The Lead-Lag Relationship between VIX Futures and SPX Futures

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Preliminary draft

Abstract

We study the lead-lag relationship between VIX futures and SPX futures on a sample of transactions time-stamped down to the millisecond and collected over the period from January 2013 to September 2020. To analyze the lead-lag relation, we consider estimators of the cross-correlation function and cross-market activity. The leadership strength is computed on a daily basis using various measures of lead-lag strength. The analysis reveals large time-variation in the lead-lag relation. Under high volatility, the markets exhibit stronger negative correlation and short-lived lead-lag with a tendency for VIX futures to lead SPX futures. We consider a regression model in order to delve further into the time-variation in the lead-lag relation. In particular, we find that the cross-market activity explains a major part of the lead-lag relation and that days of high activity are associated with a strengthened VIX futures lead over SPX futures. We argue in favor of the hypothesis that hedging activities of VIX futures dealers are an important source of cross-market activity and therefore hedging activities could be driving part of the VIX futures lead over SPX futures.

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1 Introduction

The market for VIX futures has witnessed an impressive growth since the introduction of the first VIX futures contract in 2004 on the Chicago Board of Options Exchange (CBOE) and the VIX index itself has become a widely recognized yardstick of stock market risk. Since their launch, VIX futures gained popularity as tools to hedge volatility exposure or diversify portfolios (Whaley, 2009). In 2009, the first VIX exchange-traded fund hit the market and, since then, investors have increasingly used products tied to VIX futures to speculate in future volatility outlook (Bollen et al., 2017; Bhansali and Harris, 2018). Typically, major dealers in financial markets take the other side of the VIX futures trade. Market makers and dealers are subject to strict risk requirements and profit from their flow of transactions and not from risk taking. In order to hedge their positions in volatility, dealers employ various options based hedging strategies (Chang, 2017). These strategies entail that in order to maintain the hedge after either a change in volatility or a change in their net position in volatility, the dealers have to trade the underlying index. For instance, a new position in a VIX futures can be hedged with a delta hedged position in a European option on the SPX index. Hence, trading in VIX futures leads to subsequent trading in the underlying index, which in the context of VIX futures, is typically carried out via SPX futures. Moreover, dealers tend to be in negative gamma positions for two reasons: First, as providers of liquidity to SPX options demand they tend to be short in SPX options (Baltussen et al., 2021). Second, dealers tend to be long VIX futures (Mixon and Onur, 2019; Todorov, 2019), which are typically hedged using short positions in SPX options. Hence, market makers run the risk of being part of the feedback cycle illustrated in Figure 1: An increase in volatility due to increased demand in long VIX futures leads to a decrease in SPX futures prices via the leverage effect. Being in a negative gamma position, the dealers sell SPX futures in order to rebalance hedges. This puts additional price pressure on SPX futures, which again translates into increasing VIX futures. Alternatively, dealers could turn to a more approximate hedge utilizing the negative correlation between VIX futures and SPX futures prices. Under this approach, a long position in VIX futures is simply hedged by a

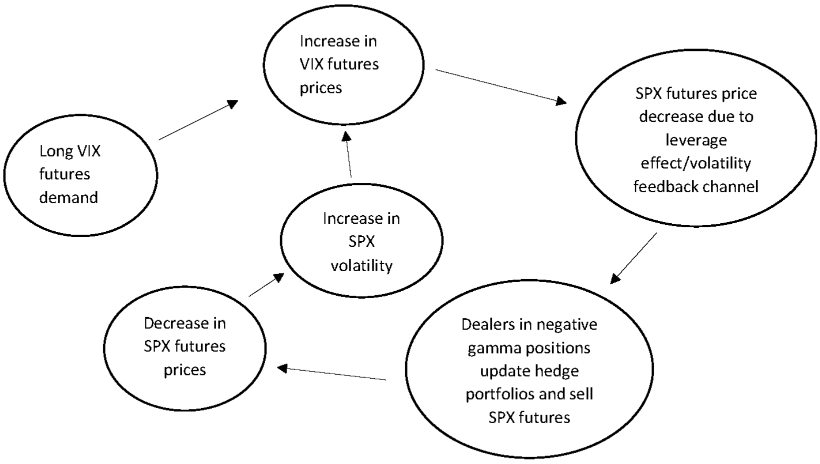

Figure 1: Feedback effect from initial long demand in VIX futures.

short position in SPX futures. In either case, the mechanisms of the hedging activities have the potential to impact the lead-lag relation between VIX futures and SPX futures.

In this paper we study the lead-lag relation between VIX futures and SPX futures on a sample of transactions time-stamped down to the millisecond and collected over the period from January 2013 to September 2020. We find that the lead-lag relation is dynamic with the VIX futures leading SPX futures on average terms. We study the determinants of the lead-lag relation and find that the level of cross-market trading has a positive and significant impact on the strength of the VIX futures lead over SPX futures. Since cross-market trading can arise from hedging by VIX futures dealers, this indicates that VIX futures hedging influences the lead-lag relationship.

VIX exchange-traded funds and notes (VIX ETP) drive a large part of the VIX futures demand and their hedging strategy is subject to a positive feedback loop where increasing VIX futures prices lead to even further hedging demand by issuers of VIX ETPs (Alexander and Korovilas, 2013; Brøgger, 2021; Todorov, 2019). The market movements on February 5 2018, where VIX ETPs played an important role for the extreme behavior of VIX futures prices, the VIX index and the SPX index, serve as anecdotal evidence in support of this belief (Bank for International Settlements, 2018; Augustin et al., 2021). If the net demand in the market for VIX futures and the resulting SPX hedging demand leads to price pressures, the hedging strategy would imply that, all else equal, the VIX futures price leads the SPX futures price relatively more in comparison to days where hedging does not affect prices.

The literature on lead-lag relations is mostly concerned with which markets first reflect new information (where informed traders prefer to trade), and hedging can be a channel through which information is transmitted to the market for the asset that is used as hedging instrument (see e.g. Kaul et al. (2004); Hu (2014)). In the context of hedging VIX futures, this can occur when the VIX futures trading is informed. However, the lead-lag relation is not necessarily the result of informed trading. Rather it might illustrate the presence of spillovers from VIX futures market makers' hedging to the stock market. Other papers have recognized that hedging can affect lead-lag relations if market makers in one market trade in the other market to hedge, such as option market makers performing delta-hedging (Easley et al., 1998; Chan et al., 2002; Schlag and Stoll, 2005). Similarly, Kao et al. (2018) find that VIX option trading has a temporary impact on changes in the VIX index and attribute this to the use of VIX options for hedging by SPX option market makers.

Lead-lag relations generally reflect some degree of inefficiency of financial markets either because all available information is not properly reflected in market prices across assets at any given point in time, or if the market is not liquid enough to absorb trading due to (uninformed) hedging activities. In either case, past price changes can be used to predict future price changes. Other studies show that even for markets that are considered large and actively traded, one market can lead the price changes of another market (Chen et al., 2016; Dao et al., 2018).

The paper is also related to the literature on the relation between lagged stock market returns and volatility: Carr and Wu (2006) study the cross-correlation function for SPX index returns and VIX index changes at a daily frequency and find marginal evidence of SPX returns having some predictive power for VIX index changes. Similarly, Bollerslev et al. (2006) find significant negative correlation between the absolute value of returns (volatility proxy) and lagged returns both sampled at a five-minute frequency. At the same time, correlations between returns and lagged absolute returns are essentially zero. Frijns et al. (2016) examine the lead-lag relation of VIX futures and the SPX index and find evidence that VIX futures lead SPX with lagged VIX futures price changes having some predictive power over SPX returns computed over intervals of 15 seconds. In a regression ananlysis performed on daily data, Lee et al. (2017) find some effect of the difference between VIX futures and the VIX index (the VIX futures basis) on future returns on SPX futures.

The findings of Frijns et al. (2016) reveal that VIX futures lead the SPX index. While it has also been shown that SPX futures lead the SPX index (Chu et al., 1999; Hasbrouck, 2003), these findings leave it unclear to which extend VIX futures lead or lag SPX futures. There is also evidence that VIX futures lead the VIX index (Shu and Zhang, 2012; Frijns et al., 2016; Bollen et al., 2017; Chen and Tsai, 2017; Kao et al., 2018) and, by the construction of the VIX index, then implicitly lead SPX options. Moreover, it has been shown by Chen et al. (2016) that SPX futures provide greater contribution to price discovery than SPX options. Thus, both VIX futures and SPX futures lead SPX options, but again the VIX futures and SPX futures lead-lag relation cannot be inferred from these studies. While Lee et al. (2017) show that the VIX futures basis has some predictable power for the SPX futures returns, though only at the daily level, we analyze the lead-lag relation of VIX futures and SPX futures from high-frequency data and furthermore examine its time-variation.

The remainder of the paper is structured as follows: Section 2 presents the data used for the analysis. Next, we introduce the methodology to quantify the lead-lag relation in Section 3. Section 4 presents the results of the empirical analysis. Finally, Section 5 concludes.

2 Data

For the analysis we collect data over the period January 2013 to September 2020. Tick-by-tick trade and quote data on E-mini S&P 500 futures (ES) and VIX futures (VX) are obtained from the Tick Data database. Daily data on the Open Interest on VIX futures is retrieved from the CBOE homepage together with daily closing prices on the VIX index and the SPX index. For each sample date, the VIX futures contract used for the analysis is the one closest to an expiry of 30 days and the SPX futures contract used is the one closest to expiry except when time to expiry is less than six days where we shift to the next contract. We focus only on

Table 1: Descriptive statistic of VIX futures and SPX futures markets

		VIX futures	SPX futures
Trading volume	Mean	78,224	1,292,320
	Median	68,311	1,199,596
	Min	18,432	286,808
	Max	386,637	3,983,301
	Std. dev.	43,221	465,899
Dollar volume (in mm)	Mean	1,355	149,368
	Median	1,113	136,824
	Min	280	29,911
	Max	10,365	567,136
	Std. dev.	941	60,032
Open interest	Mean	178,900	
	Median	168,930	
	Min	65,527	
	Max	383,927	
	Std. dev.	57,069	

Statistics are computed from daily observations over the sample period. For each sample date, the VIX futures contract used is the one with an expiry closest to 30 days. The SPX futures contract is the one closest to expiration except when this is less than six days. Trading and dollar volume are obtained over the time interval 9:30-16:15 EST.

trades during the regular trading hours of VIX futures, 9:30-16:15 EST. Any date where the exchanges closed earlier is removed from the sample. Trades with a negative price is removed while quotes are removed if bid price, ask price or bid-ask spread is negative. For the purpose of computing the lead-lag measures of Section 3, trades sharing the same time-stamp are replaced by a single trade with a price equal to the median price of the trades.

Table 1 shows trading and dollar volume for VIX futures and SPX futures as well as the open interest of VIX futures over the sample period. Clearly, SPX futures are more heavily traded than the VIX futures both when measured in terms of trading and dollar volume.

3 Lead-lag methodology

Many studies on lead-lag relations are concerned with assets that are closely linked together such that a cointegrating relation of the prices can be assumed. In those settings, the information share (Hasbrouck, 1995) or the common factor component weight approach (Gonzalo and Granger, 1995) are often applied (see e.g. Chen and Tsai (2017); Chen et al. (2016)). However, it would be inappropriate to impose the assumption of cointegration when describing the relation between VIX futures and SPX futures, so instead the lead-lag relationship is analyzed through the cross-correlation function and the cross-market activity (CMA) measure. In section 3.1 we describe how to obtain the cross-correlation function using the techniques of Hayashi and Yoshida (2005); Hoffmann et al. (2013) and section 3.2 presents three different quantifications of the lead-lag relation based on the cross-correlation function. Finally, Section 3.3 presents the methodology behind the CMA measure of Dobrev and Schaumburg (2017).

3.1 Estimation of the cross-correlation function

Based on Hayashi and Yoshida (2005) (HY) and Hoffmann et al. (2013), the cross-correlation for two assets, A and B, is estimated as

$$\hat{\rho}_{HY}(\vartheta) = \frac{\sum_{i=1}^{n_A} \sum_{j=1}^{n_B} \Delta_{t_i^A} X^A \Delta_{t_j^B} X^B 1_{\left\{(t_{i-1}^A, t_i^A] \cap (t_{j-1}^B - \vartheta, t_j^B - \vartheta] \neq \emptyset\right\}}}{\sqrt{\sum_{i=1}^{n_A} \left(\Delta_{t_i^A} X^A\right)^2} \sqrt{\sum_{j=1}^{n_B} \left(\Delta_{t_j^B} X^B\right)^2}} \tag{1}$$

which we shall refer to as the HY estimator. Here $\Delta_{t_i^k} X^k = X_{t_i^k}^k - X_{t_{i-1}^k}^k$ is the log-return of asset k, $i = 1, \ldots, n_k$ meaning that the returns of (1) are computed between each single tick, t_i^k, and the product of two returns is included in the sum whenever the time intervals over which the returns are realized is overlapping. Inspired by Dao et al. (2018), we use Figure 2 to illustrate this. By focusing on the first two line segments, we ignore the possibility of shifting the time-stamps so $\vartheta = 0$. As an example, considering the three intervals of asset B, J_1, J_2 and J_3. J_1 intersects with I_2, J_2 intersects with I_2, while J_3 intersects with I_2, I_3 and I_4. Thus, the contribution to the sum based on each of the three intervals is $\Delta_{t_1^B} X^B \Delta_{t_2^A} X^A$, $\Delta_{t_2^B} X^B \Delta_{t_2^A} X^A$ and $\Delta_{t_3^B} X^B (\Delta_{t_2^A} X^A + \Delta_{t_3^A} X^A + \Delta_{t_4^A} X^A)$, respectively. Hence, it is possible that the same return can contribute to the sum more than once as it will enter every time the interval intersects with one of the intervals of the other asset. Note that when implementing (1), the returns over I_1 will not influence the correlation as the indicator function equals zero for intervals that do not intersect with any of the intervals of the other asset.

Repeatedly adjusting the time-stamps of the second asset by an amount ϑ on some grid,

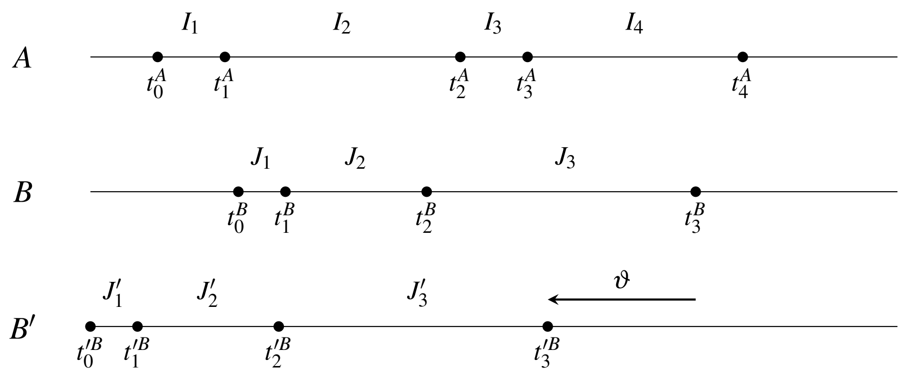

Figure 2: Illustration of the time-stamp adjustment for the cross-correlation functions.

allows us to compute the cross-correlation function. The shift of the time-stamps is illustrated in Figure 2. Note that only the time-stamps of one of the two assets is shifted while the other remain fixed. The returns $\Delta_{t_i^k} X^k$ for $k = A, B$ are invariant to the shift of the time-stamps meaning that only the indicator function changes as ϑ changes. Thus, it is the same returns that enter (1) for each ϑ but they are multiplied and summed in different ways.

3.2 Lead-lag time, lead-lag correlation and lead-lag ratio

To measure the lead-lag relation between two assets, Hoffmann et al. (2013) define the lead-lag time (LLT) as the value of ϑ that maximizes the absolute value of the cross-correlation function, $|\hat{\rho}_{HY}(\vartheta)|$, across all ϑ on some grid. If the absolute correlation is maximized at a point $\vartheta \neq 0$ then one asset is leading the other. Under certain assumptions, the point is a consistent estimator of the true LLT (Hoffmann et al., 2013).

While LLT measures the amount of time by which one asset leads the other, knowledge of the value of the cross-correlation function at the point corresponding to LLT is also informative about the nature of the lead-lag relation. The value of the cross-correlation at this point is referred to as the lead-lag correlation (LLC) (Dao et al., 2018).

Both LLT and LLC focus on a single point of the cross-correlation function. However, the rest of the cross-correlation function also contains relevant information about the strength of the lead-lag relation. The lead-lag ratio (LLR) of Huth and Abergel (2014) accounts exactly for

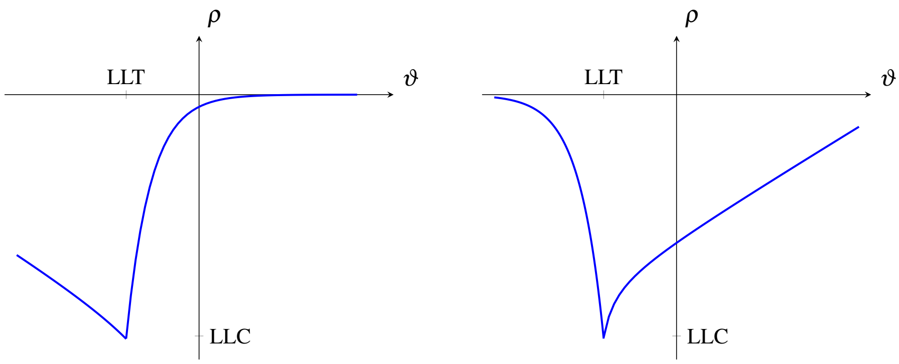

Figure 3: Illustration of cross-correlation functions.

this by compressing the entire cross-correlation function into a single measure of the lead-lag relation. Considering all the positive time-stamp adjustments $(\vartheta_1, \ldots, \vartheta_p)$, LLR is defined as

$$LLR = \frac{\sum_{i=1}^{p} \hat{\rho}_{HY}^{2}(\vartheta_i)}{\sum_{j=1}^{p} \hat{\rho}_{HY}^{2}(-\vartheta_j)} \tag{2}$$

The ratio captures the relative forecasting ability of one asset over the other. When LLR > 1 it means that the correlations at positive lags are overall larger than the correlations at negative lags. Thus, the asset for which the time-stamps are kept fixed will lead the asset for which the time-stamps are adjusted (asset A will lead asset B in Figure 2). The conclusion of the leadership is the opposite if LLR < 1 where the asset with fixed time-stamps lags the other (asset B leads asset A). Compared with LLT, LLR takes account of the overall predictive power of the returns of one asset on the returns of the other asset by summarizing the squared correlations over a specified interval. LLT is more sensitive as it reflects only one point of the cross-correlation function. For instance, consider the two cases illustrated in Figure 3 where LLT and LLC are the same but in the plot to the left, the cross-correlation function goes to zero very quickly while in the plot to the right, the cross-correlation slowly decays to zero for values of ϑ higher than LLT. Hence, the lead-lag relation to the left is much stronger than the one to the right. However, LLT and LLC will be the same while only LLR will capture this difference in the strength of the lead-lag relation. LLR could in fact lead to a different conclusion on the lead-lag relation than LLT.

A simulation study by Huth and Abergel (2014) shows that in the absence of any lead-lag relation, the LLR obtained from the HY estimator is robust to differences in the relative trading activity of the two assets. Large differences in the number of trades therefore does not falsely introduce asymmetries in the cross-correlation function.

3.3 Measuring cross-market activity

While the lead-lag measures of the above section are based on prices, we here present another measure of the lead-lag relation based on Dobrev and Schaumburg (2017) which is model-free and does not utilize prices. Instead the time-stamps of a well-defined activity, such as trading, can be used to uncover the lead-lag relation. The idea is to identify all so-called active time-stamps. Active time-stamps means that the activity, e.g. trading, takes place in both markets at that time-stamp. The total number of time-stamps with simultaneous trading is then summed over the trading day and shows how often both markets are active at the same time. Assuming that $\vartheta = 0$, this number can be obtained as

$$X_{\vartheta}^{raw} = \sum_{i=|\vartheta|}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i+\vartheta\}} \tag{3}$$

where N is the total number of time-stamps. With data at millisecond frequency, this is the total number of milliseconds over the trading day. The number of cross-active time-stamps can be scaled by the total number of active time-stamps in the least traded market. This measures cross-market activity as a proportion of the total activity.

$$X_{\vartheta}^{rel} = \frac{X_{\vartheta}^{raw}}{\min\left\{\sum_{i=|\vartheta|}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\}}, \sum_{i=|\vartheta|}^{N-|\vartheta|} 1_{\{\text{market } B \text{ active in period } i+\vartheta\}}\right\}} \tag{4}$$

In order to capture trading activity related only to cross-market trading, a further adjustment is implemented to account for the simultaneous trading which would occur simply by randomness. This gives cross-market activity in excess of what we would expect by coincidence given

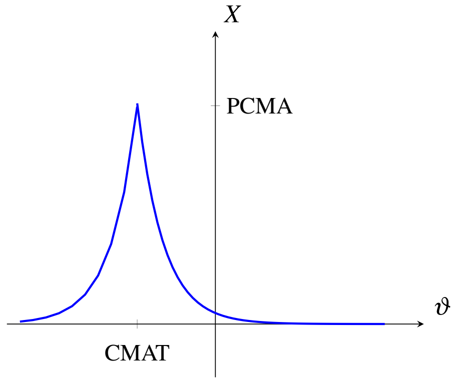

Figure 4: Illustration of cross-market activity function.

that trading in the two markets occurs independently of each other.

$$X_{\vartheta} = X_{\vartheta}^{rel} - X_{\infty}^{rel} \tag{5}$$

where the adjustment term is defined as $X_{\infty}^{rel} = 1/\left(2\left(T_2 - T_1\right)\right)\sum_{|\vartheta|=T_1+1}^{T_2} X_{\vartheta}^{rel}$ for sufficiently large $T_2 > T_1$. In addition to simultaneous trading where $\vartheta = 0$, time-stamps are shifted forward or backward in time when considering non-zero values of ϑ. For a set of different values of ϑ, a full curve for the proportion of cross-market activity can be obtained. We illustrate this in Figure 4 and denote the cross-market activity time (CMAT) as the value of the time-stamp shift corresponding to the maximum of the curve, while the peak cross-market activity (PCMA) is the value of the cross-market activity, X_{ϑ}, at the point. The value of CMAT tells us by how many milliseconds trades in one market lead trades in the other market.

4 Empirical analysis

In this section, we detail the empirical analysis of the paper. The first subsection presents the overall lead-lag relationship between VIX and SPX futures. Next, Section 4.2 presents the results of the cross-market activity analysis. In Section 4.3 we introduce the regression model for the dynamics of the lead-lag relation and present the results of the regression.

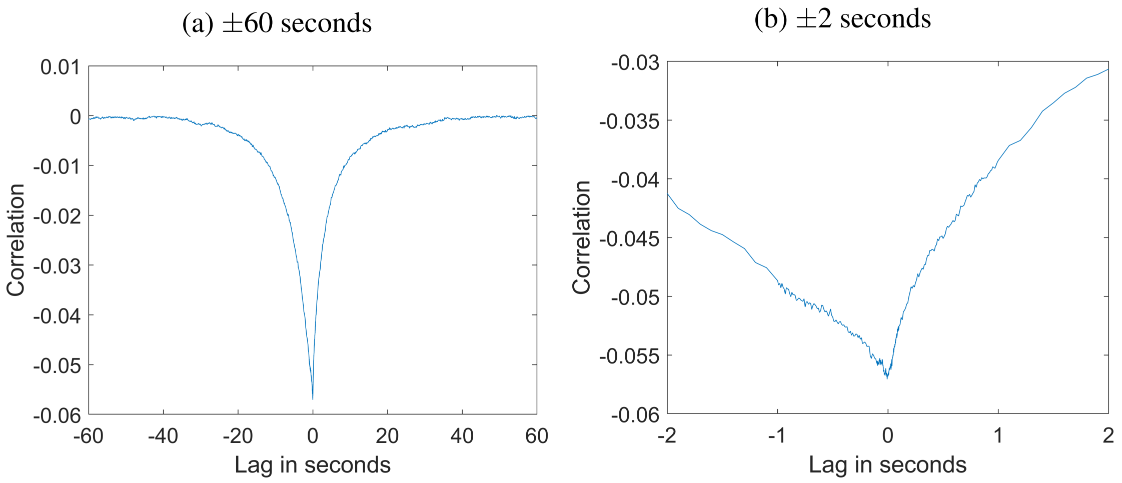

Figure 5: Median cross-correlation functions.

4.1 The lead-lag relationship based on the cross-correlation function

In this section, we present the overall results based on the computed measures of the lead-lag relation detailed in Section 3. For the computation of cross-correlations, we keep the time-stamp of the SPX futures trades fixed and shift the time-stamps of the VIX futures trades. To estimate the cross-correlation function, the grid of ϑ is chosen such that it is finer around zero and less dense as we move away from zero. This is since we expect that the lead-lag time to be small so we want to be able to capture variations in the correlation at a higher detail around zero. Hence, the grid is chosen as

$$-60,-59.9,\ldots,-1.1,-1,-0.99,\ldots,-0.11,-0.1,-0.099,\ldots,-0.001,0,0.001,\ldots,0.099,0.1,0.11,\ldots,0.99,1,1.1,\ldots,59.9,60$$

where the numbers are in seconds and where the largest value (±60 seconds) reflect the maximum allowed lead-lag.

Figure 5 depicts the median of the cross-correlation. The peak of the function is around a lag of zero and with the cross-correlation function close to zero for lags greater than approximately 20 seconds in absolute value. Zooming in on lags within 2 seconds, we observe a skewed shape of the cross-correlation function with more weight on the left part of the curve. On average, this translates into a LLR measure less than 1. Hence, on average VIX futures lead the SPX futures.

Figure 6 depicts the time series of the three measures of lead-lag strength together with

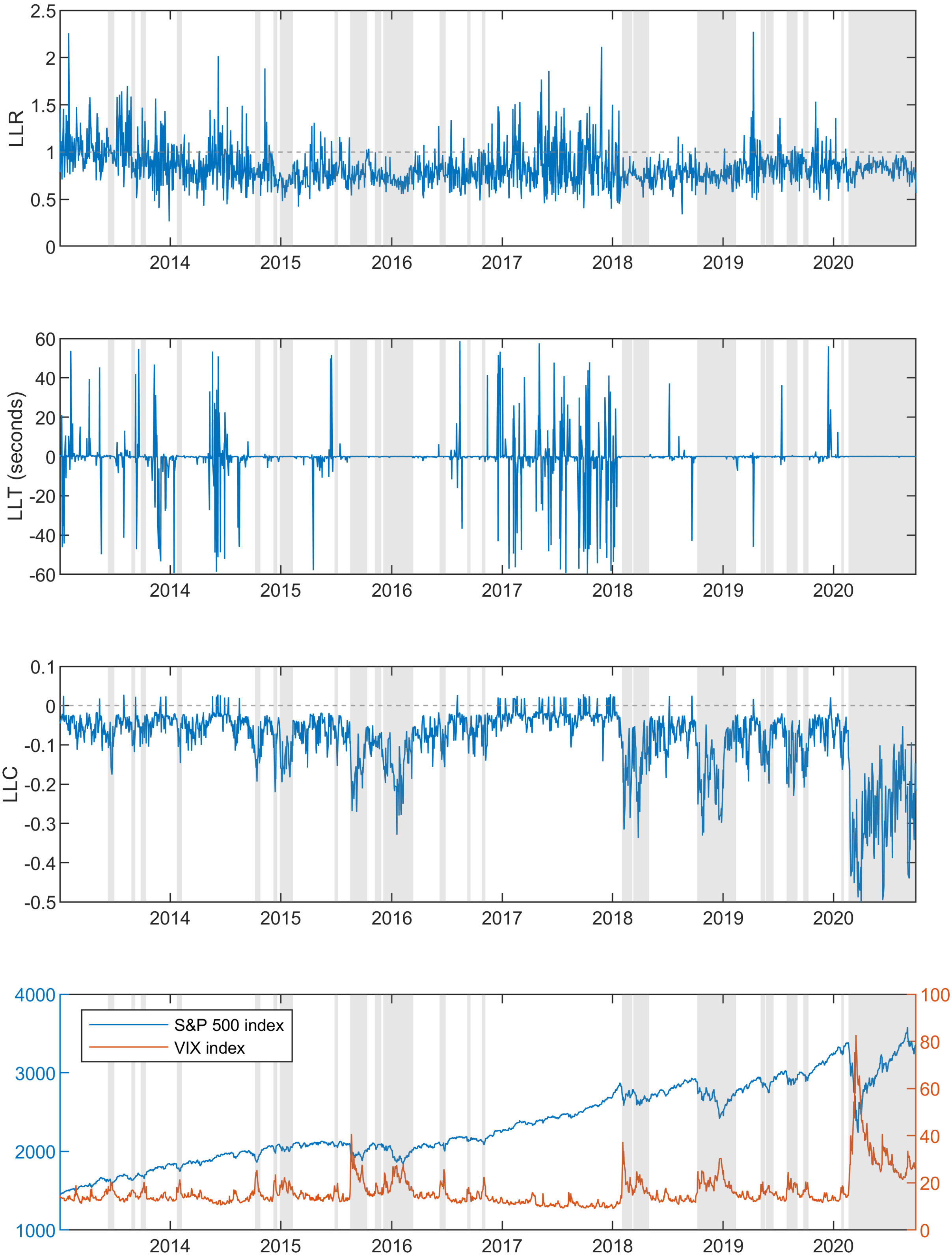

Figure 6: LLR, LLT, LLC and VIX and SPX index over the sample period. The shaded areas represent the dates with a level of the VIX index belonging to the 60% upper quantile.

time series of the VIX index and the SPX index. The shaded areas of the chart represent the dates with a volatility level belonging to the 60% upper quantile. Inspection of the upper panel of Figure 6 confirms what Figure 5 indicates, namely that on most dates the VIX futures lead the SPX futures (LLR less than one). We also notice some interesting features of the lead-lag measures on dates of high volatility: First, LLR seems to be more stable and around a level of approximately 0.8. Second, the lead-lag time is erratic when volatility is low while close to zero in high volatility regimes. Third, the lead-lag correlation gets more pronounced when volatility is high.

Table 2 reports descriptive statistics on the three lead-lag measures. Based on the observations connected to Figure 6, we also compute the statistics conditioned on the volatility being in the upper quantile. Additionally, we split the sample into the period before and after the beginning of the covid-19 crisis after February 20, 2020. In terms of the mean and median values, the LLR measure is relative stable across all four samples. However, the variability in the LLR measure is much lower conditioned on the volatility being high. Considering LLT, the picture is more extreme. On the full sample, LLT varies in the interval $[-60 \text{ seconds}, 60 \text{ seconds}]$ but with an average/median value of -0.63/-0.01 seconds. In comparison and on the high volatility sample, LLT is much more concentrated around 0 with a minimum value of 2.90 seconds and a maximum value equal to 0.66 seconds. Focusing on the two sub-periods defined by the onset of the covid-19 crisis, it seems that the covid-19 period is associated with a much tighter lead-lag time and with much lower variability in all the lead-lag measures. However, we note that with respect to the LLT measure, the covid-19 period differs from the sample conditioned on a high volatility level, where the measure based on the former sample is on average slightly positive while slightly negative in the latter.

We illustrate in Figure 7 kernel densities of the three measures of lead-lag strength computed for the sample conditioned on the VIX index being above and below its upper quantile. The figure confirms the findings reported in Table 2. The densities associated with LLT and LLR measures are more peaked with the mass more concentrated around the mean of the distribution. Focusing on the third chart, we see that LLC gets more pronounced in high volatility

Table 2: Descriptive statistic of lead-lag measures.

	Full sample			VIX upper quantile			Before covid-19			Covid-19		
	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC
Mean	0.84	-0.63	-0.09	0.78	-0.01	-0.19	0.84	-0.68	-0.07	0.82	0.01	-0.28
Median	0.80	-0.01	-0.06	0.77	0.00	-0.17	0.80	-0.02	-0.06	0.83	0.02	-0.27
Min	0.27	-59.70	-0.50	0.51	-2.90	-0.50	0.27	-59.70	-0.34	0.57	-0.59	-0.50
Max	2.27	58.70	0.03	1.16	0.66	-0.02	2.27	58.70	0.03	0.97	0.04	-0.05
Std. dev.	0.20	10.55	0.08	0.10	0.16	0.09	0.21	11.00	0.06	0.08	0.05	0.10

The covid-19 period covers all sample dates after February 20, 2020.

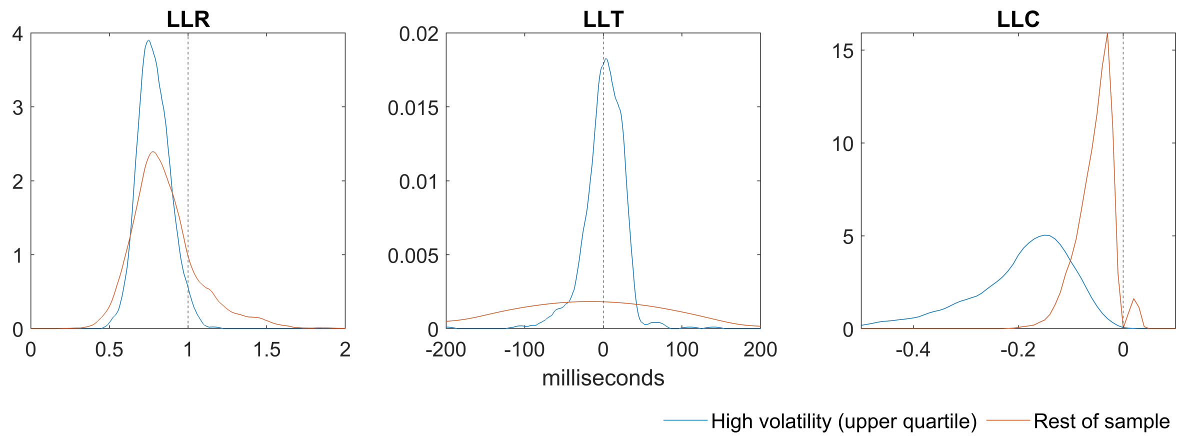

Figure 7: Kernel densities of lead-lag measures.

regimes in comparison to regimes with low VIX index values. Hence, the lead-lag relation is strengthened (measured by LLC and LLR) but is short-lived during high volatility (measured by LLT). A similar result is found in Buccheri et al. (2021), where the lead-lag correlation is found to strengthen among stocks when volatilities are high, while being more erratic in low volatility regimes. Other studies find that correlations at a daily frequency tend to increase between the VIX index and the SPX index when market movements are big (Cont and Kokholm, 2013; Todorov and Tauchen, 2011). In the context of high-frequency observations, Buccheri et al. (2021); Zhang (2010); Dobrev and Schaumburg (2017) argue that the relation between high-volatility and lead-lag relationships can be ascribed to high-frequency traders exploiting statistical dependencies across markets appearing when the volatility is high. For the VIX futures and SPX futures, this means that the stronger negative correlation in periods of high volatility is possibly exploited by high-frequency traders reducing LLT to almost zero.

4.2 Cross-market trading analysis

In this section, we quantify the cross-market activity presented in Section 3.3. For each sample date, Figure 8 plots the millisecond at which the cross-market activity peaks (CMAT) and the value of cross-market activity at the peak (PCMA) together with the VIX and SPX index. From the third panel showing PCMA, we see a clear connection with the level of the VIX index as cross-market activity increases during periods of high VIX indicated by the shaded areas. PCMA reaches its highest level at the beginning of the covid-19 pandemic. To the extend that cross-market activity is driven by high-frequency trading, the relation between cross-market activity and VIX is consistent with a feedback effect between volatility and high-frequency trading as described by Dobrev and Schaumburg (2017). Heightened levels of volatility attracts more high-frequency trading and the increased presence of high-frequency traders generates even higher levels of volatility. Comparing this with LLC in Figure 6, we see that they appear be to negatively related, and a computation of the correlation between the two time series reveals a value of -0.84. This supports that high levels of high-frequency trading, strong negative correlation and high volatility occurs simultaneously.

As shown in the first panel of Figure 8, CMAT fluctuates at a level of approximately 560 milliseconds during the first part of the sample and hereafter exhibit a clear shift to values around zero. Possibly the break can be attributed to some technological change affecting latencies or as a result of how trades are registered.[1] The second panel zooms in on CMAT to examine its fluctuations after the break. Except for a few dates, the series appear to be bounded within the range of [-20,+30] milliseconds as indicated by the dotted lines. During sub-periods, CMAT seems to be further bounded within even narrower ranges. For instance, the period from mid-2019 to the end of the sample roughly contains no values outside [-5,+30] milliseconds. There is also a tendency for the values to cluster around certain values such as -20, +20, +30 and values slight below and above zero as indicated by Figures 8 and 9. Possibly this reflects the true lead-lag time or it may be the result of fluctuations in latencies over time. As indicated by the shaded areas of Figure 8, CMAT does not show any clear pattern under periods of high

[1] Due to this observation, we exclude sample dates before August 26, 2013 in the remainder of the analysis.

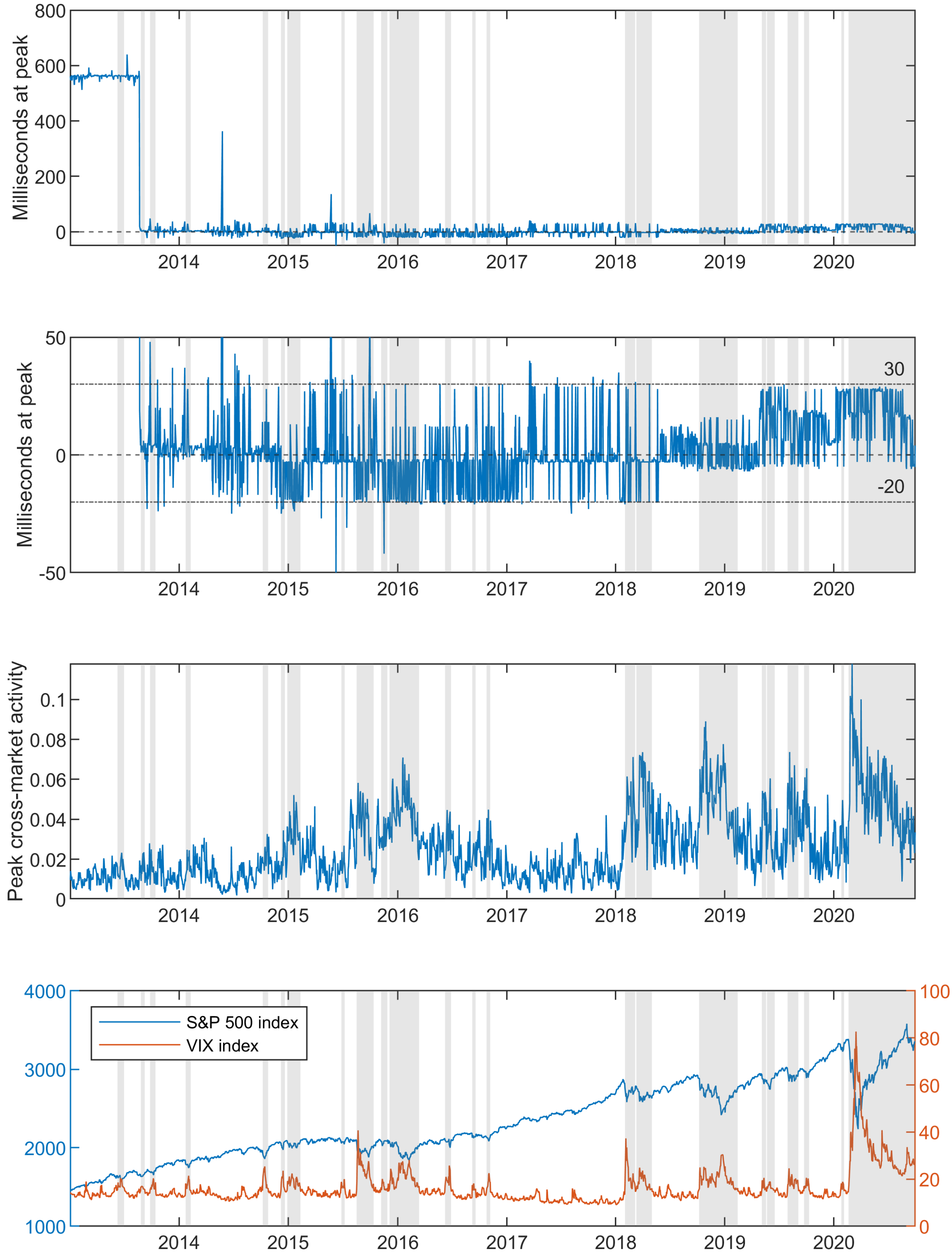

Figure 8: CMAT (milliseconds at peak), PCMA (peak cross-market activity) and VIX and SPX index over the sample period. The shaded areas represent the dates with a level of the VIX index belonging to the 60% upper quantile.

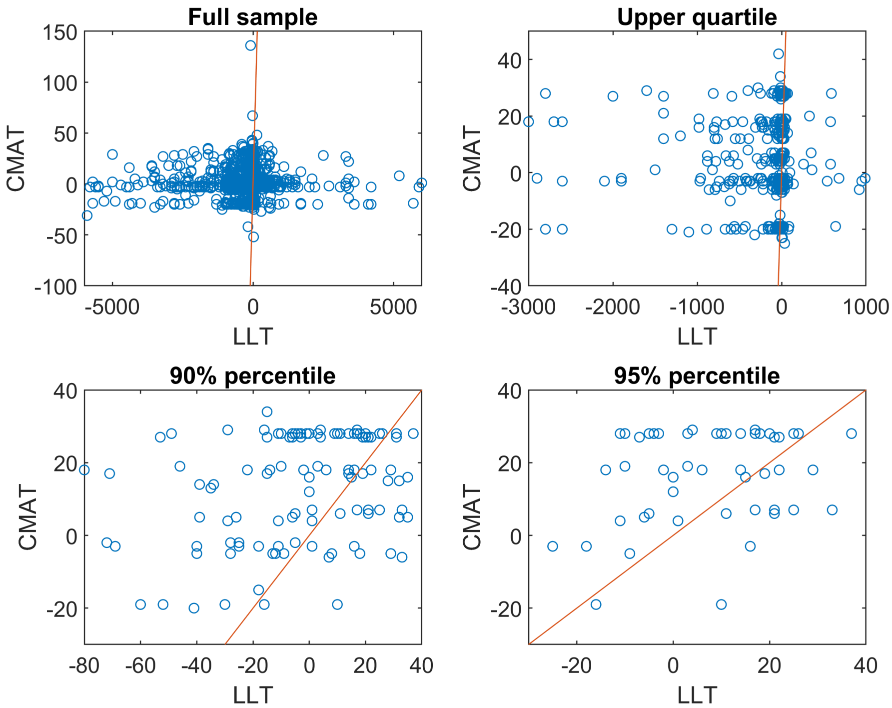

Figure 9: Comparison of LLT and CMAT. The subsamples are based on the level of the VIX index. The red line is a 45-degree line meaning that when points are above (below) the line, LLT indicates a greater (smaller) lead of the VIX futures compared to CMAT. The values of both LLT and CMAT are shown in milliseconds.

volatility. This contrasts with the LLT measure which exhibits a clear dependence on the VIX index as illustrated in Figure 6. The difference is also highlighted in Figure 9 where the range of LLT is significantly narrowed when conditioning on high VIX while the same does not occur for CMAT which continues to span the same range of values. The overweight of points above the 45-degree line indicates that on a given sample date CMAT is generally higher than LLT.

4.3 The dynamics of the lead-lag relationship

In order to understand the drivers of the lead-lag relationship, we run a regression where we choose each of the three measures of the lead-lag relation, LLR, LLT, and LLC as the dependent variable. Section 4.3.1 introduces the model and present the argumentation for inclusion of the independent variables. Next, Section 4.3.2 presents the results of the regressions.

4.3.1 Regression model

Letting LLM_t denote the chosen measure of the lead-lag relation, the regression model is

$$LLM_t = \beta_0 + \beta_1 PCMA_t^+ + \beta_2 PCMA_t^- + \beta_3 VIX_t + \beta_4 VVIX_t + \beta_5 SPX_t^+ + \beta_6 SPX_t^- + \underline{\gamma} \cdot \underline{CTR}_t + \varepsilon_t, \tag{6}$$

The $\underline{\gamma}$ coefficient is 3-dimensional reflecting the fact that we include three control variables. Below we present in detail the variables considered in the regression (6):

- Peak cross-market trading activity, $PCMA_t$: The trading activity in the two markets obviously could have some impact on the lead-lag relationship. In particular, trading activity which emerge from trading strategies involving both markets should matter. If price movements of VIX futures and SPX futures are sufficiently negatively correlated, high-frequency traders may employ trading strategies akin to statistical arbitrage. Trading in the two markets may also be linked if market makers hedge their VIX futures exposure using SPX futures. That is, after having provided liquidity in the VIX futures market, market makers trade in the SPX futures markets. To proxy the part of the trading activity which is related to these type of activities, we use the cross-market activity measure introduced by Dobrev and Schaumburg (2017) and detailed in Section 3.3. For each sample date, our measure of cross-market activity is the peak of the cross-market activity curve and we denote the location of the peak at time-t the cross-market activity time $CMAT_t$. When $CMAT_t < 0$ the maximum cross-market activity is associated with trades in VIX futures followed by trades in SPX futures and vice versa when $CMAT_t > 0$. The impact of a high level of cross-market activity on the lead-lag relationship can potentially depend on the sign of $CMAT_t$. Hence, to take into account this possible asymmetry, we include two variables associated with cross-market activity in the regression (6), namely, $PCMA_t^+ = PCMA_t 1_{CMAT_t > 0}$ and $PCMA_t^- = PCMA_t 1_{CMAT_t \leq 0}$.

  On days where VIX futures hedging is the main source for cross-market activity, we expect that $CMAT_t < 0$ and that higher cross-market activity strengthens the lead of the

VIX futures, i.e. it has a negative impact on LLR. We also expect that the hedging activities would introduce additional negative correlation between returns as buying (selling) in the VIX futures market is accompanied by selling (buying) in the SPX futures market. Hence, we should expect to see stronger negative correlation on days of sizable VIX futures hedging activities. When $CMAT_t > 0$ the cross-market trading can to a lesser extent be attributed to VIX futures hedging activities of dealers. Instead a high level of cross-market trading can be an indicator of significant amount of high-frequency traders present in the two markets.

- The VIX index, VIX_t: There are at least two reasons why the VIX index should be included as regressor. First, there is mixed evidence on whether informed trading occurs at the index level. Pan and Poteshman (2006) do not find evidence that index option trading is informative about future changes in the index while Li et al. (2017) find that informed SPX option trading take place during the financial crisis. Informed trading can arise due to better information processing skills or different views about the same publicly available information which may be more common during volatile periods (Ciner and Karagozoglu, 2008). If informed trading is present at the index level and increases with the amount of volatility then informed traders preferring to trade VIX (SPX) futures means that higher VIX will negatively (positively) impact LLR and LLT. The preferences of informed traders to trade in the two markets may be influenced by the type of the information, i.e. whether it is directional or volatility information.[2] Second, a short SPX futures position provide a hedge against stock market crashes and with a negative correlation between the SPX and VIX index, so does a long VIX futures position (Moran and Dash, 2007; Szado, 2009; Hilal et al., 2011). Thus, high volatility or uncertainty could create SPX futures selling pressure and VIX futures buying pressure. Whether investors prefer to hedge with one of the futures contracts under high volatility will be revealed by the sign of the coefficient on VIX. With investors' increasing demand for protection at

[2] Volatility information can be exploited using both SPX options (with delta-hedging) and VIX futures. However, studies showing that VIX futures now lead the VIX index (Frijns et al., 2016; Bollen et al., 2017) indicate that informed trading is more common in the VIX futures market than the SPX option market.

times of high VIX, we then expect that LLC decreases in response to an increase in VIX. Finally, inspection of Figure 6 indicates a clear pattern related to the volatility: Under high volatility, LLR is slightly below one while more erratic in periods of low volatility. The LLT measure is generally close to zero under high volatility but otherwise extremely erratic. Moreover, LLC tend to be stronger and more negative in high volatility regimes. Similarly, Chen et al. (2016) show a dependence on volatility as the relative informativeness of SPX futures and SPY is reversed under high volatility. To account for all this, we also include the level of the VIX index in the set of regressors.

- The volatility of the VIX futures, $VVIX_t$: The VVIX index provides a market based measure of the expected volatility over the next calendar month of a VIX futures contract with one month to maturity (CBOE, 2012). Hence, along the lines of the previous bullet point, we also find it relevant to include the VVIX in the analysis.

- SPX return, SPX_t: Ren et al. (2019) suggest that the lead-lag relation between index options and the index is reversed when the index is not stable or up-trending. Lee et al. (2017) show how the predictability of the VIX futures basis on SPX futures returns changes across the SPX return distribution. These results indicate the SPX return could influence the lead-lag relation. We want to allow for the sensitivity of the lead-lag relation to the SPX return to depend on whether returns are positive or negative. Hence, we include two variables in the regression: One collecting all the positive movements in the SPX index, SPX_t^+, and a second consisting of all the negative movements, SPX_t^-.

- Control variables: We control for the time to expiration of the VIX futures and SPX futures contracts used to compute the lead-lag measure. Moreover, Frino et al. (2000) show that the leadership of the futures on a stock index relative to the stock index is strengthened around the time of the macroeconomic announcements and Chen and Tsai (2017) find that VIX futures lead the VIX index more on the days of the release. Based on the possible variations in the lead-lag relationship around these announcements, we also include a dummy variable equal to one on the days of U.S. macroeconomic news release. The announcement dates are the dates of release of information on Consumer Price Index, Producer Price Index, Employment Situation or Gross Domestic Product and are obtained from Archival Federal Reserve Economic Data (ALFRED).

4.3.2 Regression results

In this section we run the regression specified in equation (6) and Table 3 reports the results of the regressions with each of the lead-lag measures as the dependent variable. In the discussion we mainly focus on the LLR measure of the lead-lag relation and the regression containing the full set of independent variables (column 5).

All the coefficients on the cross-market activity variables are significant and suggests that cross-trading in the two markets strengthen the lead of the VIX futures. When $CMAT_t < 0$ the peak of the cross-market activity reflects that VIX futures trading is followed by SPX futures trading. We take this as an indication that VIX futures hedging activities are driving the strengthening of the VIX futures lead on these days. The coefficients on the level of cross-market trading conditioned on $CMAT_t > 0$ is less intuitive. A $CMAT_t > 0$ means that the peak of the cross-market activity corresponds to SPX futures trading followed by VIX futures trading. This is an indication of SPX futures leading VIX futures. However, when the cross-market activity increases we observe a negative impact on LLR meaning that the VIX futures lead is strenghtened (or the SPX futures lead is weakened). Figure 10 depicts the average cross-correlation function conditioned on $CMAT_t > 0$ and the $PCMA_t$ being in the lower and upper quartile, respectively. Notice, when cross-market activity increases, the cross-correlation function shifts downwards and becomes more skewed to the left. In particular the change in skew is reflected in a decrease in the LLR measure. Hence, even in the case when cross-market activity is high and driven by SPX futures trades followed by VIX futures trading, the VIX futures returns are more informative about future SPX futures returns in comparison to the reverse direction.

Consistent with the pattern observed in Figure 6, column 2 shows that the VIX index has a negative impact on LLR. This indicates that during periods of high VIX, prices tend to move

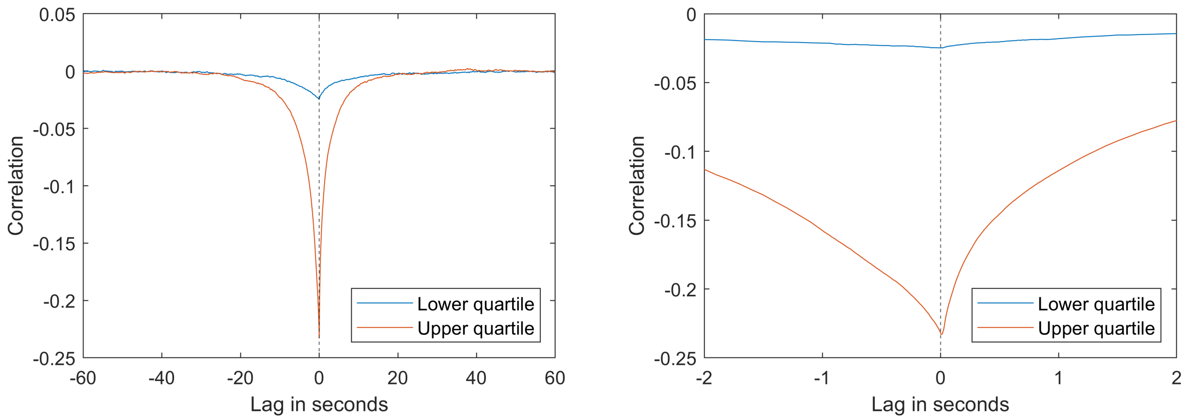

Figure 10: Average cross-correlation function for subsamples of positive CMAT-days. Quartiles of PCMA is found for the subsample where CMAT is positive. The average is taken over the cross-correlation function for the subsample where PCMA is above (below) the respective quartiles and where CMAT is positive. The two figures are identical except for the finer scale on the right figure.

first in the VIX futures market. However, column 5 reveals that once the other variables are included the effect of the level of the VIX index changes and is less significant. Similar observations can be connected to the level of the VVIX index.

We now turn the attention to the coefficients on the SPX returns. A positive (negative) change in the value of the SPX index is typically associated with a negative (positive) change in the level of the VIX. When the regression does not contain the cross-market activity variables (column 4), we see that both negative and positive SPX returns strengthen the VIX futures lead and are significant at the 1% level. Hence, when large returns in absolute value materialize, the VIX futures lead SPX futures more. When the rest of the regressors are taken into account (column 5), coefficients stay qualitatively the same.

The regression for LLT is less appealing to interpret. A quick glance at Figure 6 reveals that LLT is fluctuating wildly around zero during low volatility periods, while being close to zero when volatility is high. This can explain why the R-squared is close to zero across all the LLT regressions.

Table 3: Coefficients of the regression in equation (6).

	LLR					LLT					LLC				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Constant	0.923***	0.873***	0.949***	0.845***	0.913***	-2.923***	-2.735***	-4.281**	-1.787*	-3.669*	0.029***	0.053***	0.226***	-0.075***	0.098***
	(45.586)	(36.739)	(26.350)	(42.781)	(26.421)	(-2.664)	(-2.678)	(-2.552)	(-1.888)	(-1.723)	(3.554)	(2.775)	(9.589)	(-6.190)	(8.473)
$PCMA_t^+$	-1.589***				-1.816***	38.562***				33.571**	-4.359***				-2.454***
	(-5.981)				(-5.684)	(2.656)				(2.172)	(-20.390)				(-12.159)
$PCMA_t^-$	-3.818***				-3.988***	36.303**				31.442*	-3.568***				-2.215***
	(-12.207)				(-11.891)	(2.185)				(1.851)	(-27.378)				(-12.283)
VIX_t		-0.003***			0.002*		0.065**			0.021		-0.010***			-0.006***
		(-2.833)			(1.827)		(2.303)			(0.351)		(-8.806)			(-7.827)
$VVIX_t$			-0.001***		-0.000			0.029**		0.008			-0.004***		-0.000**
			(-3.929)		(-0.122)			(2.327)		(0.324)			(-14.380)		(-2.175)
SPX_t^+				-2.316***	-2.017***				-4.481	-47.360*				-4.180***	1.221***
				(-3.240)	(-4.010)				(-0.244)	(-1.879)				(-10.801)	(5.759)
SPX_t^-				2.263***	1.023***				-37.075**	19.464				5.207***	-1.542***
				(3.646)	(2.790)				(-2.232)	(1.380)				(6.696)	(-4.067)
News announcement	-0.001	-0.001	-0.001	0.001	0.000	-0.096	-0.086	-0.087	-0.108	-0.068	-0.000	-0.002	-0.002	0.003	-0.002
	(-0.113)	(-0.074)	(-0.081)	(0.099)	(0.009)	(-0.171)	(-0.153)	(-0.155)	(-0.193)	(-0.122)	(-0.035)	(-0.796)	(-0.485)	(0.724)	(-1.281)
Time to expiration^{VX}	-0.001*	-0.000	-0.000	-0.000	-0.001*	0.030	0.024	0.020	0.023	0.029	-0.000	0.000***	0.001***	0.001*	-0.000
	(-1.726)	(-0.592)	(-0.295)	(-0.475)	(-1.686)	(1.398)	(1.097)	(0.941)	(1.064)	(1.401)	(-1.302)	(2.587)	(3.569)	(1.779)	(-0.009)
Time to expiration^{ES}	-0.000	-0.000	-0.000	-0.000	-0.000	0.006	0.005	0.004	0.006	0.006	-0.000	0.000	0.000	-0.000	-0.000
	(-0.541)	(-0.242)	(-0.063)	(-0.295)	(-0.567)	(0.613)	(0.470)	(0.416)	(0.562)	(0.594)	(-1.528)	(0.774)	(1.089)	(-0.228)	(-0.298)
Adj. R^2	0.086	0.009	0.012	0.009	0.088	0.002	0.001	0.001	-0.001	0.000	0.724	0.720	0.541	0.254	0.865
Obs.	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772	1772

Newey-West t-statistics are in parenthesis. ***, **, * indicates 1%, 5% and 10% significance, respectively. Due to the break in CMAT shown in Figure 8, we exclude the first part of the sample making August 26, 2013 the first sample date.

The LLC measure is different from the other two measures as it does not say anything about which asset is the leader. Instead it measures the value of the cross-correlation at the peak located at LLT (see Figure 3). However, if we focus on the regression of column 15 we observe some interesting features: First, when the cross-market trading activity increases, the negative correlation between VIX futures and SPX futures gets more pronounced. Second, LLC has a negative relation to the level of the VIX index which confirms the relation between LLC and VIX from Figure 6. Third, when only including the SPX returns in the regression (column 14), we see that the correlation gets more negative when returns are high in absolute value.

Overall, we note that across all the regressions with LLR and LLC as the dependent variable, cross-market activity is the variable that provides the largest contribution to R-squared. This indicates its importance for the lead-lag relation. If hedging activity is captured by cross-market activity, this supports the role of hedging by VIX futures dealers in driving the lead-lag relation between VIX futures and SPX futures.

5 Conclusion

We study the lead-lag relationship between VIX futures and SPX futures on a high-frequency sample of transactions over the period from January 2013 to September 2020. To analyze the lead-lag relation, we consider the HY estimator of the cross-correlation function. The leadership strength is computed on a daily basis using three different measures of lead-lag strength. Namely, the lead-lag ratio, the lead-lag time, and the lead-lag correlation. The analysis reveals large time-variation in the lead-lag relation. Under high volatility, the markets exhibit stronger negative correlation and short-lived lead-lag with a tendency for VIX futures to lead SPX futures. We consider a regression model in order to delve further into the time-variation in the lead-lag relation. In particular, we find that the cross-market activity explains a major part of the lead-lag relation and that days of high activity are associated with a strengthened VIX futures lead over SPX futures. We argue in favor of the hypothesis that hedging activities of VIX futures dealers are an important source of cross-market activity and therefore hedging activities could be driving part of the VIX futures lead over SPX futures.

When the leadership of the VIX futures is partly explained by hedging activities of dealers, this may be due to two different mechanisms: If VIX futures trading is informative then the lead of the VIX futures, means that the hedging activities of VIX futures market makers help transmit information from VIX futures to SPX futures markets, i.e. information flows from one derivatives/futures market to another. On the other hand, if VIX futures trading is uninformative, which is not unlikely under the presence of large rebalancing trades by issuers of VIX ETPs, then the leadership of the VIX futures means that hedging activities reflect a potential systemic risk through its adverse effect on the index futures market at times of large-scaled SPX futures selling by VIX futures market makers. We leave it for future research to analyze the importance of these two mechanisms.

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