# A Dynamic Delegated Investment Model of SPACs*

Dan Luo† Jian Sun‡

July 18, 2022

**Abstract**

We study SPACs (Special Purpose Acquisition Companies) in a finite-horizon continuous-time delegated investment model. Due to the misalignment in incentives, the sponsor has an increasing incentive to propose unprofitable deals to the investor as the SPAC approaches its deadline. As a response, the investor redeems shares more aggressively over time. The investor's current redemption reduces the sponsor's expected payoff from proposing unprofitable deals, but future redemption reduces his expected payoff from waiting. We discuss the welfare implications of SPAC designs related to investors' redemption: 1) prohibiting the investor from redeeming shares in late periods can be a Pareto improvement; 2) coupling the investor's deal rejection with redemption benefits the sponsor; and 3) the participation of investors with behavioral biases can be a Pareto improvement.

**Keywords:** SPAC, delegated investment, dynamic delegation, moral hazard

**JEL Codes:** D82, D86, G23

*We thank Hui Chen, Peter DeMarzo, Benjamin Hebert, Andrey Malenko, Steve Grenadier, and Jeffrey Zwiebel for helpful comments.

†Booth School of Business, University of Chicago; dan.luo@chicagobooth.edu.

‡Lee Kong Chian School of Business, Singapore Management University; jiansun@smu.edu.sg

# 1 Introduction

The past two years (2020, 2021) have witnessed a remarkable rise of the special purpose acquisition company (SPAC). A SPAC is a public shell company with no operations that uses most of its offering proceeds to acquire a private operating company and takes it public. According to the calculation of Gahng et al. (2021), in 2020, "a total of 248 SPAC IPOs raised $75.3 billion," while 165 operating company IPOs raised $61.9 billion. In 2021, 679 SPAC IPOs, accounting for more than two-thirds of all IPOs, raised $286 billion. As SPAC appears to be an important way that private companies raise money and go public, a heated debate over the consequences and the future of SPACs has emerged among practitioners and the academics. Proponents praise SPACs for their agility and flexibility to accommodate financing needs better than traditional ways.[1] Opponents, citing the poor returns in the long history of blank-check companies, denounce SPACs as "bubbles" and "scams."[2] Meanwhile, it is worth noting that as an investment vehicle and a mechanism of going public, SPAC is still evolving rapidly. Practitioners are experimenting with different practices, while regulators are also pondering how to ensure healthy growth of SPACs.[3] Therefore, the goal of this paper is to understand the economic force involved in SPACs and provide guidance for future design.

SPACs do merit a special analysis in theory. SPACs can be viewed as a kind of delegated investment, but they differ from other common ones, such as private equity, hedge funds, and mutual funds, in several aspects. First, SPAC sponsors do not receive management fees, and they receive payoffs only when a deal is completed. This payoff structure makes them less willing to forgo current unprofitable deals and wait for more profitable deals. Second, SPACs feature a relatively short horizon. Typically, absent a successful merger, a SPAC will be liquidated within 24 months while it is 10 years for private equity funds. Third, SPACs allow investors to redeem shares before deal completion, so investors are heavily involved in SPACs' decision making.

This paper analyzes the economic force involved in SPACs and provides guidance for future design, focusing on the dynamic interaction between SPAC sponsors and investors. Based on the above three features, we build a finite-horizon continuous-time model of the dynamic SPAC game with one sponsor and one representative investor. In the SPAC game, the sponsor receives deals stochastically over time and decides whether to propose one to the investor in the form of a tender offer. When a deal is proposed, the investor receives information about the deal quality and can choose to either invest in it or redeem her shares.[4] In either case, the game ends, so the sponsor's

[1] See Kristin Broughton and Mark Maurer, "Why Finance Executives Choose SPACs: A Guide to the IPO Rival," Wall Street Journal, September 22, 2020

[2] "I have never found any blank-check investment vehicle attractive. No matter what the reputation or what the sponsor might be. . . . They are the ultimate in terms of lack of transparency."—Arthur Levitt, former SEC Chairman

[3] See Dave Michaels, "SEC Weighs New Investor Protections for SPACs," Wall Street Journal, May 26, 2021

[4] In our baseline model, we consider tender offers, which means the investor rejects deals by directly redeeming

opportunity to propose is unique. If no deal is proposed by a deadline, the game also ends, and the investor gets her money back. The tension between the two players rests on two points. First, the sponsor has informational advantage over the investor. He always observes the deal type, which is either good or bad, but the investor observes the type only with a probability. Second, their interests are only partially aligned. The investor, who bears the cost of investment, prefers a good deal to no deal and further to a bad deal. The sponsor, who only enjoys the payoff of investment, prefers a good deal to a bad deal and further to no deal.

We derive a unique sequential equilibrium of the SPAC game. Generically, the equilibrium consists of two stages: in the first stage, the sponsor proposes only good deals he receives, and the investor always invests in the proposed deal; in the second stage, the sponsor proposes all the good deals and a fraction of the bad ones he receives, and the investor invests or redeems contingent on the information she observes. Since the sponsor has only one chance to propose a deal, the opportunity cost of proposing a deal is his continuation value, which is the expected payoff of proposing a deal he receives in the future. Note that the sponsor can obtain a higher expected payoff from proposing a good deal than proposing a bad one, because the investment in a good deal is more profitable and also more likely to be approved by the investor. As a result, the sponsor with a good deal must propose because at best he can receive another good deal in the future. As for the sponsor with a bad deal, waiting is double-edged: he may be better off if a good deal arrives in the future and worse off otherwise. As the SPAC approaches its deadline, the downside becomes more dominant, and thus the sponsor's continuation value decreases. At a certain point, the sponsor starts to find proposing a bad deal more desirable than waiting. Concerned about the poor average quality of the proposed deal, the investor redeems shares more aggressively based on her information over time. Such redemption effectively dampens the sponsor's incentive to propose a bad deal. The equilibrium is consistent with the conventional wisdom that the incentive misalignment between the two players gives rise to a moral hazard problem of the sponsor, and it intensifies as the SPAC approaches its deadline.

Based on the equilibrium, we then analyze the sponsor's moral hazard problem—the central friction in the game. The sponsor's moral hazard is curbed by two forces. The first is the investor's redemption based on her noisy information, and the second is the sponsor's continuation value. A useful concept here is the *accumulation* of the sponsor's continuation value. From the SPAC's deadline to its beginning, the sponsor's continuation value increases. We regard the increment of the sponsor's continuation value at an instant as its accumulation rate. An important observation is that the investor's redemption reduces the possibility of investment at that instant, and thus stifles the accumulation of the sponsor's continuation value. The key insight is that the investor's redemption has two effects on the moral hazard problem: current redemption reduces the sponsor's

shares. We also consider the case in which the investor can reject deals without redeeming her shares.

incentive to propose bad deals, but future redemption dampens the accumulation of the sponsor's continuation value and increases this incentive.

However, the investor's equilibrium redemption maximizes her expected payoff after the sponsor proposes a deal, taking neither of the two effects into consideration. This opens up the possibility that the welfare outcome can be improved by altering the way the investor participates in the SPAC. To shed light on this possibility, we discuss SPAC designs related to investors' decision making and derive their welfare implications. First, we examine the investor's redemption right. Conventional wisdom dictates that the investor benefits from her redemption right because it not only allows her to avoid investment in some bad deals but also discourages the sponsor from proposing bad deals in the first place. However, we find that when the quality of the investor's information is low, the redemption right reduces the investor's welfare because the negative effect of redemption on the sponsor's continuation value dominates. In that case, accumulating from 0, the continuation value stays low for a long time. During most of the game, the investor redeems shares aggressively, misses most good deals, and earns low profit in expectation. Simply giving the investor the right to make the decision does not guarantee efficient investment outcomes. We find that a more desirable design is to make the redemption right time-varying: the investor should be allowed to redeem shares only before a specific time point, because in late periods of the SPAC, it is more efficient to facilitate the accumulation of the sponsor's continuation value by weakening redemption.

The second SPAC practice we examine is whether to allow the investor to reject without redemption. Under tender offers, the investor rejects the sponsor's proposal through redemption. An alternative is to allow the investor to reject without redemption. In that case, the sponsor forgoes the current deal and continues searching and proposing deals until the deadline. Notably, the alternative can be naturally implemented if the investor's decision making is structured as voting. On one hand, the coercive termination feature of rejection through redemption eliminates potential future investment opportunities and hurts both players. On the other hand, redemption enables the investor's rejection to curb the sponsor's moral hazard and benefits both players. We find that the sponsor's welfare is always higher under tender offers, but the investor's is ambiguous. This analysis justifies the recent transition from voting to tender offers from an equilibrium perspective.

We assume that the representative SPAC investor is fully rational in the baseline setup. However, given the novel, complicated nature of SPACs and the participation of retail investors, investor unsophistication has raised wide concerns among the public and regulators. To consider investor unsophistication, we extend the baseline setup with investors with behavioral biases. Motivated by many accounts of investor behavior, we assume that the SPAC is held by a rational investor and a behavioral investor who redeems shares less aggressively than the rational one. In response to the behavioral investor's less redemption, the rational investor redeems more aggressively. As a result, the existence of the behavioral investor reduces the aggregate redemption of the whole investor group only in late periods of the SPAC life cycle. In late periods, because of the behavioral investor's unprofitable investment, the investors' total welfare is lower. However, the sponsor's continuation value accumulates more rapidly, which mitigates moral hazard and increases the investors' total welfare in early periods. We find that when the quality of the investor's information is low, letting the behavioral investor hold a small fraction of the SPAC can be a Pareto improvement. The intuition is similar to that about the superiority of time-varying redemption right. This result implies that measures increasing investor sophistication may have unintended negative effects.

Finally, we discuss several extensions of the model. First, we explicitly incorporate entrepreneurs into the model and consider their strategic behavior. Entrepreneurs can raise funds through either the SPAC or a standard IPO. The opportunity cost of tapping the SPAC is that the deal may fail because of the investor's redemption, which delays the IPO process. Hence, the investor's redemption effectively discourages entrepreneurs from tapping the SPAC and diminishes the flow of deals the sponsor receives. Second, we find that as the SPAC approaches its deadline, the sponsor's equilibrium effort first increases due to declining continuation value and then decreases due to intensifying redemption of the investor. The two extensions further stoke our concern that the investor's redemption exacerbates the moral hazard problem and may backfire. Third, we consider the case that the sponsor can still complete the deal after the investor redeems her shares but receives a lower payoff, and show that the paper's main insights are robust. Fourth, we consider the case of long-lived deals in which the sponsor can possibly keep a deal for future proposals, showing that such possibility does not alter the equilibrium dynamics in the baseline setup.

The paper proceeds as follows. The remainder of this section reviews the related literature. Section 2 describes the baseline setup. Section 3 characterizes the equilibrium and analyzes the key forces underlying it. Section 4 discusses the design of redemption right in SPACs. Section 5 incorporates investors with behavioral biases into the model, and discusses its welfare implications. Section 6 extends the baseline setup along several dimensions. Section 7 concludes the paper. All proofs are given in Appendix.

**Related literature.** This paper contributes to the growing literature examining the development, trend, and performance of SPACs. Gahng et al. (2021) examine SPAC performance and show that SPAC investors earn positive 9.3% per year, while post-merger returns are significantly negative. They also show that SPACs have no cost advantage compared with traditional IPO. Blomkvist and Vulanovic (2020) show that the SPAC volume and SPAC share of total IPOs are negatively correlated with VIX and time-varying risk aversion, implying that market condition is a key factor in SPAC development. As a possible explanation, Alti and Cohn (2022), Bai et al. (2021), and Gryglewicz et al. (2021) consider the endogenous choice of SPACs versus traditional IPOs in rational models. Alti and Cohn (2022) focus on the signaling aspect of SPAC acquisitions.

Bai et al. (2021) provide a model with endogenous segmented markets and argue that SPACs are welfare-improving as they work as certification intermediaries for risky firms who were unserved by the traditional IPO. Gryglewicz et al. (2021) consider this question from a perspective of optimal contract. They consider two related adverse selection problems, i) adverse selection over firm type and ii) adverse selection over sponsor type, highlighting the public observability of SPAC contracts as the key feature. They argue that the SPAC is optimal under certain market conditions. Our model mainly focuses on the moral hazard problem embedded in the unique timing and organizational structure of SPACs, which is quite different from the focus of the aforementioned papers. Similar to our paper, Feng et al. (2022) focus on the preference misalignment and asymmetric information among sponsors and investors. By calibrating a model, they quantify the degree of the agency frictions and associated losses to investors. They confirm that the agency cost is pervasive and significant in the data, which is consistent with our motivation.

Chatterjee et al. (2016) and Banerjee and Szydlowski (2021) provide theoretical foundations for the use of warrants in SPACs. Chatterjee et al. (2016) consider SPACs' security design problem and argue that warrants in SPACs can mitigate the moral hazard problem in deal selection. Banerjee and Szydlowski (2021) consider security design of SPACs in a framework with behavioral investors. They show that it is optimal to include warrants in the contract when investors are overconfident in their ability to acquire information. Although our question and approach are quite different from theirs, our discussion of behavioral investors generates similar predictions. For example, we show that restricting behavioral investors' access to SPACs may not be welfare-improving, as it can exacerbate the sponsor's moral hazard problem, while Banerjee and Szydlowski (2021) obtain similar predictions in a model endogenizing sponsor's effort.

Our paper complements the theory literature by focusing on the dynamic moral hazard problem generated from the unique timing structure of SPACs and the resulting implications for SPAC designs.

The empirical literature examining SPACs provides important and interesting observations (Cumming et al. (2014), Dimitrova (2017), Jenkinson and Sousa (2011), Jog and Sun (2007), Klausner et al. (2020), Kolb and Tykvova (2016), Lin et al. (2021), Pawliczek et al. (2021), Rodrigues and Stegemoller (2012)), and our paper provides theoretical explanations for some of them. In particular, Dimitrova (2017) shows that SPAC performance is worse for deals announced near the two-year deadline, which is consistent with our theoretical prediction, as we argue that sponsors have increasing incentives to propose bad deals over time. Examining the factors that influence approval probability, Cumming et al. (2014) find that the presence of active investors in a SPAC is negatively correlated with approval probability. They also argue that the two-year life cycle is a crucial factor in determining the approval probability. Our main model and the extension with behavioral investors provide a framework to explain these results. Klausner et al. (2020) show

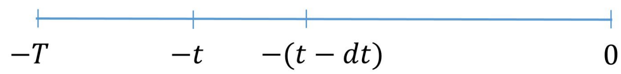


Figure 1: Time Flow

that post-merger performance is negatively correlated with dilution and cash shortfall. Although we don't model dilution explicitly, our model predicts that the post-merger performance and cash shortfall are negatively correlated. This is because when time is closer to the end of SPAC life cycle, investors are more likely to redeem their shares and sponsors are more likely to propose bad deals.

Our paper also contributes to the literature on delegation and authority in organizations (Crawford and Sobeli 1982; Aghion and Tirole 1997; Dessein 2002; Grenadier et al. 2016; Guo 2016). There are some trade-offs identified in this literature, including the trade-off between informativeness and bias (Dessein 2002) and information acquisition of different players (Aghion and Tirole 1997). We show the allocation of redemption right endogenously changes the shape of the sponsor's bias in the SPAC life cycle. This is a direct result of the dynamic nature and short investment horizon of our model, which is novel in the literature. As for dynamic setups, Grenadier et al. (2016) considers a model in which the principal exercises an option and relies on an informed but biased agent. Guo (2016) considers a dynamic delegation model with experimentation in which the principal and agent have different preferences on deal riskiness. The focus of our paper and fundamental frictions are quite different from theirs.

# 2 A Dynamic Model of SPAC

## 2.1 Model setup

Consider a SPAC with one sponsor (he) and one investor (she). They are both risk neutral and have common discount rate $r = 0$.[5] Motivated by the practice in reality, we model the SPAC as a finite-horizon continuous-time dynamic game unfolding over the period $[-T, 0]$. Figure 1 is a representation of the time flow. Both $t$ and $T$ are non-negative, and physical time moves forward as $t$ decreases from $T$ to 0. As we will show, it's easier to consider our model backward, which corresponds to $t$ increasing from 0 to $T$.

**Deals** Since this paper primarily focuses on the strategic interaction between the sponsor and the investor, we abstract away entrepreneurs' strategic behavior and assume an exogenous arrival

[5] We assume no discounting merely to simplify the exposition. The main results hold for a positive discount rate.

process of deals.[6] Per unit of time, the sponsor receives deals at the rate $\lambda$. A deal can be either good ($\omega = G$) or bad ($\omega = B$), and the probability (odds) of receiving a good deal is $\text{Prob}(G) = p_0$ ($\theta_0 = p_0/(1-p_0)$). The arrival and types of deals are independent over time. Both good and bad deals require the same investment 1 and generate gross return $R_G$ and $R_B$ respectively. We make the following assumptions on $R_G$ and $R_B$:

**Assumption 1.** *(1) $R_G > 1 > R_B > 0$; (2) (negative NPV) $p_0 R_G + (1-p_0) R_B < 1$.*

The first assumption states that good deals are efficient while bad deals are inefficient, and they both have positive deal values. The second one resonates with the concern that potential SPAC targets are of poor quality on average.

When a deal arrives, the sponsor decides whether to propose it to the investor. For the baseline setup, we assume that deals are short-lived. That is, if the sponsor does not propose the deal he receives, the deal will disappear and become unavailable immediately. With this assumption, the state of the sponsor with respect to whether he has a deal and what type of deal he has is completely independent over time.[7]

**The investor's decision making** A salient feature of SPACs is that investors can decide whether to make an investment through redemption. Traditionally, after the sponsor proposes a deal, investors vote on it. The deal succeeds if and only if a sufficient fraction of investors vote for it. However, in the recent wave of SPACs, tender offers have become the most popular way to structure investors' decision making. Shachmurove and Vulanovic (2017) highlight that "these post financial crisis SPACs are almost exclusively structured as tender offers." Motivated by this trend, we model the sponsor's proposal as a tender offer. The investor can choose to either invest one unit and receive a pre-specified fraction of shares of the deal, or redeem all her shares. Due to the nature of a tender offer, the game ends immediately after a proposal. Essentially, the sponsor has only one opportunity to propose a deal to the investor during the SPAC life cycle.

**Information** The arrival and deal type are observable to the sponsor but not to the investor.[8] When the sponsor proposes a deal, the investor receives a signal, and observes the deal type with the probability $q$ and nothing otherwise. We denote the investor's signal observation as $\{H, \varnothing, L\}$, whose probabilistic structure follows Table 1. Hence, $q$ stands for the quality of the investor's information, and $q < 1$ captures the information asymmetry between the sponsor and the investor.

[6] In Section 6.1, we explicitly model entrepreneurs' strategic behavior and examine its impact on the equilibrium dynamics.

[7] In Section 6.4, we study the case that deals are long-lived and thus the state of the sponsor is positively correlated over time.

[8] The assumption that arrivals are privately observed by the sponsor is not important. The equilibrium will be the same even if the arrivals are publicly observable.

| Observations | $H$ | $\varnothing$ | $L$ |
|---|---|---|---|
| Prob $(\cdot; G)$ | $q$ | $1-q$ | 0 |
| Prob $(\cdot; B)$ | 0 | $1-q$ | $q$ |

Table 1: The investor's observations

| Investment | $G$ | $B$ | Redeem shares |
|---|---|---|---|
| The sponsor's payoff | $v_G$ | $v_B$ | 0 |
| The investor's payoff | $u_G$ | $u_B$ | 1 |

Table 2: The payoff structure

Notice that with Assumption 1, if the sponsor proposes any deal he receives, the investor will have a negative expected profit of investing upon observing $\varnothing$.

**The payoff structure** Depending on the investment result, the sponsor and investor's payoffs follow Table 2. If the investor chooses not to redeem upon a proposal, the proposed deal succeeds. In this case, if the deal type is $\omega$, the investor receives the shares and warrants of the target firm, which are worth $u_\omega$, and the sponsor receives those worth $v_\omega = R_\omega - u_\omega$. If the investor chooses to redeem shares, the proposed deal fails. She keeps her money 1, and the sponsor receives 0. If the sponsor does not make a proposal by the SPAC deadline, the investor automatically redeems shares. We make the following assumption regarding the payoff structure:

**Assumption 2.** *(partial alignment)* $v_G > v_B > 0$, $u_G > 1 > u_B$.

This assumption stems from the contractual arrangement of SPACs: the shares granted to the sponsor are not contingent on the outcome of the investment. Typically, upon deal completion, the sponsor can obtain 20% of the deal value, and the investors obtain the other 80%. As a result, the sponsor prefers an investment in a bad deal to no investment, while the investor prefers the opposite. As recognized by both academics and practitioners, this preference misalignment underlies the fundamental moral hazard problem in SPACs.[9] On the other hand, it should not be ignored that the contractual arrangement also has an alignment side: both the sponsor and the investor prefer an investment in a good deal to an investment in a bad deal or no investment. As we will show, both sides of the partial alignment play important roles in equilibrium dynamics.

**Timeline** Although the game is in continuous time, heuristically, conditional on the game continues at time $-t$, each instantaneous "period" $[-t, -(t-dt))$ consists of events occurring in the

[9] Aware that potential agency problems may discourage investors, some SPAC sponsors try to tie their shares more closely to the ex-post value of the investment through deferred grant or clawback. It has also become more popular to let the sponsor have some skin in the game. However, these remedies are still far from eliminating the misalignment.

following order:

1. With the probability $\lambda dt$, the sponsor receives a deal and observes its type;
2. upon receiving a deal, the sponsor can propose it or not;
3. if the sponsor proposes the deal, the investor receives a signal and chooses to invest in the deal or redeem shares; then the game ends, and both players receive their payoffs;
4. if the sponsor does not propose the deal, the game continues to $-(t-dt)$.

## 2.2 Discussion of the model setup

Since the paper focuses on the strategic interaction between the sponsor and the investor, we exclude some elements common in SPACs from the baseline setup to simplify the illustration. Here, we briefly discuss these elements.

**Redemption and deal completion** The baseline setup assumes that if the investor redeems shares, the proposed deal fails and the sponsor receives nothing. However, sometimes the deal still succeeds despite massive redemption. For instance, the sponsor or non-SPAC investors may inject funds into the deal, or the target firm may intend to go public irrespective of the fund it receives. In these cases, the sponsor receives a positive payoff even if the investor redeems shares. We formally study this case in Section 6.3 and show that the paper's main insights are robust.

**The secondary market** SPACs are public companies and their shares are publicly traded. The secondary market may affect SPACs in two main ways. First, investors are able to trade their shares in the secondary market to meet their liquidity needs. Second, the secondary market aggregates investors' information and potentially affects their redemption decisions. However, investors have no liquidity demand or heterogeneous information in the baseline setup, so the secondary market has no impact on our equilibrium.

**Private Investment in Public Equity (PIPE)** SPAC sponsors frequently invite PIPE investment as part of the business combination. PIPE investment has two potential effects on the strategic interaction between the sponsor and the investor. First, upon deal completion, the sponsor may receive a transfer from PIPE investors. In that case, $v_\omega$ in the baseline setup can be interpreted as the sum of the sponsor's payoff from the SPAC and the transfer from PIPE investors. Second, PIPE investment may allow the deal to succeed even if a large fraction of investors redeem shares. This case is studied in Section 6.3.

## 2.3 Equilibrium concept

We focus on the sequential equilibria of the game. First, we characterize the players' strategies and beliefs. Since the game has a finite horizon, time is naturally a state variable on which their strategies are based. The sponsor has only one action in the game: whether to propose the deal he receives. Hence, his strategy can be characterized by $(\alpha_\omega(-t))_{\omega\in\{G,B\}}$, where $\alpha_\omega(-t)$ represents the probability that the sponsor proposes the deal of type $\omega$ at the time $-t$. The investor also has only one action in the game: whether to invest in the deal proposed by the sponsor based on her signal. Therefore, her strategy can be characterized by $(\eta_s(-t))_{s\in\{H,L,\varnothing\}}$, where $\eta_s(-t)$ represents the probability that the investor invests at the time $-t$ when observing the signal $s$.

The players' beliefs can be characterized accordingly. Let $(\tilde{\eta}_s(-t))_{s\in\{H,\varnothing,L\}}$ be the sponsor's belief about the investor's strategy. Then, by proposing a deal of type $\omega$ to the investor at $-t$, the sponsor's expected payoff is

$$F_\omega(-t)\equiv\begin{cases}[q\tilde{\eta}_H(-t)+(1-q)\tilde{\eta}_\varnothing(-t)]v_G, & \text{if } \omega=G\\ [(1-q)\tilde{\eta}_\varnothing(-t)+q\tilde{\eta}_L(-t)]v_B, & \text{if } \omega=B\end{cases}.$$

Let $\tilde{\theta}(-t)$ be the investor's prior belief of the odds of a good deal before observing the signal. As required by sequential equilibria, these beliefs should be consistent with the strategies on the equilibrium path according to Bayes' rule. However, in this model, sequential equilibria have no effective restriction on the beliefs off the equilibrium paths. Specifically, if $\alpha_G(-t)=\alpha_B(-t)=0$ at a time $-t$, $\tilde{\theta}(-t)$ can take any nonnegative values. This gives rise to a multiplicity of equilibria.[10] To obtain sharp predictions of the equilibrium, we impose D1 refinement: the investor believes that the deal must be good if it is proposed by the sponsor at a time when no deal should be proposed in equilibrium.

Below is the equilibrium concept used throughout the paper.

**Definition 1.** An (sequential) equilibrium consists of the sponsor's proposal strategy $(\alpha_\omega(-t))_{\omega\in\{G,B\}}$, the investor's investment strategy $(\eta_s(-t))_{s\in\{H,\varnothing,L\}}$, the sponsor's belief $(\tilde{\eta}_s(-t))_{s\in\{H,\varnothing,L\}}$, and investor's belief $\tilde{\theta}(-t)$, such that at any time $-t\in[-T,0]$ and conditional on no proposal before $-t$, the following conditions hold:

1. $(\alpha_\omega(-\tau))_{\omega\in\{G,B\}}$ after $-t$ maximizes the sponsor's continuation value at $-t$:

$$V(-t)=\max_{(\alpha_\omega(-\tau))_{\omega\in\{G,B\}}}\int_0^t P(-\tau;-t)\cdot\lambda\left[p_0\alpha_G(-\tau)\cdot F_G(-\tau)+(1-p_0)\alpha_B(-\tau)\cdot F_B(-\tau)\right]d\tau,$$

[10] Besides the equilibrium we derive later, another obvious equilibrium is that $\alpha_G(-t)=\alpha_B(-t)=0$ and $\tilde{\theta}(-t)=0$ all the time.

where $P(-\tau;-t) \equiv e^{-\int_\tau^t \lambda[p_0\alpha_G(-\xi)+(1-p_0)\alpha_B(-\xi)]d\xi}$ is the probability that the game still continues at time $-\tau > -t$ conditional on that the game continues at time $-t$.

2. For any $s \in \{H, \varnothing, L\}$, the investor's investment strategy $\eta_s(-t)$ maximizes her expected profit based on the prior belief $\tilde{\theta}(-t)$ and the signal $s$:

$$\eta_s(-t)\left\{\frac{\tilde{\theta}(-t)\frac{\text{Prob}(s;G)}{\text{Prob}(s;B)}}{1+\tilde{\theta}(-t)\frac{\text{Prob}(s;G)}{\text{Prob}(s;B)}}(u_G-u_B)+u_B-1\right\}.$$

3. Rational beliefs and D1 refinement:

   (a) $\tilde{\eta}_s(-t) = \eta_s(-t)$ for all $-t$ and $s \in \{H, \varnothing, L\}$;

   (b) $\tilde{\theta}(-t) = \frac{p_0}{1-p_0}\frac{\alpha_G(-t)}{\alpha_B(-t)}$ for all $-t$ satisfying $\alpha_G(-t)+\alpha_B(-t) > 0$;

   (c) $\tilde{\theta}(-t) = +\infty$ if $\alpha_G(-t) = \alpha_B(-t) = 0$.

# 3 Model Solution

## 3.1 Players' strategies

We first analyze the investor's problem. When the investor observes the signal $H$ ($L$), her posterior probability of the proposed deal being good becomes 1 (0), and her net payoff from investing in the deal is $u_G - 1 > 0$ ($u_B - 1 < 0$). Thus her equilibrium strategy must be $\eta_H(-t) = 1$ and $\eta_L(-t) = 0$ for all $-t$. To characterize the investor's equilibrium strategy, we can focus on that when she observes the signal $\varnothing$, i.e., $\eta_\varnothing(-t)$. For simplicity, we remove the subscript of $\eta_\varnothing$, and let $\eta(-t) \equiv \eta_\varnothing(-t)$. It is easy to see that the investor's problem can be reduced to

$$\max_{\eta(-t)} \eta(-t)\left\{\tilde{\theta}(-t) - \frac{1-u_B}{u_G-1}\right\},$$

where $\tilde{\theta}(-t)$ is the investor's belief of the odds of a good deal before observing her signal. Then we obtain the following lemma.

**Lemma 1.** *In equilibrium, at any time $-t$,*

1. $\eta_H(-t) = 1$, *and* $\eta_L(-t) = 0$ *;*
2. *when* $\tilde{\theta}(-t) > (<) \frac{1-u_B}{u_G-1}$, $\eta(-t) = 1$ (0)*; when* $\tilde{\theta}(-t) = \frac{1-u_B}{u_G-1}$, $\eta(-t) \in [0,1]$.

Next, we turn to the sponsor's problem. According to Lemma 1 and rational beliefs in equilibrium, if the sponsor proposes a deal of type $\omega$ at time $-t$, his expected payoff is

$$F_{\omega}(-t) = \begin{cases} [q + (1-q)\eta(-t)]\, v_G, & \text{if } \omega = G \\ (1-q)\eta(-t)\, v_B, & \text{if } \omega = B \end{cases}.$$

At any time $-t$, the sponsor's continuation value $V(-t)$ satisfies the HJB equation

$$\frac{dV(-t)}{dt} = \max_{\alpha_G(-t),\alpha_B(-t)} \lambda p_0 \cdot \alpha_G(-t) \cdot [F_G(-t) - V(-t)] + \lambda(1-p_0) \cdot \alpha_B(-t) \cdot [F_B(-t) - V(-t)]. \tag{1}$$

In addition, at the last instant of the game, it is almost certain that the sponsor will not receive a deal, so the continuation value at $-t = 0$ must be 0, i.e., $V(0) = 0$. Throughout the paper, we refer to the derivative $dV(-t)/dt$ as the accumulation rate of the sponsor's continuation value. Since

$$V(-t) = \int_0^t \frac{dV(-\tau)}{d\tau} d\tau,$$

a higher accumulation rate after $-t$ implies a higher level of the sponsor's continuation value at $-t$.

Equation (1) reflects an important feature of the game: the sponsor has at most one opportunity to propose a deal. When proposing a deal of type $\omega$ at $-t$, the sponsor has the expected payoff $F_{\omega}(-t)$. In the meantime, he also loses the opportunity to receive and propose new deals in the future, whose value amounts to $V(-t)$ in expectation. Therefore, the sponsor's equilibrium strategy $\alpha_{\omega}(-t)$ must satisfy

$$\alpha_{\omega}(-t) \begin{cases} = 1 & \text{if } F_{\omega}(-t) - V(-t) > 0 \\ \in [0,1] & \text{if } F_{\omega}(-t) - V(-t) = 0 \\ = 0 & \text{if } F_{\omega}(-t) - V(-t) < 0 \end{cases}$$

for $\omega \in \{G, B\}$.

## 3.2 Equilibrium characterization

A critical observation about the game is that the sponsor always has more incentive to propose a good deal than a bad one. On one hand, $F_G(-t) > F_B(-t)$ always holds, because a good deal not only gives the sponsor a higher payoff than a bad one but also is more likely to be approved by the investor. On the other hand, the opportunity cost of proposing a deal at time $-t$, $V(-t)$, is independent of the type of the deal that the sponsor receives. Thus, in equilibrium, it is always strictly better for the sponsor to propose a good deal than not. Further, the sponsor's continuation value is decreasing over time, i.e., $dV(-t)/dt > 0$, because as the time passes, he is less likely to receive and propose a good deal.

**Lemma 2.** *In equilibrium, for any $-t$, $F_G(-t) > V(-t)$. Further, $\alpha_G(-t) = 1$, and $V(-t)$ strictly decreases to $0$ as $-t$ increases to $0$.*

Since the sponsor always proposes the good deal he receives, the prior belief of the proposed deal's type depends on his incentive to propose bad deals. Lemma 3 implies that whenever the sponsor receives a bad deal, he must forgo it with a positive probability. This relies on the key assumption that SPACs' potential deals have a negative NPV on average, i.e.,

$$p_0 R_G + (1-p_0) R_B < 1.$$

If the sponsor proposes any bad deal he receives at a time point, the investor must redeem shares upon observing $\varnothing$, because she has a negative expected profit of investing. Given $\eta(-t) = \eta_L(-t) = 0$, the sponsor should have no incentive to propose a bad deal. Hence, this case cannot occur in equilibrium.

**Lemma 3.** *When $V(-t) < (1-q)v_B$ and $-t < 0$, $\alpha_B(-t) \in (0,1)$. When $V(-t) > (1-q)v_B$, $\alpha_B(-t) = 0$.*

Combining Lemma 1, Lemma 2, and Lemma 3, we obtain a unique equilibrium of the game, which consists of potentially two stages. For convenience, we focus on the cases in which the two stages emerge in equilibrium.

**Proposition 1.** *The unique equilibrium of the SPAC game has potentially two stages, the transition time between which is $-t^*$.*

- *The second stage spans the period $(-t^*, 0]$, in which*
  - *the sponsor's equilibrium strategy $(\alpha_\omega(-t))_{\omega \in \{G,B\}}$ satisfies $\alpha_G(-t) = 1$ and makes the investor indifferent to whether to invest or redeem shares when observing $\varnothing$, i.e.,*

$$\frac{p_0}{1-p_0}\frac{\alpha_G(-t)}{\alpha_B(-t)} = \frac{1-u_B}{u_G-1};$$

  - *the investor's equilibrium strategy $\eta(-t)$ makes the sponsor indifferent to whether to propose a bad deal or not, i.e.,*

$$V(-t) = F_B(-t) = (1-q)\eta(-t)v_B;$$

- *The first stage spans the period $[-T, -t^*)$, in which*

- *the sponsor proposes only good deals, i.e.,* $\alpha_G(-t) = 1$ *and* $\alpha_B(-t) = 0$*;*
- *the investor always invests when observing* $\varnothing$*, i.e.,* $\eta(-t) = 1$*;*

- *The transition time* $-t^*$ *satisfies* $V(-t^*) = (1-q)v_B$*, and* $V(-t)$ *is continuous at* $-t^*$*. If* $-T \geq -t^*$*, the first stage will be degenerate, and the equilibrium will be completely in the second stage.*

The misalignment of the two players' incentives is key to the equilibrium dynamics. Due to the finite horizon of the SPAC, as time passes, the sponsor has less chance to receive a deal, and thus his continuation value decreases. Note that the continuation value is also the opportunity cost of proposing a deal, which dampens the sponsor's desire to propose a bad deal. In early periods of the game, the continuation value is high enough to prevent the sponsor from proposing any bad deal, even though the investor never redeems shares upon observing the signal $\varnothing$, i.e., $\eta(-t) = 1$. Later on, when the continuation value is low, the sponsor begins to find proposing a bad deal desirable. Because of the poor average quality of potential deals, the investor is concerned about an undisciplined sponsor and spontaneously chooses to redeem shares sometimes, i.e., $\eta(-t) < 1$, which in turn helps dampen the sponsor's desire to propose bad deals. As such, moral hazard arises due to the misalignment, but is also mitigated by the investor's redemption.

However, the alignment of their incentives also plays an important role here. By waiting, the sponsor may receive good deals in the future. On one hand, according to the SPAC's payoff structure, a good deal gives the sponsor a higher payoff than a bad one. On the other hand, since the investor also prefers a good deal to a bad one, her optimal redemption automatically makes investment in the former more likely than that in the latter. For these two reasons, when the SPAC has sufficient time left, the sponsor perceives the probability of investing in a good deal in the future to be sufficiently high and thus would like to forgo the bad deal at hand. As such, the alignment mitigates moral hazard by facilitating the accumulation of the sponsor's continuation value.

Through the two channels, the investor's redemption has two effects on the sponsor's behavior: current redemption reduces the sponsor's incentive to propose bad deals, but future redemption dampens the accumulation of the sponsor's continuation value and increases this incentive. Later, we will see that the two effects have important implications for SPAC design.

## 3.3 Players' continuation value

Proposition 2 provides a characterization of the two players' continuation values. We denote the investor's continuation value at $-t$ by $U(-t)$. Then the sponsor's welfare and the investor's welfare in the SPAC game are $V(-T)$ and $U(-T)$, respectively.

**Proposition 2.**

- *The sponsor's continuation value is equal to his expected payoff if he proposes only good deals to the investor on the equilibrium path, i.e.,*

$$V(-t) = v_G \int_0^t \lambda p_0 e^{-\lambda p_0 (t-\tau)} \left[q + (1-q)\,\eta(-\tau)\right] d\tau, \tag{2}$$

*where* $\eta(-\tau) = \min\left\{1, \frac{V(-\tau)}{(1-q)v_B}\right\}$*. Further, for* $t \le t^*$*,*

$$V(-t) = \left[e^{\lambda p_0 \left(\frac{v_G}{v_B}-1\right)t} - 1\right] \frac{1}{\frac{v_G}{v_B}-1} \cdot q \cdot v_G, \tag{3}$$

*and for* $t > t^*$*,*

$$V(-t) = \left[1 - e^{-\lambda p_0 (t-t^*)}\right] v_G + e^{-\lambda p_0 (t-t^*)} V(-t^*). \tag{4}$$

- *Denote the unconditional probabilities that the sponsor proposes good deals since* $-t$ *in the first and the second stages by*

$$PG_1 = 1 - e^{-\lambda p_0 t (-\min\{t,t^*\})}$$
$$PG_2 = e^{-\lambda p_0 (t-\min\{t,t^*\})} \left(1 - e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B} \min\{t,t^*\}}\right) \frac{1}{\frac{u_G-u_B}{1-u_B}}$$

*respectively. The investor's continuation value is linear in them, i.e.,*

$$U(-t) = (u_G - 1) \cdot (PG_1 + q \cdot PG_2) + 1. \tag{5}$$

Since the sponsor is always indifferent to whether to propose a bad deal in the second stage, we can use the equilibrium path in which the sponsor never proposes a bad deal to calculate his continuation value. A useful observation about the sponsor is that his continuation value depends only on how likely a proposed good deal is invested in by the investor and consists of two parts. First, upon observing $H$, the investor invests with the probability 1, which results in investment in good deals occurring at the rate of $\lambda p_0 q$. Second, upon observing $\varnothing$, the investor invests with the probability $\eta(-t)$ at $-t$, which results in investment in good deals occurring at the rate of $\lambda p_0 (1-q)\,\eta(-t)$. In the second stage, $\eta(-t)$ makes the sponsor indifferent to whether to propose a bad deal, so it is proportional to the sponsor's continuation value as follows

$$\eta(-t) = \frac{V(-t)}{(1-q)\,v_B}.$$

Therefore, the sponsor's continuation value follows a kind of self-reinforcing dynamics: it accumulates at a higher rate when it takes a higher value. Specifically,

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot \left[ q v_G + \left( \frac{v_G}{v_B} - 1 \right) V(-t) \right].$$

The intuition is that because the investor redeems shares based on noisy information, to induce a lower probability of investing in bad ones, she unavoidably forgoes some good ones; a lower level of the sponsor's continuation value requires more redemption and thus results in more good deals being forgone.

An important implication of the self-reinforcing dynamics is that the sponsor's continuation value in the second stage is very sensitive to the quality of the investor's information $q$. As indicated by Equation (3), with a small $q$, the investor observes $H$ with a low probability, and the potential investment in good deals when she observes $\varnothing$ accounts for most of the sponsor's continuation value. As a result, the self-reinforcing dynamics largely determines its evolution: a low level of the sponsor's continuation value $V(-t)$ directly translates into a low accumulation rate $dV(-t)/dt$. Notice that in our finite-horizon SPAC game, the accumulation starts at $V(0) = 0$. The self-reinforcing dynamics traps both $V(-t)$ and $dV(-t)/dt$ at low levels for a long period. With a large $q$, the sponsor's continuation value accumulates at a substantially high rate even when its level is low; meanwhile, the rapid increase in the continuation value further accelerates its accumulation.

The statement about the investor's welfare stems from the fact that the investor earns a positive profit when she knows the proposed deal is surely good and otherwise breaks even in expectation. In the first stage, only good deals are proposed, and the probability that it happens is $PG_1$. In the second stage, the investor's expected profit is equal to 0 when observing $\varnothing$ or $L$, and $u_G - 1$ when observing $H$. With probability $PG_2$, the sponsor proposes a good deal in the second stage, and conditional on that, the investor observes $H$ with the probability $q$. A useful observation about the investor is that the length of the second stage $t^*$ and the quality of her information $q$ are important for her welfare.

## 3.4 Moral hazard and the investor's redemption

Proposition 2 illustrates the impact of the partial alignment of the two players' incentives on their welfare. In the first stage, the alignment side dominates: the sponsor proposes only good deals, and the investor does not redeem shares upon observing $\varnothing$. They both enjoy the efficient investment outcome. In the second stage, the misalignment side dominates: the sponsor proposes some bad deals, and the investor redeems shares with a positive probability upon observing $\varnothing$ as a response. Such equilibrium interaction hurts both players. Therefore, the length of the second stage, $t^*$, is an important measure of the adverse impact of moral hazard in equilibrium. Proposition 3 presents the characterization of $t^*$.

**Proposition 3.** *$t^*$ satisfies*

$$\left[e^{\lambda p_0\left(\frac{v_G}{v_B}-1\right)t^*}-1\right]\frac{1}{\frac{v_G}{v_B}-1}\cdot q\cdot v_G=(1-q)\cdot v_B. \tag{6}$$

The length of the second stage depends on two factors. The first factor is the investor's redemption at present. At any instant, upon observing $L$, the investor knows that the proposed deal is surely bad and redeems shares with probability 1. Due to such potential redemption, the sponsor's expected payoff of proposing a bad deal decreases from $v_B$ to at most $(1-q)\,v_B$. Hence, the investor's redemption shortens the second stage. The second factor is the sponsor's continuation value, which is the opportunity cost of proposing any deal. The second stage begins when the sponsor's continuation value becomes lower than $(1-q)\,v_B$. As implied by Proposition 2, the investor's future redemption reduces the sponsor's continuation value. Therefore, the investor's redemption dampens the accumulation of the sponsor's continuation value and thus lengthens the second stage.

# 4 Design of Redemption

Compared to other delegated investment, a salient feature of SPACs is that investors can decide whether to invest in a deal before deal completion through redemption. As previously discussed, the investors' redemption has two opposite effects on the sponsor's behavior. However, the investor's equilibrium redemption maximizes her expected payoff after the sponsor proposes a deal, taking neither of the two effects into consideration. This opens up the possibility that the welfare outcome can be improved by altering the way the investor participates in the SPAC. In this section, we discuss SPAC designs related to investors' redemption and derive their welfare implications.

## 4.1 Whether to allow redemption

This subsection focuses on whether to allow the investor's redemption. Due to informational disadvantage, simply giving the investor decision-making authority does not guarantee efficiency. We characterize the equilibrium when the investor cannot redeem shares and the sponsor can directly decide whether to invest. Surprisingly, we find that that when the quality of the investor's information is low, both the sponsor and the investor can be better off if the investor's redemption is not allowed. Moreover, we find that Pareto-optimal redemption schemes feature prohibiting the investor from redeeming shares after a predetermined time point.

Suppose that the investor's redemption is prohibited and the sponsor can directly decide whether to invest. Since the sponsor's proposal guarantees investment, his payoff is $v_\omega$ if he proposes a deal of type $\omega$. Let $V_s(-t)$ represent his continuation value at time $-t$. It is easy to see that as time passes, the continuation value must be weakly decreasing and always smaller than $v_G$. At the deadline of the SPAC, the continuation value must be 0. Similar to the case that the investor has the redemption right, the game is divided into two stages in equilibrium. Denote the transition time as $-t_s^*$. In the first stage, where $-t < -t_s^*$, $v_B < V_s(-t) < v_G$, and sponsor proposes only the good deals he receives. In the second stage, where $-t > -t_s^*$, $V_s(-t) < v_B < v_G$, and the sponsor proposes any deal he receives. Let $U_s(-t)$ represent the investor's continuation value at the time $-t$. We readily obtain the following properties about the equilibrium.

**Lemma 4.**

- *$t_s^*$ is finite and satisfies*

$$\left(1-e^{-\lambda t_s^*}\right)[p_0 v_G + (1-p_0)v_B] = v_B. \tag{7}$$

- *There exists $T_s^* > 0$ such that $U_s(-T) > 1$ if and only if $T > T_s^*$.*

The first point of Lemma 4 states that, although the sponsor has full discretion over investment, he only acts at odds with the investor's interest in later periods of the game. In the second stage, the sponsor will propose any deal he receives, so his continuation value at $-t$ is $\left(1-e^{-\lambda t}\right)[p_0 v_G + (1-p_0)v_B]$. The key to the existence of $t_s^*$ is that the sponsor prefers investment in a good deal to that in a bad one, as the investor does. If he expects that the remaining time allows him to receive a good deal with a sufficiently high probability, he would prefer to forgo the bad deal at hand despite the risk that he may end up with no deal.

On the other hand, in the second stage, the investor loses money in expectation because of the poor average quality of potential SPAC deals, i.e.,

$$p_0 R_G + (1-p_0)R_B < 1 \Rightarrow p_0 u_G + (1-p_0)u_B < 1.$$

In the first stage, the sponsor invests in only the good deals he receives, so the investor earns positive profit. As a result, when $T > T_s^*$, a long first stage can bring the investor sufficient profit to cover her loss in the second stage.

Next, we examine how the redemption right affects the two players' welfare.

**Proposition 4.**

- *The sponsor is always better off if the investor does not have the redemption right, i.e., $V_s(-T) > V(-T)$.*

- *For $T \leq T_s^*$, the investor is worse off without the redemption right. For $T > T_s^*$, the investor is better off without the redemption right when the quality of her information is sufficiently low, i.e., there exists $q_s^*$ such that $U_s(-T) > U(-T)$ if and only if $q < q_s^*$.*

The first point is straightforward. When the sponsor has full control over investment, his expected payoff must be higher than that in the baseline setup. The second point implies that if the quality of the investor's information is low, the investor can be better off if she cannot redeem shares and $T$ is large enough. Importantly, when $q$ is small, the investor's redemption may drastically lengthen the second stage. On one hand, the redemption does not significantly reduce the threshold of the sponsor's continuation value at which the second stage begins, so the disciplining effect is small. On the other hand, it heavily restrains the accumulation of the sponsor's continuation value. What's worse, in the second stage, the investor earns positive expected profit only upon observing $H$, which happens with probability $q$ for a good deal. As a result of a lengthy second stage and a low probability to earn positive expected profit in the second stage, the investor's expected profit could be close to 0 for a sufficiently small $q$.

Proposition 4 implies that the two effects of the investor's redemption are both important forces in the game and either of them may dominate depending on the quality of the investor's information. This prompts the following question: what design of the investor's redemption right can take advantage of the positive effect and mute the negative effect? To shed light on this question, we consider a larger design space consisting of prohibition schemes that prohibit the investor from redeeming shares in some periods. Put differently, we allow whether the investor has the redemption right to be time-varying during the SPAC life cycle. We derive Pareto-optimal prohibition schemes[11] and focus on those with the minimum prohibition.[12] Proposition 5 shows the key property of optimal prohibition schemes.

**Proposition 5.** *Optimal prohibition schemes on the redemption right can be characterized by a predetermined time point $T_R$: the investor can redeem shares only before $T_R$.*

Generically, optimal prohibition schemes feature two phases. In the first phase, the investor can redeem shares, which reduces the sponsor's incentive to propose bad deals. The main purpose of the scheme here is to take advantage of the positive effect of the investor's redemption and ensure more efficient investment outcomes in the current period. In the second phase, the investor cannot redeem shares so that the sponsor would like to propose both types of deals. The main purpose of the scheme here is to avoid the negative effect of the investor's redemption and facilitate the accumulation of the sponsor's continuation value.

[11]This means that if properties characterized here are violated, there must exist other prohibition schemes that make both players better off.

[12]That is, at the points when the redemption right does not affect the equilibrium dynamics, the schemes do not prohibit the use of the redemption right.

In the scheme, for both players, it is better to ensure more efficient investment outcomes in early periods and facilitate the accumulation of continuation value in late periods. To see the intuition, suppose that at $-t$, redemption is prohibited, and the sponsor proposes any deal he receives in equilibrium. Then due to the poor average quality of SPAC deals (see Assumption 1), the two players' total welfare must be negative if the game ends at $-t$. Consider a change that redemption is allowed at $-t$. The investor's optimal redemption decision guarantees her expected payoff to be non-negative, so the two players' total welfare now becomes positive if the game ends at $-t$. What's more, the magnitude of such welfare improvement at $-t$ depends on the sponsor's continuation value $V(-t)$. Specifically, a lower level of $V(-t)$ corresponds to more aggressive equilibrium redemption at $-t$, more good deals being forgone, and thus smaller welfare improvement. On the other hand, the cost of this change is the reduction of the sponsor's continuation value and further the total welfare in the periods before $-t$. This effect is more significant for a later period $-t$. Since the sponsor's continuation value decreases over time, the net benefit of allowing redemption is lower in later periods, and we should expect the pattern in Proposition 5 to be optimal.

## 4.2 Rejection without redemption

Motivated by the recent trend of SPACs, we model the investor's decision-making process as a tender offer for the baseline setup. Once the sponsor proposes a deal, the investor either invests or rejects through redemption. Hence, the sponsor has only one opportunity to propose deals before the deadline. An alternative is to allow the investor to reject without redemption. In that case, the sponsor forgoes the current deal and continues searching and proposing deals until the deadline. In this case, the sponsor essentially has multiple opportunities to propose.

Notably, rejection without redemption can be naturally implemented if the investor's decision making is structured as voting, which is another popular practice. Voting in SPACs proceeds as follows. After the sponsor proposes a deal, the investors vote on an investment in it. If a sufficient fraction of investors vote for it, the deal is approved. Then the investors who vote against the deal are offered the right to redeem their shares.[13] Investors who are not offered or do not exercise the redemption right will invest. If the deal is rejected, the SPAC continues, and the sponsor searches for new deals. In our single-investor setup, the investor will either approve the deal and invest, or disapprove it and let the SPAC continue.

This subsection compares the two regimes: rejection through redemption and rejection without redemption. Regarding the core mechanism of concern in this paper, this comparison can also be interpreted as the comparison between tender offers and voting. We consider a derivation of our baseline setup and assume that the sponsor can continue to search if the proposed deal is rejected,

[13] This is required by stock exchange listing rules. In many cases, SPACs offer all investors redemption rights.

keeping all other assumptions unchanged.

Denote the sponsor's and the investor's continuation value at time $-t$ by $V_v(-t)$ and $U_v(-t)$, respectively. Likewise, $V_v(-t)$ is weakly decreasing, always smaller than $v_G$, and equal to 0 at the deadline of the SPAC. By proposing a deal of type $\omega$ at $-t$, the sponsor enjoys $v_\omega$ if the investor approves the deal, and $V_v(-t)$ otherwise. Hence, given the investor's strategy, his marginal benefit of proposing a deal of type $\omega$ is proportional to $v_\omega - V_v(-t)$. To obtain sharp equilibrium predictions, we assume that the sponsor does not use weakly dominated strategies. That is, he proposes a deal of type $\omega$ at $-t$ with the probability 1 if $v_\omega - V_v(-t) > 0$ and 0 if $v_\omega - V_v(-t) < 0$. It is easy to see the game is still divided into two stages in equilibrium. Denote the transition time as $-t_v^*$. In the first stage, where $-t < -t_v^*$, $v_B < V_v(-t) < v_G$, the sponsor proposes only the good deals he receives, and the investor always approves. In the second stage, where $-t > -t_v^*$, $V_v(-t) < v_B < v_G$, the sponsor proposes any deal he receives, and the investor approves only upon observing $H$. We obtain the following results.

**Proposition 6.** *The sponsor always has lower welfare if rejection does not require redemption, i.e., $V_v(-T) < V(-T)$. But the comparison of the investor's welfare between the two regimes is ambiguous.*

Surprisingly, the sponsor is worse off under rejection without redemption. Since the first stage proceeds in the same way under both regimes, to understand the underlying economic intuition, we can focus on the second stage. On one hand, under rejection without redemption, the sponsor's continuation value accumulates at a lower rate in the second stage, which is

$$\begin{aligned}\frac{dV_v(-t)}{dt} &= \lambda p_0 \cdot q\left[v_G - V_v(-t)\right] \\ &= \lambda p_0 \cdot \left[q v_G + (1-q)V_v(-t) - V_v(-t)\right],\end{aligned}$$

as opposed to

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot \left[q v_G + (1-q)\eta(-t)v_G - V(-t)\right]$$

under rejection through redemption. The difference between the two accumulation rates is that, when the sponsor proposes a good deal but $\varnothing$ is observed, the sponsor receives $V_v(-t)$ in expectation under rejection without redemption, while he receives $v_G$ with the probability $\eta(-t)$ under rejection through redemption. Recall that $(1-q)\eta(-t)\cdot v_B = V(-t)$, so

$$\eta(-t)v_G = \frac{v_G}{(1-q)\cdot v_B}V(-t) > V(-t).$$

This simple observation relies on two points. First, since the signal $L$ has helped screen out a fraction $q$ of bad deals, the investor redeems shares less aggressively when observing $\varnothing$ under rejection through redemption. Second, the sponsor strictly prefers a good deal to a bad one.

On the other hand, the threshold of the sponsor's continuation value at which the second stage begins is higher under rejection without redemption, which is $V_v(-t) = v_B$, as opposed to $V(-t) = (1-q)v_B$ under rejection through redemption. Hence, rejection without redemption also lengthens the second stage.

The underlying economic intuition is that the coercive termination feature of rejection through redemption enables the investor's rejection to have ex-ante disciplining effects on the sponsor. Under rejection without redemption, the investor's rejection when observing $\varnothing$ or $L$ cannot suppress the sponsor's incentive to propose bad deals at all, because rejection is not worse than not proposing. To protect herself from the sponsor's undisciplined behavior, the investor must reject any investment unless she observes the signal $H$. As discussed in Section 3.4, more frequent rejection further constrains the accumulation of the sponsor's continuation value and lengthens the second stage. This intuition also helps justify the recent transition from voting to tender offers from an equilibrium perspective.

However, the comparison of the investor's welfare is ambiguous. Rejection without redemption affects her welfare in two opposite ways. On one hand, it lengthens the less efficient second stage. On the other hand, the game more likely ends with the investor earning a positive expected profit in the second stage. In both regimes, the investor can earn the positive profit $u_G - 1$ in the second stage only when she observes $H$. Under rejection without redemption, the game ends when the investor observes $H$ by the deadline, while under rejection through redemption, the game may end when she observes $\varnothing$ or $L$.

# 5 Behavioral Investors

In the baseline model, we assume that the representative SPAC investor is fully rational. She understands all the details of the SPAC contractual arrangement and correctly expects the sponsor's strategic behavior. However, given the novel and complicated nature of SPACs, even sophisticated investors may not be able to act in a fully rational way. Moreover, a significant fraction of SPAC shares are held by retail investors, whose ability to make investment decisions rationally is more dubious. Investors' unsophistication has raised many concerns and is often blamed for overpriced SPAC deals. On March 30, 2022, the Securities and Exchange Commission proposed new rules and amendments to enhance disclosure and investor protection in initial public offerings by SPACs and in business combination transactions involving SPACs and private operating companies. Motivated by the aforementioned concerns and the potential new regulations, we extend the baseline setup with behavioral investors. We focus on how investor composition changes the equilibrium dynamics and the welfare outcome, which sheds light on the impact of potential policy interventions.

## 5.1 The behavioral bias

We model behavioral investors as those who invest more often than rational investors. This pattern could arise for several reasons. First, some investors may overestimate the value of their investments because they fail to fully consider potential dilution (Klausner et al., 2020). Second, they may not understand the severe conflicts of interest between themselves and SPAC sponsors. Therefore, they tend to think that SPAC sponsors propose deals with positive expected returns. Third, when lacking clear information, investors exhibit inertia in their portfolio decisions, holding their shares more often.

To capture the behavioral bias of investing too often, we assume that all investors observe the same signal realization,[14] and behavioral investors make rational redemption decisions when receiving the precise signals $H$ or $L$ but always choose not to redeem shares when receiving the uninformative signal $\varnothing$.[15][16] We assume that a representative rational investor holds a fraction $\chi \in (0,1)$ of the SPAC, and a representative behavioral investor holds the rest. We refer to $\chi$ as the rational holdings and $1-\chi$ as the behavioral holdings. Correspondingly, we use $V_\chi(-t)$ and $U_\chi(-t)$ to represent the sponsor's continuation value and the two investors' total continuation value, respectively. For simplicity of illustration, we assume that the proposed deal succeeds if and only if at least one investor chooses not to redeem shares. Upon the success of a deal of type $\omega$, the investors' gross return is $u_\omega$, and the sponsor's payoff is $v_\omega \cdot I$ if the investors' total investment is $I$.

## 5.2 The equilibrium

We first characterize the equilibrium dynamics with the behavioral investor. The rational investor's redemption decision still follows Lemma 1 and is characterized by $\eta_\chi(-t)$, the probability that she chooses not to redeem shares upon observing $\varnothing$. Then, at the time $-t$, the sponsor's expected payoff of proposing a good deal is

$$\left[\chi q+\chi(1-q)\eta_\chi(-t)+1-\chi\right]v_G,$$

[14]The assumption that all investors' signals are perfectly correlated does not matter.

[15]Banerjee and Szydlowski (2021) make similar but slightly different assumptions about investors. In their setup, institutional investors acquire information without cost and act based on it, while retail investors do not acquire information and keep their shares by default.

[16]Our main results also hold if we assume that behavioral investors always choose not to redeem shares regardless of the signals they receive.

and that of proposing a bad deal is

$$\left[\chi\left(1-q\right)\eta_{\chi}\left(-t\right)+\left(1-\chi\right)\left(1-q\right)\right]v_{B}.$$

To better understand the equilibrium dynamics, we introduce the concept of the aggregate investment strategy, which is

$$\chi\eta_{\chi}\left(-t\right)+\left(1-\chi\right)$$

when the signal $\varnothing$ is observed, and 1 (0) when the signal $H$ ($L$) is observed. The aggregate investment strategy represents the expected investment from all investors and is comparable to the investor's strategy in the baseline setup. The following Proposition characterizes the three-stage equilibrium in this new setting and shows that the equilibrium dynamics in the first and second stages are the same as in the baseline setup.

**Proposition 7.** *The unique equilibrium of the SPAC game potentially has three stages, the transition times between which are $-t_2^*$ and $-t_1^*$:*

- *The third stage spans the period $(-t_2^*, 0]$, in which*
  - *the sponsor proposes any deal he receives, i.e., $\alpha_G(-t) = \alpha_B(-t) = 1$;*
  - *the rational investor always redeems shares upon observing $\varnothing$;*
- *The second stage spans the period $(-t_1^*, -t_2^*]$, with the same equilibrium dynamics as in the second stage of the baseline setup, i.e.,*
  - *the rational investor is indifferent between redeeming shares and not upon observing $\varnothing$,*
  - *the sponsor is indifferent between proposing bad deals and not;*
- *The first stage spans the period $[-T, -t_1^*)$, with the same equilibrium dynamics as in the first stage of the baseline model, i.e.,*
  - *the sponsor proposes only good deals, i.e., $\alpha_G(-t) = 1$ and $\alpha_B(-t) = 0$;*
  - *the rational investor never redeems shares when observing $H$ and $\varnothing$;*
- *The boundary conditions are $V_\chi(0) = 0$, $V_\chi(-t_2^*) = (1-\chi)(1-q)v_B$ and $V_\chi(-t_1^*) = (1-q)v_B$. $V_\chi(-t)$ is continuous at $-t_2^*$ and $-t_1^*$.*

Since the behavioral investor never redeems shares upon observing $\varnothing$, a direct effect of her existence is the decrease in the equilibrium redemption of the aggregate investment strategy. In response to the decrease in redemption, the rational investor redeems shares more aggressively upon observing $\varnothing$. When the sponsor's continuation value is lower than $(1-\chi)(1-q)v_B$, even if the rational investor redeems shares most aggressively, i.e., $\eta_\chi(-t) = 0$, she cannot fully cancel out the behavioral investor's less redemption. So, the sponsor proposes both types of deals because his expected payoff of proposing a bad deal is at least $(1-\chi)(1-q)v_B$. That is why a third stage emerges in equilibrium. When the sponsor's continuation value is greater than $(1-\chi)(1-q)v_B$, the rational investor can fully cancel out the behavioral investor's less redemption, so the equilibrium dynamics resemble those of the baseline setup.

## 5.3 Players' welfare

Next, we focus on the impact of behavioral investors on players' welfare. With more behavioral holdings, there is less equilibrium redemption in the third stage, so the sponsor's welfare must improve. The following Proposition confirms this result.

**Proposition 8.** *Consider a marginal increase in the behavioral investor's holding $(1-\chi)$, we must have $\frac{dV_\chi(-T)}{d(1-\chi)} > 0$.*

Regarding the investors' welfare, more behavioral holdings have two opposite effects, which are captured by Lemma 5.

**Lemma 5.** *For any $q \in (0,1)$, we must have $\frac{dt_2^*}{d(1-\chi)} > 0$ and $\frac{d(T-t_1^*)}{d(1-\chi)} > 0$.*

The direct effect of more behavioral holdings is that the behavioral investor invests more when the signal $\varnothing$ is observed at each instant in the third stage. Such investment has a negative expected profit because of the poor average quality of potential SPAC deals. Further, due to a weaker disciplining effect of redemption, the sponsor's moral hazard problem becomes more severe. This is best captured by the finding in Lemma 5 that the length of the third stage $t_2^*$ is increasing in the behavioral holdings $(1-\chi)$. Thus the existence of behavioral investor will make such unprofitable investments at a larger scale in a longer period. Therefore, the direct effect reduces the investors' welfare in later periods of the SPAC.

The indirect effect of more behavioral holdings is that the sponsor's continuation value accumulates at a higher rate in the third stage, so the continuation value in the first two stages is higher. The moral hazard problem is mitigated in the first two stages, which is best captured by the finding in Lemma 5 that the length of the first stage $T-t_1^*$ is increasing in the behavioral holdings $(1-\chi)$. Therefore, the indirect effect increases the investors' welfare in early periods of the SPAC.

Since the direct and indirect effects are in opposite directions, the net effect of the behavioral holdings on the investors' total welfare is ambiguous. Figure 2[17] presents how the investors' total welfare $U_\chi(-T)$ changes with the behavioral holdings $(1-\chi)$ under different parameters. Interestingly, we find that the investors' total welfare increases with the behavioral holdings when $q$ and $(1-\chi)$ are relatively small. This result is confirmed analytically in the following proposition.

[17] Other parameters are $T = 24$, $\lambda = 0.2$, $p_0 = 0.1$, $R_G = 3$, $R_B = 0.5$, $v_G/R_G = v_B/R_B = 0.2$, and $u_G/R_G = u_B/R_B = 0.8$.

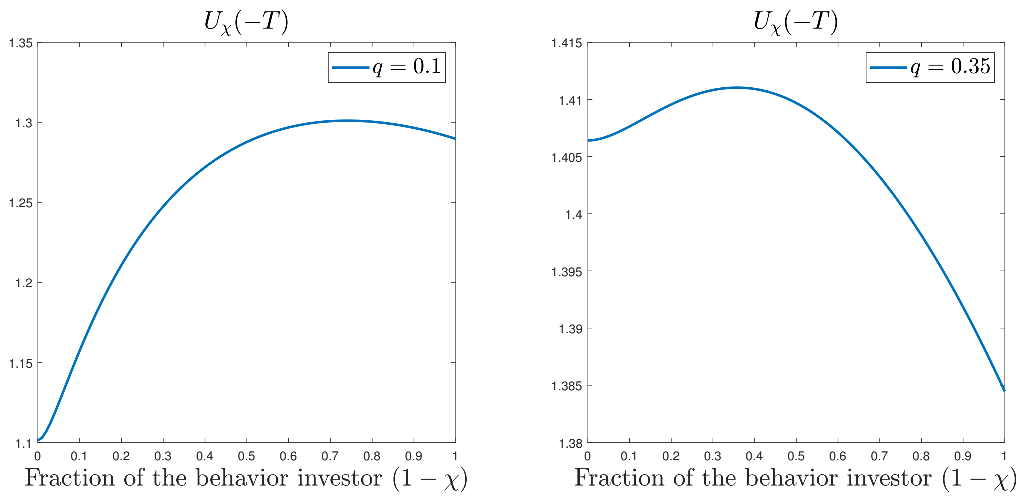


Figure 2: How Investors' Total Welfare Changes with the Participation of the Behavioral Investor.

**Proposition 9.** *Consider a marginal increase in the behavioral investor's holding* $(1-\chi)$. *For sufficiently small q, there exists* $\underline{\chi} < 1$, *such that for any* $\chi \in \left(\underline{\chi}, 1\right)$, *we have*

$$\frac{dU_\chi(-T)}{d(1-\chi)} > 0.$$

The intuition is that when $q$ and $(1-\chi)$ are small, the rational investor plays a dominant role on the investor side, and, as discussed in Section 3.3, the sponsor's continuation value accumulates at a low rate for a long time. In this case, an increase in the behavioral holdings can substantially improve the accumulation of the sponsor's continuation value and shorten the period of severe moral hazard of the sponsor. Lemma 6 demonstrates this intuition by showing that the marginal increase in the length of the first stage is higher when $q$ and $(1-\chi)$ are small.

**Lemma 6.** *For sufficiently small q,* $\frac{d^2\left(T-t_1^*\right)}{d(1-\chi)^2} < 0$.

## 5.4 Implications

Importantly, the behavioral investors who tend to invest reduces the aggregate redemption level in only the periods close to the deadline, due to the endogenous response of rational investors in equilibrium. As analyzed in Section 4.1, compared to the equilibrium in the baseline setup, it could be more efficient to facilitate the accumulation of the sponsor's continuation value by decreasing redemption in late periods. This equilibrium pattern makes the existence of behavioral investors more efficient than commonly considered.

In general, the welfare implication of behavioral investors is ambiguous. When the quality of investors' information is low, a small fraction of the behavioral holdings can significantly alleviate the sponsor's moral hazard problem, which is exacerbated by the rational investor's inefficient redemption. In this case, measures reducing the behavioral holdings or increasing investor sophistication (e.g., prohibiting retail investors' participation or emphasizing the conflicts of interest between the sponsor and investors) may have unintended negative effects on all players' welfare. Our analysis implies that the welfare outcome of regulations related to investor sophistication is not obvious, especially when investors have a severe informational disadvantage.

# 6 Extensions

## 6.1 Strategic entrepreneurs

In the baseline setup, we deliberately abstract away from entrepreneurs' strategic behavior to better focus on the strategic interaction between the sponsor and the investor. Usually, entrepreneurs can choose to bring a deal public through either a SPAC or a standard IPO; in more general settings, entrepreneurs can choose other financing strategies. In our baseline setup, the equilibrium features a (weakly) decreasing probability of investment over time for both good and bad deals. Then, in a more general setting when entrepreneurs are strategic, the choice between a SPAC and a standard IPO will be endogenous, and thus the supply of deals from entrepreneurs may not be constant in the SPAC life cycle. In this subsection, we introduce strategic entrepreneurs into the model and examine the resulting impact on the equilibrium dynamics.

There are many entrepreneurs, each of whom is endowed with one deal. Deals can be either good ($G$) or bad ($B$), and the fraction of good deals is $p_0$. Each entrepreneur observes the type of her own deal. There is only one SPAC in the market operating in the way modeled in the baseline setup. At each instantaneous "period" $[-t,-(t-dt))$, a liquidity shock arrives with probability $\lambda dt$, and a randomly chosen entrepreneur needs to raise 1 to continue her deal. If the deal is not funded instantly, it may fail, and the entrepreneur receives a lower expected payoff. The entrepreneur hit by a liquidity shock can choose to bring her deal public through either a SPAC or a standard IPO. There are three possible scenarios:

1. She chooses an IPO directly.
2. She taps the SPAC, and the deal is funded by the SPAC investor.
3. She taps the SPAC, but the deal is not funded by the SPAC investor and she turns to an IPO.

Denote an entrepreneur's payoff as $\pi_{IPO}$ if the deal is funded through an IPO directly and as $\pi_{SPAC}$ if it is funded through the SPAC. In the case that she chooses an IPO after unsuccessful SPAC financing, her payoff is $\rho \cdot \pi_{IPO}$. We assume $\rho < 1$ because preparing for the SPAC delays the IPO process, and the deal may fail or downsize due to lack of funding during that period. We do not impose a particular probabilistic structure on $\rho$, $\pi_{SPAC}$, and $\pi_{IP0}$; they can vary with deals in any reasonable way.[18]

Due to the stringent screening process and regulatory requirement of IPOs, it's more difficult to fund bad deals through IPOs, so we assume $\pi_{IPO} = 0$ for bad deals. Then it is easy to see that entrepreneurs with bad deals will always tap the SPAC. Now consider those with good deals. Suppose an entrepreneur expects that if she taps the SPAC, her deal can be funded through the SPAC with the probability $x$. Then she will tap the SPAC if and only if

$$\begin{aligned} x \cdot \pi_{SPAC} + (1-x) \cdot \rho \pi_{IPO} &> \pi_{IPO} \\ \Leftrightarrow x &> \frac{(1-\rho)\, \pi_{IPO}}{\pi_{SPAC} - \rho \pi_{IPO}}. \end{aligned} \tag{8}$$

Certainly, since unsuccessful SPAC financing causes costly delay, the entrepreneur is more willing to choose the SPAC over an IPO if a deal is more likely to be funded through the SPAC. Let $\Phi(\cdot)$ represent the cumulative distribution function of the random variable $\frac{(1-\rho)\pi_{IPO}}{\pi_{SPAC}-\rho\pi_{IPO}}$, and assume that $\Phi(\cdot)$ is strictly increasing. Then at each instant, the sponsor receives good deals at the rate $\lambda p_0 \Phi(x)$.

Now we characterize the equilibrium of the SPAC game with strategic entrepreneurs. At the time $-t$, the sponsor receives bad deals at the rate of $\lambda(1-p_0)$ and good deals at the rate of $\lambda p_0 \Phi(x(-t))$, where

$$x(-t) \equiv \alpha_G(-t) \cdot [q + (1-q)\, \eta(-t)]$$

is the probability that a good deal can be approved if the entrepreneur chooses the SPAC. Lemma 2 and Lemma 3 still hold because the deals the sponsor receives at each instant have negative NPV on average, i.e.,

$$\frac{p_0 \Phi(x(-t))\, R_G + (1-p_0)\, R_B}{p_0 \Phi(x(-t)) + 1 - p_0} \leq p_0 R_G + (1-p_0)\, R_B < 1.$$

The equilibrium is very similar to that in Proposition 1, except that the sponsor receives good deals at the rate of $\lambda p_0 \Phi(q + (1-q)\, \eta(-t))$.

**Proposition 10.** *As the SPAC approaches its deadline, a decreasing fraction of the entrepreneurs with good deals choose to tap the SPAC.*

As the SPAC approaches its deadline, the investor becomes more concerned about the sponsor's moral hazard problem and redeems shares more aggressively. The redemption effectively discourages the entrepreneurs with good deals, who can also access an IPO, from tapping the SPAC.

[18] The relationship between $\pi_{IPO}$ and $\pi_{SPAC}$ is ambiguous and depends heavily on the characteristics of the deal. $\pi_{SPAC}$ could be greater than $\pi_{IPO}$ for several reasons. For example, deals can be funded through the SPAC more quickly than a standard IPO; deals that cannot access a standard IPO may go public through the SPAC.

Therefore, the investor's redemption abates not only the sponsor's expected payoff from proposing good deals, but also the probability that he receives good deals. As analyzed in Section 4.1, this additional effect further dampens the accumulation of the sponsor's continuation value and exacerbates the sponsor's moral hazard problem. Entrepreneurs' potential strategic behavior stokes our concern that giving less-informed investors the redemption right to reassure them may backfire.

## 6.2 Endogenous effort to search for deals

In reality, the search for deals also depends on the sponsor's effort. To prepare investment proposals for the investor, the sponsor needs to spend time, energy, and money searching for and negotiating deals. Such effort can hardly be observed or enforced. Since the marginal benefit of proposing a deal is not constant over the SPAC life cycle, he may optimally exert heterogeneous amount of effort over time. In this subsection, we incorporate the sponsor's endogenous effort into the model.

At each instant $-t$, the sponsor can choose to exert a flow effort $\kappa(-t)$ to search for deals. It increases the arrival rate of deals from $\lambda$ to $\lambda+\kappa(-t)$ without changing deals' quality. Meanwhile, it incurs a private flow cost $C(\kappa(-t))$ to the sponsor. $C(\cdot)$ is a continuously differentiable, increasing, and convex function satisfying $C(0)=C'(0)=0$ and $C(\infty)=\infty$. The equilibrium is similar to that in Proposition 1, except that the sponsor receives deals at the rate $\lambda+\kappa^*(-t)$. $\kappa^*(-t)$ is chosen by the sponsor to maximize his continuation value, i.e.,

$$\kappa^*(-t) = \arg\max_{\kappa}\ (\lambda+\kappa)\, p_0 \cdot [F_G(-t) - V(-t)] - C(\kappa).$$

So, $\kappa^*(-t)$ satisfies

$$C'(\kappa^*(-t)) = p_0 [F_G(-t) - V(-t)].$$

Plugging $F_G(-t) \equiv [q+(1-q)\eta(-t)]\, v_G$ into the equation, we obtain that in the second stage,

$$C'(\kappa^*(-t)) = p_0 \left[ v_G q + \left( \frac{v_G}{v_B} - 1 \right) V(-t) \right],$$

and in the first stage,

$$C'(\kappa^*(-t)) = p_0 [v_G - V(-t)].$$

**Proposition 11.** *The sponsor exerts more effort over time in the first stage, but less effort over time in the second stage.*

At every instant, the sponsor's endogenous effort is motivated by the difference between the expected payoff of proposing a good deal and his continuation value, because proposing a bad deal is never strictly profitable. In the first stage, the expected payoff is always $v_G$, but his continuation value keeps decreasing. This implies that, failing to find a good deal, his situation deteriorates.

| Type $\omega$ | Do not redeem shares | Redeem shares | No proposal |
|---|---|---|---|
| The sponsor's payoff | $v_\omega$ | $z_\omega$ | 0 |
| The investor's payoff | $u_\omega$ | 1 | 1 |

Table 3: The payoff structure

Hence, he has stronger incentive to exert effort to search for deals over time. In the second stage, his continuation value continues to decrease, but the expected payoff of proposing a good deal also shrinks over time due to the investor's more aggressive redemption. Because a good deal is more valuable than a bad one to the sponsor, $v_G > v_B$, the decrease in the expected payoff is more pronounced than that in his continuation value in an absolute basis. As a result, his incentive to exert effort weakens over time in the second stage. Similar to that on strategic entrepreneurs, this analysis also uncovers a channel through which the investor's inefficient redemption may exacerbate the sponsor's moral hazard problem.

### 6.3 Deal completion despite redemption

The baseline setup assumes that if the investor redeems shares, the proposed deal fails and the sponsor receives 0. However, sometimes the deal can still succeed despite massive redemption, and the sponsor can receive a substantial positive payoff. To study this case, we assume that the two players' payoffs follow Table 3. The difference between this case and the baseline setup is that if the sponsor proposes a deal of type $\omega$ but the investor redeems shares, he receives a positive payoff $z_\omega < v_\omega$ instead of 0, where $z_G > z_B$; if the sponsor proposes no deal by the deadline, he still receives 0. Therefore, if the sponsor proposes a deal of type $\omega$ at time $-t$, his expected payoff is

$$F_\omega(-t) = \begin{cases} [q + (1-q)\eta(-t)](v_G - z_G) + z_G, & \text{if } \omega = G \\ (1-q)\eta(-t)(v_B - z_B) + z_B, & \text{if } \omega = B \end{cases}.$$

His continuation value $V(-t)$ satisfies Equation (1), and $V(0) = 0$. The equilibrium is characterized as follows.

**Proposition 12.** *The unique equilibrium of the SPAC game potentially has three stages, the transition times between which are $-t_2^*$ and $-t_1^*$:*

- *The third stage spans the period $(-t_2^*, 0]$, in which*
  - *the sponsor proposes any deal he receives, i.e., $\alpha_G(-t) = \alpha_B(-t) = 1$;*
  - *the investor always redeems shares upon observing $\varnothing$;*
- *The second stage spans the period $(-t_1^*, -t_2^*]$, in which*
  - *the sponsor's equilibrium strategy $(\alpha_\omega(-t))_{\omega\in\{G,B\}}$ satisfies $\alpha_G(-t) = 1$ and makes*

*the investor indifferent to whether to invest or redeem shares when observing $\varnothing$, i.e.,*

$$\frac{p_0}{1-p_0}\frac{\alpha_G(-t)}{\alpha_B(-t)} = \frac{1-u_B}{u_G-1};$$

– *the investor's equilibrium strategy $\eta(-t)$ makes the sponsor indifferent to whether to propose a bad deal or not, i.e.,*

$$V(-t) = F_B(-t) = (1-q)\,\eta(-t)(v_B - z_B) + z_B;$$

- *The first stage spans the period $[-T, -t_1^*)$, in which*
  – *the sponsor proposes only good deals, i.e., $\alpha_G(-t) = 1$ and $\alpha_B(-t) = 0$;*
  – *the investor always invests when observing $\varnothing$, i.e., $\eta(-t) = 1$;*
- *The boundary conditions are $V(0) = 0$, $V(-t_2^*) = z_B$, and $V(-t_1^*) = (1-q)(v_B - z_B) + z_B$. $V(-t)$ is continuous at $-t_2^*$ and $-t_1^*$.*

Since the sponsor can guarantee a positive payoff at least as much as $z_\omega$ irrespective of the investor's decision, he has the incentive to propose any deal when his continuation value $V(-t)$ is below $z_B$. So, a third stage emerges in equilibrium. The equilibrium dynamics in the first and the second stages are similar to those in the baseline setup. In the second stage, the sponsor's continuation value still follows self-reinforcing dynamics as follows

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot \left[z_G - z_B + q(v_G - z_G) + \left(\frac{v_G - z_G}{v_B - z_B} - 1\right)(V(-t) - z_B)\right]$$

with the two boundary conditions $V(-t_2^*) - z_B = 0$ and $V(-t_1^*) - z_B = (1-q)(v_B - z_B)$. Therefore, the two opposite effects of the investor's redemption underlying our analysis still play important roles.

Notably, the difference in the sponsor's payoffs when the investor redeems shares, $z_G - z_B$, also contributes to the accumulation of the sponsor's continuation value. This echos the intuition in the baseline setup that the alignment of the players' incentives mitigates moral hazard. On the other hand, $z_G - z_B$ is independent of $q$, so the negative effect of the investor's redemption becomes less severe in this case.

## 6.4 Long-lived deals

For the baseline setup, we assume that deals are short-lived: if the sponsor doesn't propose the deal he receives, the deal will disappear and become unavailable immediately. With this assumption, the state of the sponsor with respect to whether he has a deal and what type he has is completely independent over time. In this subsection, we explore the case of long-lived deals where the sponsor can possibly keep a deal for future proposals. We show that such a possibility does not alter the equilibrium dynamics in our setup.

The new setup is the same as the baseline one, except that the deals the sponsor has received but not yet proposed still exist. Such deals are called old deals. At each instant, the sponsor can choose to revisit one of the old deals. The revisit makes the deal ready for proposal again at a rate of $\gamma$. It is easy to see that the sponsor must choose to revisit the best deal he has received so far, so his continuation value depends on its type. Denote the sponsor's continuation value at $-t$ as $V^{\sigma}(-t)$ if the best deal he has received is of the type $\sigma \in \{G, B\}$. Then, in equilibrium, $V^{G}(-t) > V^{B}(-t)$. Heuristically, conditional on the game continues at time $-(t+dt)$, each instantaneous "period" $(-(t+dt), -t]$ consists of events occurring in the following order:

1. the initial state is $\sigma(-t-dt) \in \{G, B\}$;
2. with probability $\lambda dt$, the sponsor receives a new deal and observes its type $\omega'$, and the new deal is ready for proposal;
3. with probability $\gamma dt$, the best old deal becomes ready;
4. if there is at least one deal ready, denote the type of the best ready one as $\omega \in \{G, B\}$, the sponsor proposes the best one with probability $\alpha_{\omega}(-t)$;
5. if the sponsor proposes a deal, the investor decides whether to invest or redeem shares; if no deal is proposed, the state is updated to $\sigma(-t) = \omega$, and the game moves on to the next period.

Since the opportunity to propose is unique, a sponsor with a deal of type $\omega$ ready for proposal faces a trade-off between $V^{\sigma}(-t)$ and $F_{\omega}(-t)$, the expected payoff of proposing it right away.

Recall that a critical characteristic of the baseline setup is that the sponsor always has more incentive to propose a good deal than a bad one. It follows that his expected payoff of proposing a good deal is higher than that of proposing a bad one, but his opportunity cost is the same for both. Although the second half does not hold in the new setup (since the sponsor's continuation value depends on the type of the deals he has received so far), we can show that this critical observation still holds.

**Lemma 7.** *In equilibrium, for any $-t$, $F_G(-t) > V^{G}(-t)$, so $\alpha_G(-t) = 1$.*

Notice that if the sponsor has a good deal ready for proposal, his continuation value must be $V^{G}(-t)$. $F_G(-t) \leq V^{G}(-t)$ implies that the investor must have lower levels of redemption at some points in the future, which can compensate for the possibility that the sponsor may not have a good deal ready for proposal again. However, lower levels of redemption increase the probability of investment in a bad deal disproportionately more than the probability of investing in a good one. Hence, $F_B(-t) < V^{B}(-t) < V^{G}(-t)$ must hold. Then the rest follows the proof of Lemma 2.

**Proposition 13.** *If deals are long-lived, the equilibrium is unique and the same as that characterized by Proposition 1.*

In the baseline setup, the average quality of deals received is so low that if the sponsor proposes any bad deal he receives, the investor has a negative expected profit of investing when observing $\varnothing$. Hence, in equilibrium, the investor chooses redemption levels that induce the sponsor to propose bad deals at only the rate

$$\lambda\left(1-p_0\right)\alpha_B\left(-t\right)=\lambda p_0\frac{u_G-1}{1-u_B}.$$

If deals are long-lived, as implied by Lemma 7, the sponsor still proposes any good deal he receives right away, so revisit does not change the rate at which good deals are proposed. But revisit increases the amount of bad deals ready for proposal, which makes the investor even more concerned about the average quality of proposed deals. So, the investor will chooses the same redemption levels, and the sponsor will propose bad deals at the same rate. Notably, the sponsor does not benefit from revisit. As pointed out by Proposition 2, his expected payoff depends on only the proposals of good deals in equilibrium.

# 7 Concluding Remark

Studying SPACs from a perspective of delegated investment, this paper focuses on the strategic interaction between a sponsor and a representative investor. Consistent with conventional wisdom, the incentive misalignment of the two parties gives rise to a moral hazard problem of the sponsor. However, this is not the whole story. The alignment of the two parties' incentives helps mitigate the problem. Therefore, the investor's redemption has two opposite effects on the moral hazard problem: current redemption mitigates it, but future redemption exacerbates it. Based on these two effects, we derive various implications related to SPAC design.

In reality, a SPAC is a complicated business that involves many parties and interactions. To better illustrate our main idea, we consider a simplified setup, which renders several elements of SPACs unimportant. The following elements are potentially important in richer setups and merit more research.

1. The secondary market of SPAC shares. SPAC shares are publicly traded, which aggregates investors' information and affects their decisions.
2. Warrants. SPAC investors are offered units that consist of shares of common stock and warrants. More importantly, investors can hold the warrants even if they redeem shares.
3. Private Investment in Public Equity (PIPE). A SPAC sponsor frequently invites PIPE investment as part of the business combination, which further complicates their incentives.

4. Investors' demand for liquidity. Along with profitability, liquidity is a critical reason why some investors favor SPACs. Potentially, concern or demand for liquidity may affect investors' decisions as well.

# References


Aghion, P. and J. Tirole (1997). Formal and real authority in organizations. *Journal of Political Economy 105*(1), 1–29.

Alti, A. and J. B. Cohn (2022). A model of informed intermediation in the market for going public. *Available at SSRN 4058787*.

Bai, J., A. Ma, and M. Zheng (2021). Segmented Going-Public Markets and the Demand for SPACs. *Working Paper*.

Banerjee, S. and M. Szydlowski (2021). Harnessing the overconfidence of the crowd: A theory of SPACs. *Available at SSRN 3930346*.

Blomkvist, M. and M. Vulanovic (2020). SPAC IPO waves. *Economics Letters 197*, 109645.

Chatterjee, S., N. K. Chidambaran, and G. Goswami (2016). Security design for a non-standard IPO: The case of SPACs. *Journal of International Money and Finance 69*, 151–178.

Crawford, B. Y. V. P. and J. Sobeli (1982). Strategic Information Transmission. *Econometrica 50*(6), 1431–1451.

Cumming, D., L. H. Haß, and D. Schweizer (2014). The fast track IPO - Success factors for taking firms public with SPACs. *Journal of Banking and Finance 47*(1), 198–213.

Dessein, W. (2002). Authority and communication in organizations. *Review of Economic Studies 69*(4), 811–838.

Dimitrova, L. (2017). Perverse incentives of special purpose acquisition companies, the "poor man's private equity funds". *Journal of Accounting and Economics 63*(1), 99–120.

Feng, F. Z., T. Nohel, X. Tian, W. Wang, and Y. Wu (2022). The Incentives of Spac Sponsors When Information is Opaque. *Available at SSRN*.

Gahng, M., J. R. Ritter, and D. Zhang (2021). SPACs. *Working Paper* (352).

Grenadier, S. R., A. Malenko, and N. Malenko (2016). Timing Decisions in Organizations : Communication and Authority in a Dynamic Environment. *American Economic Review 106*(9), 2552–2581.

Gryglewicz, S., B. Hartman-Glaser, and S. Mayer (2021). PE for the Public: The Rise of SPACs. *Available at SSRN 3947368*.

Guo, Y. (2016). Dynamic delegation of experimentation. *American Economic Review 106*(8), 1969–2008.

Jenkinson, T. and M. Sousa (2011). Why SPAC investors should listen to the market. *Journal of Applied Finance (Formerly Financial Practice and Education) 21*(2).

Jog, V. M. and C. Sun (2007). Blank check IPOs: a home run for management. *Available at SSRN 1018242*.

Klausner, M., M. Ohlrogge, and E. Ruan (2020). A Sober Look at SPACs. *Working Paper*, 1–57.

Kolb, J. and T. Tykvova (2016). Going public via special purpose acquisition companies: Frogs do not turn into princes. *Journal of Corporate Finance 40*, 80–96.

Lin, C., F. Lu, R. Michaely, and S. Qin (2021). SPAC IPOs and sponsor network centrality. *Available at SSRN 3856181*.

Pawliczek, A., A. N. Skinner, and S. L. Zechman (2021). Signing blank checks: The roles of reputation and disclosure in the face of limited information. *Available at SSRN 3933259*.

Rodrigues, U. and M. Stegemoller (2012). Exit, voice, and reputation: the evolution of SPACs. *Del. J. Corp. L. 37*, 849.

Shachmurove, Y. and M. Vulanovic (2017). SPAC IPOs. *Working Paper*, 1–40.

# A Proofs

## Proof of Lemma 2

In equilibrium, suppose $F_G(-t) \leq V(-t)$ at a time $-t$. Then Lemma 1 implies that $F_B(-t) < V(-t)$, and thus we must have $\alpha_B(-t) = 0$. Then if $\alpha_G(-t) > 0$, by the investor's rational belief in equilibrium, $\tilde{\theta}(-t) = +\infty$. According to Lemma 1, we must have $\eta(-t) = 1$, so $F_G(-t) = v_G$ in equilibrium. If $t = 0$, we have $V(-t) = 0 < v_G = F_G(-t)$, a contradiction! If $t > 0$, then

$$V(-t) \leq \left(1 - e^{-\lambda t}\right) v_G + e^{-\lambda t} \cdot 0 < v_G = F_G(-t),$$

where $e^{-\lambda t}$ is the probability that the sponsor doesn't receive any deal during $[-t, 0]$, a contradiction! So we must have $F_G(-t) > V(-t)$ in equilibrium for any $-t$.

Then the sponsor's HJB equation implies $\alpha_G(-t) = 1$.

Besides, the RHS of the sponsor's HJB equation (1) must be positive, as

$$\max_{\alpha_G(-t),} \lambda p_0 \cdot \alpha_G(-t) \cdot [F_G(-t) - V(-t)] > 0$$

and

$$\max_{\alpha_B(-t)} \lambda (1 - p_0) \cdot \alpha_B(-t) \cdot [F_B(-t) - V(-t)] \geq 0$$

hold for any $-t$ in equilibrium. Then we must have

$$\frac{dV(-t)}{dt} > 0.$$

This means that the continuation value is decreasing over time.

## Proof of Lemma 3

Suppose $V(-t) < v_B(1-q)$ and $t > 0$. If $\alpha_B(-t) = 0$, following the proof of Lemma 2, we have $\eta(-t) = 1$ in equilibrium, so $F_B(-t) = (1-q)v_B > V(-t)$, contradiction! If $\alpha_B(-t) = 1$, then in equilibrium

$$\tilde{\theta}(-t) \leq \frac{p_0}{1-p_0} < \frac{1-R_B}{R_G-1} < \frac{1-u_B}{u_G-1},$$

which implies $\eta(-t) = 0$ according to Lemma 1. Then $F_B(-t) = 0 < V(-t)$, contradiction! Therefore, $\alpha_B(-t) \in (0,1)$ in equilibrium.

When $V(-t) > (1-q)v_B$, we must have $V(-t) > F_B(-t)$ in equilibrium, so $\alpha_B(-t) = 0$.

## Proof of Proposition 1

Lemma 2 implies that the sponsor's value function is decreasing over time in equilibrium. Then there must exist $t^*$ such that $V(-t) \leq (1-q)\, v_B$ if and only if $t \leq t^*$. Then for any $t \in (-t^*, 0]$, we must have $\alpha_G = 1$ and $\alpha_B \in (0,1)$. And the sponsor must be indifferent to whether to propose a bad deal or not, so we have

$$V(-t) = F_B(-t) = (1-q)\,\eta(-t)\, v_B.$$

For the investor, she must be indifferent to whether to invest or redeem shares when observing $M$, and this holds only when

$$\frac{p_0}{1-p_0}\frac{\alpha_G(-t)}{\alpha_B(-t)} = \frac{1-u_B}{u_G-1}$$

in equilibrium.

When $t > t^*$, we have $V(-t) \leq (1-q)\, v_B$, then the sponsor must choose $\alpha_B = 0$ in equilibrium. As the best response, the investor always chooses $\eta = 1$.

The sponsor's value function follows

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot [F_G(-t) - V(-t)]$$

when $t \in (-t^*, 0]$, where

$$F_G = [q + (1-q)\,\eta(-t)]\, v_G = \left[q + (1-q)\frac{V(-t)}{(1-q)\, v_B}\right] v_G$$

in equilibrium. Then $t^*$ is solved by the boundary conditions $V(0) = 0$ and $V(-t^*) = (1-q)\, v_B$. And it is clear that $t^*$ is independent of $T$.

When $t > t^*$, the sponsor's value function follows

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot [v_G - V(-t)].$$

The uniqueness of the equilibrium then is implied by the uniqueness of the solution of HJB equations.

## Proof of Proposition 2

To consider the sponsor's value function $V$, a critical observation is that he is indifferent to proposing or not when receiving a bad deal in the second stage. This means that his value function is unchanged even if he chooses to only propose good deals ex-post. Let the investor's equilibrium investment strategy be $\eta(t)$, then the sponsor's value function at time $-t$ is

$$V(-t) = v_G \int_0^t \lambda p_0 e^{-\lambda p_0 (t-\tau)} \left[q + (1-q)\eta(-\tau)\right] d\tau,$$

where $e^{-\lambda p_0 (t-\tau)}$ is the probability that the game doesn't end between time $(-t, -\tau)$, given the sponsor only proposes good deals. And $[q + (1-q)\eta(-\tau)]$ is the probability of good deals being invested in equilibrium at any time $-\tau$.

Our Proposition 1 implies that $\eta(-\tau) = \min\left\{1, \frac{V(-\tau)}{(1-q)v_B}\right\}$. Then it's clear that when $t \le t^*$, the sponsor's HJB equation becomes

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot \left[q v_G + \left(\frac{v_G}{v_B} - 1\right) V(-t)\right]$$

with boundary condition $V(0) = 0$. The unique solution to the above differential equation is

$$V(-t) = \left[e^{\lambda p_0 \left(\frac{v_G}{v_B} - 1\right) t} - 1\right] \frac{1}{\frac{v_G}{v_B} - 1} \cdot q \cdot v_G.$$

The boundary $t^*$ is solved by

$$\left[e^{\lambda p_0 \left(\frac{v_G}{v_B} - 1\right) t^*} - 1\right] \frac{1}{\frac{v_G}{v_B} - 1} \cdot q \cdot v_G = (1-q) v_B.$$

When $t > t^*$, the sponsor's HJB equation is

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot \left[v_G - V(-t)\right]$$

with boundary condition $V(-t^*) = (1-q)v_B$. It's easy to obtain the unique solution

$$V(-t) = \left[1 - e^{-\lambda p_0 (t-t^*)}\right] v_G + e^{-\lambda p_0 (t-t^*)} V(-t^*).$$

Now let's consider the investor's continuation value at time $-t$. Suppose the game continues at time $-t$. We know that she invests in all deals proposed in the first stage in equilibrium, and the net payoff she can get from the first stage is

$$(u_G - 1) \cdot PG_1$$

where $PG_1 = 1 - e^{-\lambda p_0 (t - \min\{t, t^*\})}$ is the probability that the game ends in the first stage.

In the second stage, she always gets zero net payoff when observing signal $M$ or $L$ in equilibrium. So she earns positive net payoff only when signal $H$ is observed. The probability that signal

$H$ is observed in the second stage is $q \cdot PG_2$, where

$$PG_2 = e^{-\lambda p_0 (t - \min\{t, t^*\})} \left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1 - u_B} \min\{t, t^*\}}\right) \frac{1}{\frac{u_G - u_B}{1 - u_B}}$$

is the probability that a good deal is proposed in the second stage. Then the investor's continuation value at time $-t$ is

$$U(-t) = (u_G - 1) \cdot (PG_1 + q \cdot PG_2) + 1.$$

## Proof of Proposition 3

See the proof of Proposition 2.

## Proof of Lemma 4

In the second stage where $-t > -t_s^*$, the sponsor's continuation value $V_s(-t)$ satisfies

$$\frac{dV_s(-t)}{dt} = \lambda \left[p_0 v_G + (1 - p_0) v_B - V_s(-t)\right]$$

with two boundary conditions $V_s(0) = 0$ and $V_s(-t_s^*) = v_B$. The investor's continuation value $U_s(-t)$ satisfies

$$\frac{dU_s(-t)}{dt} = \lambda \cdot \left[p_0 u_G + (1 - p_0) u_B - U_s(-t)\right]$$

with one boundary condition $U_s(0) = 1$.

In the first stage where $-t < -t_s^*$, $V_s(-t)$ satisfies

$$\frac{dV_s(-t)}{dt} = \lambda p_0 \cdot \left[v_G - V_s(-t)\right]$$

with the boundary condition $V_s(-t_s^*) = v_B$. And $U_s(-t)$ satisfies

$$\frac{dU_s(-t)}{dt} = \lambda p_0 \cdot \left[u_G - U_s(-t)\right]$$

with the boundary condition that $U_s(-t)$ is continuous at $-t_s^*$.

According to the evolution of the sponsor's continuation value, we obtain that for $-t \geq -t_s^*$,

$$V_s(-t) = \left(1 - e^{-\lambda t}\right) \left[p_0 v_G + (1 - p_0) v_B\right].$$

Then (7) is directly implied by the boundary condition $V_s(-t_s^*) = v_B$.

According to the evolution of the investor's continuation value, we obtain that for $-t \geq -t_s^*$,

$$U_s(-t) = \left(1 - e^{-\lambda t}\right)[p_0 u_G + (1-p_0)\, u_B] + e^{-\lambda t}.$$

And for $-t < -t_s^*$,

$$U_s(-t) = \left(1 - e^{-\lambda p_0 (t - t_s^*)}\right) u_G + e^{-\lambda p_0 (t - t_s^*)} U_s(-t_s^*).$$

Notice that $U_s(-t) < 1$ for $-t \geq -t_s^*$ and $U_s(-t)$ increases to $u_G$ as $t$ increases from $t_s^*$ to $+\infty$. There must exist $T_s^* > t_s^*$ such that $U_s(-T) > 1$ if and only if $T > T_s^*$.

## Proof of Proposition 4

Let's first consider the sponsor's welfare. Suppose in this setting the sponsor chooses the strategy $(\alpha_\omega(-t))_{\omega \in \{G,B\}}$ used in the baseline model. This means that he proposes all good deals he receives, and proposes bad deals only after time $-t^*$ with probability $\alpha_B(-t)$. Denote the sponsor's value function in this case as $\tilde{V}$. Then it's obvious that

$$\tilde{V}(-t^*) > V(-t^*) = (1-q)\, v_B$$

because all proposed deals are invested in this new setting. The sponsor's value function follows the same HJB equation in the baseline model and the new setting for $t \in (t^*, T)$, i.e.,

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot [v_G - V(-t)]$$

and

$$\frac{d\tilde{V}(-t)}{dt} = \lambda p_0 \cdot \left[v_G - \tilde{V}(-t)\right],$$

and their boundary conditions satisfy

$$\tilde{V}(-t^*) > V(-t^*).$$

Then we must have $\tilde{V}(-T) > V(-T)$. Since the sponsor's optimal choice $V_s(-T)$ weakly dominates $\tilde{V}(-T)$, we must have

$$V_s(-T) > V(-T).$$

Next we consider the investor's welfare. First, it's clear that when $T \leq T_s^*$, the investor is worse off without redemption rights, this is because in this case

$$U_s(-T) \leq 1 < U(-T).$$

When $T > T_s^*$, note that $U_s(-T) > 1$ and is independent of $q$, to obtain our results, we just show that the following properties hold

1. $U(-T)$ is strictly increasing in $q$;
2. $\lim_{q\to 0} U(-T) = 1$;
3. $\lim_{q\to 1} U(-T) > U_s(-T)$.

To show that $U(-T)$ is strictly increasing in $q$. According to Proposition 3, $t^*(q)$ is strictly decreasing in $q$. For any $q$ satisfying $t^*(q) \geq T$,

$$U(-T) = \left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} T}\right) \frac{1-u_B}{u_G - u_B} q \cdot (u_G - 1) + 1,$$

which is strictly increasing in $q$. For any $q > \tilde{q}$ satisfying $t^*(q) < t^*(\tilde{q}) < T$,

$$\begin{aligned} U(-T;q) &= \left(1 - e^{-\lambda p_0 (T - t^*(q))}\right) u_G + e^{-\lambda p_0 (T - t^*(q))} U(-t^*(q);q) \\ &= \left(1 - e^{-\lambda p_0 (T - t^*(\tilde{q}))}\right) u_G + e^{-\lambda p_0 (T - t^*(\tilde{q}))} U(-t^*(\tilde{q});q). \end{aligned}$$

Now we just need to show $U(-t^*(\tilde{q});q) > U(-t^*(\tilde{q});\tilde{q})$. Let

$$a = e^{\lambda p_0 (t^*(\tilde{q}) - t^*(q))} > 1$$

and

$$x = \frac{u_G - u_B}{1 - u_B} > 1.$$

Then

$$\begin{aligned} U(-t^*(\tilde{q});\tilde{q}) - 1 &= \left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} t^*(\tilde{q})}\right) \frac{1}{\frac{u_G - u_B}{1-u_B}} \tilde{q} \cdot (u_G - 1) \\ &= \left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} (t^*(\tilde{q}) - t^*(q))}\right) \frac{1}{\frac{u_G - u_B}{1-u_B}} \tilde{q} \cdot (u_G - 1) \\ &\quad + e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} (t^*(\tilde{q}) - t^*(q))} \left(U(-t^*(q);\tilde{q}) - 1\right) \\ &= \left(1 - a^{-x}\right) \frac{\tilde{q} \cdot (u_G - 1)}{x} + a^{-x} \left(U(-t^*(q);\tilde{q}) - 1\right). \end{aligned}$$

Since $a > 1$ and $x > 1$, $a^{-x} < a^{-1}$ and

$$\frac{1-a^{-x}}{x} < 1 - a^{-1}.$$

We have

$$\begin{aligned} U\left(-t^*\left(\tilde{q}\right);\tilde{q}\right) - 1 &< \left(1-a^{-1}\right)\tilde{q}\cdot\left(u_G - 1\right) + a^{-1}\left(U\left(-t^*\left(q\right);\tilde{q}\right) - 1\right) \\ &< \left(1-a^{-1}\right)\left(u_G - 1\right) + a^{-1}\left(U\left(-t^*\left(q\right);q\right) - 1\right) \\ &= U\left(-t^*\left(\tilde{q}\right);q\right) - 1. \end{aligned}$$

The intermediate result

$$U\left(-t^*\left(q\right);\tilde{q}\right) < U\left(-t^*\left(q\right);q\right)$$

comes from the explicit expression

$$U\left(-t^*\left(q\right);x\right) = \left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} t^*(q)}\right)\frac{1-u_B}{u_G - u_B} x\cdot\left(u_G - 1\right) + 1.$$

For the two limit results, as $q \to 0$, $t^* \to +\infty$, so

$$U\left(-T\right) \to \lim_{q\to 0}\left[\left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B} T}\right)\frac{1-u_B}{u_G - u_B} q\cdot\left(u_G - 1\right) + 1\right] = 1.$$

As $q \to 1$, $t^* \to 0$, so

$$U\left(-T;q\right) \to \lim_{q\to 1}\left[\left(1 - e^{\lambda p_0 (t^* - T)}\right)u_G + e^{\lambda p_0 (t^* - T)} U\left(-t^*;q\right)\right] = \left(1 - e^{-\lambda p_0 T}\right)u_G + e^{-\lambda p_0 T}.$$

Since $U_s\left(-t_s^*\right) < 1$, we have

$$\begin{aligned} U_s\left(-T\right) &= \left(1 - e^{-\lambda p_0 (T - t_s^*)}\right)u_G + e^{-\lambda p_0 (T - t_s^*)} U_s\left(-t_s^*\right) \\ &< \left(1 - e^{-\lambda p_0 (T - t_s^*)}\right)u_G + e^{-\lambda p_0 (T - t_s^*)} \\ &< \left(1 - e^{-\lambda p_0 T}\right)u_G + e^{-\lambda p_0 T}. \end{aligned}$$

So we have $\lim_{q\to 1} U\left(-T\right) > U_s\left(-T\right)$.

## Proof of Proposition 5

Without loss of generality, we consider the cases in which the switch between having the redemption right and not happens finite times. That means, the prohibition scheme can be characterized by a finite set of intervals, and within each interval, the investor either always has the redemption right (we call it $R$ interval) or always does not have (we call it $NR$ interval). For an interval starting at $-t_1$ and ending at $-t_2$, we denote it by $I(-t_1,-t_2,R)$ if the investor always has the redemption right and $I(-t_1,-t_2,NR)$ if the investor always does not.

Consider a prohibition scheme that violates the property in Proposition 5. Suppose that the original scheme's first $NR$ interval is $I(-t_0,-t_0+\tau_1,NR)$ and the next interval is $I(-t_0+\tau_1,-t_0+\tau_1+\tau_2,R)$. Since we focus on the the minimum prohibition, it is without loss of generality to assume that the sponsor proposes both types in $I(-t_0,-t_0+\tau_1,NR)$. We claim that it will be a Pareto improvement if we switch the two intervals and have $I(-t_0,-t_0+\tau_2,R)$ and $I(-t_0+\tau_2,-t_0+\tau_2+\tau_1,NR)$ instead. Denote the two players' continuation values under the original scheme as $V(-t)$ and $U(-t)$ respectively, and those under the alternative scheme as $V_1(-t)$ and $U_1(-t)$ respectively.

Note that the equilibrium dynamics during $[-t_0+\tau_1+\tau_2,0]$ under the two schemes are the same, so

$$\begin{aligned}V(-t_0+\tau_1+\tau_2)&=V_1(-t_0+\tau_1+\tau_2),\\ U(-t_0+\tau_1+\tau_2)&=U_1(-t_0+\tau_1+\tau_2).\end{aligned}$$

**Part I: We must have $V_1(-t_0)>V(-t_0)$.** For the original scheme, define $\hat{\tau}$ as

$$\hat{\tau}\begin{cases}=0, & \text{if } V(-t_0+\tau_1+\tau_2)\geq(1-q)v_B;\\ =\tau_2, & \text{if } V(-t_0+\tau_1)\leq(1-q)v_B;\\ \text{satisfies } V(-t_0+\tau_1+\tau_2-\hat{\tau})=(1-q)v_B, & \text{otherwise.}\end{cases}$$

Note that under the original scheme,

1. in $(-t_0,-t_0+\tau_1)$, the sponsor proposes both types and the investor cannot redeem shares;
2. in $(-t_0+\tau_1,-t_0+\tau_1+\tau_2-\hat{\tau})$, the two players play as if they are in the first stage of the baseline equilibrium;
3. in $(-t_0+\tau_1+\tau_2-\hat{\tau},-t_0+\tau_1+\tau_2)$, the two players play as if they are in the second stage of the baseline equilibrium.

Hence, we obtain

$$\begin{aligned}&V(-t_0)\\ =&\left(1-e^{-\lambda\tau_1}\right)\left[p_0v_G+(1-p_0)v_B\right]\\ &+e^{-\lambda\tau_1}\left\{\left(1-e^{-\lambda p_0(\tau_2-\hat{\tau})}\right)v_G+e^{-\lambda p_0(\tau_2-\hat{\tau})}\left[e^{\lambda p_0\left(\frac{v_G}{v_B}-1\right)\hat{\tau}}\left(V(-t_0+\tau_1+\tau_2)+\frac{qv_G}{\frac{v_G}{v_B}-1}\right)-\frac{qv_G}{\frac{v_G}{v_B}-1}\right]\right\}\\ \equiv&\Phi(\hat{\tau}).\end{aligned}$$

Likewise, for the alternative scheme, define $\hat{z}$ as

$$\hat{z}\begin{cases} =0, & \text{if } V_1\left(-t_0+\tau_2\right)\geq\left(1-q\right)v_B;\\ =\tau_2, & \text{if } V_1\left(-t_0\right)\leq\left(1-q\right)v_B;\\ \text{satisfies } V\left(-t_0+\tau_2-\hat{z}\right)=\left(1-q\right)v_B, & \text{otherwise.}\end{cases}$$

Note that under the alternative scheme,

1. in $(-t_0,-t_0+\tau_2-\hat{z})$, the two players play as if they are in the first stage of the baseline equilibrium;
2. in $(-t_0+\tau_2-\hat{z},-t_0+\tau_2)$, the two players play as if they are in the second stage of the baseline equilibrium;
3. in $(-t_0+\tau_2,-t_0+\tau_1+\tau_2)$, the sponsor proposes both types and the investor cannot redeem shares.

Hence, we obtain

$$\begin{aligned}
&V_1\left(-t_0\right)\\
=&\left(1-e^{-\lambda p_0\left(\tau_2-\hat{z}\right)}\right)v_G\\
&+e^{-\lambda p_0\left(\tau_2-\hat{z}\right)}\left\{e^{\lambda p_0\left(\frac{v_G}{v_B}-1\right)\hat{z}}\left[\left(1-e^{-\lambda\tau_1}\right)\left(p_0v_G+\left(1-p_0\right)v_B\right)+e^{-\lambda\tau_1}V\left(-t_0+\tau_1+\tau_2\right)+\frac{qv_G}{\frac{v_G}{v_B}-1}\right]-\frac{qv_G}{\frac{v_G}{v_B}-1}\right\}\\
\equiv&\Phi_1\left(\hat{z}\right).
\end{aligned}$$

Since $V_1\left(-t_0+\tau_2\right)>V_1\left(-t_0+\tau_1+\tau_2\right)=V\left(-t_0+\tau_1+\tau_2\right)$, $0\leq\hat{z}\leq\hat{\tau}$. Also, we can show $d\Phi\left(x\right)/dx\leq 0$ if $x\leq\hat{\tau}$ and $\hat{\tau}>0$. Therefore,

$$V\left(-t_0\right)=\Phi\left(\hat{\tau}\right)\leq\Phi\left(\hat{z}\right)<\Phi_1\left(\hat{z}\right)=V_1\left(-t_0\right).$$

**Part II: $U_1\left(-t_0\right)>U\left(-t_0\right)$.** Following the above analysis, we obtain

$$\begin{aligned}
&U\left(-t_0\right)\\
=&\left(1-e^{-\lambda\tau_1}\right)\left[p_0u_G+\left(1-p_0\right)u_B\right]+e^{-\lambda\tau_1}\times\\
&\left\{\left(1-e^{-\lambda p_0\left(\tau_2-\hat{\tau}\right)}\right)u_G+e^{-\lambda p_0\left(\tau_2-\hat{\tau}\right)}\left[\left(1-e^{-\lambda p_0\frac{u_G-u_B}{1-u_B}\hat{\tau}}\right)\left(\frac{1-u_B}{u_G-u_B}q\left(u_G-1\right)+1\right)+e^{-\lambda p_0\frac{u_G-u_B}{1-u_B}\hat{\tau}}U\left(-t_0+\tau_1+\tau_2\right)\right]\right\}\\
\equiv&\Psi\left(\hat{\tau}\right)
\end{aligned}$$

and

$$
\begin{aligned}
& U_1(-t_0) \\
= & \left(1-e^{-\lambda p_0(\tau_2-\hat{z})}\right)u_G+e^{-\lambda p_0(\tau_2-\hat{z})}\times \\
& \left\{\left(1-e^{-\lambda p_0\frac{u_G-u_B}{1-u_B}\hat{z}}\right)\left(\frac{1-u_B}{u_G-u_B}q(u_G-1)+1\right)+e^{-\lambda p_0\frac{u_G-u_B}{1-u_B}\hat{z}}\left[\left(1-e^{-\lambda\tau_1}\right)[p_0u_G+(1-p_0)u_B]+e^{-\lambda\tau_1}U(-t_0+\tau_1+\tau_2)\right]\right\} \\
\equiv & \Psi_1(\hat{z}).
\end{aligned}
$$

We can show that for any $x$, $\Psi(x)<\Psi_1(x)$. If $\hat{\tau}=0$, $\hat{z}=0=\hat{\tau}$, then

$$
U(-t_0)=\Psi(0)<\Psi_1(0)=U_1(-t_0).
$$

If $\hat{\tau}>0$, then $V(-t_0+\tau_1+\tau_2)<(1-q)v_B$, from which we can obtain a lower bound of $U(-t_0+\tau_1+\tau_2)$. Suppose that conditional on that the game does not end by $-t_0+\tau_1+\tau_2$, the game ends with a good deal being completed with probability $P_G$ and with a bad deal being completed with probability $P_B$. Note that at each instant, the sponsor proposes any good deal he receives, and a good deal is more likely to be done. So,

$$
\frac{P_G}{P_B}\geq\frac{p_0}{1-p_0}.
$$

By

$$
\begin{aligned}
(1-q)v_B>V(-t_0+\tau_1+\tau_2) &= P_Gv_G+P_Bv_B \\
&= \left(\frac{P_G}{P_G+P_B}v_G+\frac{P_B}{P_G+P_B}v_B\right)(P_G+P_B) \\
&\geq [p_0v_G+(1-p_0)v_B](P_G+P_B),
\end{aligned}
$$

we obtain

$$
P_G+P_B\leq\frac{(1-q)v_B}{p_0v_G+(1-p_0)v_B}.
$$

Then

$$
\begin{aligned}
U(-t_0+\tau_1+\tau_2) &= P_G(u_G-1)+P_B(u_B-1)+1 \\
&= \left(\frac{P_G}{P_G+P_B}u_G+\frac{P_B}{P_G+P_B}u_B-1\right)(P_G+P_B)+1 \\
&\geq [p_0u_G+(1-p_0)u_B-1](P_G+P_B)+1 \\
&\geq [p_0u_G+(1-p_0)u_B-1]\frac{(1-q)v_B}{p_0v_G+(1-p_0)v_B}+1 \\
&> (u_B-1)(1-q)+1.
\end{aligned}
$$

Next, we prove $\Psi'(\hat{\tau}) < 0$.

$$
\begin{aligned}
&\Psi'(\hat{\tau}) \times e^{\lambda \tau_1} \\
=& -e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 u_G + e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 \left[\left(1-e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B}\hat{\tau}}\right)\left(\frac{1-u_B}{u_G-u_B} q(u_G-1)+1\right) + e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B}\hat{\tau}} U(-t_0+\tau_1+\tau_2)\right] \\
&+ e^{-\lambda p_0(\tau_2-\hat{\tau})}\left[e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B}\hat{\tau}}\lambda p_0 \frac{u_G-u_B}{1-u_B}\left(\frac{1-u_B}{u_G-u_B} q(u_G-1)+1\right) - e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B}\hat{\tau}}\lambda p_0 \frac{u_G-u_B}{1-u_B} U(-t_0+\tau_1+\tau_2)\right] \\
=& e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 \left\{\left(\frac{1-u_B}{u_G-u_B} q - 1\right)(u_G-1) + e^{-\lambda p_0 \frac{u_G-u_B}{1-u_B}\hat{\tau}} \frac{u_G-1}{1-u_B}\left[\frac{1-u_B}{u_G-u_B} q(u_G-1)+1-U(-t_0+\tau_1+\tau_2)\right]\right\}.
\end{aligned}
$$

If $U(-t_0+\tau_1+\tau_2) \geq \frac{1-u_B}{u_G-u_B} q(u_G-1)+1$, $\Psi'(\hat{\tau}) < 0$. Assume that $U(-t_0+\tau_1+\tau_2) < \frac{1-u_B}{u_G-u_B} q(u_G-1)+1$.

$$
\begin{aligned}
&\Psi'(\hat{\tau}) \times e^{\lambda \tau_1} \\
<& e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 \left\{\left(\frac{1-u_B}{u_G-u_B} q - 1\right)(u_G-1) + \frac{u_G-1}{1-u_B}\left[\frac{1-u_B}{u_G-u_B} q(u_G-1)+1-U(-t_0+\tau_1+\tau_2)\right]\right\} \\
=& e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 \left\{(q-1)(u_G-1) + \frac{u_G-1}{1-u_B}\left[1-U(-t_0+\tau_1+\tau_2)\right]\right\} \\
<& e^{-\lambda p_0(\tau_2-\hat{\tau})}\lambda p_0 \left\{(q-1)(u_G-1) + \frac{u_G-1}{1-u_B}(1-u_B)(1-q)\right\} \\
=& 0.
\end{aligned}
$$

By $\Psi'(\hat{\tau}) < 0$,

$$
U(-t_0) = \Psi(\hat{\tau}) \leq \Psi(\hat{z}) < \Psi_1(\hat{z}) = U_1(-t_0).
$$

**Part III: $U_1(-T) > U(-T)$ and $V_1(-T) > V(-T)$.** For the original scheme, define $\tilde{\tau}$ as

$$
\tilde{\tau}\begin{cases} = 0, & \text{if } V(-t_0) \geq (1-q) v_B; \\ = T - t_0, & \text{if } V(-T) \leq (1-q) v_B; \\ \text{satisfies } V(-t_0-\tilde{\tau}) = (1-q) v_B, & \text{otherwise.} \end{cases}
$$

For the alternative scheme, define $\tilde{z}$ as

$$
\tilde{z}\begin{cases} = 0, & \text{if } V_1(-t_0) \geq (1-q) v_B; \\ = T - t_0, & \text{if } V_1(-T) \leq (1-q) v_B; \\ \text{satisfies } V_1(-t_0-\tilde{z}) = (1-q) v_B, & \text{otherwise.} \end{cases}
$$

Since $V_1(-t_0) > V(-t_0)$, $\tilde{z} \leq \tilde{\tau}$. Then we regard $V(-T)$ and $U(-T)$ as the functions of $\tilde{\tau}$ and $V_1(-T)$ and $U_1(-T)$ as the functions of $\tilde{z}$. The proof is largely similar to Part I and Part II.

**Part IV: Iteration** Then we can show that any scheme will take the form characterized by Proposition 5 after finite switches. Since such operation is a Pareto improvement, we obtain that the optimal prohibition schemes characterized by Proposition 5 is optimal in the Pareto sense that if it is not satisfied, there must exist another scheme that makes both players better off.

## Proof of Proposition 6

Based on our analysis, there are two stages in this new setting. In the second stage where $-t > -t_v^*$, $V_v(-t)$ satisfies

$$\frac{dV_v(-t)}{dt} = \lambda p_0 \cdot q \left[v_G - V_v(-t)\right]$$

with two boundary conditions $V_v(0) = 0$ and $V_v(-t_v^*) = v_B$. $U_v(-t)$ satisfies

$$\frac{dU_v(-t)}{dt} = \lambda p_0 \cdot q \left[u_G - U_v(-t)\right]$$

with one boundary condition $U_v(0) = 1$.

In the first stage where $-t < -t_v^*$, $V_v(-t)$ satisfies

$$\frac{dV_v(-t)}{dt} = \lambda p_0 \cdot \left[v_G - V_v(-t)\right]$$

with the boundary condition $V_v(-t_v^*) = v_B$. $U_v(-t)$ satisfies

$$\frac{dU_v(-t)}{dt} = \lambda p_0 \left[u_G - U_v(-t)\right]$$

with the boundary condition that $U_v(-t)$ is continuous at $-t_v^*$.

First, we want to show $t^* < t_v^*$. To see this, note that for $-t > \max\{-t^*, -t_v^*\}$, the sponsor's value function in the baseline setting and the new setting are

$$V(-t) = \left[e^{\lambda p_0 \left(\frac{v_G}{v_B} - 1\right) t} - 1\right] \frac{1}{\frac{v_G}{v_B} - 1} \cdot q \cdot v_G$$

and

$$V_v(-t) = \left(1 - e^{-\lambda p_0 q t}\right) v_G$$

respectively. Note that for $a > 0$ and $a \neq 1$, $\frac{a^x - 1}{x}$ is increasing in $x \in (0, \infty)$ and $\frac{1 - a^{-x}}{x}$ is decreasing in $x \in (0,\infty)$. Since $\frac{v_G}{v_B} - 1 > 0$,

$$\begin{aligned} V(-t) &> \lim_{x \downarrow 0} \left[ e^{\lambda p_0 t \cdot x} - 1 \right] \frac{1}{x} \cdot q \cdot v_G \\ &= \lambda p_0 t \cdot q \cdot v_G. \end{aligned}$$

On the other hand,

$$\begin{aligned} V_v(-t) &= \frac{1 - e^{-\lambda p_0 q t}}{\lambda p_0 q t} \lambda p_0 q t v_G \\ &< \lim_{x \downarrow 0} \frac{1 - e^{-x}}{x} \lambda p_0 q t v_G \\ &= \lambda p_0 q t v_G. \end{aligned}$$

So we must have $V_v(-t) < V(-t)$ for any $-t > \max\{-t^*, -t_v^*\}$. Besides, the boundaries $t^*$ and $t_v^*$ satisfy

$$V(-t^*) = (1-q) v_B$$

and

$$V_v(-t_v^*) = v_B.$$

Then we must have $t_v^* > t^*$.

For $t \in (0, t^*]$, we already show that $V(-t) > V_v(-t)$.

For $t \in (t^*, t_v^*]$, since $v_G > V(-t^*) > V_v(-t^*)$, we have

$$\begin{aligned} V(-t) &= \left(1 - e^{-\lambda p_0 (t - t^*)}\right) v_G + e^{-\lambda p_0 (t - t^*)} V(-t^*) \\ &> \left(1 - e^{-\lambda p_0 q (t - t^*)}\right) v_G + e^{-\lambda p_0 q (t - t^*)} V(-t^*) \\ &> \left(1 - e^{-\lambda p_0 q (t - t^*)}\right) v_G + e^{-\lambda p_0 q (t - t^*)} V_v(-t^*) \\ &= V_v(-t). \end{aligned}$$

For $t \in (t_v^*, +\infty)$,

$$\begin{aligned} V(-t) &= \left(1 - e^{-\lambda p_0 (t - t_v^*)}\right) v_G + e^{-\lambda p_0 (t - t_v^*)} V(-t_v^*) \\ &> \left(1 - e^{-\lambda p_0 (t - t_v^*)}\right) v_G + e^{-\lambda p_0 (t - t_v^*)} V_v(-t_v^*) \\ &= V_v(-t). \end{aligned}$$

Therefore, we must have $V(-T) > V_v(-T)$.

## Proof of Proposition 7

When $t$ is close to zero, the sponsor's value $V_\chi$ must be close to zero. The expected payoff from proposing a bad deal is at least

$$(1-q)(1-\chi)v_B$$

which is strictly greater than zero. So when $t$ is close to zero, we must have

$$\alpha_G(-t) = \alpha_B(-t) = 1$$

in equilibrium. Our Assumption 1 implies that the rational investor must choose

$$\eta_\chi(-t) = 0$$

in this case. The sponsor's value function satisfies

$$\begin{aligned}
\frac{dV_\chi(-t)}{dt} &= \lambda p_0 \cdot \left[F_G(-t) - V_\chi(-t)\right] + \lambda(1-p_0) \cdot \left[F_B(-t) - V_\chi(-t)\right] \\
&= \lambda p_0 \cdot \left[(\chi q + 1 - \chi)v_G - V_\chi(-t)\right] + \lambda(1-p_0) \cdot \left[(1-\chi)(1-q)v_B - V_\chi(-t)\right] \\
&= \lambda p_0 \cdot (\chi q + 1 - \chi)v_G + \lambda(1-p_0) \cdot (1-\chi)(1-q)v_B - \lambda \cdot V_\chi(-t)
\end{aligned}$$

with the boundary condition $V_\chi(0) = 0$, and

$$F_\omega(-t) = \begin{cases} (\chi q + 1 - \chi)v_G, & \text{if } \omega = G \\ (1-\chi)(1-q)v_B, & \text{if } \omega = B \end{cases}$$

represents the sponsor's expected payoff from proposing a type $\omega$ deal. The solution to the above HJB equation is

$$V_\chi(-t) = \left(1 - e^{-\lambda t}\right)\left[q p_0 v_G + (1-\chi)(1-q)(p_0 v_G + (1-p_0)v_B)\right].$$

When $V_\chi(-t) > (1-\chi)(1-q)v_B$, $\alpha_B(-t) = 1$ can not sustain an equilibrium. Then the sponsor will propose bad deals with lower probability, i.e., $\alpha_B(-t) < 1$. Similar to our discussion on the second stage in the baseline model, the rational investor chooses $\eta_\chi(-t)$ such that the sponsor is indifferent to proposing bad deals or not, i.e.,

$$V_\chi(-t) = F_B(-t),$$

and the sponsor's strategy $\alpha_B(-t)$ makes the rational investor indifferent to investing or not upon observing $\emptyset$, i.e.,

$$\frac{p_0}{1-p_0}\frac{1}{\alpha_B(-t)}=\frac{1-u_B}{u_G-1}.$$

The sponsor's value function $V_\chi$ satisfies

$$\begin{aligned}
\frac{dV_\chi(-t)}{dt} &= \lambda p_0\cdot\left[F_G(-t)-V_\chi(-t)\right]\\
&= \lambda p_0\cdot\left\{\left[\chi q+\chi(1-q)\eta_\chi(-t)+1-\chi\right]v_G-V_\chi(-t)\right\}\\
&= \lambda p_0\cdot\left\{\left[q+\frac{V_\chi(-t)}{v_B}\right]v_G-V_\chi(-t)\right\}\\
&= \lambda p_0\cdot\left\{qv_G+\left(\frac{v_G}{v_B}-1\right)V_\chi(-t)\right\}.
\end{aligned}$$

The boundary $-t_2^*$ then satisfies

$$V_\chi(-t_2^*)=(1-\chi)(1-q)v_B.$$

The solution to the above HJB equation is

$$V_\chi(-t)=e^{\lambda p_0\left(\frac{v_G}{v_B}-1\right)\left(t-t_2^*\right)}\left[V_\chi(-t_2^*)+\frac{qv_G}{\frac{v_G}{v_B}-1}\right]-\frac{qv_G}{\frac{v_G}{v_B}-1}.$$

When $V_\chi(-t)>(1-q)v_B$, proposing bad deals is dominated by waiting, and similar to the first stage in our baseline model, the sponsor now only proposes good deals. Then in this stage, $\alpha_B(-t)=0$, $\alpha_G(-t)=1$. As the best response, the rational investor chooses

$$\eta_\chi(-t)=1.$$

The sponsor's value function satisfies

$$\frac{dV_\chi(-t)}{dt}=\lambda p_0\cdot\left[v_G-V_\chi(-t)\right]$$

with boundary condition $V_\chi(-t_1^*)=(1-q)v_B$. The solution to this HJB equation is

$$V_\chi(-t)=v_G-e^{-\lambda p_0\left(t-t_1^*\right)}\left[v_G-V_\chi(-t_1^*)\right].$$

## Proof of Proposition 8

In this proof, we will show $\frac{dV_\chi(-t)}{d(1-\chi)} > 0$ for any $-t$, then Proposition 8 is just a special case of this general result.

For any $-t \in [-t_2^*, 0)$, we know

$$V_\chi(-t) = \left(1 - e^{-\lambda t}\right)[qp_0v_G + (1-\chi)(1-q)(p_0v_G + (1-p_0)v_B)].$$

So it's clear that

$$\frac{dV_\chi(-t)}{d(1-\chi)} = \left(1 - e^{-\lambda t}\right)(1-q)(p_0v_G + (1-p_0)v_B) > 0.$$

For any $-t \in [-t_1^*, -t_2^*)$, for the sake of exposition, let $y = 1-\chi$, and let

$$V_3(y,t) = V_\chi(-t)$$

for $-t \in [-t_2^*(y), 0]$ and

$$V_2(y,t) = V_\chi(-t)$$

for $-t \in [-t_1^*(y), -t_2^*(y)]$. Note the HJB equation of the sponsor's value function in the region $-t \in [-t_1^*, -t_2^*)$ is

$$\frac{dV_\chi(-t)}{dt} = \lambda p_0 \cdot \left\{qv_G + \left(\frac{v_G}{v_B} - 1\right)V_\chi(-t)\right\},$$

and this is independent of $(1-\chi)$. To show that $V_\chi(-t)$ is increasing in $(1-\chi)$ for any $-t \in [-t_1^*, -t_2^*)$, it's sufficient to show

$$\left.\frac{\partial_+ V_2(y,t)}{\partial t}\right|_{(y,t_2^*)} \cdot \left.\frac{dt_2^*}{\partial y}\right|_{(\chi,t_2^*)} < \left.\frac{\partial_+ V_3(y,t)}{\partial y}\right|_{(\chi,t_2^*)} + \left.\frac{\partial_- V_3(y,t)}{\partial t}\right|_{(y,t_2^*)} \cdot \left.\frac{dt_2^*}{\partial y}\right|_{(y,t_2^*)}. \tag{9}$$

The idea of the above condition is that by increasing $(1-\chi)$, the sponsor's value at the boundary $-t_2^*$ increases compared to the level before the increase of $(1-\chi)$. To show that condition (9) holds, first it's clear that

$$\left.\frac{\partial_+ V_3(y,t)}{\partial y}\right|_{(\chi,t_2^*)} = \left(1 - e^{-\lambda t_2^*}\right)(1-q)(p_0v_G + (1-p_0)v_B) > 0.$$

Besides,

$$\left.\frac{\partial_- V_3(y,t)}{\partial t}\right|_{(y,t_2^*)} = \lambda p_0 \cdot \left[F_G(-t_2^*) - V_\chi(-t_2^*)\right] + \lambda(1-p_0) \cdot \left[F_B(-t_2^*) - V_\chi(-t_2^*)\right]$$

and

$$\left.\frac{\partial_+ V_2(y,t)}{\partial t}\right|_{(y,t_2^*)} = \lambda p_0 \cdot \left[F_G(-t_2^*) - V_\chi(-t_2^*)\right].$$

Since

$$F_B(-t_2^*) - V_\chi(-t_2^*) = 0,$$

the condition (9) must hold.

The sponsor's value at $-t_1^*$ satisfies

$$V_\chi(-t_1^*) = (1-q)\,v_B.$$

Since we already showed that $\frac{dV_\chi(-t)}{d(1-\chi)} > 0$ for any $-t \in [-t_1^*, -t_2^*)$, we must have

$$\frac{dt_1^*}{d(1-\chi)} < 0.$$

The HJB equation of the sponsor's value function in stage 1 satisfies

$$\frac{dV(-t)}{dt} = \lambda p_0 \cdot [v_G - V(-t)]$$

which is independent of $1-\chi$, then we can conclude that $\frac{dV_\chi(-t)}{d(1-\chi)} > 0$ for any $-t \in [-T, -t_1^*)$.

In summary, we must have $\frac{dV_\chi(-t)}{d(1-\chi)} > 0$ for any $-t \in [-T, 0]$.

## Proof of Lemma 5

We already show that for any $-t \in [-t_2^*, 0)$,

$$V_\chi(-t) = \left(1 - e^{-\lambda t}\right)[qp_0v_G + (1-\chi)(1-q)(p_0v_G + (1-p_0)v_B)].$$

The boundary condition $V_\chi(-t_2^*) = (1-\chi)(1-q)v_B$ implies

$$\begin{aligned}
&\left(1 - e^{-\lambda t_2^*}\right)[qp_0v_G + (1-\chi)(1-q)(p_0v_G + (1-p_0)v_B)] = (1-\chi)(1-q)v_B \\
\Longleftrightarrow &\left(1 - e^{-\lambda t_2^*}\right) = \frac{(1-q)v_B}{\left[\frac{qp_0v_G}{1-\chi} + (1-q)(p_0v_G + (1-p_0)v_B)\right]}.
\end{aligned}$$

Then it's clear that $\frac{dt_2^*}{d(1-\chi)} > 0$.

For $-t_1^*$, the boundary condition is $V_\chi(-t_2^*) = (1-q)v_B$. In the proof of Proposition 8, we already show that $\frac{dV_\chi(-t)}{d(1-\chi)} > 0$ for any $-t \in [-T, 0]$. Then we must have $\frac{dt_1^*}{d(1-\chi)} < 0 \Longleftrightarrow \frac{d\left(T - t_1^*\right)}{d(1-\chi)} >$

0.

## Proof of Proposition 9

To simplify our exposition, let's introduce the following notations:

- $PG_i$: the unconditional probability that a good deal is proposed at stage $i$, for $i = 1,2,3$;
- $PB_i$: the unconditional probability that a bad deal is proposed at stage $i$, for $i = 1,2,3$.

For a rational investor, her utility is

$$U_R(-T) = (u_G - 1)\{PG_1 + q \cdot (PG_2 + PG_3)\} + 1.$$

Here we use the property that the rational investor always gets zero expected utility when observing signal realization $\emptyset$ in stage 2 and 3. For a behavioral investor, her utility is

$$U_B(-T) = (u_G - 1)\sum_{i=1}^{3} PG_i + (u_B - 1)\cdot(1-q)\sum_{i=1}^{3} PB_i + 1.$$

Then investors' (expected) utility is

$$U_\chi(-T) = \chi U_R(-T) + (1-\chi)U_B(-T).$$

We can show that the above probabilities are

- $PG_1 = 1 - e^{-\lambda p_0 \left(T - t_1^*\right)}$,
- $PB_1 = 0$,
- $PG_2 = e^{-\lambda p_0 \left(T - t_1^*\right)}\left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B}\left(t_1^* - t_2^*\right)}\right)\frac{1}{\frac{u_G - u_B}{1-u_B}}$,
- $PB_2 = e^{-\lambda p_0 \left(T - t_1^*\right)}\left(1 - e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B}\left(t_1^* - t_2^*\right)}\right)\frac{1}{\frac{u_G - u_B}{u_G - 1}}$,
- $PG_3 = e^{-\lambda p_0 \left(T - t_1^*\right)} e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B}\left(t_1^* - t_2^*\right)}\left(1 - e^{-\lambda t_2^*}\right) p_0$,
- $PB_3 = e^{-\lambda p_0 \left(T - t_1^*\right)} e^{-\lambda p_0 \frac{u_G - u_B}{1-u_B}\left(t_1^* - t_2^*\right)}\left(1 - e^{-\lambda t_2^*}\right)(1 - p_0)$.

Then investors' utility is

$$
\begin{aligned}
U_\chi(-T) &= (u_G-1)\left[PG_1+(\chi q+1-\chi)\cdot(PG_2+PG_3)\right]+(1-q)(u_B-1)\cdot(1-\chi)\sum_{i=1}^{3}PB_i+1 \\
&= (u_G-1)PG_1+(u_G-1)q(PG_2+PG_3) \\
&\quad +(1-\chi)(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right](PB_3+PG_3).
\end{aligned}
$$

Here we used the property $\frac{PG_2}{1-u_B}=\frac{PB_2}{u_{G-1}}$. Fix $\chi<1$, when $\chi$ changes locally, we have

$$
\begin{aligned}
\lim_{q\to 0}\frac{dU_\chi(-T)}{d\chi} &= \lim_{q\to 0}\frac{d}{d\chi}\left\{(u_G-1)PG_1+(1-\chi)(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right](PG_3+PB_3)\right\} \\
&\quad +\lim_{q\to 0}\frac{d}{d\chi}\left\{1+q(u_G-1)(PG_2+PG_3)\right\}.
\end{aligned}
$$

It's obvious that

$$
\lim_{q\to 0}\frac{d}{d\chi}\left\{1+q(u_G-1)(PG_2+PG_3)\right\}=0
$$

because all of $PG_2$, $PG_3$, $\frac{dPG_2}{d\chi}$ and $\frac{dPG_3}{d\chi}$ are locally bounded.

Let $z=e^{\lambda p_0\left(t_1^*-t_2^*\right)}$, then it's easy to show

$$
\lim_{q\to 0} z^{-\left(\frac{v_G}{v_B}-1\right)}=1-\chi.
$$

Then we have

$$
\begin{aligned}
&\lim_{q\to 0}\frac{d}{d\chi}\left\{(u_G-1)PG_1+(1-\chi)(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right](PG_3+PB_3)\right\} \\
=&\frac{d}{d\chi}\lim_{q\to 0}\left\{(u_G-1)\left(1-e^{-\lambda p_0\left(T-t_2^*\right)}z\right)+z^{-\left(\frac{v_G}{v_B}-1\right)}(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right]\left(1-e^{-\lambda t_2^*}\right)e^{-\lambda p_0\left(T-t_2^*\right)}z^{-\frac{u_G-1}{1-u_B}}\right\} \\
=&\frac{d}{d\chi}\lim_{q\to 0}\left\{(u_G-1)\left(1-e^{-\lambda p_0\left(T-t_2^*\right)}z\right)+(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right]\left(1-e^{-\lambda t_2^*}\right)e^{-\lambda p_0\left(T-t_2^*\right)}z^{1-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}\right\} \\
=&\frac{d}{d\chi}\lim_{q\to 0}\left\{u_G-1-e^{-\lambda p_0\left(T-t_2^*\right)}\left\{(u_G-1)z-(1-q)\left[(u_G-1)p_0+(u_B-1)(1-p_0)\right]\left(1-e^{-\lambda t_2^*}\right)z^{1-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}\right\}\right\} \\
=&\lim_{q\to 0}-e^{-\lambda p_0\left(T-t_2^*\right)}\lim_{q\to 0}\frac{d}{dz}\left\{(u_G-1)z+(1-q)\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(1-e^{-\lambda t_2^*}\right)z^{1-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}\right\}\frac{dz}{d\chi} \\
=&\lim_{q\to 0}-e^{-\lambda p_0\left(T-t_2^*\right)}\lim_{q\to 0}\frac{dz}{d\chi}\cdot\lim_{q\to 0}\left\{(u_G-1)+(1-q)\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(1-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)\right)\left(1-e^{-\lambda t_2^*}\right)z^{-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}\right\} \\
=&\lim_{q\to 0}-e^{-\lambda p_0\left(T-t_2^*\right)}\lim_{q\to 0}\frac{dz}{d\chi}\cdot\left\{(u_G-1)+\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(1-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}\lim_{q\to 0}z^{-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}\right\}.
\end{aligned}
$$

Here we used

$$
\lim_{q\to 0}1-e^{-\lambda t_2^*}=\frac{v_B}{p_0v_G+(1-p_0)v_B}.
$$

Since $\lim_{q\to 0}\frac{dz}{d\chi}>0$, we must have

$$\lim_{q\to 0}\frac{dU_\chi(-T)}{d\chi}\propto\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}-1\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}\lim_{q\to 0}z^{-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)} - (u_G-1).$$

Then

$$\begin{aligned}
&\lim_{q\to 0}\frac{dU_\chi(-T)}{d\chi}<0\\
\Longleftrightarrow &\lim_{q\to 0}z^{-\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}\right)}<\frac{(u_G-1)}{\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}-1\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}}\\
\Longleftrightarrow &(1-\chi)^{\frac{\frac{u_G-1}{1-u_B}v_B+v_G}{v_G-v_B}}<\frac{(u_G-1)}{\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}-1\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}}\\
\Longleftrightarrow &(1-\chi)<\left\{\frac{(u_G-1)}{\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}-1\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}}\right\}^{\frac{v_G-v_B}{\frac{u_G-1}{1-u_B}v_B+v_G}}\\
\Longleftrightarrow &\chi>1-\left\{\frac{(u_G-1)}{\left[(1-u_B)(1-p_0)-(u_G-1)p_0\right]\left(\frac{u_G-1}{1-u_B}+\frac{v_G}{v_B}-1\right)\frac{v_B}{p_0v_G+(1-p_0)v_B}}\right\}^{\frac{v_G-v_B}{\frac{u_G-1}{1-u_B}v_B+v_G}}.
\end{aligned}$$

So for sufficiently small $q$, there exists $\underline{\chi}<1$, such that for any $\chi\in\left(\underline{\chi},1\right)$, we have

$$\frac{dU_\chi(-T)}{d(1-\chi)}>0.$$

## Proof of Lemma 6

First, we have

$$\frac{d^2(T-t_1^*)}{d(1-\chi)^2}=-\frac{d^2(t_1^*-t_2^*)}{d(1-\chi)^2}-\frac{d^2t_2^*}{d(1-\chi)^2}.$$

Note that we have the following two boundary conditions for $t_1^*$ and $t_2^*$:

$$\begin{aligned}
(1-\chi)(1-q)v_B=V(-t_2^*)&=\left(1-e^{-\lambda t_2^*}\right)\left[qp_0v_G+(1-\chi)(1-q)(p_0v_G+(1-p_0)v_B)\right]\\
\Rightarrow 1-e^{-\lambda t_2^*}&=\frac{1}{\frac{qp_0v_G}{(1-\chi)(1-q)v_B}+\frac{p_0v_G+(1-p_0)v_B}{v_B}}
\end{aligned}$$

and

$$(1-q)\,v_B = V\left(-t_1^*\right) = e^{\lambda p_0\left(\frac{v_G}{v_B}-1\right)\left(t_1^*-t_2^*\right)}\left[(1-\chi)(1-q)\,v_B + \frac{qv_G}{\frac{v_G}{v_B}-1}\right] - \frac{qv_G}{\frac{v_G}{v_B}-1}$$
$$\Rightarrow \ln\frac{\frac{qv_G}{\frac{v_G}{v_B}-1}+(1-q)\,v_B}{\frac{qv_G}{\frac{v_G}{v_B}-1}+(1-\chi)(1-q)\,v_B} = \left(\frac{v_G}{v_B}-1\right)\lambda p_0\left(t_1^*-t_2^*\right).$$

Then

$$\lim_{q\to 0}\frac{d^2t_2^*}{d\,(1-\chi)^2} = \frac{d^2\lim_{q\to 0}t_2^*}{d\,(1-\chi)^2} = 0$$

and

$$-\lim_{q\to 0}\frac{d^2\left(t_1^*-t_2^*\right)}{d\,(1-\chi)^2} = -\frac{d^2\lim_{q\to 0}\left(t_1^*-t_2^*\right)}{d\,(1-\chi)^2} = -\frac{d^2\lim_{q\to 0}\left(\ln\frac{v_B}{(1-\chi)v_B}\right)}{d\,(1-\chi)^2} < 0.$$

## Proof of Lemma 7

Consider any $-t$ and suppose the type of the best deal the sponsor has received until that is $\sigma$. The sponsor's proposal strategy $\left(\alpha_\omega(\cdot)\right)_{\omega\in\{G,B\}}$[19] implies a pair of functions $\left(f_\omega(\cdot)\right)_{\omega\in\{G,B\}}$: $f_\omega(-\tau)$ represents the unconditional probability density that the sponsor proposes a deal of type $\omega$ at $-\tau$. Accordingly, the sponsor's expected payoff by adopting this strategy is

$$\tilde{V}(-t,f_G,f_B) \equiv \int_0^t F_G(-\tau)\,f_G(-\tau)\,d\tau\cdot v_G + \int_0^t F_B(-\tau)\,f_B(-\tau)\,d\tau\cdot v_B.$$

Specifically, denote the densities resulting from the sponsor's optimal proposal strategy as $f_G^\sigma$ and $f_B^\sigma$ respectively. Then

$$V^\sigma(-t) = \tilde{V}\left(-t,f_G^\sigma,f_B^\sigma\right).$$

Next, suppose $F_G(-t)\le V^G(-t)$. Then

$$[q+(1-q)\,\eta(-t)]\cdot v_G \le \int_0^t [q+(1-q)\,\eta(-\tau)]\,f_G^G(-\tau)\,d\tau\cdot v_G + \int_0^t (1-q)\,\eta(-\tau)\,f_B^G(-\tau)\,d\tau\cdot v_B.$$

Since $v_G > v_B$ and

$$q+(1-q)\,\eta(-\tau) \ge \eta(-\tau) \ge (1-q)\,\eta(-\tau),$$

it further implies

$$q+(1-q)\,\eta(-t) \le \int_0^t [q+(1-q)\,\eta(-\tau)]\left[f_G^G(-\tau)+f_B^G(-\tau)\right]d\tau.$$

[19] Note that in the new setup, the sponsor's strategy is also based on the type of the best deal he has received until then besides the time $-t$.

Since it is always possible that the sponsor may not have any deal ready for proposal in the future,

$$\int_0^t \left[f_G^G(-\tau)+f_B^G(-\tau)\right]d\tau < 1.$$

Hence,

$$\eta(-t) < \int_0^t \eta(-\tau)\left[f_G^G(-\tau)+f_B^G(-\tau)\right]d\tau.$$

We claim that $F_B(-t) < V^B(-t)$ must hold. Consider the sponsor with $\sigma = B$ at $-t$. Imagine that he mistakenly regards one of his old deals as good, always revisits it, and plays the optimal proposal strategy of the sponsor with $\sigma = G$ at $-t$. Let $f_G$ and $f_B$ represent the the true unconditional probability densities implied by this strategy. The sponsor thinks he will end up with the unconditional probability densities $f_G^\sigma$ and $f_B^\sigma$, but some "good" deals he proposes are actually bad. Therefore, for any $-\tau \in (-t, 0]$,

$$\begin{gathered}f_G(-\tau)+f_B(-\tau) = f_G^G(-\tau)+f_B^G(-\tau), \\ f_G(-\tau) \le f_G^G(-\tau).\end{gathered}$$

Note that the sponsor's optimal strategy should be no worse than this mimicking strategy. So,

$$\begin{aligned}V^B(-t) &\ge \int_0^t [q+(1-q)\eta(-\tau)]\,f_G(-\tau)\,d\tau\cdot v_G + \int_0^t (1-q)\,\eta(-\tau)\,f_B(-\tau)d\tau\cdot v_B \\ &\ge \int_0^t (1-q)\,\eta(-\tau)\,f_G(-\tau)\,d\tau\cdot v_B + \int_0^t (1-q)\,\eta(-\tau)\,f_B(-\tau)\,d\tau\cdot v_B \\ &= (1-q)\cdot v_B\cdot \int_0^t \eta(-\tau)\left[f_G^G(-\tau)+f_B^G(-\tau)\right]d\tau \\ &> (1-q)\,\eta(-t)\cdot v_B = F_B(-t).\end{aligned}$$

Following the proof of Lemma 2, we will encounter contradiction. So, $F_G(-t) > V^G(-t)$. and $\alpha_G(-t) = 1$.

## Proof of Proposition 13

First, $V^B(-t)$ strictly decreases to 0 as $-t$ increases to 0 because

$$\frac{dV^B(-t)}{dt} \ge \lambda p_0\cdot\left[F_G(-t)-V^B(-t)\right] > 0.$$

Second, following the logic similar to Lemma 3, we obtain that when $V^B(-t) < (1-q)\,v_B$ and $t > 0$, $\alpha_B(-t) \in (0,1)$; when $V^B(-t) > (1-q)\,v_B$, $\alpha_B(-t) = 0$. Combining the two, we obtain a unique equilibrium of the game.