Components of the Bid-Ask Spread and Variance:

A Unified Approach

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Abstract: We develop a structural model for the price formation and liquidity supply of an asset. Our model facilitates decompositions of both the bid-ask spread and the return variance into components related to adverse selection, inventory, and order processing costs. Furthermore, the model shows how the fragmentation of trading volume across trading venues influences inventory pressure and price discovery. We use the model to analyze intraday price formation for gold futures traded at the Shanghai Futures Exchange. We find that order processing costs explain about 50% of the futures bid-ask spread, while the remaining 50% is equally due to asymmetric information and to inventory costs. About a third of the variance in futures returns is attributable to microstructure noise. Trading at the spot market has a significant influence on futures price discovery, but only a limited impact on the futures bid-ask spread.

Keywords: Decomposition; Bid-Ask Spread, Variance; Inventory Costs; Gold Futures

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1. INTRODUCTION

Understanding of market microstructure and its influence on prices and market quality is more important than ever. The last two decades have seen securities exchanges worldwide develop from being national monopolies with quote-driven trading systems to global competitive businesses with order-driven pricing mechanisms. Technological development, competition and legislation have led to trading and quoting at frequencies unimaginable before, opening for business models in the millisecond domain spanning multiple trading platforms. When the human eye is no longer able to monitor the markets in real time, the need to understand the principles for how market microstructure influences the strategic behavior of traders (both humans and algorithms) becomes increasingly important to investors and regulators alike. In a seminal paper, Madhavan, Richardson, and Roomans (MRR; 1997) develop a theoretical model that provides such understanding.[1] Even though their model was formulated before recent market developments, the principles of market microstructure that it captures remain largely the same in today's trading environment.

In this paper, we extend the univariate structural model by MRR (1997). The original model tracks elements of both market liquidity (bid-ask spread) and volatility (return variance) to microstructure aspects such as information asymmetry, order processing costs, and stochastic rounding errors. Our first contribution to the model is that we account for inventory costs, and hence are able to distinguish inventory pressure from other types of transitory effects on prices, liquidity, and volatility. As a second contribution, we demonstrate how the model can be expanded to consider trading of the same or a similar security at a secondary market.

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[1] The model in MRR (1997) is similar to the one developed by Huang and Stoll (1997), where the latter is slightly more general than the former. Related alternatives are models by Glosten and Harris (1988) and Sadka (2006). See Kim and Murphy (2013) for a comparative analysis of these four model alternatives.

The dynamic of our model is the following. After each recorded transaction, liquidity suppliers evaluate whether it conveys any information about the value of the asset and whether any public news with bearing on asset value has been released. Based on their analysis, they choose the midpoint for their bid and ask quotes. In doing so, they also make adjustments for inventory pressure that has built up during the trading day. Finally, they submit their buy and sell orders symmetrically around the chosen midpoint, at a distance related to costs they expect to incur if a quote is hit by a market order. Our treatment of the midpoint as a choice variable for liquidity suppliers is a novelty in this branch of theoretical models. By using the midpoint we are able to identify the differences between an asset's transaction price, its midpoint price, and the corresponding underlying value. These identifications form the basis for our decompositions of liquidity and volatility.

To stay competitive in modern financial markets, liquidity suppliers have to continuously monitor their inventories to minimize their cost of capital. It is well-known that the cost of inventory influences the bid-ask spread (Amihud and Mendelson, 1980; Huang and Stoll, 1997; Stoll, 1978), but its influence on volatility has not yet been thoroughly explored. Our implementation of inventory costs in the model by MRR (1997) improves the richness of our model by allowing a three-way bid-ask spread decomposition similar to that of Huang and Stoll (1997). Furthermore, it allows us to decompose return variance into public news and four different sources of microstructure noise, including inventory costs. The strength of our unified structural approach is that we obtain closed-form solutions for the bid-ask spread and the return variance as functions of our model parameters, enabling straightforward economic interpretations of the liquidity and volatility components. The richness of our model allows us to decompose not only the variance of returns, but also the variance of midpoint quote changes and the variance of unobserved asset value changes.

The decompositions of bid-ask spread and variance are certainly interesting in isolation, but also potentially important in combination. Asparouhova, Bessembinder and Kalcheva (2010) show that microstructure noise induces a bias in common asset pricing tests, in particular with respect to liquidity. As our structural model can quantify the same microstructure elements in return variance and bid-ask spreads, it may open for further understanding of that bias.

Deregulation of financial markets has sparkled the emergence of numerous new exchanges. Recent legislation has opened for multi-lateral trading facilities (MTFs) to compete with incumbent exchanges, causing a fragmentation of trading activity. As securities trade at multiple markets, liquidity suppliers need to be active at, or at least monitor all those venues. In an extension of our model, we allow liquidity suppliers to consider trades at a secondary market as part of the information flows of the security in question. Price formation hence becomes a function of public information as well as order flow information originating from multiple trading venues. Moreover, we assume that the same liquidity suppliers are active in both markets, and take a portfolio approach to their inventory management. The model extension relates to an extensive literature about price discovery across markets (Hasbrouck, 1995; De Jong and Schotman, 2010; Korenok, Mizrach and Radchenko, 2011). The model is valid both for the case of one security trading at many markets (such as equities trading at an incumbent exchange and an MTF), and for related securities trading at the same or different venues. Our empirical application of the model is concerned with the latter, namely the futures and spot market trading of gold in Shanghai.[2]

[2] Several studies empirically model spot and futures returns on an intraday basis, most of the time in an unstructured, reduced form fashion, without an explicit parameterization of the relationship between price changes and microstructure effects.

At a time when the general interest for gold as an investment is booming, China has quickly become a dominant global player in terms of both supply and demand. Shanghai is the hub of the Chinese gold trading, hosting the largest gold spot market in the world and the fourth most traded gold futures contract (by gold volume). We contribute to previous research by being the first to perform an intraday analysis of the market microstructure at the Shanghai futures and spot markets. More generally, our study is the first to consider microstructure effects in the price formation process at any gold market in the world.

Our empirical results lend support to the relevance of our theoretical model. Using the estimated parameters in our model to decompose the gold futures bid-ask spread, we find that, on average, almost 50% of the spread can be explained by order processing costs, while the remaining 50% is roughly equally divided into one part due to asymmetric information and another part due to inventory costs. Thus, it is clearly important to account for inventory costs in the spread decomposition at the Shanghai Futures Exchange (SHFE) gold futures market. Moreover, our model-implied bid-ask spread accounts for 87% of the average observed futures spread, which is high relative the performance of competing models. The implied spread for different trading sessions throughout the day also successfully captures the intraday decline of the observed spread.

According to our variance decomposition, about a third of the return volatility in gold futures is attributable to microstructure noise. Most of the microstructure noise is due to price discreteness and order processing costs, while the influence of asymmetric information and inventory pressure is marginal. Overall, variance is much higher during the afternoon, when gold trading hubs in Europe are about to open, than during the morning and pre-lunch trading sessions at the SHFE. As can be expected for a globally traded asset, the value innovations for gold are almost entirely due to public news rather than informed trading at the SHFE, and little public news arrive when it is night in both the United States and in Europe.

Finally, our empirical application shows that the influence of the spot market gold trading on SHFE gold futures bid-ask spreads is limited. Inventory pressure is not significantly affected by the spot market, perhaps indicating that liquidity suppliers at the futures market do not trade at the spot market. Trading at the spot market does, however, influence price discovery at the futures market.

Even though MRR (1997) formulated their structural model a long time ago, recent research has extended it in several directions, indicating that the principles described in the original model are still valid. For example, Grammig, Thiessen and Wünsche (2011) extend it to study how the sharply decreasing trade durations matter for the price impact of trading. Moreover, Korenok, Mizrach and Radchenko (2011) use the model to study price discovery between different exchanges. In fact, the accuracy of the model may have improved over time. A simplification of the real world in MRR (1997), and in our model, is that it does not account for trade volumes; implicitly assuming that trade volumes do not matter or is approximately equal across trades. Kim and Murphy (2013) show that trade volume dispersion has decreased substantially as algorithms nowadays slice up orders to minimize price impact. Accordingly, Kim and Murphy (2013) show that the estimation error of bid-ask spreads generated by MRR (1997) have decreased over time.

We present our model, the bid-ask spread decomposition, the return variance decomposition, and the extension to multiple trading venues in Section 2. In Section 3, we proceed by presenting our empirical investigation of the model. In doing so, we discuss the market structure and data of the Shanghai gold market, show how we estimate the model using GMM, and finally present the results of our analysis. Section 4 concludes.

2. THEORETICAL FRAMEWORK

We model the dynamic of price and liquidity supply for a financial asset. The setting can be interpreted as either a dealer market where a market maker takes one side of every trade, or as a limit order book market where liquidity supply is an aggregate of trading decisions by a collective of strategic investors. Our model allows a three-way decomposition of the bid-ask spread into asymmetric information costs with permanent impact on price, and transitory inventory and order processing costs. The model also allows for an identification of return variance components related to public news and at least four sources of microstructure noise (asymmetric information, inventory, order processing, and stochastic rounding errors). Thus, the model encompasses both the bid-ask spread decomposition from Huang and Stoll (1997) and the variance decomposition from MRR (1997). As an extension of the model, we show how a secondary market for the same (or a similar) financial asset can influence the liquidity supply in the primary market.

Our model setup shares several features with that of MRR (1997). Let each trade in the market represent an instant of time in the model, implying a transaction time setting with non-equally spaced time indexes $t = 1, \ldots, T$. Let x_t be an indicator variable for the initiation of the trade at time t, where $x_t = +1$ if the trade is buyer initiated, $x_t = -1$ if the trade is seller initiated, and $x_t = 0$ when the trading direction is not identifiable. Assume that trade initiations from the buy and sell sides are unconditionally equally likely, and let $\lambda = \Pr[x_t = 0]$ be the unconditional probability that the trade direction is not identifiable. Hence, $E[x_t] = 0$ and $Var[x_t] = (1 - \lambda)$.

Following each trade, let μ_t denote the post-trade expected asset value, conditional upon both (i) public information flow and (ii) noisy signals originating from order flows (Glosten and Milgrom, 1985; MRR, 1997). Let ε_t represent the asset value innovation due to public information flows between $t-1$ and t, and assume that it is an independent, identically distributed, random variable with mean zero and variance σ_ε^2. Moreover, define asset value innovations related to order flow as $\theta(x_t - E[x_t|\Im_{t-1}]$, where $(x_t - E[x_t|\Im_{t-1}]$ is the unexpected order flow, $\Im_{t-1}$ denotes order flow information at, and before, time $t-1$, and θ measures the degree of information asymmetry, or the permanent impact of the order flow innovation. The update in asset value between times $t-1$ and t can then be written as:

$$\mu_t = \mu_{t-1} - \theta(x_t - E[x_t|\Im_{t-1}] + \varepsilon_t \tag{1}$$

In order to obtain an explicit expression for the expected order flow, define the conditional expectation as:

$$E[x_t|\Im_{t-1}] = \rho x_{t-1} \tag{2}$$

where ρ is the first-order autocorrelation coefficient of x_t.

So far, our model setup is exactly the same as in MRR (1997). In their model, liquidity suppliers set their bid and ask quotes regret-free around the asset value μ_t. We assume, in contrast to their model, but in line with Huang and Stoll (1997), that liquidity suppliers in addition consider their accumulated inventory when setting their price quotes. Our expression for the transaction price p_t is:

$$p_t = \mu_t + \delta \sum_{i=1}^{t} x_i + \tilde{\varphi} x_t + \xi_t \tag{3}$$

where δ is a coefficient determining the inventory influence on quoted prices; $\tilde{\varphi}$ represents other transitory effect of order flows on prices; and the error term ξ_t has a mean equal to zero and a variance of σ_ξ^2 and accounts for possible stochastic rounding errors. The coefficient $\tilde{\varphi}$ includes order processing costs and risk bearing costs, but in contrast to φ in MRR (1997), it does not include inventory costs. In our general framework, $\tilde{\varphi}$ and δ together reflect temporary effects of the order flow on the asset's price and quotes. As in Huang and Stoll (1997), we motivate this modeling decision by that liquidity suppliers seek compensation for not only concurrent costs of order flow, but also for inventory pressure from (undesired) accumulated positions since the start of the day ($t = 1$).

Our deviation from MRR (1997) is perhaps most straightforward to interpret in terms of the bid-ask spread midpoint, m_t, which is set just after the transaction in time t. Whereas liquidity suppliers in MRR (1997) set the midpoint equal to the asset value μ_t, we allow them to adjust their midpoint for accumulated inventory:

$$m_t = \mu_t + \delta \sum_{i=1}^{t} x_i \tag{4}$$

Once the midpoint is chosen, the quotes that are subject to trade in the next period are set regret-free and symmetrically around the midpoint at a distance $\tilde{\varphi} x_{t+1}$ in the same way as in MRR (1997).

Having established how prices and midpoints are determined, we derive expressions for the dynamics of asset returns and midpoint updates. By taking the first difference of Equation (3), combining it with Equations (1) and (2), we retrieve an expression for asset returns:

$$p_t - p_{t-1} = \alpha + (\theta + \tilde{\varphi} + \delta)x_t - (\tilde{\varphi} + \theta\rho)x_{t-1} + \varepsilon_t + \xi_t - \xi_{t-1} \tag{5}$$

where the constant drift term α is added for convenience.

By taking the first difference of Equation (4), and substituting for Equations (1) and (2), and adding an intercept ω, we obtain:

$$m_t - m_{t-1} = \omega + (\theta + \delta)x_t - \theta\rho x_{t-1} + \varepsilon_t \tag{6}$$

which implies that the asset's quote midpoint is updated between transactions to reflect the corresponding change in beliefs and inventory costs of the latest trade.

By estimating equations (2), (5), and (6) simultaneously, we can identify all parameters in the equations, as well as the variances σ_ε^2 and σ_ξ^2.

Decomposition of the bid-ask spread

After having adjusted the midpoint to m_t according to Equation (4), in response to the trade at time t, the liquidity suppliers prepare for the next transaction at time $t+1$ by quoting a new bid-ask spread around the midpoint. Based on Glosten and Milgrom's (1985) notion that market makers are ex-post rational, they choose regret-free ask and bid prices (conditional on $x_{t+1} = 1$ and $x_{t+1} = -1$ respectively). Hence, the ask price is $p_{t+1}^a = m_t + \theta(1 - E[x_{t+1}|\Im_t]) + \tilde{\varphi} + \delta + \varepsilon_{t+1}$, and the bid price satisfies $p_{t+1}^b = m_t - \theta(1 - E[x_{t+1}|\Im_t]) - \tilde{\varphi} - \delta + \varepsilon_{t+1}$.[3]

The difference between the ask and the bid price is the implied bid-ask spread at time $t+1$, $p_{t+1}^a - p_{t+1}^b = 2(\theta + \tilde{\varphi} + \delta)$. Evidently, the bid-ask spread corresponds to twice the sum of the asymmetric information parameter, the order processing cost parameter, and the parameter that represents the market makers' inventory costs. All these parameters are estimated within the model. As a result, we can evaluate how large a portion of the spread is due to asymmetric information, order processing costs, and inventory costs respectively.

[3] Combining these equations by insertion of x_{t+1} leads to the $t+1$ equivalent of Equation (3). For an analogous and more extensive discussion, see MRR (1997).

Decomposition of the return variance

We also decompose the asset transaction price volatility into different components by studying the variance of Equation (5):

$$Var(\Delta p_t) = (1-\lambda)[(\theta+\tilde{\varphi}+\delta)^2 + (\tilde{\varphi}+\theta\rho)^2 - 2(\theta+\tilde{\varphi}+\delta)(\tilde{\varphi}+\theta\rho)\rho] + \sigma_\varepsilon^2 + 2\sigma_\xi^2 \quad (7)$$

using the fact that $Var(x_t) = (1-\lambda)$ and $Cov(x_t, x_{t-1}) = \rho(1-\lambda)$. Following MRR (1997), we divide the variance of asset price changes into different parts. Firstly, independent of trading, variance arises from information innovations through σ_ε^2. Secondly, variance is caused by the trading process through at least four sources for microstructure noise; rounding errors in the pricing process as measured by σ_ξ^2, asymmetric information as measured by θ, order processing costs as measured by $\tilde{\varphi}$, and inventory costs as measured by δ. By rearranging terms in Equation (7), we define $A = (1-\lambda)(1-\rho)^2\theta^2$ as the part of the variance related only to asymmetric information, $B = 2(1-\lambda)(1-\rho)\tilde{\varphi}^2$ as the part related only to order processing costs, and $C = (1-\lambda)\delta^2$ as the part related only to inventory costs. In addition, we recognize three interaction terms in the variance equation; between asymmetric information and order processing costs $D = 2(1-\lambda)(1-\rho^2)\theta\tilde{\varphi}$, between asymmetric information and inventory costs $E = 2(1-\lambda)(1-\rho^2)\theta\delta$, and between order processing costs and inventory costs $F = 2(1-\lambda)(1-\rho^2)\tilde{\varphi}\delta$. Hence, the variance of Δp_t equals $\sigma_\varepsilon^2 + 2\sigma_\xi^2 + A + B + C + D + E + F$.

From the variance decomposition terms, we notice that, other things equal, each of the effects due to asymmetric information, order processing costs and inventory costs increases the variance of asset price changes. Moreover, a high probability to trade exactly at the bid-ask spread midpoint (λ) will reduce the microstructure effects on the variance of price changes.

Decomposition of the variance of midpoint quote changes

The variance of midpoint quote changes in Equation (6) can be written as:

$$Var(\Delta m_t) = (1 - \lambda)[(\theta + \delta)^2 + (\theta\rho)^2 - 2\rho^2(\theta + \delta)\theta] + \sigma_\varepsilon^2 \quad (8)$$

Like the variance of asset price changes in Equation (7), the variance of midpoint quote changes is driven by the variance of information innovations σ_ε^2. However, whereas the variance of price changes is driven by four sources for microstructure noise, we note that the variance of midpoint quote changes is affected only by two; namely asymmetric information and inventory costs. Accordingly, we can rewrite the variance of Δm_t as $\sigma_\varepsilon^2 + A + C + E$, where A, C, and E are the same expressions as above. Thus, we are able not only to decompose the variance of midpoint quote changes into different generating source, but we can also explicitly derive the difference between $Var(\Delta p_t)$ and $Var(\Delta m_t)$, where the former is exposed to rounding errors in the pricing process and order processing costs whereas the latter is not.

Decomposition of the variance of value changes

We can also obtain the variance of changes in asset value from Equation (1), after inserting the expression in Equation (2), according to:

$$Var(\Delta \mu_t) = (1 - \lambda)(1 - \rho^2)\theta^2 + \sigma_\varepsilon^2 \quad (9)$$

where the first term on the right hand side equals A. Hence, only innovations in public information and private information asymmetry have permanent impacts on the variance of changes in the unobserved asset value, while microstructure noise due to order processing, inventory and rounding errors in the pricing process only have temporary effects on transaction prices and/or quotes, and the associated variances.

Extension: One asset, two markets

We now assume that the financial asset is traded at two markets; a primary market, which is the same as above, and a secondary market. We further assume that the liquidity suppliers are providing liquidity at both markets. When that is true, they monitor information flows and inventory at both markets simultaneously.[4]

Timing is the same as in the one-market model, determined by transactions at the primary market. Trades at the secondary market never occur at exactly the same time as in the primary market, which allows us to regard secondary market trades as information flows in the intervals between primary market trades. This model feature is perfectly compatible with reality. Even in the most actively traded financial markets, primary and secondary market trades typically occur with at least a few milliseconds or microseconds in between. While primary market transactions occur at times t = 1, ..., T, we assume that either one or no trade occurs at the secondary market between two primary market trades.

Since the asset is traded on two markets, we recognize the possibility that the conditional expectation of the primary market trade initiation variable in Equation (2) is related to not only the previous own market order flow, but also to the secondary market order flow prior to $t-1$. To capture the secondary market order flow we introduce the trade indicator y_t, where y_t is $+1$ if the latest secondary market transaction occurs after $t-1$ and is buyer initiated, -1 if the latest secondary market transaction occurs after $t-1$ and is seller

[4] Ryu (2011) also extends the model in MRR (1997) and develops a cross-market model that takes order flow information from a secondary market into account when explaining intraday price formation for a primary market. However, his model does not consider inventory holdings costs.

initiated, and zero if the latest secondary market transaction occurs before $t-1$ and/or occurs at the midpoint of the prevailing bid-ask spread. Accordingly, we amend the conditional expectation function of Equation (2):

$$E[x_t|\Im_{t-1}] = \rho^x x_{t-1} + \rho^y y_{t-1} \tag{2'}$$

where y_{t-1} is equal to ± 1 if the latest secondary market trade is either buyer or seller initiated and occurs between the two primary market trades at times $t-2$ and $t-1$. The coefficients ρ^x and ρ^y determine how the previous trade at respective markets influences expectations on the direction of the next primary market trade. Thus, attention is always given to the most recent trade at the primary market, and when the second most recent trade is on the secondary market, that is considered as well.

To model inventory effects, we assume that liquidity suppliers who are active on both the primary and the secondary market adhere to a portfolio approach. For example, a liquidity supplier who buys a security at the primary market will not only lower his primary market bid and ask prices, but will also lower the corresponding secondary market bid and ask prices, as a trade there would hedge his primary market position.[5] Accordingly, we decompose the total inventory cost δ into two parts; where the first part (δ^x) reflects inventory effects from the primary market order flow, and the second part (δ^y) is associated with the order flow at the secondary market. Together, $\tilde{\varphi}$, δ^x, and δ^y reflect the temporary effects of order flows on the asset's price and quotes. Hence, in our two-market framework, the asset's transaction price and bid-ask spread midpoint are expressed as:

$$p_t = \mu_t + \delta^x \sum_{i=1}^{t} x_i + \delta^y \sum_{i=1}^{t} y_i + \tilde{\varphi} x_t + \xi_t \tag{3'}$$

[5] Our hedging argument is similar to the one used by Huang and Stoll (1997), who, however, consider market makers' inventory hedging across correlated stocks rather than one asset traded at different markets.

$$m_t = \mu_t + \delta^x \sum_{i=1}^{t} x_i + \delta^y \sum_{i=1}^{t} y_i \tag{4'}$$

where the sums of trade direction indicators x_i and y_i reflect accumulated inventory from the start of the trading day until, and including, time t. In other words, the deviation of p_t from the asset value μ_t depends on temporary order flow effects from both markets.

Analogous to the derivations in the one-market model, we retrieve testable equations for asset returns and midpoint changes:

$$p_t - p_{t-1} = \alpha + (\theta + \tilde{\varphi} + \delta^x)x_t - (\tilde{\varphi} + \theta\rho^x)x_{t-1} + \delta^y y_t - \rho^y \theta y_{t-1} + \varepsilon_t + \xi_t - \xi_{t-1} \tag{5'}$$

$$m_t - m_{t-1} = \omega + (\theta + \delta^x)x_t - \theta\rho^x x_{t-1} + \delta^y y_t - \rho^y \theta y_{t-1} + \varepsilon_t \tag{6'}$$

where we note that inventory costs of the latest trades on both the primary and the secondary market influence both asset returns and midpoint quote updates.

Finally, the cross-market inventory assumptions result in that δ in the expressions for bid and ask prices is replaced by $\delta^x + \delta^y$, implying a bid-ask spread $p_{t+1}^a - p_{t+1}^b = 2(\theta + \tilde{\varphi} + \delta^x + \delta^y)$. Evidently, the bid-ask spread corresponds to twice the sum of the asymmetric information parameter, the order processing cost parameter, and the two parameters that represents the market makers' inventory costs emanating from the primary market and the secondary market respectively.[6] Return variance decomposition is also straightforward, but the added term yields many additional terms to that equation, which is left out for brevity.

[6] It is straightforward to see that the model can be extended in the same way for more than two markets. If there are N markets and the trade direction indicator for market n (n = 1, 2, ..., N; with 1 corresponding to the primary market) is denoted z_t^n, we can write
(2'') $E[z_t^1 | \Im_{t-1}] = \sum_{i=1}^{N} \rho^i z_{t-1}^i$
(3'') $p_t = \mu_t + \sum_{i=1}^{N} (\delta^i \sum_{j=1}^{t} z_j^i) + \tilde{\varphi} z_t^1 + \xi_t$
(4'') $m_t = \mu_t + \sum_{i=1}^{N} (\delta^i \sum_{j=1}^{t} z_j^i)$
where market superscripts are adopted for ρ and δ in the same way as for z_t. This yields an identifiable bid-

3. EMPIRICAL FRAMEWORK AND RESULTS

In this section we assess our model empirically and compare its performance with the model by MRR (1997), which can be seen as a special case of our model. We use GMM to estimate the model parameters, and use these to decompose the bid-ask spread into information asymmetry cost, inventory cost, and order processing cost. We also decompose the return variance into components that are due to public news, different microstructure effects, and stochastic rounding errors. Finally, we investigate to what extent liquidity suppliers consider trading at a secondary market when submitting their quotes.

Our model suits quote-driven as well as order-driven markets. Thus, the choice of empirical setting for assessment is arbitrary. To minimize estimation error, our test market should ideally have a high trading and quote updating frequency. Furthermore, for the extension with trading at a secondary market, we need an asset with similar products traded at two markets. A venue that fits the requirements well is the Shanghai Futures Exchange (SHFE) market for gold futures. Considering that the interest for gold as an investment has increased dramatically in recent years, and that China has one of the world's largest gold markets, this market is also interesting in itself.[7] The SHFE gold futures contract is the fourth most traded gold futures contract in the world (in terms of kilograms). Furthermore, a spot-deferred contract (SDC), similar to the SHFE gold futures, is traded at the Shanghai Gold Exchange (SGE; the world's largest gold spot market). Hence, we choose the SHFE gold futures and the SGE SDC as the empirical assets for our study.

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ask spread equal to $2(\theta + \tilde{\varphi} + \sum_{i=1}^{N} \delta^{i})$.

[7] For an overview of the Chinese gold markets, see Xu, Hagströmer and Nordén (2011).

Model Estimation

The main model constitutes equation (5), representing the change in transaction prices, and equation (6), determining the change in the bid-ask spread midpoint, and equation (2), defining the conditional expectation of the trade direction variable. Equations (5) and (6) are formulated as functions of contemporaneous and lagged order flow. Accordingly, we need to estimate the parameters α (the constant price change drift parameter), θ (the asymmetric information parameter), $\tilde{\varphi}$ (the order processing cost parameter), δ (the inventory cost parameter) and ρ (the parameter reflecting how the previous trade influences expectations of the direction of the next trade). In addition, the model contains the two variance parameters σ_{ε}^2 (variance of information innovations) and σ_{ξ}^2 (variance incurred by rounding errors). These two variance components can be deduced from estimating equations (5) and (6). Furthermore, we can estimate the probability that a trade occurs exactly at the midpoint of the bid-ask spread by using the fact that $E(|x_t|) = (1-\lambda)$.

In order to estimate the equations simultaneously, and not assuming strong distributional requirements about the data generating processes, we use the generalized method of moments (GMM) as described by Hansen (1982). The idea of the method is to estimate the parameters of the model so that the measured sample moments closely match the population moments. Defining the measurable residuals in our model as:

$$u_t = p_t - p_{t-1} - (\theta + \tilde{\varphi} + \delta)x_t + (\tilde{\varphi} + \theta\rho)x_{t-1} \tag{10}$$

$$v_t = m_t - m_{t-1} - (\theta + \delta)x_t + \theta\rho x_{t-1} \tag{11}$$

enables us to exactly identify the parameters of the model. The resulting vector of population moments are as follows:

$$E\begin{pmatrix} u_t-\alpha \\ (u_t-\alpha)x_t \\ (u_t-\alpha)x_{t-1} \\ (u_t-\alpha)^2-\sigma_\varepsilon^2-2\sigma_\xi^2 \\ v_t-\omega \\ (v_t-\omega)x_t \\ (v_t-\omega)x_{t-1} \\ (v_t-\omega)^2-\sigma_\varepsilon^2 \\ x_tx_{t-1}-\rho x_{t-1}^2 \\ |x_t|-(1-\lambda) \end{pmatrix}=0$$

Likewise, we estimate the extended model using equations (2'), (5') and (6'). In this case, we define our model residuals as:

$$u_t = p_t - p_{t-1} - (\theta+\tilde{\varphi}+\delta^x)x_t + (\tilde{\varphi}+\theta\rho^x)x_{t-1} - \delta^y y_t + \rho^y\theta y_{t-1} \tag{10'}$$

$$v_t = m_t - m_{t-1} - (\theta+\delta^x)x_t + \theta\rho^x x_{t-1} - \delta^y y_t + \rho^y\theta y_{t-1} \tag{11'}$$

and the resulting vector of population moments as:

$$E\begin{pmatrix} u_t-\alpha \\ (u_t-\alpha)x_t \\ (u_t-\alpha)x_{t-1} \\ (u_t-\alpha)y_t \\ (u_t-\alpha)y_{t-1} \\ (u_t-\alpha)^2-\sigma_\varepsilon^2-2\sigma_\xi^2 \\ v_t-\omega \\ (v_t-\omega)x_t \\ (v_t-\omega)x_{t-1} \\ (v_t-\omega)y_t \\ (v_t-\omega)y_{t-1} \\ (v_t-\omega)^2-\sigma_\varepsilon^2 \\ x_tx_{t-1}-\rho^x x_{t-1}^2 \\ y_ty_{t-1}-\rho^y y_{t-1}^2 \\ |x_t|-(1-\lambda) \end{pmatrix}=0$$

To determine the direction of trade for the main market (x_t) and the secondary market (y_t) trades, the transaction price can be compared to the midpoint between the bid and ask quotes prevailing before the trade. The signing procedure is in line with Harris (1989). Trades at the prevailing midpoint are considered as neither buyer- nor seller-initiated.

Market structure and data

The SHFE gold futures market is an electronic continuous auctions market, allowing limit orders and market orders and matching such orders automatically with typical priority rules. Trading is restricted to members of the exchange, who, with few exceptions, are domestic investors. The market is open on weekdays between 9 am and 3 pm, with a lunch break between 11:30 am and 1:30 pm, local time. Trading is also halted for a brief coffee break between 10:15 am and 10:30 am. Market opening is preceded by an opening call auction that determines the opening price. The closing price is the price of the last transaction in the day, whereas the futures settlement price equals the volume-weighted average transaction price during the day.

The gold futures contract has one kilogram of gold with 99.95% purity or higher as the underlying asset. It is physically settled at maturity, and marked-to-market on a daily basis. The exchange charges a transaction fee of 0.02% or less on all transactions. Trading has increased dramatically since its commencement in 2008, and the average daily futures transaction volume is almost 18,000 contracts during our sample period.

The SDC is the most traded product at the SGE with almost 16,000 contracts traded every day. An SDC is a type of rolling futures contract where delivery can be postponed. The SDC has the same underlying asset specification as the gold futures at the SHFE and is traded in the same currency (RMB). For our sample period, trading hours overlap those of the futures market, except that the afternoon session at the SGE closes half an hour later. In addition, trading at the SGE is open during a night session between 9 pm and 2:30 am.

For our empirical analyses, we use six months of data from December 1, 2010, to May 31, 2011, which corresponds to 121 trading days. The futures contract has a monthly maturity cycle, but as shown in Xu, Hagströmer and Nordén (2011), almost all trading is concentrated to the contracts expiring in June and December. We restrict our study to those contracts, rolling over to the next contract on the day after its daily trading volume exceeds the previous contract.[8] Intraday data on trades and quotes for futures and SDCs are retrieved from the Thomson Reuters Tick History database, maintained by the Securities Research Centre of Asia-Pacific (SIRCA). The data include complete information of transactions prices and settlement prices, trading volume (the number of traded contracts), open interest, and the prices and volumes of the best buy and sell quotes. The records show the timing of each trade and quote with an accuracy down to the microsecond.

To determine the direction of trade for futures (x_t) and SDC (y_t) trades, we compare the transaction price to the midpoint between the bid and ask quotes prevailing before the trade. As both markets are fully electronic, we do not expect any lags in quote or trade reporting, so the latest quotes before each trade are used to sign the trade. The signing procedure is in line with Harris (1989). Trades at the prevailing midpoint are considered neither buyer- nor seller-initiated. To determine the bid-ask spread midpoint just following a trade m_t, we use the quote message in the data that has exactly the same time stamp as the corresponding trade.

Descriptive statistics

Table 1 presents some descriptive statistics for the SHFE gold futures transactions data. The main focus is on futures market volatility, liquidity and trading activity. We present the

[8] The tickers for the contracts considered are *SHAUM1* and *SHAUZ1*. The rollover date is April 15, 2011. The reason that volumes are switched to the next contract about two months before maturity is that SHFE requires higher margins for contracts with shorter time to maturity. The SDC has ticker *XAUTD*.

descriptive statistics for the whole day and broken down into the three intraday time periods; morning, pre-lunch, and afternoon trading session. Noteworthy is the U-shaped pattern in variance of both transaction price changes and midpoint quote changes. The variance of price changes (midpoint quote changes) starts at the level 0.0020 (0.0010) in the morning, drops to 0.0008 (0.0005) during the pre-lunch session, and increases sharply to 0.0068 (0.0053) in the afternoon. Similarly, the trading activity, measured as total number of transactions, average number of transactions per hour, and average time between trades, is clearly higher during the morning and afternoon sessions than during the pre-lunch session. However, using the average bid-ask spread as a liquidity measure, we observe a higher liquidity (narrower spread) during the afternoon trading session than during the morning or pre-lunch sessions. On average, the bid-ask spread corresponds to more than three times the minimum tick size (0.01 RMB/gram). Finally, we note that roughly a fifth of the futures transactions are preceded by an SDC transaction. Thus, although the SDC market exhibits a lower trading activity than the futures market, a futures market maker might still find it worthwhile to keep up to date with the SDC order flow developments when setting futures quotes and prices.

Results and analysis

Table 2 displays parameter estimates from the one-market model. When the full sample is considered, all parameters are positive and significantly different from zero at the 5% level. The same holds for the morning and pre-lunch sessions, but for the afternoon session the intercepts and the inventory cost parameter are not significantly different from zero. The autocorrelation parameter of the direction of trade (ρ) is stable over the day, at around 0.10 (statistically different from zero). The unconditional probability of trades occurring exactly at the bid-ask spread midpoint is very low, but statistically significant, at 0.15%.

In general, the three parameters describing microstructure effects – asymmetric information cost (θ), order processing cost ($\tilde{\varphi}$), and inventory cost (δ) – all take on economically reasonable values. The information asymmetry cost parameter increases over the day, whereas the inventory cost parameter is falling and approaches zero when the trading day draws to a close. The order processing cost parameter is higher than the other two microstructure parameters, and is slightly U-shaped over the three sub-periods.

Twice the sum of the three microstructure parameters constitute our implied bid-ask spread. The total spread, as well as the component of the spread associated with each microstructure parameter, is presented in Table 3. Over the whole day, the implied bid-ask spread is 0.0327 RMB/gram, which is in line with the observed bid-ask spread. For the whole day, our model-implied spread accounts for 87% of the average observed bid-ask spread (Table 1). Thus, our model is successful in identifying the bid-ask spread and its components. Moreover, the implied spread for the different sub-periods captures the intraday decline of the observed spread.

From Table 3 we note that about half of the bid-ask spread is constituted by order processing costs. The other half of the spread is due to asymmetric information costs and inventory costs. The distribution between the latter two costs varies over the trading day, with asymmetric costs increasing in importance over the day, and inventory costs decreasing. The wide bid-ask spread during the morning session is due primarily to transitory effects such as order processing costs and inventory costs. In the afternoon session the inventory cost is substantially lower, but the spread remains at its pre-lunch level due to increasing information asymmetry and order processing costs.

Our structural model also allows us to decompose the variance of transaction-by-transaction returns, midpoint updates, and value updates, into public news and order flow innovation components. We provide the results of this exercise in Table 4. As gold is a globally traded commodity that has little reason to feature local deviations from the world price formation, it is not surprising to find that local microstructure noise contributes little to the gold value variance. The SHFE gold futures has quickly been established as an important product for domestic Chinese gold investors (who are in general barred from foreign markets), but it does not yet contribute much to global gold price formation. When the markets in Shanghai close at 3 pm, the time at other gold trading hubs is 1 am in Chicago (CST), 7 am in London (GMT), and 8 am in Zürich (CET). This means that the afternoon gold trading in Shanghai coincides with the dawn of European markets. This can be readily seen in the value variance, which is 50 (100) times higher in the afternoon session than in the morning (pre-lunch) session. Furthermore, during the afternoon session, virtually all value variance (99.6%) comes from public news.

During the pre-lunch session, when it is night time in both the United States and Europe, order flows at the SHFE represent almost 5% of the value innovation variability, and more than 16% of the midpoint update variability. The midpoint microstructure noise during the pre-lunch session is roughly evenly distributed between asymmetric information and inventory costs. During the morning session, about 8% and 1.7% of midpoint and value update variance, respectively, are due to microstructure noise.

The transaction-by-transaction return variance composition displays high intraday variability. Similar to value and midpoint variance, the public news element is highest in the afternoon when European markets are about to open; in this case it represents three quarters of the variance. The nominal variance during the afternoon session is also three times higher than in the morning, and six times higher than in the pre-lunch session. The low-variance sessions (morning and pre-lunch), on the other hand, are dominated by microstructure noise as public news account for little more than 40%. A puzzling result is that the microstructure noise to a large extent is constituted by the variance of the stochastic rounding error term (ξ). The tick size in this market is very small, both by regulation and de facto (0.01 RMB/gram out of a price level around 300 RMB during our sample period), so the error term must be capturing more noise than that caused by price discreteness. In the morning session, this noise term represents 43% of the return variability, falling to 26% and 19% in the pre-lunch and afternoon sessions. Other microstructure noise is dominated by order processing cost, ranging from 1% (afternoon) to almost 10% (pre-lunch).

In Table 5, we present estimates for our two-market model, i.e., the application where we allow liquidity suppliers at the SHFE gold futures market to consider order flows of SDC gold contracts traded at the SGE. In general, parameter estimates of the two-market model are in line with those in the one-market model. The parameters of particular interest for this application, δ^y and ρ^y show little significance, except that ρ^y is significant in the morning session. The latter result is consistent with the idea that the SDC order flow is important for forming expectations of the futures trade direction in the morning session.

On the whole, our results imply that market makers at the futures market do not pay much attention to the spot market SDC contracts, neither for inventory pressure nor to form expectations of the futures trade direction. As the microstructure parameter estimates are virtually the same as for the one-market case, a bid-ask spread decomposition for the two-market case is redundant. However, we regard the two-market case as an interesting venue for further empirical research in other markets. For example, the two-market case can be extended to an n-market case and be applied to stocks traded at incumbent exchanges and MTFs at the equity market in Europe.

4. CONCLUDING REMARKS

The model presented in this paper shares the variance decomposition feature of MRR (1997) and the three-way bid-ask spread decomposition ability of Huang and Stoll (1997). It is unique, however, in that it fits both these features within one unified structural framework. Furthermore, our model is the first where the variance decomposition separates inventory effects from other transitory effects, and we are the first to extend the variance decomposition to midpoint returns and value innovations. As illustrated by our empirical application, the different decompositions provide a rich picture of how prices and bid-ask spreads are set in the market.

In an extension of our model, we also show how the analysis can be expanded to a secondary market. When one asset is traded at several markets, or when two similar assets exist, we argue that a rational liquidity supplier should monitor order flows at all relevant markets. Also, if the same liquidity supplier is trading in two markets, it is likely that he views his positions as one portfolio, rather than as two separate portfolios. In the current market setting, where volume is increasingly fragmented across several exchanges, we think that market makers who ignore alternative venues are likely to gather less information than competitors who monitor flows beyond their primary exchange. In our empirical study of gold markets in Shanghai, we find that even though liquidity suppliers in the futures market do not seem to consider inventory from the spot market when they set their quotes, they do assess the potential information content in spot market trades.

We see several interesting opportunities for empirical market microstructure research using our model. One empirical application that we plan to pursue is the European equity market, where recent legislation has led to the opening of several MTFs, that in turn has led to extensive cross-listing of stocks and volume fragmentation. Our model is well-suited to analyze how liquidity suppliers manage their inventories and monitor trade flows across incumbent exchanges and MTFs. Another interesting venue of research would be to expand the analysis of the Shanghai gold futures to other gold futures markets in Asia. Due to the recent development of Asian financial markets and interest in gold, new gold derivatives are now available for trading in, e.g., Hong Kong, Seoul, Singapore and Taiwan.

BIBLIOGRAPHY

Amihud, Y., and Mendelson, H. (1980). Dealership Markets: Market making with inventory. Journal of Financial Economics 8, 31-53.

Asparouhova, E. and Bessembinder, H. and Kalcheva, I. (2010). Liquidity biases in asset pricing tests. Journal of Financial Economics 96 (2), 215-237.

Glosten, L., and Harris, L. (1988). Estimating the components of the bid-ask spread. Journal of Financial Economics 21, 123-142.

Glosten, L., and Milgrom, P. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. Journal of Financial Economics 14, 71-100.

Grammig, J., Thiessen, E., and Wünsche, O. (2011). Time and the price impact of a trade: A structural approach. Working paper, University of Mannheim and University of Tübingen.

Hansen, L., (1982). Large sample properties of generalized method of moment estimators. Econometrica 50, 1029-1084.

Harris, L., (1989). A day-end transaction price anomaly. Journal of Financial and Quantitative Analysis 24, 29-45.

Hasbrouck, J. (1995). One security, many markets: Determining the contribution to price discovery. The Journal of Finance 50, 1175-1199.

Huang, R., and Stoll, H. (1997). The components of the bid-ask spread: a general approach. Review of Financial Studies 10, 995-1034.

De Jong, F., and Schotman, P. (2010). Price discovery in fragmented markets. Journal of Financial Econometrics 8, 1-28.

Kim, S., and Murphy, D. (2013). The impact of high-frequency trading on stock market liquidity measures. Working Paper, Kellogg School of Management, Northwestern University.

Korenok, O., Mizrach, B., and Radchenko, S. (2011). A structural approach to information shares. Working paper, Department of Economics, Rutgers University.

Madhavan, A., Richardson, M., and Roomans, M. (1997). Why do security prices change? A transaction-level analysis of NYSE stocks. Review of Financial Studies 10, 1035-1064.

Newey, W., and West, K. (1987). A simple positive semi-definite heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703-708.

Ryu, D. (2011). Intraday price formation and bid–ask spread components: a new approach using a cross-market model. Journal of Futures Markets 31, 1142–1169.

Sadka, R. (2006). Momentum and post-earnings-announcement drift anomalies: the role of liquidity risk. Journal of Financial Economics 80, 309-349.

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity. Econometrica 48, 817-838.

Xu, C., Hagströmer, B., and Nordén, L. (2011). Alchemy in the 21st century: hedging with gold futures. Review of Futures Markets 19, 247-280.

Table 1: Descriptive statistics for SHFE gold futures

	9:00-10:15	10:30-11:30	1:30-3:00	Whole day
Variance of price changes	0.0020	0.0008	0.0068	0.0038
Variance of midpoint quote changes	0.0010	0.0005	0.0053	0.0027
Average bid-ask spread	0.0392	0.0388	0.0360	0.0378
Average transaction volume (kg.)	5.88	5.12	5.62	5.63
Total number of transactions	136,816	65,610	147,354	349,780
Average number of transactions/hour	910.50	542.23	811.88	772.84
Average time (sec.) between transactions	3.95	6.53	4.44	4.65
Average % of trades preceded by SDC trade	18.91	22.94	17.98	19.27

Table 1 provides summary statistics on the variance of gold futures transaction price changes, variance of midpoint quote changes, the average bid-ask spread (RMB per gram gold), the average volume per transaction, the average number of transactions per hour, and the average time in seconds between transactions for three time intervals during the day, plus the whole day. Bid and ask spread and midpoint quotes are observed just after each transaction. Sample period is between December 1, 2010, and May 31, 2011.

Table 2: Parameter estimates for the one-market model

	9:00-10:15	10:30-11:30	1:30-3:00	Whole day
α				
Coefficient	0.0003	0.0003	0.0002	0.0002
p-value	0.0154	0.0060	0.3132	0.0140
ω				
Coefficient	0.0004	0.0003	0.0003	0.0003
p-value	0.0000	0.0000	0.0965	0.0000
θ				
Coefficient	0.0039	0.0044	0.0048	0.0044
p-value	0.0000	0.0000	0.0162	0.0000
$\tilde{\varphi}$				
Coefficient	0.0086	0.0070	0.0075	0.0078
p-value	0.0000	0.0000	0.0000	0.0000
δ				
Coefficient	0.0050	0.0042	0.0033	0.0041
p-value	0.0000	0.0000	0.0997	0.0000
ρ				
Coefficient	0.0814	0.1224	0.0915	0.0933
p-value	0.0000	0.0000	0.0000	0.0000
λ				
Coefficient	0.0017	0.0011	0.0015	0.0015
p-value	0.0000	0.0000	0.0000	0.0000

Table 2 displays estimated parameters in the one-market model over three intraday trading intervals, and the whole day. Sample period is between December 1, 2010, and May 31, 2011. The parameters are: α, the constant in the transaction price change equation; ω, the constant in the midpoint quote change equation; θ, the asymmetric information component; $\tilde{\varphi}$, the order processing cost component; δ, the inventory cost component; ρ, the autocorrelation coefficient of the order flow; and λ, the probability that a trade occurs at the midpoint of the bid-ask spread. The model parameters are estimated with GMM.

Table 3: Bid-ask spread decomposition

	9:00-10:15	10:30-11:30	1:30-3:00	Whole day
Implied bid-ask spread	0.0351	0.0312	0.0313	0.0327
Fraction of the bid-ask spread				
Asymmetric information	0.2224	0.2826	0.3089	0.2679
Order processing costs	0.4900	0.4462	0.4817	0.4803
Inventory costs	0.2876	0.2712	0.2094	0.2518

Table 3 presents estimated trading costs at the gold futures market over three trading intervals, and the whole day. Sample period is between December 1, 2010, and May 31, 2011. The trading cost measures are based on the estimated one-market model coefficients from Table 2. The estimated coefficients are denoted as: θ, the asymmetric information component; $\tilde{\varphi}$, the order processing cost component; δ, the inventory cost component. Accordingly, the implied bid-ask spread equals $2(\theta + \tilde{\varphi} + \delta)$ and the fraction of the implied bid-ask spread due to asymmetric information, order processing costs, and inventory costs, is equal to $\theta/(\theta + \tilde{\varphi} + \delta)$, $\tilde{\varphi}/(\theta + \tilde{\varphi} + \delta)$, and $\delta/(\theta + \tilde{\varphi} + \delta)$ respectively.

Table 4: Variance decomposition

	9:00-10:15	10:30-11:30	1:30-3:00	Whole day
Transaction price variance	0.0022	0.0009	0.0068	0.0039
Fraction of the variance				
Public information (σ_ε^2)	0.4025	0.4294	0.7589	0.6675
Discreteness ($2\sigma_\xi^2$)	0.4302	0.2602	0.1994	0.2526
Asymmetric information (A)	0.0070	0.0219	0.0034	0.0049
Order processing costs (B)	0.0628	0.0974	0.0151	0.0288
Inventory costs (C)	0.0118	0.0205	0.0016	0.0044
Interaction (D)	0.0308	0.0693	0.0106	0.0176
Interaction (E)	0.0181	0.0421	0.0046	0.0092
Interaction (F)	0.0369	0.0592	0.0066	0.0151
Midpoint quote variance	0.0009	0.0004	0.0052	0.0027
Fraction of the variance				
Public information (σ_ε^2)	0.9161	0.8355	0.9876	0.9731
Asymmetric information (A)	0.0159	0.0427	0.0044	0.0071
Inventory costs (C)	0.0268	0.0399	0.0020	0.0064
Interaction (E)	0.0412	0.0817	0.0060	0.0134
Value variance	0.0009	0.0004	0.0052	0.0026
Fraction of the variance				
Public information (σ_ε^2)	0.9829	0.9514	0.9956	0.9927
Asymmetric information (A)	0.0171	0.0486	0.0044	0.0073

Table 4 presents the estimated implied variance of futures transaction price changes, midpoint quote changes, and value changes, for three daily intervals, and the whole day. Sample period is between December 1, 2010, and May 31, 2011. Variances are based on estimated one-market model coefficients from Table 2, and are decomposed into: information innovations σ_ε^2, rounding errors or dicreteness in the pricing process σ_ξ^2, asymmetric information only $A = (1-\lambda)(1-\rho)^2\theta^2$, order processing costs only $B = 2(1-\lambda)(1-\rho)\tilde{\varphi}^2$, inventory costs only $C = (1-\lambda)\delta^2$, interaction between asymmetric information and order processing costs $D = 2(1-\lambda)(1-\rho^2)\theta\tilde{\varphi}$, interaction between asymmetric information and inventory costs $E = 2(1-\lambda)(1-$ $\rho 2\theta\delta$, and interaction between order processing costs and inventory costs $F{=}21{-}\lambda 1{-}\rho 2\varphi\delta$.

Table 5: Parameter estimates for the two-market model

	9:00-10:15	10:30-11:30	1:30-3:00	Whole day
α				
Coefficient	0.0003	0.0003	0.0002	0.0002
p-value	0.0138	0.0057	0.3157	0.0136
ω				
Coefficient	0.0004	0.0003	0.0003	0.0003
p-value	0.0000	0.0000	0.0972	0.0000
θ				
Coefficient	0.0039	0.0043	0.0048	0.0043
p-value	0.0000	0.0000	0.0165	0.0000
$\tilde{\varphi}$				
Coefficient	0.0085	0.0070	0.0075	0.0078
p-value	0.0000	0.0000	0.0000	0.0000
δ^x				
Coefficient	0.0050	0.0042	0.0033	0.0041
p-value	0.0000	0.0000	0.0995	0.0000
δ^y				
Coefficient	0.0000	-0.0000	0.0002	0.0000
p-value	0.8711	0.4962	0.6188	0.6681
ρ^x				
Coefficient	0.0819	0.1224	0.0915	0.0936
p-value	0.0000	0.0000	0.0000	0.0000
ρ^y				
Coefficient	0.0349	0.0005	0.0095	0.0174
p-value	0.0000	0.9504	0.1237	0.0000
λ				
Coefficient	0.0017	0.0011	0.0015	0.0015
p-value	0.0000	0.0000	0.0000	0.0000

Table 5 displays estimated parameters in the two-market model over three intraday trading intervals, and the whole day. Sample period is between December 1, 2010, and May 31, 2011. The parameters are: α, the constant in the transaction price change equation; ω, the constant in the midpoint quote change equation; θ, the asymmetric information component; $\tilde{\varphi}$, the order processing cost component; δ, the inventory cost component; ρ, the autocorrelation coefficient of the order flow; δ^x, the primary market inventory cost component; δ^y, the secondary market inventory cost component; ρ^x, the autocorrelation coefficient from the primary market order flow; ρ^y the autocorrelation coefficient from the secondary market order flow; and λ, the probability that a trade occurs at the midpoint of the bid-ask spread. The model parameters are estimated with GMM.