Volatility Discovery and Volatility Quoting on Markets for Options and Warrants

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Abstract

In several European and Asian countries, classical options markets coexist with markets for bank-issued options, also termed warrants. It is an open question if bank issuers adopt options markets information or if they contribute to volatility discovery by their own. We compare volatility discovery on this bivariate market system—options markets represented by the EUREX and warrants markets depicted by the EUWAX—given each of them a part of information share. The warrants market hardly makes a contribution in the process of volatility discovery, consequently the options market options market is in a clear informational leadership with an information share highly significant above 0.5 presenting the warrants as relatively uninformed market. Considering the volatility quoting behavior of issuers, we find that warrants market volatility decrease (in average) over day time, whereas options markets has a small upward trend. This leads to a possible strategy of warrants issuers exploiting intraday investors.

JEL Classification: G10, G14, C10, C22, C32, C51

Keywords: information share, volatility discovery, VDAX-NEW, warrants, options market

1 Introduction

In a perfect market, implied volatility is the market's (risk-neutral) expectation about the size of future price fluctuations (Schmalensee and Trippi (1978)). There is a huge literature dealing with the econometric modelling of time-varying volatility, seeded by Engle (1982) and Bollerslev (1986) for discrete (G)ARCH models, and Heston (1993) for continuous models of stochastic volatility, respectively. Empirical evidence for the evolution of implied volatility has been provided for single options markets. There are however parallel markets which have not been analyzed in terms of volatility discovery so far. In several European and Asian countries, classical options markets coexist with markets for bank-issued options, also termed warrants. In contrast to traditional warrants known in the US, these sort of warrants are not written by a company on its own stock, but are issued by banks and refer to various underlying securities, including stocks and stock indices. Regarding the payoff profile, these warrants are identical to classical options. Differences arise in the design and structure of the two markets.

The most important difference is the existence of a clearing house at the options exchange. By means of a margin system, the clearing house guarantees payoffs in the event of a bankruptcy of the counterpart. In contrast, warrant investors face the risk of default of the issuing bank. The second difference is the market design: While at an options exchange usually several competing market makers exist, at a warrants exchange the sole market maker is the issuing bank itself. Consequently, observed bid-ask quotes at the warrants exchange can be identified as an outcome of the price-setting policy of the respective bank. As a third difference, warrants markets are especially created for small investors. Market barriers are low, as investors can participate at this market without minimum trading lots or minimum fees. While this easy market access might be an explanation for the success and the pure co-existence of the warrants market with parallel options markets, several studies have shown that prices for warrants are systematically higher than prices for otherwise identical options (e.g., Bartram and Fehle (2007), Bartram et al. (2008), ter Horst and Veld (2008), Li and Zhang (2011), Baule and Blonski (2015)). ter Horst and Veld (2008) argue that banks have managed to make warrants look like completely different financial instruments than options, relying on the framing argument by Shefrin and Statman (1993).

For these reasons, prices at the options exchange and prices at the warrants exchange might be different to a certain extent, although they are closely related. Consequently, implied volatilities of warrants and implied volatilities of options are tied together. In this paper, we analyze a potential lead-lag relationship between these implied volatilities. Do issuers of warrants only follow the volatility information given by the options market or do they contribute to volatility discovery by their own? We calculate information shares in the spirit of Hasbrouck (1995) known from price discovery research, using the stationary information share measure developed by Baule et al. (2016).

The answer to the former question provides deeper insight into the quoting behavior of warrant issuers and the interrelation between the two markets. If issuers play a relevant role in volatility discovery, sources of information flow could be traced, as the quotes of each single issuer are observable separately. If they mainly rely on volatility information from the options market, a closer look at their volatility quoting behavior aside from information-driven pricing would be possible. Baller et al. (2016) demonstrate that banks increase their quotes for similar products, the leverage certificates, during a trading day on average. It is an open question whether such a behavior also applies to warrants.

We analyze volatility discovery at the worldwide largest warrants market, the European Warrants Exchange (EUWAX), together with the corresponding options market, the Eurex Exchange (EUREX). Instead of Black-Scholes implied volatilities, we consider model-free implied volatilities introduced by Britten-Jones and Neuberger (2000). The German volatility index VDAX-NEW, representing the the options markets in our study, is calculated from EUREX options prices with a model-free approach, very similar to the VIX for the S&P 500. We use the same approach to calculate implied volatility time series for the warrants market. As we can observe warrants quotes for each issuer, we calculate issuer-specific volatilities (ISV). Additionally, we calculate aggregated implied volatilities over the whole warrants market.

The paper examines a time period of 3.5 years from 07/2011 to 12/2014, considering options and warrants on the German market index DAX as the underlying. At the warrants market, we take the nine largest issuers into account, processing quotes from more than 189,000 different warrants. The results can be summarized as follows: The options market clearly takes the leadership in terms of volatility discovery. Warrant issuers merely follow volatility information from the options market instead of incorporating volatility innovation in their quotes first. However, some issuers do have a small but significant information share in volatility discovery. The aggregated warrants market contributes about 3.8% to the long-term variance of implied volatility, significantly different from zero.

Considering the quoting behavior of warrant issuers and the interrelation between the two markets, we show that issuer-specific volatility time series decrease (on average) during a trading day, whereas options market volatility time series has an upward trend. Such a quoting behavior could reflect a possible strategy of issuers to generate additional profit related to intraday traders.

Our paper focuses on the two parallel markets, asking *who* incorporates new information first, not *why* volatility innovations occur. Regarding the economic reasonings behind changes in volatility, for example, Guidolin and Timmermann (2003) and Vanden (2008) develop models on how corporate news and information quality affect option prices and thus implied volatilities. A different explanation is given by Garleanu et al. (2009), who rely on market imperfections such as unhedgeable risk. In the presence of such kind of risk, market makers adjust their quotes with respect to demand pressure from end users of options, resulting in implied volatility changes driven by changes in end users' demand. Volatility discovery can be seen as a natural extension of price discovery. There are some studies investigating the contribution of option markets in this regard: Booth et al. (1999) examine the intraday price discovery process among stock index, index futures and index options on German markets, finding stock index and index futures appear to be the main driving forces with 50% and 48% respectively whereas option market just contributes with 2%. Also Chakravarty et al. (2004) investigate the contribution of option markets to price discovery proofing a stronger evidence than Booth et al. (1999) of option markets part in the price discovery in US market. They look at 60 stocks listed on the NYSE that have actively traded options at the CBOE, finding a significant price discovery does occur in option markets on the order of 10% to 20%. Rourke (2013) focus on relative price discovery of near- and away-from-the-money option markets. The data for this study cover more than 50 stocks traded at NYSE and Nasdaq and a subset of the call and put options written on those stocks on high frequency basis. When option markets are considered jointly, the average combined option market information share relative to the stock market information share is 17.5%. Moreover, they find that the near- and away-from-the-money options, respectively, each contribute approximately equally to the price discovery process.

All of these studies reveal that options markets has a significant (although rather small) content of information in discovering the fundamental value. Tse et al. (2006) explore the dynamics of price discovery between Dow Jones index and its derivative products. Although, they focus on ETF Quotes, pit-traded regular futures and E-mini futures as three derivatives of the DJIA and not on option markets, it is noteworthy that multi-market trading in general ensures better pricing efficiency. Among the three derivatives of the DJIA, the results indicate that E-mini futures exhibit the highest contribution to price discovery (69.1%) while the ETF quotes also contribute significantly (28.6%). In conclusion, not only the underlying market is important for price discovery but the derivative markets as well exhibit an (although rather small related to options markets) impact of information share.

Regarding the issue of volatility discovery instead of price discovery, literature seems to be relatively underdeveloped. Some recent studies brought forward by Dias et al. (2016) examine volatility discovery, using fractional co-integration methods combined with realized variance estimators to gain insight how markets contribute to stochastic volatility process. They investigate volatility discovery for 30 simultaneously traded stocks in the U.S. market, finding that NYSE and Arca dominate Nasdaq. Within the framework of the fractional cointegration, they allow the volatility time series to be nonstationary but mean-reverting. The idea of fractional integrated processes is closely related to a long memory property of the time series leading to a relatively slow decay of long-lag autocorrelation function. The technical description of volatility time series was thus extended by Diebold and Inoue (2001) focussing in a theoretical study on the effect of long memory and regime switching. They conclude that long memory can easily be confused with structural change in the time series. Structural change on the other side can drive to misleading results considering unit-root tests. Considering different time series as e.g. exchange rates, Baillie and Bollerslev (1994) disagree to model them to be "pure" unit-root processes but rather to be them as well fractional cointegrated expressed by the typical hyperbolic decay of the autocorrelation function. According to them, exchange rate time series exhibit long memory property meaning that influence of shocks to the equilibrium exchange rates may only vanish at very long horizons. According to these studies, it is tempting to interpret volatility time series as fractional cointegrated processes. Nonetheless, volatility modelling is very often based on stationary autoregressive processes with mean reversion, as for example in the popular time-continuous model of Heston (1993). When the volatility time series are modelled as stationary processes, according to the discussed studies, they can be easily confused with unit root time series or they are treated as fractional integrated time series because of a pretended long-memory property when considering sufficient short time period. Nevertheless, stationarity is a inherent property which should be independent of considered time horizon or other testing specifications.

The remainder of the paper is organized as follows. Section 2 overviews the data set including warrant quotes, DAX, and VDAX-NEW time series, and outlines the concept of constructing issuer specific volatility time series ISV using the VDAX-NEW method. It is well known that the majority of changes in implied volatility is driven by changes in the underlying security, a phenomenon sometimes referred to as "sticky strike" effect. We therefore outline a modification of the volatility time series, eliminating the driving DAX component to concentrate on true volatility innovations. In Section 3 the econometric framework is presented, including the concept of stationary information shares. Section 4 presents the results of comparing the information share of the warrants markets and the options market in a bivariate market system and moreover the results in an intra-warrants-market system. Furthermore, in Section 5 the quoting behavior of warrant issuers and the interrelation between the two markets is investigated, analyzing a possible strategy of issuers related to intraday traders. Section 6 concludes.

2 Data and Methodology

2.1 Model-Free Implied Volatilities

The VDAX-NEW is an annualized measure expressed in percentage points of the 30-day expected volatility for the blue chip stock market index DAX traded on the Frankfurt Stock Exchange. The index replaced the former VDAX$_{old}$, which was calculated based on at-the-money options only. Instead, the VDAX-NEW covers a broad range of strike prices, aiming at calculating the variance of the implied underlying distribution at the respective time horizon as outlined by Britten-Jones and Neuberger (2000) and Demeterfi et al. (1999). Actually, the VDAX-NEW is the German analogue to the VIX, representing the volatility index of S&P 500 option prices. The VDAX-NEW construction method is described in detail in Deutsche Börse (2007). It directly relies on quotes of traded on the DAX, so its calculation is handled without a option pricing model. The index itself is an interpolating value of two subindices surrounding the 30 days horizon. These subindices and consequently the VDAX-NEW are calculated with a simple formula:

$$\sigma_t^2 = \frac{2}{T_t} \sum_j \frac{\Delta K_{t,j}}{K_{t,j}^2} \cdot R_i \cdot M(K_{t,j}) - \frac{1}{T_t} \left(\frac{F_t}{K_{t,0}} - 1 \right)^2 \tag{1}$$

with t denotes the point in time (exact to the second), T_t is the time to maturity exact to the second of option with next smaller/larger time to maturity than 30 days, F_t represents value of the Forward derived from smallest absolute distance between call and put mid quotes of the warrants, $K_{t,j}$ is the value of strike of the j'th out-of-the-money option,

$\Delta K_{t,j}$ denotes difference in strike values, $R_t = e^{r_t \cdot T_t}$ represents refinancing factor with r_t respective risk free interest rate, $K_{t,0}$ is the highest value of strike smaller than Forward F_t and $M(K_{t,j})$ denotes option value of the respective out-of-the-money calls and puts given as mid quotes. The VDAX-NEW is the square root of this model-free implied variance. In the following, we drop the extension "NEW" and refer to the VDAX for ease of speaking. As mentioned above, Deutsche Börse (2007) derives the VDAX by real traded options at the EUREX with underlying DAX, representing the options market. For the warrants market, the same technique can be used to construct an issuer specific volatility time series ISV based on warrant quotes (mid quotes) of the respective issuer. Every minute, we create a snapshot from the relevant data to construct the respective subindices and thereafter by interpolation the ISV for each issuer. As with the VDAX, for each we use the two warrant series surrounding the 30 days horizon and interpolate accordingly.

We release some of the constraints of the original VDAX method as the warrants data is not as substantial as the VDAX data base.[1] By these modification we obtain a data base big enough to manage the calculation of issuer specific volatility time series. It is worth mentioning that we do not need to follow the concept of VDAX in every detail because total level accordance of VDAX with ISV is not necessary for the following considerations. Instead, we focus on the co-movement of resulting time series.

As we want to compare the single issuers information share not only to VDAX but to each other as well, the wide span of strike values is fixed to ±25% with respect to the ATM point. So, the importance of out-of-the-money warrants is limited to our calculation

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[1] Consequently, e.g. we do not filter maximum spreads between bid and ask prices as in Deutsche Börse (2007) and the option mid quotes are just cut off when they do not reach 0.01% index points contrary to VDAX calculation where the critical value exhibits 0.5 index points.

analogously to the VDAX computation to make them comparable. Moreover, we set some constraints to receive the ISV smooth enough to be comparable.[2] The ISV are computed at a one minute frequency starting each day from the earliest possible time and running until the latest possible time. The earliest point is the instant of time when quotes of all relevant strike values are available and the latest instant of time is the last relevant mid quote. In most cases this interval lies between 09:05–22:00. From the resulting daily ISV time series we select the time series

- which include more than half of the day time (more than 300 data points),
- whose volatility (so, volatility of volatility) is less than twice the mean of volatility averaged over all issuers,
- whose range of volatility is less than twice the mean of range of volatility time series averaged over all issuers

to get a useful smooth data set of comparable volatility time series.

Even though it seems to be plausible to calculate for every issuer an ISV to extract warrants market information in the volatility discovery process, one could also focus on the issuer combined volatility as a time series of volatilities relying on the entire information brought forward by all issuers and not only based on a single issuer. For this purpose, we construct next to the ISV a volatility time series based on the VDAX method combining all information of all issuers. For sake of clarity, we subsume this volatility time series to the ISV and refer to it as "Issuer combined" in the following.

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[2] There must be at least 10 different strike values, the minimal subindex is bounded by 20 days, $|K_{t,0} - F_t| \leq 1\%$ (constraint of Equation 1) and we use put-call-parity when considered warrants are just of one sort (just calls or just puts)

2.2 Data

As mentioned above, we construct a set of issuer specific volatility time series ISV by use of more than 189,000 European warrants written on the index DAX as the underlying, traded in the time period 07/2011–12/2014. We eliminate issuers with less than 5,000 traded warrants as well as the UBS AG, because it did not issue any warrants after 2012 for the German market. There are nine (remaining) issuers or banks—BNP Paribas, Citigroup, Commerzbank, Deutsche Bank, DZ Bank, Goldman Sachs, HSBC, UniCredit and Vontobel—on which this study relies. Descriptive statistics are reported in Table 1. The data set of quotes of warrants as well as the DAX and VDAX data are delivered by SIRCA on a high frequency basis. The base data of warrants are provided by the financial data company Deriva GmbH Financial IT and Consulting. The interest rates are retrieved from Thomson Reuters. The total data set includes 93,306 call warrants and 96,071 put warrants resulting in a balanced total set of 189,377 warrants. The range of strikes covers 2,350–20,000 index points. After applying the restrictions for the use of daily ISV given above, each issuer exhibits a different number of days where volatility time series could be reasonably calculated. Table 1 also reports the number of these constructible days for each issuer. Notably, for some issuers, this number is rather small, meaning that a larger number of days had to be dropped. In most of those cases, the vol-of-vol restriction applied, that is, the volatility time series showed too large fluctuations during the trading day.

Insert Table 1 about here.

By combining the issuers information, one can construct the aggregated “Issuer combined” time series. The construction method is based on the averaged (respectively most recent) mid quotes of issuers per strike, meaning that for every strike level at every moment we obtain a mean quote averaged over all issuers. The mean quote over all issuers can be interpreted as the mean volatility priced into the quotes of the issuers. With the help of these quotes, the VDAX based on information set of ALL issuers can be constructed at every moment. This aggregation provides the possibility of a direct (bivariate) market comparison of the options exchange, given via VDAX and the warrants exchange, given via “Issuer combined”.

2.3 Modified Implied Volatilities

As a well-established empirical fact, market returns and changes in volatility exhibit a strong negative correlation. Black (1976) and Cox and Ross (1976) explain this behavior by the leverage effect, which turns a constant asset volatility into a equity volatility that increases with decreasing equity value. Additionally, for example Hibbert et al. (2008) provide a behavioral explanation. They also give a comprehensive literature overview on the co-movement of stocks and volatilities.

In particular for the German market, Masset and Wallmeier (2010) show by Granger causality tests that movements in the DAX actually imply movements in DAX implied volatility and not the other way round. As our goal is to compare movements in implied volatility on two different markets, it is therefore reasonable to adjust the volatility time series by those index-driven movements which apply to both markets. Such a dependency can be explained by a quoting behavior of options and warrant traders known as the “sticky strike” rule (see Wallmeier (2015), for example). According to this rule, traders “stick” to the quoted implied volatility of an option with a given strike, even when the underlying moves. In the presence of a negative-sloped implied volatility skew, the at-the-money point moves to a lower implied volatility when the underlying increases. As the VDAX (and thus the ISV) can be seen as a weighted implied volatility that puts more emphasis on at-the-money than on in-the-money or out-of-the-money options, the VDAX and the ISV therefore decrease with increasing DAX.

It is important no note that this short-term DAX-driven volatility movements are not based on incorporating new information by the markets, but on a simple application of the "sticky strike" quoting rule. We therefore modify the volatility time series by adjusting for these purely DAX-driven changes. For this purpose, we use linear regressions of the intra-day volatility returns at a one minute frequency on the contemporaneous DAX returns. For every trading day, we calculate minute-by-minute log returns of the respective volatility time series, r^V, and the DAX, r^{DAX}, from 09:00–17:30. Based on the coefficient estimates of the regressions,

$$r_t^V = \alpha^V + \beta^V \cdot r_t^{DAX} + \epsilon_t, \tag{2}$$

we calculate modified time series V_t^{Mod} recursively in the respective time period by

$$V_t^{Mod} = V_{t-1}^{Mod} \cdot \exp(r_t^V - \beta^V \cdot r_t^{DAX}), \tag{3}$$

where the time index t refers to a one-minute interval and the starting point of each volatility time series, V_0, is the original value at the respective day. Figure 1 shows the result of this modification for a randomly chosen day (30/10/2014).

Insert Figure 1 about here.

To get an impression of the size of the sticky strike quoting behavior, Table 2 presents average estimates of the β^V coefficients for each warrant issuer, furthermore of the aggregated time series for warrants and of the VDAX, representing the options traders. Moreover, the mean r^2_{adj} is displayed, representing the goodness-of-fit of the linear regressions on the DAX in average. The linear dependance of DAX is suddenly increased when considering the aggregated time series for warrants or the VDAX, making them more stick to the DAX than the single issuers.

Insert Table 2 about here.

The table additionally shows average bivariate differences, which are restricted to days where the data allowed for the construction of a time series for both parties. As expected, the average coefficient β^V is always negative, however with considerable differences across the issuers, ranging between -0.779 (HSBC) and -1.735 (Commerzbank): The highest negative value is reached for the aggregated time series of warrant issuer with a value of -2.3459 as the "Issuer Combined" is based on a narrower grid of strike prices resulting in a higher dependance of the underlying DAX movement as the leverage effect is enhanced over the total set of all issuers. The high r^2_{Adj} reinforces this statement. Results of t-tests for differences between different parties show that DZ Bank, Commerzbank, and also the VDAX exhibit rather higher (in absolute values) average β^V values, whereas HSBC, UniCredit and Vontobel show rather smaller values. Consequently, the volatility time series of the former react more sensitive to the DAX return than those of the latter.

3 Information Shares in Stationary Time Series

In this section, we outline the econometric model to calculate information shares for volatility time series. As these time series are stationary, we cannot apply the standard approach of Hasbrouck (1995) without modifications. Baule et al. (2016) have adapted the concept for stationary processes. We make use of this "stationary information share", which is briefly sketched in the following for the case of two volatility time series $v_t = (v_{1,t}, v_{2,t})'$. The basic idea of the (stationary) information share is to decompose the variance of the entire process referring to volatility time series of two different issuers and to calculate cross-influences of the respective issuers on each other. Starting point is a standard vector autoregression that describes the behavior of the two time series:

$$\begin{aligned}
\begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} &= A_1 \cdot \begin{pmatrix} v_{1,t-1} \\ v_{2,t-1} \end{pmatrix} + A_2 \cdot \begin{pmatrix} v_{1,t-2} \\ v_{2,t-2} \end{pmatrix} + \ldots + A_k \cdot \begin{pmatrix} v_{1,t-k} \\ v_{2,t-k} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \\
&= A(L) \cdot \begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \\
&= A(L)v_t + \epsilon_t, \qquad (4)
\end{aligned}$$

where $A_i \in \mathbb{R}^{2\times 2}$, $i = 1, \ldots, k$, are fixed coefficient matrices up to the maximum lag length k, $A(L) = A_1 L + A_2 L^2 + \ldots + A_k L^k$ represents a polynomial in the lag operator with $Lv_t = v_{t-1}$ and ϵ_t denotes an (2×1) vector representing a white-noise innovation process. These innovation terms have zero mean, $E(\epsilon_t) = 0$, and a non-singular, two-dimensional covariance matrix $\Omega = E(\epsilon_t \cdot \epsilon_t^{'}) = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$.

Under the condition of stationarity, we can transform the general VAR model (4) into a vector moving average model:

$$v_t = \Psi(L) \cdot \epsilon_t, \qquad (5)$$

with $\Psi(L) := A^{-1}(L)$ representing the inverse of the polynomial in the lag operator.[3] In general, the innovation terms are cross-correlated. To cope with this issue, we use the

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[3] Note that in contrast to $A(L)$, the inverse operator $\Psi(L) = \Psi_0 + \Psi_1 \cdot L + \Psi_2 \cdot L^2 + \cdots$ is not necessarily finite.

Cholesky decomposition $\Omega = M \cdot M'$, leaving the “pure” variance of one issuer unaffected by the correlation to the other issuer:

$$v_t = \Psi(L) \cdot M \cdot M' \cdot \epsilon_t = \Phi(L) \cdot w_t = \sum_{p=0}^{\infty} \Phi_p \cdot w_{t-p}, \tag{6}$$

with $\Phi(L) = \Phi_0 + \Phi_1 \cdot L + \Phi_2 \cdot L^2 + \cdots$ representing the modified lag operator polynomial given by the set of $\Phi_p = \Psi_p \cdot M$ for $p = 0,1,\ldots$. Applying the Cholesky matrix to the original innovation vector forces the innovation terms, $w_t = M' \cdot \epsilon_t$, to be orthonormal; that is, all shocks are uncorrelated with unit variance, $w_t \sim \mathcal{N}(0,I)$. By reason of orthonormalization and by following the concept of the well known forecast error decomposition with prediction length of ∞, it can be shown that the variance of the process can be decomposed into two kinds of innovation terms: disturbances due to a issuers's own innovations and disturbances due to the other issuers shocks, formally expressed by:

$$Var(v_{j,t}) = \sum_{p=0}^{\infty} \left(\phi_{j1}^{(p)}\right)^2 + \sum_{p=0}^{\infty} \left(\phi_{j2}^{(p)}\right)^2, \tag{7}$$

where the index $j \in \{1,2\}$ represents the respective issuer.

For the construction of information shares, the contributions of one issuer to the variance of the respective other issuer are considered:

$$\eta_{12} := \frac{\sum_{p=0}^{\infty}(\phi_{12}^{(p)})^2}{\sum_{p=0}^{\infty}(\phi_{11}^{(p)})^2 + \sum_{p=0}^{\infty}(\phi_{12}^{(p)})^2}, \qquad \eta_{21} := \frac{\sum_{p=0}^{\infty}(\phi_{21}^{(p)})^2}{\sum_{p=0}^{\infty}(\phi_{21}^{(p)})^2 + \sum_{p=0}^{\infty}(\phi_{22}^{(p)})^2}. \tag{8}$$

We can interpret η_{12} as the part of variance of the first issuer due to innovation terms of the second issuer; analogously, η_{21} represents the portion of variance of the second issuer due to the first issuer's innovation terms.

These two values do not yet represent relative impacts from one issuer to another, but relative variance impacts on the respective other issuers. Based on these relative impacts the (stationary) information share IS is constructed by taking the ratio again:

$$\lceil IS_1 \rceil := \frac{\eta_{21}}{\eta_{12} + \eta_{21}}, \qquad \lfloor IS_2 \rfloor := \frac{\eta_{12}}{\eta_{12} + \eta_{21}}. \tag{9}$$

We can interpret IS_1 as the influence of the first issuer on the second issuer, η_{21}, relative to the aggregated cross-influences of both issuers on each other, $\eta_{21} + \eta_{12}$. Vice versa, IS_2 represents the influence of the second issuer on the first issuer, relative to the aggregated cross-influences. Note that these values represent upper bounds for the first issuer and lower bounds for the second issuer, according to the definition of the Cholesky matrix M attaching a greater weight to the first issuer in the system. The respective other bounds are obtained by altering the order of the two issuers.

The stationary information share is closely related to indices used in the literature on spillover effects, see Diebold and Yilmaz (2009), Diebold and Yilmaz (2012), Koop et al. (1996), and Pesaran and Shin (1998). As Baule et al. (2016) show, it converges to the information share by Hasbrouck (1995) in the case of non-stationarity and can thus be seen as a generalization of this well-respected measure in price discovery.

4 Results: Volatility Discovery

4.1 Warrants Market vs. Options Market

In this section, we present and discuss our main results regarding volatility discovery at the warrants market with respect to the options markets. As outlined in Table 3, the aggregated warrants market (over all issuers) exhibits an information share of about 35.9%, whereas the options market, represented by the VDAX, accounts for 64.1% of information. These figures represent average values of daily information shares. The cross correlation between the two markets amounts to 0.26. Accordingly, upper and lower bound of the information share differ to a certain extent and span the range of 28% to 44%.

What do these figures tell us? Obviously, the options market is in a clear informational leadership in terms of volatility discovery with an information share highly significant above 0.5, even if all contemporaneous effects are attributed to the warrants market. This finding justifies the use of the options market as a reference market for implied volatilities in the analysis of other retail derivatives markets, as frequently done in the literature (e.g., Baule (2011), Entrop et al. (2016), Schertler (2016), Schertler and Stoerch (2015)).

However, a 35.9% share in volatility discovery for the warrants markets is seemingly a considerable contribution. But caution is needed in the interpretation of this share. Daily values for the relative information share exhibit a strong fluctuation, indicating possible statistical estimation errors. To test both the statistical significance and the economic meaning of the average information share, a bootstrapping procedure is performed.

The null hypothesis of the bootstrapping procedure consists of a zero contribution of the issuer to volatility discovery. The zero contribution can be modelled by setting the respective minor diagonal element of the fixed coefficient matrices A_i of Equation 4 to zero. By this restriction, the VDAX time series is not influenced by an ISV in the bivariate market system anymore. Following the procedure of the variance decomposition of Section 3, an information share under the null hypothesis is obtained.

Under the null hypothesis of zero contribution, the bootstrapped value of the warrants market's information share still amounts to 32.1%. Hence, the value of 35.9% overrates the actual role of the warrants market in volatility discovery. It is merely 3.8 percentage points larger than a fictitious market with zero information. Anyway, this marginal contribution is statistically highly significant. We can therefore attribute the warrants market a small, but non-zero share in volatility discovery.

To gain deeper insights in the sources of this contribution, we conduct an issuer-by-issuer analysis. As the warrants market allows the examination of single issuers as market makers, we can identify issuers who might contribute to volatility discovery and others who do not. Table 3 presents information shares of volatility discovery in a bivariate system of one issuer at the warrants market on the one hand and of the options market on the other hand.

Insert Table 3 about here.

As depicted in Section 3, the information share measure depends on the cross correlation of the markets, leading to upper and lower bounds. The correlation of the error terms ϵ in Equation 4 are rather small in the single bivariate systems so that upper and lower bounds of the computed information shares do not vary extremely. All issuer's mean information shares lie between 30% and 37%. In line with the result for the aggregated warrants market, in every bivariate market system the VDAX dominates volatility discovery with an information share above 0.5, both economically and statistically significant at a high level.[4]

To test for any significant contribution, we bootstrap the issuer's information share under the null hypothesis that they have an information share of zero, meaning there is no content in the process of discovering the fundamental value. Only half of the considered issuers exhibit information shares greater than zero at a 5%-level, so there is hardly an influence on VDAX in the respective bivariate system. We can conclude that the small

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[4] In fact, not only the mean but the lower bound of the information share of VDAX is tested above 0.5 giving an even stronger significance of VDAX taking leadership.

but non-zero contribution of the warrants market does not stem from one or two single issuers, but is indeed a contribution of the aggregated information in the market. To sum up, the warrants market hardly contributes to volatility discovery, while the options market, represented by VDAX, clearly takes the leadership. Consequently the issuers on the warrants market hardly influence the options market. It implies that the warrants market as a part of the retail market clearly behave in a different manner as the non-retail derivative markets which are proofed to have a higher content of information in discovering the fundamental value, depicted by Booth et al. (1999), Chakravarty et al. (2004), Tse et al. (2006) and Rourke (2013).

4.2 Intra warrants market

In the former subsection it was shown that the VDAX and consequently the options market exhibits a significant leadership in volatility discovery with respect to the warrants market. It remains an interesting question who of the issuers is in a relative leadership when the warrants market is considered alone. Although all issuers follow the options market to a large extent according to our previous results, they might do so at different speed. Furthermore, there are times when the options markets is closed, but the warrants market is open. Especially in those times the question of volatility discovery at the intra-warrants market is of special interest. To answer this question, we compare issuers information shares to each other in pairwise bivariate systems.

Insert Table 4 about here.

Results of mean information shares are presented in Table 4. The rows of the table refer to the information share of the respective issuer. For example, the first entry of the matrix, 36.02, is Citigroup's information share in a pairwise comparison with BNP. Each pairwise result comes with two test of significance: First, we test whether the respective smaller information share is significantly larger than zero. For this test, we apply the bootstrapping method as explained above. Results of these tests are marked with asterisks (*). Second, we test whether the respective larger information share is significantly larger than 0.5 (or, equivalently, whether the leadership of one issuer against another is significant). For this purpose, we can use a standard t-test. Results are marked with daggers (†).

Comparing all results of Table 4, we find a transitive sequence of issuer's information shares: UniCredit, placed first, is dominating every other issuer with information share values range from about 52% till almost 76% (although the leadership with respect to Commerzbank is not significant). The sequence continues with Commerzbank, BNP Paribas, Goldman Sachs, Vontobel, Citigroup, Deutsche Bank, DZ Bank, and the issuer with the lowest information shares in the bivariate systems is HSBC, exhibiting a range of information shares from about 28% till 48%.

That signifies that UniCredit is taking leadership versus all other issuers in a pairwise comparison while HSBC does not contribute to volatility discovery in a great extent. These results are of special interest for warrant traders: When they trade with issuers of the leading group—UniCredit, Commerzbank, or BNP Paribas—, they can be quite sure that their quotes reflect volatility information in a timely manner. Quotes of issuers of the slowest group in terms of volatility discovery—Deutsche Bank, DZ Bank, or HSBC—on the other hand might incorporate volatility innovations with a certain time delay of a few minutes. As a consequence, quotes might be biased to more or less favorable conditions for the counterpart. As warrants become cheaper with decreasing volatility, issuers who immediately reflect volatility innovations are preferable when volatility actually decreases.

When volatility increases on the other hand, issuers who tend to stick some more time to the previous lower volatility level are preferable. Note that these conclusions refer to high-frequent changes in volatility at a time scale of a few minutes.

5 Volatility Quoting

The previous results suggests that warrant issuers primarily follow the options market regarding volatility information. It is however known that issuers of retail derivatives engage in strategic pricing and thus in strategic volatility quoting, for example with respect to investor demand (Baule (2011)) or competitor entries (Schertler (2016)). Since we have shown that issuers rarely deviate from option market volatility to incorporate new information, we can attribute any systematic intra-day deviations to a strategically intended quoting behavior.

While there is a large strand of literature dealing with issuer pricing behavior at retail derivatives markets (Schertler (2016) provides a recent overview), only very few papers focus on intra-day patterns. For leverage certificates, a popular retail product with some similarities to bank-issued warrants, Entrop et al. (2013) show that banks increase their mark-ups with respect to the theoretical fair value during a trading day. Baller et al. (2016) present a theoretical model and an extensive empirical study of the intra-day pricing behavior of leverage certificate issuers. They outline two major effects which might influence this behavior: First, leverage certificates face an overnight jump risk of the underlying, since they may be knocked out over night with a loss in the hedge portfolio of the issuer. This risk increases to the end of the day, so it is reasonable for issuers to increase their quotes over the day. Second, investor net demand decreases over the day. While in the early trading hours, more products are sold to investors than re-sold back by them to the bank, this ratio diminishes in the late trading hours. A reasonable reaction to this change in net demand would be a decrease in issuers' mark-ups. So while these effects work in opposite directions, Baller et al. (2016) show that the first is stronger, as issuers tend to increase their mar-ups over a trading day on average.

A similar investigation for warrants does not yet exist to the best of our knowledge. Our data gives us the opportunity to investigate the average intra-day quoting behavior of warrant issuers, since deviations in quoted volatility that do not reflect contemporaneous deviations in options market volatility (that is, in the VDAX) directly transform into changes in intentionally quoted mark-ups. As there is no comparable overnight jump risk for warrants as for leverage certificates, we might observe different patterns.

In order to examine the intra-day pricing behaviour, for each issuer we average all respective volatility time series over the time interval 09:15–17:25 for each day in the considered period 07/2011–12/2014. To do so, we compute the issuer volatility time series ISV, as described in Section 2, superimpose them in the daily time interval 09:15–17:25, so cutting off the "early and late wings" and build the mean in every minute. Analogously, we average the VDAX series over day time. We use both the original volatility time series and (regarding the robustness) the adjusted time series according to the "sticky strike" effect as described above.

Results are plotted in Figures 2 and 3. Linear trends over day time are report in Table 5. Strikingly, every of the nine averaged ISV curves has a significant negative slope whereas the averaged VDAX has a significant positive gradient. This means that in average the nine issuers lower their respective volatility by pricing the warrants cheaper over day time whereas averaged VDAX is increasing over day time. The size of this effect is considerable:

The reduction in implied volatility over the day ranges from -4 base points (HSBC) to -15 base points (Vontobel), on average, while the VDAX increases by 1.5 base points at the same time.

Insert Figure 2 and Figure 3 about here.

Insert Table 5 about here.

So the intra-day quoting behaviour of warrant issuers is contrary to the case of leverage certificates: While the latter become relatively more expensive during a trading day, our data shows that implied volatilities of warrants decrease, making them cheaper over a day. As mentioned, warrant issuers do not face the overnight gap risk in a similar way as with leverage certificates, so this reason to raise the quotes when the daily closure of trading approaches does not exist. Hence, in line with the demand pattern reported by Baller et al. (2016), a decrease in volatility quoting over time might reflect the intention of issuers to exploit the trading behavior of investors: In the early hours, net demand is higher, while in the late hours more investors tend to sell their warrants back to the issuer. In particular, day trader might close their positions at the end of a day. The observed pattern in issuers' volatility quoting would generate additional profits with respect to such an investors' average behavior.

6 Conclusion

This paper investigates the non-retail options market and the retail warrants market in terms of a volatility discovery process. VDAX indicates an annualized measure expressed in percentage points of 30-day expected volatility for the blue chip stock market index

DAX traded on the Frankfurt Stock Exchange and it is based on real traded options at the EUREX. Consequently it represents an options market component. It is an open question, if issuers on the warrants market with underlying DAX are pricing their derivative products by just using information of the options market, so processing all information of EUREX unreflected, or if these issuers exhibit an own information share in the volatility discovery process.

Based on a set of nine issuers—BNP Paribas, Citigroup, Commerzbank, Deutsche Bank, DZ Bank, Goldman Sachs, HSBC, UniCredit and Vontobel—we compute via VDAX method issuer specific volatility time series ISV and compare them aggregated and single to the options market component—VDAX. As the volatility series are highly (negatively) correlated with the underlying DAX, the time series are modified by eliminating the sticky strike effect. We focus on the investigation if and how options market and warrants market contribute to volatility discovery and which issuers of warrants have an information share in a bivariate system amongst themselves in an intra-warrants market analysis. Information share measures are based on the contribution of a market's innovation to the total variance of the efficient underlying price innovations. We use a special information share measure for stationary time series adapted from idea of classic variance decomposition.

We apply the method of the stationary information share to gain insight into the interdependence between the issuer specific volatility time series and the options market component VDAX: In a bivariate system, VDAX always takes leadership compared to every single issuer of the warrants market and to the aggregated warrants market. The warrants market information share for single issuers of the warrants market is often not significant from zero. The results do not verify a high information effect from a single issuer on the warrants market to options market proofing that the issuers are "blindly" following VDAX when pricing their warrants. Although the information share volatility discovery process of the aggregated warrants market is significantly greater than zero (by about 3.8%), the options market information share still lies clearly above 50% and so the issuers on the warrants market hardly influence the options market. By comparing issuers information share to each other, we find a transitive sequence of information content: UniCredit exhibiting largest information share, followed by Commerzbank, BNP Paribas, Goldman Sachs, Vontobel, Citigroup, Deutsche Bank, DZ Bank and the issuer with the lowest information share in bivariate systems is HSBC. This rankorder might lead to quotes biased to more or less favorable conditions for the investors in times of in- or decreasing volatility.

By considering issuers pricing behaviour, we found out that VDAX exhibits a small upward trend averaged over all trading days, whereas each of the averaged issuer specific volatility time series (single and aggregated) decreases over day time. One possible interpretation in line with existing literature to the results: In the early hours, net demand is higher, while in the late hours more investors tend to sell their warrants back to the issuer. This pattern could open an arbitrage exploiting strategy for issuers when selling the warrants to intraday traders in the morning expensively (high volatility) and re-buying these warrants in the evening cheaply (low volatility).

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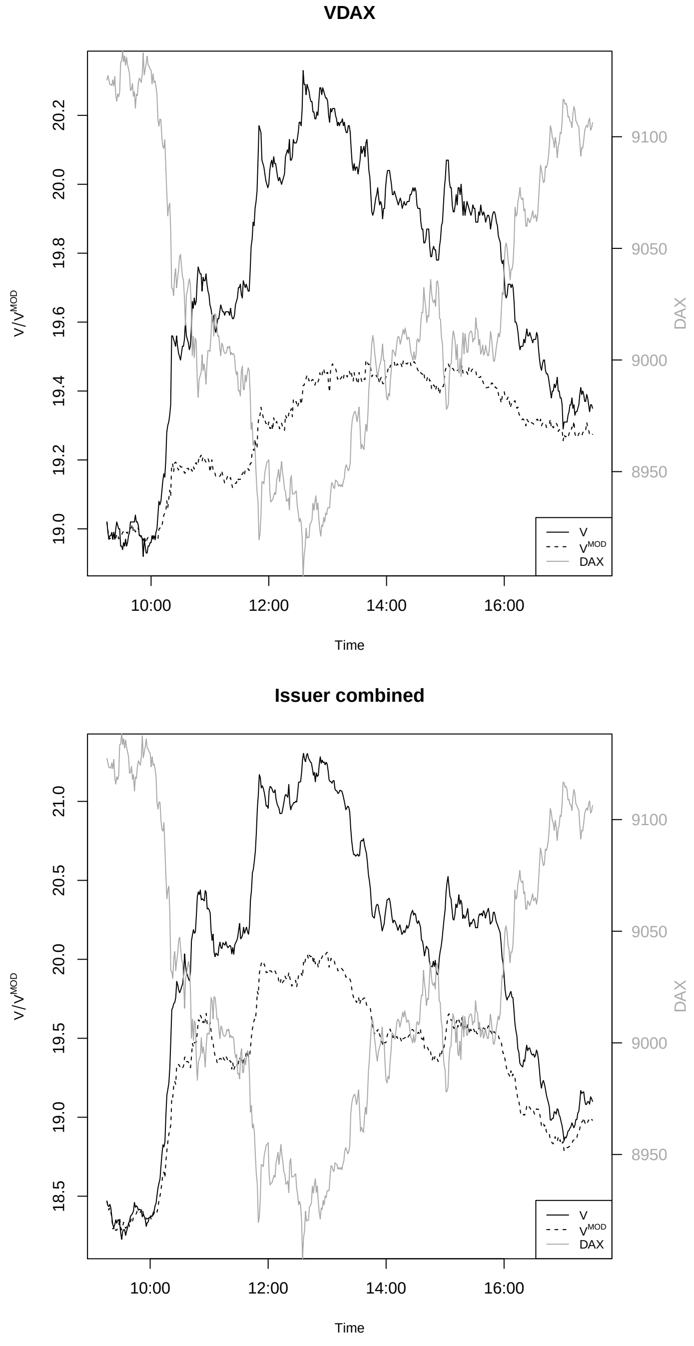

Figure 1. *Illustration of the correlation between implied volatility and underlying index for a random day (30/10/2014), for the VDAX (upper graph) and the aggregated implied volatility of warrants issuer (lower graph). Both graphs exhibit the original volatility time series (solid black), the underlying DAX time series (solid grey), and the modified volatility time series corrected for the negative correlation to the underlying by the "sticky strike" effect (dashed black).*

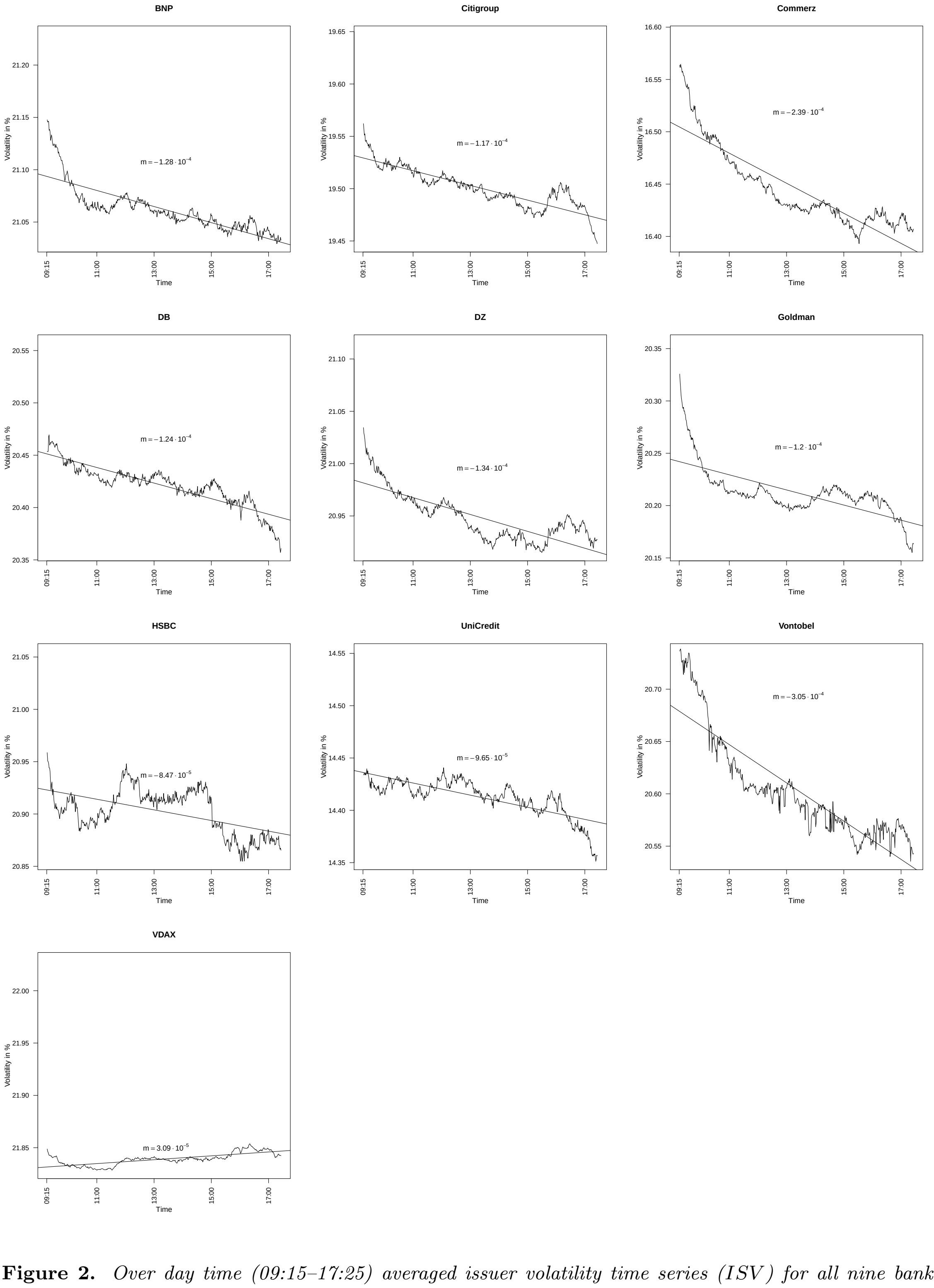

Figure 2. *Over day time (09:15–17:25) averaged issuer volatility time series (ISV) for all nine banks and averaged VDAX time series.*

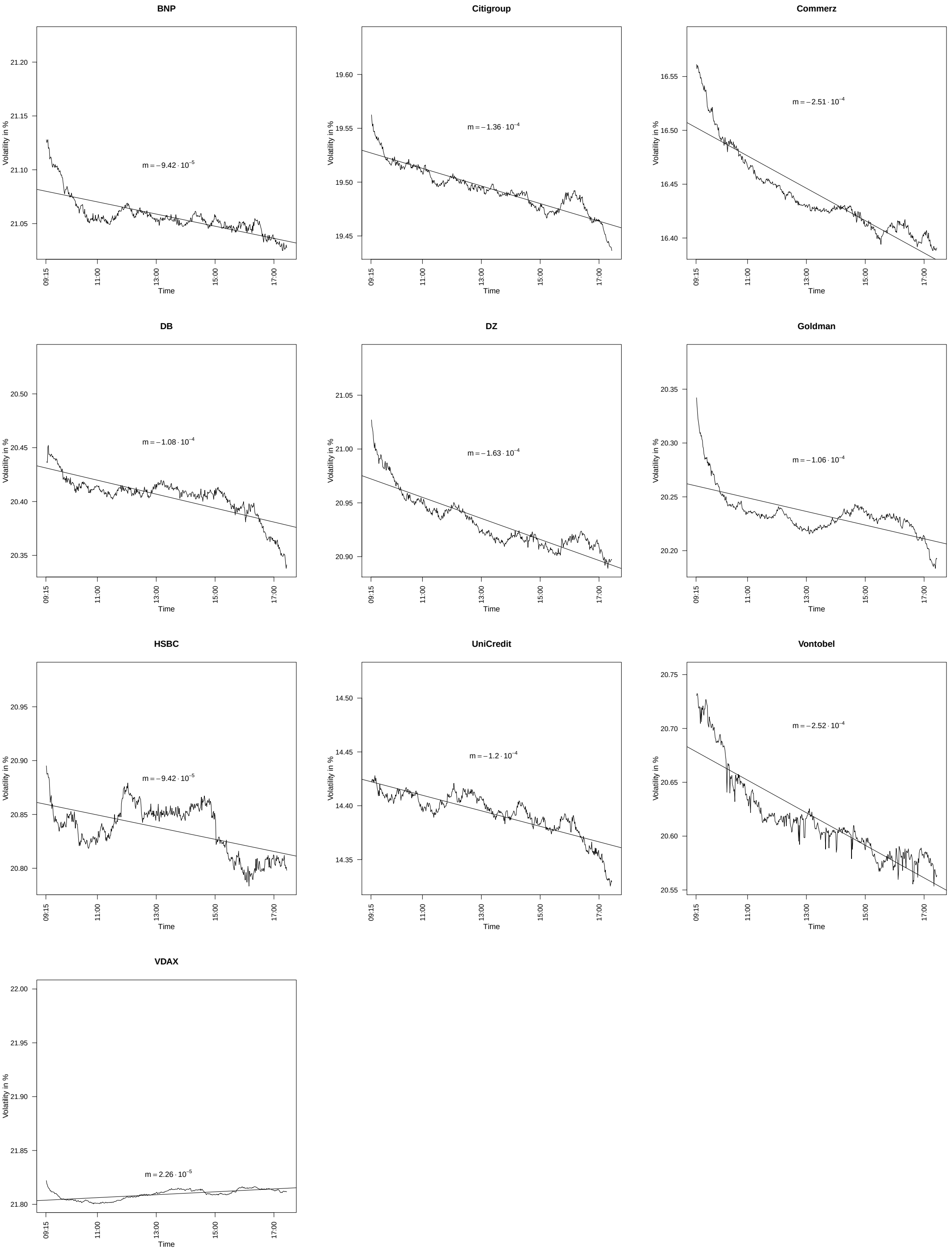

Figure 3. *Over day time (09:15–17:25) averaged modified issuer volatility time series (ISV) for all nine banks and averaged modified VDAX series.*

Issuer	Total	Calls	Puts	Min Strike	Max Strike	Days
BNP Paribas	21,549	9,604	11,945	2,500	12,900	487
Citigroup	24,502	11,969	12,533	3,000	14,000	368
Commerzbank	26,649	13,282	13,367	2,500	13,500	330
Deutsche Bank	36,172	18,367	17,805	2,350	20,000	479
DZ Bank	9,405	4,545	4,860	3,000	12,500	472
Goldman Sachs	40,191	20,807	19,384	3,100	14,000	477
HSBC	6,648	3,214	3,434	3,400	11,400	139
UniCredit	5,359	2,605	2,754	5,800	12,650	183
Vontobel	18,902	8,913	9,989	3,600	11,300	322
Total	189,377	93,306	96,071	2,500	20,000	889

Table 1. *Descriptive statistic of the nine issuers whose warrants form the database for volatility time series constructed via VDAX construction method in the time period 07/2011–12/2014. For each bank, the total number of warrants, the number of calls and the number of puts, the minimum and maximum strike price, and the number of days for which volatility time series could be constructed are reported.*

	β^V	r^2_{adj}	BNP	Citi	Commerz	Deutsche	DZ	Goldman	HSBC	UniCredit	Vontobel	VDAX
BNP Paribas	−1.389	9.61%	—									
Citigroup	−1.407	8.04%	0.034	—								
Commerzbank	−1.743	8.31%	0.455^{***}	0.471^{***}	—							
Deutsche Bank	−1.900	9.22%	0.578^{***}	0.584^{***}	−0.030	—						
DZ Bank	−1.279	5.55%	−0.031	−0.024	-0.400^{***}	-0.582^{***}	—					
Goldman Sachs	−1.210	9.54%	-0.180^{*}	−0.059	-0.521^{***}	-0.775^{***}	-0.156^{*}	—				
HSBC	−0.779	5.72%	-0.352^{***}	-0.276^{**}	-1.044^{***}	-0.856^{***}	−0.216	-0.312^{***}	—			
UniCredit	−1.052	3.48%	-0.680^{***}	-0.707^{***}	-1.135^{***}	-1.151^{***}	-0.826^{***}	-0.553^{***}	-0.530^{*}	—		
Vontobel	−1.106	3.72%	-0.369^{***}	−0.139	-0.384^{**}	-0.776^{***}	−0.200	−0.063	−0.075	0.332	—	
Issuer combined	−2.346	43.51%										-0.885^{***}
VDAX	−1.549	57.10%	0.169	0.264^{*}	0.064	-0.316^{**}	0.237^{***}	0.326^{***}	0.555^{***}	0.829^{***}	0.459^{***}	—

Table 2. *Results of "sticky strike" regressions of issuer's (single and combined) and VDAX's one-minute returns on corresponding DAX returns. In the first column, average regression coefficients* β^V *are reported for each volatility time series during their respective constructible days. The average goodness-of-fit of the linear regressions on the DAX is displayed by* r^2_{adj}. *In the following matrix, averaged* β^V*-differences are reported and t-tested against each other for each pairwise combination for the respective overlapping constructible days. For example,* 0.034 *is the average difference of BNP minus Citigroup during overlapping constructible days of BNP and Citigroup. As the* β*-coefficient are negative, a positive difference indicates an (in absolute values) higher* β*-coefficient for BNP. Significance levels are denoted by * , ** or ***, representing the 5% level, the 1%, and the 0.1% level.*

Issuer	ρ	$\lceil IS \rceil$	$\lfloor IS \rfloor$	$\overline{IS}$	$1 - \overline{IS} = \overline{IS}_{VDAX}$
BNP Paribas	0.1175	37.34	28.85	33.10^{*}	$66.90^{\dagger\dagger\dagger}$
Citigroup	0.0856	34.52	28.59	31.55^{*}	$68.45^{\dagger\dagger\dagger}$
Commerzbank	0.1442	39.49	29.87	34.68	$65.32^{\dagger\dagger\dagger}$
Deutsche Bank	0.0840	32.54	27.09	29.82^{*}	$70.18^{\dagger\dagger\dagger}$
DZ Bank	0.0876	39.88	33.70	36.79^{*}	$63.21^{\dagger\dagger\dagger}$
Goldman Sachs	0.1013	33.13	26.80	29.97	$70.03^{\dagger\dagger\dagger}$
HSBC	0.0671	38.86	34.35	36.61	$63.39^{\dagger\dagger\dagger}$
UniCredit	0.1244	41.04	32.87	36.96	$63.04^{\dagger\dagger\dagger}$
Vontobel	0.1074	37.96	29.60	33.78^{*}	$66.22^{\dagger\dagger\dagger}$
Issuers Aggregated	0.2598	43.83	27.92	35.87^{***}	$64.13^{\dagger\dagger\dagger}$

Table 3. *Average information shares (in per cent) of each of the nine issuers and the aggregated warrants market in bivariate systems with the VDAX over the entire time period 07/2011–12/2014. The first column displays the correlation between the error terms ϵ in the estimated VAR system. These correlation are rather small, so upper and lower bounds of information shares, depicted in the second and third column respectively, are close together. The fourth and fifth column display the mean information share of one particular issuer and the VDAX. Mean issuer information shares are tested versus an information share of 0 via Bootstrapping; significance at the 5% and at the 0.1% level is denoted by * and ***, respectively. Furthermore, the mean information share of the lower bound VDAX is t-tested for significance greater than 0.5. All values are significant at the 0.1% level, denoted by †††.*

	BNP	Citi	Commerz	Deutsche	DZ	Goldman	HSBC	UniCredit	Vontobel
BNP Paribas	—	$63.98^{\dagger\dagger\dagger}$	34.21^{***}	$70.16^{\dagger\dagger\dagger}$	$65.01^{\dagger\dagger\dagger}$	$61.29^{\dagger\dagger\dagger}$	$61.46^{\dagger\dagger\dagger}$	37.80^{***}	$58.93^{\dagger\dagger\dagger}$
Citigroup	36.02^{***}	—	25.14^{**}	$55.35^{\dagger}$	$61.24^{\dagger\dagger\dagger}$	35.77^{***}	$61.95^{\dagger\dagger}$	27.78^{***}	46.07^{***}
Commerzbank	$65.79^{\dagger\dagger\dagger}$	$74.86^{\dagger\dagger\dagger}$	—	$77.68^{\dagger\dagger\dagger}$	$74.76^{\dagger\dagger\dagger}$	$67.63^{\dagger\dagger\dagger}$	$64.81^{\dagger\dagger\dagger}$	48.28^{***}	$70.22^{\dagger\dagger\dagger}$
Deutsche Bank	29.84^{***}	44.65^{***}	22.32	—	$57.54^{\dagger\dagger\dagger}$	35.29^{***}	$57.08^{\dagger}$	30.97^{***}	42.87^{***}
DZ Bank	34.99^{***}	38.76^{***}	25.24^{***}	42.46^{***}	—	33.03^{***}	51.61	24.14^{***}	41.02^{***}
Goldman Sachs	38.71^{***}	$64.23^{\dagger\dagger\dagger}$	32.37^{***}	$64.71^{\dagger\dagger\dagger}$	$66.97^{\dagger\dagger\dagger}$	—	$61.49^{\dagger\dagger\dagger}$	40.75^{***}	52.18
HSBC	38.54^{**}	38.05^{*}	35.19^{*}	42.92^{***}	48.39^{***}	38.51^{*}	—	28.09	41.01^{**}
UniCredit	$62.20^{\dagger\dagger\dagger}$	$72.22^{\dagger\dagger\dagger}$	51.72	$69.03^{\dagger\dagger\dagger}$	$75.86^{\dagger\dagger\dagger}$	$59.25^{\dagger}$	$71.91^{\dagger\dagger\dagger}$	—	$71.46^{\dagger\dagger\dagger}$
Vontobel	41.07^{***}	53.93	29.78^{***}	$57.13^{\dagger\dagger\dagger}$	$58.98^{\dagger\dagger\dagger}$	47.82^{***}	58.99	28.54^{**}	—

Table 4. *Average mean information shares (in per cent) in pairwise bivariate intra warrants market systems. The rows refer to the information share of the respective issuer in a bivariate market system with the issuer given by the appropriate column. The values represent average values of the time period 07/2011–12/2014. Via bootstrapping, the respective smaller information shares are tested versus an information share of 0; significance at the 5% level is denoted by* **, at the 1% level by* ***, and at the 0.1% level by* ****. Furthermore, the respective larger information shares are t-tested versus an information share of 0.5; significance at the 5% level is denoted by* †*, at the 1% level by* ††*, and at the 0.1% level by* †††*.*

Issuer	**original**	**modified**
BNP Paribas	$-1.281 \cdot 10^{-4}$	$-0.942 \cdot 10^{-4}$
Citigroup	$-1.169 \cdot 10^{-4}$	$-1.362 \cdot 10^{-4}$
Commerzbank	$-2.394 \cdot 10^{-4}$	$-2.505 \cdot 10^{-4}$
Deutsche Bank	$-1.241 \cdot 10^{-4}$	$-1.081 \cdot 10^{-4}$
DZ Bank	$-1.344 \cdot 10^{-4}$	$-1.629 \cdot 10^{-4}$
Goldman Sachs	$-1.204 \cdot 10^{-4}$	$-1.055 \cdot 10^{-4}$
HSBC	$-0.847 \cdot 10^{-4}$	$-0.942 \cdot 10^{-4}$
UniCredit	$-0.965 \cdot 10^{-4}$	$-1.202 \cdot 10^{-4}$
Vontobel	$-3.046 \cdot 10^{-4}$	$-2.520 \cdot 10^{-4}$
Issuer combined	$-0.332 \cdot 10^{-4}$	$-0.481 \cdot 10^{-4}$
VDAX	$+0.309 \cdot 10^{-4}$	$+0.226 \cdot 10^{-4}$

Table 5. *Slope coefficients of linear trends of the averaged ISV and averaged VDAX series over day time (09:15–17:25), for the original time series and the modified time series after correcting for the "sticky strike" effect.*