The Pricing of Liquidity Factors

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Abstract

This paper examines the pricing of liquidity factors and the performance of liquidity-augmented factor models. Using the 1963-2023 US stock market data, we construct six liquidity factors based on liquidity costs (LIQ), liquidity commonality risk (COM), return sensitivity to market liquidity (RML), liquidity sensitivity to market return (LMR), liquidity sensitivity to market uncertainty (LMU), and liquidity sensitivity to macroeconomic shocks (LME). Of these factors, LIQ, COM, and LMU capture additional dimensions of risks that the Fama-French factors do not. We show that asset pricing models with a liquidity factor perform better than those with the size factor (SMB).

JEL classification: G11, G12, G14

Keywords: Liquidity, Asset pricing, Size effect, Fama-French factors, Risk-adjusted returns

1. Introduction

This paper analyses the role of liquidity in asset pricing. In particular, we examine whether adding a liquidity factor to the standard Fama-French factor model provides additional benefits. Using Fama and French's (2018) right-hand-side and left-hand-side approaches, we explore whether (1) liquidity factors pick up risks not captured by the Fama-French factors and (2) liquidity-augmented factor models explain variations in cross-sectional stock returns better than the standard factor model. We identify three liquidity factors that capture additional dimensions of risks not reflected in the Fama-French factors. These liquidity factors are highly correlated with the Fama-French size factor (SMB), and asset pricing models that include one of these liquidity factors perform better than those with SMB.

Prior research shows that illiquid assets provide higher returns than liquid assets.[1] Prior research also documents the existence and pricing of liquidity risk.[2] Pastor and Stambaugh (2003) construct a factor based on stock return sensitivity to market liquidity and find that the factor is priced in the US stock market.[3,4] Liu (2006) constructs a liquidity factor based on stock illiquidity measured by the proportion of zero-volume days and finds that the factor has strong pricing effects. Acharya and Pedersen (2005) develop a liquidity-adjusted capital asset pricing model (LCAPM), which relates stock returns to the standard CAPM beta (β_1) and three liquidity betas [i.e., the covariance between stock liquidity and market liquidity (β_2); the covariance between stock return and market liquidity (β_3), and the covariance between stock liquidity and the market return (β_4)].[5] Boyle and Hong (2020) decompose β_4 into two parts: the covariance between stock

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[1] For example, see Amihud and Mendelson (1986, 1989) and Amihud (2002).

[2] See Brennan and Subrahmanyam (1996), Jones (2002), Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Coughenour and Saad (2004), and Pastor and Stambaugh (2003).

[3] Pastor and Stambaugh's (2003) liquidity factor and β_3 in Acharya and Pedersen (2005) capture similar liquidity risk (i.e., the covariation between stock returns and market illiquidity). Pastor and Stambaugh (2003) use dollar volume traded as a proxy for liquidity, whereas Acharya and Pedersen (2005) use normalized Amihud liquidity measure (price impact).

[4] Recent studies show mixed results. In the 1963-2013 US data, Fama and French (2015, 2016) find that augmenting the Pastor-Stambaugh factor into the Fama-French factor model does not improve the model performance. Momani (2018) finds that the Pastor-Stambaugh factor is not priced in the 1966-2016 US data. Acharya and Pedersen (2005) document that the liquidity risk premium associated with β_3 (return sensitivity to market illiquidity) is not economically significant in the 1964-1999 US stock market data.

[5] Acharya and Pedersen (2005) report that the risk premium associated with β_4 is about five and ten times larger than that associated with β_2 and β_3, respectively.

liquidity and macroeconomic shock (β_{4a}) and the covariance between stock liquidity and stock market risk (β_{4b}).[6] Amihud et al. (2015) and Amihud and Noh (2021) construct the illiquidity factor (IML factor; illiquidity minus liquidity) using Amihud's (2002) illiquidity measure and find that it is priced using the US and international data.

The present study differs from prior studies in several important ways. We construct liquidity factors based on liquidity costs and risks identified in the theoretical models of Acharya and Pedersen (2005) and Boyle and Hong (2020). Furthermore, unlike Pastor and Stambaugh (2003) and Liu (2006), we use double-sorts when constructing the liquidity factors. We double-sort stocks by return volatility and liquidity risk to avoid confounding problems (Amihud et al., 2015; Amihud and Noh, 2021) that arise from a high correlation between liquidity and return volatility (Stoll, 1978) and their respective effects on asset prices (Acharya and Pederson, 2005; Ang et al., 2006, 2009).

We focus on the marginal benefit of adding a liquidity factor into the standard Fama-French factor model. Although Acharya and Pederson (2005) examine the economic significance of different liquidity risks, they do not examine the marginal benefit of adding liquidity factors into the standard factor model. As a result, their study sheds limited light on whether liquidity risks are additional pricing factors not captured by the Fama-French factors or pricing factors already reflected in the Fama-French factors. We conduct factor model performance analyses to shed further light on the issue using the nine performance metrics suggested by Barillas and Shanken (2016) and Fama and French (2015, 2016, 2018).

We construct liquidity factors using the 1963-2023 US stock market data. Specifically, we construct the following six liquidity factors: liquidity costs (LIQ), liquidity commonality (COM), return sensitivity to market liquidity (RML), liquidity sensitivity to market return (LMR), liquidity sensitivity to macroeconomic shocks (LME), and liquidity sensitivity to market uncertainty (LMU) factors. In the right-hand-side approach, we regress these six liquidity factors on the Fama-French factors, including the size factor (SMB). We find that the intercept is positive and significant when the dependent variable is liquidity

[6] Boyle and Hong (2020) show that β_{4a} is about three times more strongly priced than β_{4b}.

costs (LIQ), liquidity commonality (COM), or liquidity sensitivity to market uncertainty (LMU), indicating that the Fama-French factors cannot fully account for these factors. By contrast, the intercept is small and statistically insignificant when the dependent variable is return sensitivity to market liquidity (RML), liquidity sensitivity to market return (LMR), or liquidity sensitivity to macroeconomic shocks (LME). These results suggest that only LIQ, COM, and LMU capture risks not captured by the Fama-French factors. Given these results, we focus on LIQ, COM, and LMU in our factor model performance analyses (i.e., the left-hand-side approach).

The adjusted-R^2 for the LIQ, COM, and LMU regression models is higher than 0.64, 0.57, and 0.43, respectively. However, when we exclude the size factor (SMB), the corresponding adjusted-R^2 is lower than 0.07, 0.10, and 0.13. Therefore, much of the commonality between liquidity factors (LIQ, COM, and LMU) and the Fama-French factors comes from high correlations between the liquidity and size factors, i.e., the correlation coefficient between each liquidity factor (LIQ, COM, and LMU) and SMB is 0.77, 0.71, and 0.61, respectively. The high correlation between the liquidity and size factors does not necessarily mean the liquidity factors are redundant. The fact that the intercept is positive and significant when we use each factor as the dependent variable suggests that these liquidity factors capture risks that the Fama-French factors (including SMB) do not.

The intercept is small and statistically insignificant when we regress SMB on other Fama-French factors and a liquidity factor (or when we regress SMB only on a liquidity factor) in the right-hand-side approach. This result implies that the liquidity effect subsumes the size effect (Amihud and Mendelson, 1989). Furthermore, the insignificant intercept when the liquidity factor is included in the regression suggests that the size factor may be redundant once the liquidity factor is included in the asset pricing model.

In the left-hand-side approach, we compare the performance of the following four models: (M1) the five-factor model with the market (MKT), value (HML), profitability (RMW), investment (CMA), and momentum (MOM) factors; (M2) the Fama-French (2018) six-factor model (M1 plus SMB); (M3) M1 plus a liquidity risk factor (LIQ, COM, or LMU); and (M4) the liquidity-risk-augmented Fama-French (2018) six-factor model (M2 plus a liquidity risk factor).

We assess the marginal benefit of adding SMB into a factor model by comparing the model performance between M1 and M2. Similarly, we examine the marginal benefit of adding a liquidity factor (LIQ, COM, or LMU) by comparing the performance between M1 and M3. We compare M2 with M3 to assess the relative performance of the factor model with SMB and a liquidity factor, respectively. By comparing the performance between M2 and M4, we assess the marginal benefit of incorporating a liquidity risk factor into the factor model that includes SMB. By comparing the performance between M3 and M4, we determine the marginal benefit of incorporating SMB into the factor model that includes a liquidity risk factor.

Adding SMB improves the performance of a model that does not include a liquidity factor. We also find that adding a liquidity factor improves the performance of a model that does not include SMB. We find that M3 performs better than M2. However, M4, which includes SMB and a liquidity factor (LIQ, COM, or LMU), does not perform better than M3. Adding the size factor does not improve the model's performance when a liquidity factor is included. That is, adding a liquidity factor renders SMB redundant.

To summarize, our study contributes to the literature by documenting the following three empirical findings: (1) three liquidity factors (i.e., LIQ, COM, and LMU) capture risks not represented by the Fama-French factors; (2) each of these liquidity factors is highly correlated with the size factor (SMB); and (3) including any of these liquidity factors in the factor model makes SMB redundant, underscoring the importance of these liquidity factors in asset pricing.

The size factor has been recognized as one of the key pricing factors since the publication of Fama and French (1992). The size factor is based on the "size effect," which refers to the prior empirical finding that stocks of small companies tend to outperform those of large companies over the long term, first documented in Banz (1981). Fama and French (1992) suggest that the size factor reflects the additional risk associated with investing in smaller companies (e.g., smaller companies are riskier because they may be less diversified, face more significant financial constraints, and have less access to capital markets compared to larger, more established firms). To the extent that stocks with lower liquidity or greater liquidity betas have smaller market capitalizations, size can be considered a proxy for liquidity or liquidity beta. Our study provides evidence in support of this conjecture by showing that asset pricing models that include one of the liquidity factors perform better than those with the size factor. Including the size factor does not improve the model performance once we include the liquidity factor in the asset pricing model.

This paper is organized as follows. Section 2 describes variable measurements and the liquidity factor construction process. Section 3 presents our estimation results for the liquidity factors. Section 4 evaluates the usefulness of incorporating these liquidity factors into the factor model. Section 5 provides a summary and concluding remark.

2. **Variable measurements and factor construction**

This section first explains how we measure liquidity and liquidity betas. We then describe how we construct liquidity factors.

2.1. Liquidity and liquidity betas

We measure stock liquidity and liquidity betas using the 1963-2023 US stock market data (common stocks listed on the New York Stock Exchange and American Stock Exchange). We measure stock liquidity using the following method proposed by Amihud (2002):

$$\mathrm{ILLIQ}_{i,t} = \frac{1}{\mathrm{Days}_{i,t}} \sum_{d=1}^{\mathrm{Days}_{i,t}} \frac{|r_{i,t,d}|}{V_{i,t,d}}; \tag{1}$$

where $r_{i,t,d}$ and $V_{i,t,d}$ are stock *i*'s return and dollar trading volume (in million \$) on day *d* in month *t*, and $\mathrm{Days}_{i,t}$ is the number of trading days in month *t*. The underlying intuition of $\mathrm{ILLIQ}_{i,t}$ is that relatively more illiquid stocks will show greater price changes for a given trading volume.

Portfolio illiquidity ($\mathrm{ILLIQ}_{p,t}$) is a weighted sum of stock *i*'s illiquidity:

$$\mathrm{ILLIQ}_{p,t} = \textstyle\sum_{i \in p} w_{i,t} \mathrm{ILLIQ}_{i,t}; \tag{2}$$

where $ILLIQ_{p,t}$ is weighted illiquidity for portfolio *p* and $w_{i,t}$ is either equal or value-based weight for stock *i* in portfolio *p*. If portfolio *p* includes all sample stocks, $ILLIQ_{p,t}$ becomes the market illiquidity.

We first estimate five liquidity betas for each stock to construct liquidity factors. The five liquidity betas include the three liquidity betas in Acharya and Pederson (2005) (β_2, β_3, and β_4) and the two sub-liquidity betas (β_{4a} and β_{4b}) in Boyle and Hong (2020). The liquidity commonality beta, β_2, captures the covariation between a stock's liquidity and the market liquidity, β_3 captures the covariation of a stock's return with the market liquidity, and β_4 captures the covariation of a stock's liquidity with the market return. The two liquidity betas in Boyle and Hong (2020), β_{4a} and β_{4b}, capture the covariation of a stock's liquidity with macroeconomic shocks and financial shocks, respectively.[7] The covariation of a stock's liquidity with financial shocks (β_{4b}) is analogous to the uncertainty elasticity of liquidity (UEL) in Chung and Chuwonganant (2014), who show that market uncertainty exerts a large market-wide impact on liquidity, which gives rise to co-movements in individual asset liquidity.[8] Chung and Chuwonganant (2014) also show that UEL is greater than the combined effects of all other common determinants of stock liquidity.

The estimation of β_{4a} and β_{4b} starts with Campbell and Shiller's (1988) return decomposition. Unexpected market return for month t ($r_{M,t} - E_{t-1}[r_{M,t}]$) can be written as the sum of macroeconomic and financial shocks:

$$r_{M,t} - E_{t-1}\left[r_{M,t}\right] \approx \eta_{Mac,t} - \eta_{\pi,t}; \tag{3}$$

where $\eta_{Mac,t} = \Delta E_t\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j}\right] - \Delta E_t\left[\sum_{j=1}^{\infty} \rho^j r_{f,t+j}\right]$ and $\eta_{\pi,t} = \Delta E_t\left[\sum_{j=1}^{\infty} \rho^j \pi_{t+j}\right]$.

ΔE_t denotes the change in expectations from *t*-1 to *t*, *r* and *d* are logged stock returns and dividends, π is logged stock market excess returns, and ρ is the average ratio of the stock price to the sum of the stock price and dividend. Economic intuition behind equation (3) is that unexpected logged stock market returns

[7] By applying Merton's (1980) risk-return relationship to Campbell and Shiller's (1988) return decomposition, Boyle and Hong (2020) decomposed stock market return shocks into 'macroeconomic' and 'financial' shocks. β_{4a} and β_{4b} capture liquidity covariation with these shocks. See Boyle and Hong (2020) for details.

[8] Chung and Chuwonganant (2014) measure market uncertainty by the Chicago Board Options Exchange Market Volatility Index (VIX).

are due to macroeconomic shocks (shocks to interest rates and aggregate expected dividends), financial shocks (shocks to market risk premium), or a combination of the two. Following Boyle and Hong (2020), we assume that the expected market risk premium is constant, i.e., $E_t[\pi_{t+j}] = E_t[\pi_{t+1}]$ for all j, and apply the basic pricing equation (Cochrane, 2005), i.e., $E_t[\pi_{t+j}] \approx \gamma\sigma^2_{t+1}$. Then, equation (3) can be simplified to the following equation. [9]

$$r_{M,t} - E_{t-1}[r_{M,t}] \approx \eta_{Mac,t} + \phi(\sigma^2_{t+1} + E_{t-1}[\sigma^2_{t+1}]). \tag{4}$$

Equation (4) is immediately recognizable as a regression equation of the following form:

$$r_{M,t} - E_{t-1}[r_{M,t}] = \alpha + \phi(\sigma^2_{t+1} + E_{t-1}[\sigma^2_{t+1}]) + \epsilon_t. \tag{5}$$

We first estimate unexpected changes in market risk (i.e., $\sigma^2_{t+1} + E_{t-1}[\sigma^2_{t+1}]$) using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. We use residuals from the Auto Regressive Moving Average (ARMA) estimation for unexpected changes in the market return (i.e., $r_{M,t} - E_{t-1}[r_{M,t}]$). Therefore,

$$\hat{\eta}_{Mac,t} = \hat{\alpha} + \hat{\epsilon}_t \text{ and} \tag{6}$$

$$\hat{\eta}_{\pi,t} = \hat{\phi}(\sigma^2_{t+1} + E_{t-1}[\sigma^2_{t+1}]). \tag{7}$$

Using these estimated series from equations (6) and (7), we compute the two sub-liquidity betas discussed above – β_{4a} (liquidity sensitivity to macroeconomic shocks; covariation between liquidity costs and the estimated macroeconomic shocks in equation (6)) and β_{4b} (liquidity sensitivity to market uncertainty; covariation between liquidity costs and the estimated financial shocks in equation (7)).

2.2. Liquidity factors

Following Amihud et al. (2015) and Amihud and Noh (2021), we construct liquidity factors from portfolios formed on stock return volatility and liquidity (i.e., Amihud measure and liquidity betas).[10] We

[9] See Boyle and Hong (2020) for details.

[10] Fama and French (1992, 1996, 2015) sort stocks by size and other variables such as book-to-market ratios or profitability. However, this sorting method is less suitable for constructing a liquidity factor. If liquidity or liquidity risk is an essential determinant of investors' discount rates (Amihud and Mendelson, 1986; Acharya and Pederson, 2005), stocks with lower liquidity or greater liquidity betas would have smaller market capitalizations. Then, size can

first sort stocks into three portfolios based on return volatility.[11] We sort stocks into two portfolios within each volatility portfolio based on liquidity or liquidity beta. The liquidity factor is the difference between the value-weighted return across the high illiquidity portfolio and the value-weighted returns across the low illiquidity portfolio (e.g., value-weighted stock returns in Portfolios 1 through 3 minus value-weighted stock returns in Portfolios 4 through 6 in Figure 1).

We construct liquidity factors as follows. Using 11-month (from t−12 to t−2) daily data, we divide stocks based on return volatility into three groups. Then, within each return volatility tercile, we divide stocks into two groups based on liquidity or liquidity betas. This two-way sort yields 6 (3 × 2) portfolios in total. Then, after skipping one month, we calculate the monthly liquidity factor for the next six months (from t to t + 5). Finally, at the end of t + 5, we repeat these steps and update portfolios (every six months).

We obtain the following six liquidity factors using the above procedure: LIQ, COM, RML, LMR, LME, and LMU. LIQ is the difference in the value-weighted return between the portfolio of stocks with high and low Amihud illiquidity measures across three return volatility groups. Similarly, COM (liquidity commonality risk) is the difference in the value-weighted return between stocks with high and low liquidity commonality (β_2) across three return volatility groups, where the covariance between stock liquidity and market liquidity measures liquidity commonality. RML (return sensitivity to market liquidity) is the difference in the value-weighted return between stocks with high and low return sensitivity to market liquidity (β_3) across three return volatility groups. LMR (liquidity sensitivity to market return) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to market returns (β_4) across three return volatility groups. LME (liquidity sensitivity to macroeconomic shocks) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to

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be considered a proxy for liquidity or liquidity beta. Hence, sorting stocks by size and liquidity would be almost equivalent to sorting stocks by different illiquidity proxies. Based on these considerations and following Amihud et al. (2015) and Amihud and Noh (2021), we sort stocks by return volatility and liquidity measures.

[11] When constructing liquidity factors, we exclude stocks with prices less than $1. Harris (1994) documents that market microstructures, such as tick-size changes, affect low-priced stocks' liquidity and trading volume. Therefore, to minimize the microstructure effect, we exclude microstocks. We also exclude stocks that do not have more than 100 days of return and volume data during the portfolio formation period.

macroeconomic shocks (β_{4a}) across three return volatility groups. LMU (liquidity sensitivity to stock market uncertainty) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to stock market uncertainty (β_{4b}) across three return volatility groups.

3. Estimation results

This section presents our estimation results for the liquidity factors. Section 3.1 presents descriptive statistics for each liquidity factor. Section 3.2 investigates whether each liquidity factor captures a new dimension of risk.

3.1. Liquidity factor characteristics

Table 1 presents descriptive statistics for the Fama-French common factors and liquidity factors. The Fama-French factors include MKT (market factor), SMB (size factor), HML (value factor), RMW (profitability factor), CMA (investment factor), and MOM (momentum factor). The six columns on the right-hand side of Table 1 present monthly descriptive statistics for the six liquidity factors: LIQ, COM, RML, LMR, LMU, and LME.

The mean value of LIQ is 0.305% (3.660% per annum), and its standard deviation is 2.198%, giving a Sharpe ratio of 0.139. The mean value of LIQ and its Sharpe ratio are greater than the corresponding values for SMB. During the July 1963 to June 2023 period (720 months), LIQ is positive 55.4% of the time. The statistics for COM are similar to those for LIQ. The mean value of COM is 0.263% (3.156% per annum), and its standard deviation is 1.989%. COM is positive 55.8% of the time. The mean of RML is 0.026% (0.312% per annum), the mean of LMR is 0.154% (1.848% per annum), the mean of LMU is 0.217% (2.604% per annum), and the mean of LME is 0.048% (0.576% per annum).

All six liquidity factors have positive average returns, suggesting that stocks with greater exposure to these liquidity factors have higher average returns. Regarding the magnitude of average monthly returns and Sharpe ratio, LIQ, COM, and LMU stand out. The average monthly returns and Sharpe ratios for these liquidity factors are comparable to and even greater than those of the Fama-French factors. Figure 2 shows how \$1 invested in each factor grows from July 1963 to time T, i.e., $\sum_{t=1}^{T} \$1 \times (1 + r_t)$, where r_t is the factor return in month *t*.

Table 2 presents correlations between the Fama-French factors and the six liquidity factors. The correlations between the liquidity factors and MKT, HML, CMA, RMW, and MOM are low and of varying signs. However, the correlations between the liquidity and size factors are positive and high: 0.766 (LIQ vs. SMB), 0.711 (COM vs. SMB), and 0.607 (LMU vs. SMB), respectively. Table 2 also provides correlations between liquidity factors. The correlations between LIQ, COM, and LMU are no less than 0.778. The high correlations between the liquidity risk factors suggest that although they are supposed to capture different dimensions of liquidity risks, they have strong commonality.

3.2. Spanning regressions: The right-hand-side approach

We use spanning regressions to determine whether each factor contributes to explaining average returns. We regress each candidate factor on other factors in the model to determine whether the factor contributes to the model based on the estimated intercept (Fama, 1998). The nonzero intercept means that other factors in the model do not fully account for the variation in the candidate factor. Thus, the candidate factor could improve the model's performance. We estimate the following regression model using the 720 monthly returns from July 1963 to June 2023 to assess whether the Fama-French factors could account for LIQ:

$$\begin{aligned} LIQ = \alpha_{LIQ} + \beta_M MKT + \beta_S SMB + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM \\ + \beta_J January + \beta_{Mic} MicR_f + e; \qquad (8) \end{aligned}$$

where LIQ is the difference in the value-weighted return between the portfolio of stocks with high and low Amihud illiquidity measures across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), MOM is the momentum factor (Carhart, 1997), and RMW and CMA are the profitability and investment factors (Fama and French, 2015). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio, controlling for variation in liquidity factors due to micro-stock effects not fully captured by SMB (Amihud and Noh, 2021). The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression.

If the estimated intercept is significantly different from zero, the Fama-French factors do not fully account for the liquidity factor. In that case, including the liquidity factor could improve the performance of the asset pricing model. The first column in Table 3 shows the regression results. We find that the intercept (0.147%) is positive and significant (t-statistic = 2.962), suggesting that the eight factors do not fully account for the illiquidity premium associated with the LIQ factor. The adjusted-R^2 is 0.640, indicating that the eight factors explain much, but not all, of the variation in LIQ. Consistent with the results in Table 2, the large and statistically significant coefficient on SMB indicates a strong relation between LIQ and SMB.

Columns (2), (3), and (4) provide the results when we regress LIQ on the Fama-French (2018) six factors, Fama-French (2015) five factors, and Carhart (1997) four factors, respectively. The intercepts (0.117%, 0.119%, and 0.160%) are positive and significant, indicating that neither the Fama-French factors nor Carhart four factors fully explain LIQ. The adjusted-R^2 is all around 0.64. Again, large and statistically significant coefficients on SMB indicate a close relation between SMB and LIQ. Columns (5) and (6) show the results when we repeat the estimations in columns (2) and (3) without SMB. The results show that the intercepts (0.275% and 0.285%) are larger than those in columns (2) and (3), suggesting that the marginal benefit of including LIQ as a pricing factor in the left-hand-side approach would be larger when the original model does not include SMB as a pricing factor. When SMB is removed from the Fama-French six factors model, the adjusted-R^2 drops from 0.640 to 0.071, indicating weak relations between LIQ and other risk factors in columns (1) to (4). Column (7) provides the results when we regress LIQ only on SMB. We find that SMB accounts for about 59% of the variation in LIQ.

Motivated by the high correlation between the liquidity factors and the size factor shown in Table 2, we also examine how much of the size effect is explained by the liquidity effect using the following regression model:

$$SMB = \alpha_{SMB} + \beta_M MKT + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_L LIQ + e. \quad (9)$$

Column (9) shows the results when we use the other five Fama-French factors as explanatory variables. We find that the intercept (0.262%) is positive and significant, indicating that these factors alone do not fully account for the size premium. Column (8) repeats the estimation in column (9) after including LIQ as an additional explanatory variable. Once LIQ is added, the intercept (-0.017%) becomes small and statistically insignificant (t-statistic = -0.246). When we regress SMB on only LIQ (see column (10)), the intercept (-0.106%) is also small and not significantly different from zero (t-statistic = -1.348). The results in columns (8) and (10) show that the intercept becomes insignificant when LIQ is included in the SMB regression. In other words, there is no additional size-related return premium beyond that accounted for by the LIQ factor. These results contrast with those in columns (1), (2), (3), (4), and (7) that the intercept is always positive and significant even when we include SMB in the LIQ regression. These results suggest that the LIQ factor subsumes the SMB factor but not *vice versa*.

To assess the robustness of the results in Table 3, we replicate Table 3 using an alternative measure of the liquidity factor, IML_t, developed by Amihud and Noh (2021). Following Amihud and Noh (2021), we calculate IML_t for each month t as the average of $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$, where $IML_{ILLIQ,t}$ and $IML_{ZERO,t}$ are the differences in mean returns between the highest-illiquidity quintile portfolios and the lowest-illiquidity quintile portfolios across the three standard deviation-sorted portfolios. The illiquidity measures, ILLIQ and ZERO, are based on Amihud (2002) and Lesmond et al. (1999). Table A1 in the Appendix shows the replication results using IML instead of LIQ. The results in Table A1 are qualitatively similar to those in Table 3.[12] Notably, columns (1), (2), (3), (4), and (7) show that the intercept is always positive and significant, indicating that the eight factors (including SMB) do not fully account for the illiquidity premium associated with the IML factor. Column (10) shows that the intercept (-0.149%) is negative and significant (t-statistic = -2.020) when we regress SMB on only IML, indicating an additional size-related return premium beyond that accounted for by the IML factor.

[12] We find that the correlation coefficient between LIQ and IML is 0.911.

Table 4 replicates Table 3 using the liquidity commonality risk (COM) in place of LIQ. Similar to Table 3, columns (1), (2), (4), and (7) show that the intercept is positive and significant, indicating that the Fama-French factors and Carhart (1997) four factors (including SMB) do not fully account for the risk premium associated with the COM factor. Columns (5) and (6) show that the intercepts are large and significant when we regress COM on the Fama-French factors without SMB. As in Table 3, the adjusted-R^2 decreases when SMB is removed from the model, suggesting a small common variation between COM and other factors. Similar to Table 3, we also find in columns (8) and (10) that the intercept is small and not significantly different from zero, indicating no additional size-related return premium beyond that accounted for by the COM factor.

Table 5 replicates Table 3 using liquidity sensitivity to market uncertainty (LMU) instead of LIQ. Similar to Table 3, columns (4) and (7) show that the intercept is positive and significant, indicating that Carhart (1997) four factors (including SMB) do not fully account for the risk premium associated with the LMU factor. Columns (5) and (6) show that the intercepts are large and significant when we regress LMU on the Fama-French factors without SMB. As in Table 3, the adjusted-R^2 decreases when SMB is removed from the model, suggesting a small common variation between LMU and other factors. Similar to Table 3, we also find in columns (8) and (10) that the intercept is small and not significantly different from zero, indicating no additional size-related return premium beyond that accounted for by the LMU factor.

Tables A2, A3, and A4 in the Appendix show the results for LMR, LME, and RML, respectively. None of the intercepts are economically large or statistically significant, indicating that variations in LMR, LME, and RML are fully explained by other risk factors, especially SMB, RMW, and MKT. That is, LMR, LME, and RML do not capture additional dimensions of systematic risk. Put differently, the insignificant intercepts in the regression models of LMR, LME, and RML imply that the risks captured by these factors are only a subset of risks captured by other Fama-French factors.[13] Hence, we exclude these liquidity factors from the model performance analysis in the next section.

4. Model performance analysis

Section 3 provides evidence that LIQ, COM, and LMU capture additional dimensions of risks not captured by the Fama-French factors. This section evaluates the usefulness of incorporating these liquidity factors into the factor model.

4.1. Test statistics

We use the following nine test statistics for our model performance analysis: (1) GRS = the Gibbons-Ross-Shanken test statistic; (2) $Sh^2(f)$ = the maximum squared Sharpe ratio for model factors (f); (3) $Sh^2(\alpha)$ = the maximum squared Sharpe ratio for the intercepts (α); (4) $A(|\alpha|)$ = the average absolute value of the estimated intercepts; (5) $A(|\alpha|)/A(|r|)$ = the average absolute value of the intercept over the average absolute value of the average portfolio return minus average portfolio returns; (6) $A(\alpha^2)/A(r^2)$ = the average squared intercept over the average squared value of the average return on portfolio minus the average portfolio returns; (7) $A(s^2(\alpha))/A(\alpha^2)$ = the average of the squared sample standard errors of the intercepts over the average squared intercepts; (8) N(1%) = the number of intercepts that are statistically significant at the 1% level; and (9) $A(\overline{R}^2)$ = the average adjusted R-squared. Below, we provide a brief description of each of these metrics.

4.1.1. GRS, $Sh^2(f)$, and $Sh^2(\alpha)$

Gibbons et al. (1989) develop a test statistic where the null hypothesis is that the estimated intercepts on the portfolios are jointly zero. The GRS test statistic follows the F-distribution under the assumption that regression errors are normally distributed, homoscedastic, and uncorrelated over time. Consider the

[13] Fama and French (2015) show that the value factor becomes redundant once the profitability and investment factors are added to the factor model. They find that the risk-adjusted return for HML after controlling for RMW, CMA, and other Fama-French factors is economically small and statistically insignificant.

following regression model where 25 Fama-French excess portfolio returns, $r_{i,t} - r_{f,t}$, are regressed on L risk factors:

$$r_{i,t} - r_{f,t} = \alpha_i + F_t B_i + e_{i,t}; \quad (10)$$

where $r_{i,t}$ and $r_{f,t}$ are portfolio i's return and risk-free interest rate at time t, F_t is a vector of L risk factors ($T \times L$ matrix), B_i is the risk loading for portfolio i ($L \times 1$ matrix), and $e_{i,t}$ is an error vector ($T \times 1$ matrix).

If there are N portfolios, regression model (10) is estimated N times, generating N estimated intercepts, $\hat{\alpha} = [\hat{\alpha}_1, \hat{\alpha}_2, \dots, \hat{\alpha}_N]'$. The GRS test statistic for testing $\hat{\alpha} = 0$ is

$$\mathrm{GRS} = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left[\frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right] \sim F(N, T-N-L); \quad (11)$$

where $\hat{\alpha}$ is a vector of estimated intercepts ($N \times 1$ matrix), $\hat{\Sigma}$ is the variance-covariance matrix ($N \times N$ matrix), $\bar{\mu}$ is a vector of factor sample means ($L \times N$ matrix), and $\hat{\Omega}$ is the factor's variance-covariance matrix ($L \times L$ matrix).

The null hypothesis of the GRS test is $H_0: \alpha_i = 0 \ \forall$ i. Therefore, a large value of the GRS statistic (hence rejecting H_0) is evidence of inferior model performance. Rejecting the null hypothesis implies that the underlying asset pricing model does not effectively explain the variations in portfolio average returns. In the GRS test statistic (i.e., equation (11)), both $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ and $\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}$ are scalars. The term, $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$, is the ratio of the sum of squared sample average returns that are not explained by the factors in regression model (10) and variance-covariance matrix, which has an asymptotic χ^2 distribution. Because $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ is a ratio of squared estimated αs and the residual covariance matrix, it is interpreted as a squared Sharpe ratio for the intercepts (Gibbons et al., 1989; Fama and French, 2018). Following Gibbons et al. (1989) and Fama and French (2018), we denote $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ as $Sh^2(\alpha)$ and use it as a test metric. A better model will have smaller $Sh^2(\alpha)$. However, the values of $Sh^2(\alpha)$ vary depending on our test portfolio (left-hand-side portfolios). Hence, $Sh^2(\alpha)$ may not give consistent rankings over the competing models.

We also use the squared Sharpe ratio for the factors, $Sh^2(f) = \bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}$, as an alternative test statistic. It measures average factor returns relative to its variation. The larger value of $Sh^2(f)$ indicates superior model performance, and more importantly, the values of $Sh^2(f)$ do not change depending on the left-hand-side test portfolio returns. The value of $Sh^2(f)$ will change only if the set of risk factors on the right-hand side of equation (10) changes.

4.1.2. A(|α|), A(|α|)/A(|r|), A(α²)/A(r²), and A(s²(α))/A(α²)

If an asset-pricing model perfectly describes dispersion in cross-sectional average excess portfolio returns, the estimated intercept for each portfolio must be zero. Therefore, in addition to GRS and its sub-components, we examine four additional test statistics based on the estimated intercepts, $\hat{\alpha}_i$, and their standard errors. The first test statistic is the average absolute intercepts of portfolios, A(|α|). The letter 'A' indicates 'equally weighted average' - the same weights are given to all portfolio αs. A better model is the one with a smaller value of A(|α|).

Following Fama and French (2015, 2016, 2018), we examine the ratio of the average absolute intercept, A(|α|), and the average absolute deviations of the portfolio average returns from the average cross-sectional portfolio returns, A(|r|). If $\bar{r}_i$ is the average portfolio i return and $\bar{r}$ is the cross-sectional portfolio average return (average of $\bar{r}_i$), then the deviation of portfolio i's average return from the average cross-sectional portfolio return is $r_i = \bar{r}_i - \bar{r}$, so A(|r|) captures the average dispersion of portfolio average returns.

Intuitively, A(|α|) is the average dispersion of portfolio excess returns that a factor model does not explain, and A(|r|) is the average dispersion of portfolio excess returns. Therefore, the ratio, A(|α|)/A(|r|), measures the dispersions in the excess returns not explained by a factor model relative to the overall dispersion of portfolio returns. For example, A(|α|)/A(|r|) = 0.1 implies that 10% of the average excess returns are unexplained by a factor model. Hence, a smaller value indicates better model performance.

The average squared intercept relative to the average squared dispersion in portfolio average returns, $A(\alpha^2)/A(r^2)$, measures the unexplained dispersion of portfolio excess returns relative to their total dispersion. A smaller value of $A(\alpha^2)/A(r^2)$ indicates better model performance. The statistic, $A(s^2(\alpha))/A(\alpha^2)$, is the ratio of the average of squared sample standard errors of the intercepts and the average of squared estimated intercepts. It measures the relative size of average dispersion in the estimated intercepts arising from sampling errors, and therefore, a low value of $A(s^2(\alpha))/A(\alpha^2)$ indicates inferior model performance as it indicates that much of the dispersions in the estimated intercepts are not from sampling errors.

4.1.3. N(1%) and A($\bar{R}^2$)

If an asset pricing model perfectly describes variation in average portfolio excess returns, the estimated intercept must be zero or at least economically small and statistically insignificant. The economic size of the intercept is measured by $A(|\alpha|)$, and GRS test statistics measure the joint statistical significance of the estimated intercepts. In addition to these statistics, we consider the number of estimated intercepts statistically significant at the 1% level. For example, N(1%) = 5 for 25 Fama-French size and B/M portfolios means that of the 25 estimated portfolio $\hat{\alpha}_i$s, five are statistically significantly different from zero at the 1% level. Therefore, a smaller N(1%) indicates better model performance. Lastly, we also report the average adjusted-R^2. Higher $A(\bar{R}^2)$ indicates better model performance.

4.2. Model performance analysis results

In this section, we examine the performance of liquidity-augmented factor models using the test statistics described above. For test portfolios, we use value-weighed monthly excess returns for Fama-French portfolios sorted on various firm characteristics.[14] In particular, we use 25 Fama-French portfolios sorted on (1) size and book-to-market ratio (B/M), (2) size and operating profitability (OP), (3) size and investment (Inv), and (4) size and momentum (MOM). We also use all 100 portfolios in (1), (2), (3), and (4).

We use the following four models in the model performance analysis:

(M1) Five-Factor (Base) Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_M MKT_t + \beta_H HML_t + \beta_R RMW_t + \beta_C CMA_t + \beta_M MOM_t + e_t$$

(M2) Six-Factor Model (M1 plus the size factor)

[14] We obtained the Fama-French factors and portfolios from *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_M MKT_t + \beta_S SMB_t + \beta_H HML_t + \beta_R RMW_t + \beta_C CMA_t + \beta_M MOM_t + e_t$$

(M3) M1 plus a liquidity factor model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_M MKT_t + \beta_H HML_t + \beta_R RMW_t + \beta_C CMA_t + \beta_M MOM_t + \beta_L LIQFactor_t + e_t, \text{ where } LIQFactor \in \{LIQ_t, COM_t, LMU_t\}$$

(M4) M2 plus a liquidity factor

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_M MKT_t + \beta_S SMB_t + \beta_H HML_t + \beta_R RMW_t + \beta_C CMA_t + \beta_M MOM_t + \beta_L LIQFactor_t + e_t, \text{ where } LIQFactor \in \{LIQ_t, COM_t, LMU_t\}$$

By comparing the performance of M1 and M2, we assess the marginal benefit of adding the size factor (SMB) to the base model (i.e., M1). By comparing the performance of M1 and M3, we assess the marginal benefit of adding a liquidity factor (LIQ, COM, or LMU) to the base model. Using M2 and M3, we assess the marginal benefit of incorporating SMB into the six-factor model that includes a liquidity factor. Using M2 and M4, we examine the marginal benefit of incorporating a liquidity factor into the six-factor model that includes SMB. By comparing the performance between M3 and M4, we determine the marginal benefit of incorporating SMB into the factor model that includes a liquidity risk factor.

Panel A in Table 6 shows the test statistics estimated from 25 Fama-French portfolios sorted on size and book-to-market ratio. The first two rows provide the test statistics for M1 and M2. GRS tests indicate that both models fail to reject the null hypothesis, i.e., Fama and French's (2015, 2018) five-factor and six-factor models explain variations in the average portfolio excess returns. Incorporating SMB into the five-factor model improves the model's performance. For example, the average absolute risk-adjusted return ($A(|\alpha|)$) decreases from 19.3 basis points to 7.8 basis points. $A(|\alpha|)/A(|r|)$ also decreases from 1.283 to 0.517. The average adjusted-R^2 increases by about 10% points.

The third row in Panel A provides test statistics for M3 (the model with five factors plus LIQ). The results show that M3 exhibits superior performance compared to M2 across most test statistics (i.e., lower GRS statistic, $Sh^2(\alpha)$, $A(|\alpha|)$, $A(|\alpha|)/A(|r|)$, $A(\alpha^2)/A(r^2)$, and N(1%), and higher $Sh^2(f)$ and $A(s(\alpha)^2)/A(\alpha^2)$). The fourth row in Panel A shows the results for M4 (i.e., LIQ augmented Fama and French (2018) six-factor model). The GRS test statistic *increases* from 2.506 to 2.723, and the number of estimated intercepts that are statistically significant at the 1% level increases from 1 to 4. Consistent with the factor model regression results in Table 3, we find no improvement in $Sh^2(f)$ (0.129) when SMB is added to M3. The average absolute value of the estimated intercepts, $A(|\alpha|)$, increases from 0.074 to 0.077. These results indicate no benefit when we add SMB to the five-factor model with LIQ.

The fifth and sixth rows in Panel A replicate the third and fourth rows using COM (instead of LIQ), and the seventh and eighth rows replicate the results using LMU. The results for COM and LMU are similar to those for LIQ. The liquidity-augmented five-factor model (i.e., M3) has a lower GRS statistic, a higher $Sh^2(f)$, a lower $Sh^2(\alpha)$, and a higher $A(s^2(\alpha))/A(\alpha^2)$ compared to the corresponding values for M2 (i.e., the model with the five factors plus SMB). The performance of the model with both SMB and a liquidity factor is slightly poorer than the model with only the liquidity factor, suggesting that either SMB or the liquidity factor is redundant. After comparing the performance between the model with only SMB and the model with only a liquidity factor, we conclude that the latter has a superior model performance.

In Panels B, C, and D, we repeat Panel A with different Fama-French portfolios – 5×5 quintile sorts on (1) market capitalization and operating profitability, (2) market capitalization and investment, and (3) market capitalization and momentum. We find that the results are generally similar to those in Panel A. For instance, the GRS test statistics in Panel B indicate that the model incorporating the liquidity factor (M3) does not reject the null hypothesis that the alphas of 25 size and operating profit sorted portfolios are jointly equal to zero at the 5% level (refer to rows 3, 5, and 7), and M3 performs better (smaller GRS statistics) than the model with SMB (M2). In Panel E, we use all the 100 Fama-French portfolios in Panel A to Panel D. We find that the results are similar to those in Panel A. For instance, Panel D shows that M3 with LIQ (row 3), COM (row 5), and LMU (row 7) can explain the 25 portfolio returns, i.e., we cannot reject the null hypothesis that 25 portfolio alphas are jointly equal to zero.

In summary, our results indicate that the model incorporating the Fama-French five factors plus a liquidity factor outperforms the model that includes the Fama-French five factors and SMB across various performance measures. Due to the high correlation between SMB and the liquidity factor, incorporating both in a single model does not enhance model performance. These results suggest that once the liquidity effect is accounted for, including SMB provides minimal model improvements and may be redundant.[15]

4.3. Robustness tests

Fama and French (2015, 2016) show that HML is redundant when describing US average returns, as the large average HML return is explained mainly by the profitability and investment factors, RMW and CMA. This implies that nothing is lost even if HML is dropped from the factor model. In this section, we repeat the model performance analysis in Section 4.2 without HML and with HMLO (orthogonal HML). Similar to Fama and French (2016), we define HMLO as the sum of the intercept and residuals estimated from the regression (11).

$$\mathrm{HML}_t = \alpha + \beta_M \mathrm{MKT_t} + \beta_s \mathrm{SMB_t} + \beta_R \mathrm{RMW_t} + \beta_C \mathrm{CMA_t} + \beta_O \mathrm{MOMt} + \mathrm{e_t} \quad (11)$$

We obtain the following results (standard errors in parentheses) when we regress HML on the other factors in Fama and French's (2015) five-factor model with momentum:

$$\underset{}{\mathrm{HML_t}} = \underset{(-0.034)}{-0.003} + \underset{(0.525)}{0.010\mathrm{MKT_t}} + \underset{(3.015)}{0.085\mathrm{SMB}} + \underset{(5.235)}{0.196\mathrm{RMW_t}} + \underset{(25.228)}{1.011\mathrm{CMA_t}} - \underset{(-7.153)}{0.133\mathrm{CMA_t}} + \hat{\mathrm{e}}_{\mathrm{t}}.$$

The estimated intercept is -0.003% (0.036% per annum), and its t-statistic is -0.034. This result suggests that the large average HML return is absorbed by its exposures to the other four factors, especially CMA and RMW. HMLO is constructed from the sum of the intercept, -0.003, and the residuals, $\hat{e}_t$, in equation (11). Table 7 and Table 8 report the performance analysis results for the factor models without HML and with HMLO. The results in these tables show that (1) removing HML (or replacing HML with HMLO) does not affect model performance, consistent with Fama and French's (2015, 2016) findings about the

[15] The closest empirical results to this can be found in the work of Amihud and Mendelson (1989). After testing a joint hypothesis on the positive (or negative) relationship between expected returns and a set of variables, including the market risk (beta), residual risk, market value, and bid-ask spared, they find that expected return is an increasing function of the bid-ask spread (illiquidity). However, they find no support for the hypothesis on the effect of firm size on expected return.

redundancy of HML, and that (2) a model with a liquidity factor performs better than a model with SMB, supporting our conclusion in Section 4.2.

In addition, we repeat Table 6 with equally weighted Fama-French portfolio returns. As equally weighted returns give each stock the same importance in a portfolio, it helps to avoid the dominance of large firms in the analysis, providing a clearer picture of the average performance across all stocks (Banz, 1981). Furthermore, equally weighted portfolios tend to have different risk exposures than value-weighted portfolios. They often exhibit higher exposure to smaller firms, which can offer insights into the performance and risk characteristics of these firms that might be overlooked in value-weighted portfolios (Fama and French, 1993). Table A5 in the Appendix provides the performance analysis result.

Compared to Table 6, the explanatory power of the factors for the equally weighted returns is lower. For example, the GRS test statistics and average absolute intercepts for the 25 Fama-French size and book-to-market ratio sorted portfolios (Panel A) under M2 (M3; 5 factors with LIQ), are 4.891 (3.630) and 14 basis points (13.6 basis points) respectively. In contrast, the corresponding figures in Table 6 are 2.838 (2.506) and 7.8 basis points (7.4 basis points). Nonetheless, the relative performance between M2 and M3 remains the same as in Section 4.2.

Lastly, Table A6 provides the performance analysis results using three sets of portfolios sorted by liquidity and risk. The first set includes six portfolios sorted by the standard deviation of stock returns and liquidity costs. We formed these six portfolios when creating LIQ factor (Portfolio 1 – 6 in Figure 1). The second set includes six portfolios sorted by the standard deviation of stock returns and the covariation between stock and market liquidity (COM). The last set comprises six portfolios sorted by the standard deviation of stock returns and the covariation between stock liquidity and market uncertainty (LMU). Table A7 enables us to investigate the relative performance of the factor model with SMB compared to the model with a liquidity factor in explaining the liquidity premium rather than the size premium in Table 6.

The results show that, as anticipated, the model incorporating a liquidity factor provides superior explanatory power for liquidity portfolio returns compared to the model with the size factor. For instance, in the case of the 5-factor model (first row in Panel A), the GRS statistic is 3.399, which decreases to 2.491

when the size factor is included. However, the GRS statistic for the model with LIQ is 1.970 (with a p-value of 0.0677), indicating that the model with LIQ outperforms the model with SMB. Furthermore, when both the size factor and LIQ are included, the GRS statistic is 2.220, suggesting that incorporating SMB in addition to LIQ provides little additional explanatory power.

Overall, the model performance results in Tables A5 and A6 are consistent with Table 6. That is, the model with the five factors plus a liquidity factor performs better than the model with the five factors plus SMB.

5. **Summary and concluding remarks**

This paper examines the pricing of liquidity factors and the performance of the liquidity-augmented factor model. Using the 1963-2023 US stock market data, we construct six liquidity factors based on liquidity costs (LIQ), liquidity commonality risk (COM), return sensitivity to market liquidity (RML), liquidity sensitivity to market return (LMR), liquidity sensitivity to financial market uncertainty shocks (LMU), and liquidity sensitivity to macroeconomic shocks (LME).

We show that the liquidity factors, LIQ, COM, and LMU, capture additional dimensions of risks that the Fama-French factors do not. The risk-adjusted return for the Fama-French size factor (SMB) is economically small and statistically insignificant once the effect of the liquidity factor on returns is accounted for. To the extent that stocks with lower liquidity or greater liquidity betas have smaller market capitalizations, size can be considered a proxy for liquidity or liquidity beta. Our study provides evidence supporting this conjecture by showing that the explanatory power of the size factor weakens or disappears once the liquidity factor is included in the asset pricing model. Prior research has suggested that a firm’s market capitalization (size) is an essential determinant of its stock returns in the US and many other stock markets. Our study provides evidence that liquidity could be more relevant than size in determining stock returns in these markets.

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	Standard Deviation of Stock Returns, SD		
	High	Medium	Low
High Illiquidity	Portfolio 1	Portfolio 2	Portfolio 3
Low Illiquidity	Portfolio 4	Portfolio 5	Portfolio 6

Figure 1. Portfolio Formations – Standard Deviation and Illiquidity Sort

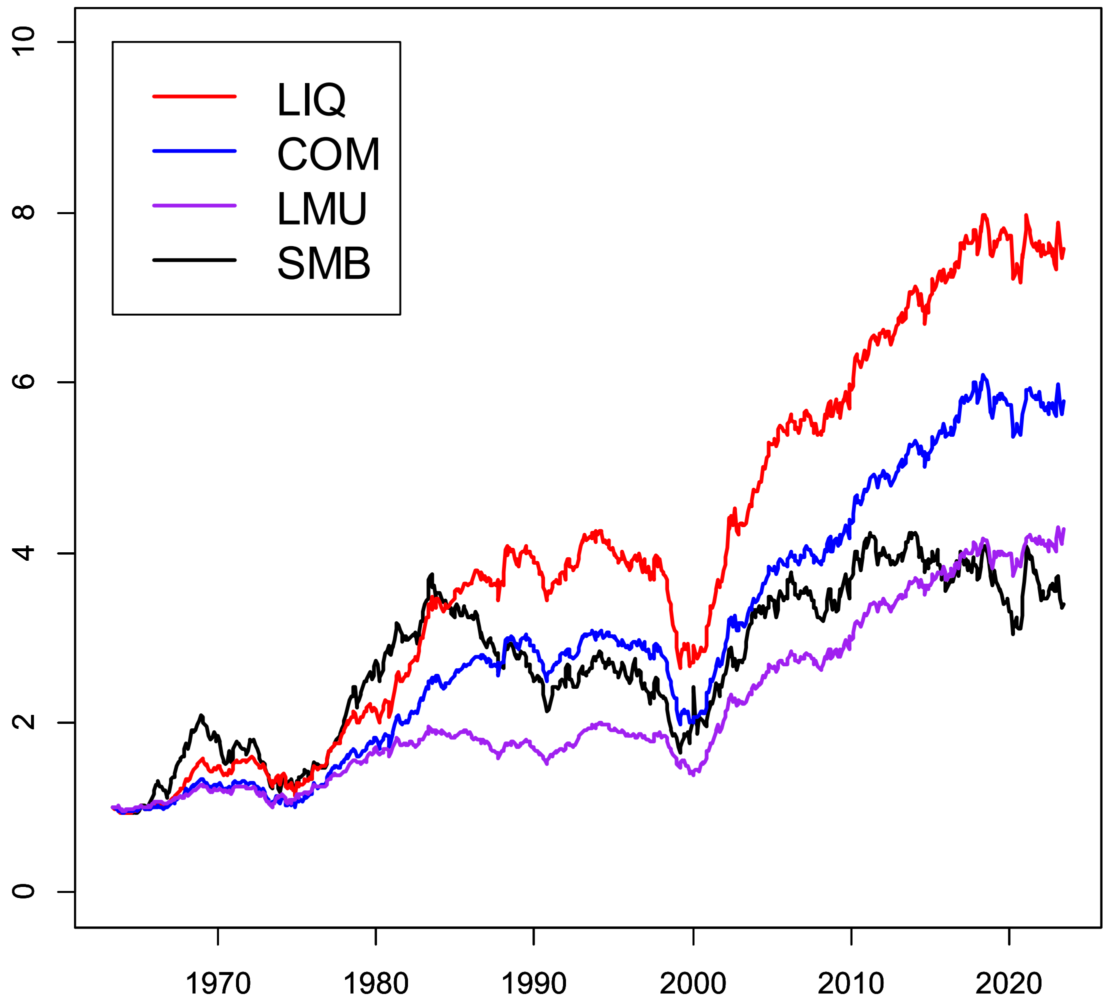

Figure 2. This figure shows how \$1 invested in each factor grows from July 1963 to time T, i.e., $\sum_{t=1}^{T}$ \$1 x (1 + r_t), where r_t is the factor return in month t.

Table 1. Descriptive Statistics

This table presents descriptive statistics for monthly Fama-French common factors and the six liquidity factors. MKT is the difference between the value weighted market (NYSE, NASDAQ and AMEX) returns and the one-month US Treasury bill rate. SMB, small minus big, and HML, high minus low, are the Fama and French's (1993) size and value factors. MOM is the momentum factor, the average return on the high prior return portfolios minus the average return on the low prior return portfolios (Carhart, 1997). CMA is the investment factor, the average return on the conservative investment portfolios minus the average return on the aggressive investment portfolios and RMW is the profitability factor, the average return on the robust operating profitability portfolios minus the average return on the weak operating profitability portfolios. LIQ is the difference in the value-weighted return between the portfolio of stocks with high and low Amihud illiquidity measures across three return volatility groups. COM (liquidity commonality risk) is the difference in the value-weighted return between stocks with high and low liquidity commonality across three return volatility groups, where the covariance between stock liquidity and market liquidity measures liquidity commonality. RML (return sensitivity to market liquidity) is the difference in the value-weighted return between stocks with high and low return sensitivity to market liquidity across three return volatility groups. LMR (liquidity sensitivity to market return) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to market returns across three return volatility groups. LME (liquidity sensitivity to macroeconomic shocks) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to macroeconomic shocks across three return volatility groups. LMU (liquidity sensitivity to stock market uncertainty) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to stock market uncertainty across three return volatility groups. The descriptive statistics are estimated over July 1963 to June 2023 (720 months).

	Fama-French Factors						Liquidity Factors					
	MKT	SMB	HML	RMW	CMA	MOM	LIQ	COM	RML	LMR	LMU	LME
Mean (%)	0.564	0.216	0.278	0.285	0.279	0.607	0.305	0.263	0.026	0.154	0.217	0.048
Median (%)	0.915	0.095	0.220	0.245	0.090	0.725	0.225	0.241	0.051	0.159	0.146	0.065
Std. Dev. (%)	4.493	3.024	2.998	2.220	2.082	4.217	2.198	1.989	1.825	1.675	1.716	1.696
Fraction of Pos.	0.599	0.517	0.539	0.567	0.531	0.625	0.554	0.558	0.513	0.532	0.543	0.517
Serial Corr	0.040	0.072	0.172	0.159	0.136	0.041	-0.043	-0.013	0.001	0.009	0.019	0.025
Sharpe Ratio	0.126	0.071	0.093	0.129	0.134	0.144	0.139	0.132	0.014	0.092	0.126	0.028

Table 2. Correlations

This table presents correlations between the Fama-French factors and the six liquidity factors. The correlations are estimated with 720 monthly returns from July 1963 to June 2023.

	Fama-French Factors						Liquidity Factors					
	MKT	SMB	HML	RMW	CMA	MOM	LIQ	COM	RML	LMR	LMU	LME
MKT	1.000											
SMB	0.278	1.000										
HML	-0.211	-0.017	1.000									
RMW	-0.179	-0.350	0.090	1.000								
CMA	-0.364	-0.097	0.688	-0.009	1.000							
MOM	-0.171	-0.066	-0.186	0.085	-0.004	1.000						
LIQ	0.098	0.766	0.182	-0.155	0.087	-0.053	1.000					
COM	0.155	0.711	0.221	-0.136	0.097	-0.088	0.946	1.000				
RML	0.328	0.070	-0.315	-0.051	-0.271	-0.039	-0.050	-0.020	1.000			
LMR	0.081	0.352	0.218	0.077	0.138	-0.060	0.578	0.625	0.011	1.000		
LMU	0.267	0.607	0.176	-0.111	0.023	-0.110	0.778	0.855	0.128	0.548	1.000	
LME	-0.043	0.173	0.223	0.109	0.243	0.016	0.431	0.446	-0.084	0.773	0.334	1.000

Table 3. Spanning Regression Results for LIQ and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: LIQ = $\alpha_{LIQ} + \beta_M MKT + \beta_S SMB + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_J January + \beta_{Mic} MicR_f + e$; where LIQ is the difference in the value-weighted return between the portfolio of stocks with high and low Amihud illiquidity measures across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = $\alpha_{SMB} + \beta_M MKT + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_L LIQ + e$. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	LIQ							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.147***	0.117**	0.119**	0.160***	0.275***	0.285***	0.185***	-0.017	0.262**	-0.106
	(2.962)	(2.289)	(2.491)	(3.056)	(3.439)	(3.592)	(3.726)	(-0.246)	(2.376)	(-1.348)
MKT	-0.051*	-0.032**	-0.033**	-0.040**	0.053**	0.051*		0.087***	0.141***	
	(-1.888)	(-2.002)	(-2.026)	(-2.470)	(1.992)	(1.859)		(4.415)	(4.251)	
SMB	0.582***	0.604***	0.604***	0.577***			0.557***			
	(14.430)	(20.767)	(21.034)	(15.954)			(14.845)			
HML	0.105***	0.104***	0.102***	0.134***	0.194***	0.187***		-0.048	0.149*	
	(2.853)	(2.867)	(2.676)	(4.647)	(2.949)	(2.830)		(-0.924)	(1.729)	
RMW	0.101***	0.110***	0.111***		-0.160***	-0.158**		-0.285***	-0.448***	
	(2.730)	(3.055)	(3.086)		(-2.701)	(-2.572)		(-3.492)	(-3.285)	
CMA	0.046	0.049	0.050		-0.060	-0.055		-0.120*	-0.181*	
	(0.953)	(1.041)	(1.052)		(-0.755)	(-0.673)		(-1.898)	(-1.726)	
MOM	0.003	0.004		0.010	0.015			0.003	0.018	
	(0.119)	(0.177)		(0.431)	(0.483)			(0.100)	(0.419)	
LIQ								1.014***		1.054***
								(22.189)		(18.895)
January	-0.294									
	(-1.270)									
MicRf	0.014									
	(0.813)									
R^2	0.640	0.640	0.640	0.630	0.071	0.072	0.587	0.680	0.176	0.587

Table 4. Spanning Regression Results for COM and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: $COM = \alpha_{COM} + \beta_M MKT + \beta_S SMB + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_J January + \beta_{Mic} MicR_f + e$; where COM (liquidity commonality risk) is the difference in the value-weighted return between stocks with high and low liquidity commonality across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: $SMB = \alpha_{SMB} + \beta_M MKT + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_L COM + e$. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels.

	COM							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.106**	0.076	0.074	0.116**	0.206***	0.210***	0.163***	0.046	0.262**	-0.069
	(2.095)	(1.442)	(1.489)	(2.225)	(2.804)	(2.881)	(3.227)	(0.599)	(2.376)	(-0.828)
MKT	0.025	0.010	0.010	0.003	0.080***	0.079***		0.058**	0.141***	
	(0.966)	(0.621)	(0.646)	(0.189)	(3.535)	(3.399)		(2.533)	(4.251)	
SMB	0.519***	0.494***	0.494***	0.469***			0.468***			
	(10.967)	(15.984)	(16.083)	(11.150)			(10.499)			
HML	0.136***	0.131***	0.133***	0.156***	0.204***	0.202***		-0.066	0.149*	
	(3.358)	(3.409)	(3.280)	(4.904)	(3.864)	(3.996)		(-0.989)	(1.729)	
RMW	0.099**	0.102**	0.101**		-0.119***	-0.118***		-0.323***	-0.448***	
	(2.242)	(2.342)	(2.328)		(-2.822)	(-2.696)		(-2.851)	(-3.285)	
CMA	0.042	0.041	0.040		-0.048	-0.046		-0.130*	-0.181*	
	(0.816)	(0.843)	(0.800)		(-0.739)	(-0.700)		(-1.674)	(-1.726)	
MOM	-0.007	-0.003		0.002	0.005			0.012	0.018	
	(-0.331)	(-0.171)		(0.094)	(0.217)			(0.370)	(0.419)	
COM								1.049***		1.081***
								(24.304)		(22.930)
January	-0.357									
	(-1.573)									
MicRf	-0.012									
	(-0.729)									
R^2	0.570	0.567	0.568	0.557	0.102	0.103	0.505	0.603	0.176	0.505

Table 5. Spanning Regression Results for LMU and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: LMU = α_{LMU} + β_MMKT + β_SSMB + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_JJanuary + β_{Mic}MicR$_f$ + e; where LMU (liquidity sensitivity to stock market uncertainty) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to stock market uncertainty across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = α_{SMB} + β_MMKT + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_LLMU + e. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	LMU							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.075	0.054	0.051	0.081*	0.144**	0.145**	0.142***	0.117	0.262**	-0.016
	(1.525)	(1.095)	(1.064)	(1.663)	(2.395)	(2.420)	(2.928)	(1.303)	(2.376)	(-0.177)
MKT	0.103***	0.061***	0.062***	0.059***	0.109***	0.109***		0.031	0.141***	
	(4.462)	(4.423)	(4.571)	(4.148)	(7.333)	(7.424)		(1.072)	(4.251)	
SMB	0.402***	0.342***	0.342***	0.322***			0.344***			
	(9.752)	(13.222)	(13.124)	(9.522)			(9.023)			
HML	0.127***	0.120***	0.123***	0.124***	0.171***	0.171***		-0.024	0.149*	
	(3.720)	(3.820)	(3.724)	(4.772)	(4.920)	(5.131)		(-0.319)	(1.729)	
RMW	0.089**	0.086**	0.085**		-0.068**	-0.068*		-0.379***	-0.448***	
	(2.323)	(2.382)	(2.361)		(-1.986)	(-1.963)		(-3.070)	(-3.285)	
CMA	0.002	-0.003	-0.005		-0.065	-0.064		-0.116	-0.181*	
	(0.036)	(-0.058)	(-0.099)		(-1.198)	(-1.216)		(-1.269)	(-1.726)	
MOM	-0.010	-0.005		-0.002	0.001			0.017	0.018	
	(-0.545)	(-0.316)		(-0.133)	(0.042)			(0.445)	(0.419)	
LMU								1.009***		1.069***
								(19.141)		(19.942)
January	-0.317									
	(-1.490)									
MicRf	-0.033**									
	(-2.180)									
R^2	0.443	0.430	0.431	0.421	0.131	0.132	0.368	0.460	0.176	0.368

Table 6. Test Statistics for Factor Model Regression Intercept

This table presents test statistics for factor model regression intercepts for 720 months (July 1963 to June 2023). The test portfolios are equally weighted 4 different sets of 25 Fama-French portfolios – 5x5 quintile sorts on market equity capitalization and independently on book-to-market ratio, operating profitability, investment and momentum. Book-to-market ratio is book value of equity to market value of equity. Operating profitability is ratio of net profit, revenues minus costs of goods sold, administrative expenses and interest expense divided by book equity. Investment is the change in total assets from year y – 2 and y – 1 divided by total assets at y – 2. Momentum is the average monthly returns from month t – 12 to t – 2. The factors examined in this table are the Fama and French (2018) six factors (MKT, SMB, HML, RMW, CMA and MOM) and the three liquidity factors (LIQ, COM and LMU). The liquidity factors are constructed from the intersection of six portfolios (3x2) sorted by standard deviation of stock returns and liquidity costs/covariances risks. The term "5 Factors" refers to the Fama and French (2018) six factors excluding SMB (MKT, HML, RMW, CMA and MOM). This table presents nine test statistics discussed in Section 4.1.

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel A: Fama-French 25 Portfolios Sorted on Size and Book-to-Market Ratio										
5 Factors	2.881	0.0000	0.112	0.115	0.193	1.283	1.562	0.208	6	0.820
5 Factors + SMB	2.838	0.0000	0.121	0.114	0.078	0.517	0.302	0.404	4	0.919
5 Factors + LIQ	2.506	0.0001	0.129	0.102	0.074	0.492	0.296	0.672	1	0.886
5 Factors + SMB + LIQ	2.723	0.0000	0.129	0.111	0.077	0.509	0.308	0.387	4	0.923
5 Factors + COM	2.606	0.0000	0.124	0.105	0.091	0.607	0.390	0.554	3	0.878
5 Factors + SMB + COM	2.799	0.0000	0.125	0.113	0.078	0.520	0.313	0.365	6	0.924
5 Factors + LMU	2.695	0.0000	0.120	0.108	0.117	0.778	0.614	0.404	2	0.862
5 Factors + SMB + LMU	2.821	0.0000	0.123	0.114	0.079	0.523	0.312	0.365	5	0.924
Panel B: Fama-French 25 Portfolios Sorted on Size and Operating Profitability										
5 Factors	1.658	0.0234	0.112	0.066	0.157	1.147	1.237	0.314	2	0.816
5 Factors + SMB	1.854	0.0071	0.121	0.075	0.053	0.385	0.190	0.736	1	0.919
5 Factors + LIQ	1.761	0.0127	0.129	0.071	0.065	0.479	0.222	1.051	0	0.885
5 Factors + SMB + LIQ	2.051	0.0020	0.129	0.083	0.061	0.445	0.201	0.681	0	0.923
5 Factors + COM	1.627	0.0281	0.124	0.066	0.055	0.400	0.187	1.341	0	0.879
5 Factors + SMB + COM	1.990	0.0030	0.125	0.080	0.058	0.426	0.195	0.676	0	0.924
5 Factors + LMU	1.554	0.0422	0.120	0.063	0.067	0.494	0.305	0.948	1	0.863
5 Factors + SMB + LMU	1.924	0.0046	0.123	0.078	0.057	0.415	0.192	0.685	0	0.924

Table 6. Continued

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel C: Fama-French 25 Portfolios Sorted on Size and Investment										
5 Factors	3.328	0.0000	0.112	0.133	0.188	1.333	1.835	0.193	9	0.827
5 Factors + SMB	3.082	0.0000	0.121	0.124	0.078	0.551	0.373	0.319	3	0.930
5 Factors + LIQ	3.113	0.0000	0.129	0.126	0.077	0.549	0.376	0.561	1	0.894
5 Factors + SMB + LIQ	3.135	0.0000	0.129	0.127	0.080	0.565	0.389	0.300	4	0.933
5 Factors + COM	3.125	0.0000	0.124	0.126	0.090	0.642	0.470	0.495	1	0.887
5 Factors + SMB + COM	3.124	0.0000	0.125	0.126	0.080	0.565	0.384	0.292	5	0.935
5 Factors + LMU	3.106	0.0000	0.120	0.125	0.111	0.786	0.709	0.381	1	0.870
5 Factors + SMB + LMU	3.037	0.0000	0.123	0.123	0.078	0.555	0.373	0.300	4	0.934
Panel D: Fama-French 25 Portfolios Sorted on Size and Momentum										
5 Factors	1.541	0.0453	0.112	0.062	0.217	1.715	2.813	0.179	7	0.803
5 Factors + SMB	1.807	0.0096	0.121	0.073	0.074	0.588	0.373	0.684	2	0.895
5 Factors + LIQ	1.125	0.3073	0.129	0.046	0.060	0.477	0.306	1.153	0	0.869
5 Factors + SMB + LIQ	1.670	0.0219	0.129	0.068	0.065	0.518	0.367	0.693	2	0.899
5 Factors + COM	1.228	0.2050	0.124	0.050	0.081	0.641	0.578	0.635	0	0.863
5 Factors + SMB + COM	1.720	0.0163	0.125	0.070	0.066	0.520	0.361	0.685	2	0.900
5 Factors + LMU	1.326	0.1332	0.120	0.053	0.120	0.949	1.055	0.379	2	0.847
5 Factors + SMB + LMU	1.758	0.0130	0.123	0.071	0.067	0.534	0.358	0.687	2	0.900
Panel E: All Portfolios in Panel A - Panel D (100 Portfolios)										
5 Factors	2.395	0.0000	0.112	0.430	0.189	1.351	1.771	0.212	24	0.816
5 Factors + SMB	2.323	0.0000	0.121	0.420	0.071	0.506	0.301	0.498	10	0.916
5 Factors + LIQ	2.309	0.0000	0.129	0.421	0.069	0.496	0.293	0.806	2	0.883
5 Factors + SMB + LIQ	2.307	0.0000	0.129	0.420	0.071	0.506	0.308	0.480	10	0.919
5 Factors + COM	2.160	0.0000	0.320	0.460	0.065	0.465	0.292	1.013	1	0.884
5 Factors + SMB + COM	2.152	0.0000	0.322	0.459	0.068	0.484	0.266	0.674	3	0.920
5 Factors + LMU	2.317	0.0000	0.124	0.420	0.079	0.569	0.391	0.649	4	0.877
5 Factors + SMB + LMU	2.311	0.0000	0.125	0.419	0.070	0.505	0.305	0.466	13	0.921

Table 7. Test Statistics for Factor Model Regression Intercept: Robustness Test I

This table presents test statistics for factor model regression intercepts for 720 months (July 1963 to June 2023). The test portfolios are 4 different sets of 25 Fama-French portfolios – 5x5 quintile sorts on market equity capitalization and independently on book-to-market ratio, operating profitability, investment and momentum. Book-to-market ratio is book value of equity to market value of equity. Operating profitability is ratio of net profit, revenues minus costs of goods sold, administrative expenses and interest expense divided by book equity. Investment is the change in total assets from year y – 2 and y – 1 divided by total assets at y – 2. Momentum is the average monthly returns from month t – 12 to t – 2. The factors examined in this table are the Fama and French (2018) six factors (MKT, SMB, RMW, CMA and MOM) and the three liquidity factors (LIQ, COM and LMU). The liquidity factors are constructed from the intersection of six portfolios (3x2) sorted by standard deviation of stock returns and liquidity costs/covariances risks. The term "4 Factors" refers to the Fama and French (2018) six factors excluding SMB and HML (MKT, RMW, CMA and MOM). This table presents nine test statistics discussed in Section 4.1.

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel A: Fama-French 25 Portfolios Sorted on Size and Book-to-Market Ratio										
4 Factors	2.880	0.0000	0.112	0.115	0.195	1.295	1.607	0.237	6	0.788
4 Factors + SMB	2.808	0.0000	0.121	0.113	0.078	0.516	0.302	0.585	1	0.891
4 Factors + LIQ	2.509	0.0001	0.129	0.102	0.078	0.515	0.309	0.838	1	0.860
4 Factors + SMB + LIQ	2.700	0.0000	0.129	0.110	0.079	0.521	0.320	0.555	1	0.897
4 Factors + COM	2.608	0.0000	0.124	0.105	0.091	0.605	0.400	0.686	1	0.854
4 Factors + SMB + COM	2.773	0.0000	0.125	0.112	0.080	0.529	0.321	0.533	2	0.899
4 Factors + LMU	2.694	0.0000	0.120	0.108	0.118	0.785	0.615	0.495	0	0.837
4 Factors + SMB + LMU	2.794	0.0000	0.123	0.113	0.080	0.532	0.316	0.532	1	0.898
Panel B: Fama-French 25 Portfolios Sorted on Size and Operating Profitability										
4 Factors	1.643	0.0256	0.112	0.066	0.160	1.173	1.289	0.313	2	0.808
4 Factors + SMB	1.854	0.0072	0.121	0.075	0.053	0.386	0.190	0.794	1	0.915
4 Factors + LIQ	1.763	0.0125	0.129	0.072	0.068	0.494	0.234	1.047	0	0.882
4 Factors + SMB + LIQ	2.056	0.0019	0.129	0.083	0.063	0.461	0.214	0.698	0	0.919
4 Factors + COM	1.622	0.0289	0.124	0.065	0.056	0.407	0.192	1.359	0	0.876
4 Factors + SMB + COM	1.993	0.0029	0.125	0.081	0.060	0.441	0.204	0.701	0	0.921
4 Factors + LMU	1.543	0.0446	0.120	0.062	0.067	0.492	0.302	0.992	1	0.860
4 Factors + SMB + LMU	1.924	0.0046	0.123	0.078	0.058	0.425	0.198	0.720	0	0.921

Table 7. Continued

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel C: Fama-French 25 Portfolios Sorted on Size and Investment										
4 Factors	3.281	0.0000	0.112	0.131	0.191	1.359	1.893	0.193	7	0.819
4 Factors + SMB	3.021	0.0000	0.121	0.122	0.077	0.550	0.373	0.344	2	0.926
4 Factors + LIQ	3.102	0.0000	0.129	0.126	0.077	0.547	0.375	0.589	1	0.892
4 Factors + SMB + LIQ	3.102	0.0000	0.129	0.126	0.079	0.561	0.386	0.328	3	0.930
4 Factors + COM	3.098	0.0000	0.124	0.125	0.089	0.634	0.465	0.521	1	0.884
4 Factors + SMB + COM	3.077	0.0000	0.125	0.124	0.079	0.560	0.379	0.320	3	0.932
4 Factors + LMU	3.061	0.0000	0.120	0.123	0.110	0.778	0.698	0.399	1	0.867
4 Factors + SMB + LMU	2.980	0.0000	0.123	0.120	0.078	0.551	0.369	0.329	3	0.931
Panel D: Fama-French 25 Portfolios Sorted on Size and Momentum										
4 Factors	1.524	0.0494	0.112	0.061	0.222	1.755	2.931	0.180	7	0.789
4 Factors + SMB	1.587	0.0350	0.121	0.064	0.074	0.586	0.370	0.735	2	0.887
4 Factors + LIQ	1.120	0.3122	0.129	0.045	0.060	0.476	0.290	1.282	0	0.863
4 Factors + SMB + LIQ	1.487	0.0603	0.129	0.060	0.064	0.508	0.341	0.802	1	0.892
4 Factors + COM	1.216	0.2152	0.124	0.049	0.080	0.631	0.550	0.699	0	0.858
4 Factors + SMB + COM	1.512	0.0528	0.125	0.061	0.064	0.504	0.337	0.788	1	0.894
4 Factors + LMU	1.310	0.1431	0.120	0.053	0.117	0.929	1.018	0.411	2	0.841
4 Factors + SMB + LMU	1.541	0.0451	0.123	0.062	0.066	0.524	0.341	0.773	1	0.893
Panel E: All Portfolios in Panel A - Panel D (100 Portfolios)										
4 Factors	2.393	0.0000	0.112	0.429	0.192	1.376	1.834	0.220	22	0.801
4 Factors + SMB	2.319	0.0000	0.121	0.419	0.071	0.505	0.300	0.580	6	0.905
4 Factors + LIQ	2.307	0.0000	0.129	0.420	0.071	0.506	0.296	0.888	2	0.874
4 Factors + SMB + LIQ	2.303	0.0000	0.129	0.419	0.071	0.510	0.308	0.564	5	0.909
4 Factors + COM	2.315	0.0000	0.124	0.420	0.079	0.565	0.388	0.720	2	0.868
4 Factors + SMB + COM	2.306	0.0000	0.125	0.418	0.071	0.506	0.304	0.552	6	0.911
4 Factors + LMU	2.336	0.0000	0.120	0.422	0.103	0.738	0.629	0.497	4	0.851
4 Factors + SMB + LMU	2.311	0.0000	0.123	0.419	0.071	0.505	0.299	0.556	5	0.911

Table 8. Test Statistics for Factor Model Regression Intercept: Robustness Test II

This table presents test statistics for factor model regression intercepts for 720 months (July 1963 to June 2023). The test portfolios are 4 different sets of 25 Fama-French portfolios – 5x5 quintile sorts on market equity capitalization and independently on book-to-market ratio, operating profitability, investment and momentum. Book-to-market ratio is book value of equity to market value of equity. Operating profitability is ratio of net profit, revenues minus costs of goods sold, administrative expenses and interest expense divided by book equity. Investment is the change in total assets from year y – 2 and y – 1 divided by total assets at y – 2. Momentum is the average monthly returns from month t – 12 to t – 2. The factors examined in this table are the Fama and French (2018) five factors (MKT, SMB, RMW, CMA and MOM), HMLO (orthogonal factor) and the three liquidity factors (LIQ, COM and LMU). The liquidity factors are constructed from the intersection of six portfolios (3x2) sorted by standard deviation of stock returns and liquidity costs/covariances risks. The term "5 Factors" refers to the Fama and French (2018) six factors excluding SMB (MKT, HMLO, RMW, CMA and MOM). This table presents nine test statistics discussed in Section 4.1.

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel A: Fama-French 25 Portfolios Sorted on Size and Book-to-Market Ratio										
5 Factors	2.879	0.0000	0.112	0.115	0.195	1.296	1.610	0.203	7	0.815
5 Factors + SMB	2.838	0.0000	0.121	0.114	0.078	0.517	0.302	0.404	4	0.919
5 Factors + LIQ	2.502	0.0001	0.129	0.102	0.074	0.493	0.296	0.673	1	0.885
5 Factors + SMB + LIQ	2.723	0.0000	0.129	0.111	0.077	0.509	0.308	0.387	4	0.923
5 Factors + COM	2.600	0.0000	0.124	0.105	0.091	0.603	0.383	0.562	3	0.878
5 Factors + SMB + COM	2.799	0.0000	0.125	0.113	0.078	0.520	0.313	0.365	6	0.924
5 Factors + LMU	2.687	0.0000	0.121	0.108	0.116	0.771	0.601	0.411	1	0.861
5 Factors + SMB + LMU	2.821	0.0000	0.123	0.114	0.079	0.523	0.312	0.365	5	0.924
Panel B: Fama-French 25 Portfolios Sorted on Size and Operating Profitability										
5 Factors	1.664	0.0226	0.112	0.066	0.161	1.175	1.292	0.304	2	0.812
5 Factors + SMB	1.854	0.0071	0.121	0.075	0.053	0.385	0.190	0.736	1	0.919
5 Factors + LIQ	1.764	0.0125	0.129	0.072	0.066	0.484	0.226	1.040	0	0.885
5 Factors + SMB + LIQ	2.051	0.0020	0.129	0.083	0.061	0.445	0.201	0.682	0	0.923
5 Factors + COM	1.628	0.0279	0.124	0.066	0.054	0.397	0.185	1.361	0	0.878
5 Factors + SMB + COM	1.990	0.0030	0.125	0.080	0.058	0.426	0.195	0.676	0	0.924
5 Factors + LMU	1.554	0.0422	0.121	0.063	0.067	0.487	0.297	0.975	1	0.862
5 Factors + SMB + LMU	1.924	0.0046	0.123	0.078	0.057	0.415	0.192	0.685	0	0.924

Table 8. Continued

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel C: Fama-French 25 Portfolios Sorted on Size and Investment										
5 Factors	3.329	0.0000	0.112	0.133	0.192	1.360	1.897	0.189	8	0.822
5 Factors + SMB	3.082	0.0000	0.121	0.124	0.078	0.551	0.373	0.318	3	0.930
5 Factors + LIQ	3.113	0.0000	0.129	0.126	0.077	0.547	0.375	0.565	1	0.894
5 Factors + SMB + LIQ	3.135	0.0000	0.129	0.127	0.080	0.565	0.389	0.301	4	0.933
5 Factors + COM	3.122	0.0000	0.124	0.126	0.090	0.638	0.464	0.503	1	0.886
5 Factors + SMB + COM	3.124	0.0000	0.125	0.126	0.080	0.565	0.384	0.292	5	0.935
5 Factors + LMU	3.101	0.0000	0.121	0.125	0.110	0.781	0.699	0.386	1	0.869
5 Factors + SMB + LMU	3.037	0.0000	0.123	0.123	0.078	0.555	0.373	0.300	5	0.934
Panel D: Fama-French 25 Portfolios Sorted on Size and Momentum										
5 Factors	1.537	0.0463	0.112	0.061	0.222	1.758	2.942	0.174	7	0.797
5 Factors + SMB	1.807	0.0096	0.121	0.073	0.074	0.588	0.373	0.684	2	0.895
5 Factors + LIQ	1.120	0.3120	0.129	0.045	0.060	0.474	0.299	1.186	0	0.868
5 Factors + SMB + LIQ	1.670	0.0219	0.129	0.068	0.065	0.518	0.367	0.693	2	0.899
5 Factors + COM	1.220	0.2116	0.124	0.049	0.080	0.636	0.568	0.651	0	0.862
5 Factors + SMB + COM	1.720	0.0163	0.125	0.070	0.066	0.520	0.361	0.685	2	0.900
5 Factors + LMU	1.317	0.1390	0.121	0.053	0.119	0.946	1.047	0.385	2	0.845
5 Factors + SMB + LMU	1.758	0.0130	0.123	0.071	0.067	0.534	0.358	0.687	2	0.900
Panel E: All Portfolios in Panel A - Panel D (100 Portfolios)										
5 Factors	2.396	0.0000	0.112	0.430	0.192	1.378	1.839	0.207	24	0.812
5 Factors + SMB	2.323	0.0000	0.121	0.420	0.071	0.506	0.301	0.498	10	0.916
5 Factors + LIQ	2.309	0.0000	0.129	0.421	0.069	0.496	0.293	0.812	2	0.883
5 Factors + SMB + LIQ	2.307	0.0000	0.129	0.420	0.071	0.506	0.308	0.480	10	0.919
5 Factors + COM	2.316	0.0000	0.124	0.420	0.079	0.565	0.385	0.661	4	0.876
5 Factors + SMB + COM	2.311	0.0000	0.125	0.419	0.070	0.505	0.305	0.466	13	0.921
5 Factors + LMU	2.337	0.0000	0.121	0.422	0.103	0.738	0.630	0.458	5	0.859
5 Factors + SMB + LMU	2.315	0.0000	0.123	0.419	0.070	0.504	0.301	0.472	12	0.920

Appendix

Table A1. Spanning Regression Results for IML and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: IML = $\alpha_{IML} + \beta_M MKT + \beta_S SMB + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_J January + \beta_{Mic} MicR_f + e$; where IML is the liquidity factor developed by Amihud and Noh (2021), MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = $\alpha_{SMB} + \beta_M MKT + \beta_H HML + \beta_R RMW + \beta_C CMA + \beta_O MOM + \beta_L IML + e$. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	IML							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.301***	0.247***	0.251***	0.294***	0.486***	0.501***	0.303***	-0.085	0.262**	-0.149**
	(4.166)	(3.341)	(3.550)	(3.879)	(3.881)	(4.012)	(4.150)	(-1.270)	(2.376)	(-2.020)
MKT	-0.194***	-0.070***	-0.071***	-0.084***	0.058	0.055		0.100***	0.141***	
	(-5.914)	(-2.925)	(-2.927)	(-3.459)	(1.469)	(1.360)		(4.638)	(4.251)	
SMB	0.748***	0.910***	0.911***	0.890***			0.851***			
	(17.625)	(23.982)	(24.195)	(23.616)			(20.736)			
HML	0.071*	0.081*	0.078*	0.142***	0.216**	0.205**		-0.006	0.149*	
	(1.842)	(1.822)	(1.674)	(4.152)	(2.316)	(2.098)		(-0.126)	(1.729)	
RMW	0.043	0.078*	0.079*		-0.330***	-0.326***		-0.212***	-0.448***	
	(1.022)	(1.717)	(1.749)		(-2.946)	(-2.969)		(-3.471)	(-3.285)	
CMA	0.108**	0.123**	0.126**		-0.042	-0.033		-0.151***	-0.181*	
	(1.879)	(1.978)	(2.019)		(-0.358)	(-0.276)		(-2.674)	(-1.726)	
MOM	0.010	0.006		0.014	0.022			0.002	0.018	
	(0.396)	(0.229)		(0.516)	(0.485)			(0.072)	(0.419)	
IML								0.713***		0.748***
								(25.524)		(21.308)
January	-0.350									
	(-1.234)									
MicRf	0.098***									
	(5.142)									
R^2	0.692	0.675	0.676	0.672	0.075	0.076	0.637	0.711	0.176	0.637

Table A2. Spanning Regression Results for LMR and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: LMR = α_{LMR} + β_MMKT + β_SSMB + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_JJanuary + β_{Mic}MicR$_f$ + e; where LMR (liquidity sensitivity to market return) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to market returns across three return volatility groups. MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = α_{SMB} + β_MMKT + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_LLMR + e. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	LMR							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.037	-0.004	-0.007	0.066	0.058	0.057	0.112*	0.223**	0.262**	0.118
	(0.648)	(-0.066)	(-0.130)	(1.082)	(0.871)	(0.888)	(1.932)	(2.383)	(2.376)	(1.092)
MKT	-0.054**	0.026	0.027	0.013	0.059***	0.059***		0.101***	0.141***	
	(-2.047)	(1.557)	(1.632)	(0.776)	(3.002)	(2.990)		(3.616)	(4.251)	
SMB	0.129***	0.234***	0.234***	0.192***			0.195***			
	(3.230)	(8.802)	(8.652)	(5.134)			(4.706)			
HML	0.074**	0.080**	0.083**	0.131***	0.115**	0.115***		0.071	0.149*	
	(2.019)	(1.979)	(2.179)	(3.983)	(2.581)	(2.850)		(0.892)	(1.729)	
RMW	0.147***	0.171***	0.170***		0.066**	0.066		-0.492***	-0.448***	
	(5.077)	(5.369)	(5.280)		(2.109)	(2.039)		(-3.800)	(-3.285)	
CMA	0.077	0.087	0.085		0.044	0.044		-0.211**	-0.181*	
	(1.511)	(1.615)	(1.584)		(0.754)	(0.755)		(-2.169)	(-1.726)	
MOM	-0.003	-0.005		0.005	-0.001			0.018	0.018	
	(-0.166)	(-0.238)		(0.194)	(-0.036)			(0.479)	(0.419)	
LMR								0.674***		0.635***
								(8.809)		(6.366)
January	-0.295									
	(-1.255)									
MicRf	0.063***									
	(3.971)									
R^2	0.237	0.212	0.213	0.170	0.066	0.067	0.122	0.305	0.176	0.122

Table A3. Spanning Regression Results for LME and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: LME = α_{LME} + β_MMKT + β_SSMB + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_JJanuary + β_{Mic}MicR_f + e; where LME (liquidity sensitivity to macroeconomic shocks) is the difference in the value-weighted return between stocks with high and low liquidity sensitivity to macroeconomic shocks across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = α_{SMB} + β_MMKT + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_LLME + e. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	LME							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	-0.052	-0.104*	-0.096	-0.020	-0.065	-0.055	0.027	0.289***	0.262**	0.201*
	(-0.845)	(-1.650)	(-1.592)	(-0.298)	(-0.969)	(-0.848)	(0.428)	(2.834)	(2.376)	(1.692)
MKT	-0.091***	0.009	0.007	-0.014	0.030	0.028		0.128***	0.141***	
	(-3.443)	(0.543)	(0.443)	(-0.820)	(1.567)	(1.451)		(4.216)	(4.251)	
SMB	0.016	0.147***	0.148***	0.107***			0.097***			
	(0.399)	(4.806)	(4.897)	(3.182)			(2.669)			
HML	0.023	0.030	0.024	0.130***	0.052	0.045		0.127	0.149*	
	(0.651)	(0.767)	(0.638)	(4.172)	(1.246)	(1.112)		(1.532)	(1.729)	
RMW	0.123***	0.153***	0.155***		0.087***	0.090***		-0.484***	-0.448***	
	(3.859)	(4.514)	(4.603)		(2.813)	(2.773)		(-3.585)	(-3.285)	
CMA	0.186***	0.198***	0.203***		0.171***	0.177***		-0.253**	-0.181*	
	(3.753)	(3.772)	(3.859)		(3.211)	(3.252)		(-2.407)	(-1.726)	
MOM	0.015	0.012		0.026	0.015			0.011	0.018	
	(0.712)	(0.565)		(1.080)	(0.653)			(0.281)	(0.419)	
LME								0.417***		0.309***
								(4.840)		(2.933)
January	-0.377									
	(-1.473)									
MicRf	0.079***									
	(5.219)									
R^2	0.170	0.130	0.131	0.082	0.075	0.075	0.029	0.225	0.176	0.029

Table A4. Spanning Regression Results for RML and SMB

This table show the results of the following regression model estimated using the 720 monthly returns from July 1963 to June 2023: RML = α_{RML} + β_MMKT + β_SSMB + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_JJanuary + β_{Mic}MicR$_f$ + e; where RML (return sensitivity to market liquidity) is the difference in the value-weighted return between stocks with high and low return sensitivity to market liquidity across three return volatility groups, MKT is the excess market return, SMB and HML are the size and value factors (Fama and French, 1993), RMW and CMA are the profitability and investment factors (Fama and French, 2015), and MOM is the momentum factor (Carhart, 1997). January is a binary variable capturing the January effect. The micro-stock effect, MicRf, is the excess value-weighed return on stocks in the smallest decile portfolio. The table also shows the results of the following regression model: SMB = α_{SMB} + β_MMKT + β_HHML + β_RRMW + β_CCMA + β_OMOM + β_LRML + e. The coefficients and standard errors are estimated using the generalized method of moments (Cochrane, 2005), and the adjusted-R^2 is from the OLS regression. The estimated coefficients are in monthly percentage points and t-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels.

	RML							SMB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.052	0.012	-0.002	0.023	0.011	-0.003	0.017	0.262**	0.262***	0.213*
	(0.718)	(0.157)	(-0.031)	(0.303)	(0.150)	(-0.042)	(0.248)	(2.374)	(2.376)	(1.778)
MKT	0.056	0.112***	0.115***	0.109***	0.111***	0.115***		0.142***	0.141***	
	(1.546)	(5.599)	(5.635)	(5.071)	(5.705)	(5.830)		(4.360)	(4.251)	
SMB	-0.073	-0.002	-0.003	-0.007			0.042			
	(-1.295)	(-0.056)	(-0.081)	(-0.209)			(1.202)			
HML	-0.177***	-0.174***	-0.164***	-0.162***	-0.174***	-0.164***		0.148	0.149*	
	(-3.071)	(-3.073)	(-2.798)	(-3.770)	(-3.072)	(-2.804)		(1.602)	(1.729)	
RMW	0.003	0.022	0.018		0.023	0.019		-0.448***	-0.448***	
	(0.056)	(0.376)	(0.307)		(0.378)	(0.317)		(-3.309)	(-3.285)	
CMA	0.015	0.022	0.014		0.022	0.015		-0.181*	-0.181*	
	(0.221)	(0.330)	(0.203)		(0.336)	(0.209)		(-1.704)	(-1.726)	
MOM	-0.020	-0.021		-0.019	-0.021			0.018	0.018	
	(-0.577)	(-0.570)		(-0.529)	(-0.578)			(0.420)	(0.419)	
RML								-0.005		0.115
								(-0.054)		(1.246)
January	-0.337									
	(-1.226)									
MicRf	0.044**									
	(1.975)									
R^2	0.175	0.167	0.166	0.168	0.168	0.167	0.003	0.174	0.176	0.003

Table A5. Test Statistics for Factor Model Regression Intercept: Robustness Test III

This table presents test statistics for factor model regression intercepts for 720 months (July 1963 to June 2023). The test portfolios are 4 different sets of 25 Fama-French equally weighted portfolios – 5x5 quintile sorts on market equity capitalization and independently on book-to-market ratio, operating profitability, investment and momentum. Book-to-market ratio is book value of equity to market value of equity. Operating profitability is ratio of net profit, revenues minus costs of goods sold, administrative expenses and interest expense divided by book equity. Investment is the change in total assets from year y – 2 and y – 1 divided by total assets at y – 2. Momentum is the average monthly returns from month t – 12 to t – 2. The factors examined in this table are the Fama and French (2018) six factors (MKT, SMB, HML, RMW, CMA and MOM) and the three liquidity factors (LIQ, COM and LMU). The liquidity factors are constructed from the intersection of six portfolios (3x2) sorted by standard deviation of stock returns and liquidity costs/covariances risks. The term "5 Factors" refers to the Fama and French (2018) six factors excluding SMB (MKT, HML, RMW, CMA and MOM). This table presents nine test statistics discussed in Section 4.1.

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel A: Fama-French 25 Portfolios Sorted on Size and Book-to-Market Ratio										
5 Factors	4.111	0.0000	0.112	0.164	0.286	1.752	3.098	0.110	11	0.815
5 Factors + SMB	4.891	0.0000	0.121	0.197	0.140	0.858	0.929	0.164	7	0.906
5 Factors + LIQ	3.630	0.0000	0.129	0.147	0.136	0.835	0.908	0.269	4	0.874
5 Factors + SMB + LIQ	4.685	0.0000	0.129	0.190	0.146	0.891	1.002	0.161	5	0.910
5 Factors + COM	3.767	0.0000	0.124	0.152	0.169	1.036	1.289	0.204	3	0.867
5 Factors + SMB + COM	4.814	0.0000	0.125	0.195	0.143	0.878	0.996	0.155	6	0.912
5 Factors + LMU	3.886	0.0000	0.120	0.156	0.207	1.263	1.779	0.156	6	0.852
5 Factors + SMB + LMU	4.943	0.0000	0.123	0.200	0.142	0.869	0.971	0.153	6	0.912
Panel B: Fama-French 25 Portfolios Sorted on Size and Operating Profitability										
5 Factors	2.927	0.0000	0.112	0.117	0.255	1.913	3.696	0.136	9	0.818
5 Factors + SMB	2.686	0.0000	0.121	0.108	0.106	0.792	0.949	0.221	6	0.917
5 Factors + LIQ	2.475	0.0001	0.129	0.100	0.101	0.757	0.907	0.377	2	0.882
5 Factors + SMB + LIQ	2.492	0.0001	0.129	0.101	0.106	0.795	0.999	0.217	6	0.920
5 Factors + COM	2.597	0.0000	0.124	0.105	0.121	0.908	1.303	0.281	2	0.875
5 Factors + SMB + COM	2.593	0.0000	0.125	0.105	0.104	0.783	0.993	0.210	6	0.922
5 Factors + LMU	2.720	0.0000	0.120	0.109	0.158	1.189	1.885	0.212	6	0.859
5 Factors + SMB + LMU	2.663	0.0000	0.123	0.107	0.103	0.769	0.972	0.209	6	0.922

Table A5. Continued

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel C: Fama-French 25 Portfolios Sorted on Size and Investment										
5 Factors	6.903	0.0000	0.112	0.276	0.287	1.649	2.725	0.099	11	0.822
5 Factors + SMB	8.127	0.0000	0.121	0.327	0.147	0.845	0.902	0.126	8	0.916
5 Factors + LIQ	6.395	0.0000	0.129	0.259	0.129	0.741	0.882	0.213	2	0.882
5 Factors + SMB + LIQ	8.050	0.0000	0.129	0.327	0.135	0.773	0.961	0.124	7	0.919
5 Factors + COM	6.526	0.0000	0.124	0.264	0.154	0.886	1.199	0.169	4	0.874
5 Factors + SMB + COM	8.122	0.0000	0.125	0.328	0.137	0.786	0.953	0.121	7	0.921
5 Factors + LMU	6.643	0.0000	0.120	0.267	0.194	1.113	1.601	0.135	7	0.859
5 Factors + SMB + LMU	8.133	0.0000	0.123	0.328	0.139	0.797	0.927	0.119	7	0.921
Panel D: Fama-French 25 Portfolios Sorted on Size and Momentum										
5 Factors	2.138	0.0011	0.112	0.085	0.253	1.883	3.842	0.122	8	0.800
5 Factors + SMB	2.820	0.0000	0.121	0.114	0.107	0.797	1.002	0.235	5	0.895
5 Factors + LIQ	1.702	0.0181	0.129	0.069	0.099	0.736	0.967	0.340	2	0.867
5 Factors + SMB + LIQ	2.659	0.0000	0.129	0.108	0.106	0.788	1.075	0.223	3	0.899
5 Factors + COM	1.828	0.0084	0.124	0.074	0.126	0.939	1.435	0.241	3	0.860
5 Factors + SMB + COM	2.746	0.0000	0.125	0.111	0.106	0.789	1.082	0.213	4	0.900
5 Factors + LMU	1.945	0.0040	0.120	0.078	0.163	1.208	2.075	0.181	5	0.843
5 Factors + SMB + LMU	2.800	0.0000	0.123	0.113	0.106	0.787	1.063	0.214	5	0.900
Panel E: All Portfolios in Panel A - Panel D (100 Portfolios)										
5 Factors	3.308	0.0000	0.112	0.593	0.271	1.793	3.191	0.115	39	0.814
5 Factors + SMB	3.482	0.0000	0.121	0.630	0.125	0.829	0.929	0.176	26	0.908
5 Factors + LIQ	3.161	0.0000	0.129	0.576	0.116	0.771	0.902	0.284	10	0.876
5 Factors + SMB + LIQ	3.425	0.0000	0.129	0.624	0.123	0.816	0.992	0.172	21	0.912
5 Factors + COM	3.203	0.0000	0.124	0.581	0.143	0.946	1.277	0.215	12	0.869
5 Factors + SMB + COM	3.456	0.0000	0.125	0.627	0.123	0.813	0.988	0.166	23	0.914
5 Factors + LMU	3.237	0.0000	0.120	0.585	0.180	1.195	1.778	0.166	24	0.853
5 Factors + SMB + LMU	3.491	0.0000	0.123	0.633	0.122	0.811	0.965	0.165	24	0.914

Table A6. Test Statistics for Factor Model Regression Intercept: Robustness Test V

This table presents test statistics for factor model regression intercepts for for 720 monthly returns from July 1963 to June 2023. The test portfolios are 3 different sets of 6 liquidity sort portfolios – 3x2 sorts on standard deviation of stock returns and independently on illiquidity (Amihud measure), covariation between stock and market liquidity, and covariation between stock liquidity and market uncertainty. The six portfolios are the ones that are created to construct the liquidity factors – LIQ, COM, and LMU. The factors examined in this table are the Fama and French (2018) six factors (MKT, SMB, HML, RMW, CMA and MOM) and the three liquidity factors (LIQ, COM and LMU). The liquidity factors are constructed from the intersection of twelve portfolios (2x3x2) sorted by equity market capitalization, standard deviation of stock returns and liquidity costs/covariances risks. The term "5 Factors" refers to the Fama and French (2018) six factors excluding SMB (MKT, HML, RMW, CMA and MOM). This table presents nine test statistics discussed in Section 4.1.

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)$/ $A(\|r\|)$	$A(\alpha^2)$/ $A(r^2)$	$A(s(\alpha)^2)$/ $A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel A: 6 Portfolios Sorted on Standard deviation of stock returns and Amihud										
5 Factors	3.399	0.0026	0.124	0.033	0.121	0.676	0.565	0.941	0	0.811
5 Factors + SMB	2.491	0.0216	0.134	0.024	0.142	0.790	0.800	0.306	0	0.898
5 Factors + LIQ	1.970	0.0677	0.139	0.019	0.131	0.732	0.788	0.366	0	0.892
5 Factors + SMB + LIQ	2.200	0.0413	0.139	0.021	0.133	0.739	0.805	0.278	0	0.917
5 Factors + COM	2.851	0.0095	0.134	0.028	0.099	0.553	0.390	0.834	0	0.880
5 Factors + SMB + COM	2.867	0.0091	0.135	0.028	0.125	0.696	0.764	0.299	0	0.913
5 Factors + LMU	2.925	0.0080	0.130	0.028	0.095	0.530	0.250	1.565	0	0.856
5 Factors + SMB + LMU	2.600	0.0169	0.134	0.025	0.136	0.757	0.779	0.306	0	0.905
Panel B: 6 Portfolios Sorted on Standard deviation of stock returns and the covariation between stock and market liquidity										
5 Factors	3.252	0.0037	0.124	0.031	0.103	0.608	0.419	1.230	0	0.829
5 Factors + SMB	2.450	0.0237	0.134	0.024	0.127	0.748	0.765	0.335	0	0.902
5 Factors + LIQ	1.960	0.0691	0.139	0.019	0.132	0.779	0.765	0.398	0	0.892
5 Factors + SMB + LIQ	2.045	0.0577	0.139	0.020	0.133	0.786	0.781	0.304	0	0.916
5 Factors + COM	2.982	0.0070	0.134	0.029	0.102	0.601	0.417	0.765	0	0.890
5 Factors + SMB + COM	3.354	0.0029	0.135	0.033	0.128	0.757	0.772	0.299	0	0.920
5 Factors + LMU	2.703	0.0134	0.130	0.026	0.081	0.477	0.220	1.709	0	0.871
5 Factors + SMB + LMU	2.480	0.0222	0.134	0.024	0.122	0.718	0.764	0.317	0	0.912

Table A6. Continued

	GRS	p(GRS)	$Sh^2(f)$	$Sh^2(\alpha)$	$A(\|\alpha\|)$	$A(\|\alpha\|)/A(\|r\|)$	$A(\alpha^2)/A(r^2)$	$A(s(\alpha)^2)/A(\alpha^2)$	N(1%)	$A(\bar{R}^2)$
Panel C: 6 Portfolios Sorted on Standard deviation of stock returns and the covariation between stock liquidity and market uncertainty										
5 Factors	1.538	0.1630	0.124	0.015	0.053	0.466	0.247	4.309	0	0.848
5 Factors + SMB	1.660	0.1280	0.134	0.016	0.133	1.172	1.662	0.366	0	0.900
5 Factors + LIQ	1.092	0.3658	0.139	0.011	0.127	1.124	1.508	0.501	0	0.884
5 Factors + SMB + LIQ	1.379	0.2203	0.139	0.013	0.128	1.134	1.543	0.386	0	0.907
5 Factors + COM	0.904	0.4913	0.134	0.009	0.103	0.910	0.891	0.858	0	0.884
5 Factors + SMB + COM	1.502	0.1748	0.135	0.015	0.131	1.159	1.664	0.349	0	0.909
5 Factors + LMU	1.122	0.3478	0.130	0.011	0.075	0.666	0.434	1.858	0	0.885
5 Factors + SMB + LMU	1.815	0.0936	0.134	0.018	0.131	1.157	1.722	0.330	0	0.915
Panel E: All Portfolios in Panel A - Panel C (18 Portfolios)										
5 Factors	2.043	0.0066	0.124	0.060	0.092	0.600	0.444	1.349	0	0.829
5 Factors + SMB	1.778	0.0242	0.134	0.053	0.134	0.868	0.908	0.333	0	0.900
5 Factors + LIQ	1.547	0.0683	0.139	0.046	0.130	0.845	0.878	0.414	0	0.889
5 Factors + SMB + LIQ	1.662	0.0412	0.139	0.049	0.131	0.853	0.897	0.317	0	0.913
5 Factors + COM	1.883	0.0147	0.134	0.056	0.101	0.658	0.473	0.817	0	0.885
5 Factors + SMB + COM	1.983	0.0089	0.135	0.059	0.128	0.831	0.895	0.314	0	0.914
5 Factors + LMU	1.890	0.0142	0.130	0.056	0.084	0.544	0.262	1.690	0	0.871
5 Factors + SMB + LMU	1.816	0.0203	0.134	0.054	0.130	0.841	0.908	0.317	0	0.910