Commodity Variance Risk Premia and Expected Futures Returns: Evidence from the Crude Oil Market

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Abstract

We develop an extended mean-variance model to investigate the relationship between variance risk premia (VRP) and expected futures returns in the commodity market. In the presence of stochastic variance, commodity producers demand both futures and option contracts to hedge their exposure to commodity price variation and volatility risk; speculators provide liquidity and ask for risk premia. This model reveals a negative relationship between VRP and expected futures returns. Empirically, we measure VRP using options and high-frequency futures data in the crude oil market. Consistent with our model, we find that VRP predict futures returns even after controlling for other predictors.

JEL Classification: G12, G13

Keywords: variance risk premia; futures return; predictability; crude oil.

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I. Introduction

Literature has identified many predictors of futures returns such as basis, inventory, hedging pressure and open interest growth. Because futures contracts are zero initial cost derivatives and expected returns only stem from futures risk premia (FRP), such predictability reflects economic relationship between a predictor and FRP. However, existing papers have not investigated the connection between variance risk premia (VRP) and FRP. This missing connection is important in futures pricing because variance of commodity futures price has been well documented as time-varying and stochastic (e.g., Trolle and Schwartz, 2009), and VRP are the premia that hedgers pay to hedge the variance risk. Since VRP capture investors' preferences and beliefs of the uncertainty of futures prices, they play a role not only in option pricing, but also in futures valuation. This creates an intrinsic linkage between VRP and FRP. In this paper, we theoretically analyze this linkage, and we show that VRP predict futures returns.

Empirically, we use crude oil options and high frequency tick-by-tick futures data from 1990 to 2010 to test our model hypothesis. Consistent with our model, we document that VRP significantly predict crude oil futures returns, even after controlling for other known predictors. This paper advances the extant literature on commodity futures returns, given the significant predictive power of VRP beyond other common predictors. It helps us understand how hedgers and speculators manage financial risk in the presence of stochastic variance, and how derivative prices, such as futures prices and option premia, are determined accordingly. This finding extends the literature on the return predictability of VRP in other financial markets such as equity and fixed income securities (e.g., Bollerslev, Tauchen and Zhou, 2009).

The main premise of this paper is that commodity hedgers have access to both futures and futures options. To mitigate commodity price risk and volatility risk, the hedgers trade not only futures, but also options. This is well supported by both the literature and industry practice. Brown and Toft (2002) and MacKay and Moeller (2007) argue that a firm should use both futures and options to increase a firm's value in corporate risk management. Adam (2009) documents that commodity producers indeed use both futures and options contracts to hedge. To hedge volatility risk, investors buy variance swaps or options and pay premia to variance sellers. On the other hand, hedgers in the commodity futures market have no initial cost, but effectively pay futures risk premia later on to recompense the counterparty for risk-sharing.

Hedging commodity price risk via futures contracts has been extensively studied in the finance literature. Many papers adapt the mean-variance preference to generate hedging and speculative demands of futures contracts.[1] One essential assumption that all these works make is that variance of commodity futures price is constant, which is contrary to the empirical evidence. We extend the mean-variance preference by allowing variance to be stochastic. In our economic setting, commodity producers trade both futures and option contracts to hedge due to the presence of stochastic variance, and speculators provide liquidity in the derivatives market. Because producers are willing to pay premia to avoid the volatility risk, VRP are negative. Furthermore, producers' simultaneous demand for both futures and option contracts generates a negative relationship between FRP and VRP. Because the zero initial cost of futures contracts implies that the expected futures returns entirely come from FRP, it follows that VRP negatively predict futures returns. Interestingly, we find that even when the hedgers' initial wealth is linear in the commodity price (i.e., vega of her initial position is zero), they do have exposure to stochastic variance. They will demand options for hedging purposes and pay the VRP. These findings are robust to the measurement error of variance and if we allow a non-zero correlation between the spot commodity price and variance.

Because of the non-linearity of option payoffs, it is not straightforward to analytically generate the simultaneous hedge demand for futures and options. For example, Brown and Toft (2002) rely on a numerical procedure to justify the simultaneous demand of both hedging instruments; Judd and Leisen (2010) and Chang and Wong (2003) use simple discrete distributions of the underlying

[1] For example, Anderson and Danthine (1980, 1981), Hirshleifer and Subramanyam (1993), Bessembinder and Lemmon (2002), Hong and Yogo (2012), and Acharya, Lochstoer and Ramadorai (2013).

security to endogenize the demand for options. We use the variance swap contracts as a theoretical proxy for the cross section of options, and derive closed-form solutions for the simultaneous hedge demand of futures and variance swap contracts. Our framework is motivated by Carr and Wu (2009), who show that the variance swap rate (VSR) can be approximated from the cross section of European options. In the absence of arbitrage opportunities, VSR is equivalent to the Q-measure expectation of realized variance (RV) or implied variance, which we will illustrate in further details in the empirical part of this paper. We therefore use the variance swap contracts and the cross section of options interchangeably.

Although our model is applicable to any storable commodity, we empirically test our model predictions using crude oil. The crude oil derivatives market is ideal to test our model for the following reasons. First, the crude oil futures contract is the largest and most liquid asset traded in the commodity derivatives market. For example, as of the year 2011, crude oil accounts for 51.4% of the entire Goldman Sachs Commodity Index (GSCI) in terms of dollar value. Furthermore, crude oil has distinguishable impacts on real output, inflation and stock markets as documented by Hamilton (1983, 2009), Kilian (2008) and Kilian and Park (2009), among others. Second, because we want to explore whether VRP contain information of real economy and can predict futures returns, it is important to have data of a long enough period to include recessions and geopolitical events which may have significant impacts on the commodity market. Third, Tang and Xiong (2012) document a significant volatility spill-over effect from crude oil to non-energy commodities in recent years, and prices of non-energy commodity futures have become increasingly correlated with crude oil. Therefore, empirical evidence of the stochastic variance and variance risk premia in the crude oil market can at least partly represent those from other commodities.

We calculate the VRP as the expected RV minus the VSR which is the same as the risk-adjusted expectation of RV. We calculate the RV from crude oil futures returns using tick-by-tick data. We further mitigate the impact of microstructure noise of high-frequency data by following Andersen, Bollerslev and Meddahi (2011). Additionally, the VSR is calculated using the non-parametric method as in Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia and Madan (2003), and Jiang and Tian (2005). We use actual option transaction prices so that our analysis is free from the wide "implied" bid-ask spread of synthetic VSR. In contrast to the conventional literature that restricts focus only to one-month maturity (e.g. Bollerslev, Tauchen and Zhou, 2009; Trolle and Schwartz, 2010), we investigate the VRP spanning various maturities, since we believe that most corporate hedging plans in the commodity market often extend much further beyond the one-month horizon. We extract the VRP for the first, the third, and the sixth closest-to-maturity contracts, which are relatively more liquid than others. We find that the VRP for all three maturities are negative on average.

We also document significant predictive power of VRP for monthly crude oil futures returns, even after controlling for other known predictors. Although literature has identified many predictors of commodity futures returns, we do not explore the universe of predictor variables. Instead, we use the prediction model of Hong and Yogo (2012) as a benchmark, and investigate the forecasting power of VRP by controlling other commonly applied predictors. In all our empirical specifications, we consider both macroeconomic variables and crude oil market-specific variables because the commodity market has not been fully integrated with the equity and bond markets, and both categories of variables are known to predict commodity futures returns (e.g. Erb and Harvey, 2006; Szymanowska, Roon, Nijman and Goorbergh, 2013). As such, our predictors include VRP, short rate, yield spread, basis, Commodity Futures Trading Commission (CFTC) position or hedging pressure, open interest growth and historical returns. At the end of each month, we use various combinations of variables to predict crude oil futures returns for the subsequent month. In our data sample, we find that one standard deviation increase in the VRP decreases the futures returns of the subsequent month by 2.46%, 2.18% and 2.01% for the 1-, 3- and 6-month futures contracts, and all Newey-West t-statistics are greater than 2.78. Notably, comparing models with and without including VRP, we find that VRP can increase adjusted R^2 from a negative number to $5-6\%$. Our out-of-sample exercises also show that models including VRP always perform better than others in terms of higher realized economic profits and lower root mean squared errors. This empirical evidence further supports our theoretical predictions.

This paper contributes to several strands of literature. First of all, it enriches the growing literature on the asset return predictability of VRP. Bollerslev, Tauchen and Zhou (2009), Zhou (2010), and Bollerslev, Marrone, Xu and Zhou (2013) demonstrate that VRP predict returns of the aggregate equity and fixed income securities within their stylized general equilibrium model. Wang, Zhou and Zhou (2013) argue that VRP are a strong predictor for firm CDS spreads within a Merton-type structural model. In this paper, we show the commodity futures return predictability of the VRP using an extended mean-variance model. Second, this paper complements existing commodity futures return literature[2] by incorporating the information contained in the VRP. Our incremental finding is that VRP can robustly predict futures returns even after controlling for other commonly used predictors. Third, this paper advances commodity VRP literature. Doran and Ronn (2008) report a negative price for stochastic variance. While they rely on a parametric model, we use the non-parametric approach to estimate VRP. Using daily futures price series, Trolle and Schwartz (2010) and Prokopczuk and Simen (2013) empirically find non-zero (negative) commodity VRP. To increase the accuracy of RV estimation, we apply Andersen, Bollerslev and Meddahi's (2011) approach using high-frequency futures price data. Finally, Prokopczuk and Simen (2013) document that the VRP inferred from Gold derivatives can predict commodity spot returns. In contrast, we investigate futures returns with various maturities. In addition they do not present any theoretical model explaining why such predictability exists and why the direction of predictability is negative.

The remainder of this paper is organized as follows. Section 2 presents our theoretical model in which producers trade futures and options simultaneously to hedge in the presence of stochastic variance. Section 3 describes the empirical methodology and data we use to measure the VRP. In section 4 we report the main empirical results that VRP predict futures returns after controlling

[2] For example, Bessembinder (1992), Bessembinder and Chan (1992), Roon, Nijman and Veld (2000), Hong and Yogo (2012), Gorton, Hayashi and Rouwenhorst (2013), and Szymanowska, Roon, Nijman and Goorbergh (2013).

for other known predictors. Section 5 concludes.

II. A Model with Variance Risk Premia in the Commodity Market

In this section we theoretically examine the hedging and speculative demands for both futures and variance swap contracts, and the linkage between FRP and VRP.

A. Economic Setting

There are two types of agents in the commodity derivative market: n_p producers and n_S speculators. Producer $i \in \{1, ..., n_p\}$ equipped with risk aversion γ_{P_i} trades futures and variance swap contracts to hedge both futures price risk and volatility risk. Speculator $j \in \{1, ..., n_S\}$ equipped with risk aversion γ_{S_j} provides liquidity and trades with the producer for one period.[3]

At the end of the period, producer i's wealth is

$$W_{P_i} \equiv Y_i \cdot S_i^* + V_F^{P_i} \cdot (S - F) + V_\sigma^{P_i} \cdot (\widetilde{\sigma}^2 - VSR)$$

where $Y_i > 0$ is her endowment of commodity, S_i^* is the spot price at producer i's physical delivery location, and S is the spot price at the delivery location of the futures contract with futures price F. The producer hedges her exposure to S_i^* by taking short positions ($V_F^{P_i} < 0$) in the futures contracts. We distinguish S_i^* and S because the producers may face S_i^*s which are not perfectly correlated to S. It is necessary to take cross hedging (Anderson and Danthine, 1981) into account in the model because of the correlation being less than one in the data. For example, the historical correlation between West Texas Intermediate (WTI) Cushing spot prices and Phillips 66 WTI (ChevronTexaco Midway Sunset California field) crude oil posted prices estimated from 1995 to 2013 is 0.80 (0.79), which is far from the unit and validates the needs for cross hedging.

$\widetilde{\sigma}^2$ is the stochastic variance of spot price, and VSR is the variance swap rate. Later, we will

[3] Even if there are two types of speculators, each specializing in one of these two derivative securities, main findings in this section do not change.

show that the producer has exposure to $\widetilde{\sigma}^2$ even though her initial wealth $Y_i \cdot S_i^*$ is linear in S_i^*, i.e., vega of her initial asset position is zero. To hedge her exposure to the stochastic variance, the producer takes $V_\sigma^{P_i} > 0$ amount of position in variance swap contracts which pay off $\left(\widetilde{\sigma}^2 - VSR\right)$ at the end of the period. Although $\widetilde{\sigma}^2$ is not directly observable, literature (e.g., Hansen and Lunde, 2006) has convincingly argued that one can accurately estimate the integrated variance using high-frequency price data. We therefore are able to explicitly model investors' exposure to the stochastic variance. In Appendix A, we discuss that even when $\widetilde{\sigma}^2$ is measured with errors, our main conclusions still hold.

For simplicity, we assume that the speculator j's wealth W_{S_j} only depends on the financial derivatives she trades and she does not hold the physical commodity asset. Her wealth at the end of the period is, $W_{S_j} \equiv V_F^{S_j} \cdot (S - F) + V_\sigma^{S_j} \cdot (\widetilde{\sigma}^2 - VSR)$, where $V_F^{S_j} > 0$ and $V_\sigma^{S_j} < 0$ are her positions in futures and variance swap. The speculators' role is to share the producer's exposure to the variation of S_i^* and $\widetilde{\sigma}^2$ and provide liquidity in futures and variance swap contracts.

B. Mean-Variance Preference in the Presence of Stochastic VarianceEconomic Setting

In the traditional mean-variance framework, we consider the Arrow-Pratt risk-aversion coefficient $\gamma > 0$ and spot price at the end of period, $S \sim N(\mu, \sigma^2)$. If the end-of-period wealth W is a linear function of S, given the utility function $u(W) \equiv -\exp\{-\gamma W\}$, the expected utility is

$$E[u(W)] = E[-\exp\{-\gamma W\}] = -\exp\{-\gamma E[W] + \frac{\gamma^2}{2} var[W]\},$$

which is equivalent to the mean-variance preference

$$MV(W) \equiv E[W] - \frac{\gamma}{2} var[W]. \tag{1}$$

In the absence of stochastic variance, (1) is exact. However, in the presence of stochastic variance, (1) is incorrect because $MV(W)$ depends *only* on the expected variance of W regardless of the variance of variance. Hence, we introduce the mean-variance preference in the presence of stochastic variance ($MVSV$). We assume that the end-of-period spot price S follows a conditional normal distribution $S|\widetilde{\sigma}^2 \sim N(\mu, \widetilde{\sigma}^2)$, where $\widetilde{\sigma}^2$ is stochastic and takes a non-negative value with its mean equal to $\overline{\sigma}^2$ and its moment generating function (hereafter, m.g.f.) equal to $M_{\widetilde{\sigma}^2}(t) \equiv E\left[\exp\left\{t\widetilde{\sigma}^2\right\}\right]$. The correlation between the hedged commodity S_i^* and the futures contract underlying S is ρ_i, i.e., $\begin{bmatrix} S_i^* \\ S \end{bmatrix} \sim N(\mu \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \widetilde{\sigma}^2 \begin{bmatrix} 1 & \rho_i \\ \rho_i & 1 \end{bmatrix})$.

Similarly with (1), we show in Appendix B that $MVSV$ of the producer i equivalent to $E[u(W_{P_i})]$ can be defined as

$$MVSV^{P_i} \equiv Y_i\mu + V_F^{P_i}(\mu - F) - V_\sigma^{P_i} VSR - \frac{1}{\gamma_{P_i}} \log M_{\widetilde{\sigma}^2}(t_{P_i}) \tag{2}$$

where

$$t_{P_i} \equiv \frac{\gamma_{P_i}^2}{2}\left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right) - \gamma_{P_i} V_\sigma^{P_i}. \tag{3}$$

It can be further expressed as a linear function of $E\left[W_{P_i}\right] = Y_i\mu + V_F^{P_i}(\mu - F) + V_\sigma^{P_i}\left(\overline{\sigma}^2 - VSR\right)$ and $var\left[W_{P_i}|\widetilde{\sigma}^2 = \overline{\sigma}^2\right] = \left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right)\overline{\sigma}^2$:

$$MVSV^{P_i} = E\left[W_{P_i}\right] - \frac{\gamma_{P_i}}{2} var\left[W_{P_i}|\widetilde{\sigma}^2 = \overline{\sigma}^2\right] - \left[\frac{1}{\gamma_{P_i}}\log M_{\widetilde{\sigma}^2}(t_{P_i}) - \left(\frac{\gamma_{P_i}}{2} var\left[W|\widetilde{\sigma}^2 = \overline{\sigma}^2\right] - V_\sigma^{P_i}\overline{\sigma}^2\right)\right]. \tag{4}$$

Observe that the first two terms of the right hand side of (4) is the same as the traditional mean-variance framework, but the third term captures the effects of stochastic variance.

From Jensen's inequality, we have

$$\log M_{\widetilde{\sigma}^2}(t_{P_i}) \geqq t_{P_i}\overline{\sigma}^2 = \left(\frac{\gamma_{P_i}^2}{2} var\left[W|\widetilde{\sigma}^2 = \overline{\sigma}^2\right] - \gamma_{P_i} V_\sigma^{P_i}\overline{\sigma}^2\right)$$

and therefore the third term of the right hand side of (4) is non-positive:

$$-\left[\frac{1}{\gamma_{P_i}}\ln M_{\widetilde{\sigma}^2}\left(t_{P_i}\right)-\left(\frac{\gamma_{P_i}}{2}var\left[W|\widetilde{\sigma}^2=\overline{\sigma}^2\right]-V_\sigma^{P_i}\overline{\sigma}^2\right)\right]\leq 0. \tag{5}$$

This extra term induced by stochastic variance decreases the expected utility. In other words, the producer dislikes the stochastic variance in this economy. The equality in (5) holds if $\widetilde{\sigma}^2$ is non-stochastic or $V_\sigma^{P_i}=\frac{\gamma_{P_i}}{2}\left(Y_i^2+2\rho Y_i V_F^{P_i}+\left(V_F^{P_i}\right)^2\right)$. In these cases, (4) degenerates to (1). $MVSV^{P_i}$ nests the traditional mean-variance preference.

Similarly, the $MVSV$ of the speculator is defined as

$$\begin{aligned} MVSV^{S_j} &= E\left[W\right]-\frac{\gamma_{S_j}}{2}var\left[W|\widetilde{\sigma}^2=\overline{\sigma}^2\right] \\ &\quad -\left[\frac{1}{\gamma_{S_j}}\log M_{\widetilde{\sigma}^2}\left(t_{S_j}\right)-\left(\frac{\gamma_{S_j}}{2}var\left[W|\widetilde{\sigma}^2=\overline{\sigma}^2\right]-V_\sigma^{S_j}\overline{\sigma}^2\right)\right], \end{aligned} \tag{6}$$

where

$$t_{S_j}\equiv\frac{\gamma_{S_j}^2}{2}\left(V_F^{S_j}\right)^2-\gamma_{S_j}V_\sigma^{S_j}. \tag{7}$$

With the similar logic applied to the producer, the speculator also dislikes the stochastic variance and (6) degenerates to (1) if $\widetilde{\sigma}^2$is non-stochastic or $V_\sigma^{S_j}=\frac{\gamma_{S_j}}{2}\left(V_F^{S_j}\right)^2$.

We have shown that both the producer and the speculator dislike the stochastic variance. Because when $V_\sigma^{P_i}\neq\frac{\gamma_P}{2}\left(Y_i^2+2\rho Y_i V_F^{P_i}+\left(V_F^{P_i}\right)^2\right)$, the stochastic variance will decrease producer's utility and have impacts on producer's optimization, we call t_{P_i} the producer i's open variance position. Similarly, we call t_{S_j} the speculator j's open variance position.

Finally, it is worth mentioning that one can obtain the explicit expression of $MVSV$ as in equations (4) and (6) for various probability distributions of $\widetilde{\sigma}^2$, such as a truncated normal distribution, a Gumbel distribution, and a discrete distribution, among others.[4]

C. Equilibrium VSR, VRP, and Futures Return PredictabilityMean-Variance Preference in the Presence of Stochastic VarianceEconomic Setting

C.1. Individual Optimality Conditions

Next we maximize the $MVSV$ utility for both agents and solve the equilibrium price and trading volume of futures and variance swap contracts. From the FOCs of producer i, we have:

$$VSR = \frac{M'_{\widetilde{\sigma}^2}\left(t_{P_i}\right)}{M_{\widetilde{\sigma}^2}\left(t_{P_i}\right)} = \frac{E\left[\widetilde{\sigma}^2 \exp\left(\widetilde{\sigma}^2 t_{P_i}\right)\right]}{E\left[\exp\left(\widetilde{\sigma}^2 t_{P_i}\right)\right]} = E\left[\widetilde{\sigma}^2 \frac{\exp\left(\widetilde{\sigma}^2 t_{P_i}\right)}{E\left[\exp\left(\widetilde{\sigma}^2 t_{P_i}\right)\right]}\right] \tag{8}$$

and her optimal trading volume of the futures contracts is:

$$V_F^{P_i} = \frac{\mu - F}{\gamma_{P_i} VSR} - \rho_i Y_i. \tag{9}$$

Observe that (9) extends the classical result

$$V_F^{P_i} = \frac{\mu - F}{\gamma_{P_i} \sigma^2} - \rho_i Y_i \tag{10}$$

where σ^2 is a constant (see Anderson and Danthine, 1981) by introducing the stochastic variance $\widetilde{\sigma}^2$ and finding that σ^2 in (10) is replaced with VSR in (9). As we will prove $VSR > \overline{\sigma}^2$ later, the first term in the right side of (9), which is induced by the bias of a futures contract, i.e., $F \neq \mu$, is smaller than that of (10).

Similarly, the individual optimality condition of speculator j is,

$$VSR = \frac{M'_{\widetilde{\sigma}^2}\left(t_{S_j}\right)}{M_{\widetilde{\sigma}^2}\left(t_{S_j}\right)} = \frac{E\left[\widetilde{\sigma}^2 \exp\left(\widetilde{\sigma}^2 t_{S_j}\right)\right]}{E\left[\exp\left(\widetilde{\sigma}^2 t_{S_j}\right)\right]} = E\left[\widetilde{\sigma}^2 \frac{\exp\left(\widetilde{\sigma}^2 t_{S_j}\right)}{E\left[\exp\left(\widetilde{\sigma}^2 t_{S_j}\right)\right]}\right] \tag{11}$$

[4] We do not report the derivations in order to manage the size of this article.

and her optimal trading volume of the futures contracts is:

$$V_F^{S_j} = \frac{\mu - F}{\gamma_{S_j} VSR}. \tag{12}$$

C.2. Futures Return Predictability of VSR and VRP

Because both derivative contracts are a zero-net-supply security, market clearing conditions are

$$\sum_{i=1,\ldots,n_P} V_F^{P_i} + \sum_{j=1,\ldots,n_S} V_F^{S_j} = 0, \tag{13}$$

$$\sum_{i=1,\ldots,n_P} V_\sigma^{P_i} + \sum_{j=1,\ldots,n_S} V_\sigma^{S_j} = 0. \tag{14}$$

Combining equations (9), (12) and (13), we obtain

$$\frac{\mu - F}{\gamma^*(VSR)} - Y^* = 0 \tag{15}$$

where the aggregate risk aversion (relative wealth) $\gamma^* \equiv \frac{1}{\sum_{i=1,\ldots,n_P}(1/\gamma_{P_i}) + \sum_{j=1,\ldots,n_S}(1/\gamma_{S_j})}$ and the effective aggregate endowment $Y^* \equiv \sum_{i=1,\ldots,n_P} \rho_i Y_i$, which is the summation of endowments hedgeable by the futures contract. We may interpret (15) as the zero net futures position of the representative agent equipped with the risk aversion γ^* and the endowment Y^*.

Immediately from (15), we have the following equation which uncovers the relationship among FRP, VSR and VRP:

$$FRP = \mu - F = (\gamma^*)(Y^*)(VSR) = (\gamma^*)(Y^*)\left(\overline{\sigma}^2 - (VRP)\right). \tag{16}$$

where $VRP \equiv E\left[\widetilde{\sigma}^2\right] - VSR = \overline{\sigma}^2 - VSR$. According to (16), FRP, which is the bias of the futures price, is positively related to the aggregate risk aversion γ^*, the effective aggregate endowment Y^*, and most importantly VSR. Hence, VRP negatively predicts the futures price return.

C.3. Volume Implications

To understand more about risk sharing of producers and speculators we study agents' optimal positions. Substituting (15) into (9), we obtain

$$V_F^{P_i} = \frac{(1/\gamma_{P_i})}{(1/\gamma^*)} Y^* - \rho Y_i. \tag{17}$$

Likewise, Substituting (15) into (12), we obtain

$$V_F^{S_j} = \frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)} Y^*. \tag{18}$$

Economic intuitions behind (17) and (18) are as follows. Producers and speculator share the price risk associated with $Y^* = \sum_{i=1,\dots,n_P} \rho_i Y_i$. Producer i's proportion for such risk sharing is $\frac{(1/\gamma_{P_i})}{(1/\gamma^*)}$ and speculator j's proportion is $\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}$. Each agent's proportion of risk sharing is determined by her "aggressiveness" $(1/\gamma_{P_i})$ or $\left(1/\gamma_{S_j}\right)$ relative to the aggregate "aggressiveness" $(1/\gamma^*) = \sum_{i=1,\dots,n_P} (1/\gamma_{P_i}) + \sum_{j=1,\dots,n_S} \left(1/\gamma_{S_j}\right)$. Unless producer i has extremely high $\left(1/\gamma_{S_j}\right)$ relative to $(1/\gamma^*)$, her hedgeable endowment ρY_i is greater than her proportion of Y^*, and therefore she takes a short position in the futures contract. Speculator j takes a long position in the futures contract by her proportion $\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}$ of Y^*.

Substituting (17) into (3), we derive producer i's open variance positions in the case that she does not take any variance swap position, i.e., $V_\sigma^{P_i} = 0$:

$$\frac{\gamma_{P_i}^2}{2} \left(1 - \rho_i^2\right) Y_i^2 + \frac{\gamma_{P_i}^2}{2} \left(\frac{(1/\gamma_{P_i})}{(1/\gamma^*)} Y^* \right)^2. \tag{19}$$

Similarly, we derive speculator j's open variance positions in the case that she does not take any variance swap position, i.e., $V_\sigma^{S_j} = 0$:

$$\frac{\gamma_{S_j}^2}{2}\left(\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}Y^*\right)^2. \tag{20}$$

We can interpret (19) and (20) as follows: In the absence of variance swap trading, the open variance positions are determined by the unhedgeable portion of the endowment $\left(1-\rho_i^2\right)Y_i^2$ and each agent's portion of risk sharing of Y^*, namely, $\left(\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}Y^*\right)^2$ or $\left(\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}Y^*\right)^2$.

However, as proved in Appendix B, the equilibrium open variance position of producer i's and speculator j is

$$t^* = \sum_{i=1,\ldots,n_P}\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{P_i}^2}{2}\left(1-\rho^2\right)Y_i^2+\frac{\gamma_{P_i}^2}{2}\left(\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}Y^*\right)^2\right]+\sum_{j=1,\ldots,n_S}\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{S_j}^2}{2}\left(\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}Y^*\right)^2\right] \tag{21}$$

at the margin if agents trade variance swap contracts. Observe that t^* in (21) is the average of open variance positions in (19) and (20) over all agents weighted by each agent's relative aggressiveness $\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}$ or $\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}$. In other words, agents trade variance swap up to the point that open variance positions are equal across all agents.

Each agent's open variance position in (19) or (20) may be different from t^* in (21). To attain equilibrium open variance position, agents should take variance swap positions. Substituting (17) into (3) and equating to (21), we calculate producer i's optimal variance swap position as

$$V_\sigma^{P_i} = \frac{\frac{\gamma_{P_i}^2}{2}\left(1-\rho^2\right)Y_i^2+\frac{\gamma_{P_i}^2}{2}\left(\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}Y^*\right)^2-t^*}{\gamma_{P_i}}.$$

Similarly, we calculate speculator j's optimal variance position as

$$V_\sigma^{S_j} = \frac{\frac{\gamma_{S_j}^2}{2}\left(\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}Y^*\right)^2-t^*}{\gamma_{S_j}}.$$

Unless producer i has extremely low γ_{P_i}, she takes a long position in the variance swap contract because she needs to hedge her open variance position induced by her endowment, namely, $\frac{\gamma_{P_i}^2}{2}\left(1-\rho^2\right)Y_i^2$. In contrast, speculator j provides liquidity to the variance swap market by taking a short position in the variance swap contract.

If agents can trade the futures contract, they share price risk of hedgeable endowments; each agent's proportion of risk sharing depends on the agent's endowment and risk aversion. Because agents may have heterogeneous endowments, risk version, and basis risk, their open variance positions before trading the variance swap contract may be also heterogeneous. Trading the variance swap contract, the agents share variance risk up to the point that their open variance positions are equalized.

C.4. More Understanding of VSR and VRP

In Appendix B, we also show that the equilibrium VSR is greater than or equal to the expected variance. Hence,

$$VRP = \ E\left[\widetilde{\sigma}^2\right] - VSR \leq 0. \tag{22}$$

The interpretation is as follows. As the previous subsubsection illustrates, some agents (mostly, speculators) share other agents' variance risk by writing the variance swap contract. To compensate additional variance risk that a variance swap rate assumes, $VSR \geq E\left[\widetilde{\sigma}^2\right]$ must hold. Therefore, VRP is negative in equilibrium.

We have derived the closed-form equilibrium volumes and prices of futures and variance swap contracts. All these results up to this point hold for an unspecified m.g.f. $M_{\widetilde{\sigma}^2}\left(t^*\right)$. In the remainder of this subsection, we consider a case of $\widetilde{\sigma}^2 \sim N(\overline{\sigma}^2, \sigma_\sigma^2)$, which implies

$$M_{\widetilde{\sigma}^2}\left(t^*\right) = \exp\left\{\overline{\sigma}^2 t^* + \frac{1}{2}\sigma_\sigma^2\left(t^*\right)^2\right\} \tag{23}$$

to gain more insights into VSR, VRP, and FRP. Even though we lose generality and negative variance is technically wrong, the gain in economic intuitions outweighs these losses.

Combining (8) and (23), we have

$$VSR = \overline{\sigma^2} + \sigma_\sigma^2 t^*$$

and

$$VRP = -\sigma_\sigma^2 t^* \tag{24}$$

which tells us that VRP is determined by the variance of variance σ_σ^2 and the equilibrium open variance position t^*.

Substituting (24) into (15), we have

$$F = \mu - (\gamma^*)(Y^*)(VSR) = \mu - (\gamma^*)(Y^*)(\overline{\sigma^2} - VRP) = \mu - (\gamma^*)(Y^*)\left(\overline{\sigma^2} + \sigma_\sigma^2 t^*\right).$$

Equivalently,

$$FRP = (\gamma^*)(Y^*)(\overline{\sigma^2} - VRP) = (\gamma^*)(Y^*)\left(\overline{\sigma^2} + \sigma_\sigma^2 t^*\right). \tag{25}$$

which implies that the five determinants of FRP are the aggregate risk aversion γ^*, the aggregate hedgeable endowment Y^*, the average variance $\overline{\sigma^2}$, the variance of variance σ_σ^2, and the equilibrium open variance position t^*. It is worth noting that VSR (VRP) succinctly summarizes the last three (two) determinants.

D. Discussion and Empirical Implication

To reduce this open variance position and the disutility induced by stochastic variance in (5), the producers buy the variance swap contract and is willing to pay extra premium to the speculator (variance swap seller). Hence, (22) holds and we formulate the following empirical hypothesis.

Hypothesis 1: VRP are negative.

Producers demand both futures contract and variance swap contracts for hedging purposes. It follows that VRP are interlinked with expected futures returns, i.e. VRP predict futures returns. To see this, consider one period beginning from $t = 0$ and ending at $t = 1$. Define $R \equiv F(1) - F(0)$. Organizing the terms of (15), we have

$$E[R] = \mu - F = \gamma^* Y^* \left(\overline{\sigma}^2 - VRP\right), \tag{26}$$

which suggests

$$\frac{\partial E[R]}{\partial VRP} = -\gamma^* Y^* < 0.$$

Because of the zero initial cost of futures contracts, the expected futures returns come from futures risk premia and are directly linked with VRP. Our second hypothesis is the following.

Hypothesis 2: VRP negatively predict futures returns.

To illustrate the negative relation between VRP and the expected futures returns E$[R]$, we visualize the model implications using a truncated normal distribution of $\widetilde{\sigma}$; for simplicity of our numerical example, we assume that there are only one producer and one speculator in our economy. We set $\rho = 0.8$ as suggested by the historical data, and $\overline{\sigma}^2 = 0.3$. Starting from initial parameters $\gamma_P = 6$, $\gamma_S = 3$, $Y = 1$, $\mu = 3$ and $\sigma_\sigma = 0.1$, we examine how the negative relation between VRP and E$[R]$s change with respect to risk aversion parameters of the producer and the speculator, the endowment level, as well as the standard deviation of stochastic variance. Figure 1 plots VRP and E[R]s for various γ_P, γ_S, Y and σ_{σ^2}. The upper, left (right) panel depicts VRP and E[R]s for different levels of γ_P (γ_S). Observe that VRP are negative and E[R]s are positive, and the response of E[R] to VRP is higher as the producer (speculator) becomes more risk averse, and the speculator requires more premia to meet the hedging demand of the producer. From the lower left panel, we observe that the greater the Y is, the greater E[R] is and the more negative VRP is. This is because the higher the endowment, the more amount of commodity the producer needs to hedge. Finally, according to the lower right panel of Figure 1, higher standard deviation of stochastic variance (σ_{σ^2}) leads to the greater E[R] and the more negative VRP. The greater the standard deviation of variance is, the higher level of volatility risk is, and the more compensation the speculator will ask for providing liquidity. In summary VRP are increasing in γ_P , γ_S, Y, and σ_{σ^2} but E[R]s are increasing in those, which indicates that VRP negatively predict futures returns. Although we only present the results for the truncated normal distribution, our conclusions hold for various distributions as long as the m.g.f. of variance is well defined.

The economic intuition of our theoretical model can be summarized as follows. In this economy, the producer demands short positions in futures to mitigate her commodity price risk. However, futures contracts alone do not eliminate the disutility induced by stochastic variance. Hence, the producer demands *both* hedging instruments, futures and variance swaps contracts, in equilibrium. Because the speculator who takes the other side of transaction requires compensations for selling the variance swap, VRP is negative. In addition, various parameters in our model such as risk aversion and stochastic variance affect *both* FRP and VRP, which are negatively related to each other. This linkage implies that VRP negatively predict futures returns.

III. Measure of Variance Risk Premia

The model we presented suggests that VRP can predict futures returns. To investigate empirical evidence for the theoretical hypotheses, we measure VRP in the crude oil derivatives market using the model-free method. We begin by defining the quadratic variation of crude oil futures prices.

On a specific trading date t, consider a crude oil futures contract with the delivery date T, and an option written on the futures contract expires on τ, which is before T.[5] The quadratic variation of futures prices from t to the option expiration date τ is,

$$V(t,\tau;T) = \frac{1}{\tau - t}\int_t^\tau \left(d\left(\log\left(F\left(s,T\right)\right)\right)\right)^2. \tag{27}$$

Note that the quadratic variation $V(t,\tau;T)$ is a random variable. The VRP are defined as,

$$VRP(t,\tau;T) = E_t^P\left[V(t,\tau;T)\right] - E_t^Q\left[V(t,\tau;T)\right]. \tag{28}$$

To obtain VRP, we essentially need to calculate the futures price variation under the P measure, which we call the realized variance $RV(t,\tau;T)$, and futures price variation under the Q measure, which we call the VSR or the implied variance $IV(t,\tau;T)$. $IV(t,\tau;T)$ can be inferred from option contracts. In our empirical analysis, we focus on the first, the third and the sixth closest to maturity futures contracts and corresponding options. These contracts are relatively liquid ones traded in the market and allow us to extend our analysis across maturities and investment horizons. We denote them by M1, M3 and M6 futures; and we denote the related VRP by VRP1, VRP3 and VRP6.

A. Realized Variance

We calculate RV from observed crude oil futures returns using tick-by-tick data. Since Andersen and Bollerslev's (1998) seminal work, literature has shown the superior property of RV estimated from intraday high frequency data. RV based on intraday data is a more accurate measure of the true return variation than estimation based on interday returns. In the commodity literature,[6] intraday data have been used to predict volatility, examine correlations and identify jumps in futures prices.

[5] CME crude oil futures contracts expire on the third business day prior to the 25th (or the precedeing bussiness day) of the month before the delivery month, if the 25th is a business day (if the 25th is not). Options written on futures expire three business days before the expiration of the futures contract.

[6] See, e.g., Martens and Zein (2004), Wang, Wu, and Yang (2008), and Halova (2011).

Meanwhile, many techniques have been developed to mitigate the impact of microstructure noise and to get robust estimation of RV. One way of estimating RV is to use squared returns of fixed sampling periods, for example, 1-minute or 5-minute. But the choice of the sampling frequency is subject to the objective function one wants to optimize, as well as the volatility model one uses. There is no consensus about the optimal sampling period in the literature.

Andersen, Bollerslev and Meddahi (2011) propose an "average" estimator by simply taking the average of the standard RV based on the fixed sampling periods. The standard RV, also known as the sparse RV, for a trading day t is defined as

$$RV_t^{sparse}(h,j) \equiv \sum_{i=1}^{N_j} (r_{j+ih}^{(t,h)})^2 \tag{29}$$

where h is the width of sample interval, such as 5 minutes; $j = 0, ..., (h-1)$ is the offset to start the RV calculation; N_j is the number of sample intervals of a trading day t with the total trading minutes D_t. More specifically, $N_j \equiv D_t/h$ if $j = 0$ and $N_j \equiv (D_t/h - 1)$ if $j = 1, ..., (h-1)$. $r_s^{(t,h)} \equiv \log(F_{t,T}(s)/F_{t,T}(s-h))$ is the log return of the futures contract with delivery time T at the trading minute s on day t. The "average" RV estimator is then calculated as

$$RV_t^{average}(h) \equiv \frac{1}{h} \sum_{j=0}^{h-1} RV_t^{sparse}(h,j). \tag{30}$$

In our following empirical analysis, we set $h = 5$ minutes, and we mainly use $RV_t^{average}(5)$.

B. Variance Swap Rate

VSR measures ex-ante risk-neutral price variation of crude oil futures, which can be inferred from crude oil option prices. Following Jiang and Tian (2005) and Carr and Wu (2009) among others, we calculate VSR from a cross-section of out-of-the-money option prices using the formula:

$$E_t^Q\left[RV(t,\tau;T)\right] = IV(t,\tau;T) = \frac{2}{B(t,\tau)(\tau-t)}\left(\int_0^{F(t,T)} \frac{P(t,\tau,T,X)}{X^2}dX + \int_{F(t,T)}^{\infty} \frac{C(t,\tau,T,X)}{X^2}dX\right), \tag{31}$$

where $B(t,\tau)$ is the time t price of a zero coupon bond maturing at τ; $F(t,T)$ is the time t futures price with the delivery date T; $P(t,\tau,T,X)$ and $C(t,\tau,T,X)$ are the time t price of a put option and a call option with strike price X and the underlying futures maturing at T. Our calculation of VSR, or option implied variance is similar to the approach of Duan and Wei (2009). More specifically, we use the trapezoidal approximation to compute the integral, and we only consider those days with at least two OTM calls and two OTM puts available for each maturity. We do not interpolate or extrapolate the options data. Since our predictability study is based on futures returns at the end of each month, we calculate VSR on the monthly basis. If VSR is not available on the last trading day of a month due to the lack of OTM option data, we compute VSR on the previous day.

C. Variance Risk Premia

VRP, the main focus of our empirical study, are defined as the difference of variance under P-measure and Q-measure as in (28). At any time t, the forward-looking variance under Q-measure or $IV(t,\tau;T)$ can be inferred from option prices. However, realized variance $RV(t,\tau;T)$ is the ex-post P-measure of price variation, and is not observable at time t. To obtain an approximation of the unobservable RV, we use historical RV to fit a linear time series regression model and forecast $\widehat{E_t}\left[RV(t,\tau;T)\right]$. This approach shares the same spirit with other VRP studies such as Bollerslev, Tauchen and Zhou (2009).

Our estimator of VRP at time t with the time horizon $(\tau-t)$ is summarized as:

$$VRP_t = \widehat{VRP}(t,\tau;T) = \widehat{E_t^P}\left[V(t,\tau;T)\right] - E_t^Q\left[V(t,\tau;T)\right]. \tag{32}$$

As such, VRP_t in equation (32) only uses information directly observable at time t. From the perspective of forecasting, it is important to have all information available at time t. Our VRP may also be interpreted as expected VRP since variations under both P-measure and Q-measure are expected values at t. In our study, we use VRP_t along with other information variables to predict futures returns for the subsequent period, R_{t+1}.

D. Data

D.1. Futures and Option Data

We use intraday crude oil futures tick data and daily crude oil option data from Jan 02, 1990 to Dec 31, 2010 traded on the CME. Our dataset covers three NBER recessions and many geopolitical events such as the first gulf war, Asian economic crisis, 9/11 terrorist attack, Iraq war on March 20, 2003 and the recent financial crisis. We directly use intraday tick-by-tick futures prices to obtain the time series of RV without any filtering. However, option data are filtered based on the following criteria. We exclude option data with open interest less than 100, or price less than $0.01, and we exclude those observations violating non-arbitrage conditions. While crude oil options traded on the CME are the American type, our formula of calculating VSR is based on European options. We convert the American option prices into European prices with the same approach as Trolle and Schwartz (2009) using the Barone-Adesi and Whaley (1987) formula. We then eliminate those observations with the Black implied volatility smaller than 1% or bigger than 200%. To check the accuracy of this method, we also investigate VSR calculated from option prices converted from a trinomial tree and a binomial tree. The difference between our method and these two methods is small.

D.2. Predictor Variables

We use the prediction model of Hong and Yogo (2012) as the benchmark, and investigate the forecasting power of the VRP of futures returns. Our definition of most predictor variables are similar with Hong and Yogo (2012). Besides our main predictor of interest, the VRP, we consider both macroeconomic variables and crude oil market specific variables that are known to predict futures returns (e.g., Bessembinder and Chan, 1992, Bessembinder, 1992, Roon, Nijman and Veld, 2000). The first category of variables include the short rate and the yield spread, which have been documented to predict commodity returns as well as equity and bond returns. The short rate is the monthly average yield of the 1-month T-Bill; the yield spread is the difference between Moody's Aaa and Baa corporate bond yields.

The other category of predictors are crude oil market specific. They are basis, the CFTC position, open interest growth and historical returns. Basis is the spread between futures prices and spot prices. It can be interpreted as the implied net convenience yield. In line with the literature, basis for a particular maturity contract is defined as

$$Basis_{t,T} = (\frac{F_{t,T}}{S_t})^{\frac{1}{T-t}} - 1, \tag{33}$$

where $F_{t,T}$ is the day t price of futures maturing at T, S_t is day t spot price. We compute basis for each maturity of contracts.

Another variable is the CFTC position,[7] which measures the supply and demand imbalance of large commercial traders documented in the CFTC reports. Roon, Nijman and Veld (2000) find that this variable has forecasting power for futures returns. In each month, we compute the imbalance variable as,

$$\text{CFTC position}_t = \frac{(\text{\# of short hedge positions})_t \text{ - } (\text{\# of long hedge positions})_t}{(\text{\# of total hedge positions})_t}. \tag{34}$$

It measures the relative net short hedge positions to the total positions of large commercial traders.

We compute open interest growth as the 12-month geometric average of open interest change for

[7] Some papers name this variable hedging pressure.

each maturity. We also compute the dollar open interest growth, which is the change of the product of the spot price times open interest. All empirical results are similar to open interest growth itself. Historical returns are the 12-month geometric average of future returns for each maturity, which essentially capture the momentum effect of futures returns.

IV. Empirical Results

A. Variance Risk Premia in the Crude Oil Market

Table 1 summarizes the monthly RV, VSR and VRP for the M1, M3 and M6 contracts of our predicting exercise period of Jan 1991 to Nov 2010. The first six rows report summary statistics for two different types of RV estimates: the average of 5-minute sparse RV ($RV^{average}$) based on Andersen, Bollerslev and Meddahi (2011) and the RV calculated from daily returns (RV^{daily}). Observe that the RV calculated from high-frequency price returns, $RV^{average}$, have lower mean and lower standard deviation than the RV calculated from daily returns, RV^{daily}. This suggests that an RV estimate using coarsely sampled daily data could be either biased or very noisy. Similar with the RV, the VSR of the 6-month maturity has a lower mean and lower standard deviation than the 1- and 3-month maturity VSR. It indicates that under both the P-measure and Q-measure, variance of futures prices tends to decrease with the maturity of the contracts, which is consistent with the well-documented Samuelson Effect in the commodity market. For all three maturities, the VSR is higher than the RV with less skewness and kurtosis. If we compare the 25%, 50% and 75% percentiles of RV and VSR, RV is always lower than VSR, suggesting VRP as defined by (28) should indeed be negative. To get the VRP, we take the difference between the expected RV and the VSR as defined in (32). Specifically, we use $RV^{average}$ to fit a linear regression model and predict the expected RV which has the same time horizon as the corresponding VSR. Results show that all VRP are negative on average; the mean of VRP1, VRP3 and VRP6 is -1.65%, -1.87% and -2.46%, respectively. We also report Newey-West t-statistics for all variables. Although the VRP of the 1-month horizon are not significant, the 3- and 6-month VRP are significant at the 1% level. We find that $\text{RV}^{average}$ is on average higher than the sparse RV calculated based on the 5-minute interval in the data sample. Since VRP are the difference between the RV and VSR, our measure of VRP could have more positive values realized than the one calculated using the 5-minute sparse RV. It will drag the mean of VRP1 up towards zero. This may be one of the reasons that our estimated VRP1 are not significantly negative.

Do VRP contain information about economic conditions? In other words, can the time variation of VRP be explained by macroeconomic fluctuations as well as by crude oil market-specific variables? To address this question, we regress VRP on variables capturing the aggregate economic and the crude oil market conditions. Instead of using individual macroeconomic variables with various release frequencies, we use two succinct economic activity indices. The first index is the Chicago Federal National Activity Index (CFNAI); the other is the Aruoba-Diebold-Scotti Business Conditions Index (ADS) published by the Federal Bank of Philadelphia. Both indices incorporate many economic indicators in the U.S. For both indices, a higher value suggests a more progressive economic condition. In addition, we include spot returns, basis, storage level and the CFTC position to reflect the contemporaneous crude oil market condition. We obtain the monthly crude oil storage level and spot prices from the web site of U.S. Energy Information Administration (EIA). Basis and the CFTC position are computed from (33) and (34).

Table 2 reports the results of regressing VRP on economic and crude oil market conditions. Newey-West t-statistics are adjusted for 12 lags. We report results for 1-, 3- and 6-month maturity in Panel A, B and C. For all three maturities, the lagged VRP are significantly positive, confirming a certain level of persistence. The storage level is positively associated with VRP and are significant for all three maturities. Recall that the VRP are the difference between the RV and the VSR (or implied variance) and are usually negative. The more negative the VRP, the higher the implied variance is relative to the RV; that is, the option buyers pay a relatively larger premium to the option writers. Positive coefficients of the storage level suggest that a lower (higher) storage level is related to more (less) negative VRP. In other words, when the market is in the state of insufficient storage of crude oil, option writers require more risk premia. Basis, which is an indirect measure of convenience yield, is negatively associated with VRP across three maturities. It suggests that when the convenience yield is high, usually when the market is in panic or worries about potential shortage, option writers collect more risk premia from option buyers. We notice that although the coefficient is significant only for the 1-month maturity, the magnitudes are not trivial for other maturities. The effect of the CFTC position or the demand-supply imbalance has to be interpreted with caution since it is an aggregate measure for all maturities, and regressions of different maturities may generate mixed results. The CFTC position is only significant for the 6-month VRP. The significant negative coefficients indicate that when there are more short positions in the futures market, VRP are more negative and risk premia to be paid is higher. The underlying logic is consistent with our theory. When producers face a market condition with more uncertainties or when they become more risk-averse, they will hedge more in the futures market (by taking the short position in the futures) and hedge more in the option market (by purchasing options). That is, more short positions in futures are associated with higher demands of options which cause higher option prices; and option writers can charge higher risk premia. Although the CFTC position is not a clean measure of hedgers' positions (or producers' hedging demand in our case), this finding is consistent with our model implications. As we show in Figure 1, when the producer has incentives to hedge more, i.e., when the market condition is more uncertain and the standard deviation of stochastic variance (σ_{σ^2}) is higher or when the risk aversion of the producer γ_P is higher, VRP become more negative.

An important pattern we observe from Table 2 is that VRP1 are statistically significantly associated with the aggregate economic activities measured by both of the two Fed indices, while VRP3 and VRP6 are not. Additionally, the coefficients of these macroeconomic variables in the VRP1 regression become even more statistically significant when both crude oil market-specific variables and macroeconomic variables are included. Since VRP1 measures the short-term expectation, it should be closely linked to the investors' perception of economic conditions in the near future, which will be well captured by the two economic activity indices. Hence, the source of predictability potentially documented by VRP1 may be ambiguous: it may be a result of either variance risk premia (the compensation for sharing variance risk) or a short-term economic outlook proxied by VRP1. For this reason, we do not use VRP1 in our predictive regression. Instead, we mainly investigate the prediction power of VRP3 and VRP6.

B. Predictability of Futures Returns

We examine if VRP can predict futures returns after controlling for other known predictors. Our predictability study focuses on monthly returns of crude oil futures. We regress futures returns on different sets of lagged predictors. We use Hong and Yogo (2012) as the benchmark and examine the incremental forecasting power of VRP. As such, our full model is

$$\begin{aligned}\text{Return}_{t+1} &= \beta_0 + \beta_1 \cdot \text{Aggregate Market Predictors}_t \\ &\quad + \beta_2 \cdot \text{Crude Oil Market-Specific Predictors}_t \\ &\quad + \beta_3 \cdot \text{VRP}_t + \varepsilon_{t+1} \end{aligned} \tag{35}$$

Aggregate market predictors are the short rate and the yield spread; crude oil market-specific predictors are basis, the CFTC position, open interest growth, and historical returns. We focus our discussion on the estimated coefficient of the VRP, $\widehat{\beta}_3$, and its robust t-statistics adjusted for heteroskedasticity and serial correlations. We also report adjusted R^2 of all candidate models and compare their forecast results.

Table 3 reports the summary statistics of the predictors and the correlation matrix of variables when we predict monthly returns of the 1-month futures contracts. Since our data period is from Jan 1990 to Dec 2010 and we predict monthly returns, this table reports statistics for our predicting exercise period of Jan 1991 to Nov 2010, when we can calculate the 12-month moving average of open interest growth and historical returns. The short rate and the yield spread are annualized numbers. Basis, open interest growth and historical returns are maturity-specific. During this period, the mean VRP at the 6-month horizon is -2.46% with a standard deviation of 5.58%; the mean VRP at the 3-month horizon is -1.87% with a standard deviation of 6.52%. Comparing with Hong and Yogo (2012), our open interest growth and historical return have lower means and higher standard deviations. It may be due to the fact that our sample period covers both the recent boom and bust of the crude oil market, but their data sample ends by the boom of 2008. In terms of persistence, the short rate, the yield spread, and historical returns are highly autocorrelated, with the autocorrelation similar to the number reported by Hong and Yogo (2012); the autocorrelation of our predictor of interest, VRP6, is 0.73. Interestingly, VRP6 and VRP3 show a low degree of correlation with other predictors, especially the other crude oil market-specific variables. It suggests that VRP may provide distinct information not contained in other variables. Although we do not report the correlation matrix for the 3- and 6-month futures return predicting exercises, due to the space limitation, the pattern is qualitatively similar.

B.1. Main Empirical Findings

Table 4 reports the main regression results testing our hypothesis that VRP negatively predict futures returns even after controlling for other predictors. Panel A is the results for monthly returns of the closest to maturity or M1 contracts; Newey-West t-statistics with 12 lags are reported in the parenthesis. Column 1 shows that in our data sample there is no significant predictability of returns, although the short rate and the yield spread have the correct negative sign as suggested by the literature. Column 2 indicates that the coefficient of VRP6 displays the desired negative sign, and the adjusted R^2 increases from a negative number to 5%. The coefficient of VRP6 is -0.44 with a t-statistic of -2.93, which means a standard deviation increase of VRP6 will decrease futures returns by $5.58\% \times 0.44 = 2.55\%$ per month. We do not find significant predictability from historical returns in column 3 either. Although all coefficients of historical returns in columns 3 to 6 are negative, indicating a certain degree of mean reversion in returns, all of them are not statistically significant. Including both open interest growth and historical return barely change the negative coefficient of VRP6 as shown in column 5. When we replace VRP6 with VRP3 in column 6, the coefficient is still negative but not statistically significant.

Panel B represents the predicting results for monthly returns of 3-month futures. All results are qualitatively similar to those of 1-month futures contracts. We find that only VRP6 can significantly predict monthly returns by increasing adjusted R^2 from a negative number in column 1 to 6% in column 2. The coefficient of VRP6 in column 2 is -0.39 with a Newey-West t-statistic of -2.94, suggesting the impact of VRP6 on futures returns is both economical and statistically significant. Other variables are unanimously silent and show no power of prediction. Not only are all coefficients insignificant, adjusted R^2s are close to zero. In a specification without VRP6 (column 1 and 3), the CFTC position has positive coefficients and t-statistics around 1.3; in a specification with VRP6 (column 2), the t-statistic of CFTC position decreases to 0.70. Panel C exhibits the predictability of monthly returns of 6-month futures. Slightly different from the other two shorter maturities, some coefficients of the short rate are significant at the 10% significance level. But all specifications without VRP6 still cannot generate a positive adjusted R^2. Inclusion of VRP6 increases the adjusted R^2 from 0 to 6%. The coefficient and significance level of VRP6 are virtually unchanged when we include both open interest growth and historical returns in the model. In addition, we find VRP3 has qualitatively similar but statistically weaker results.

We summarize our findings of monthly return prediction as follows. For the period of 1991-2010, the short rate negatively predicts monthly futures returns although the coefficients are only significant for the 6-month maturity futures contracts. For different maturities, the CFTC position has positive coefficients. The CFTC position is positive when large commercial traders initiate more short positions than long positions in the futures contracts, i.e. when producers need to hedge more. In this case, more short positions cause downward biased current futures prices and positive futures returns later on. Historical returns always have negative coefficients suggesting futures returns are mean-reverting. Most importantly, we document consistent evidence that VRP6 negatively predict monthly futures returns for various maturities. These findings are consistent with our theory. Furthermore, the robust forecasting power indicates that VRP6 may contain distinct information related to the demand and supply of futures contracts and, therefore, the equilibrium futures returns.

B.2. Out-of-Sample Analysis

Even though we have demonstrated that VRP can significantly predict futures returns, there is no guarantee that the models with VRP will forecast returns more accurately than other models out-of-sample. Our empirical framework is based on the benchmark model of Hong and Yogo (2012), in which we add the new predictor, VRP. With more predictors, our new model may actually have higher forecast variability even though it is correctly specified. To further investigate the forecasting results, we compare the out-of-sample performance of seven candidate linear regression models (Panel A of Table 5). The choice of model specifications is based on the following reasons. First, literature has shown that the commodity market was not fully segmented from other markets, both aggregate market predictors and commodity market-specific predictors forecast futures returns. A correctly specified model should have both categories of predictors. Second, we aim to compare the model specifications using VRP to those without using VRP, and to confirm the predictability of VRP after controlling for other known predictors. Models (1), (2) and (3) use predictors as suggested by Hong and Yogo (2012). The other four models include VRP; and VRP are the only crude oil market-specific predictor in models (4) and (5). This is to highlight the predicting power of the VRP as suggested by our theory. Models (6) and (7) are common predictors in model (3) plus VRP. In each model, the dependent variable is futures returns of the next month. We estimate each model based on a 9-year moving window,[8] then we predict returns for the following month using estimated parameters. We keep rolling the 9-year window over the whole time period. For

[8] We try different window sizes varying from 8 to 12 years, and findings reported here are robust.

example, we first estimate each model using data from Jan 1991 to Dec 1999, and we predict returns for Jan 2000; next we move one month forward and estimate the models with the data from Feb 1991 to Jan 2000, and we predict returns for Feb 2000, so on and so forth.

We use two horse-racing exercises to illustrate the superior forecast of futures returns and economic benefits of including VRP. The first exercise is to check if we can trade on the prediction results of different models. For each model and maturity, we set $100 USD as a buffer fund and start to invest in Jan 2000 when the model forecasts are available. Every month we long/short one barrel of crude oil futures, and roll over the positions to the end of 2010 by following the trading signal from model predictions. We assume that free cash in the buffer fund earns interests of 5% per annum. At the end of each month, if the predicted return of a particular contract of the next month is positive, we treat it as a buying signal and we take a long position; while if the prediction is negative, we short this futures contract. We keep track of the account balance. When we compare the model performances, we do not consider transaction costs and liquidity. It does not affect the conclusion because all seven competing models face the same rolling over transaction costs and liquidity, as only the same futures contracts are traded.

Figure 2 represents the growth of the $100 investment in the M1, M3 and M6 futures contracts. The solid lines represent account balance evolution using models with common predictors; dashed lines represent account evolution using models with VRP6. For the M1 contract, forecasts of various models are not very different before 2003 and accounts grow in a similar way. But from 2004, model forecasts begin to disperse and generate various trading implications. By the end of 2010, model (5) with aggregate market predictors and VRP6 shows the best performance. For the M3 contract, several models outperform others during different time periods. But overall, model (5) has the best performance most of the time. Model (7), which is the full model including all common predictors and VRP6, has the second best performance. For the M6 contracts, models with VRP always perform well, but the significant dispersion of different models happens after 2006, especially after mid-2008. Again the simple model (5), using aggregate market predictors and VRP6, achieves the highest profit; the full model (7) using all common predictors and VRP6 also beat out the other three models not using VRP. Notably, model performances start to significantly diverge from mid-2008 for all three different maturities. One possible reason is that during an extremely volatile period like 2008, both producers and speculators need to implement their hedging more strategically. Trading of futures and options contracts can be more interlinked than ever before, and VRP's informational advantage about expected futures returns becomes more salient. Another potential reason is that the predictability of futures returns may actually vary across business cycles. We will have to leave this conjecture to future research when there are enough recession samples available. Overall, we conclude that the real benefit of including VRP is significant. Trading strategies which exploit the predicting power of VRP largely outperform others. The overall performance of the full model (7) is indeed surprising. It has more predictors but better out-of-sample performance than other simpler models such as model (1), (2) and (3), indicating models not using VRP may actually be underspecified.

Of course, sophisticated investors care about not only the balance growth of a certain strategy, but also its involved risks. Panel B of Table 5 reports the ratio of average return to its standard deviation for the investing period Jan 2000 to Dec 2010. Return is defined as the account balance of the current month divided by the balance of the previous month. The two highest ratios for each maturity are boldfaced in the table. For M1 futures, the two highest ratios come from the models that only use the short rate, the yield spread, and VRP. For M3 and M6 contracts, the models with VRP3 or VPR6 also significantly outperform others and generate much higher return-to-standard deviation ratios than other models. Across three maturities, model (1) also generates very high ratios, suggesting the prediction power of historical returns.

In the second exercise, we calculate the root mean squared errors (RMSE) between predicted and observed gross returns. Return is again defined as the account balance of the current month divided by the balance of the previous month. We report the results in Panel C of Table 5. Overall, the models using VRP have more accurate predictions of futures returns than the others do. For example, the rolling M1 futures series prefers the information set with common predictors plus VRP6; for 3- and 6-month maturities, the simple models with the short rate, the spread yield, and VRP3 or VRP6 predict returns more accurately than the others do. For all three maturities, we conclude that the models with VRP, especially VRP6, generate the most accurate prediction of returns.

In summary, our out-of-sample analyses show that models using VRP can forecast returns more accurately than others using common predictors. In addition, one can achieve higher economic profits by exploiting the predicting power of VRP and by trading based on the model forecasts. It confirms that VRP have superior predictability of futures returns.

V. Conclusion

Considering a stylized derivatives market populated by commodity producers and speculators, we argue that both producers and speculators dislike stochastic variance. To maximize their utility, producers demand both futures and variance swap contracts for hedging purposes. Speculators provide liquidity and ask for premia for risk-sharing. Our theoretical model suggests that VRP are negative and negatively predict futures returns. Using crude oil options and high frequency futures data, we document empirical evidence for these two theoretical predictions. We find that VRP for various maturities are on average negative. When we include VRP in our regression models to predict futures returns, the predictability of futures returns is significantly improved controlling for other predictors of futures returns. Furthermore, we use two out-of-sample quantitative exercises to investigate the benefits of including VRP to predict futures returns. We find that predictive regression models including VRP not only generate the highest economic profits but also have the smallest prediction errors.

Our theoretical approach opens up a new revenue for future research. The approach in this paper can be flexibly extended to study, for example, the effect of heterogeneity in beliefs and uninformed trading on the pricing of commodity derivatives. Empirical findings documented in this paper also suggest some meaningful future research directions. For example, one can compare the VRP between the commodity market and other financial markets, such as the equity market and the fixed income market for various time horizons. Furthermore, it would be interesting to examine how their comovement evolves along business cycles, and to what extent the commodity market serves as a hedging vehicle, especially in the periods of recession and high inflation.

Appendix A: Extensions of the Model

Measurement Error. In reality $\widetilde{\sigma}^2$ is not observable. However, Hansen and Lunde (2006) argue that one can estimate the integrated variance very accurately using high-frequency price data, i.e., the measurement error of $\widetilde{\sigma}^2$ is small. In this subsection we argue that empirical hypothesis 2 remains the same even after assuming some measurement error of $\widetilde{\sigma}^2$.

Let $\sigma^{*2} \equiv \rho_\sigma \widetilde{\sigma}^2 + \sqrt{1-(\rho_\sigma)^2}\sigma_\varepsilon^2$ denote the observable variance consisting of the true unobservable variance $\widetilde{\sigma}^2$ and the noise σ_ε^2 where $\widetilde{\sigma}^2$ and σ_ε^2 are independently and identically distributed. ρ_σ is a signal-to-noise ratio and takes a value between 0 and 1. The higher the ρ_σ, the less noisy the σ^{*2} is. Let VSR^* denote the price for a variance swap contract settled by the noisy variance σ^{*2}. For simplicity, assume that there are only one producer and one speculator. FOCs of the producer's $MVSV = Y\mu + V_F(\mu - F) - V_\sigma VSR^* - \frac{1}{\gamma}\log M_{\sigma^2}(t_p) - \frac{1}{\gamma}\log M_{\sigma^2}(s_p)$, FOCs of the speculator's $MVSV = V_F(\mu - F) - V_\sigma VSR^* - \frac{1}{\gamma}\log M_{\sigma^2}(t_S) - \frac{1}{\gamma}\log M_{\sigma^2}(s_S)$, and market equilibrium is characterized as

$$VSR^* = \frac{M'_{\widetilde{\sigma}^2}(t_P)}{M_{\widetilde{\sigma}^2}(t_P)}\rho_\sigma + \frac{M'_{\widetilde{\sigma}^2}(s_P)}{M_{\widetilde{\sigma}^2}(s_P)}\sqrt{1-(\rho_\sigma)^2}, \tag{36}$$

$$VSR^* = \frac{M'_{\widetilde{\sigma}^2}(t_S)}{M_{\widetilde{\sigma}^2}(t_S)}\rho_\sigma + \frac{M'_{\widetilde{\sigma}^2}(s_S)}{M_{\widetilde{\sigma}^2}(s_S)}\sqrt{1-(\rho_\sigma)^2}, \tag{37}$$

and

$$E[R] = \rho Y\left(\gamma_P \frac{M'_{\widetilde{\sigma}^2}(t_P)}{M_{\widetilde{\sigma}^2}(t_P)} + \gamma_S \frac{M'_{\widetilde{\sigma}^2}(t_S)}{M_{\widetilde{\sigma}^2}(t_S)}\right) \tag{38}$$

where $t_P = \gamma_P \left[\frac{\gamma_P}{2} \left(Y^2 + \left(V_F^P\right)^2 + 2\rho Y V_F^P \right) - \rho_\sigma V_\sigma^P \right]$, $s_P = -\gamma_P \sqrt{1-(\rho_\sigma)^2} V_\sigma^P$,

$t_s = \gamma_S \left[\frac{\gamma_S}{2} \left(V_F^S\right)^2 - \rho_\sigma V_\sigma^S \right]$ and $s_S = -\gamma_S \sqrt{1-(\rho_\sigma)^2} V_\sigma^S$. As ρ_σ approaches to 1, (38) approaches to (26).

Observe that (36) and (37) have two parts. The first parts contain $\frac{M'_{\tilde{\sigma}^2}(t_P)}{M_{\tilde{\sigma}^2}(t_P)}$ and $\frac{M'_{\tilde{\sigma}^2}(t_S)}{M_{\tilde{\sigma}^2}(t_S)}$, which are "pure" VSRs. In the presence of measurement error, the "pure" VSR for the producer is not necessarily the same as the "pure" VSR for the speculator. On the other hand, the second parts contain $\frac{M'_{\tilde{\sigma}^2}(s_P)}{M_{\tilde{\sigma}^2}(s_P)}$ and $\frac{M'_{\tilde{\sigma}^2}(s_S)}{M_{\tilde{\sigma}^2}(s_S)}$, which are noises. Because the first parts are weighted by ρ_σ, the signal-to-noise ratio, and the second parts are weighted by $\sqrt{1-(\rho_\sigma)^2}$, VSR^* is the weighted average of the "true" VSR and the noise, i.e., it is a noisy estimator of "true" VSR. $\sqrt{1-(\rho_\sigma)^2}$ corresponds to the noise-to-signal ratio in Hansen and Lunde (2006) and Andersen, Bollerslev and Meddahi (2011). Specifically, Andersen, Bollerslev and Meddahi (2011) use 0.1% and 0.5% noise-to-sigma ratio, which are relatively small. Finally observe from (38) that the right hand side contains only "pure" VSR and as the noise-to-sigma ratio approaches to zero, (38) approaches to the empirical hypothesis 2.

Leverage Effect. In the main paper, we assume no correlation between S and σ^2. If such a correlation is negative (the leverage effect), the empirical hypothesis 2 remains the same. If such a correlation is positive (the reverse leverage effect) but small, the two empirical hypotheses remain the same.

Let β represent the degree of leverage and "reverse" leverage effects. Specifically, $\beta < 0$ represents the leverage effect and $\beta > 0$ represents the "reverse" leverage effect. Let spot prices follow $\left[\begin{bmatrix} S^* \\ S \end{bmatrix} \mid \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right] \sim N((\mu + \beta\left(\sigma^2 - \overline{\sigma}^2\right)) \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix})$ where $\overline{\sigma}^2$ is the the expected variance. For simplicity, assume that there are only one producer and one speculator. The producer's $MVSV = Y\left(\mu - \beta\overline{\sigma}^2\right) + V_F(\left(\mu - \beta\overline{\sigma}^2\right) - F) - V_\sigma VSR - \frac{1}{\gamma}\log M_{\sigma^2}(t_p)$ where $t_P = \gamma_P \left[\frac{\gamma_P}{2} \left(Y^2 + \left(V_F^P\right)^2 + 2\rho Y V_F^P \right) - \beta\left(Y + V_F^P\right) - V_\sigma^P \right]$. The speculator's $MVSV = V_F(\left(\mu - \beta\overline{\sigma}^2\right) - F) - V_\sigma VSR - \frac{1}{\gamma}\log M_{\sigma^2}(t_S)$,where $t_S = \gamma_S \left[\frac{\gamma_S}{2} \left(V_F^S\right)^2 - \beta V_F^S - V_\sigma^S \right]$. FOCs of these MVSVs and market clearing conditions give the required results, it holds that

$$E\left[R\right] = \gamma^{*}\rho Y \overline{\sigma}^{2} + (\beta - \gamma^{*}\rho Y)VRP. \tag{39}$$

or

$$\frac{\partial E\left[R\right]}{\partial VRP} = +(\beta - \gamma^{*}\rho Y). \tag{40}$$

The economic intuition is as follows: If β is extremely high and positive, the futures contract and variance swap contracts play the same economic role: the hedging instrument for both price risk and variance risk. Because the producer takes short positions in both the futures and variance swap contracts in equilibrium, the VRP positively predict futures returns. Unless β is extremely high, which is not the case in most of commodity markets, VRP negatively predict futures returns.

Appendix B: Proofs

MVSV of the Producer. The expectation and variance of producer i's wealth can be expressed as $E\left[W_{P_i}\right] = Y_i\mu + V_F^{P_i}(\mu - F) + V_\sigma^{P_i}\left(\overline{\sigma}^2 - VSR\right)$ and $var\left[W_{P_i} | \widetilde{\sigma}^2 = \overline{\sigma}^2\right] = \left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right)\overline{\sigma}^2$. By the law of iterated expectations,

$$E\left[u(W_{P_i})\right] = E\left[E\left[-\exp\left\{-\gamma_{P_i}\left(Y_i S^{*} + V_F^{P_i}(S - F) + V_\sigma^{P_i}(\widetilde{\sigma}^2 - VSR)\right)\right\} | \widetilde{\sigma}^2\right]\right].$$

Because $\left(S | \widetilde{\sigma}^2\right)$ and $\left(S^{*} | \widetilde{\sigma}^2\right)$ follow a bivariate normal distribution,

$$\begin{aligned} & E\left[u(W_{P_i})\right] \\ = \quad & -\exp\{-\gamma_{P_i}\left(Y_i\mu + V_F^{P_i}(\mu - F) - V_\sigma^{P_i} VSR\right)\} E\left[\exp\left\{-\gamma_{P_i}\left(V_\sigma^{P_i} - \frac{\gamma_{P_i}}{2}\left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right)\right)\widetilde{\sigma}^2\right\}\right] \\ = \quad & -\exp\left\{-\gamma_{P_i}\left(Y_i\mu + V_F^{P_i}(\mu - F) - V_\sigma^{P_i} VSR - \frac{1}{\gamma_{P_i}}\log M_{\widetilde{\sigma}^2}\left(\frac{\gamma_{P_i}^2}{2}\left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right) - \gamma_{P_i} V_\sigma^{P_i}\right)\right)\right\} \end{aligned}$$

which is monotonically increasing in

$$MVSV^{P_i} \equiv Y_i\mu + V_F^{P_i}(\mu - F) - V_\sigma^{P_i} VSR - \frac{1}{\gamma_{P_i}} \log M_{\widetilde{\sigma}^2}\left(\frac{\gamma_{P_i}^2}{2}\left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right) - \gamma_{P_i} V_\sigma^{P_i}\right).$$

Equilibrium Open Variance Position. (3), (7), (8), (11), and (14) imply

$$t^* = \sum_{i=1,\dots,n_P} \frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{P_i}^2}{2}\left(Y_i^2 + 2\rho_i Y_i V_F^{P_i} + \left(V_F^{P_i}\right)^2\right)\right] + \sum_{j=1,\dots,n_S} \frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{S_j}^2}{2}\left(V_F^{S_j}\right)^2\right]. \quad (41)$$

Substituting (17) and (18) into (41), we have

$$t^* = \sum_{i=1,\dots,n_P} \frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{P_i}^2}{2}\left(1-\rho_i^2\right) Y_i^2 + \frac{\gamma_{P_i}^2}{2}\left(\frac{\left(1/\gamma_{P_i}\right)}{(1/\gamma^*)} Y^*\right)^2\right] + \sum_{j=1,\dots,n_S} \frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)}\left[\frac{\gamma_{S_j}^2}{2}\left(\frac{\left(1/\gamma_{S_j}\right)}{(1/\gamma^*)} Y^*\right)^2\right] > 0. \quad (42)$$

The Negativity of Variance Risk Premia. Assume that $\widetilde{\sigma}^2$ is a continuous random variable. Let $f(\widetilde{\sigma}^2)$ be a p.d.f of $\widetilde{\sigma}^2$. $VSR = \frac{M'_{\widetilde{\sigma}^2}(t^*)}{M_{\widetilde{\sigma}^2}(t^*)} = E\left[\widetilde{\sigma}^2 \frac{\exp(\widetilde{\sigma}^2 t^*)}{E\left[\exp(\widetilde{\sigma}^2 t^*)\right]}\right] = \widetilde{E}\left[\widetilde{\sigma}^2\right]$ where $\widetilde{E}[\cdot]$ is the expectation under the p.d.f of $g(\widetilde{\sigma}^2) \equiv \frac{\exp(\widetilde{\sigma}^2 t^*)}{E\left[\exp(\widetilde{\sigma}^2 t^*)\right]} f(\widetilde{\sigma}^2)$. Because $t^* > 0$ from (42) and $\frac{\exp(\widetilde{\sigma}^2 t^*)}{E\left[\exp(\widetilde{\sigma}^2 t^*)\right]}$ is monotonically increasing in $\widetilde{\sigma}^2$, $g(\widetilde{\sigma}^2) \le f(\widetilde{\sigma}^2)$ for a sufficiently low $\widetilde{\sigma}^2$ and $g(\widetilde{\sigma}^2) \ge f(\widetilde{\sigma}^2)$ for a sufficiently high $\widetilde{\sigma}^2$. Hence, there exists a constant $K > 0$ such that $g(\widetilde{\sigma}^2) \le f(\widetilde{\sigma}^2)$ for $\widetilde{\sigma}^2 \le K$ and $g(\widetilde{\sigma}^2) \ge f(\widetilde{\sigma}^2)$ for $\widetilde{\sigma}^2 > K$. Because both $f(\widetilde{\sigma}^2)$ and $g(\widetilde{\sigma}^2)$ are p.d.f., $\int_0^K \left(f(\widetilde{\sigma}^2) - g(\widetilde{\sigma}^2)\right) d\widetilde{\sigma}^2 = \int_K^\infty \left(g(\widetilde{\sigma}^2) - f(\widetilde{\sigma}^2)\right) d\widetilde{\sigma}^2$. It follows that $\int_0^K \widetilde{\sigma}^2 \left(f(\widetilde{\sigma}^2) - g(\widetilde{\sigma}^2)\right) d\widetilde{\sigma}^2 \le \int_K^\infty \widetilde{\sigma}^2 \left(g(\widetilde{\sigma}^2) - f(\sigma^2)\right) d\widetilde{\sigma}^2$. Arranging the terms, we have $\int_0^K \widetilde{\sigma}^2 f(\widetilde{\sigma}^2) d\widetilde{\sigma}^2 + \int_K^\infty \widetilde{\sigma}^2 f(\widetilde{\sigma}^2) d\widetilde{\sigma}^2 \le \int_0^K \widetilde{\sigma}^2 g(\widetilde{\sigma}^2) d\widetilde{\sigma}^2 + \int_K^\infty \widetilde{\sigma}^2 g(\widetilde{\sigma}^2) d\widetilde{\sigma}^2$ and $\int_0^\infty \widetilde{\sigma}^2 f(\widetilde{\sigma}^2) d\widetilde{\sigma}^2 \le \int_0^\infty \widetilde{\sigma}^2 g(\widetilde{\sigma}^2) d\widetilde{\sigma}^2$, which means that $VSR = E\left[\widetilde{\sigma}^2 \frac{\exp(\widetilde{\sigma}^2 t^*)}{E\left[\exp(\widetilde{\sigma}^2 t^*)\right]}\right] \ge E\left[\widetilde{\sigma}^2\right]$. A proof of the case that $\widetilde{\sigma}^2$ is a discrete random variable is similar to the case of a continuous random variable.

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Table 1: Summary statistics of RV, VSR and VRP

	Mean (%)	Std. Dev (%)	Newey-West t-stat	Skew	Kurt	AR(1)	Percentile 25%	50%	75%
$RV^{average}$ (M1)	11.02	11.09	(7.68)	4.31	27.14	0.79	5.90	8.58	12.00
$RV^{average}$ (M3)	10.93	10.11	(7.99)	4.28	26.23	0.89	6.12	8.70	11.88
$RV^{average}$ (M6)	8.06	6.94	(8.13)	3.99	22.93	0.95	4.74	6.54	8.64
RV^{daily}(M1)	11.34	12.34	(7.35)	3.43	16.49	0.67	5.15	8.42	12.58
RV^{daily}(M3)	11.06	10.29	(7.92)	3.61	19.74	0.87	5.52	8.83	12.44
RV^{daily}(M6)	9.39	7.94	(8.25)	3.77	20.97	0.95	5.90	7.90	10.43
VSR (M1)	13.57	12.71	(7.88)	3.74	22.51	0.85	7.29	10.97	15.74
VSR (M3)	13.48	10.74	(9.35)	3.21	17.17	0.84	7.44	11.31	15.09
VSR (M6)	10.77	7.34	(10.25)	2.42	12.43	0.89	6.01	9.33	13.12
VRP1	-1.65	10.29	(-1.18)	-2.82	15.98	0.83	-4.10	-0.11	3.68
VRP3	-1.87	6.52	(-2.77)	2.25	35.75	0.45	-3.73	-1.09	0.91
VRP6	-2.46	5.58	(-3.78)	0.01	8.46	0.73	-4.90	-1.67	-0.07

Notes: We report the mean, standard deviation, Newey-West t-statistics, skewness, kurtosis, the first order autocorrelation and percentiles of each variable for futures with maturity of 1-, 3- and 6-month. $RV^{average}$ is the realized variance calculated using high-frequency data and the methodology of Andersen, Bollerslev and Meddahi (2011). RV^{daily} is the realized variance calculated using daily returns. VSR is the implied variance extracted from options written on 1, 3-, and 6-month futures. VRP are the variance risk premia based on the equation (25) when we use a linear regression model and historical observation of $RV^{average}$ to get the predicted RV and then get VRP. The data sample covers our predicting exercise period of Jan 1991 to Nov 2010.

Table 2: Economic information contained in VRP

Panel A: Dependent variable: VRP extracted from the closest to maturity M1 contract (VRP1)

	(1)	(2)	(3)	(4)	(5)	(6)
Lag(VRP1)	0.77***	0.72***	0.67***	0.58***	0.57***	0.57***
	(9.16)	(10.36)	(10.07)	(9.24)	(9.67)	(8.39)
Spot Return		0.07		0.05	0.05	0.04
		(0.49)		(0.34)	(0.32)	(0.31)
Basis		-0.94*		-1.23**	-1.22**	-1.23**
		(-1.86)		(-2.23)	(-2.15)	(-2.06)
Log(StorageLevel)		0.20***		0.27***	0.28***	0.25***
		(3.30)		(3.83)	(3.81)	(3.36)
CFTC Position		0.04		0.02	0.02	0.02
		(0.26)		(0.15)	(0.12)	(0.18)
Chicago Fed Index			0.02***	0.03***		
			(2.88)	(3.88)		
Chicago Fed Index_MA					0.03***	
					(3.61)	
ADS Index						0.03***
						(3.57)
Constant	-0.00	-2.48***	-0.00	-3.42***	-3.52***	-3.19***
	(-0.96)	(-3.30)	(-0.34)	(-3.83)	(-3.81)	(-3.36)
Observations	251	251	251	251	251	251
Adjusted R^2	0.60	0.62	0.62	0.65	0.64	0.65

Panel B: Dependent variable: VRP extracted from the third closest to maturity M3 contract (VRP3)

	(1)	(2)	(3)	(4)	(5)	(6)
Lag(VRP3)	0.52***	0.45***	0.51***	0.44***	0.44***	0.44***
	(8.49)	(6.32)	(7.15)	(5.95)	(5.66)	(6.03)
Spot Return		-0.10		-0.10	-0.10	-0.10
		(-1.08)		(-1.15)	(-1.20)	(-1.18)
Basis		-1.17		-1.13	-1.12	-1.12
		(-1.50)		(-1.46)	(-1.44)	(-1.43)
Log(StorageLevel)		0.25***		0.25***	0.26***	0.25***
		(2.79)		(2.81)	(2.88)	(2.63)
CFTC Position		0.10		0.09	0.09	0.10
		(0.88)		(0.91)	(0.88)	(0.93)
Chicago Fed Index			0.01	0.01		
			(0.89)	(1.27)		
Chicago Fed Index_MA					0.01	
					(1.02)	
ADS Index						0.01
						(1.29)
Constant	-0.01	-3.18***	-0.01	-3.24***	-3.27***	-3.16***
	(-1.48)	(-2.80)	(-1.54)	(-2.81)	(-2.89)	(-2.64)
Observations	251	251	251	251	251	251
Adjusted R^2	0.27	0.30	0.27	0.30	0.30	0.30

Panel C: Dependent variable: VRP extracted from the sixth closest to maturity M6 contract (VRP6)

	(1)	(2)	(3)	(4)	(5)	(6)
Lag(VRP6)	0.74***	0.68***	0.75***	0.68***	0.68***	0.68***
	(7.82)	(6.10)	(7.87)	(6.14)	(6.06)	(6.14)
Spot Return		0.11		0.10	0.10	0.10
		(1.64)		(1.62)	(1.59)	(1.63)
Basis		-0.66		-0.62	-0.60	-0.63
		(-1.48)		(-1.46)	(-1.43)	(-1.46)
Log(StorageLevel)		0.13*		0.12	0.13	0.12
		(1.72)		(1.61)	(1.57)	(1.64)
CFTC Position		-0.23**		-0.23**	-0.23**	-0.23**
		(-1.99)		(-1.99)	(-2.01)	(-1.97)
Chicago Fed Index			0.00	0.00		
			(1.15)	(0.85)		
Chicago Fed Index_MA					0.01	
					(1.22)	
ADS Index						0.00
						(0.43)
Constant	-0.00	-1.59*	-0.00	-1.57	-1.61	-1.55
	(-1.00)	(-1.73)	(-0.82)	(-1.61)	(-1.57)	(-1.64)
Observations	251	251	251	251	251	251
Adjusted R^2	0.55	0.58	0.55	0.58	0.58	0.58

Notes: This table reports linear regression results of economic variables and the lagged VRP on VRP. Spot return is the monthly log return of WTI spot prices. For each maturity of futures contract, basis is calculated by $(F_{t,T}/S)^{1/(T-t)}-1$. Log(StorageLevel) is the natural logarithm of U.S. stocks of crude oil excluding SPR published by U.S. Energy Information Administration. CFTC position is the short position minus long position of futures contracts divided by the summation of short and long positions held by the commercial traders in the CFTC report. Chicago Fed Index (and Chicago Fed Index_MA) is the Chicago Fed National Activity Index (and its three-month moving average). ADS Index is the Aruoba-Diebold-Scotti Business Condition Index available from the Federal Reserve Bank of Philadelphia. For each variable, we report the coefficient and Newey-West t-statistics with 12 lags. *, **, and *** denote the significant level of 10%, 5%, and 1% respectively.

Table 3: Summary statistics of predictors of crude oil futures returns

				Correlation Matrix						
	Mean (%)	Std. Dev. (%)	AR(1)	Short Rate	Yield Spread	Basis (M1)	CFTC Position	Open Interest Growth (M1)	Historical Return (M1)	VRP3
Short Rate	3.42	1.88	0.97							
Yield Spread	0.95	0.45	0.96	-0.53						
Basis (M1)	-0.05	0.40	0.10	-0.07	0.01					
CFTC Position	1.47	5.61	0.63	-0.07	0.03	0.10				
Open Interest Growth (M1)	0.68	1.59	0.63	-0.16	-0.19	-0.02	-0.01			
Historical Return (M1)	0.53	2.64	0.91	0.04	-0.32	0.07	0.37	0.03		
VRP3	-1.87	6.52	0.45	0.40	-0.33	-0.16	0.07	0.02	0.05	
VRP6	-2.46	5.58	0.73	0.37	-0.30	-0.12	-0.11	-0.03	0.15	0.55
Basis (M3)	-0.08	1.16	0.82							
Open Interest Growth (M3)	0.54	2.18	0.56							
Historical Return (M3)	0.54	2.43	0.91							
Basis (M6)	-0.17	1.09	0.86							
Open Interest Growth (M6)	0.51	2.88	0.53							
Historical Return (M6)	0.56	2.23	0.92							

Notes: We report the mean, standard deviation, the first order autocorrelation for predictors in all three maturities, and the correlation matrix of predictors for the 1-month maturity (M1) contract returns. Short rate is the monthly average of the one-month T-Bill rates. Yield spread is the difference between Moody's Baa and Aaa corporate yields. For each maturity of futures contract, basis is calculated by $(F_{t,T}/S)^{1/(T-t)}$-1. CFTC position is the short minus long position of futures contracts divided by the summation of short and long positions held by large commercial traders in the CFTC report. Open interest growth is the 12-month geometrical average of open interest growth of the futures contract. Historical return is the 12-month geometrical average of futures returns. This table reports statistics for our predicting exercise period of Jan 1991, when we are able to calculate open interest growth and historical returns, to Nov 2010.

Table 4: Predictability of crude oil futures returns of VRP

Panel A: Predictability of monthly returns of the closest to maturity futures contract (M1)

	(1)	(2)	(3)	(4)	(5)	(6)
Short Rate	-0.44	-0.09	-0.46	-0.08	-0.11	-0.39
	(-1.58)	(-0.29)	(-1.61)	(-0.26)	(-0.35)	(-1.19)
Yield Spread	-1.30	-2.16	-1.70	-2.15	-2.33	-2.15
	(-0.64)	(-1.20)	(-0.69)	(-1.18)	(-1.25)	(-0.92)
Basis	0.81	0.23	0.86	0.26	0.25	0.60
	(0.30)	(0.10)	(0.32)	(0.12)	(0.11)	(0.22)
CFTC Position	0.07	0.03	0.11	0.05	0.05	0.12
	(0.91)	(0.52)	(1.28)	(0.67)	(0.67)	(1.42)
Open Interest Growth	-0.09	-0.11			-0.12	-0.10
	(-0.22)	(-0.36)			(-0.38)	(-0.26)
Historical Return			-0.22	-0.06	-0.07	-0.24
			(-0.75)	(-0.30)	(-0.34)	(-0.87)
VRP6		-0.44***		-0.44***	-0.44***	
		(-2.93)		(-2.92)	(-2.87)	
VRP3						-0.11
						(-0.86)
Constant	0.04	0.02	0.04	0.02	0.03	0.04
	(1.59)	(1.00)	(1.39)	(0.99)	(1.05)	(1.52)
Observations	239	239	239	239	239	239
Adjusted R^2	-0.01	0.05	-0.01	0.05	0.05	-0.01

Panel B: Predictability of monthly returns of the third closest to maturity futures contract (M3)

	(1)	(2)	(3)	(4)	(5)	(6)
Short Rate	-0.29	0.01	-0.39	-0.05	0.00	-0.25
	(-1.12)	(0.02)	(-1.41)	(-0.18)	(0.01)	(-0.85)
Yield Spread	-1.16	-1.70	-1.67	-1.96	-1.72	-1.61
	(-0.59)	(-1.08)	(-0.72)	(-1.21)	(-1.06)	(-0.78)
Basis	0.32	0.06	0.32	0.11	0.05	0.24
	(0.76)	(0.10)	(0.81)	(0.21)	(0.08)	(0.49)
CFTC Position	0.09	0.04	0.11	0.05	0.04	0.11
	(1.32)	(0.70)	(1.37)	(0.78)	(0.73)	(1.51)
Open Interest Growth	0.18	0.14			0.14	0.16
	(0.56)	(0.52)			(0.52)	(0.50)
Historical Return			-0.12	-0.00	-0.01	-0.14
			(-0.41)	(-0.02)	(-0.05)	(-0.47)
VRP6		-0.39***		-0.39***	-0.39***	
		(-2.94)		(-2.85)	(-2.94)	
VRP3						-0.08
						(-0.87)
Constant	0.03	0.01	0.04	0.02	0.01	0.03
	(1.22)	(0.64)	(1.35)	(0.95)	(0.63)	(1.18)
Observations	239	239	239	239	239	239
Adjusted R^2	-0.01	0.06	-0.01	0.05	0.05	-0.01

Panel C: Predictability of monthly returns of the sixth closest to maturity futures contract (M6)

	(1)	(2)	(3)	(4)	(5)	(6)
Short Rate	-0.37*	-0.09	-0.42*	-0.09	-0.09	-0.35
	(-1.68)	(-0.37)	(-1.67)	(-0.38)	(-0.38)	(-1.37)
Yield Spread	-1.16	-1.73	-1.46	-1.75	-1.75	-1.54
	(-0.68)	(-1.26)	(-0.72)	(-1.21)	(-1.23)	(-0.81)
Basis	-0.02	-0.20	-0.11	-0.20	-0.20	-0.11
	(-0.06)	(-0.46)	(-0.30)	(-0.45)	(-0.45)	(-0.28)
CFTC Position	0.08	0.05	0.10	0.05	0.05	0.10
	(1.48)	(0.85)	(1.50)	(0.93)	(0.87)	(1.64)
Open Interest Growth	0.08	0.00			0.00	0.08
	(0.36)	(0.02)			(0.02)	(0.35)
Historical Return			-0.15	-0.01	-0.01	-0.15
			(-0.48)	(-0.07)	(-0.07)	(-0.49)
VRP6		-0.36***		-0.36***	-0.36***	
		(-2.84)		(-2.78)	(-2.87)	
VRP3						-0.06
						(-0.76)
Constant	0.03	0.02	0.04	0.02	0.02	0.03
	(1.52)	(1.04)	(1.42)	(1.04)	(1.03)	(1.41)
Observations	239	239	239	239	239	239
Adjusted R^2	-0.01	0.06	-0.01	0.06	0.06	-0.01

Notes: We report the predictability of monthly returns of 1-, 3- and 6-month maturity futures contracts. All predictor variables are lagged by one month. We report the coefficients and Newey-West t-statistics with 12 lags. *, **, and *** represent the significant level at 10%, 5% and 1%. The predictor data is from Jan 1991 to Nov 2010. Definition of variables is same as Table 3.

Table 5: Rolling window realized profits and root mean squared errors

Panel A: Predictor variables we used in each model.

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Short Rate	+	+	+	+	+	+	+
Yield Spread	+	+	+	+	+	+	+
Basis	+	+	+			+	+
CFTC Position	+	+	+			+	+
Historical Return	+		+			+	+
Open Interest Growth		+	+			+	+
VRP3				+		+	
VRP6					+		+

Panel B: The ratio of average returns to their standard deviations in each model.

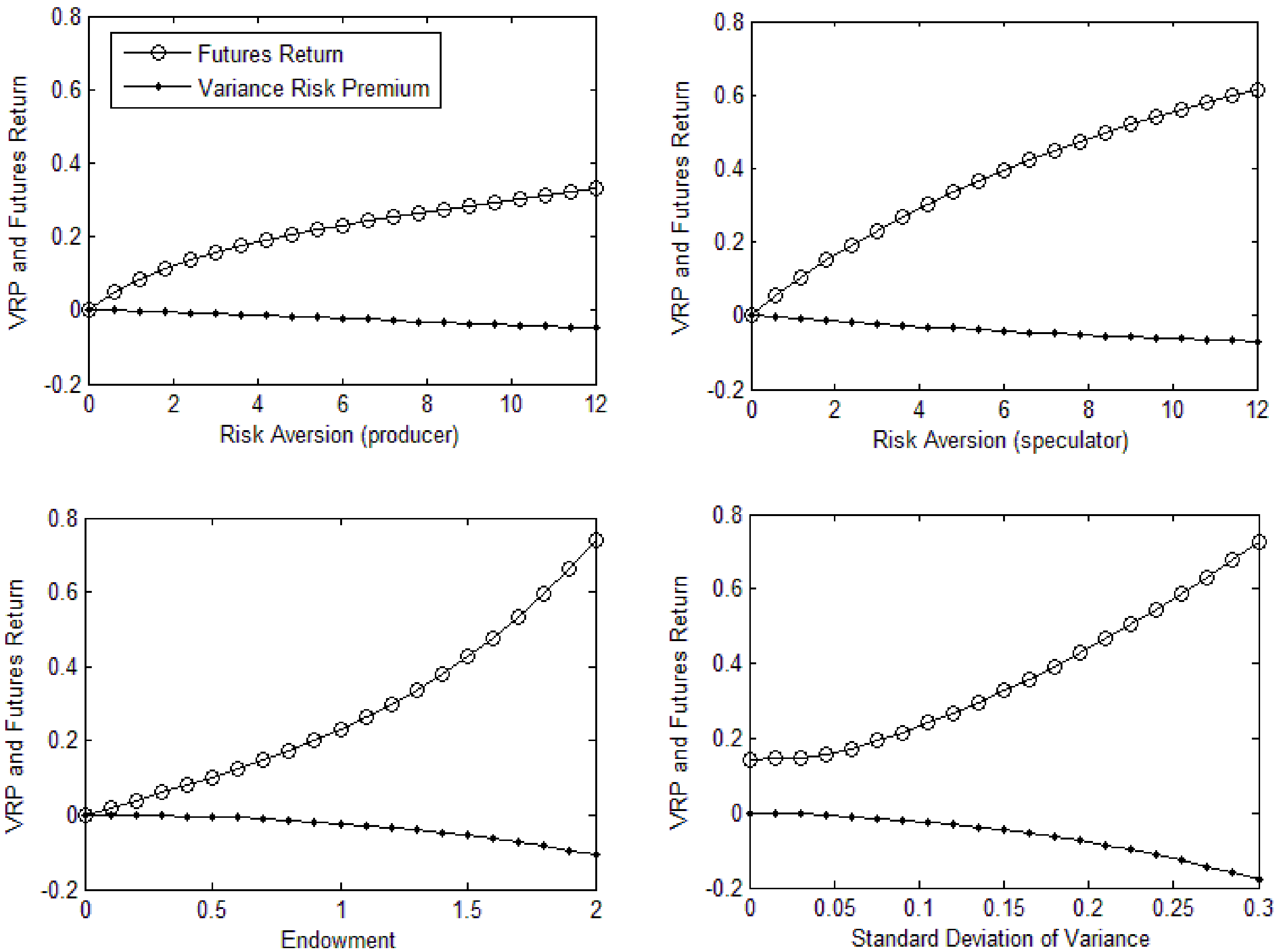
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Rolling M1 Futures	0.17	0.08	0.09	**0.18**	**0.27**	0.14	0.11
Rolling M3 Futures	0.25	0.20	0.20	0.25	**0.28**	**0.28**	0.23
Rolling M6 Futures	0.28	0.24	0.26	0.30	**0.36**	0.27	**0.34**

Panel C: Root mean squared errors in each model (in %).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Rolling M1 Futures	10.08	10.14	10.11	10.09	**9.76**	10.36	**10.03**
Rolling M3 Futures	9.08	9.11	9.18	**9.04**	**8.79**	9.30	9.09
Rolling M6 Futures	8.12	8.12	8.18	**8.01**	**7.81**	8.27	8.07

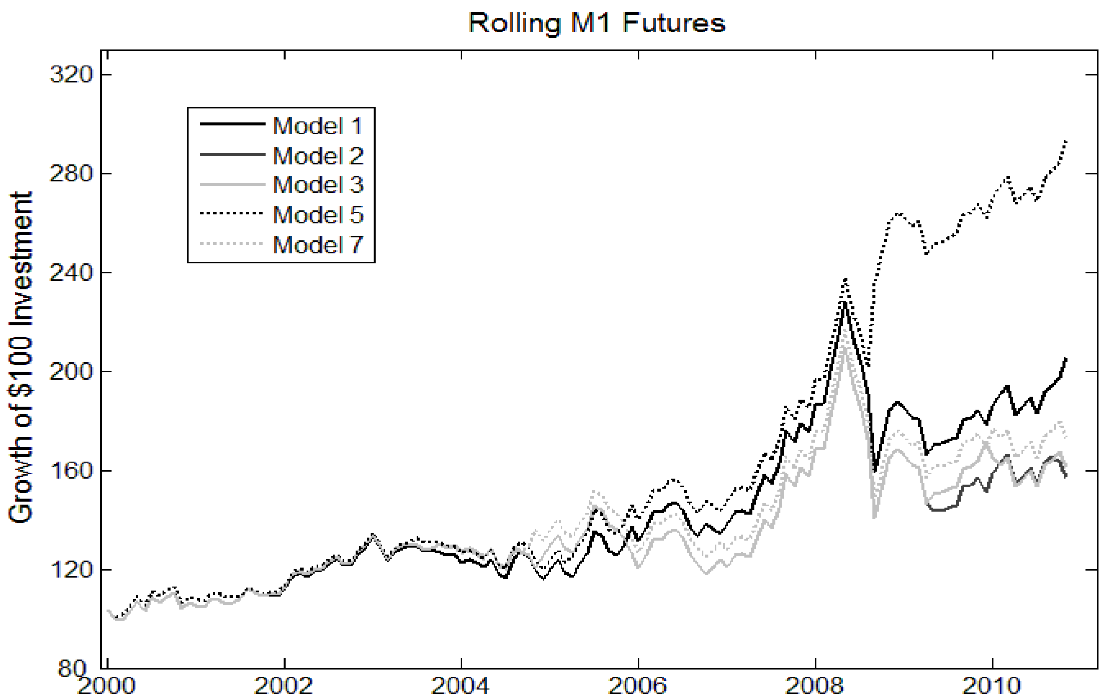
Notes: We report the out-of-sample performance of seven candidate models including various predictors. We estimate our model based on a 9-year window (the first window is from Jan 1991, when we can calculate the 12-month geometrical average of open interest growth and historical returns, to Dec 1999) and we predict returns for the following month. We start to trade in Jan 2000 and each time we roll our window one month forward till the end of 2010. Panel A shows the predictors we use in each model. We set $100 USD as a buffer fund in Jan 2000, and each month we long/short a barrel of crude oil futures based on the trading signals predicted by the model. We assume that free cash in the buffer fund earns interest of 5% per annum. Reported numbers in Panel B are the ratio of average monthly returns divided by their standard deviation from Jan 2000 to Dec 2010. The two highest ratios for each maturity are boldfaced. Panel C reports root mean squared errors (RMSE) between predicted and observed gross returns from Jan 2000 to Dec 2010. The two lowest RMSEs for each maturity are boldfaced.

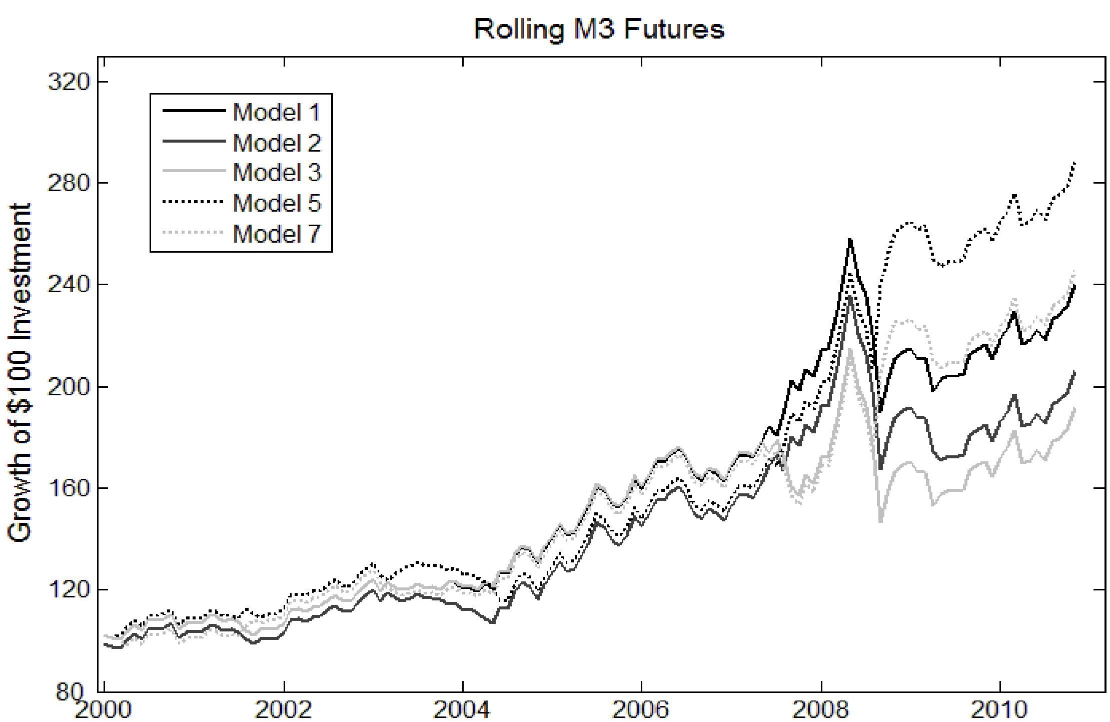
Figure 1: Negative relation of expected futures return and variance risk premium in the model

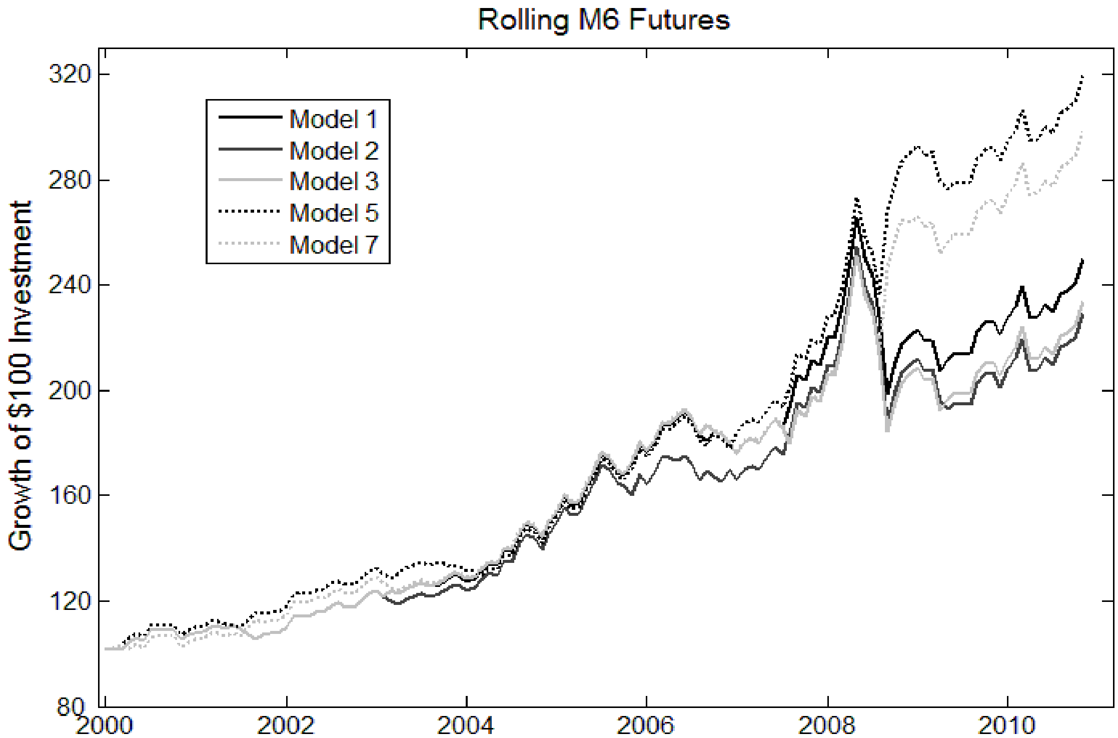


Notes: We plot equilibrium expected futures return E[R] and variance risk premia VRP as implied by the model. The two top graphs show how the relation between E[R] and VRP changes when we increase the risk aversion of the producer and the speculator; the two bottom graphs show how the relation between E[R] and VRP changes if we increase producer's endowment and the standard deviation of variance ($\sigma_{\sigma 2}$). In each graph, we set the initial value of parameters as γ_P =6, γ_S =3, Y=1, μ=3, $\boldsymbol{\sigma}_{\sigma^2}$= 0.1, ϱ=0.8 and the mean of σ^2 = 0.3.

Figure 2: Account balance evolution if we trade crude oil futures based on model predictions.







Notes: We plot the account balance evolution of trading crude oil futures based on the buying/selling signals from model predictions. Model specifications are described in the Panel A of Table 5. Model 5 and 7 use VRP as a predictor and other three do not. We estimate the models based on a 9-year window (the first window is from Jan 1991, when we can calculate the 12-month geometrical average of open interest growth and historical returns, to Dec 1999). Then we predict returns for the following month and invest accordingly in Jan 2000. At the end of each month we buy/sell M1, M3 and M6 crude oil futures and we track the account growth. Every time we roll our window one month forward till the end of 2010. For each model the account starts from $100. We assume free cash in the account earns interest rate of 5% per annum.