Sequential Learning, Asset Allocation, and Bitcoin Returns

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Abstract

For optimal asset allocation, mean-variance investors must learn about the joint dynamics of new and existing assets, not only their profitability. Bitcoin's *digital gold* narrative provides a unique laboratory to test this hypothesis. We find that a decrease in investors' estimate on correlation between Bitcoin and the US stock markets strongly predicts higher Bitcoin returns next day. The same empirical pattern universally appears in out-of-sample predictions, global equity markets, and other cryptocurrencies. Our stylized model and empirical proxy for Bitcoin demand quantitatively explain the return predictability pattern in light of asset allocation practices and investors' learning on time-varying correlation.

Key words: Bitcoin, Uncertainty, Learning, Time-Varying Correlation, Return Predictability

JEL Classification: G12, G15, D83

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1 Introduction

Investment is a never-ending battle against risk and uncertainty. Investors routinely quantify risk in existing assets while dealing with uncertainty in unexpected new assets. Then what would investors do if a new asset never belongs to any existing asset class? What if the asset's narrative (Shiller 2020) suddenly changes like a mythical shape-shifting creature? How will investors' response to a new narrative affect asset prices?

Bitcoin is a perfect laboratory to answer those questions. Bitcoin begins as a decentralized peer-to-peer *payment system* (Nakamoto 2008), and it explodes as a speculative digital *currency* until reaching a peak in late 2017. Since then, many Bitcoin advocates promote a new narrative, digital *gold*, that diverts attention from slow and expensive Bitcoin transactions (Hinzen et al. 2020).[1] After Bitcoin's series of survivals through image transformation (Zajonc 1968), the *digital gold* narrative successfully persuades traditional investors, such as pension funds and sovereign wealth funds, to include Bitcoin in their portfolios.[2] However, investors still face unprecedented uncertainty in Bitcoin's correlations with other assets. Such uncertainty urges investors to learn about the correlations instead of Bitcoin's unfathomable valuation.

We hypothesize that rational investors keep rebalancing their Bitcoin-stock portfolios by tracking the time-varying correlation of Bitcoin and their existing well-diversified stock portfolios.[3] Therefore, investors' learning, or equivalently changes in their beliefs, systematically affects Bitcoin demands and prices.

The conditional correlation between Bitcoin and the US stock markets begin to fluctuate since the inception of Bitcoin futures markets in late 2017.[4] Such a structural break in correlation dynamics allows us to test our hypothesis on investors' learning behavior and its impact on Bitcoin

[1]See Hardle et al. (2020) for an extensive literature review.

[2]See news articles at bit.ly/3qxaVmt, bloom.bg/3gl5zG2, bloom.bg/3qCcoIc, on.wsj.com/3htc369, and https://on.wsj.com/37clZgh.

[3]We find that including Bitcoin in a portfolio improves its ex-ante daily Sharpe ratio 84% of the time, even with the most conservative belief on Bitcoin's risk premium. Literature finds mixed evidence on the role of this new asset: as a safe haven, hedge, or diversifier. Early studies such as Dyhrberg (2016a) and Dyhrberg (2016b) find supportive evidence for Bitcoin's hedging ability. In contrast, studies with more recent data show opposite results: Conlon et al. (2020), Klein et al. (2018), Smales (2019), Conlon and McGee (2020), Baur et al. (2018), Feng et al. (2018), and Bouri et al. (2017b). Furthermore, hedging or safe haven property of Bitcoin appears only with high economic or policy uncertainty: Bouoiyour et al. (2019), Demir et al. (2018), Bouri et al. (2017a), Fang et al. (2019), Nygaard et al. (2019), and Paule-Vianez et al. (2020). Others show the results vary across investment horizons and countries. See Shahzad et al. (2019), Shahzad et al. (2020), Corbet et al. (2018), and Bouri et al. (2017b).

[4]This is ironic because Bitcoin's diversification benefit becomes suddenly uncertain as soon as traditional investors participate in the markets while recognizing Bitcoin as a mainstream asset. See Yae et al. (2021) for detail.

prices. Using simple predictive regressions, we find that an increase (decrease) in investors' estimate on correlation between Bitcoin and S&P500 strongly and robustly predicts lower (higher) Bitcoin returns next day.[5] However, the level of correlation fails to predict subsequent Bitcoin returns.

This predictability evidence is puzzling in two aspects. First, in many cases, predictability appears to exist only by levels rather than changes because of a bias in regressions with persistent predictor variables (Stambaugh 1999). Second, the sign of Bitcoin return predictability contradicts conventional wisdom. A high correlation means a small diversification benefit; therefore, the risk premium should be high enough to compensate it. However, we observe lower returns right after an increase in correlation.

A simple intuition on trading practices and investors' learning can explain this puzzle. Suppose a fund manager runs a portfolio of Bitcoin and an S&P500 index fund. She revises her estimates on inputs for portfolio optimization at market closing. She requests in-house traders or external trading firms to rebalance her portfolio accordingly. Then the traders would split and delay orders during the next trading day hoping for executing at better prices or hiding the fund manager's intentions (Kyle 1985, Admati and Pfleiderer 1989).[6] Therefore, an increase in correlation estimate today leads to a downward price pressure tomorrow. In this way, *asynchronous portfolio rebalancing* creates return predictability, at daily frequency, that is observable but negligible to traders who face huge intra-day volatility.[7]

To verify our empirical findings and intuition, we construct empirical proxies for investors' daily Bitcoin demand due to their learning on correlation in the joint dynamics of Bitcoin and S&P500. We fit a model of Dynamic Conditional Correlation (DCC) Generalized Autoregressive Conditional Heteroskedasticity (GARCH) introduced by Engle (2002). Then we compute Bitcoin demand proxies as conditional optimal portfolio weights on Bitcoin from the standard mean-variance portfolio theory (Markowitz 1952). Our proxies for Bitcoin demand uncover important empirical facts that conditional correlation estimates alone cannot reveal.

Using the proxies for Bitcoin demand, we find that a one-standard-deviation increase (decrease) in the daily Bitcoin-demand-change due to learning on correlation can predict 0.5% higher (lower)

[5]We have 765 observations, R^2 is above 1%, and Bitcoin's daily return volatility is 5%, all of which are comparable to the market return predictability results in the literature with 60 years of monthly data.

[6]See an article related to this Bitcoin trading behavior at https://yhoo.it/3i9ctPO.

[7]See empirical evidence on stealth trading (Barclay and Warner 1993, Chakravarty 2001), order imbalance (Chan and Fong 2000, Barber et al. 2008), and institutional investors (Sias and Starks 1997, Keim and Madhavan 1995).

subsequent Bitcoin returns with Newey-West adjusted t-statistics above 4.0.[8] Other existing predictors or extensive control variables barely affect the magnitude or the statistical significance of the coefficient estimate. In addition, popular Machine Learning algorithms show that our predictor is the most dominant among the predictors documented in the literature even if non-linearity and interactions are considered.[9] Finally, out-of-sample R^2 is up to 2.33% at daily frequency (3.52% before COVID-19) using a robust linear model.

Our intuition on Bitcoin's return-predictability produces five groups of testable predictions. We find supportive evidence in all the predictions. First, the same empirical patterns of Bitcoin's return-predictability universally appear in seven major international equity markets plus Ethereum (ETH) and Ripple (XRP). Also, aggregating global Bitcoin demands across countries improves predictability. Second, signals from time-varying volatilities are too noisy for investors to use for portfolio optimization. This finding explains why changes in volatility estimates fail to predict Bitcoin returns. Third, changes in Bitcoin demand proxies are superior Bitcoin return predictors to changes in correlation estimates. Fourth, Bitcoin's return-predictability is conditional on the levels of volatilities and risk premia. Lastly, our predictor variable shows no predictability in stocks and in Bitcoin when correlation is constant before Bitcoin futures markets.

Furthermore, by extending our empirical proxy construction, we build a stylized static model of Bitcoin prices to quantitatively investigate the Bitcoin return predictability. With reasonably calibrated parameters, the model can simultaneously match the observed coefficient estimates on the changes and levels of correlations in predictive regressions. Therefore, all evidence supports our intuition that investors' learning on correlation can explain the puzzling patterns of Bitcoin return predictability in the presence of asynchronous portfolio rebalancing.

The contribution of the paper is threefold. First, this paper proves that Bitcoin and potentially other cryptocurrencies are closely linked to mainstream assets, as opposed to the literature that implicitly isolates such a new asset class from existing ones. We highlight (1) the ex-ante diversification benefits of Bitcoin for stock investors and (2) the importance of uncertainty and investors' learning in the joint dynamics with stock markets, not only in the Bitcoin's own dynamics.[10] Sec-

[8]By contrast, Pollet and Wilson (2010) find return predictability in stock market, not in Bitcoin, by the average of cross-sectional correlations across individual stocks.

[9]LASSO, Group Lasso, Elastic Net, Random Forest, Gradient Boosting Machine, and Neural Network with one hidden layer. The last four algorithms consider non-linearity and all possible interactions among predictors.

[10]Several researches estimate Bitcoin's time-varying correlations and suggest an effective portfolio strategy to

ond, this paper discovers a rational aspect of Bitcoin prices, unlike others that focus mainly on Bitcoin's irrational side.[11] We find, both empirically and theoretically, that the puzzling sign of Bitcoin return-predictability can arise from investors' rational behavior: portfolio optimization with learning. Lastly but most importantly, we answer a general economic question: how does investors' learning affect asset prices?

The paper proceeds as follows. Section 2 presents motivating and puzzling empirical findings on Bitcoin return predictability. Section 3 provides an intuitive explanation for the puzzle and constructs an empirical proxies for Bitcoin demand. Section 4 validates Bitcoin return predictability from various angles. Section 5 investigates testable predictions from our intuitive explanation. Section 6 develops a stylized static model of Bitcoin prices and verifies our intuition on the Bitcoin's retur-predictability puzzle quantitatively. Section 7 concludes.

2 Motivation: Time-Varying Correlations and Bitcoin Returns

This section presents a motivating empirical finding. First, we explain how to estimate the time-varying correlation between Bitcoin and stock market returns. Next, we show a structural break of the correlation dynamics. Finally, we establish a novel but puzzling evidence of Bitcoin return predictability.

2.1 Estimation of Time-Varying Correlations

We choose a parsimonious time-series model to describe the joint dynamics of Bitcoin and the stock markets. Our workhorse model for time-varying correlations is DCC(1,1)-GARCH(1,1), Dynamic Conditional Correlation – Generalized Autoregressive Conditional Heteroskedasticity, by Engle (2002). Nevertheless, we find that our results are robust to different model choices.

Let a 2×1 vector $\boldsymbol{r}_t = [r_{b,t} \;\; r_{m,t}]^\top$ denote the daily log returns of Bitcoin and S&P500 index

investors in practice. See, for example, Klein et al. (2018), Guesmi et al. (2019), and Deng et al. (2020). Also, Gao and Nardari (2018) perform a similar task on commodities. However, none of them study how investors actually react to time-varying correlations and how the investors' reactions affect Bitcoin prices.

[11]For example, consider retail traders' behavioral mistakes or big whales' price manipulations. See Cheng et al. (2019), Makarov and Schoar (2020), and Griffin and Shams (2020) among many others.

at time t.

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \mathbf{e}_t \quad \text{where} \quad \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t),$$

where $\boldsymbol{D}_t = diag\{\sigma_{b,t}, \sigma_{m,t}\}$ is a diagonal matrix of the time-varying conditional volatilities, modeled by univariate GARCH(1,1):

$$\sigma_{i,t}^2 = \omega_i + \alpha_i e_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \quad \text{for} \quad i = b, m,$$

while $\boldsymbol{R_t}$ is the conditional correlation matrix of $\mathbf{e}_t$, modeled by DCC(1,1):

$$\boldsymbol{R}_t = \boldsymbol{Q}_t^{*-1} \boldsymbol{Q}_t \boldsymbol{Q}_t^{*-1} \quad \text{where} \quad \boldsymbol{Q}_t = (1 - a - b)\overline{\boldsymbol{Q}} + a \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}^\top + b \boldsymbol{Q}_{t-1}, \tag{1}$$

where $\overline{\boldsymbol{Q}}$ is the unconditional covariance matrix of $\boldsymbol{\epsilon}_t = \boldsymbol{D}_t^{-1} \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{R}_t)$ and $\boldsymbol{Q}_t^*$ is a diagonal matrix whose diagonal elements are the square roots of the diagonal elements of $\boldsymbol{Q}_t$.[12] The model reduces to the Constant Conditional Correlation (CCC) GARCH model of Bollerslev (1990) if $\boldsymbol{R}_t$ is time-invariant $\boldsymbol{R}_t = \overline{\boldsymbol{R}}$ and so $\boldsymbol{Q}_t = \overline{\boldsymbol{Q}}$, which will produce $a \approx 0$ in estimation.

We also consider alternative GARCH models with conditional correlations. The Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model (Nelson 1991) can capture the leverage effects in asset returns — asymmetric responses of conditional variances to negative or positive shocks. On the other hand, an asymmetric response to joint shocks in conditional correlations can be captured by the Asymmetric Generalized Dynamic Conditional Correlation (AG-DCC) model (Cappiello et al. 2006). As a special case, an Asymmetric DCC (A-DCC) model modifies the DCC equation as follows.

$$\boldsymbol{Q}_t = (\bar{\boldsymbol{R}} - a^2 \bar{\boldsymbol{R}} - b^2 \bar{\boldsymbol{R}} - g^2 \bar{\boldsymbol{N}}) + a^2 \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}' + g^2 \boldsymbol{n}_{t-1} \boldsymbol{n}_{t-1}' + b^2 \boldsymbol{Q}_{t-1}$$

where a, b, and g are scalar parameters.

[12] To ensure model validity, parameters a and b satisfy the restrictions $a \geq 0$, $b \geq 0$, $a + b < 1$, and $\boldsymbol{Q}_t$ should be positive definite.

2.2 A Structural Break in Correlations Dynamics

In late 2017, Bitcoin experiences the most iconic event in its history; Bitcoin futures markets begin at Chicago Board Options Exchange (CBOE) on December 11 and at Chicago Mercantile Exchange (CME) on December 18, 2017.[13] Removing short-selling constraints by futures contracts immediately results in a crash of Bitcoin prices, consistent with Miller (1977). However, the crash itself is not the only consequence of the futures markets. Since then the nature of uncertainty in Bitcoin changes from idiosyncratic to systematic (Pástor and Veronesi 2009), and Bitcoin trading and price dynamics change significantly, as documented in the literature (for example, Kim et al. 2020 and Hardle et al. 2020).

To see how the correlation dynamics change around this iconic event, we split the full sample (from January 2015 to December 2020) into two so that the second period sample starts on Dec 18, 2017.[14] The resulting sample sizes for two sub-samples are 746 and 765, respectively.

As shown in Yae et al. (2021), the estimated conditional correlations in our dataset display a dramatic structural break in the correlation dynamics. Table 1 shows the parameter estimates of different multivariate GARCH models for three periods: 1) from January 2015 to December 2020, 2) from January 2015 but prior to the Bitcoin futures markets (pre-futures sample, henceforth), and 3) from the beginning of Bitcoin futures markets to December 2020 (post-futures sample, henceforth). In the table, the parameter a in DCC(1,1)-GARCH(1,1) model is zero in the pre-futures sample but 0.030 (t-stat 2.01) in the post-futures sample. The same pattern appears in the other multivariate GARCH specifications. The estimated conditional correlations (dashed line) in Figure 1 are practically zero and flat for the pre-futures sample while the correlations fluctuate for the post-futures sample. That is, statistical evidence on time-varying correlation comes entirely from the post-futures sample.[15]

Co-occurrence of the structural break and futures trading is interesting and deserves deeper investigations with both theoretical and empirical approaches. Even at first glance, we can find various forces in action. For example, availability of futures trading possibly implies investors' perception on Bitcoin as a mainstream investment asset class. If so, changes in overall money

[13]CBOE delisted Bitcoin futures in 2019 temporarily but CME never did.
[14]Our results are robust to different choices on split date around this date.
[15]See Yae et al. (2021) for more analysis and discussion on the structural break.

supply or fund flows can generate positive correlations across different asset classes. Co-varying risk appetites or speculative sentiments can do the same. Yet, a rising demand of safe haven assets can generate negative correlations. If each of these factors acts dominantly in different times, the correlation between Bitcoin and the stock market can be time-varying. Furthermore, investors' learning on the correlation itself might create the time-variation of correations as well. Therefore, we admit that finding the root cause of the structural break in correlation dynamics is beyond the scope of this paper. Instead, we focus on the Bitcoin return predictability for the period when correlation is time-varying.

If investors constantly monitor the correlation dynamics, they should notice the structural break at certain point. To see when the investors first notice the structural change, we perform rolling-window estimations starting May 2015. At each day, we fit a DCC(1,1)-GARCH(1,1) model using the log returns of Bitcoin and SP&500 index for the past two years. For example, on May 1, 2015, we use daily log returns data from May 1, 2013 to April 30, 2015 for estimation.

Figure 2 shows how investors' perception on correlation dynamics changes in real-time. The dashed lines show the time-variation of the 99% intervals—the 0.5th and 99.5th percentiles— of the estimated conditional correlations using a past-looking window of two years whereas the solid line shows their medians.[16] Therefore, a wide gap between dashed lines represents a huge time-variation of the conditional correlations in the past two years, which is identified by investors on a given day. If this rolling-window estimation approximates investors' sequential learning of the joint dynamics of Bitcoin and stocks, then the pattern in the graph implies that at least some investors must recognize the structural change in early January of 2018. Therefore, if investors' reaction to the time-varying correlation affects Bitcoin prices, the effect should appear but not prior to 2018.

2.3 Bitcoin's Return-Predictability Puzzle

The introduction of futures markets is Bitcoin's big leap toward a mainstream asset. Institutional investors who are fueled by the *digital gold* narrative of Bitcoin start to include Bitcoin in their portfolios. Such passive investors' main goal is diversification rather than price speculation, yet they still need to estimate correlations for portfolio optimization. Therefore, the correlation affects

[16]To smooth out the graphs, we compute the 0.5th and 99.5th percentiles from an extended set of conditional correlations, which is a union of the original two-year-long estimate sets measured on each of 40 trading days (two months) around the given date, that is, the percentiles of $40 \times 252 \times 2 = 2,016$ conditional correlation estimates.

passive investors' demand and possibly equilibrium Bitcoin prices. Such effects can be empirically detectable because institutional investors become major players in the Bitcoin markets lately. The time-varying correlations in the post-futures period provide us with an opportunity to test this idea.

In fact, correlations between asset returns are key elements in canonical asset pricing models. The time-varying correlation can imply a time-varying CAPM beta in a Conditional CAPM or a state variable in a Intertemporal CAPM. Therefore, the traditional asset pricing models suggest that time-varying CAPM betas or correlation levels should explain the time-variation of Bitcoin risk premium and predict subsequent Bitcoin returns.

However, we find that none of these variables explain Bitcoin's risk premium. Table 2 shows that these variables barely predict subsequent Bitcoin returns when we construct the variables from the DCC-GARCH estimation at daily frequency.[17] Instead, changes in correlation predict well. The estimated slope coefficient is -0.153, and its Newey-West adjusted t-statistic is -3.7 with an R^2 of 1.1%. This predictability is robust to various specifications: excluding samples after the COVID-19 outbreak, using robust linear models such as LAD (Least Absolute Deviation) and Rank regressions, transforming the data by Inverse Normal Transformation, or trimming the data. In all specifications, the predictability evidence is stable or even stronger. Also, our *post-futures* sample size is large enough to provide reliable evidence. We have 765 observations, R^2 is above 1%, and Bitcoin's daily return volatility is 5%, all of which are comparable to the market return predictability results in the literature with 60 years of monthly data. [18]

This predictability evidence is puzzling. First, the level of correlation cannot predict subsequent Bitcoin returns even if changes in correlation can. In many empirical cases, the opposite is often observed because of the bias in regressions with persistent predictor variables. Second, even if we attribute the lack of predictability to data or econometric issues, the sign of predictability by changes in correlation contradicts conventional wisdom and what canonical models suggest. High correlation means a small diversification benefit and so the risk premium should be high enough to compensate it. However, we observe lower returns right after an increase in correlation.[19] We

[17]Time-varying CAPM beta estimates of Bitcoin are constructed from DCC-GARCH estimates on conditional correlation and volatilities.

[18]Bitcoin's high volatility can be also interpreted as a subordinated process: Bitcoin behaves as if the clock for Bitcoin runs fast. This interpretation also supports the use of daily returns data for a short period.

[19]A level of correlation tends to be high when changes in correlation is high because $cov(\rho_t, \Delta\rho_{(t-1):t}) = cov(\rho_t, \rho_t -$

provide a simple explanation for this puzzle in the next section while discussing more on this matter in Section 6.5.

3 Asset Allocation with A Mean-Variance Perspective

The previous section shows that Bitcoin return predictability looks less puzzling when we consider the following: (1) institutional investors' daily Bitcoin demands from portfolio optimization and (2) trading practices which are absent in traditional economic models. This section provides a simple intuition on the predictability and its sign. Then we show how to construct an empirical proxy for such a Bitcoin demand due to time-varying correlations. While doing so, we also confirm empirically that Bitcoin provides diversification benefits to passive institutional investors.

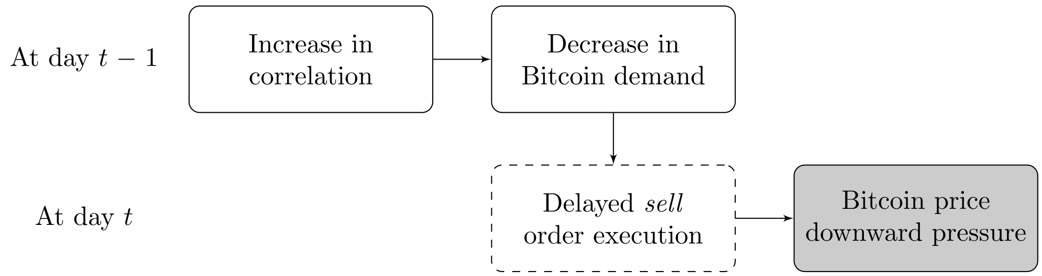
3.1 Intuition: Asynchronous Portfolio Rebalancing as Trading Practice

Our intution is simple. Suppose a *passive* fund manager revises her estimates on inputs for portfolio optimization at market close. Then she requests in-house traders or external trading firms to rebalance her portfolio. The traders in practice rarely execute orders immediately. Instead, they split and delay orders during the next trading session while hoping for better execution prices without revealing the fund manager's intentions (Kyle 1985, Admati and Pfleiderer 1989).[20] At an aggregate level, such delayed rebalancing demands are highly correlated across passive investors who observe the same public information such as historical prices. Therefore, an increase in correlations today suppresses the demand for Bitcoin next day and Bitcoin prices. Likewise, the same intuition can explain Bitcoin price's upward pressure next day if today's correlation decreases. This simple intuition explains the puzzling sign of Bitcoin return predictability which is observable to econometricians but ignorable to traders who face huge intra-day volatility. This one-day time lag in rebalancing is so negligible at monthly frequency that conventional empirical tests of canonical asset pricing models often overlook this short-term predictability.[21] The following flowchart visualizes the intuition.

$\rho_{t-1}) = var(\rho_t) - cov(\rho_t, \rho_{t-1}) = var(\rho_t)(1 - cor(\rho_t, \rho_{t-1})) > 0$

[20] See an article related to this Bitcoin trading behavior at https://yhoo.it/3i9ctPO.

[21] See discussion in Section 6.5 for the validity of assumption on daily rebalancing.



This intuition on the daily return predictability is supported by not only the theories in market micro-structure literature but also rich empirical evidence. Stealth trading is prevalent in the market (Barclay and Warner 1993, Chakravarty 2001), order imbalance predicts daily returns (Chan and Fong 2000, Barber et al. 2008), and institutional investors' behaviors create return autocorrelation at daily level (Sias and Starks 1997, Keim and Madhavan 1995). Furthermore, this learning-(asynchronous)-rebalancing mechanism is empirically valid even if investors rebalance less frequently or our Bitcoin demand proxy is incomplete and noisy.

Our intuition is more than a plausible story. It produces multiple predictions that are empirically testable. Section 5 investigates the predictions in detail. Finally, we formalize this intuition and develop a static model of Bitcoin prices in Section 6.

Alternative explanation #1: time-varying risk premium The Bitcoin return predictability puzzle is difficult to explain by the time-varying risk premium in a traditional asset pricing framework with investors' learning. First, it is empirically and conceptually problematic to construct an economically meaningful proxy for risk premium at daily frequency. Even if we can, market micro-structure noises dilute or dominate the effect of time-varying risk premium implied by equilibrium pricing models. Yet, using monthly returns leaves us with only 36 observations because we have only three years of data since the correlations start to fluctuate. Second, more importantly, Bitcoin does not belong to a traditional economic framework. Diversity of Bitcoin pricing models in the literature is a proof. Numerous non-standard components are used in the recent Bitcoin pricing literature, e.g., gradual adoption, Bitcoin mining cost, payment transaction volume, size of network, halving, and sentiments.[22] If we would build a dynamic equilibrium model with time-varying risk premium, ignoring these exotic forces is unrealistic, yet unifying all of them is challenging. We provide relevant discussion in Section 6.5.

[22]See Athey et al. 2016), Biais et al. (2020), and Cong et al. (Forthcoming) among many others.

Alternative explanation #2: time-varying sentiment One might be tempted to suggest that changes in correlation is a proxy for other variables such as investors' sentiment level. This type of conjecture is hard to prove or disprove, but we find this is unlikely the case. Variables like common sentiments, excessive liquidity, or degree of disagreement are highly persistent at daily frequency, yet changes in correlation are barely serially-correlated, by construction.

3.2 Empirical Demand Proxy: Construction and Decomposition

Our simple intuition is based on the assumption that passive institutional investors recognize Bitcoin as an investment asset that provides additional diversification benefits. Since those investors are often short-sale constrained, they must exclude Bitcoin if the optimal weight on Bitcoin in their portfolios is negative. Therefore, our intuition looses ground if the weights in our sample period are negative most of time. To rule out such a possibility, we first compute the time-varying optimal weights on Bitcoin in a passive portfolio as an empirical proxy for Bitcoin demand. These optimal weights turn out mostly positive implying that passive institutional investors' demands for Bitcoin exist and vary over time. Also, using these weights as an empirical proxy helps us understand and explain the return predictability through a lens of simple mean-variance framework

3.2.1 Construction of a Proxy

Our intuition says that Bitcoin price increases with lagged Bitcoin demands among passive institutional investors. We construct an empirical proxy for such Bitcoin demands to quantify the intuition empirically. Our construction of a demand proxy requires additional assumptions as follows.

Suppose there are mean-variance investors who want to maximize the Sharpe ratio of their investment portfolios.[23] They mix a new asset, Bitcoin, with their existing well-diversified risky portfolio such as stock market portfolios, say, S&P500 index funds. We let $\mu_{b,t}$ and $\mu_{m,t}$ denote the conditional (subjective) risk premia of Bitcoin and the stock market, respectively. Similarly, $\sigma_{b,t}$ and $\sigma_{m,t}$ denote the conditional (subjective) volatilities of Bitcoin and the stock market, respectively.

[23] That is, a riskfree asset exists in the market without a borrowing constraint so that the investors can maximizes their utility function with their own risk aversion by mixing the tangency portfolio and the riskfree asset.

Then the investors' ex-ante optimal weight on Bitcoin in the new tangency portfolio is given by

$$w_{b,t} = \frac{\sigma_{m,t}^2 \mu_{b,t} - \rho_t \sigma_{b,t} \sigma_{m,t} \mu_{m,t}}{\sigma_{m,t}^2 \mu_{b,t} - \rho_t \sigma_{b,t} \sigma_{m,t} \mu_{m,t} + \sigma_{b,t}^2 \mu_{m,t} - \rho_t \sigma_{b,t} \sigma_{m,t} \mu_{b,t}} = \frac{\mu_t^* - \rho_t \sigma_t^*}{(\mu_t^* - \rho_t \sigma_t^*) + (\sigma_t^* - \rho_t \mu_t^*)\sigma_t^*}, \quad (2)$$

where ρ_t is the correlation coefficient between Bitcoin and the stock market, and where $\mu_t^* = \mu_{b,t}/\mu_{m,t}$ and $\sigma_t^* = \sigma_{b,t}/\sigma_{m,t}$ are Bitcoin's risk premium ratio and volatility ratio relative to the stock market, respectively.[24]

We regard this optimal weight $w_{b,t}$ as an empirical proxy for an investors' Bitcoin demand, which is a function of $(\mu_t^*, \sigma_t^*, \rho_t)$, that is, $w_{b,t} = w_b(\mu_t^*, \sigma_t^*, \rho_t)$. Intuitively, $w_{b,t}$ monotonically decreases with ρ_t and σ_t^* within empirically reasonable ranges. We provide formal conditions for demand monotonicity in Section A.2. Note that all the parameter values in the equation above are the investor's point estimates under her subjective belief, rather than true values. Also, we assume that the investors have the anticipated utility (Kreps 1998, Cogley and Sargent 2008) with which an agent sequentially estimates and updates the parameters but treat them as true values at the moment without uncertainty for decision making. Therefore, true parameter values of $(\mu_t^*, \sigma_t^*, \rho_t)$ are irrelevant to proxy construction as long as we can approximate investors' belief on them. Lastly, we emphasize that we are not building an equilibrium model here although the assumptions made above sound similar to common assumptions in asset pricing models. Instead, they serve only for construction of empirical proxies.

3.2.2 Decomposition of a Proxy

Our proxy for Bitcoin demand depends on investors' subjective learning about the inputs for portfolio optimization, that is, $(\mu_t^*, \rho_t, \sigma_t^*)$. In Bitcoin markets, not all investors fully perform Bayesian learning. Furthermore, what investors would learn depends on their investment motives because attention and resources are limited in the real world. If an investor trades Bitcoin for price speculation (speculative investors, hereafter), she would barely care about diversification. The investor rather directs all her attention and resources to learn about μ_t^*, not (ρ_t, σ_t^*). If an investor manages a passive portfolio (non-speculative investors, hereafter), her focus is to learn about (ρ_t, σ_t^*) rather than μ_t^* because she believes that predicting highly volatile Bitcoin's price is a difficult task and

[24] Refer to any college level textbook on investments for (2).

only adds extra errors to the portfolio optimization.

This simple view on two investors with distinct investment motives helps us refine our analysis by decomposing our proxy for Bitcoin demand. First, inspired by speculative investors' motive, we define $w_{b,t}^{(mean)} \triangleq w_{b,t}(\mu_t^*, \overline{\sigma}^*, \overline{\rho})$ as the optimal weight on Bitcoin when investors learn only on μ_t^* while substituting the unconditional estimates $(\overline{\sigma}^*,\ \overline{\rho})$ for $(\sigma_t^*,\ \rho_t)$. On the other hand, $w_{b,t}^{(c+v)} \triangleq w_b(\overline{\mu}^*, \sigma_t^*, \rho_t)$ is the optimal weight on Bitcoin when passive (non-speculative) investors learn only on $(\sigma_t^*,\ \rho_t)$ while substituting the unconditional estimate $\overline{\mu}^*$ for μ_t^*. Next, we decompose the conditional Bitcoin demand $w_{b,t}$ as follows, based on a discrete-time-version analogy to a total derivative of $w_{b,t}$ with respect to (ρ_t, σ_t^*) and μ_t^*.

$$w_{b,t} - \overline{w}_b \approx \underbrace{[w_{b,t}^{(c+v)} - \overline{w}_b]}_{\text{Non-speculative}} + \underbrace{[w_{b,t}^{(mean)} - \overline{w}_b]}_{\text{Speculative}}, \tag{3}$$

where $\overline{w}_b = w_b(\overline{\mu}^*, \overline{\sigma}^*, \overline{\rho})$ is the unconditional Bitcoin demand. The first term is related to diversification benefits whereas the second term is mainly about the subjective outlook on Bitcoin prices. Thus we interpret the first term as non-speculative demand for Bitcoin by investors' learning while the second term as speculative. Next, we can further decompose the first term in Equation (3), according to its source of learning:

$$w_{b,t}^{(c+v)} - \overline{w}_b \approx \underbrace{[w_{b,t}^{(cor)} - \overline{w}_b]}_{\text{from correlation}} + \underbrace{[w_{b,t}^{(vol)} - \overline{w}_b]}_{\text{from volatility}}, \tag{4}$$

where $w_{b,t}^{(cor)} = w_{b,t}(\overline{\mu}^*, \overline{\sigma}^*, \rho_t)$ and $w_{b,t}^{(vol)} = w_{b,t}(\overline{\mu}^*, \sigma_t^*, \overline{\rho})$ are the optimal weights on Bitcoin when investors learns either correlation only or volatility ratio only, respectively. For example,

$$\Delta w_{b,(t-1):t}^{(cor)} = w_{b,t}^{(cor)} - w_{b,t-1}^{(cor)}, \tag{5}$$

where $w_{b,t}^{(cor)} = w_{b,t}(\overline{\mu}^*, \overline{\sigma}^*, \rho_t)$. That is, the first term is the Bitcoin demand due to correlation changes while the second term is due to volatility changes. Later, this further decomposition allows us to empirically identify what passive non-speculative investors mainly learn. Finally, we decompose the Bitcoin demand change $\Delta w_{b,(t-1):t}$ as follows by combining Equation (3) and (4).

$$\Delta w_{b,(t-1):t} \triangleq w_{b,t} - w_{b,t-1} \approx \Delta w^{(cor)}_{b,(t-1):t} + \Delta w^{(vol)}_{b,(t-1):t} + \Delta w^{(mean)}_{b,(t-1):t}, \tag{6}$$

where the last three terms are time differences of the three decomposed optimal weights. Later we find that the correlation coefficient between $\Delta w^{(c+v)}_{b,(t-1):t}$ and $\Delta w^{(cor)}_{b,(t-1):t} + \Delta w^{(vol)}_{b,(t-1):t}$ is 0.99, implying that our Bitcoin demand decomposition based on Equation (4) produces negligible approximation errors.

Our decomposition of Bitcoin demand proxy is built on a simple binary classification of investors. This classification is only conceptual rather than physical. For example, an investor might have both types of motives for her investments. If so, our decomposed proxies captures the total demand for each motive, aggregated across investors.

Is this an empirical proxy or equilibrium model? We stress that we are only building empirical proxies rather than an equilibrium asset pricing model here although our approach relies on several assumptions that are often used in equilibrium models. After all, constructing a demand proxy provides us with a quantitative framework to investigate the Bitcoin return predictability puzzle. In this section, we confirm that (1) passive investors are willing to include Bitcoin in their well-diversified portfolio and (2) Bitcoin's high volatility helps us detect the return predictability. The other benefits of our Bitcoin demand proxy are explained in the remaining sections.

3.3 Proxy Estimation and Evidence for Diversification Benefits

Speculative investors only care about Bitcoin valuation based on their private information, subjective interpretation, and a Bitcoin pricing model of their own choice. We have no access to speculative investors' information sets or subjective belief. Therefore, constructing a demand proxy for speculative investors is challenging. Instead, we focus on the non-speculative Bitcoin demand in Equation (4) due to learning of correlations and volatilities.

We estimate (ρ_t, σ^*_t) using a multivariate GARCH model, DCC(1,1)-GARCH(1,1), as described in Section 2. Therefore, our approach of proxy estimation is based on two assumptions. First, we restrict non-speculative investors' information set to historical returns of Bitcoin and the market portfolio. This is the smallest possible information set to estimate (ρ_t, σ_t^*); this parsimonious approach is the most popular among both academics and practitioners. Furthermore, the approach is immune to manipulation; it prohibits us from cherry-picking certain information set. Second, we assume that non-speculative investors believe the multivariate GARCH model used in our proxy construction is a true data generating model. Yet, we find that a choice of model does not alter our results and conclusion.

By contrast, non-speculative investors do not learn about the conditional risk premium ratio μ_t^*. The signal-to-noise ratio for a mean parameter is generally large because of its statistical nature. Furthermore, since a mean parameter estimate usually follows a Normal distribution, the ratio of two mean parameter estimates follows a Cauchy distribution whose mean and any other moments are indefinite. These challenges also applies to non-speculative investors, let alone econometricians. Therefore, it is reasonable to assume that non-speculative investors do not learn about μ_t^* at daily frequency. Throughout this paper, we simply fix $\overline{\mu}^*$ at a specific value, $\overline{\mu}^* = 1$, to avoid cherry-picking an arbitrary value.[25]

In fact, $\overline{\mu}^* = 1$ means that Bitcoin's risk premium is as low as the market risk premium. Since Bitcoin has high volatility and its fundamental value is extremely uncertain, no investors will purchase Bitcoin when they believe Bitcoin has lower risk premium than the market portfolio. $\overline{\mu}^* = 3.26$ means that Bitcoin's Sharpe ratio is as high as that of S&P500 for the post-futures period. Since the CAPM implies that the market portfolio has the highest Sharpe ratio, reasonable values for $\overline{\mu}^*$ should lie in this range $[0, 3.26]$. Furthermore, we suspect that passive investors are so cautious and conservative that their belief on $\overline{\mu}^*$ is in the lower side of the range.

Other unconditional estimates $(\overline{\sigma}^*, \overline{\rho})$ are fixed at their medians of (ρ_t, σ_t^*) from the multivariate GARCH estimation, respectively. Since we eventually use changes in demand as a main predictor rather than its level, our main results remain practically invariant to our choice on ($\overline{\mu}^*$, $\overline{\sigma}^*$, $\overline{\rho}$) values and multivariate GARCH models, as shown in Section 4.4.1.

Non-speculative investors' demand proxy $w_{b,t}^{(c+v)}$ quantitatively proves that Bitcoin improves diversification benefits even when the correlation becomes overall positive and time-varying since late 2017. First, we compute the optimal weights on Bitcoin by mixing Bitcoin and a S&P500 index fund to maximize Sharpe ratio. Then Figure 3 Panel (a) shows that the proportion of

[25]We show later that our empirical results are not sensitive to $\overline{\mu}^*$.

positive (daily) optimal weights $w_{b,t}^{(c+v)}$ on Bitcoin, for each value of $\overline{\mu}^*$. That is, each point on the graph is based on a unique value of $\overline{\mu}^*$ in the horizontal axis. The proportion starts with 84% at $\overline{\mu}^* = 1$ (the most conservative estimate on risk premium in Bitcoin) and gradually increases with $\overline{\mu}^*$ until it reaches almost 100% at $\overline{\mu}^* = 2$. The median optimal weight $w_{b,t}^{(c+v)}$ at $\overline{\mu}^* = 1$ is about 2.5% and increases with $\overline{\mu}^*$. Positive optimal weights on Bitcoin imply that Sharpe ratio is improved and the efficient frontier moves up by including Bitcoin in the portfolio.

Figure 3 Panel (b) shows the median and (25^{th}, 75^{th}) percentiles of daily ex-ante Sharpe-ratio-improvement metric for the post-futures period with $\overline{\mu}^*$ ranging from 1 to 3.26.

$$\text{(Sharpe ratio improvement)} = \frac{SR_t(r_{p,t+1})}{SR_t(r_{m,t+1})} - 1,$$

The bottom 25^{th} percentile of this metric is positive even at $\overline{\mu}^* = 1$ and increases with $\overline{\mu}^*$. Unlike the existing literature, we compute ex-ante Sharpe ratios rather than ex-post versions based on realized excess returns. We find that Sharpe ratios computed by ex-post returns are unreliable because of Bitcoin's dramatic price swings; by contrast, our results are robust to different choices on sample period.

Finally, Figure 5 shows the decomposed *excess* non-speculative demands $w_{b,t}^{(cor)} - \overline{w}_b$ and $w_{b,t}^{(vol)} - \overline{w}_b$ in the post-futures sample. Bitcoin price levels appear positively associated with the excess non-speculative demand due to correlation changes, but not due to volatility ratio changes. Summary statistics can be found in Table A.2.

4 Bitcoin Return Predictability at Full Scale

Using our decomposed demand proxies, we present a list of empirical results in Bitcoin return predictability: Granger causality, in-sample estimations with linear models, variable comparisons with machine learning methods, out-of-sample predictions, and robustness tests. All results confirm the Bitcoin return predictability truly exists.

4.1 Data

We download Bitcoin and S&P500 daily closing price data from *coinmarketcap.com* and *investing.com*, respectively.[26] Then, we combine multi-day Bitcoin returns to one-period return whenever the US stock market is not traded. We select the data from January 2015 to December 2020 (henceforth, 'full sample') since our focus is traditional investors rather than early adopters. As a result, the full sample includes 1,511 daily returns of Bitcoin and S&P500 index. Similarly, we obtain NYSE composite and NASDAQ composite indexes plus international equity indexes, ranked by nominal GDP as of 2019: Shanghai Composite (China), Nikkei 225 (Japan), BSE Sensex (Germany), FTSE 100 (India), DAX (UK), CAC 40 (France), and FTSE Italia All-Share (Italy) indexes.[27] We adjust the international data according to their daily foreign exchange rates from the futures markets.[28] Other cryptocurrency Ethereum (ETH) and Ripple (XRP) are downloaded from *coinmarketcap.com*. In addition, SKEW, VIX, and S&P500 trading volume data are acquired from *investing.com*.

To isolate the effect of learning, we collect other variables that are known to predict Bitcoin returns in literature (e.g., Liu and Tsyvinski 2021): USD index returns and gold returns from *investing.com*, daily treasury yield rates from the US Department of Treasury, blockchain-related attributes including total hashrates, global block difficulty, unique address count, total Bitcoin quantity, unique transactions count, and trading volume on major Bitcoin exchanges from *blockchain.com*, Wikipedia Bitcoin pageviews from Wikipedia *Pageviews Analysis*, and Google search trends for Bitcoin from *trends.google.com*. The Economic Policy Uncertainty Index data based on daily news are obtained from *policyuncertainty.com*. See Table 4 for the complete list of predictors we consider.

[26]The US stock market closes at 4pm Eastern Standard Time (EST) whereas the Bitcoin market closes at 12am Coordinated Universal Time (UTC). Bitcoin closing price is recorded 3 or 4 hours later than the S&P500 index on a given day depending on daylight saving. For simplicity, we ignore this time difference and use closing price of both assets in our analyses.

[27]International equity market have different daily closing times. We use forward market open index levels for China's and Japan's markets while using closing index levels for other equity markets, to mitigate non-synchronous data issue.

[28]We use forex open exchange rates for Chinese Yuan, Japanese Yen, European Euro, and British Pound whereas forex close exchange rates for Indian Rupee to mitigate non-synchronous data issue.

4.2 In-Sample Predictability

4.2.1 Time-Dependency in Returns and Granger Causality Tests

To lay the groundwork, we first investigate how lagged Bitcoin and stock market returns predict Bitcoin returns. Each column of Table 3 Panel A shows OLS (ordinary least squares) estimates on $a_1^{(L)}$ and their Newey-West t-statistics of the following univariate predictive regression for the post-futures sample:

$$r_{b,t+1} = a_0 + a_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns and z_{t+1-L} is a lagged independent variable: Bitcoin demand changes due to correlation $\Delta w_{b,(t-L):t}^{(cor)}$ (standardized), Bitcoin returns $r_{b,t}$, or S&P500 returns $r_{m,t}$ for $L = 1, ..., 4$. We observe that the coefficient is large (0.51) and significant (t-statistics = 4.15) only for $\Delta w_{b,(t-1):t}^{(cor)}$, that is, the lagged Bitcoin demand change due to correlations with lag $L = 1$. The coefficient reduces to less than its half (0.21) at the second lag $L = 2$ but its size is still the second largest among all in Panel A. The decreasing pattern in coefficients is consistent with the sequential learning effect that gradually fades.

We conduct the Granger causality tests (Granger 1969) in multivariate time-series setting:

$$r_{b,t+1} = a_0 + \sum_{L=1}^{4} a_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

Table 3 Panel B reports three different regression results. First, we include lagged Bitcoin demand changes due to correlations and lagged Bitcoin returns. Second, we add lagged S&P500 returns to the first case. Finally, we repeat the second case using the weighted least squares (WLS) with weights that equal the inverses of time-varying variance estimates from DCC(1,1)-GARCH(1,1). We confirm that the same pattern as the univariate regressions in Panel A appears in all cases of the Granger causality tests in Panel B. Lagged Bitcoin returns and lagged S&P500 returns fail to predict Bitcoin returns in any case.

4.2.2 Predictive Regressions

Inspired by the learning-rebalancing mechanism, we express Bitcoin's demand-return relationship in the following predictive regression model:

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t} + Z_t \gamma + \varepsilon_{t+1}, \tag{7}$$

where b_1 and b_2 measure how subsequent Bitcoin price reacts to the previous Bitcoin demand changes due to learning of correlation and volatilities, respectively. We include a large set of control variables Z_t to mitigate endogeneity issues, if any, in the predictive regression. Note our main predictor $\Delta w^{(cor)}_{b,(t-1):t}$ is a time difference and almost serially-uncorrelated; the first-order autocorrelation is -0.06 for the post-futures sample (-0.01 before COVID-19). Thus, our predictive regression is free from the common bias in predictive regressions (Stambaugh 1999).

Table 5 is the main table of the paper. The table shows that a one-standard-deviation increase in the daily Bitcoin-demand-change $\Delta w^{(cor)}_{b,(t-1):t}$ (due to learning about correlation) predicts about 0.51% higher Bitcoin returns next day in the post-futures sample.[29] R^2 in a univariate regression is 1.15% at daily frequency, which is larger than $R^2 = 1.05\%$ by $\Delta\rho_t$. The coefficient b_1 estimates remains stable around 0.5 and statistically significant in various specifications: with control variables, with rank-based estimation, without the sample after the COVID-19 outbreak, and with different stock market proxies (Table 7).

In contrast, Bitcoin-demand-changes due to learning about volatilities fail to predict Bitcoin returns. The coefficient b_2 estimates are small (ranging from 0.03 to 0.06 in the post-futures period) and not statistically significant. R^2 is only 0.01% in a univariate regression. Furthermore, time-varying daily CAPM β_t, constructed by DCC-GARCH estimates, barely affects next-day Bitcoin returns. The coefficient on lagged Bitcoin returns is small, -0.42, and insignificant (t-statistics between -1.74 and -1.79). While Bianchi and Dickerson (2019) show that trading volume plays a role in predicting Bitcoin returns, we find that lagged trading volume of Bitcoin appears to negatively predict Bitcoin returns but goes statistically insignificant when Bitcoin attributes controls are included in regression. Economic Policy Uncertainty Index with a coefficient estimate around 0.63 is positively associated with subsequent Bitcoin returns but shy of statistical significance (t-statistics

[29] For this interpretation to make sense, we standardize $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$.

between 1.68 and 1.76). Above all, Bitcoin daily return predictability largely comes from investors' learning about correlation rather than volatilities or other predictors concerned.

4.2.3 Relative Importance of Predictors

Changes in Bitcoin demand due to time-varying correlations certainly predict subsequent daily Bitcoin returns. However, linear regression models might suffer from over-fitting and do not clearly show how strong the predictability evidence is relative to other commonly used predictors.

To address these concerns, we apply several Machine Learning algorithms to fit the data: LASSO, Group LASSO, Elastic Net, Random Forest (RF), Gradient Boosting Machine (GBM), and Neural Network with one hidden layer (NN1). In particular, the last three algorithms consider non-linearity and all possible interactions among predictors. Table 4 shows a list of all predictors simultaneously used in training. To avoid over-fitting, we perform a five-fold cross-validation to tune hyper-parameters for each algorithm by a grid search.[30]

To quantify the importance of each predictor, we compute a reduction in R^2, MSE (Mean-Squared-Errors), or MAE (Mean-Absolute-Errors) when a given predictor is set to zero in a trained model, following Gu et al. (2020). We report only the results based on R^2 since all three measures produce very similar outcomes in importance metrics. Two heat maps in Figure 4 visualize this variable-importance metric for several machine learning algorithms. In Panel (a), the variable importance metric is scaled by the column-wise sum so that the total sum for each column can be unity. Darker colors represent greater importance in prediction. Panel (b) repeats the Panel (a) but scales the importance metric by normalizing the variable importance metric to be between zero and one for each algorithm.

Finally, Table 6 reports the in-sample importance metric numbers for the most and the second most important predictors for each algorithm using post-futures data. The most important variable (MIV) is dominated by $\Delta w_{b,t}^{(cor)}$ and lagged trading volume; $\Delta w_{b,t}^{(cor)}$ is the top predictor in Group LASSO (GLASSO), tree-based models (RF and GBM), and an artificial neural network (NN1) while lagged trading volume is only dominant in certain linear models, LASSO and Elastic Net.

[30]We tune the shrinkage factors in LASSO, Group LASSO, and Elastic Net. For Random Forest, we tune the feature sampling size and the minimum size of terminal nodes. For Gradient Boosting Machine, we tune the number of trees, the shrinkage factor, the maximum depth of trees, the number of minimum observation in each node, and the fraction of sub-sampling. For Neural Network, we tune the L1 shrinkage factor and the learning rate. For all algorithms, we use a mean absolute error loss function for performance evaluation.

It is noticeable that $\Delta w_{b,t}^{(cor)}$ is always among the top two important predictors whereas all other predictors fall short.

From this Machine Learning exercise, we conclude that our proposed predictor is not only orthogonal to other predictors but also a dominant predictor for daily Bitcoin returns, especially when non-linearity and possible interactions are considered.

4.3 Out-of-Sample Predictability

We perform out-of-sample tests to address potential issues in the in-sample estimation: over-fitting, look-ahead biases, coefficient instability, and investors' learning on parameters in addition to correlations and volatilities. We re-estimate the DCC(1,1)-GARCH(1,1) and fit predictive regression models every trading day in real time to forecast sequentially Bitcoin returns using $\Delta w_{b,(t-1):t}^{(cor)}$ or $\Delta \rho_t$ without any look-ahead bias. We increase the estimation (training) window after initial training with one year of data such that our prediction period starts from the beginning of 2019. In Table 8, we report out-of-sample R^2 statistics, suggested by Campbell and Thompson (2008):

$$R_{OS}^2 = 1 - \frac{SSE(p)}{SSE(h)}, \tag{8}$$

where $SSE(p)$ is the sum of squared forecast errors by our real-time predictive regression model whereas $SSE(h)$ is by the historical average of the past Bitcoin returns as a benchmark forecast. Positive R_{OS}^2 is regarded as evidence for return-predictability beyond that by the benchmark forecast.

While R_{OS}^2 uses historical average returns as a benchmark, Gu et al. (2020) argue that this benchmark is flawed when analyzing individual stock returns with noisy historical averages. To deal with that, they propose a modified out-of-sample performance measure, denoted as R_{OS}^{2*}, that compares prediction against a forecast value of zero:

$$R_{OS}^{2*} = 1 - \frac{SSE(p)}{SSE(0)} \tag{9}$$

In other words, $SSE(0)$ in R_{OS}^{2*} uses flat zero as a benchmark forecast rather than historical average returns. In our test sample, R_{OS}^{2*} is lower than R_{OS}^2, which means the historical average performs worse than *zero as a forecast* for Bitcoin returns.

In general, R^2_{OS} is typically lower than in-sample R^2. We find the same pattern in the ordinary least-squares estimation (OLS) in Table 8. However, our robust alternatives (WLS, LAD and Rank regressions) even outperform the in-sample predictions. Even though R^{2*}_{OS} is generally lower than R^2_{OS}, it largely outperforms in-sample R^2 for median and rank-based estimation (LAD and Rank). Therefore, concerns about potential over-fitting and coefficient instability are well cleared. Also, outperforming out-of-sample forecasts suggest that investors may sequentially learn about the parameters in the time-series model in addition to the time-varying correlations and volatilities. Strong out-of-sample predictability by robust linear regression models also suggests that our return predictability is not driven by outliers or influential points since such methods are less sensitive to the extreme observations.

We find that the predictability is stronger with globally aggregated Bitcoin demand weighted by GDP, producing $R^{2*}_{OS} = 1.74\%$ at daily frequency with LAD regressions.[31] Excluding the COVID-19 period sample barely changes the predictability. This out-of-sample predictability evidence is impressive even to practitioners. Campbell and Thompson (2008) argue that even a small R^2_{OS}, such as 0.5% at monthly frequency, can deliver economically meaningful return predictability to investors, let alone at daily frequency. Therefore, this out-of-sample Bitcoin return predictability offers a valuable trading opportunity to investors in practice.

4.4 Robustness Tests

4.4.1 Bitcoin Demand Construction

We derive non-speculative demand for Bitcoin in Section 3 relying on a few approximations. First, we use $\mu^*_t = \frac{\mu_{b,t}}{\mu_{m,t}} = 1$ for simplicity but it can be actually any value between 1 and $\frac{\sigma_{b,t}}{\sigma_{m,t}} = 3.26$ from the inequality (A.4). Second, computing correlation-related conditional demand $w^{(cor)}_{b,(t-1):t}$ requires $\overline{\sigma}^*$ which we replace by the sample median of time-varying volatility ratios from the DCC(1,1)-GARCH(1,1) estimation. Similarly, computing volatility-ratio-related conditional demand $w^{(vol)}_{b,(t-1):t}$ needs $\overline{\rho}^*$ which we replace by the sample median of time-varying correlation coefficients from the DCC(1,1)-GARCH(1,1) estimation.

[31] We align observations from global aggregate predictors with those from S&P500-based predictor such that predictability between S&P500-based and global-based predictors can be compared.

Figure A.2 Panel (a) displays how the Newey-West t-statistics of the coefficients b_1 and b_2 in the predictive regression $r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t} + e_{t+1}$ will vary with different choices on μ^*_t. The Newey-West t-statistics of the coefficient b_1 are stable around 4 even with different μ^*_t values ranging from 0 to 3.26 whereas the Newey-West t-statistics of the coefficient b_2 are always below 1 and mostly close to zero. Additionally, we repeat the same procedure the using different values for $\overline{\sigma}^*$ and $\overline{\rho}^*$. The Newey-West t-statistics of the coefficient b_1 are almost invariant even if we replace $\overline{\sigma}^*$ by 25% or 75% percentiles of σ_t from the DCC-GARCH estimation. On the other hand, the Newey-West t-statistics of the coefficient b_2 are still lower than 1 and mostly close to zero even if we replace $\overline{\rho}^*$ by -0.5 or $+0.5$. Hence, our results are robust for different values of $\overline{\sigma}^*$ and $\overline{\rho}^*$. This is not surprising given that our predictor is the change in demand, not the level.

Figure A.2 Panel (b) repeats the main results in Panel (a) but using different DCC-GARCH models and distribution assumptions. We estimate DCC(1,1)-GARCH(1,1), asymmetric DCC(1,1)-GARCH(1,1) (namely, aDCC-GARCH), and DCC(1,1)-EGARCH(1,1). For distribution assumptions, we explore t-distribution in addition to normal distribution. Similarly to Panel (a), Newey-West t-statistics of the coefficient b_1 are stable around 4 while those of b_2 are less than 1 and mostly close to zero. Therefore, our main findings are robust to various DCC-GARCH and distribution assumptions.[32]

4.4.2 Quantile Regression

We repeat our main predictive regression with quantile objective functions to confirm robustness against non-normality and extreme observations. The quantile regression satisfies the following:

$$Q(\tau) = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t}$$

where $Q(\tau)$ is $(\tau \times 100)^{th}$ percentile of Bitcoin's daily log returns $r_{b,t+1}$. $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$ refer to non-speculative demand changes due to correlation and volatility ratio changes from $t-1$ to t, respectively. When $\tau = 0.5$, we have least absolute deviations (LAD) regressions. Figure A.3 shows how coefficient estimates b_1 and b_2 with their 95% confidence intervals varies depending on the choice of the quantile criteria τ in Panel (a) and (b), respectively.

[32] We do not report here, but adding autoregressive components in the mean function of DCC-GARCH model strengthen our results.

In Figure A.3, the coefficient on $\Delta w^{(cor)}_{b,(t-1):t}$, i.e., b_1, varies around the previous OLS estimate 0.5 while keeping its statistical significance for the most part, except $\tau < 0.1$. On the other hand, the coefficients on $\Delta w^{(vol)}_{b,(t-1):t}$, i.e., b_2, are rather flat around zero and insignificant over all quantiles. The quantile regression results confirm robustness of Bitcoin return predictability.

4.4.3 Trimming and Winsorizing

Bitcoin returns are highly volatile and fat-tailed. A few extreme observations can distort simple OLS estimates. Here, we trim or winsorize the data to see how the extreme values affect our results in the predictive regression $r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1}$. For trimming, we omit observations whenever $r_{b,t+1}$ is outside the interval $(c, 100-c)$ percentiles. For winsorizing, we replace both dependent and independent variables outside their own intervals $(c, 100-c)$ percentiles by their nearest boundary values. Then we fit the predictive regression and visualize the results for each cutoff point c (ranging from 0 to 5 percentiles) in Figure A.4 and A.5, for the post-futures period with and without the COVID-19 era samples. In Figure A.4, we observe that the coefficient b_1 from trimmed data is around 0.5 and statistically significant even if we omit total 10% of the sample at tails. In Figure A.5, the coefficient b_1 from the winsorized data even increases with the cutoff c.

4.4.4 OOS Binary Prediction - Confusion Matrix

As another robustness check, we provide a confusion matrix and related tests for a binary classifier (price up/down) rather than regressions. With the intuition behind our Bitcoin demand proxy, we create a naive classification rule: predict 'up' if $\Delta w^{(cor)}_{b,(t-1):t} > 0$ and 'down' otherwise. We re-estimate the DCC(1,1)-GARCH(1,1) every trading day to construct $\Delta w^{(cor)}_{b,(t-1):t}$ in a truly out-of-sample fashion. Note that classifier is not trained from forecast errors for better return predictability. We intentionally avoid optimizing the prediction and construct a conservative classifier to test robustness of our return predictability. We only use DCC-GARCH estimation results to construct $\Delta w^{(cor)}_{b,(t-1):t}$. The evaluation sample is from 01/02/2019 to 12/31/2020 and we sequentially increase the training sample period.

In Table A.5 Panel A, the confusion matrix shows that Sensitivity = 0.615, Specificity = 0.496, Positive Predicted Value (PPV) = 0.589, Negative Predictive Value (NPV) = 0.523, and Accuracy (ACC) = 0.560, which indicates that the binary prediction is informative. If the classifier is purely random, both 'Sensitivity + Specificity' and 'PPV + NPV' should be approximately one in population. Panel B shows that the 95% confident intervals for both 'Sensitivity + Specificity' and 'PPV + NPV' have the lower bound at 1.025, which is greater than 1, also suggesting that the classifier is significantly informative.

We perform three formal tests for the binary classifier: Fisher's Exact test, Chi-square test, and Wilcoxon Rank-sum Test.[33] In all three tests, no p-value exceeds 5%, suggesting that the classifier well predicts the Bitcoin price movement direction at 5% significance level. As a sensitivity check, we try different cutoff values for the classifier between -0.002 and 0.002, which are the 15^{th} and the 80^{th} percentiles of $\Delta w^{(cor)}_{b,(t-1):t}$, as shown in Figure A.6 Panel (a). We also try different ending dates of the evaluation period, as in Panel (b). The p-values of all three tests in both panels fluctuate but do not exceed 5%. All the evidence with this binary classifier supports Bitcoin return predictability from $\Delta w^{(cor)}_{b,(t-1):t}$.

5 Testable Predictions of Learning-Rebalancing Explanation

The previous section validates the evidence of Bitcoin's puzzling return predictability from various angles. The remaining question is whether passive investors' learning and asynchronous rebalancing actually create daily return predictability of Bitcoin as suggested in Section 3.1. To answer this question, we provide five groups of testable predictions implied by our explanation on Bitcoin return predictability. This section shows that our decomposed demand proxies can uncover relevant empirical evidence that conditional correlation estimates alone cannot reveal.

5.1 Prediction #1: Global Equity and Cryptocurrency Markets

Bitcoin is not limited to the US investors. Any international investors who optimize their portfolios using Bitcoin and their own equity markets contribute to Bitcoin demand. This global aspect of Bitcoin provides two testable predictions: (1) Bitcoin's return predictability should appear also with international equity markets if passive investors' learning followed by rebalancing is the main reason for the Bitcoin predictability shown with the US equity markets, and (2) if such learning

[33] Fisher's Exact test and Chi-square test share the null hypothesis H_0: true (predicted) positives and true (predicted) negatives are equally likely to be predicted (true) positives. Wilcoxon Rank-sum Test has the null hypothesis H_0: the median return of the predicted positives is equal to the median return of the predicted negatives.

and rebalancing affect Bitcoin prices through Bitcoin demand, return predictability should appear stronger when the Bitcoin demand proxy is constructed from the aggregated global Bitcoin demand instead of the US demand only. This second prediction differentiates the first prediction from a statistical artifact of correlated global equity markets. On the other hand, Bitcoin investors often include other cryptocurrencies in their portfolios for diversification within the cryptocurrency class. Hence, Bitcoin demand is naturally correlated with other cryptocurrency demands. If our demand proxy truly captures such a common component among cryptocurrency demands rather than idiosyncratic ones, our Bitcoin demand proxy should also predict other cryptocurrency returns.

Global Equity Markets We test the first prediction by repeating our analysis in Section 4.2.2 with seven major global equity markets ranked by GDP in 2019: Shanghai Composite (China), Nikkei 225 (Japan), BSE Sensex (Germany), FTSE 100 (India), DAX (UK), CAC 40 (France), and FTSE Italia All-Share (Italy) indexes. International evidence turns out comparable to the domestic one. In Table 7, the median R^2 of the eight countries (including the US) is 1.04% which is slightly lower than 1.15% from S&P500. The median coefficient is 0.49, close to 0.51 from S&P500. The median Newey-West t-statistics is 2.70, smaller than 4.15 from S&P500 possibly due to higher volatilities in global equity markets. International evidence here confirms that our learning-rebalancing mechanism is a universal phenomenon, not a lucky outcome only in the US.

Globally Aggregated Demand Our analysis also confirms the second prediction. We aggregate the total Bitcoin demand changes of above eight countries using four different weighting schemes, as explained in Table 7. Note aggregating Bitcoin demand proxies across countries is conceptually more natural than aggregating correlation.[34] The globally aggregated proxy for demand change produces overall higher coefficients (0.53 to 0.62) vs. 0.51 for S&P500, similar t-statistics (3.55 to 4.91) vs. 4.15 for S&P500, and higher R^2 (1.25% to 1.71%) vs. 1.15% for S&P500.[35] The superior prediction performance of globally aggregated demand proxies proves that the evidence in global equity markets for the first prediction is not a mere artifact of correlated equity markets. Furthermore, we find that the performance is stronger when the demand is aggregated with economically

[34] This is one of the benefits of constructing a demand proxy.

[35] Given that countries have different holidays, at each date a combined predictor is calculated according to available entries and only skipped if no equity markets are open at that date.

sensible weights such as GDP. Also, the globally aggregated demand proxy with GDP outperforms a combination forecast (equal-weighted average of forecasts from individual regressions) implying that this global aggregation is beyond a statistical trick.

Other Cryptocurrencies To test the prediction on other cryptocurrencies, we repeat the predictive regression analysis while replacing the dependent variable (i.e., subsequent daily Bitcoin returns) by Ethereum (ETH) or Ripple (XRP) returns.[36] The median coefficients are 0.68 for ETH and 0.1 for XRP comparing to 0.49 for Bitcoin. The median t-statistics are 2.98 for ETH and 2.82 for XRP comparing to 2.70 for Bitcoin. The median R^2 are 1.18% for ETH and 0.94% for XRP comparing to 1.04% for Bitcoin. All these results suggest that our demand proxy predicts the common components in the cryptocurrency market rather than idiosyncratic parts.[37]

5.2 Prediction #2: Expected Errors in Optimal Portfolio Weights

Section 4.2.2 shows that changes in Bitcoin demand due to learning about volatilities fail to predict Bitcoin returns (See $\Delta w_{b,t}^{(vol)}$ in Table 5). This empirical result implies that investors do not learn about volatilities for portfolio optimization even if our learning-rebalancing mechanism truly generates the Bitcoin predictability. If so, it must be because investors' volatility forecasts are too noisy to be inputs for portfolio optimization.[38]

Investors understand that the calculated portfolio weights are only ex-ante optimal, not ex-post. Even if investors are doing their best in learning time-varying correlations and volatilities, investors' estimates on them can be inaccurate next day when such hidden state variables dramatically change overnight. Such fundamental tracking errors in optimal portfolio add extra volatilities to their portfolios. In fact, this is why a naïve "1/N" equal-weight portfolio often outperforms a portfolio with optimized weights. Therefore, rational investors who understand the impact of this tracking errors would suppress or even ignore noisy rebalancing signals.

We test this prediction by comparing the degrees of time-variation in correlation, volatilities, and optimal portfolio weights. The first two columns in Table 9 show that the estimated volatility ratios

[36] We do not report here due to space limits but Litecoin shows similar results. We exclude Tether because it is a stable coin that lacks uncertainty in our mechanism.

[37] We add extensive robustness tests related to Table 7. For example, we find similar significant results when we construct a cryptocurrency demand proxy using Ethereum and Ripple instead of Bitcoin.

[38] It is well known that return volatilities are time-varying. Therefore we ignore the case in which investors falsely believe volatilities are constant.

σ_t^* is less persistent and much more volatile than the estimated correlations ρ_t. The fitted AR(1) models imply that investors face 3.48 times greater errors in volatility ratio than in correlation, in terms of the forecasting mean-squared-error. Such a difference in errors explodes when it transfers to corresponding errors in optimal portfolio weights. Investors face 30 times greater expected errors in $w_{b,t}^{(vol)}$ than in $w_{b,t}^{(cor)}$, as shown in the last two columns in the table. Given this dramatic difference in expected errors, rational investors would direct all their attention to correlation changes while ignoring changes in volatility ratio. This analysis also confirms that constructing a demand proxy is key to understand Bitcoin return predictability.

5.3 Prediction #3: Bitcoin Demand Proxy vs. Correlation Estimates

If passive investor's learning and rebalancing affect Bitcoin prices through Bitcoin demand, the return predictability should be stronger when the predictor is constructed from the demand proxy rather than correlation estimates. This prediction is true in our analysis in Section 4.2.2, but the evidence is weak because two predictors are highly (negatively) correlated. In this section, we refine the Bitcoin demand proxy and provide stronger empirical evidence consistent with the prediction.

A Conditional Proxy for Bitcoin Demand Changes Section 3.2 defines a proxy for Bitcoin demand changes as follows: $\Delta w_{b,(t-1):t}^{(cor)} = w_{b,t}^{(cor)} - w_{b,t-1}^{(cor)}$ where $w_{b,t}^{(cor)} = w_{b,t}(\overline{\mu}^*, \overline{\sigma}^*, \rho_t)$. Therefore, this proxy is independent of the time-variation of volatility ratio σ_t^*. Thanks to such an unconditional aspect, the resulting proxy is conservative and reliable but loses conditional information in volatility ratio σ_t^*. Investors would ignore a change in volatility ratio because of its noisiness, yet they can still utilize the level of volatility ratio as an input for optimal portfolio weight because estimates on levels are less noisy than changes in such estimates. Hence, we newly define a conditional version of a proxy for Bitcoin demand changes as follows.

$$\Delta w_{b,(t-1):t}^{(cor|vol)} = w_{b,t}(\overline{\mu}^*, \sigma_{t-1}^*, \rho_t) - w_{b,t-1}(\overline{\mu}^*, \sigma_{t-1}^*, \rho_{t-1}) \tag{10}$$

Note the level of volatility ratio is fixed at σ_{t-1}^* at both $t-1$ and t. Therefore, investors still ignore changes in volatility ratio as confirmed in Section 5.2 and do not revise the optimal weight because of potential changes in volatility ratio. However, the overall level of volatility ratio still affects the calculations of optimal portfolio weights. Given the same level of correlation, the optimal portfolio weight depends on the level of volatility ratio.[39]

In-Sample Predictability Comparison Table 10 compares Bitcoin return predictability by three different predictors. The first column shows the result by changes in correlation $\Delta\rho_t$ from Table 2. The second and third columns show the results by original unconditional proxy $\Delta w^{(cor)}_{b,(t-1):t}$ in Equation (5) from Table 5 and by the new conditional proxy $\Delta w^{(cor|vol)}_{b,(t-1):t}$ in Equation (10), respectively. As expected by the learning-rebalancing channel, the R^2's of three univariate regressions are in the right sequence: 1.05% (changes in correlations), 1.15% (unconditional proxy), and 1.59% (conditional proxy). The next four columns show that the conditional proxy makes other two predictors insignificant when they are included together in multivariate regressions. The last column shows the Pearson correlation coefficients between three predictors. The original unconditional proxy is highly negatively correlated with changes in correlations, $Corr(\Delta\rho_t, \Delta w^{(cor)}_{b,(t-1):t}) = -0.99$. Therefore, they do not differ much in return predictability. However, the new conditional proxy is less correlated with changes in correlations, $Corr(\Delta\rho_t, \Delta w^{(cor|vol)}_{b,(t-1):t}) = -0.93$, so, return predictability can be greatly improved.

Out-of-Sample Predictability Comparison Table 11 shows out-of-sample return prediction performances by three predictors: $\Delta\rho_t$, $\Delta w^{(cor)}_{b,(t-1):t}$, and $\Delta w^{(cor|vol)}_{b,(t-1):t}$. We repeat Table 8 using two robust linear models: least absolute deviation (LAD) and Rank regressions. Consistent with the in-sample evidence in Table 10, the R^2's of univariate predictive regressions remain in the right order: $\Delta\rho_t$ generates the smallest R^2 while $\Delta w^{(cor|vol)}_{b,(t-1):t}$ does the largest. This pattern is universal whether LAD or Rank is used and whether the proxies are constructed from the US equity markets (S&P500) or globally aggregated equity markets. Out-of-sample R^2 is maximized by Rank regression with the globally aggregated conditional proxy, up to $R^2_{OS} = 3.40\%$ and $R^{2*}_{OS} = 2.29\%$ at daily frequency.[40]

[39]This new proxy is still an approximation. Investors might use a moving average of volatility ratios in the past few days instead. However, we find that the empirical results remain qualitatively similar regardless of how we approximate the overall level of volatility ratio.

[40]See Section 4.3 for the difference between R^2_{OS} and R^{2*}_{OS}.

5.4 Prediction #4: Conditional Predictability of Bitcoin Demand Proxy

The results in Section 5.3 imply that the prediction performance of $\Delta\rho_t$ and $\Delta w^{(cor)}_{b,(t-1):t}$ are conditional on volatility ratio. Those variables predict Bitcoin returns better at certain levels of volatility ratio but worse at other levels. If so, the pattern of such conditional predictability should be consistent with the pattern of Bitcoin demand implied by passive investor's learning-rebalancing mechanism. Using Bitcoin demand proxies, we first show that the learning-rebalancing mechanism theoretically implies relatively weaker predictability when volatility ratio is too high or too low. Then we show that this prediction is empirically true. Furthermore, the Bitcoin demand proxy should be more informative showing stronger return predictability when the proxy is constructed with reasonable values of risk premium ratio $\overline{\mu}^*$. We find that this prediction is also true.

5.4.1 Conditional on Volatility Ratio

Figure 6 Panel (a) shows the impacts of correlation on Bitcoin demand at different volatility ratio levels. We compute the time-series of $\Delta w^{(cor)}_{b,(t-1):t}$ and its sample standard deviation with $\overline{\sigma}^*$ fixed at each σ^*_t for $t = 1, 2, ..., T$. Panel (a) shows the sample standard deviation for each choice of $\overline{\sigma}^*$ in the horizontal axis, expressed as its empirical quantile in the set $\{\sigma^*_t\}^T_{t=1}$. The sample standard deviation of $\Delta w^{(cor)}_{b,(t-1):t}$ measures the sensitivity of Bitcoin demand changes to correlation changes. A small standard deviation means that correlation changes have only a minimal impact on Bitcoin demand changes. Therefore, return predictability should be weak when the standard deviation is small. In Panel (a), the standard deviation is low when volatility ratio is extremely high or low. Therefore, return predictability should be weak in those cases. If so, removing such observations in a predictive regression should improve return predictability.

Figure 6 Panel (b) confirms this prediction empirically. We gradually trim the observations with extremely high or low volatility ratio based on the graph in Panel (a). For example, we first pick a threshold of the standard deviation in the vertical axis of Panel (a). Then remove observations with lower standard deviation than the threshold. As a result, only the observations in the middle of Panel (a) remain for a predictive regression. We repeat this procedure for different thresholds to remove more observations gradually. Panel (b) reports the Newey-West t-statistics of three different predictors $\Delta\rho_t$, $\Delta w^{(cor)}_{b,(t-1):t}$, and $\Delta w^{(cor|vol)}_{b,(t-1):t}$, from univariate predictive regressions with different trimming schemes, i.e., different thresholds.[41] As observations with an extreme volatility ratio are trimmed, the t-statistic and return predictability increase in all three predictors.[42]

The degree of improvement differs in three predictors. For the conditional proxy $\Delta w^{(cor|vol)}_{b,(t-1):t}$, the improvement is almost negligible. This is consistent with our prediction. By construction, the conditional proxy already incorporates the information in Panel (a). Therefore, we should not observe a further improvement in return predictability by removing observations with weak predictability. By contrast, the other predictors $\Delta \rho_t$ and $\Delta w^{(cor)}_{b,(t-1):t}$ show a large improvement in predictability. In particular, the t-statistic of the unconditional proxy $\Delta w^{(cor)}_{b,(t-1):t}$ increases and eventually converges to that of the conditional proxy $\Delta w^{(cor|vol)}_{b,(t-1):t}$. Therefore, the predictability improvement by the conditional proxy in Section 5.3 is not a coincidence but truly driven by the conditional predictability implied by the optimal portfolio weight calculation (Equation 2), as shown in Figure 6 Panel (a).

5.4.2 Conditional on Risk Premium Ratio

Another variable in the optimal portfolio weight is risk premium ratio $\overline{\mu}^*$. Since passive investors belief on $\overline{\mu}^*$ is unobservable and difficult to estimate, we first suggest its reasonable values between 1 and 3.26 (Section 3.3). Then we fix $\overline{\mu}^*$ mostly in this paper to avoid cherry-picking an arbitrary value. We suspect that passive investors are so cautious and conservative that their belief on $\overline{\mu}^*$ is in the lower side of the range. Therefore, the Bitcoin demand proxy should be more informative and show stronger return predictability when the proxy is constructed with reasonable values of risk premium ratio $\overline{\mu}^*$, in particular, in the lower half of the range. We find that this prediction is also true.

Figure 7 shows how return predictability varies depending on an input value of $\overline{\mu}^*$ for optimal portfolio weight calculation. As expected, the conditional proxy $\Delta w^{(cor|vol)}_{b,(t-1):t}$ outperforms the unconditional proxy $\Delta w^{(cor)}_{b,(t-1):t}$ only when $\overline{\mu}^*$ is fixed roughly between 1 and 2, in terms of Newey West t-statistics in Panel (a) and R^2 in Panel (b). On the other hand, the unconditional proxy $\Delta w^{(cor)}_{b,(t-1):t}$ shows stable prediction performances regardless of $\overline{\mu}^*$. The conditional proxy $\Delta w^{(cor|vol)}_{b,(t-1):t}$ extracts

[41] Here, R^2 is not a proper measure for return predictability comparisons, because removing observations alters baseline R^2.

[42] Note trimming here is based on extreme volatility ratios, not on extreme returns. Therefore, the improvement in return predictability is not a result of removing outliers (extreme returns).

information on Bitcoin demand more aggressively. Therefore, the resulting proxy becomes noisy when $\overline{\mu}^*$ is fixed at unreasonable values. By contrast, the original unconditional proxy $\Delta w^{(cor)}_{b,(t-1):t}$ is less efficient in capturing information on Bitcoin demand but more robust to extreme values of $\overline{\mu}^*$ in calculations. Based on these results, we argue that the passive investors belief on $\overline{\mu}^*$ is probably between 1 and 2.

5.5 Prediction #5: Placebo Tests

If the Bitcoin predictability truly arises from our learning-rebalancing mechanism rather than from a statistical artifact, the following placebo tests should be rejected. We confirm such predictions.

5.5.1 Return Predictability for Pre-Futures Sample?

Prior to the Bitcoin futures markets, investors perceive risk in Bitcoin as purely idiosyncratic, and the conditional correlation between Bitcoin and the stock markets is practically fixed at zero (Figure 1 and Table 1). Nevertheless, we can still generate illusory time-varying correlations (the gray region in Figure 1) for the pre-futures sample, by using the DCC-GARCH estimates from the full-sample (or the post-futures sample). This process creates an illusion that allows us to perform a placebo test; Bitcoin return predictability should disappear for the pre-futures sample. It is because investors do not recognize these virtual-correlation-based signals for rebalancing in real time prior to the Bitcoin futures markets (Figure 2).

This hypothesis is tested in Table A.4 in the online appendix. The first three columns (1)-(3) repeat the predictive regressions in Table 5 for the full sample, splitting the independent variables into two pieces so that the coefficient on $\Delta w^{(cor)}_{b,(t-1):t}$ or $\Delta w^{(vol)}_{b,(t-1):t}$ can take different values for two subsample periods. The next three columns (4)-(6) are the placebo tests to predict only for the pre-futures sample using the illusory Bitcoin demand proxies estimated from the full sample. Finally, the columns (7)-(9) repeat the columns (1) and (2) in Table 5 for the post-futures sample as a benchmark.

Table A.4 in the online appendix confirms our hypothesis. The coefficient on $\Delta w^{(cor)}_{b,(t-1):t}$ for the pre-futures sample is small and insignificant, ranging from -0.07 to -0.10 with t-statistics around -0.6. Yet, the same coefficient for the post-futures sample is about 0.5 and statistically significant with t-statistics around 4. On the other hand, The coefficient on $\Delta w^{(vol)}_{b,(t-1):t}$ is small and insignificant regardless of sample periods, ranging from -0.21 to 0.06.

5.5.2 Return Predictability in Stock Markets?

We argue that return predictability in stocks is difficult to find even if we apply the same approach in Section 3 to the stock market returns instead of Bitcoin. First, most of Bitcoin investors also invest in stocks, but only a fraction of stock investors consider Bitcoin seriously and learn their joint dynamics. Thus, the learning-rebalancing channel is less important in the demand for stocks. Second, fluctuating weights on the Bitcoin-stock portfolio affect asset demand asymmetrically because of their market capitalization difference. For example, when the weight on Bitcoin jumps eight times from 0.3% to 2.4%, the weight on stocks only drops from 99.7% to 97.6%. Therefore, the effect of learning should be negligible in stocks, compared to Bitcoin. As expected, we do not find evidence for stock return predictability using our demand proxy. In a univariate predictive regression for S&P500 index returns, the coefficient on $\Delta w^{(cor)}_{b,(t-1):t}$ is 0.11 while its t-statistic is 1.07.

6 A Static Model of Bitcoin Prices

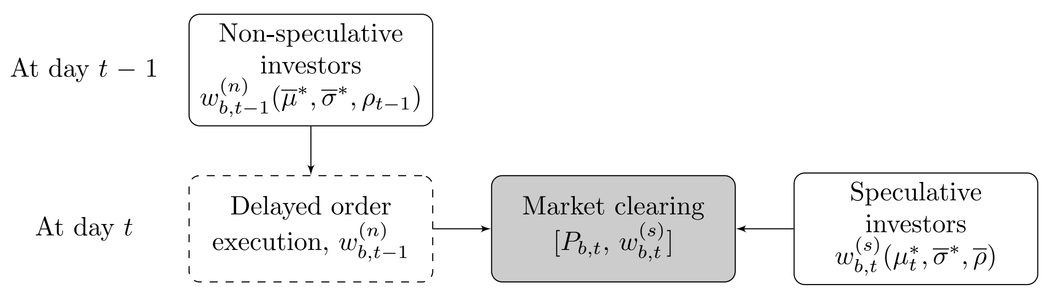
The previous section proves various benefits of constructing an empirical proxy for Bitcoin demand. Our proxy is built on model-like assumptions such as mean-variance portfolio optimizations, yet we stress that the proxy is only to enhance empirical evidence rather than test a theoretical model. In this section, we develop a static model of equilibrium Bitcoin prices, based on our intuitive explanation. The model can explain Bitcoin's return predictability puzzle in Table 2 by producing the observed coefficients in the predictive regressions with the changes and levels of correlation as a predictor.

6.1 Bitcoin Demands and Timeline

Following Section 3, we split investors (or investors' motives) into speculative and non-speculative. Then we interpret both investors' trading decisions in a mean-variance framework to construct Bitcoin demands, similarly to Section 3.2. If an investor trades Bitcoin for price speculation, she would barely cares about diversification. The investor rather directs all her limited attention and resources to learn about the risk premium. If an investor manages a passive portfolio, she believes that predicting highly volatile Bitcoin's price is a difficult task and only adds unnecessary noises to the portfolio optimization. Then she would focus on learning about correlations rather than the risk premium.[43]

The optimal weights on Bitcoin in both types of investors' portfolios are determined by Equation (2) with their own subjective input values, respectively. For non-speculative investors, the optimal weight is denoted by $w_{b,t-1}^{(n)}$ which equals $w_{b,t-1}^{(cor)} = w_{b,t-1}(\overline{\mu}^*, \overline{\sigma}^*, \rho_{t-1})$. In other words, non-speculative investors care about only time-varying correlations. For speculative investors, the optimal weight on Bitcon at day t is denoted by $w_{b,t}^{(s)}$ which equals $w_{b,t}^{(mean)} = w_{b,t}(\mu_t^*, \overline{\sigma}^*, \overline{\rho})$. That is, speculative investors concern about only the risk premium of assets, or equivalently, only the current asset prices given the expected future prices. Therefore, even if speculative investors in the real world do not maximize Sharpe ratio, we can still rely on the mean-variance framework to explain their speculative investment decisions simply by adjusting μ_t^* accordingly.[44]

Unlike the approach in Section 3, the evolution of investors beliefs is disregarded in the equilibrium model. Technically, this is not a learning model but a simple static model. The model starts with a snapshot of investors' beliefs at day $t-1$ as exogenous variables and then completes an equilibrium at day t. The following flowchart visualizes how Bitcoin's market clearing occurs by the two types of investors, as a high-level summary.



The key feature of the model comes from the intuition in Section 3.1: *asynchronous portfolio rebalancing*. A non-speculative fund manager revises her estimate on time-varying correlation at market close. Then the fund manager requests traders to rebalance her portfolio.[45] Instead of

[43]Section 5.2 explains why Bitcoin investors are likely to ignore time-variation in volatilities.

[44]The distinction between two types of investors are not physical. One investors can have both speculative and non-speculative motives, and we can interpret her Bitcoin demand as a sum of the demands of two different agents inside her.

[45]See discussion in Section 6.5 for the validity of assumption on daily rebalancing.

executing orders immediately, the traders in practice split and delay orders during the next trading session while hoping for better execution prices without revealing the fund manager's intentions.[46] Therefore, their optimal weight on Bitcon at day t is $w^{(n)}_{b,t-1}$ rather than $w^{(n)}_{b,t}$, and this is not an error in subscripts. Bitcoin price at day t does not affect non-speculative investors' Bitcoin demand, in terms of portfolio weight $w^{(n)}_{b,t-1}$, which is already set on the previous day.

Unlike our empirical proxy construction, our equilibrium model puts speculative investors into play to induce Bitcoin's market clearing in an equilibrium. Speculative investors' demand, by contrast, is concurrent as they instantly react to the real-time prices. Furthermore, the Bitcoin price is directly linked to speculative investors' subjective Bitcoin risk premium $\overline{\mu}^*$ and thus to their Bitcoin demand $w^{(s)}_{b,t}$. Instead of calibrating their subjective Bitcoin risk premium $\overline{\mu}^*$, we exogenously set speculative investors' subjective expectation on future Bitcoin prices.

6.2 Definition and Calibration of Variables

First, we let $Q^{(s)}_{b,t-1}$ and $Q^{(n)}_{b,t-1}$ denote Bitcoin quantity held by speculative and non-speculative investors at day $t-1$, respectively. Second, to keep notation simple, we normalize both the total Bitcoin quantity and Bitcoin price at day $t-1$ to unity, i.e., $P_{b,t-1} = 1$ and $Q_{b,t-1} = Q^{(s)}_{b,t-1} + Q^{(n)}_{b,t-1} = 1$, respectively. We also set the daily riskfree rates as zero, that is, $R_{f,t-1} = R_{f,t} = 1$ as a gross return.

We calibrate the other exogenous variables in the model, following Section 3.2. Two unconditional estimates $(\overline{\rho}, \overline{\sigma}^*)$ are fixed at (0.0946, 4.993%) which are their medians of (ρ_t, σ^*_t), from the multivariate GARCH estimation, respectively. For non-speculative investors, we set $\overline{\mu}^* = 1$ and $\rho_{t-2} = \overline{\rho}$, and the optimal weight $w^{(n)}_{b,t-2} = 2.11\%$ is then determined by $(\overline{\mu}^*, \overline{\sigma}^*, \rho_{t-2})$ through Equation (2).[47] Therefore, non-speculative investors' portfolio value is $A^{(n)}_{t-1} = Q^{(n)}_{b,t-1}/w^{(n)}_{b,t-2}$ since $P_{b,t-1} = 1$. Likewise, the optimal weight $w^{(n)}_{b,t-1}$ is determined by $(\overline{\mu}^*, \overline{\sigma}^*, \rho_{t-1})$.

For speculative investors, we express their portfolio value at day $t-1$ as $A^{(s)}_{t-1} = Q^{(s)}_{b,t-1}/w^{(s)}_{b,t-1}$ in which timing is slightly different from non-speculative investors' case. Then we back out μ^*_{t-1} from Equation (2), given the values of exogenous variables $w^{(s)}_{b,t-1}$ and $(\overline{\sigma}^*, \overline{\rho})$. By setting speculative

[46] See Kyle (1985), Admati and Pfleiderer (1989)), and an article related to this Bitcoin trading behavior at https://yhoo.it/3i9ctPO.

[47] See a WSJ news article at https://on.wsj.com/37clZgh. Workers at participating companies can invest up to 5% of their account balances in cryptocurrency.

investors' belief on daily market risk premium as $\mu_{m,t-1} = 0.06/252$, we can compute their Bitcoin price forecast $E[P_{b,t}|\mathcal{F}^{(s)}_{t-1}]$ from the definition of μ^*_{t-1}:

$$\mu^*_{t-1} = \frac{\mu_{b,t-1}}{\mu_{m,t-1}} = \left(\frac{E[P_{b,t}|\mathcal{F}^{(s)}_{t-1}]}{P_{b,t-1}} - R_{f,t-1} \right) \Big/ \left(E[R_{m,t}|\mathcal{F}^{(s)}_{t-1}] - R_{f,t-1} \right). \tag{11}$$

Next, to shut down other channels that affect Bitcoin prices, we set the realized market portfolio gross return as $R_{m,t} = 1$ and assume that Bitcoin quantity does not change overnight, i.e., $Q_{b,t} = Q_{b,t-1} = 1$. Also, speculative investors' belief on market risk premium and future Bitcoin price remains unchanged from $t-1$ to t, that is, $\mu_{m,t} = \mu_{m,t-1}$ and $E[P_{b,t+1}|\mathcal{F}^{(s)}_t] = E[P_{b,t}|\mathcal{F}^{(s)}_{t-1}]$. The following table organizes main variables in the model.

Day	Non-speculative	Speculative	Markets	
$t-1$	ρ_{t-1}	$w^{(s)}_{b,t-1}$, $Q^{(s)}_{b,t-1}$		Exogenous
	$Q^{(n)}_{b,t-1} = 1 - Q^{(s)}_{b,t-1}$			
	$A^{(n)}_{t-1} = Q^{(n)}_{b,t-1}/w^{(n)}_{b,t-2}$	$A^{(s)}_{t-1} = Q^{(s)}_{b,t-1}/w^{(s)}_{b,t-1}$		
	$w^{(n)}_{b,t-1}$	$E[P_{b,t}\|\mathcal{F}^{(s)}_{t-1}]$		
t		$E[P_{b,t+1}\|\mathcal{F}^{(s)}_t]$	$R_{m,t} = 1$, $Q_{b,t} = 1$	
	$Q^{(n)}_{b,t} = 1 - Q^{(s)}_{b,t}$	μ^*_t, $w^{(s)}_{b,t}$, $Q^{(s)}_{b,t}$		Endogenous
			$P_{b,t}$	

Given the variables in the first row, we can compute the other variables at day $t-1$ and also those in the first row at day t. Therefore, the first five rows show exogenous variables outside an equilibrium since this model explains the equilibrium Bitcoin price only at day t.

6.3 A Static Equilibrium

Bitcoin price $P_{b,t}$ is endogenously determined at day t where two types of investors trade with each other as follows. Given a market price $P_{b,t}$ of Bitcoin, μ^*_t is computed by the day-t version of Equation (11). Then μ^*_t determines speculative investors' weight $w^{(s)}_{b,t}$ via Equation (2). Both types of investors' Bitcoin demands, $Q^{(s)}_{b,t}$ and $Q^{(n)}_{b,t}$, in terms of quantity, are set by Bitcoin demands,

$w_{b,t}^{(s)}$ and $w_{b,t}^{(n)}$, in terms of portfolio weights, given Bitcoin price $P_{b,t}$. Therefore, Bitcoin price $P_{b,t}$ should result in market clearing at the equilibrium:

$$1 = Q_{b,t} = Q_{b,t}^{(s)} + Q_{b,t}^{(n)} = \frac{w_{b,t}^{(s)} A_t^{(s)}}{P_{b,t}} + \frac{w_{b,t-1}^{(n)} A_{t-1}^{(n)}}{P_{b,t}}, \tag{12}$$

where the timing difference between two types of investors comes from non-speculative investors' *asynchronous portfolio rebalancing*. Given the Bitcoin price $P_{b,t}$ and weight on Bitcoin $w_{b,t-1}^{(s)}$, the value of the speculative investor' portfolio evolves as follows.

$$A_t^{(s)} = A_{t-1}^{(s)} \left(w_{b,t-1}^{(s)} \frac{P_{b,t}}{P_{b,t-1}} + (1 - w_{b,t-1}^{(s)}) R_{m,t} \right), \tag{13}$$

By plugging Equation (13) into (12), we rewrite the market clearing condition (12) in the following.

$$\begin{aligned} P_{b,t} &= w_{b,t}^{(s)} A_t^{(s)} + w_{b,t-1}^{(n)} A_{t-1}^{(n)} \\ &= w_{b,t}^{(s)} A_{t-1}^{(s)} \left(w_{b,t-1}^{(s)} \frac{P_{b,t}}{P_{b,t-1}} + (1 - w_{b,t-1}^{(s)}) R_{m,t} \right) + w_{b,t-1}^{(n)} A_{t-1}^{(n)}, \end{aligned} \tag{14}$$

where we set $P_{b,t-1} = 1$ and $R_{m,t} = 1$. We numerically find the equilibrium Bitcoin price $P_{b,t}$ that satisfies the market clearing condition (14).[48]

6.4 Model Implications on Predictive Regressions

Table 12 shows the model-implied coefficient $a_1^{(c)}$ in predictive regressions of Bitcoin returns $r_{b,t+1}$ on $(\rho_t - \rho_{t-1})$, i.e., changes in correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(c)} + a_1^{(c)} (\rho_t - \rho_{t-1}) + \varepsilon_{t+1} \quad \text{where} \quad a_1^{(c)} = \frac{1}{P_{b,t}} E\left[\frac{\partial P_{b,t+1}}{\partial \rho_t}\right] = E\left[\frac{\partial \log P_{b,t+1}}{\partial \rho_t}\right],$$

where we deviate ρ_{t-1} from $\rho_{t-2} = \overline{\rho}$ holding others constant. In Table 12, we compute $a_1^{(c)}$ using a numerical derivative for each combination of $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)}$ ranging from 0.1 to 0.9. The model-implied coefficient $a_1^{(c)}$ is negative regardless of $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)}$ as expected. The pattern in its magnitude is straightforward. Given $w_{b,t-1}^{(s)}$, predictability (or magnitude of the

[48] Alternatively, we can plug the $w_{b,t}^{(s)}$ expression into Equation (14) and solve a polynomial equation by brute force.

coefficient) decreases with speculative investors' share in Bitcoin, $Q^{(s)}_{b,t-1}$. High $Q^{(s)}_{b,t-1}$ means that non-speculative investors are only minor players in Bitcoin markets. Therefore, their demand changes barely affect Bitcoin prices. On the other hand, Given $Q^{(s)}_{b,t-1}$, predictability increases with speculative investors' portfolio concentration in Bitcoin, $w^{(s)}_{b,t-1}$. Low $w^{(s)}_{b,t-1}$ means that speculative investors see relatively low expected returns on Bitcoin. Therefore, small percentage changes in the price can affect more dramatic percentage changes in Bitcoin's expected return and thus in their Bitcoin demand if $w^{(s)}_{b,t-1}$ is low. In other words, speculative investors are willing to absorb non-speculative investors' demand changes even with small price adjustments. Such a behavior of speculative investors leads to relatively weak predictability evidence when $w^{(s)}_{b,t-1}$ is low.

This calibration exercise has the following implications. First, we can check if the observed predictability evidence is quantitatively consistent with our intuition. Since the model in this section is based on our intuitive explanation, we need to raise a red flag if the model cannot replicate the coefficient $a^{(c)}_1$ similar to its empirical estimate. The point estimate on $a^{(c)}_1$ of the regression using our data is -0.154 where its 95% confidence interval implied by Newey-West standard errors is $(-0.236, -0.072)$. Therefore, our model can easily produce the empirical estimate on $a^{(c)}_1$ with reasonable parameter values. For example, calibrating $w^{(s)}_{b,t-1} = 0.6$ and $Q^{(s)}_{b,t-1} = Q^{(s)}_{b,t-1}/Q_{b,t-1} = 0.3$ produces exactly $a^{(c)}_1 = -0.154$. This exercise does not prove that our intuition is an absolute truth, yet it provides a good indirect test for whether the intuition is completely off. Conversely, if this model is true, then we can estimate $w^{(s)}_{b,t-1}$ and $Q^{(s)}_{b,t-1} = Q^{(s)}_{b,t-1}/Q_{b,t-1}$ from the data although additional variables are possibly required for complete parameter identification.

Figure 8 visualizes how changes in non-speculative investors' correlation estimates and also demand can affect Bitcoin returns next day. Inspired by the result of Table 12, we set $w^{(s)}_{b,t-1} = 0.6$ and $Q^{(s)}_{b,t-1} = Q^{(s)}_{b,t-1}/Q_{b,t-1} = 0.3$ and inherit other parameter values from the table. The graph is vertically shifted so that zero return means no change in correlation. Consistent with our intuition, Bitcoin returns decrease with correlation changes but increase with non-speculative investors' demand changes.

We repeat Table 12 by comparing expected Bitcoin returns by different correlation levels, rather than its changes. Table 13 shows the model-implied coefficient $a^{(l)}_1$ in predictive regressions of Bitcoin returns $r_{b,t+1}$ on ρ_t, i.e., the level of correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(l)} + a_1^{(l)}\rho_t + \varepsilon_{t+1} \quad \text{where} \quad a_1^{(l)} = \frac{E[r_{b,t+1}^{(+)}] - E[r_{b,t+1}^{(-)}]}{\rho_t^{(+)} - \rho_t^{(-)}},$$

where $\rho_t^{(+)} = \overline{\rho}^* + SD(\rho_t)$, $\rho_t^{(-)} = \overline{\rho}^* - SD(\rho_t)$, $(\overline{\rho}^*, \rho_t)$ are the unconditional and conditional correlations from the DCC-GARCH estimation, respectively, and $SD(\rho_t) \approx 0.15$. On the other hand, $E[r_{b,t+1}^{(+)}]$ and $E[r_{b,t+1}^{(-)}]$ are the model-implied Bitcoin log returns corresponding to the correlation level $\rho_t^{(+)}$ and $\rho_t^{(-)}$, respectively. To compute $E[r_{b,t+1}^{(+)}]$, we first find $E[\rho_{t+1}|\rho_t^{(+)}]$ from Equation (1). Then compute the equilibrium Bitcoin prices at $t+1$ when $\rho_t = E[\rho_{t+1}|\rho_t^{(+)}]$ and $\rho_t = \rho_t^{(+)}$, respectively, holding others constant. Finally, we have a gross return that equals the ratio of those two prices, and its logarithm is $E[r_{b,t+1}^{(+)}]$. We compute $E[r_{b,t+1}^{(-)}]$ in a similar fashion.

The point estimate on $a_1^{(l)}$ of the regression using our data is practically zero, ($\hat{a}_1^{(l)} = 0.0006$), where its 95% confidence interval implied by Newey-West standard errors is $(-0.0946, +0.0957)$. 80.2% of the model-implied coefficient $a_1^{(l)}$ in Table 13 fall between zero and 0.001. Most importantly, our model can simultaneously generate both $a_1^{(c)}$ and $a_1^{(l)}$ matching their empirical estimates with a common choice of $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)} = Q_{b,t-1}^{(s)}/Q_{b,t-1}$. The model confirms that the correlation level cannot predict Bitcoin returns while its change can. Therefore, these two tables quantitatively support our explanation on the Bitcoin return predictability puzzle.

6.5 Discussion: What is this model about?

Our model is simple but prone to misunderstanding. In this section, we clarify a few important aspects of the model and discuss validity of the model assumptions.

Is this just another variant of CAPM? No, it is not. Both our model and the CAPM exploit an idea of mean-variance portfolio optimization. Such a similarity might mislead readers into regarding the model as a variant of the CAPM. That said, our model has no implications on how the risk premium of an asset should be determined in an equilibrium with respect to its true covariance to the market portfolio. Speculative and non-speculative investors have their own subjective beliefs, which do not have to coincide with the true objective values. Also, the parameters $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)}$ are not easy to estimate or calibrate. Therefore, it is challenging to test this model as we do for other standard asset pricing models. Instead, we show model-implied coefficients to quantitatively validate the intuitive explanation in Section 3.1 for the Bitcoin return predictability puzzle.

Is this a learning model? No, it is not; our model is static. Our intuition in Section 3.1 is built on the idea that investors learn about the joint dynamics of Bitcoin and S&P500. The idea develops into the core assumption in the model: two different subjective learning of speculative and non-speculative investors. However, investors' learning plays no active role in the model. The model is static and starts with investors' beliefs as an outcome of learning. The model neither assumes nor explains how investors have such beliefs on risk premium, volatility and correlation although, by contrast, we rely on the DCC-GARCH model for the empirical proxy construction in Section 3. The only thing that happens within a model is the trading by two types of investors with different beliefs which are given as exogenous variables. That is, our model captures only the snapshot of Bitcoin price in an equilibrium, given other variables, similarly to a simple supply-demand model in a textbook. Therefore, this Bitcoin pricing model in this section is silent on how investors' beliefs evolve in response to shocks and signals. We find that internalizing investors' learning in a dynamic asset pricing model is beyond the purpose of this paper because of the challenges discussed in the last paragraph of Section 3.1. We therefore leave it for future research.

Is it time-varying risk premium or trading practice? Time-varying risk premium is a popular rational explanation for return predictability in the equity markets (Cochrane 2008). However, we argue that the time-varying risk premium may not be the best underlying mechanism for Bitcoin return predictability evidence at daily frequency. First, it is empirically and conceptually difficult to construct an economically meaningful proxy for risk premium at daily frequency. Second, market micro-structure noise can dilute or dominate the effect of time-varying risk premium implied by equilibrium pricing models. Therefore, even if the true risk premium changes every day, it is unlikely to measure it accurately and detect Bitcoin return predictability at daily frequency. Above all, Bitcoin might not belong to a traditional economic framework. Various non-standard components are used in the recent Bitcoin pricing literature, e.g., gradual adoption, Bitcoin mining cost, payment transaction volume, size of network, halving, and sentiments.[49] It is unrealistic to build a

[49]See Athey et al. 2016), Biais et al. (2020), and Cong et al. (Forthcoming) among many others.

model with time-varying risk premium incorporating all these exotic features. Therefore, trading practice of *asynchronous portfolio rebalancing* is arguably a simple answer for the Bitcoin return predictability puzzle.

Is daily rebalancing realistic? It does not matter much. First, we argue that our model can be interpreted in many different ways. The investors in the model equilibrium can be only a subset of investors. For example, non-speculative investors can be divided into h different subgroups. Then the investors in each subgroup rebalance their stock-and-Bitcoin portfolios every h trading days so that only one subgroup can rebalance on a given day. Our model works exactly in the same way even if we change our assumption on non-speculative investors this way, or even if we let non-speculative investors have heterogeneous or even random rebalancing frequencies without altering the model outcomes. Likewise, speculative investors can periodically or randomly participate in the market. Such changes in assumptions affect only the dynamic aspects of the model if any, yet our model is static. Therefore, those alternative assumptions only result in a different interpretation on the model: the model is about the investors who participate in trading on a given day rather than all investors. Therefore, the quantitative implications remain the same in the tables and figures. Furthermore, this alternative assumption on less frequent rebalancing explains why non-speculative investors do not mind one-day lag between rebalancing decisions and following order executions.

Can investors trade during a day or only at closing? Yes, they can do so without altering our model implications. Non-speculative investors (or their traders) aim to match the dollar amount of Bitcoin in the portfolios to $w_{b,t-1}^{(n)} A_{t-1}^{(n)}$, which is predetermined at $t-1$ independently of the Bitcoin closing price $P_{b,t}$ at t, when they complete the last order at closing of day t. That is, non-speculative investors are free to trade during a day. Likewise, speculative investors also can freely trade during a day. Since the non-speculative investors will complete their final order at closing, variables such as $P_{b,t}$, μ_t^*, $w_{b,t}^{(s)}$, and $Q_{b,t}^{(s)}$ are also endogenously determined in the equilibrium at closing on day t. Therefore, whether investors can trade during a day does not affect our model outcomes.

7 Conclusion

The image of Bitcoin has transformed many times since its inception.[50] The recent COVID-19 episode only adds to it and we expect more in the future. Also, Bitcoin opens a gate for new class of assets, e.g., NFT (non-fungible token). Therefore, it is important to understand how a new narrative affects investors' behaviors and market equilibriums (Shiller 2020).

We present two empirical findings regarding this issue: (1) the ex-ante diversification benefit of Bitcoin and (2) strong and robust Bitcoin return predictability that contradicts conventional wisdom. Then we explain such a puzzle by intuitive reasoning, empirical proxies, and an equilibrium model in light of trading practices and investors' learning, all of which reflect rational aspects of Bitcoin unlike most literature focusing on irrational retail traders' behaviors or price manipulation by Bitcoin whales.

This paper delivers two key messages beyond predicting Bitcoin returns. First, investors' learning plays a crucial role in asset pricing, in particular, when uncertainty is high. Second, Bitcoin's next transformation may force investors to face another kind of uncertainty and direct their attention to it.[51] Either decaying interest in time-varying correlations or changing investors landscape can weaken the Bitcoin return predictability in the future. However, the main idea of the paper is not about correlation *per se* but about general asset transformation by a new narrative (Yae et al. 2021) and investors' learning on it. Investors still learn about some characteristics of Bitcoin, although it is not necessarily the correlation, to reduce uncertainty of new kind. Such a learning behavior can create another research opportunity.

We admit that our analysis has some limitations. First, the stylized model in Section 6 is purely static; it does not internalize investors' learning dynamics in equilibrium. Building a full dynamic model is a challenge because the model should also internalize all other non-conventional aspects of Bitcoin pricing, such as gradual adoption, mining cost, Fork, halving, and sentiments. Such a unified model would be too complicated to analyze and test, however. Second, we do not explain why correlations start to fluctuate as soon as Bitcoin futures trading begins. We only use such a structural break to split the sample for a main analysis (with post-futures sample) and a placebo

[50]See Biais et al. 2019 for an example of technical transformation.

[51]For example, check out Bitcoin Vault at https://yhoo.it/2Xh2Uoe. See Yae (2019) for an intuition about why Bitcoin can survive long without whales' dumping at a peak.

test (with pre-futures sample).[52] Third, our empirical proxies and theoretical model are based on the assumption of mean-variance investors. However, Bitcoin is well known for its high skewness and kurtosis (Conlon and McGee 2020). The first two moments have certainly a major effect, yet considering higher moments might lead to other interesting findings.

Our predictability results suggest a new dominant Bitcoin return predictor uncorrelated with existing ones. For practical purposes, one can build a more sophisticated prediction model based on our predictor. We leave this trading strategy idea for future research, let alone the limitations of the paper.

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[52]We only use the historical event as a guidance to a potential structural break, avoiding a pitfall of multiple tests and data mining.

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Figure 1: Time-Varying Correlations Between Bitcoin and S&P500 Index

The figure shows the time-varying correlation estimates from DCC(1,1)-GARCH(1,1) models with daily log returns of Bitcoin and S&P500 Index: the gray regions (estimates from the full-sample) and the dashed lines (from two subsamples). The dashed line in the pre-futures period (before 12/18/2017) is practically flat and zero but swings across zero many times in the post-futures period (on and after 12/18/2017).

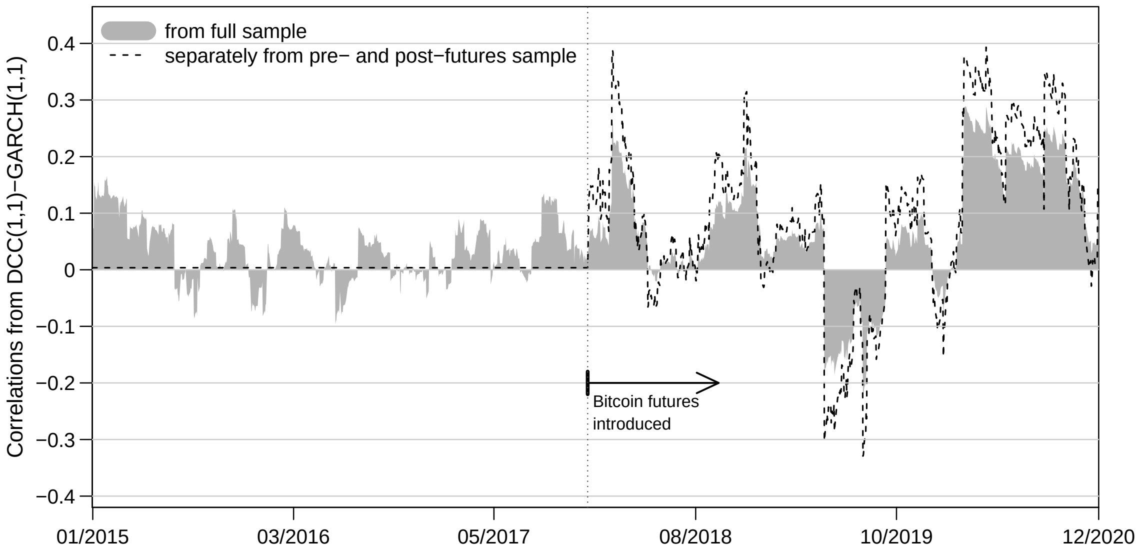

Figure 2: Time-Variations of Sequentially Estimated Correlation Dynamics

The plot illustrates how investors' perception on correlation dynamics changes in real-time. The dashed lines show the time-variation of the 99% intervals—the 0.5th and 99.5th percentiles— of the estimated conditional correlations using a past-looking window of two years whereas the solid line shows their medians. To smooth out the graphs, we compute the 0.5th and 99.5th percentiles from an extended set of conditional correlations, which is a union of the original two-year-long estimate sets measured on each of 40 trading days (two months) around the given date, that is, the percentiles of $40 \times 252 \times 2 = 2{,}016$ conditional correlation estimates. Therefore, a wide gap between dashed lines implies that investors on a given day notice a huge time-variation in their conditional correlation estimates from DCC(1,1)-GARCH(1,1) fitting with the past two years of data.

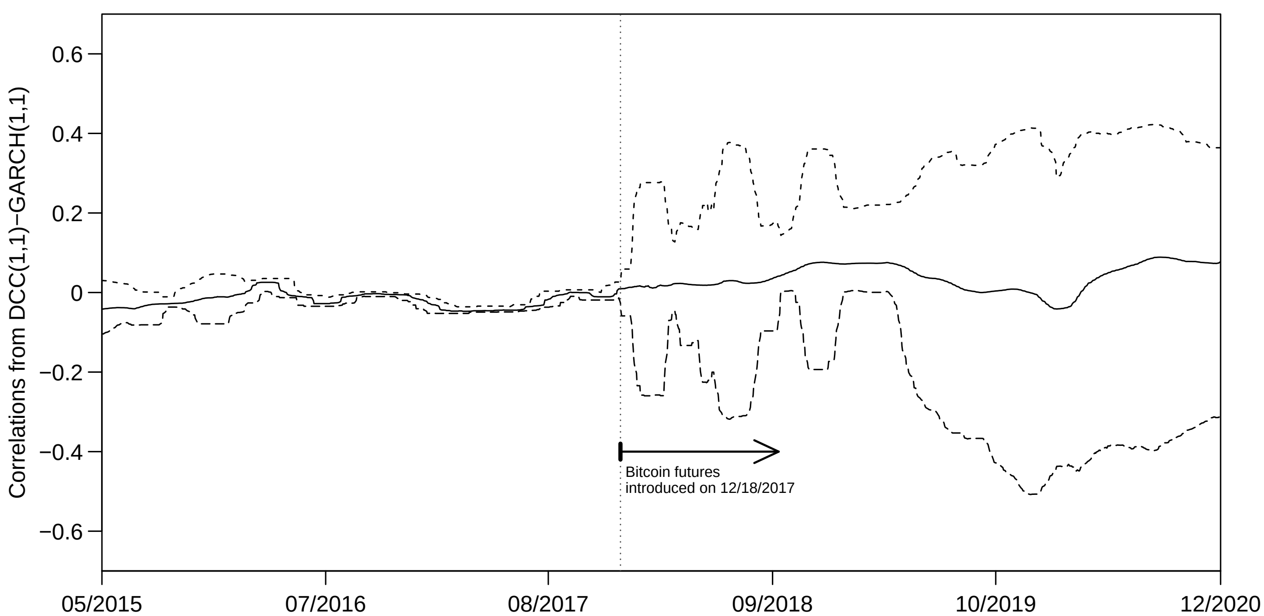

Figure 3: Optimal Weights on Bitcoin in Passive Portfolios

Panel (a) shows the proportion of *positive* optimal weights $w_{b,t}$ on Bitcoin among 765 days in the post-futures sample for different levels of $\overline{\mu}^* = RP(r_{b,t})/RP(r_{m,t})$, that is, the ratio of Bitcoin' risk premium over S&P500's on a horizontal axis. For Panel (b), we first define 'ex-ante Sharpe-ratio improvement by including Bitcoin' as follows:

$$\text{(Sharpe ratio improvement)} = \frac{SR_t(r_{p,t+1})}{SR_t(r_{m,t+1})} - 1,$$

where $r_{p,t+1}$ is a return of a portfolio that optimally mixes Bitcoin and S&P500 with a short-sale constraint while $r_{m,t+1}$ is a return of S&P500. Then Panel (b) repeats Panel (a) but draws the median (sold line) and (25^{th}, 75^{th}) percentiles (dashed lines) of 'ex-ante Sharpe ratio improvement by including Bitcoin', among 765 days in the post-futures sample.

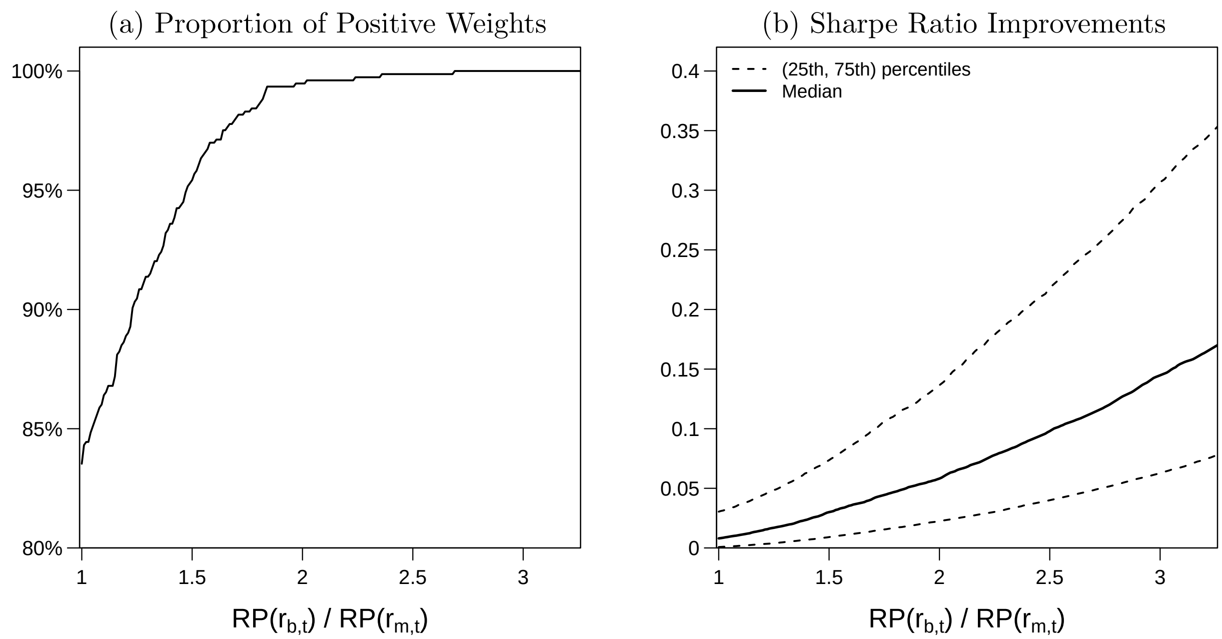

Figure 4: Variable Importance Heat Map

Two heat maps show variable importance metrics, which are reductions in R^2 when a given predictor is set to zero in a trained model, following Gu et al. (2020). The graphs show the relative importance of each predictor in each of the following machine learning algorithms: LASSO, Group LASSO, Elastic Net, Random Forest, Gradient Boosting Machine, and Neural Network with one hidden layer. In Panel (a), the variable importance metric (Metric I) is scaled by the column-wise sum so that the total sum for each column can be unity. Darker colors represent greater importance in prediction. Panel (b) repeats Panel (a) but scales the importance metric (Metric II) by normalizing the variable importance metric to be between zero and one for each algorithm. For Group LASSO, we use the groupings specified in Table 4 column 'LG' such that predictors with similar characteristics are grouped together.

(a) Metric I (b) Metric II

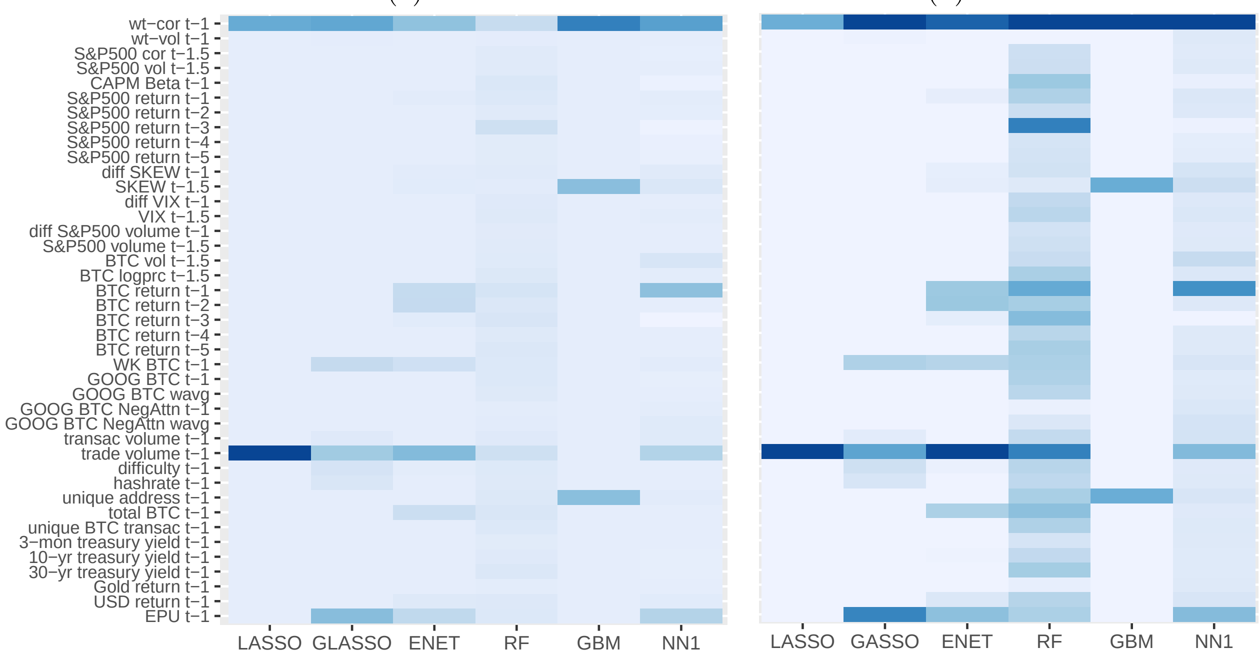

Figure 5: Bitcoin Demand due to Time-Varying Correlation and Volatility Ratio

The figure displays estimated excess non-speculative demands for Bitcoin for the post-futures sample: solid lines with round markers ($w_{b,t}^{(cor)} - \overline{w}_b$) and dashed line ($w_{b,t}^{(vol)} - \overline{w}_b$). The gray regions show Bitcoin's daily closing prices (USD).

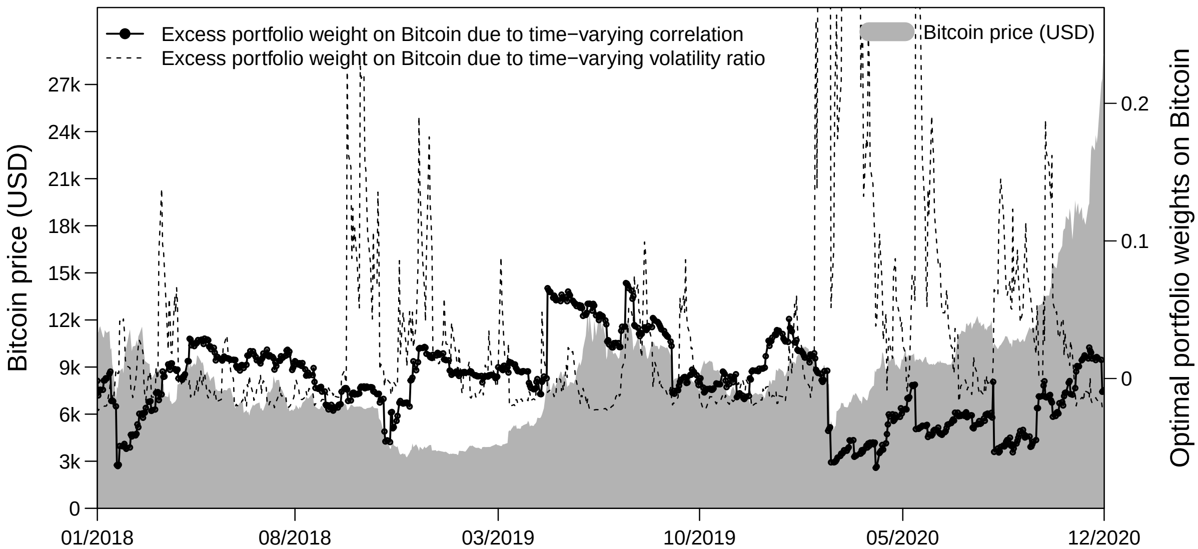

Figure 6: Return Predictability Conditional on Volatility Ratio

Panel (a) shows the sample standard deviation of $\Delta w_{b,(t-1):t}^{(cor)}$ for each choice of $\overline{\sigma}^*$ in the horizontal axis, expressed as its empirical quantile in the set $\{\sigma_t^*\}_{t=1}^{T}$. Panel (b) reports the Newey-West t-statistics of three different predictors $\Delta\rho_t$, $\Delta w_{b,(t-1):t}^{(cor)}$, and $\Delta w_{b,(t-1):t}^{(cor|vol)}$, from univariate predictive regressions with different trimming thresholds. The ratio of Bitcoin's risk premium over S&P500's, $\overline{\mu}^* = RP(r_{b,t})/RP(r_{m,t})$, is fixed at one.

(a) $SD(\Delta w_{b,t}^{(cor)})$—Unconditional

(b) Newey-West T-stats after Trimming

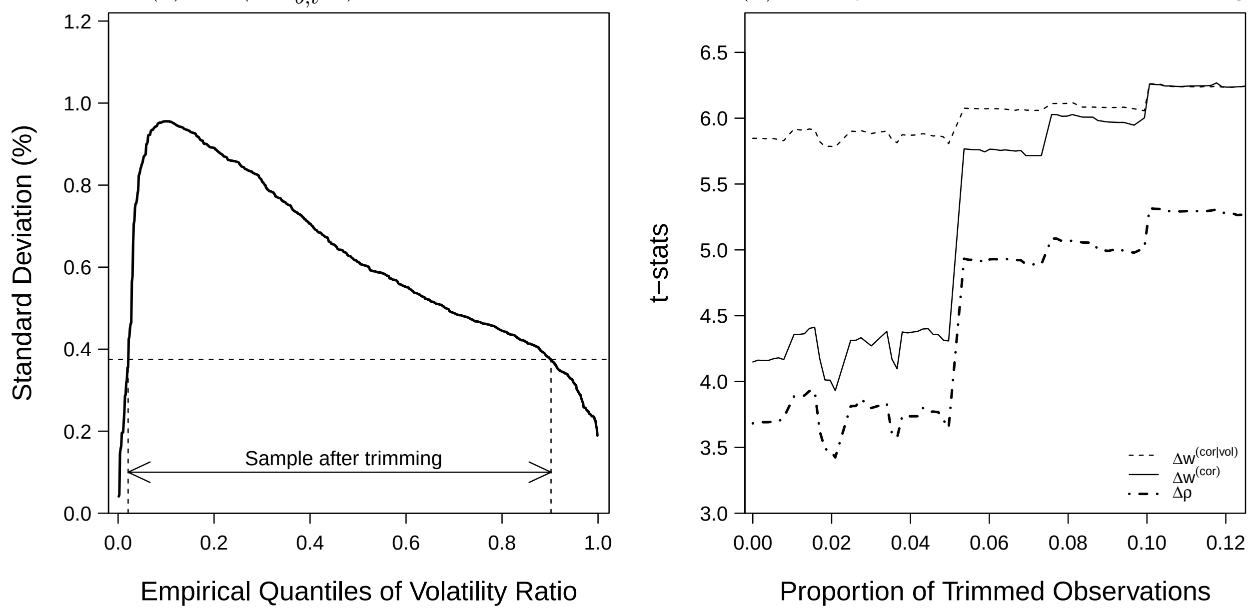

Figure 7: Return Predictability Conditional on Risk Premium Ratio

Two panels show how return predictability varies depending on an input value of $\overline{\mu}^{*}$ for optimal portfolio weight calculation, in terms of Newey West t-statistics in Panel (a) and R^2 in Panel (b) from univariate predictive regressions. The predictor variables are $\Delta w^{(cor)}_{b,(t-1):t}$, $\Delta w^{(cor|vol)}_{b,(t-1):t}$, $\Delta w^{(vol)}_{b,(t-1):t}$, and $\Delta w^{(vol|cor)}_{b,(t-1):t}$ where $\Delta w^{(vol|cor)}_{b,(t-1):t}$ is defined in a similar fashion to $\Delta w^{(cor|vol)}_{b,(t-1):t}$ in Equation (10).

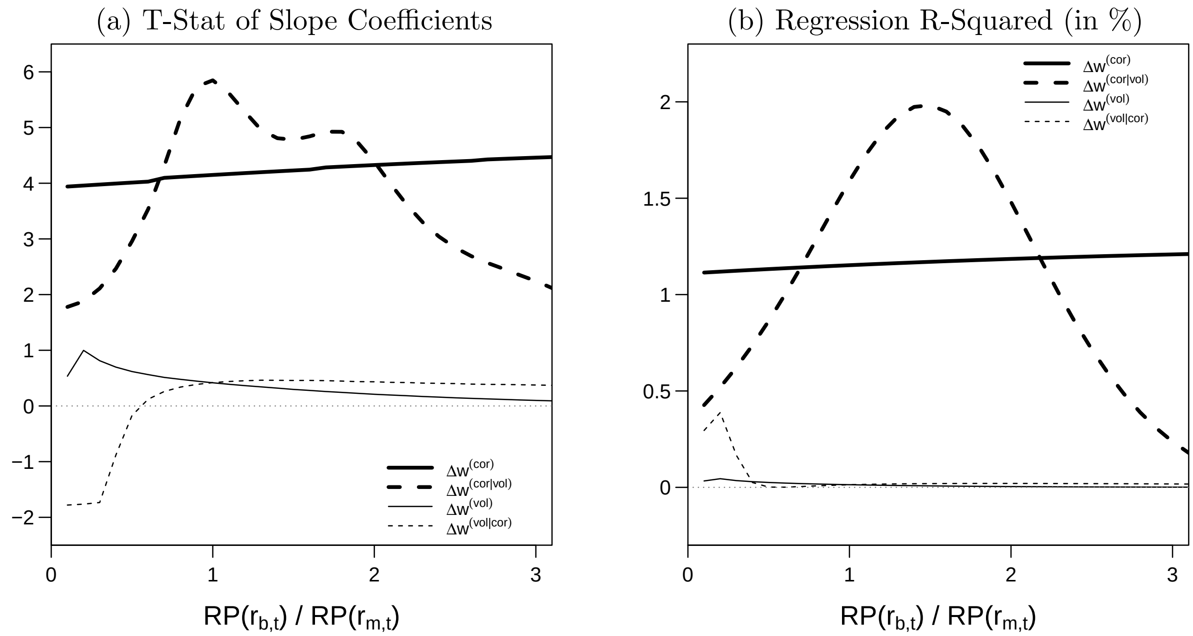

Figure 8: Comparative Statics Analysis of the Bitcoin Pricing Model

Panel (a) shows how Bitcoin price changes when the non-speculative investors' estimate on the correlation changes at static equilibrium as a comparative static analysis. The vertical axis is the subsequent log returns (%) of Bitcoin, in excess of log returns without correlation changes. Panel (b) repeats Panel (a) but replaces the horizontal axis by changes in non-speculative investors' optimal portfolio weights (%) on Bitcoin, corresponding to the correlation changes in Panel (a). We calibrate model parameters following Table 12, including $w^{(s)}_{b,t-1} = 0.6$ and $Q^{(s)}_{b,t-1}/Q_{b,t-1} = 0.3$.

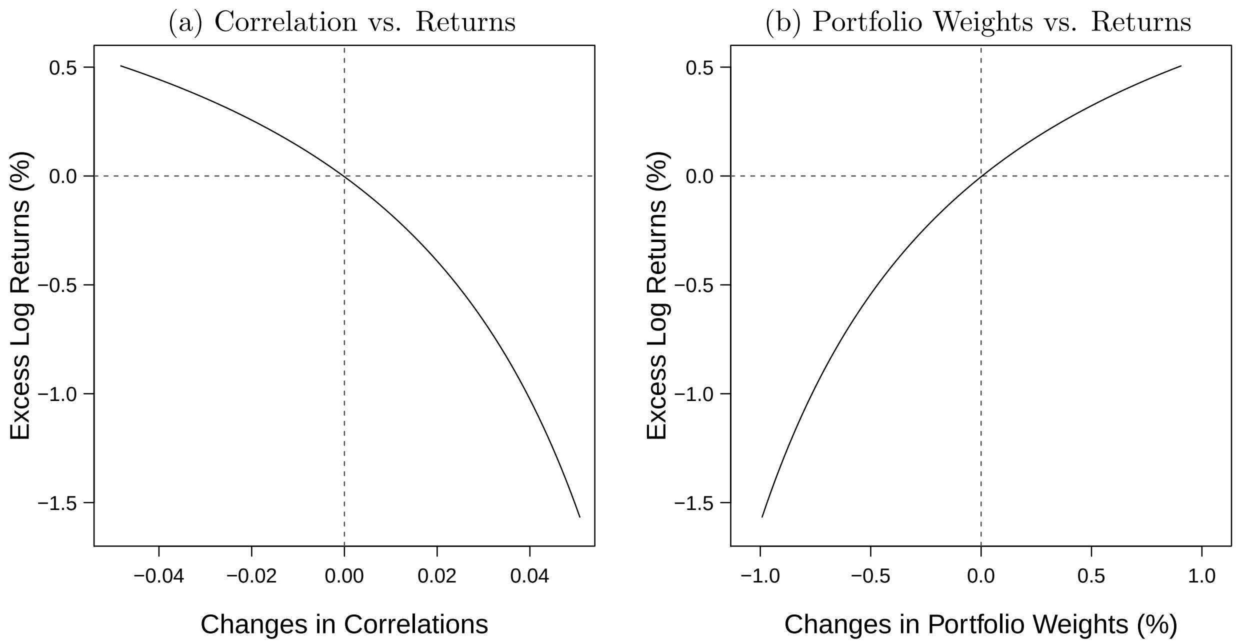

Table 1: Selected Parameter Estimates of DCC-GARCH Models

This table shows the estimates of parameters a and b and associated t-statistics in parenthesis by DCC(1,1)-GARCH(1,1) model for the pre-futures, the post-futures, and the full sample period. A full set of parameter estimates are reported in the online appendix A.1.

Sample	Pre-futures	Post-futures	Full
from	01/01/2015	12/18/2017	01/01/2015
to	12/17/2017	12/31/2020	12/31/2020
DCC(1,1)			
a	0.000	0.030	0.016
	(0.00)	(2.01)	(1.52)
b	0.925	0.948	0.964
	(2.94)	(30.84)	(28.84)

Table 2: Bitcoin Return Predictability

This table shows the results of univariate predictive regressions using data from post-futures period:

$$r_{b,t+1} = a_0 + a_1 x_t + \varepsilon_{t+1}$$

where $r_{b,t+1}$ is daily Bitcoin log returns and x_t can be any variable in the following. ρ_t is lagged Bitcoin-S&P500 correlation level estimated from DCC(1,1)-GARCH(1,1), β_t is lagged CAPM β, cov_t is lagged Bitcoin-S&P500 covariance, and $\Delta\rho_t$ is a lagged change in correlation. We compute β_t and cov_t using correlation and volatility estimates from DCC(1,1)-GARCH(1,1). All predictor variables x_t are not standardized. 'pre-COVID' indicates that regression is performed with data from 12/18/2918 to 02/29/2020. 'LAD' is short for least absolute deviations regression and 'Rank' refers to a rank-based estimation (Hettmansperger and McKean 2010). INT means both dependent variable and predictor adopt Inverse Normal Transformation (INT) based on Beasley et al. (2009). 'Trimmed' indicates that both dependent variable and predictor are trimmed at 2.5% and 97.5%. All t-statistics are Newey West t-statistics except for LAD and Rank where original t-statistics are used. All predictors are scaled by their standard deviations.

Predictor x_t	coefficient a_1	t-statistic (NW)	R^2 (%)
ρ_t	0.01	0.05	0.00
β_t	-0.12	-0.74	0.06
cov_t	0.12	0.66	0.06
$\Delta\rho_t$	-0.49	-3.68	1.05
$\Delta\rho_t$ (pre-COVID)	-0.49	-2.89	1.08
$\Delta\rho_t$ (LAD)	-0.42	-3.82	1.02
$\Delta\rho_t$ (Rank)	-0.46	-4.09	1.04
$\Delta\rho_t$ (INT)	-0.13	-3.60	1.79
$\Delta\rho_t$ (Trimmed)	-0.79	-2.82	1.26

Table 3: Time-Dependency in Returns and Granger Causality Tests

Panel A shows OLS (ordinary least squares) estimates on the slope coefficient $a_1^{(L)}$ and Newey-West t-statistics of the following univariate predictive regression for the post-futures sample:

$$r_{b,t+1} = a_0 + a_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns (%) and z_{t+1-L} is a lagged independent variable: Bitcoin demand changes due to correlation $\Delta w_{b,(t-L):t}^{(cor)}$ (standardized), Bitcoin returns $r_{b,t}$, or S&P500 returns $r_{m,t}$ for $L = 1, ..., 4$. Each column in Panel A is from an individual univariate regression. On the other hand, Panel B shows three Granger causality test results in multivariate time-series setting:

$$r_{b,t+1} = a_0 + \sum_{L=1}^{4} a_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

WLS refers to weighted least squares with weights that equal the inverses of time-varying variance estimates from DCC(1,1)-GARCH(1,1) fitting.

	$\Delta w_{(t-L):(t-L+1)}^{(cor)}$				Bitcoin returns				S&P500 returns			
L (*lag*)	1	2	3	4	1	2	3	4	1	2	3	4
Panel A: Univariate time-series regression												
$a_1^{(L)}$	0.51	0.21	-0.13	0.01	-0.07	0.08	0.05	-0.01	-0.15	0.10	0.02	-0.02
t-stat	4.15	0.48	-0.66	0.06	-1.49	1.87	1.31	-0.20	-1.00	1.07	0.11	-0.20
Panel B: Granger causality tests												
OLS coef.	0.43	0.21	-0.14	-0.07	-0.08	0.06	0.05	0.00				
t-stat	2.68	0.47	-0.82	-0.36	-1.76	1.16	1.25	0.07				
OLS coef.	0.42	0.21	-0.13	-0.08	-0.07	0.05	0.05	0.00	-0.09	0.03	0.00	-0.01
t-stat	2.64	0.49	-0.75	-0.42	-1.59	1.07	1.39	0.02	-0.67	0.30	0.01	-0.08
WLS coef.	0.65	0.25	-0.07	-0.06	0.01	0.04	0.07	0.02	0.13	0.06	-0.02	-0.07
t-stat	3.23	1.26	-0.33	-0.32	0.21	0.76	1.53	0.55	0.99	0.42	-0.19	-0.57

Table 4: A List of Predictors

The table shows a list of predictors used in Figure 4 and Table 5 and 6. When including both lagged change and level of the same variable, we compute the lagged level of the variable as the average of its one-day- and two-day-lagged values. Column 'CG' categorizes each variable into one of the followings: Group M (market attributes), Group B (blockchain attributes), and Group L (extra lagged returns). Column 'LG' specifies groupings of predictors for Group LASSO with those sharing an integer index belonging to a same group.

CG	LG	Predictor Name	Definition
	1	wt-cor t-1	change in optimal weight on Bitcoin from t-2 to t-1 due to time-varying correlation
	2	wt-vol t-1	change in optimal weight on Bitcoin from t-2 to t-1 due to time-varying volatility
	3	BTC return t-1	log return of Bitcoin at time t-1
	4	CAPM Beta t-1	CAPM Beta at time t-1
	5	trade volume t-1	Bitcoin trade volume on major exchanges at time t-1
	6	EPU t-1	Economic Policy Uncertainty Index at time t
M	1	S&P500 cor t-1.5	averge of BTC-S&P500 correlation at time t-1 and t-2
M	2	S&P500 vol t-1.5	averge of S&P500 volatility at time t-1 and t-2
M	7	S&P500 return t-1	log return of S&P500 index at time t-1
M	8	diff SKEW t-1	change of COBE SKEW index from t-2 to t-1
M	8	SKEW t-1.5	average of COBE SKEW index at time t-1 and t-2
M	9	diff VIX t-1	the difference between COBE VIX index at t-1 and that at t-2
M	9	VIX t-1.5	average of COBE VIX index at time t-1 and t-2
M	10	diff S&P500 volume t-1	change of CBOE Volume for S&P 500 Index options (as % of market cap) from t-2 to t-1
M	10	S&P500 volume t-1.5	average of CBOE Volume for S&P 500 Index options at time t-1 and t-2
M	2	BTC vol t-1.5	average of Bitcoin volatility at time t-1 and t-2
M	3	BTC logprc t-1.5	average of Bitcoin log price at time t-1 and t-2
M	11	3-mon treasury yield t-1	3-month treasury yield at time t-1
M	11	10-yr treasury yield t-1	10-year treasury yield at time t-1
M	11	30-yr treasury yield t-1	30-year treasury yield at time t-1
M	12	Gold return t-1	log return of Gold at time t-1
M	13	USD return t-1	log return of US Dollar at time t-1
B	14	WK BTC t-1	Wikipedia Bitcoin search counts at time t-1
B	14	GOOG BTC t-1	Google search counts for *Bitcoin* at time t-1
B	14	GOOG BTC wavg	average Google search counts for *Bitcoin* over the past week
B	14	GOOG BTC NegAttn t-1	ratio of Google search counts for *Bitcoin hack* to *Bitcoin* at t-1
B	14	GOOG BTC NegAttn wavg	average ratio of Google search counts for *Bitcoin hack* to *Bitcoin* over the past week
B	15	transac volume t-1	Bitcoin transaction volumn at time t-1
B	16	difficulty t-1	Bitcoin mining diffficulty at time t-1
B	16	hashrate t-1	Bitcoin hashrate at time t-1
B	17	unique address t-1	number of Bitcoin unique address at time t-1
B	18	total BTC t-1	total number of Bitcoin at time t-1
B	15	unique BTC transac t-1	number of unique Bitcion transaction at time t-1
L	7	S&P500 return t-2	Daily log return of S&P500 index from t-2 to t-1
L	7	S&P500 return t-3	Daily log return of S&P500 index from t-3 to t-2
L	7	S&P500 return t-4	Daily log return of S&P500 index from t-4 to t-3
L	7	S&P500 return t-5	Daily log return of S&P500 index from t-5 to t-4
L	3	BTC return t-2	Daily log return of Bitcoin from t-2 to t-1
L	3	BTC return t-3	Daily log return of Bitcoin from t-3 to t-2
L	3	BTC return t-4	Daily log return of Bitcoin from t-4 to t-3
L	3	BTC return t-5	Daily log return of Bitcoin from t-5 to t-4

Table 5: In-Sample Predictive Regressions for Post-Futures Sample

The table shows the coefficient estimates and associated robust t-statistics in the predictive regression (7):

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t} + Z_t \gamma + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns (%). $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$ refer to (standardized) changes in non-speculative Bitcoin demand due to correlation and volatility ratio changes from $t-1$ to t, respectively. Robust t-statistics in parenthesis are calculated from Newey and West (1987) and Newey and West (1994). Other predictor variables Z_t include lagged Bitcoin returns $r_{b,t} = \log(P_{b,t}) - \log(P_{b,t-1})$, lagged CAPM beta β_t, lagged Bitcoin trade volume $Volume_{b,t}$, Economic Policy Uncertainty index (policyuncertainty.com) $EPU_{b,t}$, and additional control variable groups. In Table 4 column 'CG', we classify each additional control variable into one of the following groups: Group M (market attributes), Group B (blockchain attributes), and Group L (extra lagged returns). When including a control variable group in regression, we use the first two principal components from that group. Note that investor attention variables related to Google search trends for Bitcoin are based on Liu and Tsyvinski (2021). Group L includes $r_{b,t-L}$ for $L = 1, ..., 4$ and $r_{m,t-L}$ for $L = 0, ..., 4$, implying Granger causality test.

Predictor	Post-futures (12/18/2017 to 12/31/2020)					Post-futures before COVID-19 (12/18/2017 to 02/29/2020)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\Delta w^{(cor)}_{b,(t-1):t}$	0.51		0.47	0.47	0.47	0.50		0.48	0.47	0.48
	(4.15)		(2.89)	(2.91)	(2.80)	(3.16)		(2.78)	(2.63)	(2.37)
$\Delta w^{(vol)}_{b,(t-1):t}$		0.06	0.03	0.04	0.03		-0.19	-0.23	-0.22	-0.20
		(0.42)	(0.28)	(0.30)	(0.20)		(-1.27)	(-1.66)	(-1.67)	(-1.56)
$r_{b,t}$			-0.42	-0.42	-0.42			-0.22	-0.23	-0.22
			(-1.79)	(-1.76)	(-1.74)			(-0.83)	(-0.83)	(-0.82)
β_t			0.04	0.06	0.07			0.06	0.07	0.09
			(0.22)	(0.31)	(0.31)			(0.28)	(0.32)	(0.40)
$Volume_{b,t}$			-0.58	-0.55	-0.55			-0.74	-0.91	-0.87
			(-2.59)	(-1.35)	(-1.39)			(-3.52)	(-1.93)	(-1.78)
$EPU_{b,t}$			0.63	0.64	0.63			0.21	0.20	0.21
			(1.71)	(1.68)	(1.76)			(0.98)	(0.93)	(0.96)
Controls			M	MB	MBL			M	MB	MBL
R^2 (%)	1.15	0.01	3.94	3.99	4.00	1.17	0.16	4.04	4.14	4.38
$Adj.R^2$	1.02	-0.12	2.92	2.71	2.47	0.99	-0.02	2.62	2.36	2.24

Table 6: Importance of Predictor Variables

The table shows variable importance metrics (Metric I in Figure 4), which are reductions in R^2 when a given predictor is set to zero in a trained model (Gu et al. 2020), for $\Delta w_{b,(t-1):t}^{(cor)}$ and the second important predictor under different Machine Learning algorithms: LASSO, Group LASSO (GLASSO), Elastic Net (ENET), Random Forest (RF), Gradient Boosting Machine (GBM), and Neural Network with one hidden layer (NN1). The variable importance metric is scaled by the sum of all predictors so that the total sum for each algorithm can be unity. 'MIV' represents the most important variable whereas 'SIC' means the second most important variable. 'VI score' is a variable importance score implied by the variable importance metric.

	LASSO	GLASSO	ENET	RF	GBM	NN1
MIV	lagged trading volume	$\Delta w_{b,(t-1):t}^{(cor)}$	lagged trading volume	$\Delta w_{b,(t-1):t}^{(cor)}$	$\Delta w_{b,(t-1):t}^{(cor)}$	$\Delta w_{b,(t-1):t}^{(cor)}$
VI score of MIV	0.67	0.35	0.27	0.08	0.50	0.38
SIV	$\Delta w_{b,(t-1):t}^{(cor)}$	lagged EPU	$\Delta w_{b,(t-1):t}^{(cor)}$	lagged trading volume	lagged market skewness	lagged Bitcoin returns
VI score of SIV	0.33	0.26	0.24	0.06	0.25	0.24

Table 7: Predictive Regressions with Global Equities and Other Cryptos

This table repeats column (1) of Table 5 for alternative US stock market indexes, international equity markets, and three major cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), and Ripple (XRP). We report coefficient estimates b_1, their Newey-West statistics, and R^2 (%) of the predictive regressions with the post-futures sample from 12/18/2017 to 12/31/2020:

$$r_{c,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1} \quad \text{for} \quad c \in \{BTC, ETH, XRP\},$$

where $r_{c,t+1}$ is daily log returns (%) and $\Delta w^{(cor)}_{b,(t-1):t}$ is (standardized) changes in non-speculative Bitcoin demand due to correlation from $t-1$ to t. In DCC(1,1)-GARCH(1,1) estimation for each row, we use the following data: 1) S&P500, NYSE composite, and NASDAQ composite indexes for the US stock markets and 2) Shanghai Composite, Nikkei 225, BSE Sensex, FTSE 100, DAX, CAC 40, and FTSE Italia All-Share indexes for international equity markets. Once estimating $\Delta w^{(cor)}_{b,(t-1):t}$ individually from each international equity market, we aggregate such demands over different countries in the following ways. 'Equal-weighted' calculates a simple average of $\Delta w^{(cor)}_{b,(t-1):t}$ from S&P500 and the aforementioned seven international equity market indexes. 'GDP-weighted' and 'Volume-weighted' compute the averages, weighted by GDP (as of 2018) and cumulative Bitcoin trading volumes by country (as of July 2018 from https://coin.dance), respectively. Finally, 'Combination forecast' unites (with equal weights) the average predicted values of the $r_{c,t+1}$ from each individual predictive regression.

	Coefficient estimates			Newey-West t-stat			R^2 (%)		
	BTC	ETH	XRP	BTC	ETH	XRP	BTC	ETH	XRP
S&P500	0.51	0.81	0.68	4.15	4.10	2.75	1.15	1.75	0.83
NYSE	0.48	0.75	0.57	3.45	3.73	1.91	1.02	1.51	0.57
NASDAQ	0.50	0.86	0.61	4.20	4.32	2.61	1.07	1.99	0.66
China	0.55	0.80	0.78	3.29	3.23	2.97	1.30	1.64	1.08
Japan	0.59	0.68	0.86	2.83	2.37	2.80	1.42	1.13	1.22
India	0.96	1.15	0.86	2.05	1.83	1.97	4.00	3.47	1.32
UK	0.46	0.62	0.56	3.20	3.16	3.29	0.93	1.04	0.56
Germany	0.45	0.67	0.74	2.37	3.26	3.03	0.92	1.23	1.04
France	0.40	0.60	0.65	2.57	2.79	2.84	0.73	0.95	0.77
Italy	0.21	0.32	0.43	1.21	1.28	1.89	0.19	0.29	0.36
Equal-weighted	0.53	0.79	0.75	3.55	3.96	3.25	1.25	1.65	1.01
GDP-weighted	0.62	0.94	0.86	4.58	4.74	3.68	1.71	2.38	1.34
Volume-weighted	0.61	0.92	0.80	4.91	4.85	3.50	1.62	2.24	1.16
Combination forecast	0.60	0.86	0.85	3.97	4.39	3.64	1.56	1.98	1.28

Table 8: Out-Of-Sample Predictive Regressions

This table repeats column (1) of Table 5 in a completely out-of-sample fashion. The reported numbers are in- and out-of-sample performance measures of the predictive regression:

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1}.$$

At each day t, we re-estimate $\Delta x_{b,(t-1):t}$ (from DCC-GARCH) and (b_0, b_1) while expanding a training window without any look-ahead biases. We compare four different estimation methods: ordinary least squares (OLS), weighted least squares (WLS), least absolute deviation (LAD, median), and rank-based estimation (Rank) from Hettmansperger and McKean (2010). For WLS weights, we use reciprocals of real-time time-varying variance estimates from DCC(1,1)-GARCH(1,1). R^2_{IS} is in-sample R^2. R^2_{OS} is defined in Equation (8) following Campbell and Thompson (2008) while R^{2*}_{OS} is defined in Equation (9) following Gu et al. (2020).

Evaluation period	1/2/2019 to 12/31/2020				1/2/2019 to 2/29/2020			
	OLS	WLS	LAD	Rank	OLS	WLS	LAD	Rank
$\Delta w^{(cor)}_{b,(t-1):t}$, from S&P500								
R^2_{IS} (%)	1.29	1.17	0.92	0.97	1.47	1.36	1.07	1.07
R^2_{OS} (%)	1.07	1.54	2.32	2.33	1.49	2.07	3.49	3.52
R^{2*}_{OS} (%)	-0.08	0.40	1.19	1.20	-0.55	0.04	1.49	1.52

Table 9: Expected Errors in Portfolio Weights

The table shows the following AR(1) model estimation:

$$z_t = b_0 + b_1 z_{t-1} + e_t,$$

where z_t is either DCC(1,1)-GARCH(1,1) estimate (σ_t^* or ρ_t) or the portfolio weight ($w_{b,t}^{(vol)}$ or $w_{b,t}^{(cor)}$) due to learning, for the post-futures sample. Standard errors are reported in parenthesis. In the last row, we report $var(e_t^{(\sigma)})/var(e_t^{(\rho)})$ as a measure of uncertainty ratio. For comparison, σ_t^* and ρ_t are standardized.

z_t	DCC-GARCH est.		Portfolio weights	
	σ_t^*	ρ_t	$w_{b,t}^{(vol)}$	$w_{b,t}^{(cor)}$
b_1	0.92 (0.01)	0.98 (0.01)	0.92 (0.01)	0.98 (0.01)
$var(e_t)$	0.16	0.05	11.13	0.37
$\frac{var(e_t^{(\sigma)})}{var(e_t^{(\rho)})}$		3.48		30.12

Table 10: Comparison of Predictability by Different Proxies (In-Sample)

The table shows the coefficient estimates and associated robust t-statistics in univariate predictive regressions by OLS (ordinary least squares):

$$r_{b,t+1} = b_0 + b_1 \Delta x_t + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns (%) and x_t can be $\Delta\rho_t$, $\Delta w_{b,(t-1):t}^{(cor)}$, or $\Delta w_{b,(t-1):t}^{(cor|vol)}$. All predictor variables are standardized by their sample means and standard deviations, respectively. Robust t-statistics in parenthesis are calculated from Newey and West (1987) and Newey and West (1994). The estimations are based on the post-futures sample (12/18/2017 to 12/31/2020). The last column shows the correlation coefficients of a proxy to $\Delta\rho_t$.

Predictor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Corr. to $\Delta\rho_t$
$\Delta\rho_t$	-0.49 (-3.68)			1.11 (1.04)	0.51 (1.04)		0.10 (0.09)	1.00
$\Delta w_{b,(t-1):t}^{(cor)}$		0.51 (4.15)		1.62 (1.55)		-0.63 (-0.98)	-0.52 (-0.36)	-0.99
$\Delta w_{b,(t-1):t}^{(cor\|vol)}$			0.60 (5.85)		1.08 (2.26)	1.20 (1.93)	1.19 (1.78)	-0.93
R^2 (%)	1.05	1.15	1.59	1.24	1.75	1.76	1.76	
$Adj.R^2$ (%)	0.92	1.02	1.46	0.98	1.49	1.50	1.37	

Table 11: Comparison of Predictability by Different Proxies (Out-Of-Sample)

This table repeats out-of-sample prediction in Table 8 for three predictors, $\Delta\rho_{(t-1):t}$, $\Delta w^{(cor)}_{b,(t-1):t}$, and $\Delta w^{(cor|vol)}_{b,(t-1):t}$ in Table 10 using LAD and Rank regressions. The predictor variables are either estimated with S&P500 or aggregated from individual estimations with global equity indexes with GDP weights ('GDP-weighted' in Table 7). R^2_{OS} and R^{2*}_{OS} are as defined in Equation (8) and (9), respectively. R^2_{IS} is in-sample R^2.

	LAD			Rank		
Predictors	$\Delta\rho_{(t-1):t}$	$\Delta w^{(cor)}_{b,(t-1):t}$	$\Delta w^{(cor\|vol)}_{b,(t-1):t}$	$\Delta\rho_{(t-1):t}$	$\Delta w^{(cor)}_{b,(t-1):t}$	$\Delta w^{(cor\|vol)}_{b,(t-1):t}$
from S&P500						
R^2_{IS}	0.78	0.92	1.15	0.78	0.97	1.29
R^2_{OS}	2.13	2.32	1.46	2.17	2.33	2.56
R^{2*}_{OS}	1.00	1.19	1.34	1.04	1.20	1.43
Globally aggregated (GDP)						
R^2_{IS}	1.01	1.26	1.71	0.99	1.16	1.81
R^2_{OS}	2.46	2.86	3.27	2.55	2.84	3.40
R^{2*}_{OS}	1.34	1.74	2.15	1.42	1.71	2.29

Table 12: Model-Implied Coefficients in Predictive Regressions (Corr. Changes)

This table shows the model-implied coefficient:

$$a_1^{(c)} = \frac{1}{P_{b,t}} E\left[\frac{\partial P_{b,t+1}}{\partial \rho_t}\right] = E\left[\frac{\partial \log P_{b,t+1}}{\partial \rho_t}\right],$$

which corresponds to the slope coefficient $a_1^{(c)}$ in the predictive regression of Bitcoin returns $r_{b,t+1}$ on $(\rho_t - \rho_{t-1})$, i.e., changes in correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(c)} + a_1^{(c)}(\rho_t - \rho_{t-1}) + \varepsilon_{t+1}$$

We compute $a_1^{(c)}$ using a numerical derivative for each combination of $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(n)}/Q_{b,t-1}$. At day $t-1$, speculative investors have the optimal weight on Bitcoin $w_{b,t-1}^{(s)}$ at $t-1$. Also, at $t-1$, speculative and non-speculative investors who participate in trading at t hold $Q_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(n)}$ units of Bitcoin, respectively, where $Q_{b,t-1} = Q_{b,t-1}^{(s)} + Q_{b,t-1}^{(n)}$. The point estimate on $a_1^{(c)}$ of the regression using our data is -0.154 where its 95% confidence interval implied by Newey-West standard errors is $(-0.236, -0.072)$. Therefore, we can reverse-engineer two parameters $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)}/Q_{b,t-1}$ by matching the point estimate to the numbers in the table. For example, calibrating $w_{b,t-1}^{(s)} = 0.6$ and $Q_{b,t-1}^{(n)}/Q_{b,t-1} = 0.3$ produces $a_1^{(c)} = -0.154$ if other parameters are calibrated at their empirically representative values as in Section 6.2.

	$Q_{b,t-1}^{(s)}/Q_{b,t-1}$								
$w_{b,t-1}^{(s)}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	-0.053	-0.024	-0.014	-0.009	-0.006	-0.004	-0.003	-0.001	-0.001
0.2	-0.117	-0.053	-0.031	-0.020	-0.013	-0.009	-0.006	-0.003	-0.001
0.3	-0.195	-0.088	-0.052	-0.033	-0.022	-0.015	-0.010	-0.006	-0.002
0.4	-0.291	-0.133	-0.078	-0.051	-0.034	-0.023	-0.015	-0.009	-0.004
0.5	-0.408	-0.189	-0.112	-0.072	-0.048	-0.032	-0.021	-0.012	-0.005
0.6	-0.554	-0.260	-0.154	-0.100	-0.067	-0.045	-0.029	-0.017	-0.008
0.7	-0.736	-0.350	-0.209	-0.136	-0.091	-0.061	-0.040	-0.023	-0.010
0.8	-0.964	-0.467	-0.281	-0.184	-0.124	-0.083	-0.054	-0.031	-0.014
0.9	-1.253	-0.621	-0.377	-0.248	-0.167	-0.113	-0.073	-0.043	-0.019

Table 13: Model-Implied Coefficients in Predictive Regressions (Corr. Levels)

This table repeats Table 12 but instead shows the model-implied coefficient:

$$a_1^{(l)} = \frac{E[r_{b,t+1}^{(+)}] - E[r_{b,t+1}^{(-)}]}{\rho_t^{(+)} - \rho_t^{(-)}},$$

where $\rho_t^{(+)} = \overline{\rho}^* + SD(\rho_t)$, $\rho_t^{(-)} = \overline{\rho}^* - SD(\rho_t)$, $(\overline{\rho}^*, \rho_t)$ are the unconditional and conditional correlations from the DCC-GARCH estimation, respectively, and $SD(\rho_t) \approx 0.15$. On the other hand, $E[r_{b,t+1}^{(+)}]$ and $E[r_{b,t+1}^{(-)}]$ are the model-implied Bitcoin log returns corresponding to the correlation level $\rho_t^{(+)}$ and $\rho_t^{(-)}$, respectively. The coefficient $a_1^{(l)}$ corresponds to the slope coefficient in the predictive regression of Bitcoin returns $r_{b,t+1}$ on ρ_t, i.e., levels of correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(l)} + a_1^{(l)} \rho_t + \varepsilon_{t+1}$$

To compute $E[r_{b,t+1}^{(+)}]$, we first find $E[\rho_{t+1}|\rho_t^{(+)}]$ from Equation (1). Then compute the equilibrium Bitcoin prices at $t+1$ when $\rho_t = E[\rho_{t+1}|\rho_t^{(+)}]$ and $\rho_t = \rho_t^{(+)}$, respectively, holding others constant. Finally, we have a gross return that equals the ratio of those two prices, and its logarithm is $E[r_{b,t+1}^{(+)}]$. We compute $E[r_{b,t+1}^{(-)}]$ in a similar fashion. The point estimate on $a_1^{(l)}$ of the regression using our data is practically zero, 0.0006, where its 95% confidence interval implied by Newey-West standard errors is $(-0.0946, +0.0957)$. Therefore, the model can simultaneously generate both $a_1^{(c)}$ and $a_1^{(l)}$ matching their empirical estimates with a common choice of $w_{b,t-1}^{(s)}$ and $Q_{b,t-1}^{(s)} = Q_{b,t-1}^{(s)}/Q_{b,t-1}$.

	$Q_{b,t-1}^{(s)}/Q_{b,t-1}$								
$w_{b,t-1}^{(s)}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0006	0.0003	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
0.3	0.0011	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
0.4	0.0017	0.0007	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000	0.0000
0.5	0.0027	0.0010	0.0006	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000
0.6	0.0040	0.0015	0.0008	0.0005	0.0003	0.0002	0.0001	0.0001	0.0000
0.7	0.0059	0.0021	0.0011	0.0007	0.0004	0.0003	0.0002	0.0001	0.0000
0.8	0.0085	0.0030	0.0016	0.0010	0.0006	0.0004	0.0003	0.0001	0.0001
0.9	0.0120	0.0042	0.0022	0.0013	0.0008	0.0006	0.0003	0.0002	0.0001

Online Appendix

(Not Intended for Publication)

for

Sequential Learning, Asset Allocation, and Bitcoin Returns

James Yae and George Zhe Tian

July 15, 2021

A.1 DCC-GARCH Models

Table A.1: Parameter Estimates of DCC-GARCH Models

The table shows parameter estimates and associated t-statistics in parenthesis for different DCC-GARCH models. E(1,1) refers to EGARCH(1,1) and a(1,1) refers to Asymmetric Dynamic Conditional Correlation a-DCC(1,1). Panel A, B, and C are for full, pre-futures, and post-futures sample periods, respectively. Panel A and B only report DCC parameters a and b while Panel C also includes GARCH parameter estimates and information criteria. DCC-a estimates are all zeros in the pre-futures period and become positive and overall significant in the post-futures period across all DCC-GARCH models. This result implies a constant correlation in the pre-futures period and time-varying correlations in the post-futures period.

DCC order	(1,1)	(1,1)	(1,1)	(1,1)	a(1,1)
GARCH order	(1,1)	E(1,1)	(2,1)	(1,2)	(1,1)
Panel A: Full sample period, 01/01/2015–12/31/2020					
DCC					
a	0.016 (1.52)	0.014 (1.61)	0.016 (1.43)	0.016 (1.43)	0.015 (1.94)
b	0.964 (28.84)	0.970 (37.78)	0.963 (26.55)	0.963 (26.24)	0.960 (27.13)
Panel B: Pre-futures period, 01/01/2015–12/17/2017					
DCC					
a	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
b	0.925 (2.94)	0.926 (2.85)	0.924 (3.77)	0.926 (3.51)	0.942 (5.53)
Panel C: Post-futures period, 12/18/2017–12/31/2020					
DCC					
a	0.030 (2.01)	0.025 (1.88)	0.034 (1.94)	0.032 (1.99)	0.028 (1.88)
b	0.948 (30.84)	0.957 (33.67)	0.941 (26.86)	0.944 (28.54)	0.948 (30.47)
g					0.007 (0.37)
GARCH-Bitcoin					
α_1	0.192 (1.96)	-0.127 (-1.17)	0.037 (0.76)	0.223 (1.80)	0.192 (1.96)
α_2			0.266 (1.66)		
β_1	0.703 (10.86)	0.826 (18.74)	0.522 (4.59)	0.1621 (0.96)	0.703 (10.85)
β_2				0.480 (3.52)	
γ_1		0.353 (1.98)			
GARCH-S&P500					
α_1	0.285 (3.55)	-0.152 (-3.06)	0.231 (1.76)	0.285 (2.14)	0.285 (3.56)
α_2			0.093 (1.09)		
β_1	0.714 (9.88)	0.936 (53.32)	0.675 (4.85)	0.715 (1.08)	0.714 (9.88)
β_2				0.000 (0.00)	
γ_1		0.450 (4.93)			
Information Criteria					
AIC	-9.78	-9.80	-9.81	-9.78	-9.78
BIC	-9.72	-9.72	-9.73	-9.70	-9.71

A.2 Demand Monotonicity

Intuitively, portfolio diversification implies that the demand for Bitcoin $w_{b,t}$ should monotonically decreases with the correlation between Bitcoin and the stock (ρ_t), and also with Bitcoin-stock volatility ratio σ_t^*. In particular, if $\mu_t^* = 1$ and $\rho_t = 0$, then $w_{b,t} = \left[\{\sigma_t^*\}^2 + 1\right]^{-1}$ decreases with σ_t^*. If $\mu_t^* = 1$ and $\sigma_t^* = 2$, then $w_{b,t} = \frac{1}{2} - \frac{3}{8}\left[\frac{5}{4} - \rho_t\right]^{-1}$ decreases with ρ_t. Here, we formalize conditions for demand monotonicity to elucidate our Bitcoin demand estimation procedure.

First, note the denominator in Equation (2) is positive if and only if

$$\rho_t < \frac{\mu_t^* + \{\sigma_t^*\}^2}{(\mu_t^* + 1)\sigma_t^*} \tag{A.1}$$

The optimal weight $w_{b,t}$ diverges when its denominator is zero, but we find that reasonable values of inputs generate positive denominator, as shown later in Figure A.1 Panel (a).

Proposition A.1. (Demand Monotonicity) *Suppose that the condition (A.1) is satisfied. Then we have the following necessary and sufficient conditions.*

(a) The optimal portfolio weight $w_{b,t}$ on Bitcoin monotonically decreases with Bitcoin-Stock correlation coefficient ρ_t if and only if

$$\frac{\mu_t^*}{\sigma_t^*} = \frac{\text{Sharpe Ratio}_t(\text{Bitcoin})}{\text{Sharpe Ratio}_t(\text{Market})} < 1 \tag{A.2}$$

(b) The optimal portfolio weight $w_{b,t}$ on Bitcoin monotonically decreases with Bitcoin-to-Stock volatility ratio σ_t^* if and only if

$$\rho_t < \frac{2\mu_t^*\sigma_t^*}{\{\mu_t^*\}^2 + \{\sigma_t^*\}^2} \tag{A.3}$$

The condition (A.2) should be satisfied in the CAPM world where the market portfolio has the highest possible Sharpe ratio. Even if CAPM is not true, it is unlikely that highly volatile Bitcoin has higher Sharpe ratio than the well-diversified stock market.[A.1] Also, the risk premium of Bitcoin should be higher than the stock market. Otherwise, no one will invest in Bitcoin with such a high

[A.1] Note the historical realized Sharpe ratio is not the right measure in Proposition A.1. We need ex-ante Sharpe ratios which we cannot observe directly.

volatility. That is, we presume that the condition (A.2) and the following condition are satisfied.

$$1 \leq \mu_t^* = \frac{\mu_{b,t}}{\mu_{m,t}} \leq \frac{\sigma_{b,t}}{\sigma_{m,t}} \tag{A.4}$$

The daily conditional risk premium or the risk premium ratio μ_t^* is difficult to estimate. Instead, we investigate later how μ_t^*, within the inequality condition (A.4), affect the demand monotonicity condition (in Figure A.1) and our main empirical results (in Figure A.2).

We notice that Bitcoin demand monotonicity conditions are well satisfied in the post-futures period. In Table A.3, the correlation coefficient between $w_{b,t}^{(cor)}$ and $Corr_t(r_b, r_m)$ is -0.997, implying that they are truly monotonically associated, as expected by Proposition A.1. Also, Figure A.1 visualizes how well the condition (5) and the condition (A.3) in Proposition A.1 (b) are satisfied by the conditional correlation and volatility ratio estimates from the DCC(1,1)-GARCH(1,1) model fitting with the post-futures sample. Panel (a) confirms that all the correlation and volatility ratio estimates are well located within the gray region, implying the condition (A.1) is met. Therefore, the demand for Bitcoin monotonically decreases with the correlation (ρ_t) between Bitcoin and the stock market in the post-futures period. Similarly, Panel (b) shows that most of the correlation and volatility ratio estimates are located inside the gray region even in the most restricted case $\mu_t^* = 1$, that is, the case where the risk premia of Bitcoin and the stock market are identical. Hence, the demand for Bitcoin monotonically decreases also with Bitcoin-stock volatility ratio (σ_t^*). Nonetheless, our analysis does not require the demand monotonicity conditions, because we look into how Bitcoin prices react to different components in the Bitcoin demand proxy $w_{b,t}$ rather than the conditional correlation and volatilities.

A.3 Proof of Proposition A.1

The optimal portfolio weight on Bitcoin is

$$w_{b,t} = \frac{\mu_t^* - \rho_t \sigma_t^*}{(\mu_t^* - \rho_t \sigma_t^*) + (\sigma_t^* - \rho_t \mu_t^*)\sigma_t^*} = \frac{\mu_t^* - \rho_t \sigma_t^*}{g(\mu_t^*, \sigma_t^*, \rho_t)} \tag{A.5}$$

1) Proof of Proposition A.1(a): Suppose both conditions (A.1) and (A.2) are satisfied. The partial derivative of $w_{b,t}$ with respect to ρ_t can be calculated as follows and will be negative.

$$\frac{\partial w_{b,t}}{\partial \rho_t} = \frac{-\sigma_t^* g(\mu_t^*, \sigma_t^*, \rho_t) + (\mu_t^* - \rho_t \sigma_t^*)(\sigma_t^* + \sigma_t^* \mu_t^*)}{[g(\mu_t^*, \sigma_t^*, \rho_t)]^2} = \frac{\{\mu_t^*\}^2 - \{\sigma_t^*\}^2}{[g(\mu_t^*, \sigma_t^*, \rho_t)]^2} \sigma_t^* < 0$$

The last inequality comes from $\sigma_t^* > 0$ and the condition (A.2) that implies $0 < \mu_t^* < \sigma_t^*$. Note we need the condition (A.1) to guarantee the denominator of the weight positive: $g(\mu_t^*, \sigma_t^*, \rho_t) > 0$. It is because $w_{b,t}$ diverges at $g(\mu_t^*, \sigma_t^*, \rho_t) = 0$. Therefore, $w_{b,t}$ monotonically decreases with ρ_t ■

2) Proof of Proposition A.1(b) Suppose both conditions (A.1) and (A.3) are satisfied. The partial derivative of $w_{b,t}$ with respect to σ_t^* can be calculated as follows and will be negative.

$$\frac{\partial w_{b,t}}{\partial \sigma_t^*} = \frac{-\rho_t g(\mu_t^*, \sigma_t^*, \rho_t) - (\mu_t^* - \rho_t \sigma_t^*)(-\rho_t + 2\sigma_t^* - \rho_t \mu_t^*)}{[g(\mu_t^*, \sigma_t^*, \rho_t)]^2} = \frac{\rho_t(\{\mu_t^*\}^2 + \{\sigma_t^*\}^2) - 2\mu_t^* \sigma_t^*}{[g(\mu_t^*, \sigma_t^*, \rho_t)]^2} < 0 \tag{A.6}$$

The last inequality comes from the condition (A.3). Note we need the condition (A.1) to guarantee the denominator of the weight positive: $g(\mu_t^*, \sigma_t^*, \rho_t) > 0$. It is because $w_{b,t}$ diverges at $g(\mu_t^*, \sigma_t^*, \rho_t) = 0$. Therefore, $w_{b,t}$ monotonically decreases with σ_t^* ■

Figure A.1: Demand Monotonicity Conditions

Three curves in Panel (a) draw the right-hand-side of the inequality (A.1) at three different μ_t^* levels at 1, 2, and 3.26 which equals the estimate on σ_t^* from the data, consistent with the inequality (A.4). The areas below the curves are therefore where the condition (A.1) is met. Similarly, three curves in Panel (b) draw the right-hand-side of the inequality (A.3) at three different μ_t^* levels at 1, 2, and 3.22. In both panels, horizontal axis is the ratio of Bitcoin and S&P500's conditional volatilities while the vertical axis is their conditional correlations. The black dots in both panels represent corresponding observations in the post-futures period.

(a) Condition (A.1) (b) Condition (A.3)

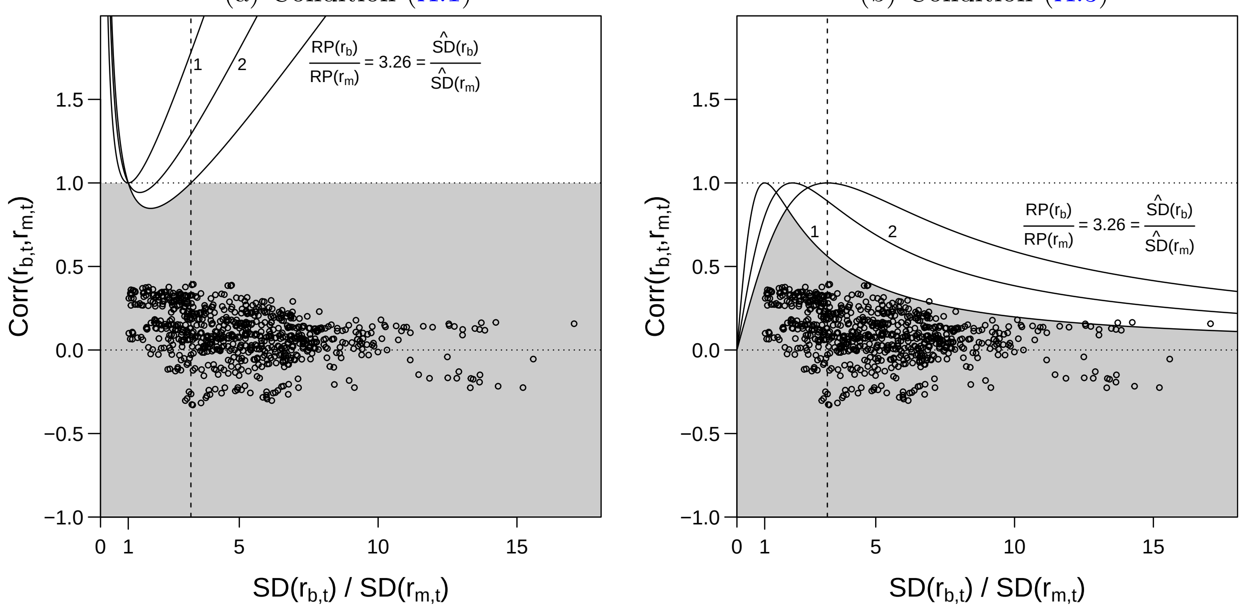

Table A.2: Summary Statistics of Data & Non-Speculative Bitcoin Demand

The table shows summary statistics and the non-speculative Bitcoin demand estimates implied by DCC(1,1)-GARCH(1,1) for the post-futures sample (12/18/2017-12/31/2020) and the post-futures sample before the COVID-19 period (12/18/2017-02/29/2020). All measures of returns, standard deviations, and weights are expressed in percentages whereas other estimates are unit-free. r_m and r_b denote daily log returns of S&P500 index and Bitcoin (USD), respectively. $w_{b,t}^{(cor)}$ is the non-speculative demand due to correlation changes while $w_{b,t}^{(vol)}$ is the non-speculative demand due to volatility ratio changes. $w_{b,t}^{(c+v)}$ is the non-speculative demand due to both correlation and volatility ratio changes. All variables are at daily frequency.

	Post-Futures					Post-Futures before COVID-19				
	Mean	S.D.	Skew	Min	Max	Mean	S.D.	Skew	Min	Max
Asset returns										
Bitcoin (r_b, %)	0.06	4.79	-1.3	-46.47	20.30	-0.13	4.65	-0.3	-23.87	20.30
S&P500 (r_m, %)	0.04	1.47	-1.0	-12.77	8.97	0.02	0.96	-0.8	-4.52	4.84
DCC-GARCH est.										
$Corr_t(r_b, r_m)$	0.10	0.15	-0.4	-0.33	0.39	0.04	0.13	-0.5	-0.33	0.39
$SD_t(r_b)$	4.68	1.67	3.6	3.18	21.00	4.68	1.41	1.7	3.19	11.89
$SD_t(r_m)$	1.17	0.98	4.0	0.44	9.33	0.93	0.45	1.7	0.44	3.13
$SD_t(r_b)/SD_t(r_m)$	5.25	2.68	1.0	1.02	17.06	5.90	2.63	1.1	1.33	17.06
CAPM β_t	0.37	0.76	-1.1	-3.43	2.68	0.24	0.83	-0.8	-3.43	2.68
Bitcoin demand est.										
$w_{b,t}^{(cor)}$, (%)	1.89	2.85	0.1	-4.38	9.05	2.94	2.37	0.2	-4.25	9.05
$w_{b,t}^{(vol)}$, (%)	5.56	8.57	2.9	-0.21	48.95	3.16	4.42	2.9	-0.21	34.63
$w_{b,t}^{(c+v)}$, (%)	5.01	8.08	2.9	-4.24	48.94	3.89	5.01	2.0	-4.24	34.97
$\Delta w_{b,t-1:t}^{(cor)}$	-0.00	0.61	-0.3	-5.09	6.55	0.00	0.57	1.9	-4.27	6.55
$\Delta w_{b,t-1:t}^{(vol)}$	-0.01	3.41	0.4	-37.65	30.82	0.02	2.41	3.3	-12.23	22.66
$\Delta w_{b,t-1:t}^{(c+v)}$	-0.01	3.66	-1.0	-45.63	28.74	0.02	2.51	3.0	-12.21	22.26

Table A.3: Correlation Matrix of Bitcoin Demand Estimates

The table shows a correlation matrix of the implied variables from DCC-GARCH estimates based on the post-futures period 12/18/2017 - 12/31/2020. r_m and r_b are daily log returns of S&P500 index and Bitcoin (USD), respectively. β_t and $\rho_{bm,t}$ are daily CAPM beta and conditional correlation estimates from DCC-GARCH fitting. $w_{b,t}^{(cor)}$, $w_{b,t}^{(vol)}$, and $w_{b,t}^{(c+v)}$ denote the non-speculative demands due to correlation changes, volatility ratio changes, and both correlation and volatility ratio changes, respectively. All variables are at daily frequency.

	β_t	$\rho_{bm,t}$	$\frac{SD_t(r_b)}{SD_t(r_m)}$	$w_{b,t}^{(cor)}$	$w_{b,t}^{(vol)}$	$w_{b,t}^{(c+v)}$	$\Delta w_{b,t}^{(cor)}$	$\Delta w_{b,t}^{(vol)}$	$\Delta w_{b,t}^{(c+v)}$	$\Delta w_{b,t}^{(cor)} + \Delta w_{b,t}^{(vol)}$
β_t										
$\rho_{bm,t}$	0.82									
$\frac{SD_t(r_b)}{SD_t(r_m)}$	-0.06	-0.35								
$w_{b,t}^{(cor)}$	-0.79	-0.99	0.36							
$w_{b,t}^{(vol)}$	0.01	0.37	-0.65	-0.39						
$w_{b,t}^{(c+v)}$	-0.26	0.01	-0.55	-0.03	0.92					
$\Delta w_{b,t}^{(cor)}$	-0.09	-0.09	-0.03	0.09	-0.05	-0.02				
$\Delta w_{b,t}^{(vol)}$	-0.04	-0.03	-0.07	0.03	0.14	0.17	0.02			
$\Delta w_{b,t}^{(c+v)}$	-0.04	-0.04	-0.06	0.04	0.12	0.16	0.19	0.98		
$\Delta w_{b,t}^{(cor)} + \Delta w_{b,t}^{(vol)}$	-0.05	-0.04	-0.07	0.04	0.13	0.16	0.19	0.98	0.99	

A.4 Placebo Tests

Table A.4: Predictive Regression in Sub-Periods

This table repeats columns (1) and (2) of Table 5 for three different sample periods: full (01/01/2015 to 12/31/2020), pre-futures (01/01/2015 to 12/17/2017) and post-futures (12/18/2017 to 12/31/2020). The reported numbers are coefficients and associated Newey-West t-statistics (in parenthesis). $\mathrm{I}^{(1)}$ is an indicator function whose value takes one for the pre-futures sample and zero otherwise whereas $\mathrm{I}^{(2)}$ is similar but for the post-futures sample. For columns (1)-(6), $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$ are estimated using the full sample while using only post-futures sample for columns (7)-(9).

Predictor	Full			Pre-futures			Post-futures		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta w^{(cor)}_{b,(t-1):t}\mathrm{I}^{(1)}$	-0.10		-0.07	-0.10		-0.07			
	(-0.72)		(-0.51)	(-0.71)		(-0.52)			
$\Delta w^{(cor)}_{b,(t-1):t}\mathrm{I}^{(2)}$	0.52		0.52				0.51		0.51
	(4.35)		(4.11)				(4.15)		(4.09)
$\Delta w^{(vol)}_{b,(t-1):t}\mathrm{I}^{(1)}$		-0.21	-0.19		-0.21	-0.19			
		(-1.21)	(-0.92)		(-0.97)	(-0.95)			
$\Delta w^{(vol)}_{b,(t-1):t}\mathrm{I}^{(2)}$		0.04	0.02					0.06	0.05
		(0.20)	(0.17)					(0.42)	(0.42)
R^2 (%)	0.68	0.10	0.77	0.06	0.22	0.24	1.15	0.01	1.16

A.5 Robustness Tests

Figure A.2: Robustness of Return Predictability

This figure displays how the Newey-West t-statistics of the coefficients b_1 and b_2 in the following regression vary with different choices on μ_t^* in Bitcoin demand construction in Section 3.

$$r_{b,t+1} = b_0 + b_1 \Delta w_{b,(t-1):t}^{(cor)} + b_2 \Delta w_{b,(t-1):t}^{(vol)} + \varepsilon_{t+1}$$

The horizontal axis $\mu_t^* = \frac{\mu_{b,t}}{\mu_{m,t}}$ ranges from 0 to 3.22, consistent with the inequality (A.4). Then different lines in Panel (a) show Newey-West t-statistics of b_1 and b_2 with different values for $\overline{\sigma}^*$ and $\overline{\rho}^*$ at demand construction. We replace $\overline{\sigma}^*$ by 25% or 75% percentiles of σ_t from DCC-GARCH estimation. Also we replace $\overline{\rho}^*$ by −0.5, 0, or +0.5. Panel (b) repeats the main result in Panel (a) but using different DCC-GARCH specifications and distribution assumptions such as t-distribution instead of Normal distribution.

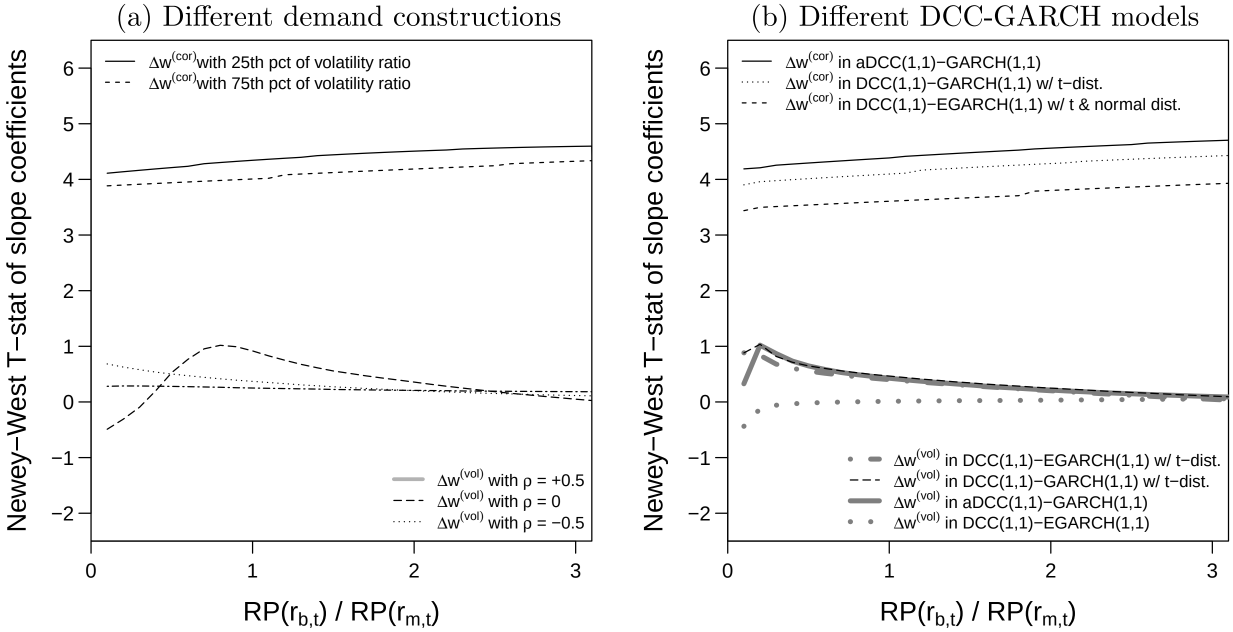

Figure A.3: Quantile Predictive Regressions

This figure shows coefficients (solid lines with circle markers) and their 95% confidence intervals (dash-dotted lines) from the quantile regressions satisfying the following:

$$Q(\tau) = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t}$$

where $Q(\tau)$ is $(\tau \times 100)^{th}$ percentile of Bitcoin's daily log returns $r_{b,t+1}$. $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$ refer to non-speculative demand changes due to correlation and volatility ratio changes from $t-1$ to t, respectively. When $\tau = 0.5$, we have least absolute deviations (LAD) regressions.

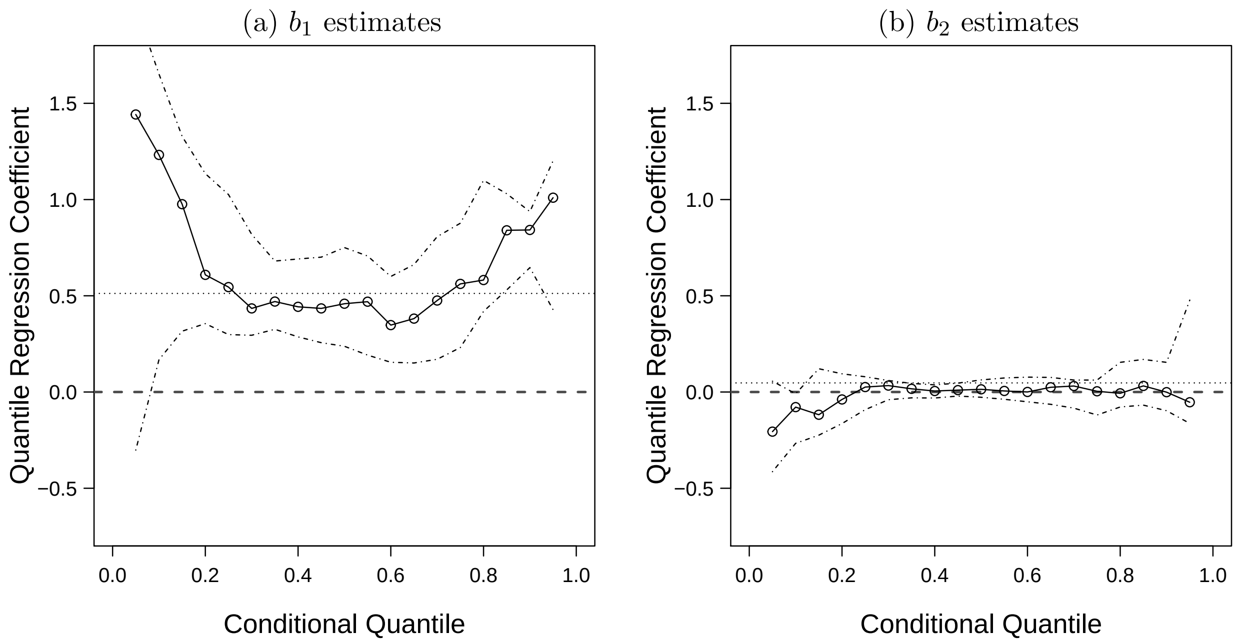

Figure A.4: Predictive Regressions with Trimmed Data

The figure shows how different trimming schemes change OLS estimates on b_1 (solid lines with circle markers) and associated 95% confidence intervals (dash-dotted lines) from Newey-West standard errors, for two different sample periods.

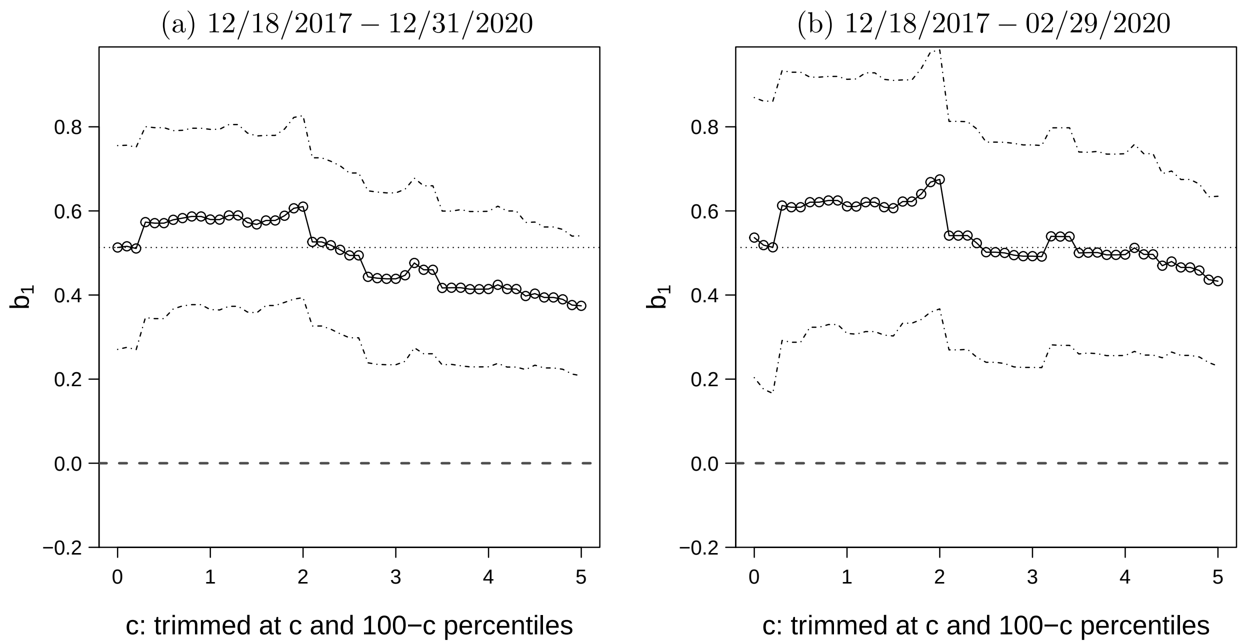

Figure A.5: Predictive Regressions with Winsorized Data

The figure shows how different winsorizing schemes change OLS estimates on b_1 (solid lines with circle markers) and associated 95% confidence intervals (dash-dotted lines) from Newey-West standard errors, for two different sample periods.

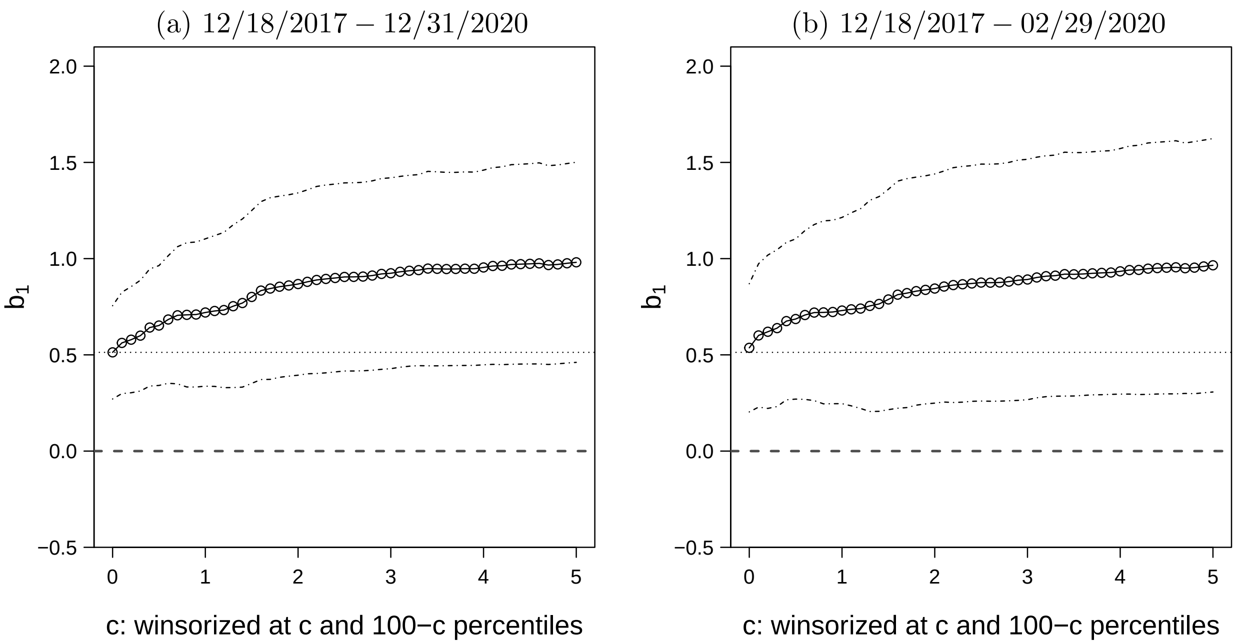

Table A.5: Evaluation of Out-of-Sample Prediction

This table evaluates the performance of out-of-sample binary prediction using the predictive period from 01/02/2019 to 12/31/2020. Panel A shows a confusion matrix based on predicted binary outcomes: positive or negative Bitcoin returns. Sensitivity, or True Positive Rate (TPR), is calculated as the number of true positives divided by the number of real positives. Specificity, or True Negative Rate (TNR), is computed as the number of true negatives over the number of real negatives. PPV stands for Positive Predicted Value, calculated as the number of true positives divided by the number of predicted positives. NPV, or Negative Predictive Value, is computed as the number of true negatives divided by the number of predicted negatives. Accuracy is equal to the number of correctly predicted as a fraction of the total number of the sample observations. Panel B reports the 95% confident intervals for 'Sensitivity + Specificity' and 'PPV + NPV', computed analytically as follows.

$$TPR + TNR \pm 1.96 \times \sqrt{(TPR \times (1 - TPR)/n_1 + TNR \times (1 - TNR)/n_2)}$$

where n_1 is the number of real positives and n_2 is the number of real negatives. Similarly,

$$PPV + NPV \pm 1.96 \times \sqrt{(PPV \times (1 - PPV)/n_3 + NPV \times (1 - NPV)/n_4)}$$

where n_3 is the number of predicted positives and n_4 is the number of predicted negatives. Bootstrapping methods produce practically the same results.

Panel A: Confusion Matrix

	Real Positive	Real Negative	
Predicted Positive	168	117	PPV = 0.589
Predicted Negative	105	115	NPV = 0.523
	Sensitivity = 0.615	Specificity = 0.496	Accuracy = 0.560

Panel B: 95% Confidence Intervals

	Lower Bound	Upper Bound
Sensitivity + Specificity	1.0246	1.1975
PPV + NPV	1.0249	1.1995

Figure A.6: P-Values of Three Tests for Confusion Table

This figure displays how the p-values of Wilcoxon Rank-sum tests, Fisher's Exact tests, and Chi-square tests for the summation of sensitivity and specificity greater than one vary with respect to the different prediction cutoffs in Panel (a) and ending dates of evaluation sample in Panel (b). Low p-values implies high levels of informativeness of the classifier.