is an average of the previous n data.

returns and there is no reason why investors have to commit to any trading rule. Our approach offers a number of useful insights about technical analysis. First, instead of testing whether the slope parameter is statistically significant, we can incorporate the prediction model in a portfolio choice problem to study economic relevance of technical analysis—how much would a utility maximizing investor trust a "buy" signal and adjust his allocations to stocks? Second, we derive an approximated solution for the optimal allocation to stocks, which is explicit up to the investors' moment estimates and investment horizon. Third, we show quantitatively how the uncertainty in predictive power of technical analysis affects market timing, while such uncertainty concern is omitted from most previous studies. Fourth, we show that investors with shorter investment horizons do not bear much utility loss even if they ignore such uncertainty. By contrast, such utility loss is sizable for investors with longer horizons. This implication helps to explain why short-term speculators seem to favor technical trading even though the usefulness of technical analysis is often questioned.

Although, to the best of our knowledge, there is no direct study addressing our topic, we do get some useful insights from the current state of literature. The effect of uncertain stock return predictability on portfolio choice is studied by, for example, Stambaugh (1999), Barberis (2000), and Xia (2001). They show that when a predictive variable has uncertain predictive power, a sensible investor would choose his portfolio allocation according to his current assessment of the conditional mean and the variance of his estimate for the slope parameter. Further, the portfolio allocation may be sensitive to the investment horizon. The authors also show that, in general, ignoring estimation risk results in a substantial opportunity cost to the investor. We, however, consider technical signals rather than the dividend yield which is commonly studied in the literature. We shall show that it is slightly more complicated to account for estimation risk when the predictive variable is a technical signal. The reason is that the law of motion of the technical signal (as a function of past stock prices) is determined by the prediction model itself but not an external model. Therefore, the same source of estimation risk, namely having unknown parameters in the prediction model, concurrently affects the forecasts of stock returns and technical signals. We use a relatively simple, but otherwise standard, Bayesian approach to account for estimation risk—selecting the most appropriate econometric approach is beyond the scope of this paper.[6] Zhu and Zhou (2009) develop a model to justify why technical analysis, specifically the moving average trading rule, can improve an investor's opportunity set. Instead of constructing unconditional forecasts of stock returns, we assume the investor continuously infers information from technical signals to make conditional forecasts[7]. Based on a similar conceptual framework to that of Zhu and Zhou (2009), Zhou, Zhu, and Qiang (2012) test their one-period moving average strategies and conclude that the proposed strategies outperform other strategies that completely disregard the usefulness of moving averages. We instead consider a long-horizon investor and study the effects of estimation risk on portfolio choice and investor welfare, which are understudied in the context of technical analysis.

Beyond academic studies, technical analysis has been widely used by sophisticated practitioners and is considered an important tool for stock investment. Menkhoff (2010) carries out a survey of fund managers[8] in 2003/2004. The respondents are 692 fund managers in five countries: United

[6] There are alternative approaches to address estimation risk, for example, out-of-sample testing, correction factors, and randomization methods such as bootstrapping and Monte Carlo simulation.

[7] We use "conditional forecast" to mean any approach that goes beyond using a single overall average, or unconditional mean, for forecasting.

[8] The author justifies his sample choice by two reasons: first, fund managers have evolved as the most important group in modern financial markets; second, fund managers are highly qualified market participants compared to typical individual investors.

States, Switzerland, Germany, Italy, and Thailand and the majority of the surveyed fund managers specialize in stock markets. The survey indicates that 87% of respondents use technical analysis and a major group (18%) prefers technical analysis to other tools for investment decisions, including learning fundamental information from the market. This survey shows that a substantial proportion of technical analysis users has high reasoning power. Therefore, it is relevant to consider a more realistic use of technical analysis through a portfolio choice model and examine its economic relevance by studying the model implications.

The paper is organized as follows. The next section discusses some issues about technical analysis and portfolio choice with a prediction model. Section II introduces our model. Section III describes the data and calibrates the model. Section IV studies the model implications. Section V summarizes and concludes the paper.

2 Background

There are four issues about technical analysis that are worth clarifying. First, if stock returns are predictable[9], then some technical signals may have predictive power; if stock returns are independent and identically distributed, then no technical signal can have predictive power. By analogy, no accounting information or macro variable can have predictive power either. At minimal, technical analysis summarizes some time-series properties of stock prices, although their statistical properties are not well studied.

Second, technical analysis is a study of market price adjustments. The equilibrium price adjusts to new information every day. The adjustment reflects shifts in demand and supply as a result of the reactions of all market participants

[9] For example, Bossaerts and Hillon (1999) provide strong evidence for in-sample return predictability using an international stock market dataset, although they fail to demonstrate out-of-sample predictability. However, Cochrane (2008) points out that poor out-of-sample performance is not a test against predictability.

to new information. However, this adjustment can be sluggish due to various reasons such as noise, market frictions, and investors' herding behavior. Noisy rational expectations models and feedback models, among others, attempt to provide theoretical justification for sluggish price adjustments. For example, see Working (1958), Brown and Jennings (1989), and Wang (1993) for noisy rational expectations models; De Long et al. (1990) and Shleifer and Summers (1990) for feedback models. The common implication of these models is that systematic adjustments in stock prices induce short-term serial correlations. That is, the current price need not fully reveal all available information. Therefore, technical signals may indicate the likely direction of price adjustment.

Third, showing predictive power of a trading rule does not itself rebut market efficiency[10]. Forecast errors can be large regardless of the quality of parameter estimation. Therefore, the investor's portfolio need not generate any significant abnormal return after adjustment for risk and transaction costs. However, an investor may disadvantage himself if he completely ignores technical signals. For example, Han, Yang, and Zhou (2013) show that some technical signals contain unique economic information that is not already contained in other information sources.

Fourth, investors may well recognize that technical signals need not be the most powerful class of predictive variables[11] for stock returns, but other variables are typically not observed frequent enough for real-time trading. In particular, technical signals can be valuable to investors who engage in high frequency trading, for example, investment banks, pension funds, mutual funds, and other buy-side institutional traders.

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[10] Fama (1970) emphasizes that the notion of market efficiency does not necessarily imply successive price changes to be statistically independent.

[11] There are many well-studied predictive variables for stock returns, for example, dividend yields, earnings-price ratio, term spreads, and expected inflation.

There are two features about portfolio choice with a prediction model that are worth emphasizing. First, the true values of the model parameters are not necessarily relevant to an investor with a finite horizon. To be precise, consider the following linear prediction model for stock returns: $R_{t+1} = \beta_0 + \beta_1 X_t + \varepsilon_t$, where X_t is a binary technical signal[12]. Given a sample of T observations on R_{t+1} and X_t, let $(\hat{\beta}_0^T, \hat{\beta}_1^T)$ denote some estimators of (β_0, β_1). The true values of the model parameters are often justified by the probability limit given by $(\beta_0, \beta_1) = \text{plim}_{T\to\infty}(\hat{\beta}_0^T, \hat{\beta}_1^T)$. However, the finite sample estimates $(\hat{\beta}_0^T, \hat{\beta}_1^T)$ are the parameter values directly relevant for constructing forecasts, even though their values need not be close to the hypothetical true values.

Second, a sensible investor who uses technical signals for forecasting stock returns would take into account estimation risk when determining the optimal allocation. Indeed, even if the estimators have good statistical properties, the sample evidence need not fully reflect the future predictive power because of the random variation of sampling. Under the Bayesian approach, the investor integrates his expected utility over the unknown parameter space for portfolio optimization. The resulted optimal portfolio strategy incorporates the degree of uncertainty about the unknown parameters, often measured by the covariances of the estimators $(\hat{\beta}_0^T, \hat{\beta}_1^T)$. Studies sharing this idea include Bawa, Brown, and Klein (1979), Kandel and Stambugh (1996), Brennan (1998), Barberis (2000), Jacquier, Kane, and Marcus (2005), and Xia (2001).

[12] For example, a "buy" signal is represented by $X_t = 1$ and $X_t = 0$ for a "sell" signal.

3 The Model

3.1 The Basic Setting

Consider an investor with a long horizon who trades continuously in a two-asset economy in which a risk-free asset pays an instantaneous rate of interest r, and a risky stock represents the aggregate equity market.

We fix a finite horizon $[0,T]$. The cum-dividend stock price grows according to the following process:

$$\frac{\mathrm{d}P_t}{P_t} = \mu_t \,\mathrm{d}t + \sigma \,\mathrm{d}B_t^*,$$

where the percentage volatility σ is a known parameter; B_t^* is a Brownian motion defined on the probability space $(\Omega, \mathbb{P}^*, \mathfrak{F}^*)$ with a standard filtration $\mathfrak{F}^* = \{\mathfrak{F}_t^* : t \le T\}$. However, the instantaneous percentage drift, μ_t, is unknown to the investor.

The investor observes a technical signal X_t, a function of past stock prices, that is potentially useful to estimate μ_t, or equivalently, to forecast $\mathrm{d}P_t / P_t$. Let S and L, with $0 < S < L < T$, be two lookback periods. We define

$$X_t = \begin{cases} 1 & \text{if } D_t^{S,L} > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$D_t^{S,L} \equiv \frac{1}{S}\int_{t-S}^{t} P_\tau \,\mathrm{d}\tau - \frac{1}{L}\int_{t-L}^{t} P_\tau \,\mathrm{d}\tau$$

is the difference of two moving averages of stock prices. We assume the history $\{P_t : -L \le t \le 0\}$ is known. It is also useful to note that the dynamics of $D_t^{S,L}$ are given by

$$\mathrm{d}D_t^{S,L} = \left[\frac{1}{S}(P_t - P_{t-S}) - \frac{1}{L}(P_t - P_{t-L})\right]\mathrm{d}t.$$

Let $\mathbb{P}$ denote the investor's subjective probability measure, and $\mathfrak{F}_t = \{P_\tau : \tau \le t\}$, with $\mathfrak{F}_t \subset \mathfrak{F}_t^*$, denote the investor's information set at time t. Under the reference model probability $\mathbb{P}$, the investor conjectures a linear predictor $\beta_0 + \beta_1 X_t$ of μ_t, and thus the cum-dividend stock price is represented by the following "linear prediction model":

$$\frac{\mathrm{d}\,P_t}{P_t} = (\beta_0 + \beta_1 X_t)\,\mathrm{d}\,t + \sigma\,\mathrm{d}\,B_t,$$

where (β_0, β_1) are unknown model parameters; B_t is a standard $\mathbb{P}$-Brownian motion adapted to the investor's information set $\mathfrak{F}_t$. While the same linear prediction model is also used in Xia (2001), there are two structural differences for our predictive variable: first, the technical signal X_t is a binary variable; second, it is a function of past stock prices only with no assumption on its law of motion required. The common assumption that the predictive variable follows a (univariate) Markovian mean-reverting process is therefore not applicable. Here, we also relax the assumption that the intercept parameter is known.

For any fixed point of time t in $[0,T)$, given an initial wealth W_t and the investment horizon T, the investor chooses a portfolio allocation ξ to maximize his expected utility of wealth,

$$\max_{\xi} \mathrm{E}[U(W_T) \mid \mathfrak{F}_t],$$

given the wealth dynamics

$$\frac{\mathrm{d}\,W_\tau}{W_\tau} = r\,\mathrm{d}\,\tau + \xi(\beta_0 + \beta_1 X_\tau - r)\,\mathrm{d}\,\tau + \xi\sigma\,\mathrm{d}\,B_\tau.$$

We use the notation τ above because the letter t is already used to denote the fixed chosen point t. Note that ξ is a constant to solve for each point of time t.

We call the sequence of solutions to ξ a "rolling strategy" because it is essentially a "buy-and-hold strategy" with continuous rebalancing.[13] Since the conditional distribution of W_T varies over time due to continuously updated information, so does the maximizer

$$\xi_t^* \equiv \underset{\xi}{\operatorname{argmax}} \, \mathrm{E}[U(W_T) \mid \mathfrak{F}_t].$$

We call ξ_t^* the "optimal allocation"[14] conditional on current information. Our behavioral assumption that the investor follows this rolling strategy may be strong. A more realistic model would allow for a dynamic strategy to account for the possibility of learning the model parameters in the future, as in Brennan (1998) and Xia (2001). However, we believe that this "rolling" assumption can also give us useful implications for portfolio choice and investor welfare because it allows us to obtain a traceable solution for optimal stock allocations.

In this paper, we assume the power-utility function

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \quad \gamma > 1,$$

where γ is the investor's risk-aversion parameter. We shall only consider investors with a risk-aversion parameter greater than the logarithmic case.

3.2 The Investor's Optimization Problem

It is useful to rewrite the utility function as

$$U(W_T) = \frac{\exp[(1-\gamma)\log W_T]}{1-\gamma}$$

and apply Ito's rule to obtain $\log W_T$,

[13] The buy-and-hold strategy is considered by, for example, Stambaugh (1999), Barberis (2000), and Avramov (2002).

[14] The solution is "optimal" subject to our model assumptions and specifications.

$$\log W_T = \log W_t + [r + \xi(\beta_0 + \beta_1 \bar{\mathcal{X}}_t^T - r) - \xi^2 \tfrac{\sigma^2}{2}](T-t) \\ + \xi\sigma(B_T - B_t),$$

where we define the variable

$$\bar{\mathcal{X}}_t^T \equiv \frac{1}{T-t}\int_t^T X_\tau \, \mathrm{d}\tau,$$

which can be interpreted as the average value of the technical signals over the period $[t,T]$. Since $(\beta_0, \beta_1, \bar{\mathcal{X}}_t^T)$ are not adapted to $\mathfrak{F}_t$, they are considered random variables at time t.

Let $\theta_t \equiv \beta_0 + \beta_1 \bar{\mathcal{X}}_t^T$ denote the "average percentage drift" over the remaining investment horizon. By the law of total expectation, we have

$$\mathrm{E}[U(W_T) \mid \mathfrak{F}_t] = \int_{\mathbb{R}} \mathrm{E}[U(W_T) \mid \mathfrak{F}_t, \theta_t] \phi(\theta_t \mid \mathfrak{F}_t) \, \mathrm{d}\theta_t, \tag{1}$$

where $\phi(\theta_t \mid \mathfrak{F}_t)$ is the conditional density of θ_t and the integral is taken over the real line $\mathbb{R}$. By log-normality, the expectation inside the integral is

$$\mathrm{E}[U(W_T) \mid \mathfrak{F}_t, \theta_t] = e^{r(1-\gamma)(T-t)} U(W_t) \\ \times \exp\{(1-\gamma)\xi(\theta_t - r - \xi \tfrac{\gamma\sigma^2}{2})(T-t)\}.$$

Suppose that θ_t follows a conditional Gaussian distribution with mean $m_t \equiv \mathrm{E}[\theta_t \mid \mathfrak{F}_t]$ and variance $v_t \equiv \mathrm{Var}[\theta_t \mid \mathfrak{F}_t]$, then equation (1) satisfies:

$$\mathrm{E}[U(W_T) \mid \mathfrak{F}_t] \\ \propto \exp\{(1-\gamma)\xi\{(m_t - r) - \tfrac{\xi}{2}[\gamma\sigma^2 + (1-\gamma)v_t(T-t)]\}(T-t)\}, \tag{2}$$

where the notation "$\propto$" indicates that the expression holds as an equality subject to a multiplicative term independent of ξ.

Even if θ_t is non-Gaussian, we can approximate it by a Gaussian process that matches the first and the second moments. We take (2) as the true expression and optimize it with respect to ξ to obtain the optimal allocation

$$\xi_t^* = \frac{m_t - r}{\gamma\sigma^2 + (\gamma-1)v_t(T-t)}. \tag{3}$$

3.3 The Investor's Inference Problem

Let us use the notation $\beta \equiv (\beta_0, \beta_1)$. By the law of total expectation, we can evaluate m_t as follows,

$$\begin{aligned} m_t \equiv \mathrm{E}[\theta_t \mid \mathfrak{F}_t] &= \mathrm{E}^{\beta}\{\mathrm{E}^{\bar{\mathcal{X}}}[\beta_0 + \beta_1 \bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta] \mid \mathfrak{F}_t\} \\ &= \mathrm{E}[\beta_0 \mid \mathfrak{F}_t] + \mathrm{E}[\beta_1 \mathcal{E}_t^{\bar{\mathcal{X}}}(\beta) \mid \mathfrak{F}_t], \end{aligned}$$

where we define the function $\mathcal{E}_t^{\bar{\mathcal{X}}} : \mathbb{R}^2 \to \mathbb{R}$ by

$$\mathcal{E}_t^{\bar{\mathcal{X}}}(\beta) \equiv \mathrm{E}[\bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta],$$

which is a function of β to be found. The operator $\mathrm{E}^{\beta}\{\cdot\}$ indicates that the expectation is taken over the distribution of β, and similarly, $\bar{\mathcal{X}}_t^T$ for $\mathrm{E}^{\bar{\mathcal{X}}}[\cdot]$. In the reference model $\mathbb{P}$, the random process $\{X_\tau : t < \tau \le T\}$ is determined by $\{P_\tau : t < \tau \le T\}$, which depends on β, thus the expectation $\mathrm{E}[\bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta]$ also depends on β and cannot be factored out from the outer expectation $\mathrm{E}^{\beta}\{\cdot\}$. By contrast, if the law of motion of X_t were not determined by the prediction model itself, we would simply have $m_t = \mathrm{E}[\beta_0 \mid \mathfrak{F}_t] + \mathrm{E}[\beta_1 \mid \mathfrak{F}_t] \cdot \mathrm{E}[\bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t]$, which would be linear in the conditional expectations of (β_0, β_1).

Similarly, by the law of total variance, we can evaluate v_t as follows,

$$\begin{aligned} v_t \equiv \mathrm{Var}[\theta_t \mid \mathfrak{F}_t] &= \mathrm{E}^{\beta}\{\mathrm{Var}^{\bar{\mathcal{X}}}[\beta_0 + \beta_1 \bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta] \mid \mathfrak{F}_t\} \\ &\quad + \mathrm{Var}^{\beta}\{\mathrm{E}^{\bar{\mathcal{X}}}[\beta_0 + \beta_1 \bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta] \mid \mathfrak{F}_t\} \\ &= \mathrm{E}[\beta_1^2 \mathcal{V}_t^{\bar{\mathcal{X}}}(\beta) \mid \mathfrak{F}_t] + \mathrm{Var}[\beta_0 + \beta_1 \mathcal{E}_t^{\bar{\mathcal{X}}}(\beta) \mid \mathfrak{F}_t], \end{aligned}$$

where we define the function $\mathcal{V}_t^{\bar{\mathcal{X}}} : \mathbb{R}^2 \to \mathbb{R}^+$ by

$$\mathcal{V}_t^{\bar{\mathcal{X}}}(\beta) \equiv \mathrm{Var}[\bar{\mathcal{X}}_t^T \mid \mathfrak{F}_t, \beta],$$

which is a function of β to be found. The operator $\mathrm{Var}^{\beta}\{\cdot\}$ indicates that the variance is taken over the distribution of β, and similarly, $\bar{\mathcal{X}}_t^T$ for $\mathrm{Var}^{\bar{\mathcal{X}}}[\cdot]$.

Let $m_t^\beta \equiv \mathrm{E}[\beta \mid \mathfrak{F}_t]$ and $v_t^\beta \equiv \mathrm{V}[\beta \mid \mathfrak{F}_t]$ denote the conditional mean and the variance-covariance matrix of the investor's estimate of β. We assume that the investor has a bivariate Gaussian prior probability distribution over β, with mean vector m_0^β and variance-covariance matrix v_0^β. Following Liptser and Shiryaev (2001), the distribution of β conditional on $\mathfrak{F}_t$ is also Gaussian with mean vector m_t^β and variance-covariance matrix v_t^β.

Proposition 1. *Given the prior probability distribution of β is Gaussian with mean vector m_0^β and variance-covariance matrix v_0^β, the solutions to m_t^β and v_t^β are*

$$m_t^\beta = \left[I + \frac{v_0^\beta}{\sigma^2} \int_0^t (\vec{X}_\tau \vec{X}_\tau^\top) \mathrm{d}\tau \right]^{-1} \left[m_0^\beta + \frac{v_0^\beta}{\sigma^2} \int_0^t \vec{X}_\tau \frac{\mathrm{d}P_\tau}{P_\tau} \right],$$

$$v_t^\beta = \left[I + \frac{v_0^\beta}{\sigma^2} \int_0^t (\vec{X}_\tau \vec{X}_\tau^\top) \mathrm{d}\tau \right]^{-1} v_0^\beta,$$

where $\vec{X}_t = (1, X_t)^\top$ is a 2×1 vector; I is a 2×2 identity matrix.

We use the following notation to denote the elements of m_t^β and v_t^β,

$$m_t^\beta = \begin{bmatrix} m_t^{\beta_0} \\ m_t^{\beta_1} \end{bmatrix}, \quad v_t^\beta = \begin{bmatrix} v_t^{\beta_0} & v_t^{\beta_0,\beta_1} \\ v_t^{\beta_0,\beta_1} & v_t^{\beta_1} \end{bmatrix}.$$

Also, we use $\partial_i[f] \equiv \partial f / \partial \beta_i$ and $\partial_{i,j}[f] \equiv \partial^2 f / \partial \beta_i \partial \beta_j$, $i, j = \{0,1\}$, to denote the partial derivatives of a given function f.

Proposition 2. *Suppose that the functions $\mathcal{E}_t^{\bar{X}}$ and $\mathcal{V}_t^{\bar{X}}$ are continuously differentiable around m_t^β, then m_t can be expressed as the following Taylor series*

$$
\begin{aligned}
m_t = [m_t^{\beta_0} + m_t^{\beta_1}\mathcal{E}_t^{\bar{X}}] &+ (\tfrac{1}{2} m_t^{\beta_1}\partial_{0,0}[\mathcal{E}_t^{\bar{X}}])v_t^{\beta_0} \\
&+ (\partial_1[\mathcal{E}_t^{\bar{X}}] + \tfrac{1}{2} m_t^{\beta_1}\partial_{1,1}[\mathcal{E}_t^{\bar{X}}])v_t^{\beta_1} \\
&+ (\partial_0[\mathcal{E}_t^{\bar{X}}] + m_t^{\beta_1}\partial_{0,1}[\mathcal{E}_t^{\bar{X}}])v_t^{\beta_0,\beta_1} + H_t,
\end{aligned} \tag{4}
$$

and v_t *can be expressed as the following Taylor series*

$$
\begin{aligned}
v_t = [(m_t^{\beta_1})^2\mathcal{V}_t^{\bar{X}}] &+ \left\{\tfrac{1}{2}(m_t^{\beta_1})^2\partial_{0,0}[\mathcal{V}_t^{\bar{X}}] + (1 + m_t^{\beta_1}\partial_0[\mathcal{E}_t^{\bar{X}}])^2\right\} v_t^{\beta_0} \\
&+ \left\{\mathcal{V}_t^{\bar{X}} + 2m_t^{\beta_1}\partial_1[\mathcal{V}_t^{\bar{X}}] + \tfrac{1}{2}(m_t^{\beta_1})^2\partial_{1,1}[\mathcal{V}_t^{\bar{X}}] \right. \\
&\qquad \left. + (\mathcal{E}_t^{\bar{X}} + m_t^{\beta_1}\partial_1[\mathcal{E}_t^{\bar{X}}])^2\right\} v_t^{\beta_1} \\
&+ \left\{2m_t^{\beta_1}\partial_0[\mathcal{V}_t^{\bar{X}}] + (m_t^{\beta_1})^2\partial_{0,1}[\mathcal{V}_t^{\bar{X}}] \right. \\
&\qquad \left. + 2(1 + m_t^{\beta_1}\partial_0[\mathcal{E}_t^{\bar{X}}])(\mathcal{E}_t^{\bar{X}} + m_t^{\beta_1}\partial_1[\mathcal{E}_t^{\bar{X}}])\right\} v_t^{\beta_0,\beta_1} + O_t,
\end{aligned} \tag{5}
$$

where $\mathcal{E}_t^{\bar{X}}$, $\mathcal{V}_t^{\bar{X}}$, *and their partial derivatives are evaluated at* m_t^{β}; H_t *and* O_t *are remainder terms.*

We give the proofs of these propositions in Appendix A.

To our knowledge, closed-form solutions for $\mathcal{E}_t^{\bar{X}}$ and $\mathcal{V}_t^{\bar{X}}$ are not feasible. Therefore, we develop estimators and use Monte Carlo simulation to estimate these functions. Given any arbitrary positive integer n, we define the quantities $\Delta t \equiv T/n$, $\ell \equiv \lfloor nL/T \rfloor$, and $s \equiv \lfloor nS/T \rfloor$, where "$\lfloor \cdot \rfloor$" is the floor function. For a sufficiently large n, we have $L \simeq \ell\Delta t$ and $S \simeq s\Delta t$. The notation "$\simeq$" means "approximately equal to or equal to". We partition the time interval $[-\ell\Delta t, T]$ into $\ell + n$ equidistant subintervals,

$$-L \simeq t_{-\ell} < \cdots < -S \simeq t_{-s} < \cdots < 0 = t_0 < \cdots < t_n = T,$$

and approximate any t in $[-L, T]$ by t_j if $t_{j-1} < t \le t_j$, $j = -\ell, \ldots, n$. Next, we consider the following Euler approximation scheme:

$$\hat{P}_{t_{j+1}} = \hat{P}_{t_j} + \hat{P}_{t_j}\left[(\beta_0 + \beta_1\hat{X}_{t_j})\Delta t + \sigma\epsilon_{t_j}\sqrt{\Delta t}\,\right], \tag{6}$$

$$\hat{D}_{t_{j+1}}^{S,L} = \hat{D}_{t_j}^{S,L} + \left[\frac{1}{s}(\hat{P}_{t_j} - \hat{P}_{t_{j-s}}) - \frac{1}{\ell}(\hat{P}_{t_j} - \hat{P}_{t_{j-\ell}})\right], \tag{7}$$

$$\hat{X}_{t_{j+1}} = \begin{cases} 1 & \text{if } \hat{D}^{S,L}_{t_{j+1}} > 0, \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

with $\hat{P}_{t_j} = P_{t-j\Delta t}$ if $j \leq i,\ i = 0,\ldots,n-1$; ϵ_{t_j} are independent standard Gaussian random variables.

To simulate a trajectory of $\{\hat{X}_{t_j} : j = i,\ldots,n-1\}$, $i = 0,\ldots,n-1$, we start from the initial values $(\hat{P}_{t_i} = P_t, \hat{X}_{t_i} = X_t)$ and proceed recursively according to the Euler scheme (6)-(8). We then calculate the sum

$$\hat{\bar{\mathcal{X}}}_t^T \equiv \frac{1}{T-t} \sum_{j=i}^{n-1} \hat{X}_{t_j} \Delta t$$

to approximate the stochastic integral $\bar{\mathcal{X}}_t^T$. Let $\hat{\bar{\mathcal{X}}}_{t,k}^T$ denote the sum calculated using the k-th trajectory, $k = 1,\ldots,K$. To estimate the moment $\mathcal{E}_t^{\bar{\mathcal{X}}}(\beta)$, we use the estimator

$$\hat{\mathcal{E}}_t^{\bar{\mathcal{X}}}(\beta; \vec{\epsilon}_t) \equiv \frac{1}{K} \sum_{k=1}^{K} \hat{\bar{\mathcal{X}}}_{t,k}^T,$$

and similarly,

$$\hat{\mathcal{V}}_t^{\bar{\mathcal{X}}}(\beta; \vec{\epsilon}_t) \equiv \frac{1}{K} \sum_{k=1}^{K} (\hat{\bar{\mathcal{X}}}_{t,k}^T)^2 - \left(\frac{1}{K} \sum_{k=1}^{K} \hat{\bar{\mathcal{X}}}_{t,k}^T \right)^2$$

for the moment $\mathcal{V}_t^{\bar{\mathcal{X}}}(\beta)$, where $\vec{\epsilon}_t = (\{\epsilon_{t_j,1}\}_{j=i}^{n-1}, \ldots, \{\epsilon_{t_j,K}\}_{j=i}^{n-1})$ is an $(n-i) \times K$ matrix of independent standard Gaussian random numbers. We use the argument $(\cdot; \vec{\epsilon}_t)$ to emphasize that the values of $\hat{\mathcal{E}}_t^{\bar{\mathcal{X}}}$ and $\hat{\mathcal{V}}_t^{\bar{\mathcal{X}}}$ depend on $\vec{\epsilon}_t$. By fixing $\vec{\epsilon}_t$ for each t, the estimators $\hat{\mathcal{E}}_t^{\bar{\mathcal{X}}}$ and $\hat{\mathcal{V}}_t^{\bar{\mathcal{X}}}$ are smooth in β. Thus, we can evaluate the partial derivatives in the Taylor series of m_t and v_t, given by (4)-(5), using finite difference approximation. We present the required formulas in Appendix B.

4 Data, Model Calibration, and Some Empirical Facts

We measure the horizon T in years and consider daily intervals (with step size $\Delta t = \frac{1}{252}$) small enough for good discrete approximations to the continuous-time processes in the model. We use the inflation adjusted CRSP index (including distributions) on a value-weighted basket of stocks listed in NYSE, AMEX, and NASDAQ as an example of the risky stock. We obtain the nominal daily index data for the period 2nd January 1980 to 31st December 2014 from the CRSP VWRETD file under the folder named "Index/Stock File Indexes". Then, we make a month-by-month inflation adjustment by dividing the stock index series by the consumer price index (CPI), using January 1980 as the base month. The CPI data are obtained from the CRSP inflation file under the folder named "US Treasury and Inflation Indexes". The standard deviation of the daily inflation adjusted index returns is 1.083%. Dividing this figure by $\sqrt{\Delta t}$ gives $\sigma = 0.172$. We also obtain the nominal monthly interest rate data from the CRSP 30-day Treasury bill file. The monthly average interest rate is 0.382%, and 0.267% for the monthly average of the rate of change in CPI. Thus, the implied daily real interest rate is 0.004%. Dividing this figure by Δt gives $r = 0.01$. For the Euler approximation scheme, we use $K = 2000$ trajectories, although we find that even $K = 500$ would give similar results.

As an example, we use the moving average crossover rule with 1-day and 100-day moving averages. In terms of the model notation, we have $s = 1$ and $\ell = 100$. To obtain reasonable priors (m_0^β, v_0^β), we use the inflation adjusted index data for the subsample period 2nd January 1980 to 22nd December 1994 (leaving 20×252 days for the out-of-sample period) to run the regression:

$$\frac{P_{t_{j+1}} - P_{t_j}}{P_{t_j}} = (\beta_0 + \beta_1 X_{t_j})\Delta t + \varepsilon_{t_j},$$

where $X_{t_j} = 1$ if the 1-day moving average is above the 100-day moving average at time t_j, and zero otherwise. We then take the ordinary least squares estimates as the priors:

$$m_0^\beta = \begin{bmatrix} 0.058 \\ 0.061 \end{bmatrix}, \quad v_0^\beta = \begin{bmatrix} 0.004 & -0.004 \\ -0.004 & 0.006 \end{bmatrix}.$$

Figure 1 plots the time series of the intercept and the slope estimates when the investor starts investing on 23rd December 1994 with the prior Gaussian distribution with mean vector and variance-covariance matrix (m_0^β, v_0^β). These time series end on the last trading day of year 2014. Observe that the slope estimates roughly follow a downward trend over the horizon but clearly stay above zero. The estimates of the intercept loosely form a mirror image of that of the slope estimates. Although the intercept estimates are larger than the slope estimates in general, it is evident that the slope estimates have a clear impact on the conditional forecasts of stock returns. In summary, the data may encourage some investors to believe that the technical signals possibly have predictive power on stock returns (even if the true but unknown value of β_1 is zero).

5 Analysis and Results

5.1 Mean and Variance Decompositions

We express the expected value and the variance of the average percentage drift, (m_t, v_t), as the following linear forms:

$$m_t = \tilde{m}_t + \pi_t^0 v_t^{\beta_0} + \pi_t^1 v_t^{\beta_1} + \pi_t^{01} v_t^{\beta_0,\beta_1} + H_t, \tag{9}$$

$$v_t = \tilde{v}_t + \lambda_t^0 v_t^{\beta_0} + \lambda_t^1 v_t^{\beta_1} + \lambda_t^{01} v_t^{\beta_0,\beta_1} + O_t. \tag{10}$$

The remainders H_t and O_t correspond to the ones in Proposition 2, and the definitions of the coefficients follow accordingly: $\tilde{m}_t \equiv m_t^{\beta_0} + m_t^{\beta_1}\mathcal{E}_t^{\bar{X}}$, $\tilde{v}_t \equiv (m_t^{\beta_1})^2 \mathcal{V}_t^{\bar{X}}$, and so on. We call $(\tilde{m}_t, \tilde{v}_t)$ the "basic component" of (m_t, v_t). We also call $(\tilde{m}_t, \tilde{v}_t)$ the "estimation-risk ignorant estimates" because they disregard any estimation risk adjustment. For simplicity, we approximate m_t and v_t by summing only the first four terms, although one could improve the approximation by including higher-order moments and derivatives.

We divide equation (9) by m_t to obtain

$$\begin{aligned} \frac{m_t}{m_t} &= \frac{\tilde{m}_t}{m_t} + \frac{\pi_t^0 v_t^{\beta_0}}{m_t} + \frac{\pi_t^1 v_t^{\beta_1}}{m_t} + \frac{\pi_t^{01} v_t^{\beta_0,\beta_1}}{m_t}, \\ 1 &= \tilde{M}_t + M_t^0 + M_t^1 + M_t^{01}, \end{aligned}$$

where $\tilde{M}_t$ is the percentage composition of the basic component $\tilde{m}_t$; M_t^0, M_t^1, and M_t^{01}, for the estimation-risk adjustment components. Similarly, we divide equation (10) by v_t to obtain

$$\begin{aligned} \frac{v_t}{v_t} &= \frac{\tilde{v}_t}{v_t} + \frac{\lambda_t^0 v_t^{\beta_0}}{v_t} + \frac{\lambda_t^1 v_t^{\beta_1}}{v_t} + \frac{\lambda_t^{01} v_t^{\beta_0,\beta_1}}{v_t}, \\ 1 &= \tilde{V}_t + V_t^0 + V_t^1 + V_t^{01}, \end{aligned}$$

where $\tilde{V}_t$ is the percentage composition of the basic component $\tilde{v}_t$; V_t^0, V_t^1, and V_t^{01}, for the estimation risk adjustment components.

We consider four investment horizons, $T = \{5, 10, 15, 20\}$. For each horizon, we perform the mean and the variance decompositions at five points of time, $t = \delta T$, $\delta = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{251}{252}\}$. Panel A of Table I displays the results of the mean decomposition. We observe that the basic component accounts for roughly 100% of m_t most of the time while the estimation risk adjustment components are relatively small. Our results suggest that ignoring estimation risk does not materially bias the expected value of average percentage drift θ_t.

Panel B of Table I displays the results of the variance decomposition. We observe that the size of the estimation-risk ignorant component is materially smaller than that of the estimation risk adjustment components. Notably, the estimation-risk ignorant component becomes essentially 0% towards the end of the horizon. Our results suggest that ignoring estimation risk materially underestimates the variance of the average percentage drift.

Note that both (m_t, v_t) and $(\tilde{m}_t, \tilde{v}_t)$ depend on the technical trading rule and the stock price data. Therefore, their relative sizes can only be determined empirically. Without actual data, we would have an inconclusive theoretical discussion.

5.2 Horizon Effect and Hedging Demand

Let us define the "suboptimal allocation" by

$$\tilde{\xi}_t \equiv \frac{\tilde{m}_t - r}{\gamma\sigma^2 + (\gamma - 1)\tilde{v}_t(T - t)}. \tag{11}$$

This is the allocation that would be made if the investor used the estimation-risk ignorant estimates, $(\tilde{m}_t, \tilde{v}_t)$. Equivalently, we obtain $\tilde{\xi}_t$ by setting $(v_t^{\beta_0}, v_t^{\beta_1}, v_t^{\beta_0,\beta_1})$ to zero in the optimal allocation ξ_t^* given by (3). Both the optimal and the suboptimal allocations depend on the remaining investment horizon, $T - t$: directly through the denominators; and indirectly through the mean and the variance of average percentage drift $(m_t, \tilde{m}_t, v_t, \tilde{v}_t)$. The influence of the remaining horizon on the portfolio allocation is commonly called the "horizon effect". Also, we call the difference of the two allocations, $\Delta_t \equiv \xi_t^* - \tilde{\xi}_t$, the investor's "hedging demand". We can interpret this quantity as an allocation to hedge against estimation risk.

We consider three values of the risk-aversion parameter, $\gamma = \{5, 7, 9\}$. For each investment horizon, $T = \{5, 10, 15, 20\}$, we calculate the optimal allocation, the suboptimal allocation, and the hedging demand at five points of time, $t = \delta T$, $\delta = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{251}{252}\}$. Table II displays our results. Consider time $t = 0$. Observe first that, for all values of γ, the optimal allocation monotonically decreases as the horizon increases while the suboptimal allocation is not sensitive to the horizon. This is because the empirical values of $\tilde{v}_t$ are sufficiently small that the adjustment term $\tilde{v}_t(T-t)$ in (11) does not have a strong effect on the suboptimal allocations. Second, the optimal allocation decreases more sharply for higher value of γ as the horizon increases. Overall, the hedging demand is increasing in horizon but decreasing in risk-aversion parameter.

Next, we look at other points of time along the investment horizon. Our results show that the horizon effect is clear. At time $t = \frac{3}{4}T$, with only a quarter of horizon remaining, the values of the hedging demand are still above 2%. However, the hedging demand declines with the remaining horizon. The reason is that as the estimates become more precise and the remaining horizon decreases, the compounding effect of estimation risk becomes less severe.

The notion that investors bearing estimation risk may allocate less to stocks for longer horizon is not new. For example, in the context of an IID model for returns, Brennan (1998) finds that the dynamic allocation declines monotonically as the horizon is reduced when the investor is more risk tolerant than a logarithmic investor; in the context of a predictive VAR model for returns, Barberis (2000) finds that the allocation can decrease with horizon when the investor uses a buy-and-hold strategy. Stambaugh (1999) and Xia (2001) also show that the allocation can decline eventually when the horizon becomes sufficiently long, although the allocation can first increase with horizon. Our results are consistent with these findings.

However, in these studies, the portfolio allocation equation is not characterized in a form that signifies the role of remaining horizon, $T-t$. Jacquier, Kane, and Marcus (2005) derive a closed-form solution for a buy-and-hold strategy that incorporates estimation risk in the context of an IID model for returns. Their findings on horizon effect are qualitatively consistent to that of Brennan (1998). Similarly, we make the role of remaining horizon more tractable in the context of a prediction model. Because our conclusion on horizon effect depends on our model assumptions, model specifications, data, and estimation strategy, it is not a direct consequence of known results.

5.3 Market Timing Effect

When stock returns are possibly predictable, the portfolio allocation can depend on the current value of the predictive variable. This is called the "market timing effect". For our model, when the estimate of the slope parameter is positive, then a buy signal implies a higher expected stock return. This attracts the investor to increase his allocation in the risky stock. However, since the portfolio allocation also depends on other state variables, not just the technical signal, it is difficult to study the market timing effect conditional on all other state variables. As a compromise, for any given portfolio allocation strategy ξ, we calculate the averages of $\xi_t \mid X_t = 1$ and $\xi_t \mid X_t = 0$ in a fixed time interval and interpret their difference as the unconditional average market timing effect.

We again consider the optimal and the suboptimal strategies for four investment horizons, $T = \{5,10,15,20\}$, and three values of risk-aversion parameters, $\gamma = \{5,7,9\}$. We compute the averages using the subsample period from 23rd December 1994 to 20th December 1999 (5×252 trading days) because it is the widest time interval in which our portfolios are built using the same data. During this subsample period, we observe a reasonable mix of buy and sell signals. Our analysis tries to study how the market timing effect varies with horizons.

We display the results in Table III. Firstly, observe that the average market timing effect is positive and it has a range of 1% to 5.5% (per trading day). While we consider this effect economically significant, it is far from 100% for all portfolios considered in this paper. In this regard, it is hard to justify the use of all-or-nothing strategy commonly assumed in the literature. Secondly, we observe two patterns of average market timing effect: it decreases as investment horizon increases; and it also decreases as the risk-aversion parameter increases. Thirdly, we note that the average market timing effect associated with the optimal strategy decreases more notably with horizon compared to that of the suboptimal strategy. We highlight the differences in average market timing effect between the optimal and the suboptimal strategies in Figure 2. We see that, except for the shortest horizon considered ($T = 5$), the average market timing effect is stronger for the suboptimal strategy. We also see that the differences are more notable as the horizon increases and the risk-aversion parameter decreases. Overall, our results show that market timing strongly depends on risk aversion, investment horizon, and attitude towards estimation risk.

Our findings relate to that of Zhu and Zhou (2009) in the following sense. Although our models differ in how the investor infers information from the technical signals, we show that it is optimal for the investor to allocate an additional proportional of his wealth to the stock when he observes a buy signal. Our model, however, allows the investor to continuously assess the predictive power of the signals based on sample evidence and he adjusts his portfolio allocation accordingly. In particular, while the estimates of the slope parameter remain positive in our sample period, it is possible to become negative. In that case, the investor would actually decrease his allocation to the risky stock even though a “buy” signal is observed. This is possible because if the current price is well above the moving averages, the chance of a price reversal is also high. Therefore, the sign of the slope estimate depends on whether the data suggest that the buy signals are more often related to new upward trends or price reversals on average. When the statistical relationships are mixed, the slope estimate can be close to zero and the signals become uninformative. Our idea differs from the conventional assumption that the investor necessarily increases his allocation (and by the same proportion of wealth) whenever he observes a buy signal.

5.4 Welfare Costs of Ignoring Estimation Risk

Given an allocation ξ_t to the risky stock, we define a function J by

$$J(t, W_t, m_t, v_t; \xi_t)$$
$$\equiv U(W_t)\exp\left\{(1-\gamma)\xi_t\left\{(m_t - r) - \tfrac{\xi_t}{2}[\gamma\sigma^2 + (1-\gamma)v_t(T-t)]\right\}(T-t)\right\}.$$

This is the investor's maximized expected utility at time *t*. Next, we define the "welfare cost" of using the suboptimal allocation $\tilde{\xi}_t$ by the quantity ∇_t that satisfies the equality

$$J(t, 1, m_t, v_t; \xi_t^*) = J(t, 1+\nabla_t, m_t, v_t; \tilde{\xi}_t).$$

The welfare cost ∇_t measures the percentage wealth compensation required to leave the investor, who has \$1 today to invest up to the horizon, indifferent between the optimal and the suboptimal allocation strategies. Similar measures are used in the literature, for example, Campbell and Viceira (1999), Xia (2001), and Han, Yang, and Zhou (2013). By construction, the value of J is maximized at all time *t* if ξ_t is the optimal allocation ξ_t^*. Hence, the welfare cost ∇_t is always non-negative. We are interested to see the size of this value to study how costly is it to an investor if he ignores estimation risk.

We again consider four investment horizons, $T = \{5, 10, 15, 20\}$, and three values of risk-aversion parameters, $\gamma = \{5, 7, 9\}$. For each horizon, we compute the welfare cost at five points of time, $t = \delta T$, $\delta = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{251}{252}\}$. Table IV displays the results. We see two patterns of welfare cost. First, it increases substantially as the horizon increases. For the shortest horizon considered $(T = 5)$, at time $t = 0$, its range is 0.352% to 0.530%. By contrast, for the longer horizons, $T = \{15, 20\}$, the range is 6.394% to 21.474%, and the welfare cost remains above 3% for at least one quarter of the horizon. Our results imply that it can be costly to ignore estimation risk for investors with a longer horizon. This is because the suboptimal strategy results in more severe overinvestment for longer horizons. Relatively speaking, estimation risk may be less of a concern for investors with shorter horizons. To further signify the magnitude of welfare cost, we fix the time at $t=0$ and plot it as a function of horizon and risk-aversion parameter in Figure 3. The results show that the welfare cost increases with horizon at an increasing rate. The second pattern we observe is that the welfare cost increases as the risk-aversion parameter decreases and the increase is stronger for longer horizons. For example, for $T=10$, the welfare cost increases from 2.245% to 3.434% as γ decreases from 9 to 5 at time $t=0$; for $T=20$, the welfare cost increases from 13.469% to 21.474%.

To summarize, using historical data on stock returns and technical signals, we gain some insight about the opportunity cost to an investor if he ignores the uncertain predictive power of the technical signals generated by the moving average trading rule. Our results complement studies using other predictive variables, prominently the dividend yield. Although the general effects of neglecting estimation risk in portfolio choice are well studied, without an actual application to technical analysis, it would be hard to speculate the magnitude of these effects in real world.

6 Conclusion and Future Work

We have analyzed an investor's portfolio choice problem in which he forecasts stock returns using technical signals. We calibrate our model with CRSP stock index data and find that if an investor considers the uncertainty in predictive power of technical signals, he allocates substantially less of his wealth to stocks and time the market less aggressively, the longer his investment horizon. Moreover, we find that the opportunity cost of ignoring such uncertain predictive power increases sizably with horizon at an increasing rate. Our model implications contrast with the common assumption that investors allocate 100% of wealth to stocks whenever they observe a buy signal but nothing otherwise.

This study is only an initial attempt to incorporate information from technical signals in a Bayesian portfolio choice problem. Although we have considered the role of estimation risk, we have assumed that the investor ignores the possibility of learning the model parameters in the future. An important extension is to consider a dynamic portfolio strategy as in Brennan (1998) and Xia (2001). Also, we have only considered the moving average trading rule. Given our data and this particular trading rule, our results suggest that ignoring estimation risk does not materially bias the expected average drift of returns but materially underestimates its variance. As a consequence, the optimal portfolio allocation is sensitive to the remaining horizon. Other more elaborate rules, such as those depend on charting patterns considered by Lo, Mamaysky, and Wang (2000), may give us a different insight. Our model can be easily modified for these rules.

Appendix A. Proofs of Propositions

Proof of Proposition 1: This is a direct application of Theorem 12.7 in Liptser and Shiryaev (2001, p. 36). Q.E.D.

Proof of Proposition 2: Equation (4) is the second-order Taylor series expansion of $m_t = \mathrm{E}[\beta_0 \mid \mathfrak{F}_t] + \mathrm{E}[\beta_1 \mathcal{E}_t^{\bar{X}}(\beta) \mid \mathfrak{F}_t]$ around the point $(m_t^{\beta_0}, m_t^{\beta_1})$. To obtain equation (5), we find the second-order and the first-order Taylor series expansions of the first and the second terms of

$$v_t = \mathrm{E}[\beta_1^2 \mathcal{V}_t^{\bar{X}}(\beta) \mid \mathfrak{F}_t] + \mathrm{Var}[\beta_0 + \beta_1 \mathcal{E}_t^{\bar{X}}(\beta) \mid \mathfrak{F}_t],$$

respectively. Q.E.D.

Appendix B. Formulas for Finite Difference Approximation

We use the following partial derivative approximations:

$$\partial_0 \mathcal{E}_t^{\bar{x}}(m_t^{\beta}) \approx \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}+\delta, m_t^{\beta_1}) - \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}-\delta, m_t^{\beta_1})}{2\delta},$$

$$\partial_1 \mathcal{E}_t^{\bar{x}}(m_t^{\beta}) \approx \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}+\delta) - \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}-\delta)}{2\delta},$$

$$\partial_{0,0} \mathcal{E}_t^{\bar{x}}(m_t^{\beta}) \approx \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}+\delta, m_t^{\beta_1}) - 2\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}) + \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}-\delta, m_t^{\beta_1})}{\delta^2},$$

$$\partial_{1,1} \mathcal{E}_t^{\bar{x}}(m_t^{\beta}) \approx \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}+\delta) - 2\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}) + \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}-\delta)}{\delta^2},$$

$$\begin{aligned}\partial_{0,1} \mathcal{E}_t^{\bar{x}}(m_t^{\beta}) \approx\ & \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}+\delta, m_t^{\beta_1}+\delta) - \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}+\delta, m_t^{\beta_1}) - \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}+\delta)}{2\delta^2} \\ & - \frac{\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}-\delta, m_t^{\beta_1}) + \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1}-\delta) - \mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}-\delta, m_t^{\beta_1}-\delta)}{2\delta^2} \\ & + \frac{2\mathcal{E}_t^{\bar{x}}(m_t^{\beta_0}, m_t^{\beta_1})}{2\delta^2}.\end{aligned}$$

Similar formulas are used for $\mathcal{V}_t^{\bar{x}}$. We set $\delta = 0.01$.

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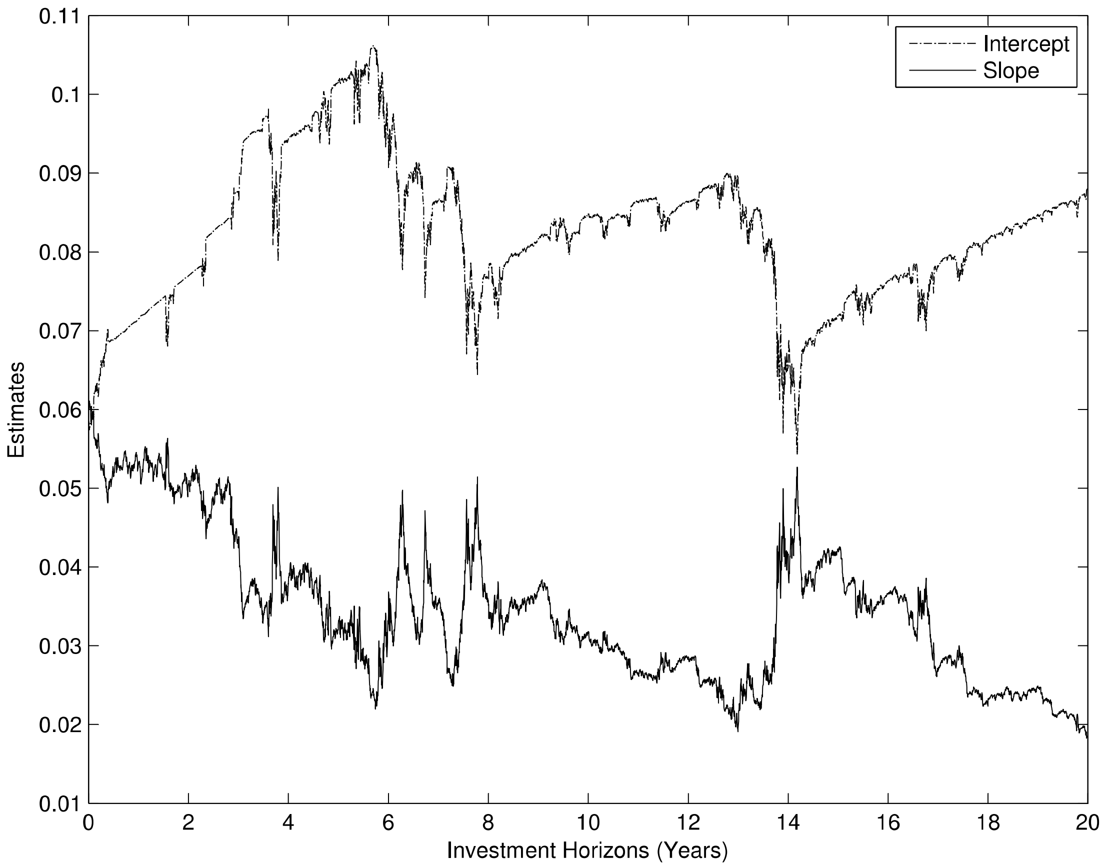

Figure 1. Estimates of intercept and slope parameters of the linear prediction model. This figure plots the time series of the intercept and the slope estimates when the investor starts investing on 23rd December 1994 until 31st December 2014 (20×252 trading days). The ordinary least squares estimates for the subsample period 2nd January 1980 to 22nd December 1994 are used as the priors.

Table I

Mean and Variance Decompositions of Average Percentage Drift

This table presents the results of the mean and the variance decompositions of the average percentage drift. The expected value of the average percentage drift, θ_t, is denoted by m_t; $\tilde{M}_t$ denotes the percentage composition of the basic component $\tilde{m}_t$; (M_t^0, M_t^1, M_t^{01}) denote the percentage compositions of the estimation risk adjustment components associated with $(v_t^{\beta_0}, v_t^{\beta_1}, v_t^{\beta_0,\beta_1})$. The variance of the average percentage drift is denoted by v_t; $\tilde{V}_t$ denotes the percentage composition of the basic component $\tilde{v}_t$; (V_t^0, V_t^1, V_t^{01}) denote the percentage compositions of the estimation risk adjustment components associated with $(v_t^{\beta_0}, v_t^{\beta_1}, v_t^{\beta_0,\beta_1})$. The results show that ignoring estimation risk does not materially bias the expected value of average percentage drift but materially underestimates its variance. The figures are measured in percent.

		Panel A: Mean Decomposition					Panel B: Variance Decomposition				
		m_t	$\tilde{M}_t$	M_t^0	M_t^1	M_t^{01}	v_t	$\tilde{V}_t$	V_t^0	V_t^1	V_t^{01}
$t=0$											
T	5	9.246	100.996	-0.028	4.523	-5.492	0.170	2.394	272.706	141.971	-317.070
	10	9.218	101.522	-0.247	4.430	-5.705	0.164	1.238	283.437	147.039	-331.714
	15	9.239	101.352	-0.206	4.523	-5.668	0.162	0.833	286.983	148.586	-336.401
	20	9.259	101.173	-0.164	4.590	-5.599	0.161	0.630	289.006	149.615	-339.251
$t=\frac{1}{4}T$											
T	5	10.483	100.703	-0.027	3.867	-4.543	0.158	2.502	273.141	158.191	-333.834
	10	11.248	100.650	-0.101	3.450	-3.999	0.141	1.174	293.917	171.028	-366.119
	15	11.148	100.650	-0.027	3.303	-3.926	0.133	0.627	299.737	160.516	-360.881
	20	12.099	100.529	-0.061	2.936	-3.405	0.120	0.285	309.165	179.205	-388.656
$t=\frac{1}{2}T$											
T	5	11.377	100.508	-0.076	3.315	-3.747	0.151	3.132	273.353	175.882	-352.368
	10	12.134	100.487	-0.076	2.892	-3.303	0.124	0.818	299.676	181.614	-382.108
	15	9.997	100.822	-0.099	2.918	-3.642	0.115	0.771	276.053	144.729	-321.552
	20	10.358	100.418	-0.027	2.748	-3.138	0.102	0.503	280.635	161.321	-342.459
$t=\frac{3}{4}T$											
T	5	10.814	100.470	0.231	2.780	-3.480	0.167	3.766	234.774	103.979	-242.518
	10	9.845	101.110	-0.140	2.587	-3.557	0.126	2.045	250.804	121.513	-274.361
	15	10.234	100.277	0.049	2.704	-3.031	0.101	0.908	272.862	158.706	-332.477
	20	9.742	100.204	0.021	2.579	-2.804	0.094	1.999	265.410	154.560	-321.969
$t=\frac{251}{252}T$											
T	5	13.348	100.000	0.000	0.000	0.000	0.157	0.000	219.977	319.977	-439.954
	10	11.645	100.000	0.000	0.000	0.000	0.139	0.000	191.320	291.320	-382.640
	15	11.412	100.000	0.000	0.000	0.000	0.122	0.000	185.458	285.458	-370.916
	20	10.649	100.000	0.000	0.000	0.000	0.105	0.000	198.146	298.146	-396.292

Table II

Horizon Effect and Hedging Demand against Estimation Risk

This table presents the optimal stock allocation, the suboptimal stock allocation, and their difference as the hedging demand against estimation risk. The optimal allocation is defined by $\xi_t^* \equiv (m_t - r)/[\gamma\sigma^2 + (\gamma-1)v_t(T-t)]$ and the suboptimal allocation is defined by $\tilde{\xi}_t \equiv (\tilde{m}_t - r)/[\gamma\sigma^2 + (\gamma-1)\tilde{v}_t(T-t)]$, where γ and T are the investor's risk-aversion parameter and investment horizon. The hedging demand is defined by $\Delta_t \equiv \xi_t^* - \tilde{\xi}_t$. We consider four investment horizons, $T = \{5, 10, 15, 20\}$, and present $(\xi_t^*, \tilde{\xi}_t, \Delta_t)$ at five points of time, namely $t = \delta T$, $\delta = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{251}{252}\}$. The results show that, at time $t = 0$, the optimal allocation monotonically decreases with horizon, while the suboptimal allocation is not sensitive to the horizon. As the remaining horizon decreases, the hedging demand decreases and eventually becomes close to zero. The figures are measured in percent.

		Optimal Strategy				Suboptimal Strategy				Difference		
T		5	10	15	20	5	10	15	20	5	10	15	20
$t=0$													
γ	5	45.606	38.736	33.829	30.049	56.392	56.531	56.565	56.589	-10.787	-17.796	-22.736	-26.540
	7	32.147	27.075	23.499	20.774	40.264	40.364	40.388	40.405	-8.118	-13.289	-16.890	-19.631
	9	24.821	20.810	18.001	15.874	31.310	31.387	31.406	31.419	-6.489	-10.577	-13.405	-15.545
$t=\frac{1}{4}T$													
γ	5	55.548	54.121	49.116	50.645	64.691	69.889	69.263	75.714	-9.143	-15.768	-20.146	-25.069
	7	39.290	38.052	34.377	35.346	46.195	49.909	49.465	54.076	-6.905	-11.857	-15.088	-18.730
	9	30.394	29.341	26.442	27.146	35.924	38.813	38.469	42.057	-5.530	-9.472	-12.027	-14.911
$t=\frac{1}{2}T$													
γ	5	63.969	64.753	49.594	49.833	70.665	75.917	61.607	63.805	-6.696	-11.164	-12.013	-13.972
	7	45.392	45.782	34.952	35.053	50.464	54.221	43.999	45.571	-5.072	-8.439	-9.047	-10.518
	9	35.176	35.408	26.985	27.035	39.244	42.170	34.219	35.442	-4.068	-6.761	-7.234	-8.407
$t=\frac{3}{4}T$													
γ	5	63.103	55.407	56.918	52.733	66.888	60.765	62.899	59.421	-3.784	-5.359	-5.981	-6.688
	7	44.901	39.355	40.387	37.366	47.770	43.399	44.925	42.436	-2.868	-4.043	-4.538	-5.070
	9	34.849	30.515	31.297	28.934	37.151	33.752	34.940	33.002	-2.302	-3.237	-3.643	-4.069
$t=\frac{251}{252}T$													
γ	5	83.809	72.292	70.716	65.557	83.837	72.314	70.734	65.572	-0.028	-0.022	-0.018	-0.015
	7	59.862	51.636	50.510	46.826	59.883	51.653	50.524	46.837	-0.022	-0.016	-0.014	-0.011
	9	46.559	40.161	39.285	36.420	46.576	40.174	39.297	36.429	-0.017	-0.013	-0.011	-0.009

Table III

Average Market Timing Effect

This table presents the difference in the averages of $\xi_t \mid X_t = 1$ and $\xi_t \mid X_t = 0$ for the optimal and the suboptimal allocations, $\xi_t = (\xi_t^*, \tilde{\xi}_t)$, using the subsample period from 23rd December 1994 to 20th December 1999 (5×252 trading days), where $X_t = 1$ if the 1-day moving average is above the 100-day moving average at time t and $X_t = 0$ otherwise. We use this difference as a measure of the average effect of a buy signal $(X_t = 1)$ on stock allocation. The results show that, on average, the investor allocates an additional proportion of his wealth to the risky stock when a buy signal is observed. The average market timing effect is mild but economically significant. The figures are measured in percent.

		Optimal Strategy				Suboptimal Strategy				Difference			
	T	5	10	15	20	5	10	15	20	5	10	15	20
γ	5	5.819	3.068	2.549	2.218	5.485	3.333	3.176	3.115	0.334	-0.265	-0.627	-0.897
	7	4.151	2.169	1.789	1.547	3.917	2.381	2.268	2.225	0.234	-0.212	-0.480	-0.678
	9	3.226	1.677	1.377	1.188	3.047	1.852	1.764	1.730	0.180	-0.175	-0.387	-0.543

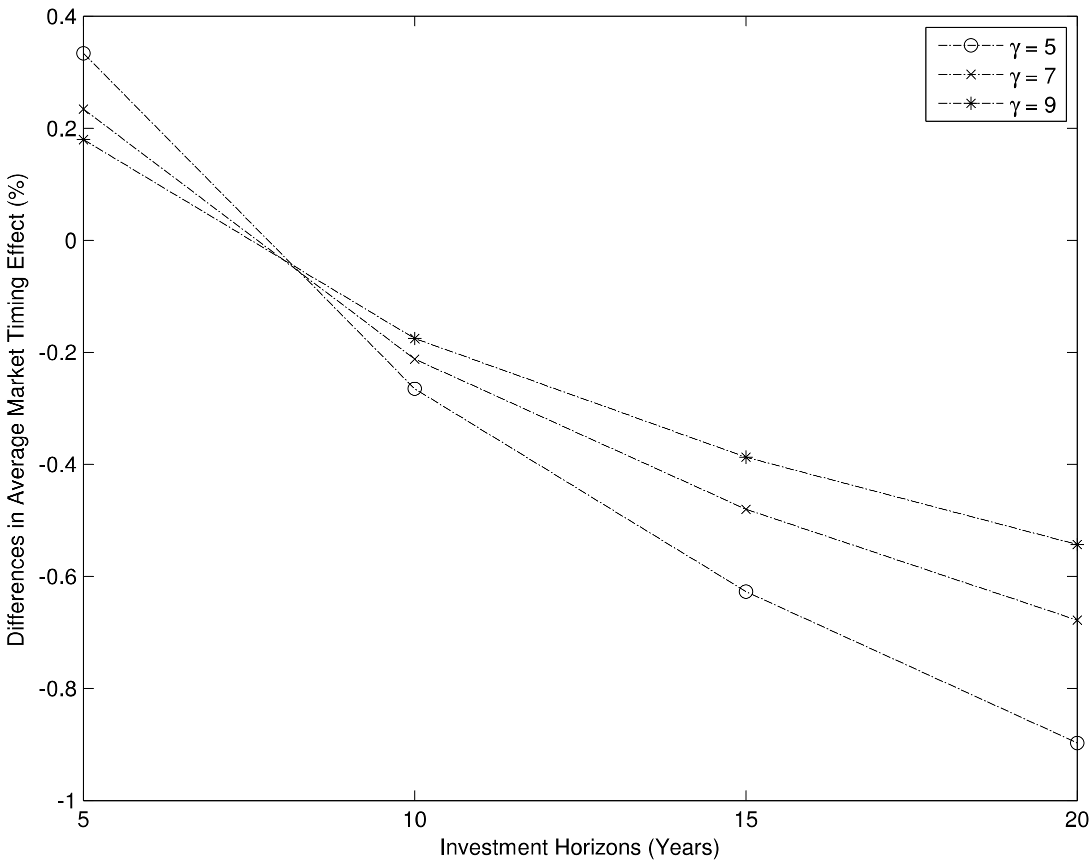

Figure 2. Differences in average market timing effect between the optimal and the suboptimal allocation strategies. This figure plots the difference in the average market timing effect between the optimal and the suboptimal allocations as a function of the investment horizon T and the risk-aversion parameter γ, using the subsample period from 23rd December 1994 to 20th December 1999 (5×252 trading days). The results show that, except for the shortest horizon considered ($T=5$), the average market timing effect is stronger for the suboptimal strategy. The differences are more notable as the horizon increases and the risk-aversion parameter decreases. The figures are measured in percent.

Table IV

Welfare Cost for Ignoring Estimation Risk

This table presents the welfare cost which measures the percentage wealth compensation required to leave the investor, who has \$1 today to invest up to the horizon, indifferent between the optional and the suboptimal allocation strategies. We consider four investment horizons, $T = \{5,10,15,20\}$, and present the results at five points of time, namely $t = \delta T$, $\delta = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{251}{252}\}$. At time $t = 0$, the welfare cost is strongly increasing in horizon. For the longer horizons, $T = \{15,20\}$, the welfare cost remains above 3% for at least one quarter of the horizons. The results show that it is costly for a long-horizon investor to ignore estimation risk. The figures are measured in percent.

T		5	10	15	20
$t=0$					
γ	5	0.530	3.434	9.954	21.474
	7	0.426	2.730	7.829	16.644
	9	0.352	2.245	6.394	13.467
$t=\frac{1}{4}T$					
γ	5	0.269	1.789	4.851	10.927
	7	0.217	1.436	3.869	8.648
	9	0.180	1.187	3.186	7.084
$t=\frac{1}{2}T$					
γ	5	0.091	0.540	0.992	1.859
	7	0.074	0.436	0.797	1.495
	9	0.061	0.362	0.660	1.237
$t=\frac{3}{4}T$					
γ	5	0.014	0.058	0.109	0.187
	7	0.011	0.046	0.089	0.151
	9	0.009	0.038	0.074	0.126
$t=\frac{251}{252}T$					
γ	5	0.000	0.000	0.000	0.000
	7	0.000	0.000	0.000	0.000
	9	0.000	0.000	0.000	0.000

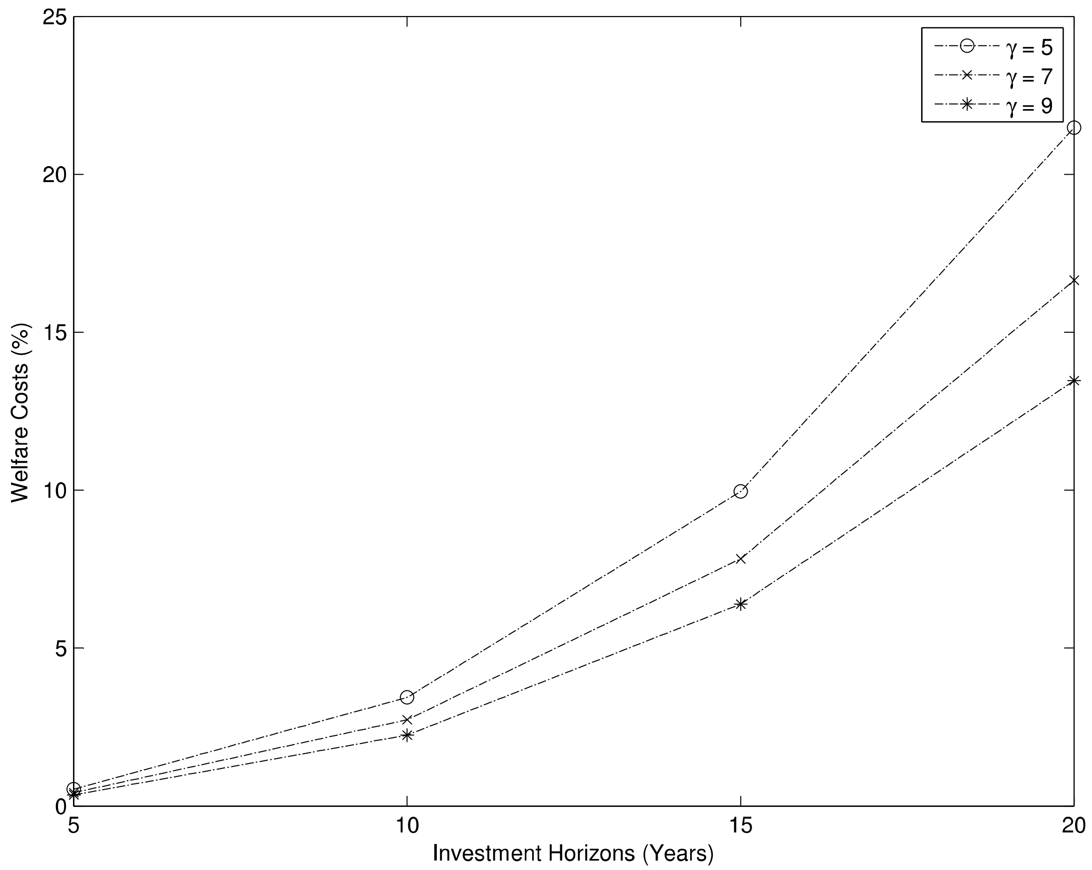

Figure 3. Welfare cost for ignoring estimation risk at time zero. This figure plots the welfare cost as a function of the investment horizon T and the risk-aversion parameter γ at time $t = 0$. The results show that the welfare cost increases with horizon at an increasing rate. Besides, the welfare cost increases as the risk-aversion parameter decreases and the increase is stronger for longer horizons. The figures are measured in percent.