Time Variation in Options Expected Returns

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Abstract

The paper studies the theoretical effect of various factors on option expected returns, and also specifically sheds light on one of the more well-documented anomalies in the empirical option returns literature: the very low average returns to out-the-money call options as an asset class. Existing theory regarding the expected returns of call options states that the expected returns should be greater than the expected return of the underlying asset and increasing in the strike price in the cross-section. Empirical analysis of average option returns consistently finds a completely different picture: call option average returns are decreasing in the strike price and out-the-money calls have negative returns on average. Recent research has focused on trying to explain these "anomalous" empirical results. In this paper I revisit the theory of expected returns. I show that a derivation of expected returns which relies solely on no-arbitrage principles and makes no assumptions about investor preferences leads to call expected returns that are increasing over some ranges of strike prices and decreasing over other ranges of strike prices (typically increasing over in-the-money calls, decreasing over out-the-money calls). My derivation includes the prediction that deep out-the-money calls have negative expected returns. Within this framework the existing empirical results appear consistent with traditional asset pricing assumptions. I show that option contracts have inherently unstable betas which vary significantly over time, and that models of expected option returns must account for this. I furthermore show that options held to expiry are fundamentally different from a risk-return perspective than options held to some point prior to expiry.

Existing literature contends that theoretical expected returns on call option contracts are higher than the expected return on the underlying asset, and increasing in the strike price (higher for contracts initially out-the-money than contracts initially in-the-money). By contrast, empirical literature has consistently found that call option returns, on stock or indices, are decreasing in the strike price and significantly negative for contracts initially out-the-money[1]. Much recent literature has focused on explaining this difference between theoretical predictions and empirical results[2].

In this paper, I revisit the theory of call option expected returns. I introduce, and illustrate with numerical methods, a derivation that relies strictly on no-arbitrage principles and assumes nothing about investor preferences. This framework for theoretical call expected returns produces a prediction of call option expected returns that are decreasing in the strike price for the range of contracts with initially out-the-money strikes, including a prediction of negative expected returns for some plausible specifications of the risk-return statistics of the underlying asset. These theoretical predictions are significantly more consistent with the existing empirical results than previous theory, and in this context the empirical work on call options appears significantly less "anomalous" than previously believed.

I further study the effect of implied volatility and time to maturity on call expected returns. These are studied separately as *ceteris paribus* relationships, in the spirit of the well-known option Greeks, but with respect to option expected return rather than price. I find that call expected returns can be increasing or decreasing in implied volatility, depending on the moneyness of the option.

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[1] See, for example, O'Brien and Shackleton (2005), Li (2007), Wilkens (2007), Ni(2009), McKeon (2013) and Constantinides et al. (2013).

[2] Broadly, these explanations fall into the following categories: non-standard investor preferences (notably, preference for skewness), market frictions, time-varying or priced volatility, and tail events. The literature will be reviewed more specifically later in the paper.

The same result holds for time to maturity. My finding with respect to implied volatility is in contrast with Hu and Jacobs (2017), but consistent with Chaudhury (2017). I discuss and reconcile these different results.

It is important to stress that my results are obtained in a purely theoretical framework, with no empirical market data employed. Therefore, existing explanations for this pattern of expected returns across call option strike prices do not apply here. I employ a controlled environment in which underlying asset returns are normal, no skewness or tail events, investor preferences are consistent with traditional asset pricing, and there are no market frictions. These factors are all undoubtedly important to empirical option returns, but the result in my paper shows that they are not necessary to the pattern of returns decreasing in call option strike price. In other words, some factor from within traditional asset pricing theory is at least a part of the story in explaining the empirical results which have been documented.

I discuss the reasons why my framework produces different predictions than previous theories. Previous theoretical predictions of call option expected returns have typically relied on one of two frameworks: derivation based on stochastic discount factor (SDF), or derivation based on analytical derivations of option beta. The issue with derivations based on the SDF is that a unique SDF only exists in complete markets, defined as markets where the number of assets is greater than or equal to the number of future scenarios. When future scenarios are defined by distributions of prices, then clearly there are an infinite number of future scenarios and markets are incomplete. Existing theoretical predictions based on the SDF all assume complete markets, and would not be valid under the assumption of incomplete markets.

The issue with theoretical derivation based on beta and the CAPM framework is that option contracts have inherently unstable betas. That is to say, even with no changes to the information

set, the beta of an option contract will vary over time. To see this easily, consider that a two-month at-the-money call will be a one-month in-the-money call a month later if the asset price has changed in accordance with an expected return greater than the risk-free rate. Since moneyness and maturity are two factors which strongly influence the leverage and therefore risk of the option contract, this same option contract has two very different risk profiles at the two- and one-month timeframes, even if nothing unexpected has happened in the month in between.

Therefore, expected returns theory derived from calculations of "instantaneous" beta for the option contract are limited because they do not account for the fact that the beta obtained is known to be unstable and certain to change over time. I demonstrate in the paper that this time variation in beta is often highly significant in magnitude.

However, while noting the limitations of existing theoretical frameworks above, it must be said that the predictions which have been derived have an intuitively appealing feature to them. Intuitively, call options are leveraged investments on the underlying asset, and so it seems unsurprising when theory predicts expected returns that are greater than or equal to those of the underlying asset and increasing in strike price (since leverage increases with strike price). Therefore, theoretical predictions which are in conflict with this prediction require a sound economic explanation.

I discuss possible explanation for my results for option expected returns, given that previous results seemed intuitively consistent with well-established asset pricing principles. I suggest that "market timing" properties of option contracts may be influencing the expected returns. While other assets, stocks and bonds for example, may have time-varying expected returns, this variation is always driven by changing market conditions or changes to the nature of the asset. Options are unique in having time-varying expected returns *ceteris paribus*. In other words, even

with no changes to market environment and assuming everything unfolds as expected, the expected return on an option contract is still expected to change over time. This is the factor which can reconcile theoretical predictions and empirical results.

Theory of Call Option Expected Returns

The expected gross return of a call option can be written as:

$$1 + E(R)_C = \frac{CF}{BS} \tag{1}$$

Where *CF* is the probability-weighted future cash flows (the area under the probability distribution of $S_T - X$, where S_T is the terminal stock price and *X* is the option strike price), and *BS* is the Black-Scholes price of the call option[3].

This is consistent in form with Coval and Shumway (2001), except they write the price of the option in the denominator as the probability-weighted future cash flows multiplied by the stochastic discount factor (SDF). In other words, their setup includes a more general form, while Equation (1) above includes a specific options pricing model, Black-Scholes (although, as I explain later, my results can be generalized to any pricing model that obeys standard no-arbitrage principles). In the Coval-Shumway setup, the SDF is determined jointly with the future stock prices. This is the assumption will be discussed further later on in the paper.

To analyze how option expected returns vary in the cross-section by strike price (assuming the same underlyer, constant volatility and risk-free rate, and the same maturity) I differentiate Equation (1) with respect to strike price:

$$\frac{d[1+E(R)_C]}{dX} = \frac{CF.BS'(X) - CF'(X).BS}{[BS]^2} \tag{2}$$

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[3] See Black and Scholes (1973).

The above result is obtained directly from application of the quotient rule, where *BS'(X)* is the derivative of Black-Scholes with respect to X, and *CF'(X)* the derivative of the probability-weighted future cash flows with respect to X.

The elements of the equation are easy to sign:

- *CF* and *BS* are obviously both positive.

- *BS'(X)* and *CF'(X)* are both negative (the Black-Scholes price of a call decreases with an increase in strike price, as does the area under the distribution of future cash flows. This can be shown formally by differentiating the two formulae, but is also intuitively clear.)

Therefore, the sign of the derivative of call expected returns with respect to strike price is:

$$\frac{[negative] - [negative]}{[positive]}$$

…and the resulting sign far from obvious absent more specific information.

In fact, the Black-Scholes option price is not specifically required to obtain the above result. The fact that call option prices should be decreasing in the strike price is well-established and inviolate, based on simple no-arbitrage principles. Therefore, *any* option pricing model which produces lower call prices for higher strike prices (as any valid option pricing model should) will produce the above result[4].

Since the price is the discounted value of the futures cash flows, it may seem intuitive to write the following:

- *CF > BS*,

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[4] A similar setup is employed in Hu and Jacobs (2015) to study changes in option expected returns with respect to implied volatility. They find the sign to be unambiguously negative for calls, while Chaudhury (2017) finds the sign to be ambiguous. This issue is discussed in greater detail later in the paper.

- and *CF'(X) > BS'(X)*

This does not lead us to a consistent sign for the fraction, though, since the relative size of the two terms in the numerator depends on the starting X itself, in addition to the associated discount rate of the original call and the new call. Furthermore, the above inequalities impose the condition that the expected return must be positive, a fact we have not established *ex ante.* Of course, we cannot impose *any* structure on these discount rates when the rates themselves are what we are solving for.

To summarize, the key result is that the change in option expected return over strike price is ambiguous. This is in contrast to existing literature on the theory of options expected returns which finds that expected return is unambiguously increasing in strike price for both call and put options. Furthermore, I demonstrate later in this paper that for plausible calibrations of Equation (2) option expected returns can be *decreasing* in the strike price for some ranges of strike prices.

Use of Stochastic Discount Factor to Derive Call Expected Returns

The question becomes: are there assumptions that we can make about investor preferences that would impose some structure on Equation 2 and lead to the numerator always being positive (which would imply |CF'(X).BS| > |CF.BS'(X)|)?

In Coval and Shumway (2001) the assumption made to generate a positive value for $d[E(R)_C]/dX$ is that the price of the underlying asset and the stochastic discount factor (SDF) are negatively correlated. The SDF is the set of state-dependent values which price the underlyer and the call option. This assumption has been widely noted in the literature, but there is a further assumption in the Coval-Shumway derivation of $d[E(R)_C]/dX$ that is easy to overlook. The setup

implicitly assumes that the SDF which prices the stock (or, is jointly determined with the stock prices) can also price the call option.

However, a unique SDF only exists in complete markets, defined as markets where the number of assets is greater than or equal to the number of future scenarios. When future scenarios are defined by distributions of prices, then clearly there are an infinite number of future scenarios and markets are incomplete. Therefore, the SDF derived from the cash flows of a call option is not a unique SDF which can price all other assets too. This means that the simplifications employed in the Coval-Shumway derivation would not hold in an incomplete market, and therefore neither would the predictions regarding call option expected returns.

Use of Beta to Derive Call Expected Returns

Use of beta, from the Capital Asset Pricing Model framework of risk and expected return[5], to derive predictions regarding expected returns of options dates back to at least the seminal Black-Scholes paper of 1973. Black and Scholes noted that call option beta, often referred to as "instantaneous beta", could be written as:

$$\beta_C = \frac{\Delta.S_0}{C_0}\beta_S \qquad (3)$$

where β_C is the call option beta, β_S is the underlying asset beta, and $[\Delta.S_0 / C_0]$ is considered the elasticity of the option price with respect to stock price. Δ is the call option delta, the sensitivity of option price with respect to stock price, S_0 is the current stock price and C_0 the current call option price. Black-Scholes (1973) note that this elasticity will always be greater than or equal to one, implying call option beta that is always greater than or equal to the beta of the underlying

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[5] See Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966).

asset. This is an alternative way of stating that the call option is a leveraged investment on the underlying asset, and therefore should have greater expected return than the underlying asset.

However, a limitation of using beta in this way is that it is an instantaneous beta. This would be fine for assets with relatively stable betas, where beta is expected to only change when significant new information enters the information set for pricing the asset. However, options are unique in having inherently unstable betas, which are expected to change over time even if nothing unexpected happens. This issue is explored in greater detail in the later section "Explanations".

Rubinstein (1984) also studies option expected returns, and comes up with results that are consistent with Coval-Shumway. Rubinstein writes the expected return on a call option as (notation modified from the original paper):

$$E[R_C] = \left[\frac{E(C_t)}{C_0}\right]^{\frac{1}{t}} - 1$$

where, C_0 is the initial price of the Call and C_t is the price of the Call later at time t (not necessarily the expiry date in this framework). The initial value of the Call is given by the Black-Scholes price:

$$C_0 = BS\{S_0, K, T, r_f, \sigma\}$$

and the later value of the Call at time t is also given by the Black-Scholes price:

$$E(C_t) = BS\{S_0 \mu t, K r_f, T, r_f, \sigma\}$$

In other words, at time t the expected price of the call is based on the stock price having increased at the expected return, the strike price having increased at the risk-free rate and the time to maturity having remained the same.

The intuition for this is far from clear. Clearly, what actually happens when holding an option contract is that time passes and the strike price remains constant (and, it is true, the stock

price would change and we could model that change by the expected return). A more intuitive value for the expected future price of the Call option C_t would be:

$$E(C_t) = BS\{S_0\mu t, K, (T-t), r_f, \sigma\}$$

The above would reflect the fact that as time passes the stock price changes and the strike price remains constant (one might also model changes to implied volatility or the risk-free rate, but neither is considered here.) Using the above representation of $E(C_t)$ leads back to results consistent with the empirics and my later results reported in this paper: expected returns that are decreasing in the strike for out-the-money calls, with expected returns being negative for some plausible specifications.

Derivation of Expected Returns by Numerical Methods

As noted above, an analytical solution to Equation 2 is far from straightforward. Furthermore, it would require imposing conditions on the relative values of BS, CF, BS'(X) and CF'(X). Any conditions imposed would amount to imposing structure on the option expected returns, and therefore the solution would simply reflect whatever structure is imposed. To establish call option expected returns by strike price I adopt a numerical procedure.

I determine expected return by running a simulation that generates 100,000 returns for each call option contract, based on future stock prices generated under the assumption of Brownian motion. Future stock prices are generated according to the following equation consistent with Black-Scholes:

$$S_T = S_0 + \mu T.S_0 + \sigma\sqrt{T}.S_0\epsilon \tag{4}$$

where S_T is the future stock price at expiry of the option contract, S_0 is the original stock price at time 0, μ is the annual expected return on the stock, T is the maturity of the option contract as a

fraction of a year, σ is the annual volatility (standard deviation) in returns of the stock, and ε is a random term with mean zero and standard deviation 1.

The equation is used to generate 100,000 future stock prices, with the simulation using S_0 = \$100, μ and σ of various levels as reported, and T of one month (1/12). For each option contract considered (I include a range of different strike prices in the analysis) 100,000 returns are generated based on the original Black-Scholes price and the final intrinsic value at time T. Formally, the return for each cycle of the simulation is:

$$Return = \frac{MAX(0, S_T - X) - BS(S_0, X, r_f, T, \sigma)}{BS(S_0, X, r_f, T, \sigma)} \tag{5}$$

The results of the simulation are presented in Figure 1. The general picture is one of expected return increasing in the strike price in the range of strike prices that constitute in-the-money (ITM) options. Out-the-money (OTM) calls are generally characterized by decreasing expected returns, with some specifications of the stock return characteristics generating negative expected returns.

This pattern of expected returns is consistent with empirical results which have been reported for average returns on call options and how they vary by strike price. Empirically, only deep ITM calls exhibit reliably positive average returns, with the negative average returns on OTM calls being a well-established, robust, empirical phenomenon.

An interesting feature of the pattern of expected returns across strike prices is the following: for a given expected return μ, the scenario with higher volatility σ in the stock leads to higher call expected returns for both deep in- and deep out-the-money contracts. Call contracts that are mildly in- or out-the-money, or at-the-money, exhibit higher expected returns for low stock-volatility scenarios.

In other words, the pattern of expected returns in the call option reflects the pattern of returns in the stock. If the stock has high volatility, in other words the returns distribution is a flat, spread-out picture, then the call expected returns across strike prices is a flatter picture. If the stock has low volatility, in other words the returns distribution is a thin, peaked picture, then the call expected returns across strike prices is a more curved picture.

This simulation does not directly address the question of whether certain assumptions about investor preferences would provide the structure to produce expected returns that are increasing in the strike price and always greater than the expected return on the underlying asset. However, they do indirectly provide evidence on the question, or at least allow us to re-phrase it in a way which makes it easier for us to answer yes or no.

Let's take one of the simulations as an example: the scenario where the underlying asset is assumed to have normally distributed expected returns of 8% and volatility of 20%. The other assumption of the simulation is that the options are originally priced by Black-Scholes. This setup results in a clear picture of OTM call expected returns decreasing in the strike price (starting with the call option with strike price $102) and becoming negative (for strike prices of $110 and above). It is important to note that the strike price ranges which see decreasing or negative expected returns are not strike prices that are implausibly far away from the market price of the underlyer – this is not a result that can be explained by options that have almost zero chance of finishing in-the-money (the call with strike $110, for example, finishes in-the-money in approximately 5.3% of scenarios).

If we believe that assumptions about investor preferences can rule out this result, that implies that assumptions about investor preferences would rule out the possibility of an asset with the aforementioned characteristics (normal distribution of returns, mean 8%, volatility 20%). This

seems unlikely, since the numbers used here are plausible and consistent with actual data from financial markets. The numbers used also obey the most direct implication of standard assumptions about investor preferences: that the expected return of the stock is greater than the risk-free rate.

Figure 2 reveals that the maturity of the option is not relevant to the pattern of expected returns. Figure 2 repeats the previous analysis with $T = 1$, in other words the maturity of the contracts is one year. The specific expected returns differ to those in Figure 1, but the basic picture is still evident. The results still indicate that for many specifications OTM calls have expected returns that are decreasing in the strike price.

Table 1 re-enforces the result reported earlier in Figure 1. Table 1 reports the Black-Scholes call option price C_0 and the probability weighted future cash flows of various option contracts, denoted CF. CF is generated by evaluating the following integral:

$$\int_{R^*}^{\infty} \left\{ 100 \times (1 + R^*) \times \left[\frac{1}{\sqrt{2\pi}\sigma} * e^{\left[\frac{-1}{2}\left(\frac{R^*-\mu}{\sigma} \right)^2 \right]} \right] \right\} \tag{5}$$

where 100 is the current stock price S_0, R^* is the minimum return required for the call option to finish in-the-money $X/100 - 1$, and other variables are as previously defined. This integral gives the value of each future cash flow of the option, conditional on the option finishing in-the-money, weighted by the probability of that cash flow based on the stock return required to produce it and the further assumptions that the stock returns follow a normal distribution with mean μ and standard deviation σ.

The results reported in Table 1 obviously match up with the picture shown in Figure 1, since evaluating the integral is simply a different way of generating the probability-weighted future cash flows and calculating a return. In spirit, the procedure does the same thing as the simulation

used to generate Figure 1. However, the information in Table 1 allows for a comparison of future cash flows versus original Black-Scholes price for various option contracts.

Situations where CF is calculated to be lower than the Black-Scholes price are scenarios where the expected return is negative. Table 1 reveals that these are scenarios where the option price is originally relatively low in dollar value, typically less than $1. Although the negative expected returns are concentrated in these very low-priced contracts, positive but decreasing expected returns are observed at higher prices. For example, for an underlying asset with expected return 15% and volatility 40% there is a notable decrease in expected return as we look from the $100-strike contract to the $106-strike contract. This decrease in expected return occurs as the price of the option decreases from $4.68 for the $100-strike to $2.38 for the $106-strike.

Effect of Implied Volatility on Call Option Expected Returns

Although several papers, notably Bakshi and Kapadia (2003a, 2003b), Goyal and Saretto (2009) and Cao and Han (2013), examine the pricing of underlyer volatility in option returns from an empirical perspective, less research exists on the theory of why volatility is priced. The pricing of volatility is largely viewed as an empirical phenomenon, driven by the notion that option contracts are a "hedge" against volatility. In other words, the pricing of implied volatility in option returns falls into the category of non-standard investor preferences.

However, Hu and Jacobs (2017) and Chaudhury (2017) produce work on options expected returns which challenges this. Both papers study the effect of different implied volatilities on option expected returns, and in both cases the papers make no non-standard assumptions regarding investor preferences. Hu and Jacobs (2017) derives a negative comparative static relationship between implied volatility and call option expected return by specifying the distribution function

for underlyer returns and then solving the first derivative with respect to implied volatility for the returns equation. Chaudhury (2017) uses numerical methods to calibrate the returns derived from the Black-Scholes model. The papers find conflicting results: Hu-Jacobs report un unambiguous negative relation between implied volatility and call option expected return, Chaudhury finds the relation ambiguous.

Figure 3 illustrates the variation in call option expected returns with respect to implied volatility. This chart illustrates different expected returns for call options with different levels of moneyness and implied volatility. The expected returns are produced with numerical methods, with realized return generated for 100,000 simulations for each option contract. Implied volatility (σ) is stable over the holding period for each simulation and is reported on the horizontal axis. The original asset price (S_0) is assumed to be \$100, and the call option strike prices considered are \$80 (deep ITM), \$90 (ITM), \$100 (ATM), \$110 (OTM) and \$120 (deep OTM). The contracts are all assumed to be 1-month contracts (T=1/12), and are held to expiry with the final price of the underlying asset (S_T) being determined randomly by the equation $S_0 + \mu T . S_0 + \sigma \sqrt{T} . S_0 \epsilon$, where μ is the expected return of the underlying asset, assumed to be 15% annual rate in this analysis, and other elements are as previously defined. The initial price of the call options is derived from Black-Scholes.

Figure 4 reports a similar analysis with further details. This chart illustrates different expected returns for call options with different levels of moneyness and implied volatility. Expected returns are generated as per the procedure in Figure 3, but in this case the holding period is varied. While Figure 3 included only 1-month contracts held to expiry, Figure 4 includes various holding periods as detailed (6m-1m, for example, is an initial 6-month contract held for 5 months until the 1-month to expiry mark. Expected returns are expressed as monthly rates.

What these results show is that the pattern of returns across implied volatility is highly dependent on whether the contract is held to expiry or not. Expected returns for call options are consistently decreasing in the implied volatility, except for some cases in which the contract is held all the way up to the expiry date. When held to expiry, contracts initially out-the-money, and some contracts initially in-the-money, exhibit expected returns that are increasing in implied volatility. This demonstrates a consistent theme in call option expected returns: contracts held all the way to expiry are fundamentally different in terms of expected returns than contracts held to points prior to expiry.

Explanations

In summary so far, a purely theoretical derivation of option expected returns shows that variation in expected returns across strike price is of ambiguous sign. Calibration of the theoretical model for expected returns reveals that expected returns are generally decreasing in the strike price for OTM calls and puts, with some OTM calls exhibiting negative expected returns. These results are consistent with empirical results for call options, but in stark contrast to existing theory of option expected returns which predicts that expected returns are increasing in option strike price for calls and puts.

The significance of the results so far is the following: explanations for the empirical results obtained for option returns has focused on behavioral explanations, or at least explanations which relax traditional asset pricing assumptions and assumptions about investor preferences. For example, one possibility considered is that investors may view OTM calls as "lottery tickets" and exhibit risk-seeking behavior when chasing the highly positively-skewed returns offered by these contracts. However, the results presented in this paper suggests that the empirical results reported

for call options may be consistent with traditional asset pricing assumptions, and do not require explanations with non-standard assumptions about investor preferences.

Some specific questions or issues raised by the analysis are now considered.

- Can the volatility smile explain the results?

No, since the results are derived under the assumption of constant volatility. Therefore, implied volatility is constant across strike prices in all simulations.

- Can non-standard investor preferences explain the results?

An avenue often followed when attempting to explain empirical call returns is that of non-standard investor preferences. Standard investor preferences center on risk aversion, with the implication that risky assets will have expected returns higher than the risk-free rate.

Non-standard preferences allow for lower expected returns amongst assets if investors exhibit a preference for positive skewness in asset returns, or a preference for assets which hedge against market volatility. In the case of the simulations in this paper, neither set of non-standard preferences can explain the results since they were not specifically imposed in the simulations. The simulations simply impose that the expected return of the stock is greater than the risk-free rate, consistent with the implication of standard risk aversion.

However, more complex explanations center around the future states of the economy and the willingness of investors to pay a premium for assets which pay out in bad states of the economy. In many ways this is simply consistent with standard assumptions of risk aversion, but has led to examination of non-standard versions of the stochastic discount factor (SDF) as an explanation for call option returns.

The SDF is what ties the current asset price to future states of the economy. A line of literature attempts to explain call option returns with non-standard SDFs. Standard SDFs are linear, but U-shaped SDFs have been considered in recent literature. The exact implication of U-shaped SDFs for investor preferences is still debated, but in Figure 7 and Table 5 I illustrate that linear SDFs are still capable of pricing call options with negative expected returns.

Figure 7 maps out a tree of future stock prices for an example stock. Table 5 reports the time T cash flows, as well as the SDF, for the stock introduced in Figure 7 and a call option on the stock with strike price $103 (originally out-the-money). The SDF is solved by numerical methods so as to produce the current stock price of $100 when applied to the future cash flows. The same SDF then prices the call option, which has a premium of $0.67. When this current premium is compared to the probability-weighted average of the future cash flows, the expected return on the call is strongly negative. Again, this is the empirical result for OTM call option reproduced in a controlled setting in which all standard asset pricing assumptions are obeyed. (An assumption for the SDF is that it is negatively correlated with the stock cash flows. This is the technical manifestation of strong cash flows being valued less in good states of the economy).

In a second calibration of the SDF, I force the expected return on the call option to match that of the stock (theory predicts it should be higher, but I use equality as a threshold). It is still possible to derive an SDF which prices both stock and option. This new SDF gives a significantly lower price to the strong states of the economy. Intuitively, this makes sense because the option is capable of delivering a $0 cash flow in good states of the economy (when the stock price increases, but not enough to drive the option into-the money).

A couple of comments are important here. The fact that more than one SDF can price the stock at $100 indicates that the SDF is not unique, consistent with incomplete markets. It

furthermore underscores the importance of determining an SDF jointly with the cash flows from all assets. In complete markets the SDF derived for one asset prices all other assets too, but clearly this does not work in this case.

- <u>Is there sufficient data to expect the simulation to produce results for expected returns? Are the results driven by the specific parameters selected?</u>

A variety of plausible specifications, all of which are consistent with conventional notions of asset pricing, produce decreasing expected returns in the strike price for OTM calls. Some plausible specifications even produce decreasing expected returns for at- or slightly in-the-money options. In all cases, unless otherwise specified, the simulations enforce the most basic implication of conventional investor preference theory: the expected return on the underlying stock is greater than the risk-free rate of return.

- <u>Are the results driven by "tail" events that are unlikely in practice, or option contracts that are either illiquid or priced at zero in practice?</u>

It is the case, as evident in Table 1 for example, that most of the negative expected returns and many of the positive but decreasing (compared to contracts with lower strike prices) expected returns are coming from relatively low-priced options. However, since the analysis in the paper is strictly within the realm of theory, with no market frictions playing a role at all, the results for these relatively low-priced options are still legitimate and interesting. Furthermore, expected returns are decreasing in the strike price for ranges of strikes that are well within reasonable distance of the original market price of the underlying stock. Many of the option strike prices for

which these results are obtained are within a one standard deviation return of the original stock price.

- Can the returns be explained intuitively as investors paying for favorable market timing properties in an investment? Does this imply that gamma is a priced factor in expected returns?

I suggest that one feature of options that makes the expected return so difficult to model and capture is that there's no reason to believe that expected returns are stable over time. Indeed, some simple examples will illustrate the exact opposite: not only are option expected returns varying through time, but this is in fact expected *ex ante*. It is careful to distinguish this from the notion of time-varying expected returns in other financial assets. Some evidence exists, for example, that stock returns may be time-varying, but this variation is driven by changes in the fundamental nature of the stock itself, or by changing market conditions. This is different to option contracts, where expected return is expected to be unstable and change over time *even if the future unfolds exactly as expected with no unexpected shocks to the market environment or the fundamental nature of the underlying asset.*

Table 2 sheds light on this interesting issue. The option gamma can be thought of as capturing the "market timing" properties of the option contract. Call options have the appealing feature that as they move further and further in-the-money delta increases. In other words, the sensitivity of the call option price to changes in the price of the underlyer increases exactly when the value of the underlying asset increases. This change in delta is measured by gamma, the second derivative of the option price with respect to the price of the underlying asset.

Since this paper is concerned with option returns, it is not so much the gamma we are interested in, but rather the gamma as a percentage of the original call option price. This metric provides a measure of the effect on the option return rather than the option price.

Table 2 reports results for option gamma as a percentage of the original option price for various call option contracts. The main point to note is that this metric is monotonically increasing in the strike price for call options. Therefore, call options exhibit a positive market timing feature, which investors might be willing to pay for. These "market timing" elements, as captured by the gamma ratio, is an interesting area to explore for an explanation of the pattern of expected returns.

- Time-Varying Betas

Figure 5 illustrates different betas for call options with different levels of moneyness. Call option beta is calculated as:

$$\beta_C = \frac{\Delta.S_0}{C_0} \beta_S$$

where β_C is the call option beta, β_S is the underlying asset beta, and $[\Delta.S_0 / C_0]$ is considered the elasticity of the option price with respect to stock price. Δ is the call option delta, the sensitivity of option price with respect to stock price, S_0 is the current stock price and C_0 the current call option price. In this analysis the underlying asset beta is assumed to be 1, matching a market level of beta. Time to maturity, express in days, is reported on the horizontal axis. The original stock price (S0) with 30 days to expiry is assumed to be $100, with each subsequent stock price determined in accordance with an expected return of 12% annual rate, adjusted to the 1-day timeframe.

This chart dramatically illustrates the inherent time-variation in beta for call option contracts, particularly those which are initially out-the-money. In this chart the time-to-expiry and

the price of the underlying asset are both changing over time, but in completely expected ways. The expiry date and the expected return of the underlyer remain constant over the entire timeframe depicted. In other words, no new information enters the information set during the holding period shown.

Despite this, beta changes significantly during the timeframe of the call option contracts, with initially out-the-money calls exhibiting an explosion in beta as they near the expiry date. This instability in beta provides a fresh avenue for study, and helps explain why theoretical returns have had so much trouble capturing empirical returns in options research. The inherent instability in risk-expected return profile is a factor which must be accounted for in any theory of option expected returns.

Table 4 summarizes the changes in beta over time for various different call option contracts. Call option beta is calculated as:

$$\beta_C = \frac{\Delta . S_0}{C_0} \beta_S$$

where β_C is the call option beta, βS is the underlying asset beta, and $[\Delta.S_0 / C_0]$ is considered the elasticity of the option price with respect to stock price. Δ is the call option delta, the sensitivity of option price with respect to stock price, S_0 is the current stock price and C_0 the current call option price. In this analysis the underlying asset beta is assumed to be 1, matching a market level of beta. Time to maturity, express in days, is reported on the horizontal axis. The original stock price (S_0) with 30 days to expiry is assumed to be $100, with the future stock price (S_t) 5 days prior to expiry being determined in accordance with an expected return of 8% annual rate, adjusted to the 25-day timeframe. The change in beta is calculated as:

$$\Delta \beta_C = \beta_{C,5} - \beta_{c,30}$$

where $\beta_{C,5}$ is the call option beta at T-5 (calculated in accordance with the equation above for call option beta) and $\beta_{C,30}$ is the call option beta at T-30. The results summarized below are for 100,000 different simulations of this change in beta, based on different values for S_t, where S_t is randomly generated by standard random walk with drift ($\mu = 8\%$, σ as indicated in the table)

The results show that the range of future betas is significantly wider for OTM call options than ITM call options. This significant instability in beta matches up closely with the option contracts which have expected returns departing from the predictions of standard asset pricing theory. Figure 6 graphically illustrates the distributions of future betas for a variety of different options, clearly illustrating the wider range of the OTM option distribution relative to the ATM and ITM calls.

Conclusion

This paper revisits the theory of expected option returns and finds that the cross-section of expected returns across strike prices is of ambiguous sign. Plausible calibrations of the expected return model generate expected returns that are increasing in the strike price for ITM contracts, but decreasing in the strike price for at- to out-the-money options. This is consistent with existing empirical results, but a result not previously anticipated by option expected returns theory.

The main point of the paper is that non-traditional investor preferences or non-normal distributions of returns in the underlyer are not required to explain the empirical results that OTM calls have average returns that are decreasing in the strike price and even negative for deep OTM contracts. This paper argues that these results are to be expected under expected returns theory which enforces all traditional asset pricing assumptions. Behavioral factors and non-normality on

asset returns may still be factors in option returns, but are not required to explain the basic pattern of expected returns across different strike prices.

Other questions which arise and are worthy of further study include: How are the option "Greeks" priced as factors in option expected returns? Is time value a cost of trading which represents a 'drag' on returns or a risk factor that increases expected return? How should we model the inherent instability in option beta over time, given that models of risk and expected return generally rely on a stable risk-expected return structure over time?

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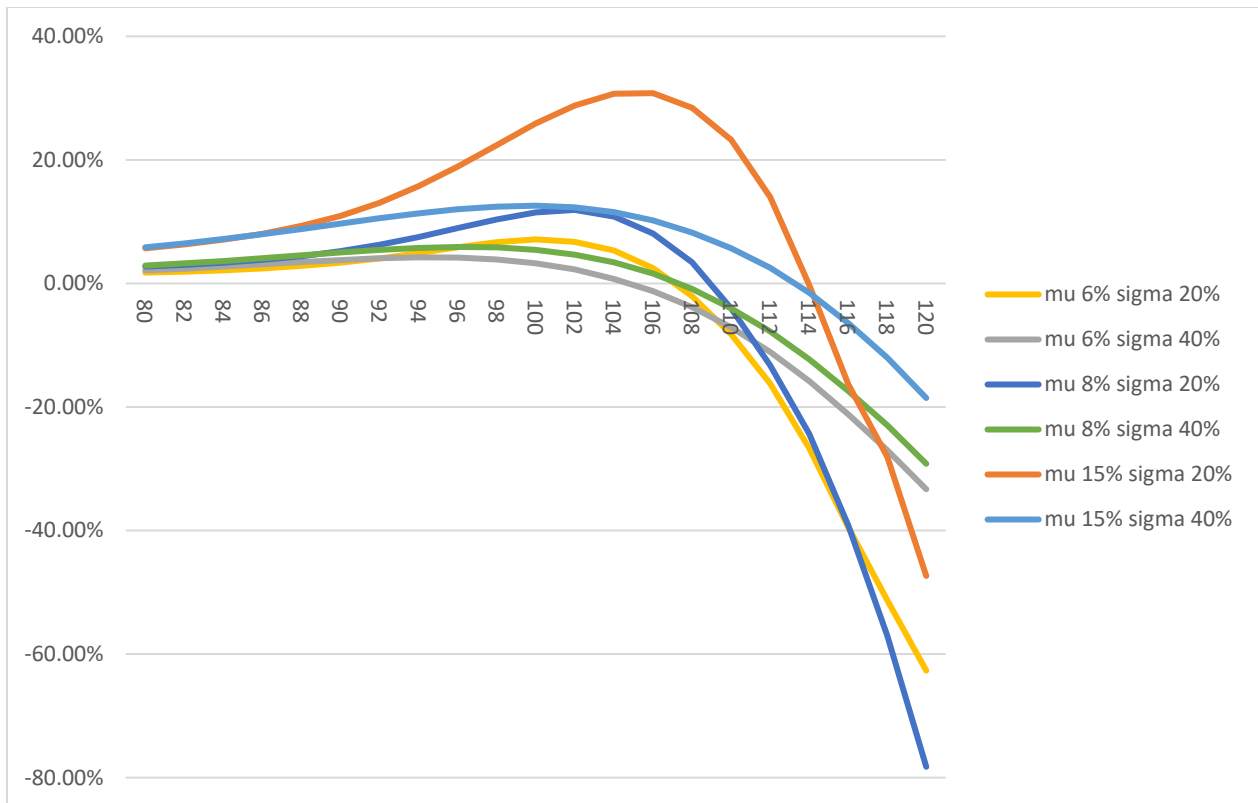
Figure 1: Expected returns of call options for various strike prices.

The above chart is generated with 100,000 future stock prices based on an original stock price of $100, expected return and volatility in returns as indicated. The future stock prices are generated under the assumption of Brownian motion.

The option expected return is the arithmetic average of the 100,000 call returns generated by comparing the original Black-Scholes price of the option $C(X,\sigma,T,r_f,S_0)$, where X is as indicated on the horizontal of the chart, σ is as indicated, T is 1 month (1/12), r_f is 2% as an annualized rate, and S_0 is $100.
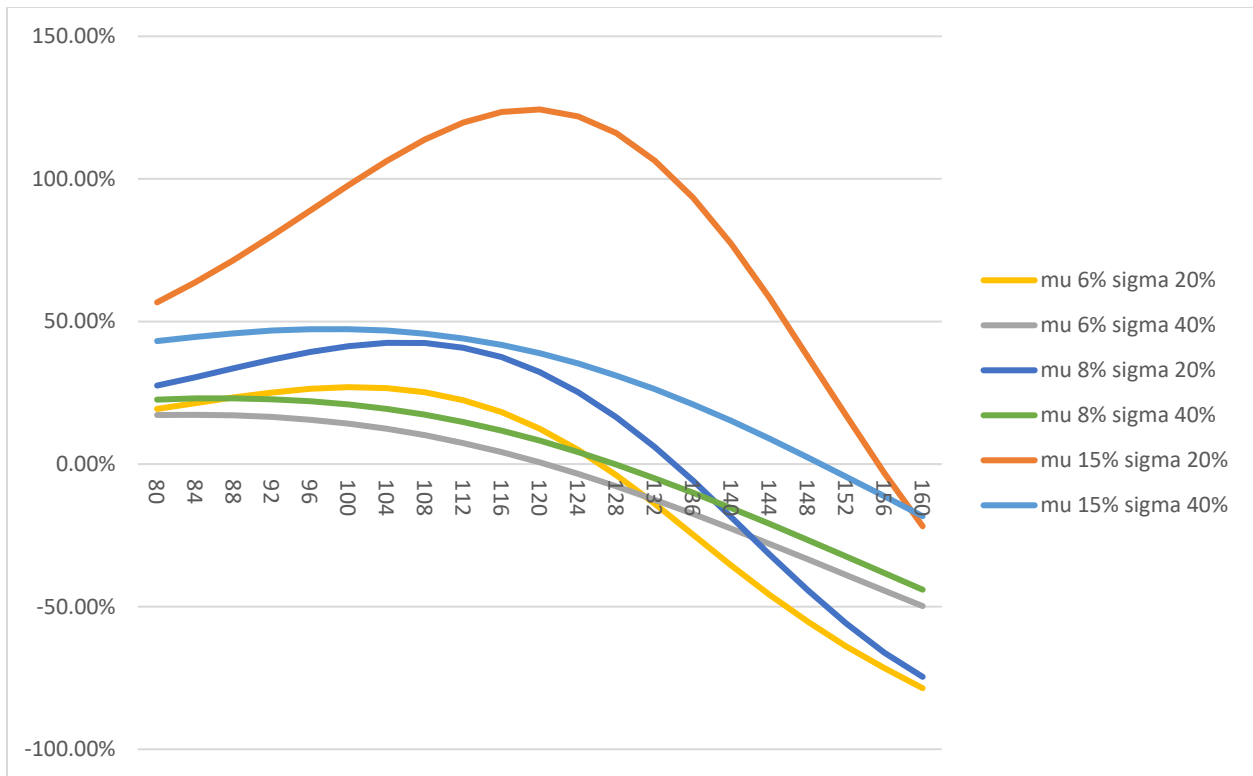
Figure 2: Expected returns of 1-year call options for various strike prices.
The above chart is generated with 100,000 future stock prices based on an original stock price of $100, expected return and volatility in returns as indicated. The future stock prices are generated under the assumption of Brownian motion.

The option expected return is the arithmetic average of the 100,000 call returns generated by comparing the original Black-Scholes price of the option $C(X, \sigma, T, r_f, S_0)$, where X is as indicated on the horizontal of the chart, σ is as indicated, _T is 1 year_, r_f is 2% as an annualized rate, and S_0 is $100.

<u>Table 1: Comparison of probability-weighted future cash flows and Black-Scholes prices for various options</u>

The table reports the Black-Scholes call option price C_0 and the probability weighted future cash flows of the option CF. μ is the expected return on the underlying asset, σ is the volatility in returns of the underlying asset and X is the call option strike price. In all cases, the holding period of the option, and therefore the period over which future cash flows are generated, is one month. *Return* is calculated as $CF/C_0 - 1$. *CF* is generated by evaluating the following integral:

$$\int_{R^*}^{\infty} 100 \times (1 + R^*) \times \left[\frac{1}{\sqrt{2\pi}\sigma} * e^{\left[\frac{-1}{2}\left(\frac{R^*-\mu}{\sigma} \right)^2 \right]} \right]$$

where 100 is the current stock price S_0, R^* is the minimum return required for the call option to finish in-the-money $X/100 - 1$, and other variables are as previously defined. This integral gives the value of each future cash flow of the option, conditional on the option finishing in-the-money, weighted by the probability of that cash flow based on the stock return required to produce it and the further assumptions that the stock returns follow a normal distribution with mean μ and standard deviation σ.

$\mu = 6\%, \sigma = 20\%$

	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.133	$14.150	$4.919	$2.385	$0.513	$0.011	$0.002
CF	$20.500	$14.511	$5.220	$2.562	$0.526	$0.006	$0.001
Return	1.82%	2.55%	6.10%	7.41%	2.44%	-41.11%	-63.80%

$\mu = 6\%, \sigma = 40\%$

	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.234	$14.609	$6.890	$4.684	$2.384	$0.601	$0.319
CF	$20.675	$15.077	$7.202	$4.861	$2.369	$0.480	$0.217
Return	2.18%	3.21%	4.53%	3.78%	-0.59%	-20.10%	-31.93%

$\mu = 8\%, \sigma = 20\%$

	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.133	$14.150	$4.919	$2.385	$0.513	$0.011	$0.002
CF	$20.667	$14.677	$5.351	$2.652	$0.555	$0.007	$0.001
Return	2.65%	3.72%	8.77%	11.18%	8.09%	-35.33%	-59.83%

(Table continued overleaf)

$\mu = 8\%, \sigma = 40\%$

	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.234	$14.609	$6.890	$4.684	$2.384	$0.601	$0.319
CF	$20.836	$15.227	$7.311	$4.948	$2.423	$0.495	$0.225
Return	2.97%	4.23%	6.12%	5.63%	1.64%	-17.58%	-29.51%

$\mu = 15\%, \sigma = 20\%$

	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.133	$14.150	$4.919	$2.385	$0.513	$0.011	$0.002
CF	$21.250	$15.257	$5.820	$2.982	$0.667	$0.010	$0.001
Return	5.55%	7.83%	18.31%	25.02%	29.83%	-10.74%	-41.05%

$\mu = 15\%, \sigma = 40\%$

|	In-the-money			At-the-money		Out-the-money	
X	80	86	96	100	106	116	120
C(0)	$20.234	$14.609	$6.890	$4.684	$2.384	$0.601	$0.319
CF	$21.399	$15.753	$7.700	$5.259	$2.616	$0.551	$0.254
Return	5.76%	7.83%	11.76%	12.27%	9.74%	-8.23%	-20.44%

This table reports the call option gamma as a percentage of the original Black-Scholes option price for a variety of different options. The reported metric is Γ/C_0, where Γ is the option gamma and C_0 the original Black-Scholes price of the option. X indicates the option strike price and σ the volatility of the underlying asset (gamma is independent of the expected return of the underlyer). Gamma is given by differentiating the Black-Scholes equation for a call option twice with respect to the underlying stock price S_0 to yield:

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}} \text{ where } N'(d_1) = \frac{1}{\sqrt{2\pi}}e^{-(d_1)^2/2} , d_1 = \frac{In(S_0/X)+(r_f+\sigma^2/2)T}{\sigma\sqrt{T}}$$

and r_f is the risk-free rate, assumed to be 2% annualized and T is the time to maturity, assumed to be one month.

X	90	92	94	96	98	100	102	104	106	108	110	Difference [90 - 100]
$\sigma = 15\%$												
Γ/C_0	0.0004	0.0016	0.0048	0.0123	0.0267	0.0508	0.0870	0.1370	0.2019	0.2822	0.3779	-0.3774
$\sigma = 30\%$												
Γ/C_0	0.0019	0.0031	0.0047	0.0068	0.0096	0.0130	0.0172	0.0221	0.0278	0.0343	0.0416	-0.0397
$\sigma = 60\%$												
Γ/C_0	0.0014	0.0017	0.0020	0.0024	0.0028	0.0033	0.0038	0.0043	0.0049	0.0055	0.0062	-0.0048
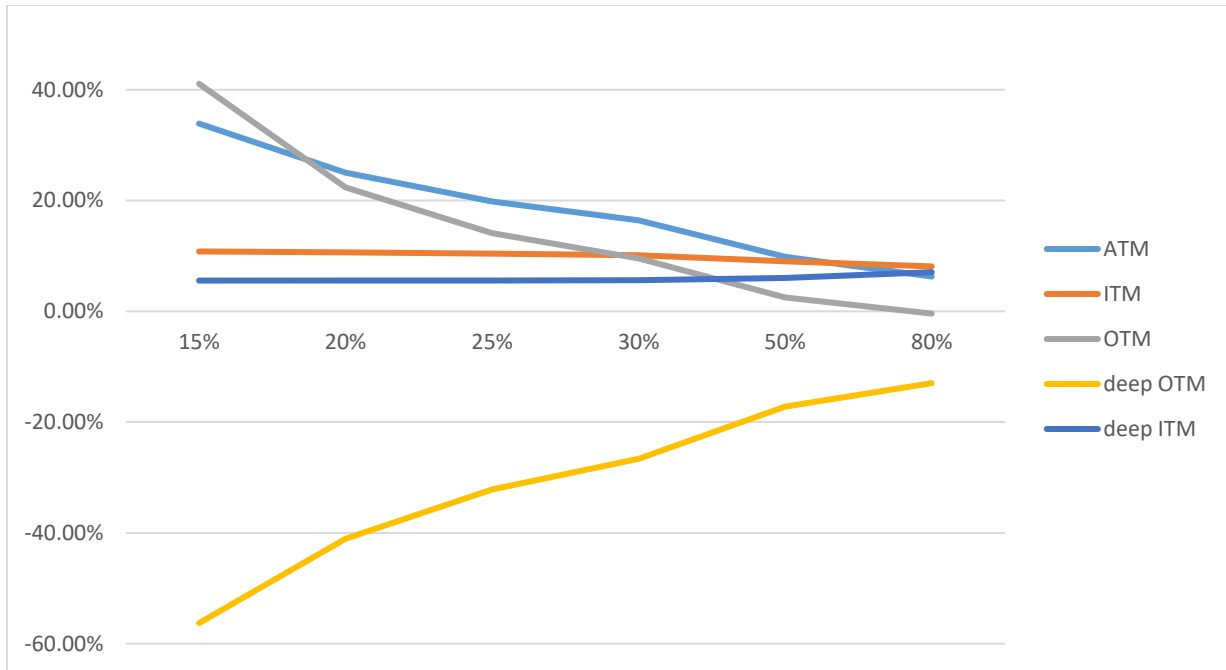
Figure 3: Variation in Call Option Expected Returns with Implied Volatility
This chart illustrates different expected returns for call options with different levels of moneyness and implied volatility. The expected returns are produced with numerical methods, with realized return generated for 100,000 simulations for each option contract. Implied volatility (σ) is stable over the holding period for each simulation and is reported on the horizontal axis. The original asset price (S_0) is assumed to be \$100, and the call option strike prices considered are \$80 (deep ITM), \$90 (ITM), \$100 (ATM), \$110 (OTM) and \$120 (deep OTM). The contracts are all assumed to be 1-month contracts (T=1/12), and are held to expiry with the final price of the underlying asset (S_T) being determined randomly by the equation $S_0 + \mu T. S_0 + \sigma \sqrt{T}. S_0 \epsilon$, where μ is the expected return of the underlying asset, assumed to be 15% annual rate in this analysis, and other elements are as previously defined.

Figure 4: Variation in Call Option Expected Returns with Implied Volatility

This chart illustrates different expected returns for call options with different levels of moneyness and implied volatility. Expected returns are generated as per the procedure in Figure 3, but in this case the holding period is varied. While Figure 3 included only 1-month contracts held to expiry, Figure 4 includes various holding periods as detailed (6m-1m, for example, is an initial 6-month contract held for 5 months until the 1-month to expiry mark. Expected returns are expressed as monthly rates.

Table 3: Summary of Results

	Effect on Call Option Expected Return (Holding period ends **prior to expiry**)		
Factor	In-the-money	At-the-money	Out-the-money
Volatility	Decreasing	Decreasing	Decreasing

	Effect on Call Option Expected Return (Holding period ends **at expiry**)		
Factor	In-the-money	At-the-money	Out-the-money
Volatility	Increasing	Decreasing	Increasing
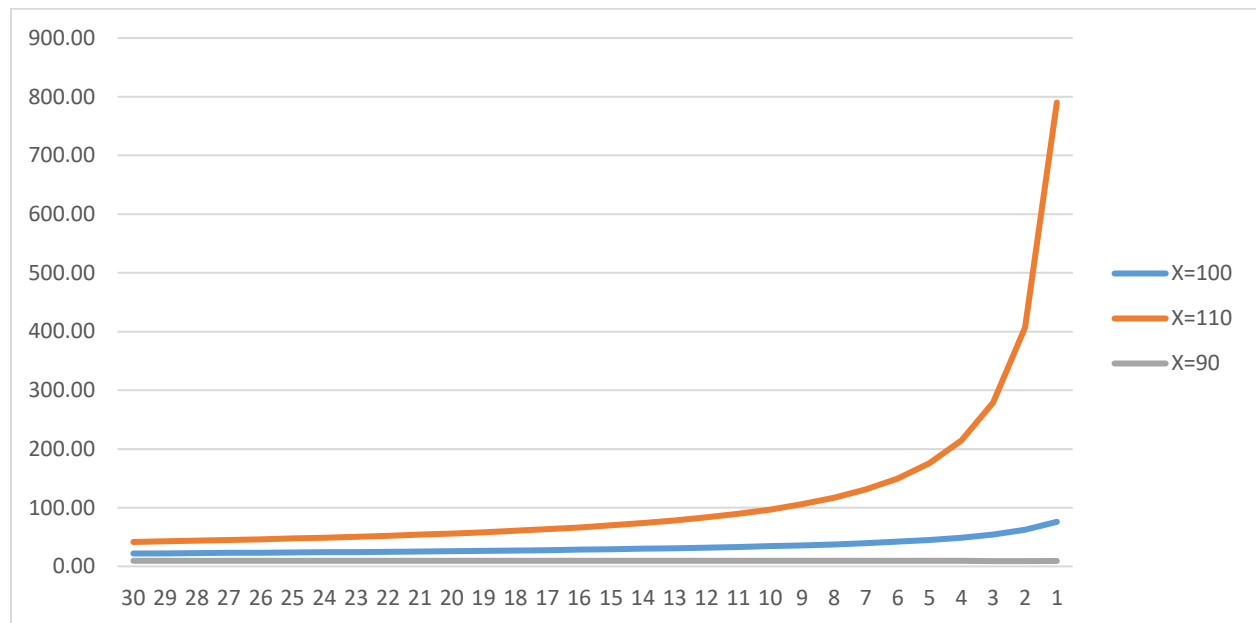
Figure 5: Call Option Beta over Time

This chart illustrates different betas for call options with different levels of moneyness. Call option beta is calculated as:

$$\beta_C = \frac{\Delta.S_0}{C_0}\beta_S$$

where β_C is the call option beta, β_S is the underlying asset beta, and $[\Delta.S_0 / C_0]$ is considered the elasticity of the option price with respect to stock price. Δ is the call option delta, the sensitivity of option price with respect to stock price, S_0 is the current stock price and C_0 the current call option price. In this analysis the underlying asset beta is assumed to be 1, matching a market level of beta. Time to maturity, express in days, is reported on the horizontal axis. The original stock price (S_0) with 30 days to expiry is assumed to be $100, with each subsequent stock price determined in accordance with an expected return of 12% annual rate, adjusted to the 1-day timeframe.

<u>Table 4: Changes in option beta over time</u>

This table summarizes the changes in beta over time for various different call option contracts. Call option beta is calculated as:

$$\beta_C = \frac{\Delta.S_0}{C_0}\beta_S$$

where β_C is the call option beta, β_S is the underlying asset beta, and $[\Delta.S_0 / C_0]$ is considered the elasticity of the option price with respect to stock price. Δ is the call option delta, the sensitivity of option price with respect to stock price, S_0 is the current stock price and C_0 the current call option price. In this analysis the underlying asset beta is assumed to be 1, matching a market level of beta. Time to maturity, express in days, is reported on the horizontal axis. The original stock price (S_0) with 30 days to expiry is assumed to be $100, with the future stock price (S_t) 5 days prior to expiry being determined in accordance with an expected return of 8% annual rate, adjusted to the 25-day timeframe. The change in beta is calculated as:

$$\Delta\beta_C = \beta_{C,5} - \beta_{c,30}$$

where $\beta_{C,5}$ is the call option beta at T-5 (calculated in accordance with the equation above for call option beta) and $\beta_{C,30}$ is the call option beta at T-30. The results summarized below are for 100,000 different simulations of this change in beta, based on different values for S_t, where S_t is randomly generated by standard random walk with drift ($\mu = 8\%$, σ as indicated in the table)

	X = $80		X = $100		X = $110	
	$\sigma = 8\%$	$\sigma = 56\%$	$\sigma = 8\%$	$\sigma = 56\%$	$\sigma = 8\%$	$\sigma = 56\%$
Mean	-0.038	3.394	85.931	14.729	775.585	23.079
Std.dev.	0.412	7.846	115.132	15.121	191.716	16.667
Skewness	0.667	3.223	1.496	1.023	-0.026	0.474
Kurtosis	0.903	13.016	2.378	0.515	-0.035	-0.475
Minimum	-1.253	-2.412	-40.547	-4.648	-10.659	-5.552
Maximum	3.098	80.244	912.365	78.878	1594.138	77.232

X = $80, σ = 8.00%   X = $80, σ = 56.00%

X = $100, σ = 8.00%   X = $100, σ = 56.00%
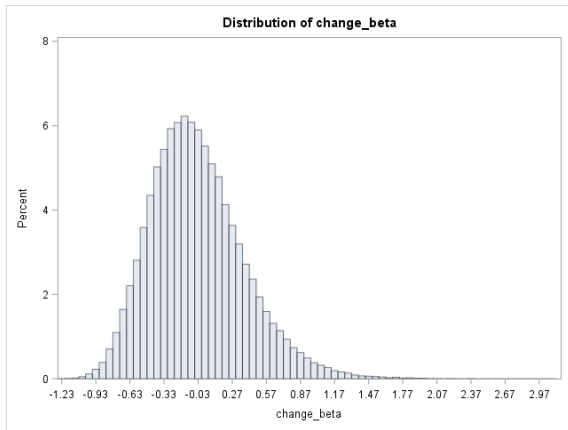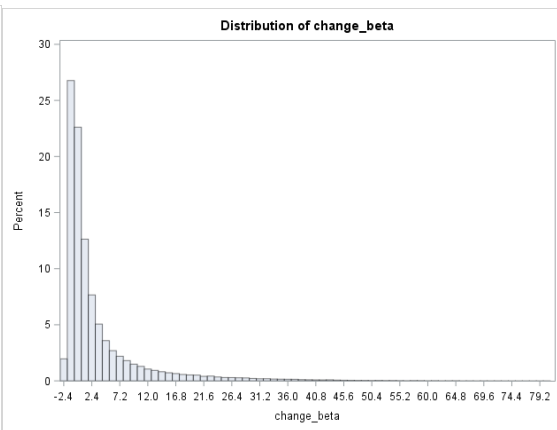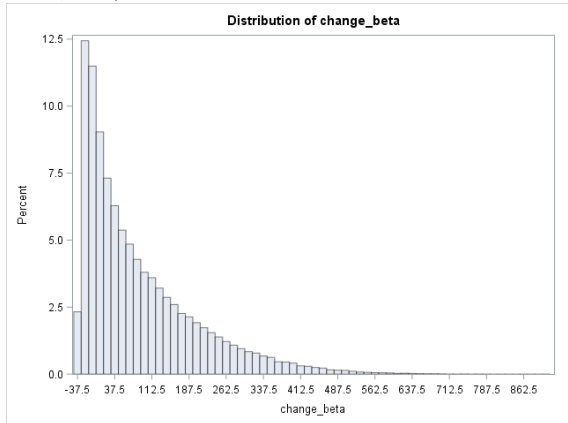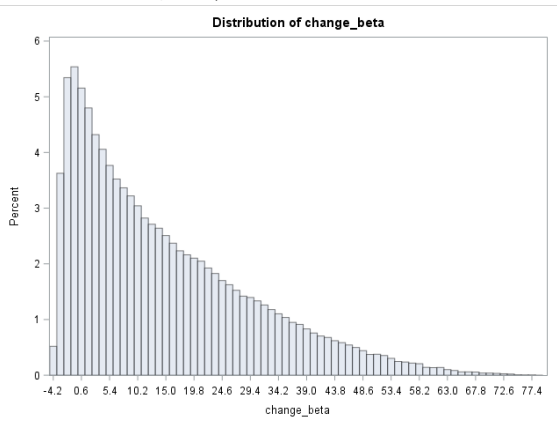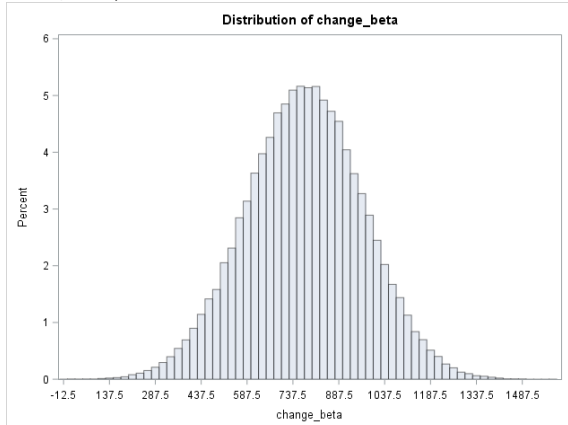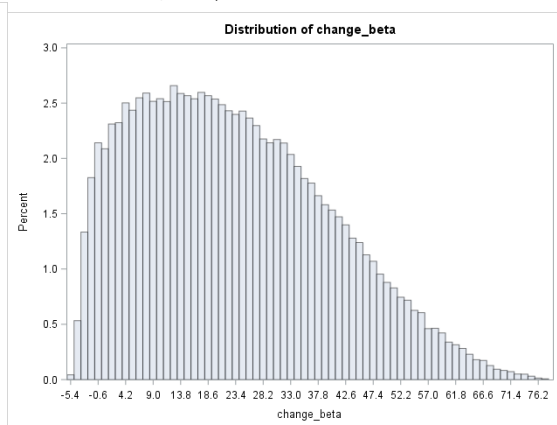

X = $110, σ = 8.00%   X = $110, σ = 56.00%

Figure 6: Distributions of future option betas.
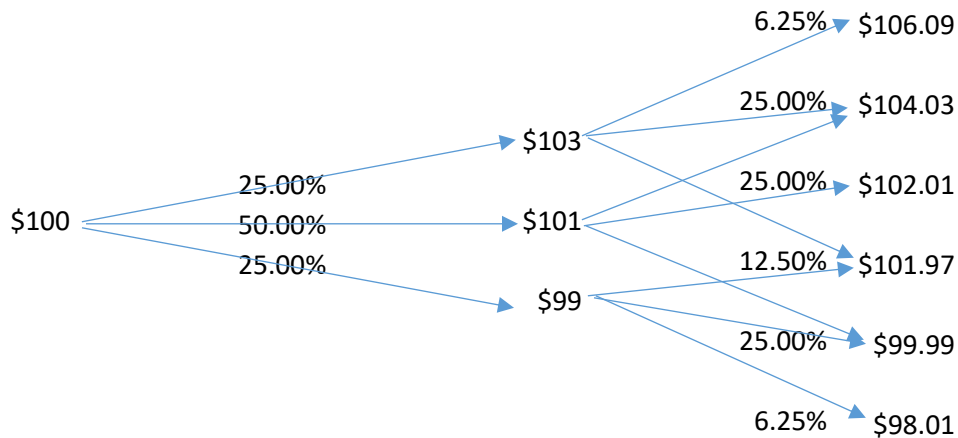These charts depict the distributions of future betas for various options, as detailed in Table 4.

Figure 7: This follows the price tree of a stock with current price $100.

Returns per period over the next two periods are as follows:
1% with 50% probability,
3% with 25% probability,
and -1% with 25% probability.

Final prices after period two are:

Price [S_T]	Probability
$106.09 [$100 x 1.03 x 1.03]	with 6.25% probability [25% x 25%]
$104.03 [$100 x 1.01 x 1.03 or $100 x 1.03 x 1.01]	with 25% probability [25% x 50% x 2]
$102.01 [$100 x 1.01 x 1.01]	with 25% probability [50% x 50%]
$101.97 [$100 x 0.99 x 1.03 or $100 x 1.03 x 0.99]	with 12.50% probability [25% x 25% x 2]
$ 99.99 [$100 x 0.99 x 1.01 or $100 x 1.01 x 0.99]	with 25% probability [25% x 50% x 2]
$ 98.01 [$100 x 0.99 x 0.99]	with 6.25% probability [25% x 25%]

<p style="text-align:center">Table 5: Pricing of stock and call option with stochastic discount factors</p>

This table reports the time T cash flows, as well as the stochastic discount factor, for the stock introduced in Figure 7 and a call option on the stock with strike price $103 (originally out-the-money). *Prob.* indicates the probability of the future scenario at time T, *X* indicates the cash flow on the asset in that scenario, and *M* indicates the stochastic discount factor in that scenario. S_0 indicates the current price of the stock and C_0 the current price of the call option, both derived from the stochastic discount factor. $E(R)_{C,0,T}$ is the expected return of the call option at time 0 for the timeframe time 0 to time T.

Stock, cash flows, $S_0 = \$100$

Prob.	X	M
6.25%	$106.09	**0.1619**
25.00%	$104.03	**0.1627**
25.00%	$102.01	**0.1634**
12.50%	$101.97	**0.1634**
25.00%	$ 99.99	**0.1641**
6.25%	$ 98.01	**0.1648**

Call, strike = $103, **$C_0$ = $0.67**

Prob.	X	M
6.25%	$ 3.09	**0.1619**
25.00%	$ 1.03	**0.1627**
25.00%	$ 0.00	**0.1634**
12.50%	$ 0.00	**0.1634**
25.00%	$ 0.00	**0.1641**
6.25%	$ 0.00	**0.1648**

$E(R)_{C,0,T} = -32.53\%$

Stock, cash flows, $S_0 = \$100$

Prob.	X	M
6.25%	$106.09	**0.0916**
25.00%	$104.03	**0.1540**
25.00%	$102.01	**0.1844**
12.50%	$101.97	**0.1844**
25.00%	$ 99.99	**0.1849**
6.25%	$ 98.01	**0.1853**

Call, strike = $103, **$C_0$ = $0.44**

Prob.	X	M
6.25%	$ 3.09	**0.0916**
25.00%	$ 1.03	**0.1540**
25.00%	$ 0.00	**0.1844**
12.50%	$ 0.00	**0.1844**
25.00%	$ 0.00	**0.1849**
6.25%	$ 0.00	**0.1853**

$E(R)_{C,0,T} = 2.01\%$