Volatility-of-volatility Risk in the Crude Oil Market

Yahua Xu [*] Tai-Yong Roh [†] Yang Zhao [‡]

July 3, 2019

Abstract

Under the stochastic volatility-of-volatility framework, we show that it is a significant risk factor for cross-sectional delta-hedged gains, and has negative market price. Moreover, oil volatility-of-volatility can significantly and negatively predict one-period ahead delta-hedged option gains. The findings are robust after implementing several tests including adding jump risk measures, using another measure of oil volatility-of-volatility and changing to 1-week forecasting horizon. The information content of oil volatility-of-volatility is also distinctive from its equity counterpart, which can contribute to predict the future real personal consumption expenditure along with oil volatility and equity volatility-of-volatility.

JEL Classification: G1; C5; Q3; Q4

Keywords: Crude oil market; Stochastic volatility-of-volatility risk; Delta-hedged gains; Jump risks; Pricing implications

[*] China Economics and Management Academy, Central University of Finance and Economics

[†] Corresponding author: China Economics Management Academy, Central University of Finance and Economics, No. 39 South College Road, Haidian District, 100081 Beijing, China. Phone: +86 10 62888376. Fax: +86 10 62888376. Email: yahua.xu@cufe.edu.cn.

[‡] Chinese Academy of Finance and Development, Central University of Finance and Economics

1 Introduction

Aggregate volatility-of-volatility risk in the crude oil market can have important implications for pricing and investment decisions. It can be a state variable in an economy with time-varying investment opportunities. However, even though there is a well-established strand of literature about the impacts of stochastic volatility in asset pricing from both theoretical and empirical perspectives (e.g., Ang et al., 2006; Campbell et al., 2018; Cao and Han, 2013; Heston, 1993; Hull and White, 1987), much less is known about the role of volatility-of-volatility risk, which characterizes the distribution of stochastic volatility, in pricing and investing decisions. Crude oil is used as a major input for production, thus fluctuations in oil market have substantial impacts on real economy (e.g., Christoffersen and Pan, 2018; Ferderer, 1996; Hamdi et al., 2019; Herrera et al., 2019). It is natural to imply that uncertainty of oil market volatility may also be an important risk factor affecting consumption and production decisions. In this paper, we aim to analyze whether aggregate volatility-of-volatility risk in crude oil market is priced delta-hedged gains of USO options, and whether oil volatility-of-volatility contains distinct information from their equity counterpart.[1]

Our research is related to the growing body of literature in asset pricing with volatility-of-volatility risk, the economic implications of which has been much less examined until recently. Bollerslev et al. (2009) extend the long-run risk model of Bansal and Yaron (2004) by introducing the stochastic volatility-of-volatility risk and further show that for a representative agent equipped with Epstein-Zin-Weil recursive preferences, volatility-of-volatility risk drives the time-varying variance risk premium and contributes to predict the future equity index market returns. Later on Chen et al. (2017) develop a macroeconomic model that incorporates market risk, volatility risk and volatility-of-volatility risk as pricing factors, and confirm the important role of volatility-of-volatility as a state variable which is significantly priced in the cross-section of stock returns. Furthermore,

[1] USO stands for the United States Oil Fund, which is one of the largest and most liquid crude oil Exchange Traded Funds (ETFs).

there are some empirical papers exploring the cross-sectional implications of volatility-of-volatility risk in various markets, such as the individual stocks (e.g., Baltussen et al., 2018), the equity index market (e.g., Hollstein and Prokopczuk, 2018) and the hedge fund market (e.g., Agarwal et al., 2017). Latest research connects the volatility-of-volatility risk with option returns: Huang et al. (2018) extend the analysis of volatility-of-volatility as a systematic risk factor into S&P 500 and VIX option markets; Cao et al. (2018) investigate the cross-sectional effects of volatility-of-volatility risk in future delta-hedged equity index option returns. So far, it remains unknown whether volatility-of-volatility is also a significant risk factor in the crude oil market.

We extract the volatility-of-volatility risk factor from commodity-linked securities, namely, USO options. After the financialization of commodity markets in 2004-2005, the commodity futures and options become greatly welcomed as a result of regulatory changes and the availability of new commodity derivatives, which in turn brought dramatic changes to commodity prices and volatility dynamics.[2] Since the energy crisis in 1970s, fluctuations of the oil market, one of the most important energy markets, have gained substantial attention from the industry and academia. Inspired by the frequent upward and downward movement of crude oil market, especially the sharp drop during 2014-2015, we investigate the time-varying volatility-of-volatility risk factor extracted from the crude oil market. We restrict our sample to the post-financialization period, for utilizing USO options.

Our analysis extends the discussion of oil market to the volatility-of-volatility risk, while most previous work is confined to the stochastic volatility. For instance, Chiang et al. (2015) show that latent stochastic volatility factor is significantly related with key macro-variables and thus it is a significant pricing factor. Prokopczuk et al. (2017) uncover the existence of negative oil volatility risk premium by looking at the average of synthetic variance swap rate constructed from oil futures options. Christoffersen and Pan (2018) confirm that oil volatility is a significant state variable which links the real economy and

[2]Research on the impacts and structural break caused by financialization of commodities includes Basak and Pavlova (2016).

the cross section of stock returns. Even though recent studies have identified the important role of volatility-of-volatility risk in the equity market, the question of whether volatility-of-volatility risk remains the same significant in the crude oil market is unanswered.

Our paper contributes to the extant literature in several aspects. First, we construct oil market volatility-of-volatility measure by taking the scaled standard deviation of recent 1-month historical volatility, which is proxied by OVX, by following Baltussen et al. (2018).[3] Second, we examine how oil volatility-of-volatility risk is priced in the delta-hedged option gains. Research closest in spirit to our analysis is Baltussen et al. (2018), and Agarwal et al. (2017), who find that volatility-of-volatility is an important factor in the cross section of stocks and hedge funds, respectively. Third, we find that oil volatility-of-volatility negatively predicts future delta-hedged gains of USO options, which is consistent with the similar findings of Huang et al. (2018) in equity index market. To our best knowledge, our work represents the first effort to examine volatility-of-volatility risk in the crude oil market, by providing both cross-sectional and time-series evidence.

To test our hypothesis, we first construct the volatility-of-volatility measure by using the Exponentially Weighted Moving Average (EWMA) model with OVX index. We also make a comparison with the equity index volatility and volatility-of-volatility risks. The equity market volatility and volatility-of-volatility index are surrogated by VIX and VVIX respectively.[4] The correlation between oil and equity index volatility-of-volatility is much lower than the oil and equity market volatility, which indicates that oil and equity volatility-of-volatility risks contain more distinct information than their volatility counterparts.

To the best of our knowledge, our research is the first attempt to investigate whether uncertainty about oil market volatility is priced in the cross section of option returns. Bakshi et al. (2003) implement similar study about whether volatility risk is cross-sectionally

[3] OVX is the "oil VIX", and it measures the next 30-day volatility of crude oil market.

[4] The CBOE Volatility Index, known as VIX, is a forward measure of the next 30-day stock market's expectation of volatility. The CBOE volatility-of-volatility index, the VVIX for short, represents the expected volatility of the 30-day forward price of the VIX.

priced in delta-hedged gains of S&P 500 options. Huang et al. (2018) analyze the equity market volatility-of-volatility and option returns associated with S&P 500 and VIX, and they also provide a testable model in which the expected delta-hedged gains can be expressed as a sum of market price of risk, and compensations for the volatility and volatility-of-volatility risks. Hollstein and Prokopczuk (2018) also confirm that the aggregate volatility-of-volatility risk is priced in the cross section of equity returns and contains negative predictive power for future volatility and volatility-of-volatility. We adopt the methodology of Huang et al. (2018) to check whether oil volatility and volatility-of-volatility risks are cross-sectionally priced in the delta-hedged USO option returns.[5]

We further investigate the predictive power of volatility-of-volatility risk in crude oil market. To achieve this goal, we run a set of time-series regressions, controlling for various jump measures. We find that both oil market volatility and volatility-of-volatility risks significantly negatively predict one-period-ahead delta-hedged gains of USO options, which is consistent with previous study in equity index and VIX markets such as Huang et al. (2018). We also run several robustness tests by: (i) taking a different construction method for volatility-of-volatility risk measure; (ii) considering the 1-week forecasting horizon by using 1-week USO options and running the time-series predictive regressions.

We also identify the distinct information content of oil volatility-of-volatility risk by investigating the predictive power of oil volatility, oil volatility-of-volatility and equity volatility-of-volatility on real personal consumption expenditure.[6] We find that oil volatility-of-volatility can significantly and negatively predict one-month forward log real personal consumption expenditure, controlling for oil volatility and equity volatility-of-volatility. Moreover, the contemporaneous analysis shows that both oil and equity volatility-of-volatility significantly impacts the log real personal consumption expenditure.

[5]As ETF shares similar structure of equity, USO options can provide equity-like features, which are simpler compared to options on West Texas Intermediate (WTI) crude oil futures.

[6]Personal consumption expenditure measures changes of prices in consumer goods and services. It is the component statistic for consumption in gross domestic product (GDP) and collected by the United States Bureau of Economic Analysis (BEA).

The remainder of this paper proceeds as follows. Section 2 explains the model framework that relates oil volatility-of-volatility risk and expected delta-hedged option payoff. Section 3 describes implementation details and the results about testing the market price of oil volatility-of-volatility risk. Section 4 reports a set of robustness tests in which we consider jump fears, other measure of oil volatility-of-volatility and 1-week USO delta-hedged gains. Finally, Section 5 summarizes the findings and concludes.

2 Model setup

2.1 Stochastic volatility-of-volatility model

In this section, we follow the framework of Huang et al. (2018) to feature the dynamics of underlying returns by incorporating the time-varying volatility and volatility-of-volatility risks. Assume that under the physical measure $\mathbb{P}$, the underlying price S_t follows

$$\frac{dS_t}{S_t} = \mu(S_t, v_t, \eta_t)dt + \sqrt{v_t}d\omega_t^1, \tag{1}$$

$$dv_t = \theta(v_t)dt + \sqrt{\eta_t}d\omega_t^2, \tag{2}$$

$$d\eta_t = \gamma(\eta_t)dt + \phi\sqrt{\eta_t}d\omega_t^3, \tag{3}$$

where $(\omega_t^1, \omega_t^2, \omega_t^3)_{t\geq 0}$ is a three-dimensional Brownian motion with correlation coefficient $corr(d\omega_t^i, d\omega_t^j) = \rho_{ij}$ for all $i \neq j$. v_t denotes the stochastic variance of instantaneous returns and η_t denotes the variance of innovation in v_t, which follows an autonomous stochastic process.

Correspondingly, under the risk-neutral probability measure $\mathbb{Q}$, the dynamics for the underlying price S_t is

$$\frac{dS_t}{S_t} = rdt + \sqrt{v_t}d\tilde{\omega}_t^1, \tag{4}$$

$$dv_t = (\theta(v_t) - \lambda_t^v)dt + \sqrt{\eta_t}d\tilde{\omega}_t^2, \tag{5}$$

$$d\eta_t = (\gamma(\eta_t) - \lambda_t^\eta)dt + \phi\sqrt{\eta_t}d\tilde{\omega}_t^3, \tag{6}$$

where $(\tilde{\omega}_t^1, \tilde{\omega}_t^2, \tilde{\omega}_t^3)_{t\geq 0}$ represents the corresponding three-dimensional Brownian motion under the risk-neutral probability measure $\mathbb{Q}$. Moreover, λ_t^v represents the compensation for the stochastic variance and λ_t^η represents the compensation for the stochastic variance of variance.

Consider a call option written on the underlying S_t. By Itô's Lemma, its price at time $t+\tau$ can be expressed as

$$C_{t+\tau} = C_t + \int_t^{t+\tau} \frac{\partial C_u}{\partial S_u} dS_u + \int_t^{t+\tau} \frac{\partial C_u}{\partial v_u} dv_u + \int_t^{t+\tau} \frac{\partial C_u}{\partial \eta_u} d\eta_u + \int_t^{t+\tau} b_u du, \tag{7}$$

where b_t is defined as

$$\begin{aligned} b_t = \frac{\partial C_t}{\partial t} + \frac{1}{2} v_t S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} + \frac{1}{2}\eta_t \frac{\partial^2 C_t}{\partial v_t^2} + \frac{1}{2}\phi^2 \eta_t \frac{\partial^2 C_t}{\partial \eta_t^2} + \rho_{12}\sqrt{v_t}\sqrt{\eta_t} S_t \frac{\partial^2 C_t}{\partial S_t \partial v_t} \\ + \rho_{13}\phi\sqrt{v_t}\sqrt{\eta_t} S_t \frac{\partial^2 C_t}{\partial S_t \partial \eta_t} + \rho_{23}\phi\eta_t \frac{\partial^2 C_t}{\partial v_t \partial \eta_t}. \end{aligned}$$

Meanwhile, the valuation equation that determines the price of the call option C_t is

$$rS_t \frac{\partial C_t}{\partial S_t} + (\theta(v_t) - \lambda_t^v) \frac{\partial C_t}{\partial v_t} + (\gamma(\eta_t) - \lambda_t^\eta) \frac{\partial C_t}{\partial \eta_t} + b_t - rC_t = 0. \tag{8}$$

Rearranging Eq.(8), we obtain that

$$b_t = r\left(C_t - S_t \frac{\partial C_t}{\partial S_t}\right) - (\theta(v_t) - \lambda_t^v) \frac{\partial C_t}{\partial v_t} - (\gamma(\eta_t) - \lambda_t^\eta) \frac{\partial C_t}{\partial \eta_t}. \tag{9}$$

Recall the delta-hedged gain (i.e., $\Pi_{t,t+\tau}$) defined by Eq.(16). Consider the dynamics of v_t and η_t defined by Eq.(2) and (3), respectively, it can be simplified by substituting the expression of b_t in Eq.(9) into Eq.(7)

$$\Pi_{t,t+\tau} = \int_t^{t+\tau} \lambda_u^v \frac{\partial C_u}{\partial v_u} du + \int_t^{t+\tau} \lambda_u^\eta \frac{\partial C_u}{\partial \eta_u} du + \int_t^{t+\tau} \sqrt{\eta_u} \frac{\partial C_u}{\partial v_u} d\omega_u^2 + \int_t^{t+\tau} \phi\sqrt{\eta_u} \frac{\partial C_u}{\partial \eta_u} d\omega_u^3. \tag{10}$$

Therefore, the expected value of the delta-heged gain (i.e., $\Pi_{t,t+\tau}$) under physical measure $\mathbb{P}$ is given by considering the martingale property of the Itô integral

$$E_t\left[\Pi_{t,t+\tau}\right] = \int_t^{t+\tau} E_t\left[\lambda_u^v \frac{\partial C_u}{\partial v_u}\right] du + \int_t^{t+\tau} E_t\left[\lambda_u^\eta \frac{\partial C_u}{\partial \eta_u}\right] du, \tag{11}$$

where λ_t^v and λ_t^η denotes the compensations for stochastic volatility and volatility-of-volatility risks, respectively. Moreover, $\frac{\partial C_u}{\partial v_u}$ and $\frac{\partial C_u}{\partial \eta_u}$ represents the vega and volga of the call option, respectively. The above equation shows that the expected delta-hedged gains depend on the risk premiums of volatility and volatility-of-volatility.

2.1.1 Testable implications

For tractability, we further assume that the risk premiums have linear structure

$$\lambda_t^v = \lambda^v v_t, \quad \lambda_t^\eta = \lambda^\eta \eta_t, \tag{12}$$

where λ^v and λ^η represent constant market prices of volatility and volatility-of-volatility risks, respectively.

By applying Itô-Taylor expansions on Eq.(11), the scaled expected delta-hedged gain is linear with respect to the fundamental factors

$$\frac{E_t\left[\Pi_{t,t+\tau}\right]}{S_t} = \lambda^v \beta_t^v v_t + \lambda^\eta \beta_t^\eta \eta_t. \tag{13}$$

In the above equation, β_t^v and β_t^η represent the options' exposure to the volatility and volatility-of-volatility risk, respectively, and are expressed as

$$\begin{aligned} \beta_t^v &= \sum_{n=0}^{\infty} \frac{\tau^{1+n}}{(1+n)!} \Phi_{t,n}^v > 0, \\ \beta_t^\eta &= \sum_{n=0}^{\infty} \frac{\tau^{1+n}}{(1+n)!} \Phi_{t,n}^\eta > 0, \end{aligned} \tag{14}$$

where $\Phi^v()$ and $\Phi^\eta()$ are positive functions depending on option moneyness, vega and volga (i.e. m, $\frac{\partial C_u}{\partial v_u}$ and $\frac{\partial C_u}{\partial \eta_u}$), respectively.

3 Data and methodology

3.1 Data information

The empirical analysis spans from August, 2010 to June, 2018. We use the USO option data which are obtained from Thomson Reuters Ticker History (TRTH) of SIRCA to construct the delta-hedged gains.[7] Before computation, we filter out the option data with incomplete or incorrect information. Specifically, we remove options with a zero close bid, options with close ask greater than close bid, options with prices violating the standard no-arbitrage condition and options with Black-Scholes implied volatility greater than 100% and less than 1%. We

[7] Compare to options written on crude oil futures, ETF option data have the advantage of equity-option-like structure, which is much simpler.

mainly focus on delta-hedged gains obtained from 1-month options in the following analysis. Note that weekly USO options are launched in July 2010, and we will also construct delta-hedged gains by using weekly options to make a comparison with their counterpart computed from 1-month options. We use the 5-min intraday prices of USO ETF, obtained from Thomson Reuters Datascope Select, to calculate the model-free jump measures which are proposed by Barndorff-Nielsen et al. (2004) and Barndorff-Nielsen and Shephard (2006). The risk-free rates are proxied by Libor rates, obtained from Bloomberg.

3.2 Key variable definitions

3.2.1 Volatility-of-volatility (vov) measures

We adopt the CBOE Crude Oil Volatility Index (OVX) as proxy of crude oil market volatility, which measures the forward 30-day volatility of the market.[8] We then construct the volatility-of-volatility measure by using the Exponentially Weighted Moving Average (EWMA) model with OVX index[9]:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) u_{t-1}^2, \tag{15}$$

where u_t is the log return of OVX, σ_t is the conditional volatility, and λ is the degree of weighting decrease which we take the value of 0.94 here.

3.2.2 Delta-hedged option gains

A delta-hedged option portfolio consists of longing an option, and shorting delta, implied by option, units of its underlying stock, with the net income investing at risk-free rate. Therefore, the gains of such a portfolio are insensitive to the underlying stock.

Let $C(t, \tau, K)$ denote the time-t price of a call option with time-to-maturity τ and strike price K and the corresponding option delta is represented by $\Delta(t, \tau, K)$, then the delta-hedged gains,

[8]It is the "oil VIX", and the method to construct it is similar to that of VIX methodology, that is, extracting market volatility information from option data.

[9]EWMA model is widely used in practice because of its simplicity and non-strict requirement for data.

denoted by $\Pi_{t,t+\tau}$, can be expressed as

$$\Pi_{t,t+\tau} = C_{t+\tau} - C_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r_u(C_u - \Delta_u S_u)du, \tag{16}$$

where S_t denotes the time-t price of the underlying stock and r_t denotes the risk-free rate at time t. For compactness, we replace $C(t, \tau, K)$ and $\Delta(t, \tau, K)$ with the shorthand notation C_t and Δ_t (i.e., $\Delta_t = \frac{\partial C_t}{\partial S_t}$) , respectively.

3.3 Summary statistics of the key variables

3.3.1 Volatility-related variables

Table I reports the descriptive statistics for the crude oil market volatility (i.e., *vol*), crude oil market volatility-of-volatility (i.e., *vov*), equity market volatility (i.e., VIX) and equity market volatility-of-volatility (i.e., VVIX). [10] Notably *vov* exhibits much larger skewness and kurtosis than other variables, which suggests that oil volatility-of-volatility is highly asymmetric.

[Insert Table I here]

Table II reports the cross-sectional correlations among the variables. The correlation between oil market volatility (i.e., *vol*) and volatility-of-volatility (i.e., *vov*) is 0.250, suggesting the comovement between volatility and volatility-of-volatility exists in the crude oil market but is limited to some extent. It is also consistent with the time-series plots of *vol* and *vov*, as depicted by Figure 2. Notably in Figure 2, the spikes exhibited by *vol* and *vov* reveal quite different patterns. Moreover, the correlation between oil market volatility (i.e., *vol*) and equity market volatility (i.e., VIX) is 0.493, much higher than the correlation between oil market volatility-of-volatility (i.e., *vov*) and equity market volatility-of-volatility (i.e., VVIX), which is around 0.169. It highlights the fact that volatility and volatility-of-volatility risks in the crude oil and equity markets contain distinct information.

[Insert Table II here]

[Insert Figure 2 here]

[10] We also incorporate the equity market volatility and volatility-of-volatility risks here for making a comparison with the crude oil market counterparts.

3.3.2 Statistical properties of delta-hedged gains

In practice, we consider the discrete-time counterpart of Eq.(16), with a daily rebalancing

$$\Pi_{t,t+\tau} = C_{t+\tau} - C_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} r^f_{t_n} \left(C_t - \Delta_{t_n} S_{t_n}\right) \frac{\tau}{N}, \tag{17}$$

where Δ_{t_n} is the option delta, implied by the Black and Scholes (1973) model, on day t_n and N is the total number of trading days from time t to $t + \tau$.

Table III reports the descriptive statistics for the unscaled delta-hedged gains (i.e., $\Pi_{t,t+\tau}$) and scaled delta-hedged gains (i.e., $\frac{\Pi_{t,t+\tau}}{S_t}$) constructed on 1-month USO call and put options, grouped by moneyness (i.e., $\frac{S_t e^{r\tau}}{K}$). On average, both delta-hedged gains, unscaled and scaled, are significantly negative over all moneyness, suggesting that the strategy loses money, which is consistent with previous findings in the equity market (see, for example, Bakshi and Kapadia, 2003; Cao and Han, 2013; Carr and Wu, 2009).

[Insert Table III here]

Additional empirical observations that appear broadly consistent with volatility and volatility-of-volatility risks can be made. The magnitudes of delta-hedged gains become smaller when getting deeper in- and out-of-the-money for most of the cases. Consider put options with moneyness $m \in [0.900, 0.925]$ versus put options with moneyness $m \in [0.975, 1.000]$, from which we can observe that the unscaled delta-hedged gains are \$-0.068 versus \$-0.071, and the scaled delta-hedged gains are -0.19% versus -0.21%. The curve for vega is (i.e., $\frac{\partial C_t}{\partial \sigma_t}$) concave: it reaches the maximum point for at-the-money strikes, and tends to decrease for away-from-the-money strikes, as depicted by Figure 1, which suggests that the impact of volatility risk is getting smaller for away-from-the-money options. However, we also observe that for put options with moneyness $m \in [0.925, 0.950]$, its unscaled delta-hedged gains are \$-0.076 and scaled delta-hedged gains are -0.20% on average, where the former is greater than and the latter is similar to their at-the-money counterparts. It is inconsistent with the decreasing magnitude of vega for away-from-the-money options. It may be noted that the curve of volga (i.e., $\frac{\partial^2 C_t}{\partial \sigma_t^2}$) is convex around the range of near-the-money, as presented by Figure 1, which reaches the lowest point for at the money strikes, and tends to increase when getting away from the moneyness equal to 1. It suggests the increasing volatility-of-volatility risk for away-from-the-money options.

[Insert Figure 1 here]

Moreover, we find that the losses of delta-hedged portfolios increase with the extending hedging horizons. It can be easily observed from the delta-hedged gains constructed on 1-week USO options, represented by Table IV. For example, for the call option groups with moneyness $m \in [0.975, 1.000]$, the unscaled delta-hedged gains (i.e. $\Pi_{t,t+\tau}$) are \$-0.031 and \$-0.056 for the 1-week and 1-month options, respectively. Similar observations can also be made for options in other moneyness groups. Thus, the delta-hedged gains tend to increase (i.e., decrease with respect to the magnitudes) when the hedging horizons are extending.

[Insert Table IV here]

Overall, the delta-hedged gains are negative for all options in the crude oil market, and the non-monotonous changes of delta-hedged gains with respect to moneyness indicate the possible impacts of volatility and volatility-of-volatility risks in the crude oil market.

4 Empirical analysis

Previous studies find that the delta-hedged strategy underperforms zero in the equity market (see, for example, Bakshi and Kapadia, 2003; Carr and Wu, 2009). Bakshi and Kapadia (2003) confirm the significant impact of volatility risk on the negative delta-hedged gains in the equity index market. Recent work of Huang et al. (2018) proves that volatility-of-volatility risk also plays a significant role in pricing the delta-hedged gains. However, the question of how delta-hedged strategy performs in crude oil market and whether volatility and volatility-of-volatility risks significantly affect the portfolio gains remain unanswered.

4.1 Delta-hedged gains and stochastic volatility-of-volatility risk

4.1.1 Cross-sectional analysis

The stochastic volatility-of-volatility model implies that options with higher exposures to volatility and volatility-of-volatility risks should have more negative gains. To empirically verify this justification, we follow previous literature (see, e.g., Bakshi and Kapadia, 2003; Huang et al., 2018) to employ option Greeks (i.e. vega and volga) implied by Black and Scholes (1973) model to proxy the options' volatility and volatility-of-volatility risk betas. Then we adopt the following econometric specification to check the cross-sectional implications of delta-hedged option gains on the volatility and volatility-of-volatility risks

$$\text{Gains}^i_{t,t+\tau} = \Phi_1 \text{Vega}^i_t + \Phi_2 \text{Volga}^i_t + \gamma_t + \epsilon^i_{t,t+\tau}, \tag{18}$$

where we define the scaled delta-hedged gains as $\text{Gains}^i_{t,t+\tau} \equiv \frac{\Pi^i_{t,t+\tau}}{S_t}$. We also denote the Black and Scholes (1973) implied option vega and volga by Vega^i_t and Volga^i_t. Let i represent the moneyness range ($i = 1, 2, \ldots, I$) corresponding to option groups shown in Table III. We use γ_t to denote the time-fixed effects.

Table V reports the cross-sectional evidence from the regressions of average scaled delta-hedged gains (i.e., Gains^i) on options' volatility and volatility-of-volatility risk betas. The univariate estimate for Vega is significantly negative, with a value of 0.058 and t-statistic of 4.80. The negative volatility premiums in crude oil market is consistent with previous findings about negative volatility risk premium (see, Bakshi and Kapadia, 2003; Bardgett et al., 2019; Bollerslev et al., 2011). The second row shows the univariate estimate for Volga, which is insignificant. The third row presents the multivariate regression in which both Vega and Volga are incorporated. Both estimates are negative and statistically significant at 1% level: the slope of Vega is -0.119 and the corresponding t-stat is -7.22; the slope of Volga is -0.020 and the corresponding t-stat is -5.38. [11] The negative price of volatility-of-volatility risk in crude oil market is consistent with that in equity index market, as reported by Huang et al. (2018). One notable thing is that the volatility risk is not statistically significantly priced in Huang et al. (2018). In sum, the cross-sectional evidence indicates that both volatility risk and volatility-of-volatility risks are negatively and significantly priced in the crude oil market.

[Insert Table V here]

[11] We scale up both slopes by 1000 here.

4.1.2 Time-series analysis

Under the stochastic volatility-of-volatility model, we can see that time variations of the expected delta-hedged gains are driven by the time-varying volatility and volatility-of-volatility risks. Thus, we adopt the following econometric specification to examine the impacts of both risks on the time variations of scaled delta-hedged gains

$$\text{Gains}^i_{t,t+\tau} = \Phi_0 + \Phi_1 vol_t^2 + \Phi_2 vov_t^2 + \text{Gains}^i_{t-\tau,t} + u^i + \epsilon^i_{t+\tau}, \tag{19}$$

where we include fixed effect u^i to account for the heterogeneity in the sensitivity of options in various moneyness bins to the underlying volatility and volatility-of-volatility risks. Moreover, vol_t represents the oil market volatility, vov_t represents the oil market volatility-of-volatility and $\text{Gains}^i_{t-\tau,t}$ represents the 1-month lagged delta-hedged gains, with i indexing the moneyness ranges the USO options belonging to.

[Insert Table VI here]

The results are reported in Table VI. The first column of Table VI shows the results for the case that vov^2 is a single predictor. It significantly predicts 1-period-ahead delta-hedged gains, with a negative coefficient of -0.894 (t-stat: -7.83), which is consistent with the results of cross-sectional analysis. The second column of Table VI is a joint prediction model incorporating both vol^2 and vov^2, which shows that both the coefficients of vol^2 and vov^2 are negative and highly significant. The findings for crude oil market are very similar to those of equity index market as demonstrated by Bakshi and Kapadia (2003) and Huang et al. (2018). Overall, the time-series evidence also indicates that both volatility and volatility-of-volatility risks are negatively and significantly priced in the crude oil market.

4.2 Information content of the oil volatility-of-volatility risk

So far, we have confirmed that both oil volatility and volatility-of-volatility risks are significantly and negatively priced in the USO option market, and this part of work builds on previous study of Huang et al. (2018) which focuses on equity volatility and volatility-of-volatility risks and delta-hedged S&P 500 option gains. In this section, we further explore whether the information content of oil volatility-of-volatility risk is different from its equity market counterpart, as it has been proved that volatility risk of oil and equity markets are different from each other and both are important pricing factors (e.g., Christoffersen and Pan, 2018; Gao et al., 2018).

To achieve this goal, we analyze the contemporaneous and predictive relationships between oil market volatility-of-volatility (i.e., *vov*) and real personal consumption expenditure (i.e., PRCE), by including a set of volatility-related control variables. Specifically, we run the predictive regression as

$$\frac{1}{h}\sum_{j=1}^{h} \ln \mathrm{RPCE}_{t+j} = \alpha_h + \beta_h vov_t + \gamma_h^1 vol_t + \gamma_h^2 \mathrm{VVIX}_t + \gamma_h^3 \ln \mathrm{RPCE}_{t-1} + \epsilon_t, \tag{20}$$

where the dependent variable $\ln \mathrm{RPCE}$ is the log of real personal consumption expenditure, *vov* is the oil market volatility-of-volatility, *vol* is the oil market volatility and VVIX is the equity market volatility-of-volatility. The forecasting horizon h ranges from 1 month to 12 months.

We also consider the contemporaneous relationship between the real personal consumption expenditure and the oil volatility-of-volatility, controlling for other volatility-related variables.

$$\ln \mathrm{RPCE}_t = \alpha + \beta vov_t + \gamma^1 vol_t + \gamma^2 \mathrm{VVIX}_t + \gamma^3 \ln \mathrm{RPCE}_{t-1} + \epsilon_t. \tag{21}$$

The results are reported in Table VII. The slope of oil volatility-of-volatility (i.e., *vov*) is significant and negative for contemporaneous and 1-month predictive regressions, even controlling for oil volatility (i.e., *vol*) and equity volatility-of-volatility (i.e., VVIX). The oil volatility-of-volatility can affect real personal consumption expenditure lies on the fact that oil industry is closely related to real economy (e.g., Arezki et al., 2017; Maghyereh et al., 2019). In all, oil volatility-of-volatility contains specific information, which can be convered by the oil market volatility and equity market volatility-of-volatility.

[Insert Table VII here]

5 Further empirical analysis

6 Robustness

6.1 Robustness check: Control for jump risks

Under the stochastic volatility-of-volatility model, as specified by Eq.(11)-(3), jump risk is not considered, however, as demonstrated by previous research, adding jumps can improve the performance of pricing models (e.g., Bakshi et al., 1997; Bates, 1996; Duffie et al., 2000). In this section, we introduce jump risk into the model, which considers the impacts of stochastic volatility, stochastic volatility-of-volatility and jump risks, thus we can further investigate whether the pricing role of oil volatility-of-volatility is affected by jumps.

6.1.1 Stochastic volatility-of-volatility with jumps model

Under the physical measure $\mathbb{P}$, we assume the underlying price S_t follows the dynamic process

$$\frac{dS_t}{S_t} = \mu(S_t, v_t, \eta_t)dt + \sqrt{v_t}d\omega_t^1 + (e^x - 1)dq_t - \mu_J \Lambda_J \sqrt{v_t}dt, \tag{22}$$

$$dv_t = \theta(v_t)dt + \sqrt{\eta_t}d\omega_t^2, \tag{23}$$

$$d\eta_t = \gamma(\eta_t)dt + \phi\sqrt{\eta_t}d\omega_t^3. \tag{24}$$

Note that this framework allows random jumps, along with stochastic volatility and volatility-of-volatility, to play a role in pricing the underlying S_t. The jump term follows a Poisson process with volatility-independent intensity $\Lambda_J\sqrt{v_t}$. We use x to denote the jump size and q to represent its corresponding physical density. We further assume that the mean of $e^x - 1$ is μ_J, then the compensator is $\mu_J\Lambda_J\sqrt{v_t}dt$.

Assume that under the risk-neutral probability measure $\mathbb{Q}$, the density of the jump size (i.e., x) is q, and the mean of $e^x - 1$ is μ_J^*, then the dynamics of the underlying S_t can be written as

$$\frac{dS_t}{S_t} = rdt + \sqrt{v_t}d\tilde{\omega}_t^1 + (e^x - 1)dq_t^* - \mu_J^* \Lambda_J \sqrt{v_t}dt, \tag{25}$$

$$dv_t = (\theta(v_t) - \lambda_t^v)dt + \sqrt{\eta_t}d\tilde{\omega}_t^2, \tag{26}$$

$$d\eta_t = (\gamma(\eta_t) - \lambda_t^\eta)dt + \phi\sqrt{\eta_t}d\tilde{\omega}_t^3. \tag{27}$$

By Itô's Lemma, the call option price at time $t+\tau$ can be written as

$$\begin{aligned} C_{t+\tau} = C_t &+ \int_t^{t+\tau} \frac{\partial C_u}{\partial S_u} dS_u + \int_t^{t+\tau} \frac{\partial C_u}{\partial v_u} dv_u + \int_t^{t+\tau} \frac{\partial C_u}{\partial \eta_u} d\eta_u \\ &+ \Lambda_J \int_t^{t+\tau} \sqrt{v_u} \int_{-\infty}^{\infty} \left[C(S_t e^x) - C(S_t)\right] q(x) dx du + \int_t^{t+\tau} b_u du, \end{aligned} \tag{28}$$

where b_t is the same as defined in Eq.(2.1).

Moreover, the valuation equation is given by

$$\begin{aligned} (r - \mu_J^* \Lambda_J \sqrt{v_t})\, S_t \frac{\partial C_t}{\partial S_t} + (\theta(v_t) - \lambda_t^v)\, \frac{\partial C_t}{\partial v_t} + (\gamma(\eta_t) - \lambda_t^\eta)\, \frac{\partial C_t}{\partial \eta_t} \\ + \Lambda_J \sqrt{v_t} \int_{-\infty}^{\infty} \left[C(S_t e^x) - C(S_t)\right] q(x)^* dx + b_t - rC_t = 0. \end{aligned} \tag{29}$$

Rearranging Eq.(29), we obtain that

$$\begin{aligned} b_t = r\left(C_t - S_t \frac{\partial C_t}{\partial S_t}\right) &- (\theta(v_t) - \lambda_t^v)\, \frac{\partial C_t}{\partial v_t} - (\gamma(\eta_t) - \lambda_t^\eta)\, \frac{\partial C_t}{\partial \eta_t} \\ &+ \mu_J^* \Lambda_J \sqrt{v_t} S_t \frac{\partial C_t}{\partial S_t} - \Lambda_J \sqrt{v_t} \int_{-\infty}^{\infty} \left[C(S_t e^x) - C(S_t)\right] q(x)^* dx. \end{aligned} \tag{30}$$

Therefore, the delta-heged gain (i.e., $\Pi_{t,t+\tau}$) under physical measure $\mathbb{P}$ is given

$$\begin{aligned} E_t\left[\Pi_{t,t+\tau}\right] = &\int_t^{t+\tau} E_t\left[\lambda_u^v \frac{\partial C_u}{\partial v_u}\right] du + \int_t^{t+\tau} E_t\left[\lambda_u^\eta \frac{\partial C_u}{\partial \eta_u}\right] du + \mu_J^* \Lambda_J \int_t^{t+\tau} E_t\left[\sqrt{v_u} S_u \frac{\partial C_u}{\partial S_u}\right] du \\ &- \Lambda_J \int_t^{t+\tau} \sqrt{v_u} \left[\int_{-\infty}^{\infty} C(S_u e^x) q(x)^* dx - \int_{-\infty}^{\infty} C(S_u e^x) q(x) dx\right] du. \end{aligned} \tag{31}$$

Compared to the expected delta-hedged gain under stochastic volatility framework in Eq.(11), if jump fears are considered, Eq.(31) indicates that the delta-hedged gain is also affected by jump risk.

6.1.2 Jump risk measures

If the jump risk is considered, as shown by Eq.(24), the delta-hedged gain is also attributed to the large, discontinuous jumps in the underlying prices, apart from compensations for the volatility-of-volatility risk. To address the concern whether the effect of volatility-of-volatility risk is robust to the presence of jump risk, we adopt two measures of jump risks.

The first jump measure is the risk-neutral skewness and kurtosis as the jump fears can be proxied by skewness and kurtosis of the underlying return distribution (see, for example, Bakshi et al., 2003; Bates, 2000; Jackwerth and Rubinstein, 1996), where the former is closely related to jump size and the latter is closely related to jump intensity. We construct the risk-neutral skewness and kurtosis by using the model-free approach of Bakshi et al. (2003). Specifically, the higher-order risk-neutral moments can be spanned by a set of out-of-money calls and puts,

$$\begin{aligned} \text{skew}_{t,t+\tau} &= \frac{e^{r\tau} W_{t,t+\tau} - 3\mu_{t,t+\tau} e^{r\tau} V_{t,t+\tau} + 2\mu^3_{t,t+\tau}}{\left[e^{r\tau} V_{t,t+\tau} - \mu^2_{t,t+\tau}\right]^{3/2}}, \\ \text{kurt}_{t,t+\tau} &= \frac{e^{r\tau} X_{t,t+\tau} - 4\mu_{t,t+\tau} e^{r\tau} W_{t,t+\tau} + 6e^{r\tau}\mu^2_{t,t+\tau} V_{t,t+\tau} - 3\mu^4_{t,t+\tau}}{\left[e^{r\tau} V_{t,t+\tau} - \mu^2_{t,t+\tau}\right]^2}, \end{aligned} \tag{32}$$

where $V_{t,t+\tau}$, $W_{t,t+\tau}$ and $X_{t,t+\tau}$ are based on the prices of volatility, cubic and quartic contract, and $\mu_{t,t+\tau}$ is a function of them. The detailed expressions can be found in the Appendix A.1.

The second jump measure is constructed by applying the model-free methodology of Barndorff-Nielsen et al. (2004) and Barndorff-Nielsen and Shephard (2006) to high-frequency USO prices. According to Barndorff-Nielsen et al. (2004), the realized variance, the sum of squared intraday returns, is a consistent estimate for the integrated volatility as long as there are no jumps

$$RV_t = \sum_{i=1}^{M_t} r^2_{t,i}, \tag{33}$$

where M_t is the total number of observations, and $r_{t,i}$ is the ith 5-min return during the trading day t. Barndorff-Nielsen et al. (2004), Barndorff-Nielsen and Shephard (2006) as well as Huang and Tauchen (2005) further show that the bipower variation, the sum of the product of adjacent absolute intraday returns, is a consistent estimate of the integrated volatility, even in the presence of jumps

$$BV_t = \frac{\pi}{2} \frac{M}{M-1} \sum_{i=2}^{M} |r_{t,i}||r_{t,i-1}|\,. \tag{34}$$

Therefore, the jump measure can be constructed based on the realized variance and bipower variation defined above

$$J_t = \text{sign}\left(r_t\right) \times \sqrt{\left(RV_t - BV_t\right) \times \mathbb{I}\left(ZJ_t \geq \Phi^{-1}_{\alpha}\right)} \tag{35}$$

where Φ^{-1}_{α} denotes the inverse cumulative distribution function (CDF) of the standard normal distribution, and we set $\alpha = 99.9\%$ here. $\mathbb{I}()$ is an indicator function. The specific expression for ZJ_t can be found in Appendix A.2.

Furthermore, the model-free jump risk measure (i.e., J) can be grouped into positive and negative jumps based on the sign of market return of the day

$$\begin{aligned} J_t^+ &= \mathbb{I}\left(r_t \geq 0\right) J_t \\ J_t^- &= \mathbb{I}\left(r_t < 0\right) J_t, \end{aligned} \tag{36}$$

where r_t denotes the daily return of day t.

6.1.3 Empirical analysis

So far, we have identified the effects of volatility and volatility-of-volatility risks on the delta-hedged gains. The losses on the delta-hedged portfolios might also be attributed to the fear of market crashes, namely jump risks, as demonstrated by the stochastic volatility-of-volatility with jumps model in Section 6.1.1. In this section, we empirically verify the joint effects of volatility, volatility-of-volatility and jump risks on the scaled delta-hedged gains, by including the jump risk measures. Specifically, we consider two sets of jump measures as described in Section A.2.

The first set of jump measures considered is the risk-neutral skewness and kurtosis extracted from the whole cross section USO option prices, by using the model-free approach of Bakshi et al. (2003). The econometric specification is as follows

$$\text{Gains}_{t,t+\tau}^i = \Phi_0 + \Phi_1 vol_t + \Phi_2 vov_t + \Phi_3 skew_t + \Phi_4 kurt_t + \text{Gains}_{t-\tau,t}^i + u^i + \epsilon_{t+\tau}^i, \tag{37}$$

where $skew_t$ and $kurt_t$ denote the risk-neutral skewness and kurtosis of trading day t, respectively.

Column 3, 4, and 5 display of Table VI display the results after controlling for risk-neutral skewness (i.e., *skew*) and kurtosis (i.e., *kurt*). The predictive power of volatility (i.e., *vol*) and volatility-of-volatility (i.e., *vov*) risks is not affected by the risk-neutral jump measures, and the corresponding slopes Φ_1 and Φ_2 remain largely unchanged. Moreover, regarding the predictability of risk-neutral jumps, higher jump fear risk predicts more negative delta-hedged gains in the future: *kurt* significantly and negatively predicts future gains, no matter alone or combining with *skew*. This finding is consistent with similar study in the equity index market (e.g., Bakshi et al., 2003; Huang et al., 2018).

The second set of jump measure is constructed on the 5-min intraday USO data. We further divide the jump measure into positive/negative components based on market moving directions.

Thus, we consider the following two sets of regressions

$$\begin{aligned}\text{Gains}^i_{t,t+\tau} &= \Phi_0 + \Phi_1 vol_t + \Phi_2 vov_t + \Phi_3 J_t + \Phi_4 \text{Gains}^i_{t-\tau,t} + u^i + \epsilon^i_{t+\tau}, \\ \text{Gains}^i_{t,t+\tau} &= \Phi_0 + \Phi_1 vol_t + \Phi_2 vov_t + \Phi_3 J^+_t + \Phi_4 J^-_t + \Phi_5 \text{Gains}^i_{t-\tau,t} + u^i + \epsilon^i_{t+\tau},\end{aligned} \tag{38}$$

where J_t represents the jump measure at trading day t; J^+_t and J^-_t stands for the positive and negative jumps, respectively.

We control for the realized jump measures, both decomposed and undecomposed, with the results reported in Column 6 and 7 of Table VI. *vol* and *vov* still strongly and negatively predict future delta-hedged option gains, even after including the control variables. Similar to the case of controlling for risk-neutral jumps, the coefficients of realized jump measures (i.e. J, J^+, and J^-) are highly significant. Overall, the predictability of oil volatility-of-volatility risk is robust after controlling for jump risks.

6.2 Robustness check: Other volatility-of-volatility measure

We build the second measure of oil volatility-of-volatility by following Baltussen et al. (2018). Specifically, we scale the standard deviation of oil market volatility which is still proxied by OVX on day t with the average volatility over the past month as follows

$$vov^B_t = \frac{\sqrt{\frac{1}{22}\sum_{j=t-21}^{t}(vol_t - \overline{vol}_t)^2}}{\overline{vol}_t}, \tag{39}$$

where vov^B is the second measure of oil volatility-of-volatility, *vol* is the oil volatility and $\overline{vol}_t = \sum_{j=t-21}^{t} vol_j$.

[Insert Table VIII here]

The results are displayed by Table VIII. Similar to our previous findings, the univariate regression of delta-hedged USO option gains on oil volatility-of-volatility (i.e., vov^B) generates a highly significant and negative coefficient. Moreover, in multivariate regressions by adding oil volatility and jump measures, vov^B remains highly significant. It suggests that oil volatility-of-volatility is significantly priced in the delta-hedged gains of USO options, even though we construct it using an alternative method.

6.3 Impacts of maturities: weekly options

Weekly options, or called "weeklies ", have firstly been introduced by the Chicago Board Options Exchange (CBOE) in October 2005. They are around one week to expiration, while traditional options normally have a life of months or years before expiration. The addition of weekly options produces benefits such as greater trading flexibility and precision timing, thus, the trading of weeklies grows rapidly.[12] It is worth noting that USO weekly options are introduced in July, 2010. Therefore, we also construct delta-hedged gains by using 1-week USO options. We then implement similar analysis as 1-month delta-hedged gains to investigate the impacts of volatility, volatility-of-volatility and jump risks on them.

There are mainly two advantages of using USO weeklies. First, as Andersen et al. (2017) stated, the pricing of short-dated at-the-money (ATM) options mainly depends primarily on diffusive component whereas deep OTM options mainly reflect the characteristics of the risk-neutral jump process. Using weekly ATM options can be an effective way to control the jump fears effects and focus primarily on diffusive risks. Second, using weekly options can increase sample size and complement our main results obtained from use of monthly options. As we reported in Section 3, volatility (i.e., *vol*) and volatility-of-volatility (i.e., *vov*) risks show sizable time variations.

[Insert Table IX here]

Column 1, 2, and 3 of Table IX show our results associated with 1-week delta-hedge option gains. *vol* and *vov* remain significant and negative predictive power 1-week forward delta-hedged option gains. Specifically, the t-stat for the coefficient of *vov* is -5.01 for predicting 1-week option gains, whereas the corresponding t-stat is -3.26 for the predicting 1-month option gains. We also consider the model specifications with jump fears proxies. Even after controlling for jump fears, the coefficient of *vov* is still negative and significant; however, the coefficient of *vol* is not significant anymore. Consistent with Andersen et al. (2017) that jump fear effect is lower for weekly ATM options, the predictive powers of jump fear proxies are weaker than those associated with 1-month delta-hedged option returns. Specifically, the coefficients of risk-neutral skewness (i.e., *skew*), risk-neutral kurtosis (i.e., *kurt*), and downside realized jump risk (i.e., J^-) are not

[12]For example, according to Andersen et al. (2017), the trading shares of S&P 500 weeklies have reached by 40%.

significant whereas those associated with realized jump (i.e., J) and upside realized jump (i.e., J^+) are statistically significant.

7 Conclusion

Based on the theoretical framework featuring stochastic volatility-of-volatility risk, which shows that the expected delta-hedged option gains depend on the compensations for volatility and volatility-of-volatility risks, we provide a detailed empirical analysis for oil volatility-of-volatility risk.

We demonstrate that oil volatility-of-volatility is a risk factor significantly priced in the cross section of USO option delta-hedged gains, beyond the oil volatility risk. Such finding is consistent with the descriptive statistics that can be directly observed: (i) The negative value of average delta-hedged USO option gains suggests that investors require compensations to undertake oil volatility and volatility-of-volatility risks; (ii) The low correlation between oil volatility and oil volatility-of-volatility, with the value of 0.250, indicates that the information contained by the two risk factors are different.

We have also verified that oil volatility-of-volatility can contribute to predict one-period ahead delta-hedged gains, along with oil volatility. The predictive power of oil volatility-of-volatility remains significant after implementing several robustness checks. The first check is incorporating some jump risk measures, such as the risk-neutral skewness, risk-neutral kurtosis and realized jumps extracted from high-frequency USO data. In the second robustness check we adopt the methodology proposed by Baltussen et al. (2018) to construct the oil volatility-of-volatility risk measure, and find that it remains significant in predicting one-period ahead delta-hedged gains. In the third robustness check, we change the forecasting horizon to be one week by utilizing USO weekly option delta-hedged gains.

We also emphasize the unique information content of oil volatility-of-volatility risk. The limited correlation between oil and equity market volatility-of-volatility risks, with the value of 0.169, highlights their distinctive nature. We show that oil volatility-of-volatility can significantly predict the log of real personal consumption expenditure, after controlling for oil volatility and equity volatility-of-volatility risks.

References

Agarwal, V., Arisoy, Y., and Naik, N. (2017). Volatility of aggregate volatility and hedge fund returns. *Journal of Financial Economics*, 125(3):491–510.

Andersen, T., Fusari, N., and Todorov, V. (2017). Shortterm market risks implied by weekly options. *Journal of Finance*, 72(3):1335–1386.

Ang, A., Hodrick, R., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *Journal of Finance*, 61(1):259–299.

Arezki, R., Jakab, Z., Laxton, D., Matsumoto, A., Nurbekyan, A., Wang, H., and Yao, J. (2017). Oil prices and the global economy. *IMF Working Paper*.

Bakshi, G., Cao, C., and Chen, Z. (1997). Empirical performance of alternative option pricing models. *Journal of Finance*, 52(5):2003–2049.

Bakshi, G. and Kapadia, N. (2003). Delta-Hedged Gains and the Negative Market Volatility Risk Premium. *Review of Financial Studies*, 16(2):527–566.

Bakshi, G., Kapadia, N., and Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies*, 16(1):101–143.

Baltussen, G., Bekkum, S., and Grient, B. (2018). Unknown unknowns: Uncertainty about risk and stock returns. *Journal of Financial and Quantitative Analysis*, 53(4):1615–1651.

Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4):1481–1509.

Bardgett, C., E., G., and Leippold, M. (2019). Inferring volatility dynamics and risk premia from the S&P 500 and VIX markets. *Journal of Financial Economics*, 131(3):593–618.

Barndorff-Nielsen, O., Neil, S., O., and Neil, S. (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics*, 2:1–37.

Barndorff-Nielsen, O. E. and Shephard, N. (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics*, 4(1):1–30.

Basak, S. and Pavlova, A. (2016). A model of financialization of commodities. 71(4):1511–1556.

Bates, D. (1996). Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies*, 1(9):69–107.

Bates, D. (2000). Post-'87 Crash Fears in the S&P 500 Futures Option Market. *Journal of Econometrics*, 94:181–238.

Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81:637–659.

Bollerslev, T., Gibson, M., and Zhou, H. (2011). Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities. *Journal of Econometrics*, 160(1):235–245.

Bollerslev, T., Tauchen, G., and Zhou, H. (2009). Expected stock returns and variance risk premia. *Review of Financial Studies*, 22(11):4463–4492.

Campbell, J., Giglio, S., Polk, C., and Turley, T. (2018). An intertemporal capm with stochastic volatility. *Journal of Financial Economics*, 128(2):207–233.

Cao, J. and Han, B. (2013). Cross section of option returns and idiosyncratic stock volatility. *Journal of Financial Economics*, 108(1):231–249.

Cao, J., Vasquez, A., Xiao, X., and Zhan, X. (2018). Volatility uncertainty and the cross-section of option returns. *working paper*.

Carr, P. and Wu, L. (2009). Variance risk premiums. *Review of Financial Studies*, 22(3):1131–41.

Chen, T., Chordia, T., Chung, S., and Lin, J. (2017). Volatility-of-volatility risk in asset pricing. *working paper*.

Chiang, E., Hughen, K., and Sagi, J. (2015). Estimating oil risk factors using information from equity and derivatives markets. *The Journal of Finance*, 70(2):769–804.

Christoffersen, P. and Pan, X. (2018). Oil volatility risk and expected stock returns. *Journal of Banking and Finance*, 95:5–26.

Duffie, D., Pan, J., and Singleton, K. (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusion. *Econometrica*, 68(6):1343–1376.

Ferderer, J. (1996). Oil price volatility and the macroeconomy. *Journal of Macroeconomics*, 18(1):1–26.

Gao, L., Hitzemann, S., Shaliastovich, I., and Xu, L. (2018). Oil volatility risk. *working paper*.

Hamdi, B., Aloui, M., Alqahtani, F., and Tiwari, A. (2019). Relationship between the oil price volatility and sectoral stock markets in oil-exporting economies: Evidence from wavelet nonlinear denoised based quantile and granger-causality analysis. *Energy Economics*, 80:536–552.

Herrera, A., Karaki, M., and Rangaraju, S. (2019). Oil price shocks and u.s. economic activity. *Energy Policy*, 129:89–99.

Heston, S. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2):327–343.

Hollstein, F. and Prokopczuk, M. (2018). How aggregate volatility-of-volatility affects stock returns. *The Review of Asset Pricing Studies*, 8(2):253–292.

Huang, D., Schlag, C., Shaliastovich, I., and Thimme, J. (2018). Volatility-of-volatility risk. *forthcoming in Journal of Financial and Quantitative Analysis*.

Huang, X. and Tauchen, G. (2005). The relative contribution of jumps to total price variance. *Journal of Financial Econometrics*, 3(4):456–499.

Hull, J. and White, A. (1987). The Pricing of Options on Assets with Stochastic Volatilities. *The Journal of Finance*, 42(2):281–300.

Jackwerth, J. and Rubinstein, M. (1996). Recovering probability distributions from option prices. *Journal of Finance*, 51(5):1611–1631.

Maghyereh, A., Awartani, B., and Sweidan, O. (2019). Oil price uncertainty and real output growth: new evidence from selected oil-importing countries in the middle east. *Empirical Economics*, 56(5):16011621.

Newey, W. and West, K. (1987). A simple positive semidefinite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708.

Prokopczuk, M., Symeonidis, L., and Wese Simen, C. (2017). Variance risk in commodity markets. *Journal of Banking and Finance*, 81:136–149.

A Some mathematical expressions

A.1 Construction of risk-neutral moments

In Bakshi et al. (2003), the prices of the volatility, cubic and quartic contract can be expressed by option prices as follows:

$$
\begin{aligned}
V_{t,t+\tau} &= \int_{S_t}^{\infty} \frac{2\left(1-\ln\left(\frac{K}{S_t}\right)\right)}{K^2} C(t,\tau,K)dK \\
&+ \int_{0}^{S_t} \frac{2\left(1+\ln\left(\frac{S_t}{K}\right)\right)}{K^2} P(t,\tau,K)dK, \\
W_{t,t+\tau} &= \int_{S_t}^{\infty} \frac{6\ln\left(\frac{K}{S_t}\right) - 3\left(\ln\left(\frac{K}{S_t}\right)\right)^2}{K^2} C(t,\tau,K)dK \\
&- \int_{0}^{S_t} \frac{6\ln\left(\frac{S_t}{K}\right) + 3\left(\ln\left(\frac{S_t}{K}\right)\right)^2}{K^2} P(t,\tau,K)dK, \\
X_{t,t+\tau} &= \int_{S_t}^{\infty} \frac{12\left(\ln\left(\frac{K}{S_t}\right)\right)^2 - 4\left(\ln\left(\frac{K}{S_t}\right)\right)^3}{K^2} C(t,\tau,K)dK \\
&+ \int_{0}^{S_t} \frac{12\left(\ln\left(\frac{S_t}{K}\right)\right)^2 + 4\left(\ln\left(\frac{S_t}{K}\right)\right)^3}{K^2} P(t,\tau,K)dK,
\end{aligned}
\tag{40}
$$

where $C(t,\tau,K)$ stands for the price of a call option with time-to-maturity τ and strike price K at time t.

Note that in practice we take discretization of the above integrals by defining the increment ΔK as

$$
\Delta I\left(K_i\right) = \begin{cases} \frac{K_{i+1}-K_{i-1}}{2}, 0 \le i \le N & (\text{ with } K_{-1} = 2K_0 - K_1, K_{N+1} = 2K_N - K_{N-1}) \\ 0, \quad \text{otherwise} & \end{cases}
\tag{41}
$$

A.2 Expressions in the model-free jump measure

The detailed expression for ZJ in the model-free jump measure (i.e. J), shown by Eq.(35), is as following

$$
ZJ_t = \frac{\frac{RV_t - BV_t}{RV_t}}{\sqrt{\left[(\pi/2)^2 + \pi - 5\right]/m \times \max\left(1, TP_t/BV_t^2\right)}},
\tag{42}
$$

where

$$
\begin{aligned}
TP_t &= m\mu_{4/3}^{-3} \frac{m}{m-2} \sum_{i=3}^{m} \left|r_{t,i-2}\right|^{4/3} \left|r_{t,i-1}\right|^{4/3} \left|r_{t,i}\right|^{4/3}, \\
\mu_k &= 2^{k/2} \frac{\Gamma[(k+1)/2]}{\Gamma[1/2]}, k > 0.
\end{aligned}
\tag{43}
$$

where $\Gamma()$ denotes the gamma function.

B Tables

Table I: Descriptive statistics of key variables

	Mean	Std. dev.	Skew	Kurt	Q1	Median	Q3
vol	33.040	10.850	0.823	3.604	25.590	31.360	39.300
vov	0.002	0.002	4.681	36.711	0.001	0.002	0.003
VIX	16.560	5.559	1.872	7.604	12.900	15.200	18.510
VVIX	90.670	12.912	1.146	7.050	82.340	89.180	97.570

Note: The table reports descriptive statistics such as mean, standard deviation, skewness, kurtosis, the 25th percentile, median, and the 75th percentile for the key variables: crude oil market volatility (*vol*), crude oil market volatility-of-volatility (*vov*), CBOE Volatility Index (VIX), CBOE VIX Volatility Index (VVIX) and equity market volatility-of-volatility (vov^{VIX}). The data sample ranges from August 2010 to June 2018.

Table II: Correlations

	vol	*vov*	VIX	VVIX
vol	1.000	0.250	0.493	0.427
vov		1.000	0.424	0.169
VIX			1.000	0.529

Note: The table reports correlations among the key variables: crude oil market volatility (*vol*), crude oil market volatility-of-volatility (*vov*), CBOE Volatility Index (VIX), and CBOE VIX Volatility Index (VVIX). The data sample ranges from August 2010 to June 2018.

Table III: Delta-hedged gains for 1-month USO options

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\Pi_{t,t+\tau}$(in \$)					$\frac{\Pi_{t,t+\tau}}{S_t}$(in %)			
Moneyness	Mean	t-Stat	% < 0	Std.	AR(1)	Mean	t-Stat	Std.	AR(1)
Call options									
0.900 − 0.925	−0.050	−2.36	59%	0.16	0.15	−0.13%	−1.86	0.01	0.05
0.925 − 0.950	−0.059	−2.19	61%	0.19	0.19	−0.17%	−1.99	0.01	0.06
0.950 − 0.975	−0.058	−1.83	60%	0.21	0.21	−0.19%	−1.96	0.01	0.15
0.975 − 1.000	−0.056	−1.86	59%	0.20	0.21	−0.17%	−1.65	0.01	0.12
1.000 − 1.025	−0.065	−2.06	64%	0.21	0.11	−0.18%	−1.83	0.01	−0.03
1.025 − 1.050	−0.050	−2.06	63%	0.21	−0.02	−0.15%	−2.03	0.01	−0.10
1.050 − 1.075	−0.058	−2.39	68%	0.20	−0.13	−0.17%	−2.19	0.01	−0.13
1.075 − 1.100	−0.044	−2.79	69%	0.13	0.01	−0.14%	−2.84	0.00	−0.07
Put options									
0.900 − 0.925	−0.068	−2.90	68%	0.18	0.13	−0.19%	−2.37	0.01	0.11
0.925 − 0.950	−0.076	−2.27	63%	0.23	0.22	−0.20%	−1.95	0.01	0.10
0.950 − 0.975	−0.070	−2.07	60%	0.23	0.13	−0.19%	−1.97	0.01	0.00
0.975 − 1.000	−0.071	−2.18	66%	0.21	0.27	−0.21%	−1.95	0.01	0.23
1.000 − 1.025	−0.069	−2.01	64%	0.23	0.17	−0.18%	−1.78	0.01	0.03
1.025 − 1.050	−0.060	−2.50	66%	0.19	0.06	−0.16%	−2.20	0.01	−0.07
1.050 − 1.075	−0.084	−3.61	70%	0.20	−0.13	−0.24%	−3.07	0.01	−0.12
1.075 − 1.100	−0.038	−1.85	68%	0.20	−0.12	−0.11%	−1.89	0.01	−0.15

Note: We compute the gain on a portfolio of a long position in a 1-month USO option, hedged by a short position in the underlying, such that the net investment earns the risk-free interest rate. The data sample ranges from ranging from August 2010 to June 2018. The delta-hedged gain is rebalanced at a daily basis, and computed according to Eq.(17). We report (i) the unscaled delta-hedged gain (i.e. $\Pi_{t,t+\tau}$), (ii) the delta-hedged gain scaled by the underlying (i.e. $\frac{\Pi_{t,t+\tau}}{S_t}$). Both are averaged over the specified moneyness ranges, where moneyness is defined as $m \equiv \frac{Se^{r\tau}}{K}$. The t-Stat tests the null hypothesis that the delta-hedged gain is equal to 0. The $\% < 0$ column shows the percentage of negative delta-hedged gains within the corresponding moneyness range.

Table IV: Delta-hedged gains for 1-week USO options

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\Pi_{t,t+\tau}$(in \$)					$\frac{\Pi_{t,t+\tau}}{S_t}$(in %)			
Moneyness	Mean	t-Stat	% < 0	Std.	AR(1)	Mean	t-Stat	Std.	AR(1)
Call options									
0.900 − 0.925	−0.015	−2.99	65%	0.06	0.01	−0.05%	−2.21	0.00	−0.14
0.925 − 0.950	−0.018	−2.96	62%	0.09	−0.05	−0.05%	−2.15	0.00	−0.09
0.950 − 0.975	−0.028	−3.86	63%	0.12	−0.02	−0.09%	−3.84	0.00	−0.06
0.975 − 1.000	−0.031	−3.46	61%	0.13	0.03	−0.09%	−2.73	0.01	0.04
1.000 − 1.025	−0.024	−2.72	61%	0.15	−0.07	−0.06%	−1.91	0.01	0.00
1.025 − 1.050	−0.014	−1.30	68%	0.17	−0.03	−0.07%	−2.09	0.01	−0.07
1.050 − 1.075	−0.026	−3.09	61%	0.11	0.04	−0.08%	−2.97	0.00	−0.06
1.075 − 1.100	−0.015	−1.58	59%	0.14	0.02	−0.08%	−2.34	0.00	0.02
Put options									
0.900 − 0.925	−0.012	−1.43	54%	0.12	0.02	0.00%	−0.07	0.01	0.02
0.925 − 0.950	−0.033	−3.62	59%	0.12	0.00	−0.10%	−3.30	0.00	−0.07
0.950 − 0.975	−0.034	−3.77	60%	0.12	0.10	−0.10%	−3.48	0.00	0.02
0.975 − 1.000	−0.033	−3.60	62%	0.13	0.02	−0.10%	−3.05	0.01	0.02
1.000 − 1.025	−0.028	−3.34	63%	0.14	−0.12	−0.07%	−2.13	0.01	−003
1.025 − 1.050	−0.025	−3.23	68%	0.12	0.00	−0.09%	−3.33	0.00	−0.08
1.050 − 1.075	−0.023	−3.49	74%	0.09	0.11	−0.08%	−3.25	0.00	0.02
1.075 − 1.100	−0.007	−0.87	68%	0.11	−0.02	−0.03%	−1.06	0.00	−0.03

Note: We compute the gain on a portfolio of a long position in a 1-week USO option, hedged by a short position in the underlying, such that the net investment earns the risk-free interest rate. The data sample ranges from ranging from August 2010 to June 2018. The delta-hedged gain is rebalanced at a daily basis, and computed according to Eq.(17). We report (i) the unscaled delta-hedged gain (i.e. $\Pi_{t,t+\tau}$), (ii) the delta-hedged gain scaled by the underlying (i.e. $\frac{\Pi_{t,t+\tau}}{S_t}$). Both are averaged over the specified moneyness ranges, where moneyness is defined as $m \equiv \frac{Se^{r\tau}}{K}$. The t-Stat tests the null hypothesis that the delta-hedged gain is equal to 0. The $\% < 0$ column shows the percentage of negative delta-hedged gains within the corresponding moneyness range.

Table V: Delta-hedged gains by volatility and volatility-of-volatility risks

	(1)	(2)	(3)	(4)
	Vega		Volga	
	Slope	t-Stat	Slope	t-Stat
$\text{Gains}_{t,t+\tau}$	-0.058^{***}	(-4.80)		
			-0.002	(-0.61)
	-0.119^{***}	(-7.22)	-0.020^{***}	(-5.38)

Note: The table reports the cross-sectional regressions of 1-month USO option gains on the Vega (i.e., $\frac{\partial C_t}{\partial \sigma_t}$) and Volga (i.e., $\frac{\partial^2 C_t}{\partial \sigma_t^2}$) of options, taking into account of the time fixed effects. Note that all the slopes are scaled up by 1000. The option Greek are implied by Black and Scholes (1973). The data sample ranges from August 2010 to June 2018. The numbers in brackets are Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by ***, **, and *.

Table VI: Predictability of delta-hedge gains

	$\Pi_{t,t+\tau}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
vol^2		-2.450e-06***	-2.310e-06***	-2.450e-06***	-2.550e-06***	-5.090e-06***	-5.640e-06***
		(-7.19)	(-6.58)	(-7.25)	(-7.26)	(-12.48)	(-13.40)
vov^2	-0.894***	-0.610***	-0.553***	-0.548***	-0.578***	-0.410***	-0.313***
	(-7.83)	(-5.17)	(-4.53)	(-4.65)	(-4.77)	(-3.64)	(-2.76)
$skew$			3.201e-04*		-2.283e-04		
			(1.87)		(-1.08)		
$kurt$				-2.751e-05***	-3.237e-05***		
				(-4.58)	(-4.31)		
J						1.037***	
						(10.48)	
J^+							0.623***
							(4.36)
J^-							1.390***
							(11.24)
$\Pi_{t-\tau,t}$	0.060*	0.031	0.033	0.047	0.047	0.070**	0.049
	(1.87)	(0.96)	(1.01)	(1.47)	(1.45)	(2.31)	(1.63)

Note: The table shows the predictability of 1-month forward delta-hedged gains by volatility (vol), volatility-of-volatility (vov) and jump (i.e., $skew$, $kurt$, J, J^+ and J^-) risks. The data sample ranges from August 2010 to June 2018. The numbers in brackets are Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by ***, **, and *.

Table VII: Predictability of real personal consumption expenditure

	ln RPCE					
	0m	1m	2m	3m	6m	12m
vol^2	-1.551e-07	1.200e-07	-3.524e-08	-6.163e-08	-8.520e-08	-1.703e-07
	(0.81)	(0.48)	(-0.17)	(-0.27)	(-0.47)	(-1.02)
vov^2	-0.203***	-0.116**	-0.008	0.022	0.004	-0.027
	(-4.96)	(-2.33)	(-0.16)	(0.49)	(0.15)	(-1.14)
VVIX	-1.447e-07**	-4.523e-09	-8.432e-09	2.031e-09	3.555e-08	6.569e-09
	(-2.08)	(-0.05)	(-0.13)	(0.03)	(0.71)	(0.17)
$\ln \text{RPCE}_{t-1}$	-0.188**	-0.148	0.015	0.036	0.052	0.071**
	(-2.08)	(-1.52)	(0.23)	(0.55)	(1.05)	(2.10)
Adj. R^2	7.27%	-0.69%	-3.99%	-3.55%	-1.91%	5.09%

Note: The table shows the predictability of log real personal consumption expenditure (ln RPCE) by oil volatility-of-volatility (vov), controlling for oil volatility (vol), equity volatility-of-volatility (VVIX) and its lag term. The data sample ranges from August 2010 to June 2018. The numbers in brackets are Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by ***, **, and *.

Table VIII: Robustness check: another volatility-of-volatility measure

				$\Pi_{t,t+\tau}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
vol^2		-2.770***	-2.570***	-2.740***	-2.810***	-5.380***	-5.830***
		(-8.31)	(-7.41)	(-8.28)	(-8.09)	(-13.70)	(-14.64)
$(vov^B)^2$	-0.122***	-0.075***	-0.061**	-0.066***	-0.069***	-0.056***	-0.060***
	(-5.25)	(-3.26)	(-2.55)	(-2.86)	(-2.93)	(-2.60)	(-2.80)
$skew$			41.404**		-14.280		
			(2.41)		(-0.67)		
$kurt$				-2.948***	-3.257***		
				(-4.89)	(-4.29)		
J						1.077***	
						(10.98)	
J^+							0.581***
							(4.05)
J^-							1.458***
							(12.16)
$\Pi_{t-\tau,t}$	0.095***	0.048	0.047	0.063*	0.064*	0.086***	0.067**
	(2.76)	(1.41)	(1.38)	(1.87)	(1.89)	(2.70)	(2.13)

Note: The table shows the predictability of 1-month forward delta-hedged gains by volatility (vol), volatility-of-volatility (vov^B) and jump (i.e., $skew$, $kurt$, J, J^+ and J^-) risks. The data sample ranges from August 2010 to June 2018. The numbers in brackets are Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by ***, **, and *.

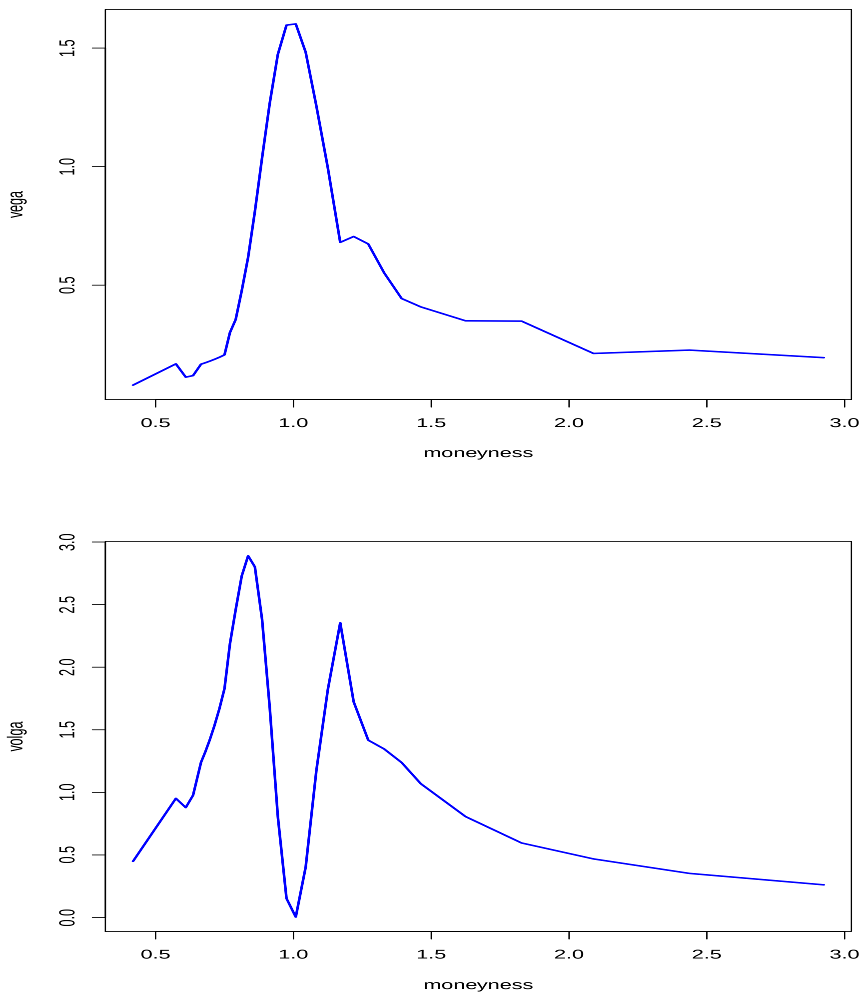
Table IX: Robustness check: predictability of 1-week delta-hedge gains

	$\Pi_{t,t+\tau}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
vol^2		-8.817e-07***	-9.915e-07***	-9.921e-07***	-1.030e-06***	-1.087e-07	-7.013e-08
		(-7.43)	(-8.01)	(-8.04)	(-8.26)	(-0.26)	(-0.17)
vov^2	-0.218***	-0.109**	-0.077	-0.081*	-0.083*	-0.403***	-0.401***
	(-5.36)	(-2.53)	(-1.60)	(-1.68)	(-1.72)	(-4.82)	(-4.86)
$skew$			-1.380e-04		-3.523e-04**		
			(-0.93)		(-1.99)		
$kurt$				-2.123e-05	-4.117e-05**		
				(-1.37)	(-2.23)		
J						0.337***	
						(4.00)	
J^+							0.596***
							(5.60)
J^-							0.129
							(1.25)
$\Pi_{t-\tau,t}$	-0.024	-0.025	-0.024	-0.026	-0.025	-0.026	-0.037
	(-1.25)	(-1.31)	(-1.21)	(-1.32)	(-1.27)	(-0.63)	(-0.88)

Note: The table shows the predictability of 1-week forward delta-hedged gains by volatility (vol), volatility-of-volatility (vov) and jump (i.e., $skew$, $kurt$, J, J^+ and J^-) risks. The data sample ranges from August 2010 to June 2018. The numbers in brackets are Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by ***, **, and *.

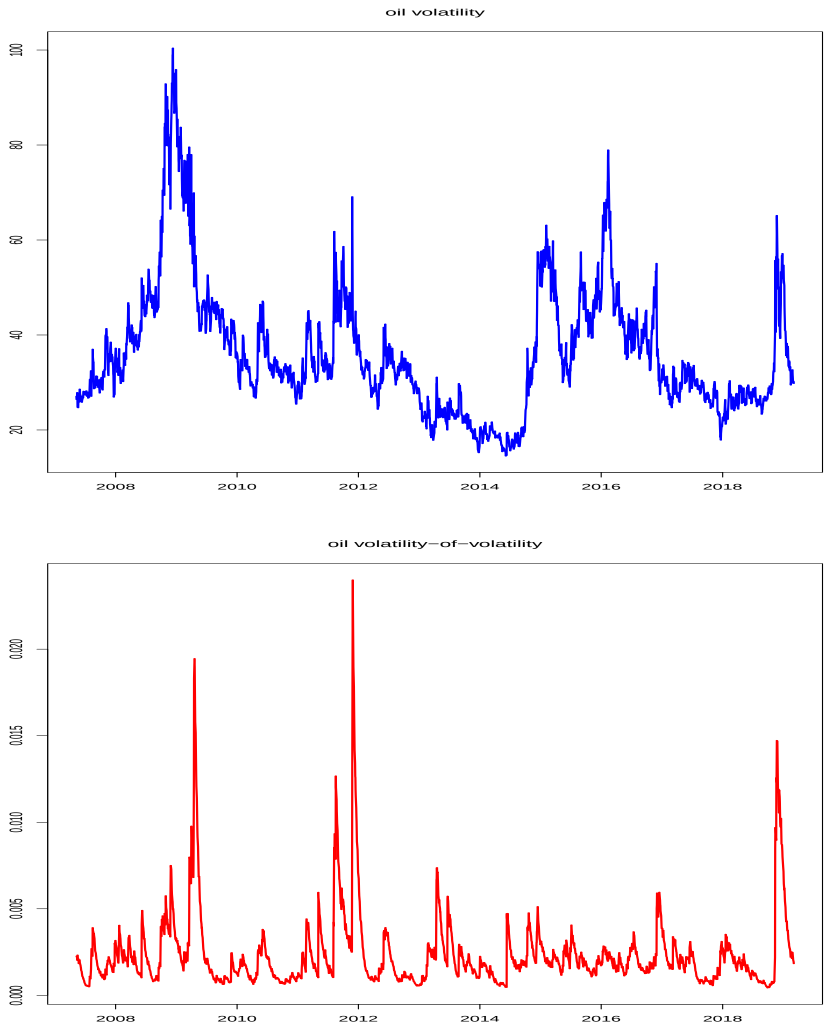
C Figures

Figure 1: Call vega and volga



Note: The figures show plots of the Black-Scholes implied vega (i.e., $\frac{\partial C_t}{\partial \sigma_t}$, upper figure) and volga (i.e., $\frac{\partial^2 C_t}{\partial \sigma_t^2}$, lower figure) for USO call option on Sep 18, 2015 with 1-month time to maturity with respect to its moneyness (i.e. $m = Se^{r\tau}/K$). The vega curve is concave with respect to the moneyness, while the volga curve is convex around near-the-money range.

Figure 2: Oil volatility and volatility-of-volatility



Note: The figures show the time series of volatility (the upper one) and volatility-of-volatility (the lower one) of the crude oil market. The data sample ranges from August 2010 to June 2018.