To Rebalance or Not to Rebalance: Portfolio risk may be larger than you think![*]

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Abstract

We show that significant portfolio return and variance biases arise when adopting a rebalancing strategy rather than using a buy-and-hold approach in dealing with portfolios spanning across multiple periods. We extend the result in Liu and Strong [2008] for bias in average portfolio returns, and derive bias in variance of portfolios. We show that the magnitude of portfolio variance bias defined as a difference between the variance of portfolio constructed using rebalanced returns and the decomposed buy-and-hold returns depends on average portfolio returns and average returns of its constituents as well as autocovariances of the portfolio and its individual stocks. Empirical evidence based on S&P500 constituents for the period 2003-2011 confirms that bias in variance of portfolios can become significant. In particular, we observe significant negative bias during 2003, 2005 and 2010 and significantly positive bias in more turbulent 2008 and 2011. The existence of portfolio variance biases have important implications not only in evaluating the risk of such portfolios, but also in measuring their performance (e.g., when using Sharpe ratio). We vary the frequency of price quotations and estimated average return and variance biases for 5-minute, daily, weekly and monthly data. Our findings indicate that one should exercise caution when assuming multiperiod rebalanced portfolio returns, as resulting biases can lead to spurious results when analyzing investment strategies or testing asset pricing models.

1 Introduction

A common methodology used in the finance literature when calculating multiperiod portfolio returns is to adopt a rebalancing strategy, keeping the weights of each as-

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set in a portfolio constant at every period. A few notable works that apply rebalancing method to calculate multiperiod portfolio returns include Fama and French [1996], Carhart [1997], Daniel et al. [1997], Lee and Swaminathan [2000], along with more recent ones of Chan et al. [2002], Ahn et al. [2003], Teo and Woo [2004], Cohen et al. [2005], Nagel [2005], Diether et al. [2009], Huang et al. [2010], Hou et al. [2011]. However, common sense would suggest that investors managing their own investments would not consider rebalancing their portfolios on a 5-mitute, daily, weekly or even monthly basis. Furthermore, such rebalancing strategy does not lead to returns that investors would typically achieve under the alternative buy-and-hold assumption, unless they rebalance their portfolios back to the initial weights each data sampling period. The latter, however, appears to be unrealistic due to continuous information flow, that will require the investors to adjust the portfolio weights resulting in prohibitive transaction costs, see e.g. Liu and Strong [2008].

As a matter of fact, in 2009, Warren Buffett told PBS "I read a book, what is it, almost 60 years ago roughly, called The Intelligent Investor and I really learned all I needed to know about investing from that book, and in particular chapters 8 and 20. . . I haven't changed anything since". Chapter 8 of Benjamin Graham's *The Intelligent Investor* entitled "The Investor and Market Fluctuations" discusses the benefits of a buy-and-hold approach. It reads "...The true investor scarcely ever is forced to sell his shares, and at all other times he is free to disregard the current price quotation. He need pay attention to it and act upon it only to the extent that it suits his book, and no more. Thus the investor who permits himself to be stampeded or unduly worried by unjustified market declines in his holdings is perversely transforming his basic advantage into a basic disadvantage. That man would be better off if his stocks had no market quotation at all, for he would be spared the mental anguish caused him by other persons' mistakes of judgment." (Graham and Zweig, 2003, pp.106-107).

Although the rebalancing strategy has become popular due to its simplicity and tractability, authors using these methods seem to ignore the associated biases. Approximation with rebalancing strategy "...may suffice for a quick and coarse comparison of investment performance across many assets, but for finer calculations in which the volatility of returns plays an important role ... the approximation may break down." [Campbell et al., 1997, p.10]. Starting from the earlier studies by Roll [1984], Blume and Stambaugh [1983] and Conrad and Kaul [1993] that outline the presence of market microstructure biases, and recommend to use buy-and-hold returns, the recent paper by Liu and Strong [2008] discusses in details the existence of biases resulting from using the rebalancing method. Authors formalize the results on the calculation of single-period portfolio returns over a multiperiod holding horizon, and compute the bias of the *portfolio mean return* in each month as the difference between the average rebalanced return and the decomposed buy-and-hold return. Liu and Strong [2008] show that rebalancing can lead to spurious statistical inference (the two methods produce a difference of 8% per year), and document that rebalancing impacts an upward bias to the size premium and a downward bias to the momentum effect.

In this article, we present calculation of decomposed portfolio returns and compute the bias in *portfolio variance*s, which extends the result presented in Liu and Strong [2008] for the bias in the *portfolio mean returns.* We formally derive the portfolio variance bias for the two-period example, and generalize it to a multiperiod case. Similarly to the bias in portfolio mean returns, bias in portfolio variance can take significant (either positive or negative) values, depending on the time-series properties of portfolio returns and the returns of its constituents, as well as the time period under consideration. We apply the proposed methodology to portfolio returns computed using constituents of the S&P 500 index. Stocks in our constructed portfolios are selected randomly without replacements and the number of stocks in portfolios varies from 1 to 80. The time period under consideration is from 2 January 2003 to 30 December 2011. We show how periods of increased volatility in the financial market occurring in our sample impact the estimated portfolio variance resulting in large biases.

Our empirical evidence based on S&P500 constituents for the period 2003-2011 confirms that bias in portfolios can become significant. We note that variance bias becomes more stable as the number of assets in portfolios increases. In particular, in portfolios of 50 assets we observe significant negative biases during 2003, 2005 and 2010 and significantly positive bias in more turbulent 2008 and 2011. The existence of portfolio variance biases in these particular time periods have important implications not only in evaluating the risk of such portfolios, but also in assessment of their performance (for example when using coefficient of variation, Sharpe or signal-to-noise ratios). Our results indicate that one should exercise caution when assuming multiperiod rebalanced portfolio returns, as resulting biases can lead to spurious results when analyzing investment strategies or testing asset pricing models.

The remainder of the paper is organized as follows. Section 2 presents derivations for the multiperiod portfolio returns, corresponding to the buy-and-hold method as well as the rebalancing method, and derives biases that arise in variance of portfolios. Section 3 deals with empirical analysis, where stocks selected randomly from the S&P500 index, form portfolios according to the decomposed by-and-hold or rebalancing method and the empirical biases are contrasted. Section 4 concludes the paper and provides final remarks.

2 Derivations

2.1 Buy-and-hold versus Rebalanced Returns

We begin by focusing on a distinction between decomposed buy-and-hold portfolio returns and rebalanced portfolio returns, assuming that rebalancing is performed every period according to the data sampling frequency.[1] We assume that the investor holds a portfolio of N stocks and denote individual stock i's simple return ($i = 1, ..., N$) in month τ by $r_{i,\tau}$. Furthermore, when constructing our portfolios we adopt the most popular approach - an equally weighted portfolio with $w_i = 1/N$. We note, however, that the results derived below can be generalized to arbitrary weights w_i with $\sum_{i=1}^{N} w_i = 1$.[2]

For the *rebalanced* portfolio, portfolio returns in each holding period $\tau = 1, ..., T$ can be computed as an average of the individual stock returns in that period:

$$r_{reb,\tau} = \frac{1}{N}\sum_{i=1}^{N} r_{i,\tau}. \tag{1}$$

As documented in Liu and Strong [2008], the rebalancing method is inaccurate in reflecting investor's wealth in individual periods over a multiperiod holding horizon, unless portfolio is rebalanced back to the initial weight at the beginning of each new period. This, in turn appears to be unrealistic from the investor's perspective, since revisions of portfolio weights are unlikely to occur at regular intervals, especially when taking into account the prohibitive transaction costs of regular periodic rebalancing. In practice, new information flow will determine when revision of weights should take place (we adhere to the point made in Graham and Zweig, 2003, pp.106-107 and mentioned earlier in the Introduction).

For the *buy-and-hold* portfolio, which is a standard and accurate method of measuring the investment performance of buy-and-hold investors, the return in each period τ can be computed as

$$r_{bh,1} = \frac{1}{N}\sum_{i=1}^{N} r_{i,1} \tag{2}$$

for the first month $\tau = 1$ and

$$r_{bh,\tau} = \frac{1}{\sum_{j=1}^{N}\prod_{t=1}^{\tau-1}\left(1 + r_{j,t}\right)}\sum_{i=1}^{N}\prod_{t=1}^{\tau-1}\left(1 + r_{i,t}\right) r_{i,\tau} \tag{3}$$

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[1] That is the portfolio is rebalanced every month when monthly data are used, every week when using weekly data, every day when using daily data, etc.

[2] The $1/N$ strategy is often used in practice and its out-performance across a wide range of different asset allocation strategies is documented in DeMiguel et al. [2009].

for $\tau = 2, ..., T$. Thus, in the first period the individual buy-and-hold portfolio return corresponds to the average of the individual stock returns in this period, and is equivalent to the return on the rebalanced portfolio. For periods $\tau = 2, ..., T$, buy-and-hold portfolio returns are computed as weighted average of period τ stock returns with weights determined by the performance over previous periods. If one to assume no auto- and cross-autocorrelation in individual stock returns, one would note that for rebalanced portfolios the returns in any two periods are independent, whereas for buy-and-hold portfolios the returns are dependent in any two periods.

2.2 Bias in Portfolio Returns

To derive bias in portfolio returns and variance, we make the following notations. We denote the average return on the rebalanced portfolio by

$$\bar{r}_\tau = \frac{1}{N}\sum_{i=1}^{N} r_{i,\tau} \tag{4}$$

and thus, the expected average return of the rebalanced portfolio is given by

$$E\left(r_{reb,\tau}\right) = E\left[\frac{1}{N}\sum_{i=1}^{N} r_{i,\tau}\right] = E\left[\bar{r}_\tau\right]. \tag{5}$$

First, we derive the return bias for $\tau = 2$, and generalize it for an arbitrary τ.[3] Further to that, we use the approximation $1/(1+\bar{r}_\tau) \approx 1 - \bar{r}_\tau$, ignoring higher order terms in the Taylor series expansion. The bias between the expected return of the rebalanced and the buy-and-hold portfolio is given by

$$Bias_2^E = E(r_{reb,2}) - E(r_{bh,2}), \tag{6}$$

and using Eq. (5) and Eq. (3) for $\tau = 2$, we can write

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[3] We note that there is no bias if the holding period corresponds to a single period ($\tau = 1$). However, one would not consider an investment strategy based on a single period as it is unattractive due to transaction costs, see Liu and Strong [2008]; or simply not adequate for constructing a sufficient sample of decomposed portfolio returns for testing of asset pricing models.

$$\begin{aligned}
Bias_2^E &= E\left[\bar{r}_2\right] - E\left[\frac{1}{\sum_{j=1}^{N}\left(1+r_{j,1}\right)}\sum_{i=1}^{N}\left(1+r_{i,1}\right)r_{i,2}\right] \\
&= E\left[\bar{r}_2\right] - E\left[\frac{1}{N\left(1+\bar{r}_1\right)}\left\{\sum_{i=1}^{N}r_{i,2}+\sum_{i=1}^{N}r_{i,1}r_{i,2}\right\}\right] \\
&\approx E\left[\bar{r}_2\right] - E\left[\left(1-\bar{r}_1\right)\frac{1}{N}\left\{\sum_{i=1}^{N}r_{i,2}+\sum_{i=1}^{N}r_{i,1}r_{i,2}\right\}\right] \\
&= E\left[\bar{r}_2\right] - E\left[\frac{1}{N}\sum_{i=1}^{N}r_{i,2}+\frac{1}{N}\sum_{i=1}^{N}r_{i,1}r_{i,2}-\bar{r}_1\frac{1}{N}\sum_{i=1}^{N}r_{i,2}-\bar{r}_1\frac{1}{N}\sum_{i=1}^{N}r_{i,1}r_{i,2}\right] \\
&= E\left[\bar{r}_2\right] - E\left[\bar{r}_2+\frac{1}{N}\sum_{i=1}^{N}r_{i,1}r_{i,2}-\bar{r}_1\bar{r}_2-\bar{r}_1\frac{1}{N}\sum_{i=1}^{N}r_{i,1}r_{i,2}\right] \\
&= E\left[\bar{r}_1\bar{r}_2-\left(1-\bar{r}_1\right)\frac{1}{N}\sum_{i=1}^{N}r_{i,1}r_{i,2}\right] \\
&= E\left[\bar{r}_1\bar{r}_2\right]-\frac{1}{N}\sum_{i=1}^{N}E\left[\left(1-\bar{r}_1\right)r_{i,1}r_{i,2}\right] \qquad (7)
\end{aligned}$$

Assuming that $\bar{r}_1$ is uncorrelated with individual returns $r_{i,1}$ and $r_{i,2}$, Eq. (7) can further be rewritten as

$$\begin{aligned}
Bias_2^E &= E(\bar{r}_1)E(\bar{r}_2)+Cov(\bar{r}_1,\bar{r}_2)-\frac{1}{N}\sum_{i=1}^{N}E\left(1-\bar{r}_1\right)\left[E(r_{i,1})E(r_{i,2})+Cov(r_{i,1},r_{i,2})\right] \qquad (8) \\
&= E(\bar{r}_1)E(\bar{r}_2)+\underbrace{Cov(\bar{r}_1,\bar{r}_2)}_{>0}-\underbrace{E\left(1-\bar{r}_1\right)}_{>0}\frac{1}{N}\sum_{i=1}^{N}E(r_{i,1})E(r_{i,2}) \\
&\quad+\underbrace{\left[-\underbrace{E\left(1-\bar{r}_1\right)}_{>0}\frac{1}{N}\sum_{i=1}^{N}\underbrace{Cov(r_{i,1},r_{i,2})}_{<0}\right]}_{>0}.
\end{aligned}$$

Eq. (8) indicates that even if returns are independent, the return bias is non-zero. Furthermore, it depends on the expected average portfolio returns of the rebalanced portfolio, expected individual stock returns, as well as the autocovariance in the portfolio returns and the average autocovariance in the individual stock returns. Following empirical evidence documented in Lo and Mackinlay [1990], Mech [1993] and Liu and Strong [2008], portfolio returns are positively autocorrelated, that is, $Cov(\bar{r}_1,\bar{r}_2) > 0$ for the rebalanced portfolio, contributing to a positive bias.[4] Individual returns, on the contrary, are negatively autocorrelated, that is, $Cov(r_{i,1}, r_{i,2}) < 0$, see Fisher [1966], Roll

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[4] In fact, transaction costs cause portfolio return autocorrelation by delaying price adjustment.

[1984], Lo and Mackinlay [1990], Jegadeesh and Titman [1995].[5] This negative autocorrelation is more pronounced in the small and low-price stocks, see Lo and Mackinlay [1990] and Liu and Strong [2008]. Hence, one would most likely find this negative serial correlation particularly in small and low-price stocks.[6] Thus, one would expect to observe a positive bias in portfolios constructed of small and low-price stocks. On the other hand, Kaul and Nimalendran [1990] document positive autocorrelation between stock returns once the bid-ask spread is extracted; which may lead to negative bias constructed of large and high-price stocks.

Using Eq. (3) and Eq. (5), we can write the general form bias for the portfolio returns in period $\tau = 2, ..., T$ as

$$\begin{aligned} Bias_{\tau}^{E} &= E(r_{reb,\tau}) - E(r_{bh,\tau}) \\ &= \sum_{i=1}^{N} \left[\frac{1}{N} E(r_{i,\tau}) - E\left(\frac{1}{\sum_{j=1}^{N} \prod_{t=1}^{\tau-1} (1 + r_{j,t})} \sum_{i=1}^{N} \prod_{t=1}^{\tau-1} (1 + r_{i,t})\, r_{i,\tau} \right) \right]. \end{aligned} \tag{9}$$

Generalizing the discussion above to an arbitrary τ, and referring to Liu and Strong [2008] for further details and illustrative example, we conclude that positive bias is most likely to be observed in small and low-price stock portfolios and negative bias may be observed in large and high-price stock portfolios. Liu and Strong [2008] also note that negative bias can arise when expected stock returns are constant over time but vary cross-sectionally; that is when high (low) expected returns are associated with high (lower) expected weights in the buy-and-hold return (second term of Eq. (9)), while rebalancing (first term of Eq. (9)) reverses this effect.

2.3 Bias in Variance of Portfolio

Similarly to the computation of the bias in portfolio returns, we begin with the calculation of the bias in the portfolio variance for $\tau = 2$, and generalize it thereafter to an arbitrary τ. The bias between the rebalanced portfolio and the buy-and-hold portfolio is given by

$$Bias_{2}^{V} = Var(r_{reb,2}) - Var(r_{bh,2}), \tag{10}$$

where the variance of the rebalanced portfolio is given by

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[5] Negative autocorrelation in individual returns is caused by nonsynchronous trading (Fisher [1966]) or transaction costs and bid-ask spreads (Roll [1984], Jegadeesh and Titman [1995]).

[6] For instance, Liu [2006] documents high correlation between the returns of infrequently traded stocks and size, as well as a bid-ask spread; and Branch and Freed [1977], Conrad and Kaul [1993] find a negative relationship between price and bid-ask spread.

$$Var(r_{reb,2}) = Var\left[\bar{r}_2\right] = E\left[\bar{r}_2^2\right] - E\left[\bar{r}_2\right]^2 \tag{11}$$

and the variance of the buy-and-hold portfolio can be written as

$$\begin{aligned}
Var(r_{bh,2}) &= Var\left[\frac{1}{\sum_{j=1}^{N}\left(1+r_{j,1}\right)}\sum_{i=1}^{N}\left(1+r_{i,1}\right)r_{i,2}\right] \\
&= Var\left[\frac{1}{N\left(1+\bar{r}_1\right)}\left\{\sum_{i=1}^{N} r_{i,2} + \sum_{i=1}^{N} r_{i,1}r_{i,2}\right\}\right] \\
&\approx Var\left[\left(1-\bar{r}_1\right)\frac{1}{N}\left\{\sum_{i=1}^{N} r_{i,2} + \sum_{i=1}^{N} r_{i,1}r_{i,2}\right\}\right] \\
&= Var\left[\frac{1}{N}\sum_{i=1}^{N} r_{i,2} + \frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2} - \bar{r}_1\frac{1}{N}\sum_{i=1}^{N} r_{i,2} - \bar{r}_1\frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right] \\
&= Var\left[\bar{r}_2 + \frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2} - \bar{r}_1\bar{r}_2 - \bar{r}_1\frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right] \\
&= Var\left[\bar{r}_2(1-\bar{r}_1) + (1-\bar{r}_1)\frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right] \\
&= Var\left[(1-\bar{r}_1) + (\bar{r}_2 + \frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2})\right], \qquad (12)
\end{aligned}$$

where again in the third equality we applied an approximation $1/(1+\bar{r}_\tau) \approx 1 - \bar{r}_\tau$, ignoring higher order terms in the Taylor series expansion. We can further rewrite Eq. (12) as follows:

$$\begin{aligned}
Var_{bh,2} &= Var(\bar{r}_1) + Var(\bar{r}_2) + Var\left(\frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right) \qquad (13) \\
&- 2Cov(\bar{r}_1,\bar{r}_2) - 2Cov\left(\bar{r}_1, \frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right) + 2Cov\left(\bar{r}_2, \frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right).
\end{aligned}$$

Similarly to the above, we can assume that the portfolio return $\bar{r}_{\tau-1}$ is uncorrelated with the individual stock returns, and thus, the last two terms in Eq. (13) disappear, and $Bias_2^V$ for the variance reduces to

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\begin{aligned}
Bias_2^V &= Var(r_{reb,2}) - Var(r_{bh,2}) \\
&= Var\left(\bar{r}_2\right) - Var(\bar{r}_1) - Var(\bar{r}_2) - Var\left(\frac{1}{N}\sum_{i=1}^{N} r_{i,1}r_{i,2}\right) + 2Cov(\bar{r}_1, \bar{r}_2) \\
&= 2\underbrace{Cov(\bar{r}_1, \bar{r}_2)}_{>0} - \underbrace{Var(\bar{r}_1)}_{>0} - \underbrace{\frac{1}{N^2}Var\left(\sum_{i=1}^{N} r_{i,1}r_{i,2}\right)}_{>0}. \qquad (14)
\end{aligned}
$$

From (14) we observe that the bias in variance of portfolio is not zero; it depends on the autocovariance of portfolio returns, the variance of the first period return, as well as variance of the sum of product of individual portfolio returns. Similarly to the bias in the portfolio returns, bias in variance of portfolio can take either positive or negative value, depending on the properties of the portfolio returns as well as the individual stock returns. Using the same argument as above, the portfolio return are more likely to be positively autocorrelated (see Lo and Mackinlay [1990], Mech [1993] and Liu and Strong [2008]), that is, $Cov(\bar{r}_1,\bar{r}_2) > 0$ for the rebalanced portfolio. Hence, positive autocovariance in portfolio returns will be contributing to a positive bias.

Equation (14) can be generalized to $\tau = 2, ..., T$

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\begin{aligned}
Bias_\tau^V &= Var(r_{reb,\tau}) - Var(r_{bh,\tau}) \\
&= Var\left(\bar{r}_\tau\right) - Var\left[\frac{1}{\sum_{j=1}^{N}\prod_{t=1}^{\tau-1}\left(1+r_{j,t}\right)}\sum_{i=1}^{N}\prod_{t=1}^{\tau-1}\left(1+r_{i,t}\right)r_{i,\tau}\right]. \qquad (15)
\end{aligned}
$$

3 Empirical Analysis

3.1 Data

In this section we put our theoretical results derived in Section 2 to the test. We construct equally weighted rebalanced and buy-and-hold portfolios of various sizes from S&P 500 constituents over a 9 year sample period from January 2, 2003 to December 30, 2011. We let the number of stocks in each portfolio vary between 1 and 80, and select stocks randomly without replacement. The period includes the financial crisis associated with the bankruptcy of Lehman Brothers in September 2008 and the subsequent period of turmoil in US and international financial markets. The underlying data are 5 minute, daily, weekly and monthly observations on prices for 501 stocks drawn from the constituent stocks of the S&P500 index during the sample period obtained from SIRCA Thompson Reuters Tick History. This data set was constructed by Dungey et al. [2012] and does not purport to be all the stocks listed on the S&P500 index, but has drawn from that population of stocks to select those with sufficient coverage and data availability for high frequency time series analysis of this type. The original dataset consisting of over 900 stocks was taken from the 0#.SPX mnemonic provided by SIRCA. This included a number of stocks that are traded OTC and on alternative exchanges. Some stocks that altered currency of trade during the period were excluded from the analysis. We adjusted the dataset for changes in RIC code[7] during the period through mergers and acquisitions, stock splits and trading halts. We also removed some stocks with insufficient observations during the sample period. The data handling process is fully documented in the web-appendix to Dungey et al. [2012]. In the dataset for this paper we force the inclusion of Lehman Brothers until their bankruptcy in September 2008, but drop Fannie Mae and Freddie Mac from the analysis. The final data set contains 501 individual stocks. Full list of included stocks is provided in the appendix.

3.2 Results

We allow for the diversification effect in the portfolio, that is, the relationship between the decreasing risk in the portfolio when the number of securities in that portfolio increases.[8] Figure 1 represents variance bias in portfolios by year (2003-2011). In calculating biases in portfolios we retain the exact sample of stocks randomly drawn without replacement from the S&P500 constituents list when contrasting rebalanced and buy-and-hold approaches. The number of stocks $n = 1, ..., 80$ is shown on the x-axis.[9] We perform 10,000 random draws and compute the median variance bias (blue solid line), the mean variance bias (blue dotted line) as well as the 90% confidence band (shaded region between the 5th and 95th percentile). It can be observed that the mean and the median biases fall into specified confidence region for all years under consideration. The sign of the variance bias depends on the year under consideration, and will be discussed below. Variance biases, in most cases, stabilize for portfolios in excess of 50 assets depending on the time period chosen.

For brevity we choose to analyze more closely a well diversified portfolio consisting of 50 stocks that were randomly selected without replacement. The number of draws remains 10,000.[10] We trace biases in portfolio returns, variances and signal-to-noise ratios[11] for each month in 2003-2011 using one year of past data. These are shown in the

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[7] A Reuters instrument code, or RIC, is a ticker-like code used by Thomson Reuters to identify financial instruments and indices.

[8] We note that one can obtain most of the benefits of diversification by holding a relatively small number of stocks; see, e.g., Elton and Gruber [1977].

[9] It is obvious that variance bias for a single stock portfolio is always zero.

[10] The total number of possible combinations of 50 stocks out of 501 is 2.57×10^{69}.

[11] Bias in signal-to-noise ratio will be equal to bias in Sharpe ratio if the risk free rate remains constant through the entire holding period. For small infrequent changes in risk free rate, the two biases will be approximately equal. In our empirical analysis of monthly biases as well as biases over one year in the case of rolling 1 year window estimation in Figure 2, signal-to-noise bias approximates Sharpe ratio bias well.

left panel of Figure 2 for the variance bias (top panel), return bias (middle panel) and Sharpe ratio (low panel). In our computations we use one month moving window over the entire period from 2003 to 2011 to obtain the biases for each month (panels on the left). Although our main focus in this section is on the analysis of daily returns, we also use 5-minute, weekly and monthly sampling frequencies to estimate variance biases. We use the same dataset with prices only sampled at different frequencies and the same sample of assets in each simulated portfolio. Overnight returns for 5-minute data have been included. Panels on the right show biases for 2008, our year of interest, across randomly selected portfolios of size $n = 1...80$ stocks. Shaded region represents the 90% confidence interval around daily biases. As expected, the higher is the frequency of the data, and thus the frequency of rebalancing to maintain the equal weight in the rebalanced portfolio, the larger is the bias in returns (middle right panel). One observes from Figure 1 and the left panel of Figure 2 that for portfolios of 50 assets significant negative biases occur during 2003, 2005 and 2010 indicating that rebalancing of the portfolio leads to the lower variance than the buy-and-hold strategy, and thereby, rebalancing strategy underestimates portfolio variance. Significantly positive bias is attributed to more turbulent 2008 and 2011, indicating that the rebalancing strategy overshoots the buy-and-hold strategy. The existence of portfolio variance biases in these particular time periods have important implications not only in evaluating the risk of such portfolios, but also in measuring their performance (e.g. using signal-to-noise ratio).

In Table 1 we present a brief summary of the results for rebalanced and buy-and-hold portfolios, obtained using daily data and 10,000 randomly constructed portfolios of 50 stocks that are equally weighted at the beginning of each year. We assume that from the first trading day of the year the investor either follows a rebalanced portfolio approach and calculates portfolio returns using Eq. (1) or adheres to buy-and-hold strategy using Eq. (3) to calculate portfolio returns. For each given year we estimate averages of portfolio returns (columns 1 and 4), standard deviations (columns 2 and 5) and signal-to-noise ratios (columns 3 and 6) based on daily returns within that year. The results are reported in annualized terms.[12] For the bias results in columns (7) through (9), * denotes 90% significance, that is when the range from 5th percentile to 95th percentile of estimated biases in portfolio statistics for a given year does not contain zero. We emphasize that the average over 10,000 portfolios for the bias statistics is computed as a matched difference, i.e. not as a difference in means. In other words, we compute bias at the end of year for each of the 10,000 portfolios, and then average across these portfolios. We notice that portfolio returns in 2006, and especially in 2008-2009, were

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Despite its similarities, we distinct the two, but note that that they can be used interchangeably.

[12] Daily estimates have been annualized using a factor of 250 for average returns, and $\sqrt{250}$ for standard deviation and signal-to-noise ratio.

overstated by the rebalanced approach. This overstatement of portfolio returns have been observed in at least 90% of the 10,000 randomly constructed portfolios. On the other hand, the variance has been significantly understated in 2003, 2005, and 2010. The largest overstatement of variances has been observed in 2008, with another significant exaggeration in 2011. The overstatement of the Sharpe ratio by the rebalancing strategy occurs in 2003, 2006, 2008 and 2009.

Table 2 reports the largest positive (negative) biases in portfolio returns and portfolio variances in panel A (panel B). We observe that the largest significant biases in portfolio returns occur during the years 2008-2009. The results for the largest significant biases in portfolio variance are mixed; however 6 out of 9 significant biases occur between November 2007 and August 2011. This period corresponds to the turbulent period of financial crisis, followed by the global recession. We can confirm previous results that the rebalancing method tends to overshoot the expected returns and the variances during this period. The results for the lowest biases indicate than none of the return biases are significant at 90% level; however all the variance biases are significant, with the largest significant biases occurring in 2009. Researchers might fall into a trap when relying on biases for the portfolio returns only, ignoring the second moments. This is especially pertinent for the situation at hand: although biases in portfolio returns appear insignificant due to an increased variance, biases in the variance of portfolios are significant for all years under consideration.

Liu and Strong [2008] and Canina et al. [1998] discuss common time-series characteristics of individual stocks and portfolios and implications these characteristics may exhibit on the portfolio return bias. We follow Canina et al. [1998] and calculate the cross-sectional average first-, second-, and third-order autocorrelation of each stock's daily return ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$); the first-, second-, and third-order autocorrelation for the equally weighted 50-stock rebalanced portfolio (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}); and the cross-sectional variance of the average returns, $Var\left(\overline{r_i}\right)$. To calculate the first three variables ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$), each security's autocorrelation for month t is calculated from daily returns and averaged cross-sectionally. As there are 21 trading day in most month, we use 20, 19 and 18 observations to compute the first-, the second-, and the third-order autocorrelation, respectively. Calculations of the other three variables (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}) are performed similarly, using daily equally portfolio returns in place of asset returns. The third variable $Var\left(\overline{r_i}\right)$ is obtained by averaging the returns of each individual stock through time in a given month, and then computing cross-sectional variance of those returns. The results are summarized in Figure 3: upper panel corresponds to the autocorrelation in the stock returns ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$); middle panel corresponds to the autocorrelation in the portfolio returns (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}); and low panel shows the cross-sectional variance of the average returns $Var\left(\overline{r_i}\right)$. We show central locations for the computed variables as well as the 90% confidence band (shaded area between the 5th and 95th percentile) computed using 10,000 random draws. Lo and Mackinlay [1990] document that average daily autocorrelation in returns is mostly negative. Empirical literature shows that individual stock returns are negatively autocorrelated because of non-synchronous trading (e.g., Fisher [1966]) or bid-ask spreads (e.g., Roll [1984], Jegadeesh and Titman [1995]). Our evidence precludes us from drawing the same conclusion. Given that our sample consists of the largest 501 stocks in the US financial markets, non-synchronous trading or bid-ask spreads might not be an issue at least for daily or lower frequencies. Furthermore, consistent with the previous literature (Lo and Mackinlay [1990], Mech [1993], Canina et al. [1998]) we observe, on average, positive first-order autocorrelations in portfolios for the first half of our sample.[13] However, following the financial crisis associated with the bankruptcy of Lehman Brothers in September 2008 and the subsequent period of turmoil in the US, we observe negative first-order autocorrelations in portfolio returns. The second- and third-order autocorrelations in portfolios are negative on average, which is in line with the results reported in Canina et al. [1998]. The cross-sectional variance of the average returns, $Var\left(\overline{r_i}\right)$, is stable for the first half of our sample, and becomes volatile starting from 2007, which corresponds to the global financial crisis and subsequent period of global recession.

In Table 3 we present the results of several multiple regressions that are based on variables described above, and aim to explain the bias in portfolio average returns and variance. The dependent variable is the bias in portfolio mean returns, $Bias^E$, and the bias in portfolio variance, $Bias^V$. The independent variables are the cross-sectional average first-, second-, and third-order autocorrelation of each stock's daily return ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$); the first-, second-, and third-order autocorrelation of the daily equally weighted rebalanced portfolio computed monthly (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}); and the cross-sectional variance of the average returns, $Var\left(\overline{r_i}\right)$. The sample contains 108 monthly observations (from January 2003 to December 2011). Parameter estimates are reported in columns (1), (3) and (2), (4) for bias in portfolio mean returns and bias in portfolio variance, respectively. Absolute values of t-statistics are reported in parentheses with ***,**, and * denoting significance at 1%, 5% and 10% level, respectively. Our results indicate that the effect of the cross-sectional variance of the average returns is mostly[14] positive and significant at 1% level in all regressions. This finding indicates that higher cross-sectional variability in the average portfolio returns results in higher biases. Another significant (and positive) variable is $\overline{\rho_{i3}}$, explaining bias in portfolio returns (significance at 5% level) and portfolio variances (significance at 10% level). Second order autocorrelation in

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[13] Transaction costs cause portfolio return autocorrelation by delaying price adjustment.

[14] The effect is only negative (with regression coefficient of nearly zero) in one regression with $Bias^V$ as a dependent variable (column 2). However, the effect becomes positive once autocorrelations in portfolios are removed from the regression (column 4).

stock returns $\overline{\rho_{i2}}$ contributes positively only to explaining $Bias^E$ at 10% significance, and appears insignificant when explaining the bias in portfolio variance, $Bias^V$. The only portfolio autocorrelation that has an effect on $Bias^E$ is the third-order autocorrelation ρ_{ew3}, that appears to be negatively related to $Bias^E$ in the first regression.

4 Conclusion

This paper examines bias in the variance of portfolios when considering multiperiod horizons. Motivated by the results in Liu and Strong [2008] who compute bias in the portfolio means, we examine biases that occur in the variance of portfolios. Biases arise as a consequence of adopting a rebalancing strategy for the portfolio returns rather than using the buy-and-hold returns. Variance bias is computed as a difference between the variance of portfolio constructed using rebalanced returns and the decomposed buy-and-hold returns. It turns out that although decomposing multiperiod portfolio returns into a series of single-period returns via the rebalancing strategy is a convenient and widely-adopted method in academic literature, it is unrealistic to assume that in practice investors would consider reshuffling their portfolios back to the initial weights at regular and frequent intervals. This is due to the fact that continuous information flow will determine the portfolio weight and time intervals at which rebalancing occurs.

We examine biases arising in the means and variances of portfolios empirically using equally weighted rebalanced and buy-and-hold portfolios of various sizes constructed from S&P 500 constituents over the 9 year sample period from January 2, 2003 to December 30, 2011. We allow the number of stocks in each portfolio to vary between 1 and 80, and select stocks randomly without replacement. We show that the results (significance and sign of the bias) depend on the time period under consideration and the properties of portfolio returns as well as individual stock returns. In particular, we find that negative variance biases tend to occur during 2003, 2005 and 2010 indicating that rebalancing of the portfolio understates portfolio variance. Significantly positive biases are attributed to more turbulent 2008 and 2011, indicating that the rebalancing strategy overstates the buy-and-hold strategy during these times. We observe the largest significant biases in portfolio returns between 2007 and 2011, corresponding to the turbulent period of financial crisis, followed by the global recession. When trying to explain bias in portfolio returns and variances, we find that higher cross-sectional variability in the average portfolio returns results in higher biases and that other variables contributing to the explanation of biases include autocorrelations of individual stock's return.

Overall, our results indicate that one should exercise caution when assuming multiperiod rebalanced portfolio returns, as biases in portfolio variances as well as portfolio mean returns can lead to spurious results when analyzing investment strategies or testing asset pricing models. We emphasize that researches might fall into a trap when observing (possibly insignificant) biases in portfolio returns and ignoring second moments, that might in fact include large biases. The existence of portfolio variance biases, particularly during the more turbulent period of financial crises and global recession, might have important implications not only in evaluating the risk of such portfolios, but also in measuring their performance.

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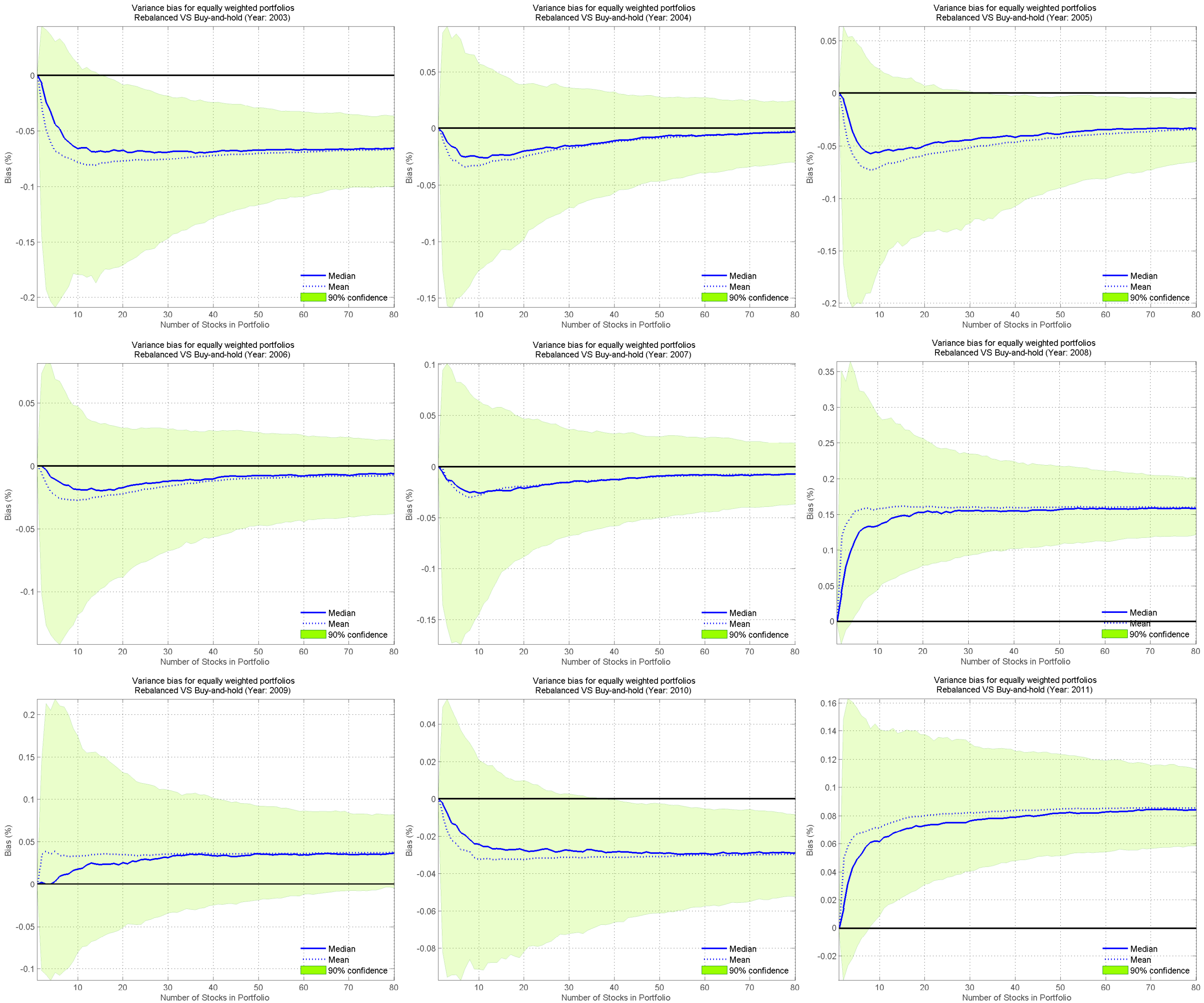
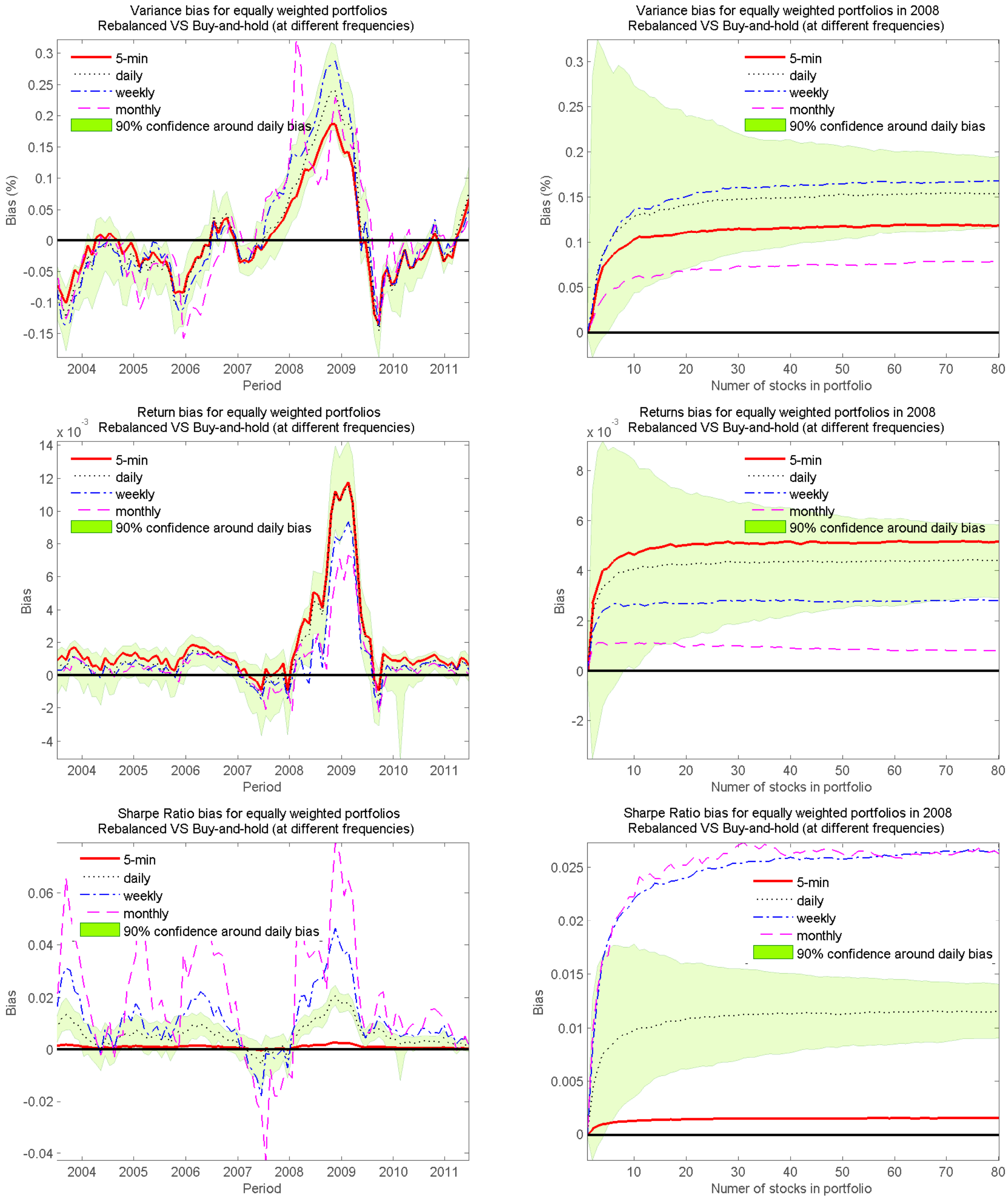
Figure 1: Variance Bias in portfolios by year.



Shaded region represents 5th and 95th percentile of estimated biases for 10,000 random draws. To construct portfolios n-stock are selected randomly, where $n = 1..80$ and $\frac{501!}{n!(501-n)!} \gg 10,000$. It is obvious that variance bias for a single stock portfolio is always zero.

Figure 2: Bias in portfolios at different frequencies.



Variance, average return and signal-to-noise ratio biases are constructed based on returns of randomly selected portfolios of 50 assets using the past one year of data. One month moving window over the entire period from 2003 to 2011 was used to obtain the biases for each month (panels on the left). Using 5-minute, daily, weekly and monthly sampling frequencies, we estimate variance biases for 2008, our year of interest, across randomly selected portfolios of sizes $n = 1...80$ stocks (panels on the right). Shaded region represents 5th and 95th percentile of estimated daily biases for 10,000 random draws. To construct portfolios n-stock are selected randomly based on daily data where $\frac{501!}{n!(501-n)!} \gg 10,000$. We uses the same dataset but sample prices at different frequencies to obtain returns. We keep the same sample of assets in each simulated portfolio across estimations for different frequencies. Overnight returns for 5-minute data have been included.

Table 1: Mean portfolio return, variance and signal-to-noise ratio for rebalanced and buy-and-hold portfolios using daily data.

	Rebalanced portfolio			Buy-and-hold portfolio			Bias		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Avg. return (%)	St.Dev (%)	Signal-to-noise	Avg. return (%)	St.Dev (%)	Signal-to-noise	$Bias^E$	$\frac{Bias^V}{Var(r_{bh})}$ (%)	$Bias^S$
2003	33.5	17.3	1.94	33.2	17.9	1.85	0.36	-6.97*	0.09*
2004	16.3	13.2	1.24	15.6	13.2	1.18	0.66	-0.87	0.06
2005	7.2	12.2	0.59	6.2	12.5	0.49	0.97	-4.26*	0.09
2006	12.6	12.4	1.02	11.1	12.4	0.90	1.43*	-0.99	0.12*
2007	2.9	17.0	0.17	4.8	17.1	0.28	-1.85	-1.10	-0.10
2008	-36.1	45.1	-0.80	-41.6	41.8	-0.99	5.43*	16.35*	0.19*
2009	48.9	35.5	1.38	45.3	34.8	1.30	3.57*	3.72	0.08*
2010	21.0	21.3	0.99	20.5	21.6	0.95	0.51	-2.88*	0.04
2011	-1.3	27.3	-0.04	-1.8	26.3	-0.07	0.48	8.43*	0.02

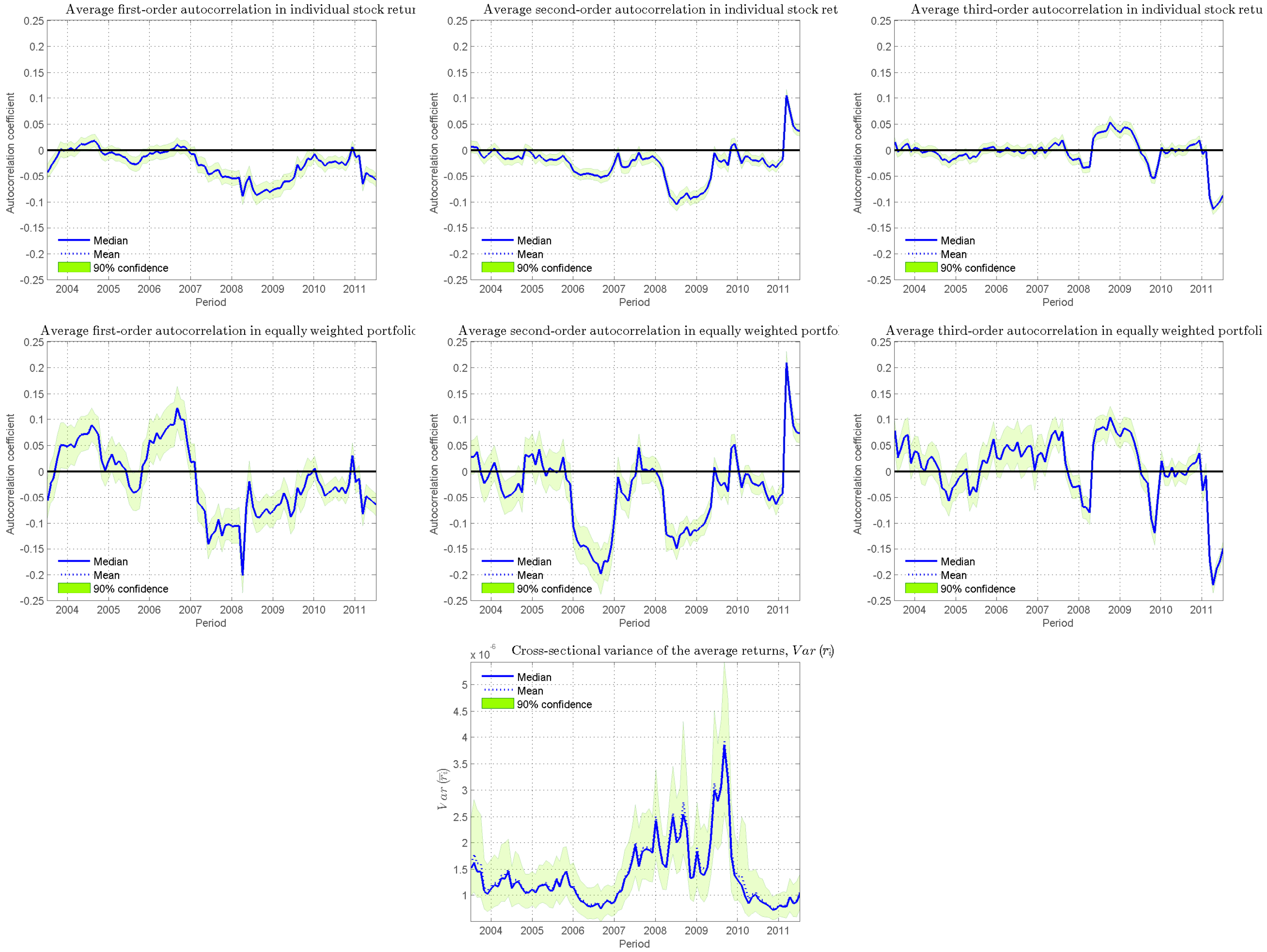
Using daily frequency we compute annualized averages of portfolio returns (columns 1 and 4), standard deviation (columns 1 and 5) and signal-to-noise ratio (columns 3 and 6) based on 10,000 randomly constructed portfolios of 50 stocks that are equally weighted at the beginning of each year. Daily estimates have been annualized using a factor of 250 for average returns; $\sqrt{250}$ for standard deviation and signal-to-noise ratio. For the bias results in columns (7) through (9), "*" denotes 90% significance, that is when the range from 5th percentile to 95th percentile of estimated biases in portfolio statistics for a given year does not contain zero. For presentation purposes, bias in variance is presented as a percentage.

Table 2: The 20 largest biases.

Rank	Year	Month	$Bias^E$	Rank	Year	Month	$\frac{Bias^V}{Var(r_{bh})}$ (%)
Panel A: Months with highest bias							
1	2008	October	15.84*	1	2008	November	8.51*
2	2009	March	13.17*	2	2008	October	7.51*
3	2008	November	11.36*	3	2009	January	4.47*
4	2008	December	6.31	4	2006	July	4.40*
5	2008	September	4.29	5	2009	February	4.20*
6	2008	July	3.49*	6	2004	July	4.01*
7	2009	February	3.27	7	2008	September	3.80
8	2009	May	3.06	8	2006	June	3.32*
9	2008	January	3.02	9	2011	August	3.15*
10	2009	January	2.91	10	2007	November	2.93*
Panel B: Months with lowest bias							
108	2009	April	-4.47	108	2009	April	-10.53*
107	2008	June	-2.63	107	2009	May	-5.53*
106	2009	August	-2.36	106	2009	August	-4.91*
105	2011	September	-1.68	105	2004	January	-3.57*
104	2006	January	-0.89	104	2008	August	-3.26*
103	2003	April	-0.81	103	2003	August	-3.16*
102	2003	May	-0.79	102	2003	October	-2.83*
101	2009	December	-0.75	101	2010	April	-2.34*
100	2007	December	-0.74	100	2003	July	-2.29*
99	2004	April	-0.70	99	2011	October	-2.29*

The largest positive (negative) biases in portfolio returns and portfolio variances are reported in panel A (panel B). We observe that the largest significant biases in portfolio returns occur during the years 2008-2009. The results for the largest significant biases in portfolio variance are mixed; however 6 out of 9 significant biases occur between November 2007 and August 2011. "*" denotes 90% significance, that is when the range from 5th percentile to 95th percentile of estimated biases in portfolio statistics for a given year does not contain zero. For presentation purposes, bias in variance is presented as a percentage.

Figure 3: Time series properties of individual stocks' and portfolio returns.



Figures below show the cross-sectional average first-, second-, and third order autocorrelation of each stock's daily return ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$); the first-, second-, and third-order autocorrelation of the daily equally weighted rebalanced 50-stock portfolio computed monthly (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}); and the cross-sectional variance of the average returns, $Var\left(\overline{r_i}\right)$. Estimations obtained for every months using the past one year of daily data and averaged cross-sectionally. Since we construct 10,000 random portfolios, we present only the mean, median and 5th and 95 percentiles of estimates above.

Table 3: Explaining the bias in portfolio average returns and variance.

Variable	$Bias^E$ (1)	$Bias^V$ (2)	$Bias^E$ (3)	$Bias^V$ (4)
Intercept	-0.000026	-0.000009	-0.00027	-0.000010*
	(1.57)	(1.63)	(1.80)	(1.94)
$\overline{\rho_{i1}}$	-0.000379	0.000049	-0.000073	0.000066
	(1.27)	(0.48)	(0.55)	(1.49)
$\overline{\rho_{i2}}$	0.000276	0.000047	0.000180*	0.000032
	(1.02)	(0.51)	(1.68)	(0.91)
$\overline{\rho_{i3}}$	0.000579**	0.000109	0.000188*	0.000049
	(2.34)	(1.31)	(1.67)	(1.31)
ρ_{ew1}	0.000130	0.000007		
	(1.13)	(0.19)		
ρ_{ew2}	-0.000043	-0.000008		
	(0.37)	(0.19)		
ρ_{ew3}	-0.000171*	-0.000008		
	(1.76)	(0.81)		
$Var\left(\overline{r_i}\right)$	3.166255***	-0000026***	3.232233***	0.834733***
	(4.79)	(3.71)	(5.04)	(3.92)
R^2	0.2377	0.0981	0.2281	0.1185

The dependent variable is the bias in portfolio mean returns, $Bias^E$, and the bias in portfolio variance, $Bias^V$. The independent variables are the cross-sectional average first-, second-, and third order autocorrelation of each stock's daily return ($\overline{\rho_{i1}}$, $\overline{\rho_{i2}}$, and $\overline{\rho_{i3}}$); the first-, second-, and third-order autocorrelation of the daily equally weighted rebalanced portfolio computed monthly (ρ_{ew1}, ρ_{ew2}, and ρ_{ew3}); and the cross-sectional variance of the average returns, $Var\left(\overline{r_i}\right)$. The sample contains 108 monthly observations (January 2003 - December 2011). Parameter estimates are reported in columns (1), (3) and (2), (4) for bias in portfolio mean returns and bias in portfolio variance respectively. Absolute values of t-statistics are in parentheses. ***,**, and * denote significance at 1%, 5% and 10% respectively.

5 Appendix

Algorithm 1 CONSTRUCTING SIMULATED PORTFOLIOS AND OBTAINING RESULTS.

1. Randomly select n stocks out of available N without replacement;
2. Given daily price quotes calculate simple return for each stock selected in Step (1), $r_{i,\tau}$, $\tau = 1..T$;
3. Given selection in Step (1) and using Eqs. (1) and (3), calculate decomposed portfolio returns for rebalanced and buy-and-hold approaches respectively;
4. Find expected value and variance for the two portfolios obtained in Step (3) and calculate associated biases using Eqs. (7) and (12) for $\tau = T$.
5. Find additional statistics for individual stock and constructed portfolios:
    (a) first-, second-, and third order autocorrelation of returns of each stock selected in Step (1);
    (b) first-, second-, and third-order autocorrelation of the portfolios constructed in Step (3);
6. Repeat Steps (1)-(5) $M = 10,000$ times;
7. Based on results of Step (6) obtain mean, median, 5th and 95 percentiles for return and variance biases in Step (4) and for additional statistic in Step (5);
8. Repeat Steps (1)-(7) for the next period by:
    (a) (overlapping one year rolling windows) moving the one year data window one month ahead; used for Figures 2 and 3.
    (b) (non-overlapping annual windows) by selecting price quotes from the last trading day of a previous year and to the last trading day of the current year for which the analysis is performed; used for Figure 1 and Table 1.
    (c) (non-overlapping monthly windows) by selecting price quotes from the last trading day of a previous month and to the last trading day of the current month for which the analysis is performed; used for Tables 2 and 3.
9. Repeat Steps (2)-(8) for other data frequencies (but track the selection of stocks in Step (1) in portfolios to avoid sample selection bias).
10. Repeat Steps 1-9 for each $n = 1..N$.

RIC Code	Company Name	RIC Code	Company Name
A.N	Agilent Technologies Inc	AA.N	Alcoa Inc
AAPL.OQ	Apple Inc	ABC.N	AmerisourceBergen Corporation
ABT.N	Abbott Laboratories	ACAS.OQ	American Capital Ltd
ACE.N	ACE Limited	ACN.N	Accenture plc
ADBE.OQ	Adobe Systems Inc	ADI.N	Analog Devices Inc
ADM.N	Archer Daniels Midland Company	ADP.OQ	Automatic Data Processing Inc
ADSK.OQ	Autodesk Inc	AEE.N	Ameren Corporation
AEP.N	American Electric Power Co Inc	AES.N	The AES Corporation
AET.N	Aetna Inc	AFL.N	AFLAC Inc
AGN.N	Allergan Inc	AIG.N	American International Group Inc
AIV.N	Apartment Investment & Management Co	AIZ.N	Assurant Inc
AKAM.Oq	Akamai Technologies Inc	AKS.N	AK Steel Holding Corporation
ALL. N	The Allstate Corporation	ALTR.OQ	Altera Corp
AM.N	American Greetings Corp	AMAT.OQ	Applied Materials Inc
AMCC.OQ	Applied Micro Circuits Corp	AMD.N	Advanced Micro Devices Inc
AMGN.OQ	Amgen Inc	AMT.N	American Tower Corporation
AMZN.OQ	Amazoncom Inc	AN.N	AutoNation Inc
ANF.N	Abercrombie & Fitch Co	APA.N	Apache Corp
APC.N	Anadarko Petroleum Corporation	APD.N	Air Products & Chemicals Inc
APH.N	Amphenol Corporation	APOL.OQ	Apollo Group Inc
ARG.N	Airgas Inc	ASH.N	Ashland Inc
ATI.N	Allegheny Technologies Inc	AVB.N	Avalonbay Communities Inc
AVP.N	Avon Products Inc	AVY.N	Avery Dennison Corporation
AXP.N	American Express Company	AZO.N	AutoZone Inc
BA.N	Boeing Co	BAC.N	Bank of America Corporation
BAX.N	Baxter International Inc	BBBY.OQ	Bed Bath & Beyond Inc
BBT.N	BB&T Corporation	BBY.N	Best Buy Co Inc
BC.N	Brunswick Corporation	BCR.N	CR Bard Inc
BDX.N	Becton Dickinson and Company	BEN.N	Franklin Resources Inc
BHI.N	Baker Hughes Incorporated	BIIB.OQ	Biogen Idec Inc
BK.N	The Bank of New York Mellon Corporation	BLK.N	BlackRock Inc
BLL.N	Ball Corporation	BMC.OQ	BMC Software Inc
BMS.N	Bemis Company Inc	BMY.N	Bristol-Myers Squibb Company
BRCM.OQ	Broadcom Corp	BSX.N	Boston Scientific Corporation
BUT.N	Peabody Energy Corp	BWA.N	BorgWarner Inc
BXP.N	Boston Properties Inc	C.N	Citigroup Inc
CA.OQ	CA Technologies	CAG.N	ConAgra Foods Inc
CAH.N	Cardinal Health Inc	CAM.N	Cameron International Corporation
CAT.N	Caterpillar Inc	CB.N	The Chubb Corporation
CBE.N	Cooper Industries plc	CBG.N	CBRE Group Inc
CCE.N	Coca-Cola Enterprises Inc	CCL.N	Carnival Corporation
CEG.N	Constellation Energy Group Inc	CELG.OQ	Celgene Corporation
CERN.OQ	Cerner Corporation	CHK.N	Chesapeake Energy Corporation
CHRQ.OQ	CH Robinson Worldwide Inc	CI.N	Cigna Corp
CIEN.OQ	CIENA Corp	CINF.OQ	Cincinnati Financial Corp
CL.N	Colgate-Palmolive Co	CLF.N	Cliffs Natural Resources Inc
CLX.N	The Clorox Company	CMA.N	Comerica Incorporated
CME.OQ	Comcast Corporation	CMI.N	CME Group Inc
CMS.N	Cummins Inc	CMSCSA.OQ	CMS Energy Corp
CNP.N	CenterPoint Energy Inc	CNX.N	CONSOL Energy Inc
COF.N	Capital One Financial Corp	COG.N	Cabot Oil & Gas Corporation
COH.N	Coach Inc	COL.N	Rockwell Collins Inc
COP.N	ConocoPhillips	COST.OQ	Costco Wholesale Corporation
CPB.N	Campbell Soup Co	CPWR.OQ	Compuware Corporation
CR.N	Crane Co	CRM.N	Salesforcecom
CSC.N	Computer Sciences Corporation	CSCO.OQ	Cisco Systems Inc
CSX.N	CSX Corp	CTAS.OQ	Cintas Corporation
CTB.N	Cooper Tire & Rubber Co	CTL.N	CenturyLink Inc
CTSH.OQ	Cognizant Technology Solutions Corporation	CTXS.OQ	Citrix Systems Inc
CVC.N	Cablevision Systems Corporation	CVG.N	Convergys Corporation
CVH.N	Coventry Health Care Inc	CVS.N	CVS Caremark Corporation
CVX.N	Chevron Corporation	D.N	Dominion Resources Inc
DD.N	E I du Pont de Nemours and Company	DDR.N	DDR Corp
DDS.N	Dillards Inc	DE.N	Deere & Company
DELL.OQ	Dell Inc	DF.N	Dean Foods Company
DGX.N	Quest Diagnostics Inc	DHI.N	DR Horton Inc
DHR.N	Danaher Corp	DIS.N	Walt Disney Co
DLTR.OQ	Dollar Tree Inc	DLX.N	Deluxe Corp
DNB.N	Dun & Bradstreet Corp	DNR.N	Denbury Resources Inc
DO.N	Diamond Offshore Drilling Inc	DOV.N	Dover Corp

RIC Code	Company Name	RIC Code	Company Name
DOW.N	The Dow Chemical Company	DRI.N	Darden Restaurants Inc
DTE.N	DTE Energy Co	DTV.OQ	DIRECTV Inc
DUK.N	Duke Energy Corporation	DV.N	DeVry Inc
DVA.N	DaVita Inc	DVN.N	Devon Energy Corporation
DYN.N	Dynegy Inc	EA.OQ	Electronic Arts Inc
EBAY.OQ	eBay Inc	ECL.N	Ecolab Inc
ED.N	Consolidated Edison Inc	EFX.N	Equifax Inc
EIX.N	Edison International	EL.N	Estee Lauder Companies Inc
EMC.N	EMC Corporation	EMN.N	Eastman Chemical Co
EMR.N	Emerson Electric Co	EOG.N	EOG Resources Inc
EP.N	El Paso Corp	EQR.N	Equity Residential
EQT.N	EQT Corporation	ESRX.OQ	Express Scripts Inc
ESV.N	Ensco plc	ETFC.OQ	E_TRADE Financial Corporation
ETN.N	Eaton Corporation	ETR.N	Entergy Corporation
EW.N	Edwards Lifesciences Corp	EXC.N	Exelon Corporation
EXPD.OQ	Expeditors International of Washington Inc	EXPE.OQ	Expedia Inc
F.N	Ford Motor Co	FAST.OQ	Fastenal Company
FCX.N	Freeport-McMoRan Copper & Gold Inc	FDO.n	Family Dollar Stores Inc
FDX.N	FedEx Corporation	FE.N	FirstEnergy Corp
FFIV.OQ	F5 Networks Inc	FHN.N	First Horizon National Corporation
FII.N	Federated Investors Inc	FISV.OQ	Fiserv Inc
FITB.OQ	Fifth Third Bancorp	FLIR.OQ	FLIR Systems Inc
FLR.N	Fluor Corporation	FLS.N	Flowserve Corp
FMC.N	FMC Corp	FMCC.OB	Federal Home Loan Mtg
FNMA.OB	Fannie Mae	FRX.N	Forest Laboratories Inc
FTI.N	FMC Technologies Inc	GAS.N	AGL Resources Inc
GCI.N	Gannett Co Inc	GD.N	General Dynamics Corp
GE.N	General Electric Company	GGP.N	Gilead Sciences Inc
GILD.OQ	General Mills Inc	GIS.N	Corning Inc
GLW.N	GameStop Corp	GME.N	Genworth Financial Inc
GNW.N	Google Inc	GPC.N	Genuine Parts Company
GPS.N	Gap Inc	GR.N	Goodrich Corp
GS.N	The Goldman Sachs Group Inc	GT.N	Goodyear Tire & Rubber Co
GWW.N	WW Grainger Inc	HAL.N	Halliburton Company
HAR.N	Harman International Industries Inc	HAS.O	Hasbro Inc
HBAN.OQ	Huntington Bancshares Incorporated	HCBK.OQ	Hudson City Bancorp Inc
HCN.N	Health Care REIT Inc	HCP.N	HCP Inc
HD.N	The Home Depot Inc	HIG.N	Hartford Financial Services Group Inc
HMA.N	Health Management Associates Inc	HNZ.N	H J Heinz Company
HON.N	Honeywell International Inc	HOT.N	Starwood Hotels & Resorts Worldwide Inc
HP.N	Helmerich & Payne Inc	HPQ.N	Hewlett-Packard Company
HRB.N	H&R Block Inc	HRL.N	Hormel Foods Corp
HRS.N	Harris Corp	HSP.N	Hospira Inc
HSY.N	Hershey Co	HUM.N	Humana Inc
IACI.O	IAC_InterActiveCorp	IBM.N	International Business Machines Corp
IFF.N	International Flavors & Fragrances Inc	IGT.N	International Game Technology
INTC.OQ	Intel Corporation	INTU.OQ	Intuit Inc
IP.N	International Paper Co	IPG.N	The Interpublic Group of Companies Inc
IR.N	Ingersoll-Rand Plc	IRM.N	Iron Mountain Inc
ISRG.OQ	Intuitive Surgical Inc	ITT.N	ITT Corporation
ITW.N	Illinois Tool Works Inc	JBL.N	Jabil Circuit Inc
JCI.N	Johnson Controls Inc	JCP.N	J C Penney Company Inc
JDSU.OQ	JDS Uniphase Corporation	JEC.N	Jacobs Engineering Group Inc
JNJ.N	Johnson & Johnson	JNPR.K	Juniper Networks Inc
JNS.N	Janus Capital Group Inc	JNY.N	The Jones Group Inc
JOY	Joy Global Inc	JPM.N	JPMorgan Chase & Co
JWN.N	Nordstrom Inc	K.N	Kellogg Company
KBH.N	KB Home	KEY.N	KeyCorp
KFT.N	Kraft Foods Inc	KIM.N	Kimco Realty Corporation
KLAC.OQ	KLA-Tencor Corporation	KMB.N	Kimberly-Clark Corporation
KMX.N	CarMax Inc	KO.N	The Coca-Cola Company
KR.N	The Kroger Co	KSS.N	Kohls Corp
L.N	Loews Corporation	LEG.N	Leggett & Platt Incorporated
LEH.N	Lehman Brothers	LEN.N	Lennar Corp
LH.N	Laboratory Corp of America Holdings	LIFE.OQ	Life Technologies Corporation
LIZ.N	Liz Claiborne Inc	LLL.N	L-3 Communications Holdings Inc
LLTC.OQ	Linear Technology Corp	LLY.N	Eli Lilly & Co
LM.N	Legg Mason Inc	LMT.N	Lockheed Martin Corporation
LNC.N	Lincoln National Corp	LOW.N	Lowes Companies Inc
LPX.N	Louisiana-Pacific Corp	LSI.N	LSI Corporation

RIC Code	Company Name	RIC Code	Company Name
LTD.N	Limited Brands Inc	LUK.N	Leucadia National Corp
LUV.N	Southwest Airlines Co	LXK.N	Lexmark International Inc
MAR.N	Marriott International Inc	MAS.N	Masco Corporation
MAT.O	Mattel Inc	MBI.N	MBIA Inc
MCD.N	McDonalds Corp	MCHP.OQ	Microchip Technology Inc
MCK.N	McKesson Corporation	MCO.N	Moodys Corp
MDP.N	Meredith Corp	MDT.N	Medtronic Inc
MET.N	MetLife Inc	MHP.N	The McGraw-Hill Companies Inc
MHS.N	Medco Health Solutions Inc	MKC.N	McCormick & Co Inc
MMC.N	Marsh & McLennan Companies Inc	MMM.N	3M Co
MO.N	Altria Group Inc	MOLX.OQ	Molex Inc
MON.N	Monsanto Co	MOS.N	The Mosaic Company
MRK.N	Merck & Co Inc	MRO.N	Marathon Oil Corporation
MS.N	Morgan Stanley	MSFT.OQ	Microsoft Corporation
MTB.N	M&T Bank Corporation	MTG.N	MGIC Investment Corp
MTW.N	Manitowoc Co Inc	MU.OQ	Micron Technology Inc
MUR.N	Murphy Oil Corporation	MWV.N	MeadWestvaco Corporation
MWW	Monster Worldwide Inc	MYL.OQ	Mylan Inc
NBL.N	Noble Energy Inc	NBR.N	Nabors Industries Ltd
NCR.N	NCR Corp	NDAQ.OQ	Nasdaq OMX Group Inc
NE.N	Noble Corp	NEM.N	Newmont Mining Corp
NFLX.OQ	Netflix Inc	NFX.N	Newfield Exploration Co
NI.N	NiSource Inc	NKE.N	Nike Inc
NOC.N	Northrop Grumman Corporation	NOV.N	National Oilwell Varco Inc
NRG.N	NRG Energy Inc	NSC.N	Norfolk Southern Corp
NTAP.OQ	NetApp Inc	NTRS.OQ	Northern Trust Corporation
NU.N	Northeast Utilities	NUE.N	Nucor Corporation
NVDA.OQ	NVIDIA Corporation	NVLS.OQ	Novellus Systems Inc
NWL.N	Newell Rubbermaid Inc	NWSA.O	News Corp
NYT.N	The New York Times Company	ODP.N	Office Depot Inc
OI.N	Owens-Illinois Inc	OKE.N	ONEOK Inc
OMC.N	Omnicom Group Inc	OMX.N	OfficeMax Incorporated
ORCL.OQ	Oracle Corporation	ORLY.OQ	OReilly Automotive Inc
OXY.N	Occidental Petroleum Corporation	PAYX.OQ	Paychex Inc
PBCT.OQ	Peoples United Financial Inc	PBI.N	Pitney Bowes Inc
PCAR.OQ	PACCAR Inc	PCG.N	PG&E Corp
PCL.N	Plum Creek Timber Co Inc	PCLN.OQ	pricelinecom Incorporated
PCP.N	Precision Castparts Corp	PDCO.OQ	Patterson Companies Inc
PEG.N	Public Service Enterprise Group Inc	PEP.N	Pepsico Inc
PFE.N	Pfizer Inc	PFG.N	Principal Financial Group Inc
PG.N	Procter & Gamble Co	PGN.N	Progress Energy Inc
PGR.N	Progressive Corp	PH.N	Parker Hannifin Corporation
PHM.N	PulteGroup Inc	PKI.N	PerkinElmer Inc
PLD.N	Prologis Inc	PLL.N	Pall Corp
PMCS.OQ	PMC-Sierra Inc	PMTC.OQ	Parametric Technology Corporation
PNC.N	PNC Financial Services Group Inc	PNW.N	Pinnacle West Capital Corporation
POM.N	Pepco Holdings Inc	PPG.N	PPG Industries Inc
PPL.N	PPL Corporation	PRGO.OQ	Perrigo Co
PRU.N	Prudential Financial Inc	PSA.N	Public Storage
PWER.OQ	Power-One Inc	PWR.N	Quanta Services Inc
PX.N	Praxair Inc	PXD.N	Pioneer Natural Resources Co
QCOM.OQ	QUALCOMM Incorporated	QLGC.OQ	QLogic Corp
R.N	Ryder System Inc	RAI.N	Reynolds American Inc
RDC.N	Rowan Companies Inc	RF.N	Regions Financial Corp
RHI.N	Robert Half International Inc	RIG.N	Transocean Ltd
RL.N	Ralph Lauren Corporation	ROK.N	Rockwell Automation Inc
ROP.N	Roper Industries Inc	ROST.OQ	Ross Stores Inc
RRC.N	Range Resources Corporation	RRD.OQ	RR Donnelley & Sons Company
RSG.N	Republic Services Inc	RSH.N	RadioShack Corp
RTN.N	Raytheon Co	S.N	Sprint Nextel Corp
SANM.OQ	Sanmina-SCI Corp	SBUX.OQ	Starbucks Corporation
SCG.N	SCANA Corp	SE.N	Spectra Energy Corp
SEE.N	Sealed Air Corporation	SHLD.OQ	Sears Holdings Corporation
SHW.N	The Sherwin-Williams Company	SIAL.OQ	Sigma-Aldrich Corporation
SJM.N	The J M Smucker Company	SLB.N	Schlumberger Limited
SLE.N	Sara Lee Corp	SLM.O	SLM Corporation
SNA.N	Snap-on Inc	SNDK.OQ	SanDisk Corp
SNV.N	Synovus Financial Corp	SO.N	Southern Company
SPG.N	Simon Property Group Inc	SPLS.OQ	Staples Inc
SRCL.OQ	Stericycle Inc	SRE.N	Sempra Energy

RIC Code	Company Name	RIC Code	Company Name
SSP.N	The E W Scripps Company	STI.N	SunTrust Banks Inc
STJ.N	St Jude Medical Inc	STR.N	Questar Corporation
STT.N	State Street Corp	STZ.N	Constellation Brands Inc
SUN.N	Sunoco Inc	SVU.N	SUPERVALU Inc
SWK.N	Stanley Black & Decker Inc	SWN.N	Southwestern Energy Co
SWY.N	Safeway Inc	SYK.N	Stryker Corp
SYMC.OQ	Symantec Corporation	SYY.N	Sysco Corp
T.N	AT&T Inc	TAP.N	Molson Coors Brewing Company
TE.N	TECO Energy Inc	TER.N	Teradyne Inc
TEX.N	Terex Corp	TGT.N	Target Corp
THC.N	Tenet Healthcare Corp	TIE.N	Titanium Metals Corporation
TIF.N	Tiffany & Co	TIN.N	Temple-Inland Inc
TJX.N	The TJX Companies Inc	TLAB.OQ	Tellabs Inc
TMK.N	Torchmark Corp	TMO.N	Thermo Fisher Scientific Inc
TNB.N	Thomas & Betts Corp	TROW.OQ	T Rowe Price Group Inc
TSN.N	Tyson Foods Inc	TSO.N	Tesoro Corporation
TSS.N	Total System Services Inc	TUP.N	Tupperware Brands Corporation
TWX.N	Time Warner Inc	TXN.N	Texas Instruments Inc
TXT.N	Textron Inc	TYC.N	Tyco International Ltd
UIS.N	Unisys Corporation	UNH.N	Unitedhealth Group Inc
UNM.N	Unum Group	UNP.N	Union Pacific Corporation
UPS.N	United Parcel Service Inc	URBN.OQ	Urban Outfitters Inc
USB.N	US Bancorp	UTX.N	United Technologies Corp
VAR.N	Varian Medical Systems Inc	VFC.N	VF Corporation
VLO.N	Valero Energy Corporation	VMC.N	Vulcan Materials Company
VNO.N	Vornado Realty Trust	VRSN.OQ	VeriSign Inc
VTR.N	Ventas Inc	VZ.N	Verizon Communications Inc
WAG.N	Walgreen Co	WAT.N	Waters Corp
WDC.N	Western Digital Corp	WEC.N	Wisconsin Energy Corp
WFC.N	Wells Fargo & Company	WFR.N	MEMC Electronic Materials Inc
WFT.N	Weatherford International Ltd	WHR.N	Whirlpool Corp
WLP.N	WellPoint Inc	WM.N	Waste Management Inc
WMB.N	Williams Companies Inc	WMT.N	Wal-Mart Stores Inc
WOR.N	Worthington Industries Inc	WPI.N	Watson Pharmaceuticals Inc
WPO.N	The Washington Post Company	WY.N	Weyerhaeuser Co
WYNN.OQ	Wynn Resorts Ltd	X.N	United States Steel Corp
XEL.N	Xcel Energy Inc	XL.N	XL Group plc
XLNX.OQ	Xilinx Inc	XOM.N	Exxon Mobil Corporation
XRAY.OQ	DENTSPLY International Inc	XRX.N	Xerox Corp
YHOO.OQ	Yahoo! Inc	YUM.N	Yum! Brands Inc
ZION.OQ	Zions Bancorp	ZMH.N	Zimmer Holdings Inc