Corporate Policy when Equity and Bond Holders Price Risk Differently*

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Abstract

In a dynamic investment and financing model, we account for differences between equity and corporate bond holders' pricing of macroeconomic risk. In line with anecdotal and empirical evidence, we calibrate the bond investor's price of risk to be unconditionally higher than the equity investor's, as well as volatile and independent of the macroeconomy. Relative to a counterfactual scenario where both investors price risk identically, average market (book) leverage is 2.8 (3.3) percentage points lower, which reveals a new quantitatively significant channel to address the under-leverage puzzle. Also, in the scenario with heterogenous risk pricing, firms issue equity more frequently and invest less.

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1. Introduction

A well-known puzzle in the capital structure literature is that firms appear to have suboptimally low leverage ratios.[1] Recent theoretical studies introduce aversion to macroeconomic risk to explain why leverage ratios appear to be low (Bhamra, Kuehn & Strebulaev, 2010; Chen, 2010). In these models, distress costs associated with corporate bond defaults are more likely to occur during economic downturns. Therefore, in the valuation of corporate bonds, macroeconomic risk aversion generates a premium for the extent to which distress costs correlate with the macroeconomy. This pushes the cost of debt higher, which results in lower optimal leverage.

However, these models adopt a single representative investor framework, which constrains bond and equity investors to be identical in terms of how they price macroeconomic risk, whereas empirical and anecdotal evidence suggests that risk is priced differently across corporate bond and equity markets (e.g.: Titman, 2002; He & Xiong, 2013; Choi & Kim, 2016). Furthermore, the corporate finance literature emphasises the existence of conflicts between equity and corporate bond holders' interests, which implicitly subsumes their heterogeneity. Therefore, it is only natural to ask: how does heterogeneity in investors' pricing of risk affect corporate policy? More importantly, what is the quantitative impact on the optimal leverage choice? To answer these questions, we employ a dynamic financing and investment model and allow two different investors to price the risk in equity and corporate bonds separately. We show that heterogeneity in the two investors' pricing of macroeconomic risk has a non-trivial effect on corporate policy, especially optimal leverage.

Our two-investor framework follows logically from the implicit separation between equity and bond holders, as observed in the numerous studies that examine the agency costs of equity and bond holder conflicts.[2] These conflicts arise as equity holders (or their appointed managers) choose to maximise the value of equity, whereas bond holders prefer firm value maximisation. In this context, equity holders

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[1] More specifically, the expected tax shield benefits of debt for the average US firm outweigh the expected distress costs (e.g. deadweight costs associated with bankruptcy proceedings or the fire sale of assets during liquidation), such that an increase in leverage could improve firm value. For example, Graham (2000) finds that the average US firm could double the total tax shields benefits and improve firm value by as much as 9.7%, if the firm were to increase leverage to the point where the marginal tax shield benefit equals the marginal cost of distress.

[2] See, for example: Childs, Mauer & Ott (2005); Titman & Tsyplakov (2007); Gamba, Aranda & Saretto (2015); Chen & Manso (2017).

execute corporate policy and take bond prices as given. On the other hand, bond investors price corporate bonds and take equity holders' decision as given. There is no coordination to maximise firm value, as a single representative investor would.[3] Therefore, our two-investor framework is more consistent with the conflicting objectives of equity and bond holders. More importantly, it allows us to consider differences in how risk is priced across equity and corporate bond markets when investors' aversion to macroeconomic risk is heterogenous.

In our framework, heterogeneity in aversion to macroeconomic risk means that, for each unit of equivalent risk, investors demand different expected rates of return. We thus implicitly relax the standard assumption that corporate bond and equity markets are integrated.[4] The relaxation of this assumption is justified by ample empirical and anecdotal evidence and we consider two key dimensions that describe the lack of integration. First, there is evidence that risk premia in the corporate bond market are, on average, larger than in the equity market, as discussed by Titman (2002).[5] Second, Titman also suggests that the difference in risk premia between the two markets varies through time. If the difference were constant, it would require strong co-movement in risk premia, which is not what Collin-Dufresne, Goldstein & Martin (2001) observe. Instead, they find that a significant proportion of the variation in credit spread changes in the corporate bond market is unrelated to the equity market.[6]

For our baseline model, we adopt a partial equilibrium dynamic setup similar to Gomes & Schmid (2010), in which firms make investment, financing and default decisions. Our key distinguishing feature is that we have two separate investors for bonds and equity, each with a different level of risk aversion

[3] In the literature on the costs of agency conflicts between equity and bond holders, firm value maximisation produces the first best outcome. Therefore, a single representative household with claims to both the equity and debt of a firm would not harm their interests by pursuing equity-only maximisation.

[4] For two markets to be integrated, an investor must earn the same expected return in each market for an equivalent amount of risk (Titman, 2002). This assumption implies that a single representative investor is sufficient to capture how risk is priced in both markets. However, if risk is priced differently in each market (i.e. they are not integrated), the use of a single representative investor is no longer valid.

[5] Titman (2002) references a 2001 working paper version of Huang & Huang (2012). He suggests that an equity risk premium of 15% is needed to justify the credit spread on BBB-rated bonds, whereas the observable equity risk premium is only 9%. There are other recent studies which are unable to reconcile risk premia in bond and equity markets, such as Choi & Kim (2016).

[6] Collin-Dufresne et al. (2001) allude that this is related to supply conditions specific to the corporate bond market. This is consistent with Titman's (2002) suggestion that capital supply conditions are a potential source of time variation in the degree of integration between equity and corporate bond markets.

with respect to aggregate macroeconomic risk, such that average risk premia can differ across equity and corporate bond markets. Furthermore, the bond holder's risk aversion can vary independently of the aggregate state, which leads to time-varying divergence in risk premia between equity and corporate bond markets, consistent with Collin-Dufresne et al. (2001). We solve our model numerically and calibrate parameters such that our simulated cross-section of firms yields moments consistent with empirical observations. Importantly, the equity holder's risk aversion is set to match the equity Sharpe ratio, which leaves the bond holder's risk aversion process to be calibrated to match leverage dynamics.

We find that the bond holder's aversion to macroeconomic risk must be unconditionally higher, as well as volatile,[7] in order to match the target moments and obtain lower optimal leverage.[8] We perform counterfactual analysis to gauge the magnitude of the reduction in leverage. Relative to an economy with integrated markets, if the bond holder's risk aversion is unconditionally three times that of the equity holder, average optimal market (book) leverage is 2.1 (2.4) percentage points lower. Additionally, if we introduce variation in the bond holder's risk aversion, average optimal market (book) leverage is reduced a further 0.7 (0.9) percentage points. In both instances, there is also a modest increase in the frequency of equity issuance. All these counterfactual results are consistent across multiple simulations.

To understand the mechanism behind these results, note that firms are more likely to default on corporate bonds when the macroeconomy is weak. As the bond holders become more risk averse with respect to macroeconomic risk, they demand relatively more compensation for default risk. For each additional unit of debt, the marginal cost of borrowing rises at a significantly greater rate than the marginal tax shields benefits. Therefore the optimal level of debt is lower.

The decrease in optimal leverage is more pronounced in the weaker economic states. In these states, the increase in the bond investor's price of risk matters more since financial distress risk is higher. When economic productivity is below its long-run mean, average optimal market (book) leverage is 3.0 (3.2) percentage points lower relative to the case where markets are integrated. When above the long-run

[7] We calibrate the risk aversion of the bondholder to vary independently of the aggregate state. Therefore, while its unconditional mean is fixed at 150, there is 1.56% chance each period it will rise to 915 (worst possible outcome for the cost of debt) or fall to 11 (best possible outcome for the cost of debt).

[8] The effect of the unconditional level of risk aversion is more important. In Table 3, we show that the magnitude of the effect of variation in bond holder's risk aversion is sensitive to the unconditional level of risk aversion.

aggregate productivity mean, average optimal market (book) leverage is 1.7 (2.4) percentage points lower[9].

We also find that investment does not necessarily decrease as the bond holder's aversion to macroeconomic risk increases. In our model, equity issuance is costly and firms prefer to finance investment with debt to secure interest tax shields. All else equal, firms with a low debt capacity can only finance low levels of capital through borrowing. However, due to decreasing returns to scale, the marginal benefit of investment is higher for low levels of capital, such that it is worthwhile to incur equity issuance costs to fund further investment. Other firms with higher debt capacity (but still not enough to match the investment expenditure of the low debt capacity firms) will forego investment to avoid equity issuance costs. As the bond holder's price of risk increases, debt capacity decreases. Some firms will now find it optimal to issue equity and will invest more than before. Others' debt capacity will reduce to a point where debt finances less investment than before, but it is not optimal to issue equity to make up the shortfall in investment. In our counterfactual analysis, we observe that, when markets are integrated, firms expand their productive capacity in 6.4% of our firm-year observations. This drops to 5.7% when markets are not integrated, which suggests that the decrease in investment is the stronger effect.

The paper is organised as follows. In section 2, we discuss related literature and our contribution. In section 3, we present a simple two-period model for intuition. The dynamic model follows in section 4. We present results in section 5 and conclude in section 6.

2. Related Literature

To our knowledge, we are the first to examine the theoretical implications of heterogeneity in the pricing of macroeconomic risk for capital structure and gauge its potential to address the under-leverage puzzle. Our work is related to the literature which incorporates macroeconomic risk to model firms' optimal financing decisions (Hackbarth, Miao, & Morellec, 2006; Bhamra et al., 2010; Chen, 2010). Our main

[9] For brevity, we are reporting the combined the effects of a higher as well as volatile price of risk for the bond holder. Separated results are shown in Table 3.

contribution is that we account for the evidence that equity and corporate bond markets are not integrated, such that bond and equity investors can demand different return premia for macroeconomic risk. With this feature we provide a new angle to study the under-leverage puzzle.

We do note that our model has some shortcomings with respect to some concerns raised by Bhamra et al. (2010). They argue that optimal leverage at refinancing points in the model is not equivalent to observed leverage in the data, since firms rarely restructure their debt in the data[10]. Therefore, while their model generates aggregate market leverage of 28% at refinancing points, this figure increases to 40% when they consider all their simulated data points[11]. With our one-period bond specification, firms are always at their refinancing points, hence we are unable to address their concern. Nevertheless, our key contribution is to gauge the differential effect of heterogeneous risk pricing on optimal leverage, rather than to match aggregate leverage.

The independent variability in risk premia in the corporate bond market is a feature that is associated with dynamic market-specific supply conditions (Collin-Dufresne et al., 2001; Titman, 2002). Therefore, one can interpret our results as evidence that credit supply conditions have an impact on capital structure decisions. Hugonnier, Malamud & Morellec (2015) take the supply-side approach to theoretically examine the impact of credit market frictions on capital structure. Their approach differs from ours, in that they focus on individual firms' access to credit. In their model, credit market access is random and firm-specific, while our credit supply shock is aggregate and directly reflected in the debt investors' price of risk.

Our work also has important implications for theoretical studies which examine agency conflicts between equity and bondholders (Childs, Mauer & Ott, 2005; Titman & Tsyplakov, 2007; Gamba, Aranda & Saretto, 2015; Chen & Manso, 2017). These studies focus on the consequences of equity holders' pursuit of equity value maximisation, as opposed to firm value maximisation, which is preferable

[10] Leary & Roberts (2005) suggest that restructuring costs prevent firms from refinancing frequently.

[11] In their model, firms are more financially healthy in good economic states when equity value rises and market leverage falls. They can thus more readily overcome refinancing costs and adjust their leverage upwards. In bad economic states, firms must wait for equity value to drift downwards more before it is optimal to refinance, such that observed market leverage increases more in bad states than in good states.

to bond holders. Our framework has the capacity to assess these consequences in the context of an economy where equity and corporate bond markets are not integrated.

Finally, we note that we take the evidence on equity and corporate bond market integration as given. Our motivation is primarily based on asset pricing studies which investigate the behaviour of prices in the corporate bond market relative to the equity market. Huang & Huang (2012) find that credit spreads appear too large relative to equity premia. Building on Collin-Dufresne et al. (2001), Kapadia & Pu (2012) confirm that co-movement between debt and equity markets is weak, which they attribute to frictions that limit arbitrage. Choi & Kim (2016) find that, for risk factors common to corporate bond and equity markets, return premia are higher in the bond market. Other evidence which suggests the two markets are segmented is centred around institutional frictions that limit capital mobility (He & Xiong, 2013; Bodnaruk & Rossi, 2016). At best, we provide some indirect evidence against market integration, given our counterfactual analysis shows that target moments are better matched when equity and corporate bond investors price risk differently.

3. Static Model

3.1 Setup

To illustrate the key mechanism of our dynamic model, we first present a simple two-period model with one equity and one bond investor. Suppose that, at time 0, the equity investor sets up a firm and can choose an amount of capital K that generates the following payoff at time 1:

$$\Pi(K, x) = (1 - \tau)e^x K^\alpha \tag{S1}$$

Here, τ is the corporate tax rate, $x \sim N(0, \sigma_x^2)$ is a productivity shock and α determines the production function curvature. We assume $\alpha < 1$, such that there are decreasing returns to scale. Capital fully depreciates at time 1.

The equity investor can finance the capital expenditure with a combination of her own money and debt. To obtain debt finance, she can issue a one-period bond of face value B, which promises a tax-deductible coupon payment c at time 1. If the firm cannot meet its debt obligations, the equity holder defaults and receives zero payoff. Therefore, the equity holder's claim is:

$$P_e(K,B,x) = \max\{0, \Pi(K,x) - (1+c)B + \tau cB\} \tag{S2}$$

For any combination of K and B, we define the default boundary value of x as:

$$x^*\colon \Pi(K,x^*) - (1+c)B + \tau cB = 0 \tag{S3}$$

The bond investor's payoff is contingent on the realised value of x relative to x^*. If $x < x^*$, the equity holder defaults and the bond investor receives a fraction $(1-\xi)$ of the after-tax operating profits $\Pi(K,x)$, where $\xi \in [0,1]$ represents the deadweight loss from the cost of bankruptcy proceedings. Otherwise, the bond investor receives the promised face value amount plus the coupon:

$$P_b(K,B,x) = (1+c)B\mathbf{1}_{x\geq x^*} + (1-\xi)\Pi(K,x)\mathbf{1}_{x<x^*} \tag{S4}$$

The equity holder chooses K and B to maximise the value of equity:

$$V_e = \max_{\mathrm{K,B}}\{-K + B + E[M(x)P_e(K,B,x)]\} \tag{S5}$$

For now, $M(x)$ is the pricing kernel of a representative investor sensitive to the firm's productivity risk. Suppose we also use the representative investor's pricing kernel to price the bond claim at par:

$$B = E[M(x)P_b(K,B,x)] \tag{S6}$$

For (S6) to hold, the coupon rate must be endogenous and satisfy:

$$1 + c = \frac{1 - E[M(x)R\mathbf{1}_{x<x^*}]}{E[M(x)\mathbf{1}_{x\geq x^*}]} \tag{S7}$$

Here R is the proportion of face value recovered upon default i.e. $R = (1-\xi)\Pi(K,x)/B$. Note that c and x^* are simultaneously determined for any given choice of K and B. This reflects the fact that c affects the equity holders' ability to meet the debt obligation, which in turn determines x^*. At the same time, x^* determines the level of default risk for which the bond holder demands compensation via c. It can be shown that, so long as $R < (1 + E[M(x)]^{-1})$, c is increasing in x^*.

When we substitute (S2), (S4) and (S6) into (S5), the equity maximisation problem can be expressed as follows:

$$V_e = \max_{\mathrm{K,B}}\{-K + E[M(x)\Pi(K,x)] + \tau cBE[M(x)\mathbf{1}_{x\geq x^*}] - \xi E[M(x)\Pi(K,x)\mathbf{1}_{x<x^*}]\} \qquad (S8)$$

This can be broken down into three components:

$V_e = \max_{\mathrm{K,B}} \ldots$

$-K + E[M(x)\Pi(K,x)]$	*V1: Firm value maximisation*
$+\tau cBE[M(x)\mathbf{1}_{x\geq x^*}]$	*V2: Tax shield benefits*
$-\xi E[M(x)\Pi(K,x)\mathbf{1}_{x<x^*}]$	*V3: Deadweight loss from bankruptcy*

If there are no taxes (i.e. $\tau = 0$) and no deadweight bankruptcy costs (i.e. $\xi = 0$), then corporate structure is irrelevant and the optimisation depends only on K, as would be the case in a Modigliani & Miller (1958) framework. Given that in the real world corporations pay taxes and bankruptcy is costly, many studies have attempted to explain capital structure as a trade-off between tax benefits and distress costs, but with limited success (Graham, 2000). Therefore, more recent models introduce a representative investor who is averse to aggregate risk (e.g.: Chen, 2010). In this simple framework, we capture this feature with the pricing kernel $M(x)$, which allocates more weight to payoffs when $x < x^*$, such that distress costs become relatively larger on a risk-adjusted basis and optimal B is lower.

However, with a single representative investor, the price of risk for equity and bonds is the same which means that markets are implicitly integrated. In order to do away with this assumption, we specify a pricing kernel function similar to that of Berk, Green & Naik (1999), that allows heterogeneity in the pricing of risk:

$$M_i(x) = \exp\left\{-r_f - \gamma_i x - \frac{1}{2}\gamma_i^2\sigma_x^2\right\} \qquad (S9)$$

Subscript i differentiates the equity holder (e) from the bond holder (b). Both investors have the same intertemporal discount rate i.e. $E[M_i(x)] = \exp\{-r_f\}$, where r_f is the risk-free rate. They differ in how they price risk factor x via risk aversion parameter γ_i.

For a given combination of K and B, we discuss the impact of an increase in γ_b relative to the scenario where markets are integrated (i.e. $\gamma_b = \gamma_e$):

1. All else fixed, the bondholders demand a higher coupon rate
2. The higher coupon rate reduces the firm's ability to meet debt obligations such that x^* increases
3. The coupon rate further increases with x^*
4. Adjustments in steps 2 and 3 repeat until the coupon rate is consistent with the new x^*

We denote the new coupon rate c_b and the new default boundary x_b^* to differentiate them from their values under integrated markets. We now restate the breakdown of the equity value maximisation problem when $\gamma_b > \gamma_e$:

$$V_e = \max_{\text{K,B}} \ldots$$

$-K + E[M_e(x)\Pi(K,x)]$	*V1: Firm value maximisation*
$+\tau c_b B E\left[M_e(x)\mathbf{1}_{x \geq x_b^*}\right]$	*V2: Tax shield benefits*
$-\xi E\left[M_b(x)\Pi(K,x)\mathbf{1}_{x < x_b^*}\right]$	*V3: Deadweight loss from bankruptcy*
$+(1 + c_b)BE\left[(M_b(x) - M_e(x))\mathbf{1}_{x \geq x_b^*}\right]$	*V4: Difference in valuation of repayment*
$+E\left[(M_b(x) - M_e(x))\Pi(K,x)\mathbf{1}_{x < x_b^*}\right]$	*V5: Difference in valuation default payout*

This breakdown illustrates how corporate policy determinants change when bond holders price risk differently, namely when $\gamma_b > \gamma_e$.[12] Relative to the case where $\gamma_e = \gamma_b$, we have, as noted before, a higher coupon rate c_b and higher default boundary x_b^*. The deadweight loss from bankruptcy is now priced from the perspective of the bond holder, which is also reflected in the higher coupon rate. Most important, there is a valuation wedge with respect to the contingent claims of the bond holder (i.e. components *V4* and *V5*). A more risk averse bond holder attaches relatively less weight to scenarios where the principal is repaid and relatively more weight to the firm's profit in the event of default.

[12] We focus on this case because it consistent with evidence that risk premia are higher in the bond market. The $\gamma_b < \gamma_e$ case would simply have the opposite implications.

3.2 Analysis

Since the two-period model is only designed for intuition, we solve it with arbitrary parameter values.[13] For our first analysis, we fix the choice of K and focus on the choice of optimal B under three scenarios:

1. Risk neutrality i.e. $\gamma_b = \gamma_e = 0$
2. Risk averse investors with integrated markets i.e. $\gamma_b = \gamma_e > 0$
3. Investors price risk differently i.e. $\gamma_b > \gamma_e$ (with the value of γ_e unchanged)

In Figure 1, we show how the marginal benefit and cost curves of debt change under each scenario. In each of the three panels, we observe that, for low levels of leverage, the marginal cost of debt is near zero, while that marginal tax shield of debt is positive. Once leverage is sufficiently high, debt becomes costlier and bond holders demand a higher coupon rate. Although the marginal interest tax shield rises with the coupon rate, the marginal cost of debt increases at a higher rate still. In panel II, we observe the effect of risk averse investors under integrated markets. Relative to panel I where the investors are risk neutral, the marginal cost curve is steeper, which results in lower optimal leverage.[14]

In panel III of Figure 1, markets are no longer integrated and the bond holder is now more risk averse, hence the marginal cost curve is even steeper such that optimal leverage is lower relative to panel II. A noteworthy feature is that the marginal cost of debt is no longer purely determined by the risk of financial distress. Part of it is now attributed to the net effect of the wedge in the valuation of the bond holder's claims (more specifically the combined effect of the partial derivatives of *V4* and *V5* with respect to B).

[Figure 1]

We next consider the sensitivity of the optimal choice for both K and B across different ratios of γ_b to γ_e. In the first panel of Figure 2, we observe that, as the bond holder becomes more risk averse, optimal B falls, yet K is largely unchanged. With the aid of panels II, III and IV, we can offer some

[13] The model still needs to be solved numerically with discrete grids for B and K, due to the complication that $x^*(K, B)$ and $c(K, B)$ need to be simultaneously determined and a closed form solution is not possible. Nevertheless, relative to the dynamic model, we can use much finer grids and provide a richer intuition.

[14] This is a simplified illustration of the key leverage-reduction mechanism in models such as Chen's (2010).

insight. We show what happens to equity value, credit spreads[15] and default probability[16] as policies are optimally adjusted to accommodate the more risk averse bond holder. We compare this against the counterfactual scenario where the equity holder fails to adjust and instead maintains the same optimal policy as when $\gamma_b = \gamma_e$. The key insights are as follows:

- As the bond holder demands more compensation for the risk of distress, a lower debt level is required to reduce the firm's default probability.
- Credit spreads change very little when corporate policy is optimally adjusted. One might think credit spreads should fall with the lower risk of default. However, since the bond holder's price of risk is increasing, a lower level of default risk does not necessarily imply a lower credit spread.
- Equity value is protected when the equity holder adjusts optimal policy. However, it decreases in γ_b and is lower relative to where it was when $\gamma_b = \gamma_e$. A lower optimal level of debt limits the amount of value the equity holder can extract from interest tax shields.
- As there are no equity issuance cost frictions, the equity holder can simply replace the shortfall in debt finance with equity and thus maintain the same optimal investment policy. However, in the process, interest tax shield benefits are foregone. Less value can be created through the financing decision, while the investment decision is unchanged.

[Figure 2]

Given equity issuance is costly in reality, we next consider how this would impact the outcomes shown in Figure 2. We therefore repeat the analysis shown in Figure 2, but with the addition of fixed and linear costs of equity issuance (we use also this specification in the dynamic model). If $K > B$, the firm must pay the fixed cost of equity issuance, as well as a cost proportional to $(K - B)$. The outcome is illustrated in Figure 3.

[Figure 3]

As equity becomes relatively costly to issue, the firm prefers debt even more. Essentially, debt now has the added benefit in that it allows the firm to avoid equity issuance costs. In theory, this would be

[15] Credit spreads are the difference between the endogenous coupon rate $c(K, B)$ and the riskless rate r_f.

[16] Default probability is computed as $\Pr(x < x^*(K, B))$ i.e. the likelihood that x will be below the default boundary.

reflected in an upward shift in debt's marginal benefit curve in Figure 1. Nevertheless, the optimal level of debt still decreases as γ_b/γ_e rises. The extent to which debt capacity acts as a constraint on investment depends on the following considerations:

- When the level of capital that can be financed with debt is already high, the marginal benefit of additional units of capital is lower, due to decreasing returns to scale.
- The decision to issue equity to finance further investment depends on whether the additional units of capital can generate enough expected profit to justify the costs of equity issuance.[17]

In the first panel of Figure 3, the optimal choice is to fully finance investment with debt when the ratio γ_b/γ_e is relatively low. Here, the optimal level of debt is higher and allows the firm to finance a relatively higher level of capital. The marginal benefit of additional capital is too low to incentivise the firm to raise equity for extra investment. Once the bond holder is sufficiently risk averse, the firm can only finance smaller amounts of capital with debt. Therefore, the firm issues equity to capture the higher marginal benefit of capital. Relative to Figure 2, we also observe that the firm incurs higher credit spreads, as debt now has an extra issuance-cost-saving benefit to trade off against the costs of distress risk i.e. the firm can tolerate a higher level of default probability, which leads to a higher coupon rate.

Overall, the message from Figure 3 suggests that, if the bond holder's price of risk is higher, the level of investment should decrease, so long as we account for the cost of equity issuance. However, before we move on to the dynamic model, we must cross one last bridge of intuition with regard to investment. The dynamic model will allow us to consider a heterogenous cross section of firms which, every period, will be faced with corporate decisions that are very similar in essence to the ones considered in the static model. Part of this heterogeneity will be reflected in differences in net worth i.e. the sum of current profits, assets and liabilities. We can capture this in the static model by adding net worth variable A to the equity maximisation problem:

$$V_e = \max_{\text{K,B}} \{A - K + B + E[M_e(x)P_e(K, B, x)]\} \tag{$S10$}$$

[17] This is a simplified version of events, but it captures the essence of the trade-off. Further consideration must also be given to the fact that the coupon rate is decreasing in K. So as the firm invests more, there is also a decrease in the slope of the marginal cost curve of debt. However, this is not sufficient to offset the effect of the increase in γ_b on the marginal cost of debt.

We now consider how optimal corporate policies vary across different values of A and we continue to assume that equity issuance incurs a fixed and proportional cost. We compare the case where markets are integrated ($\gamma_b = \gamma_e$) with the case where they are not ($\gamma_b > \gamma_e$). Figure 4 shows the comparative static results.

[Figure 4]

In the first panel of Figure 4, we observe that, conditional on A, optimal K with integrated markets is not always higher. This pattern can be attributed to the financing decisions shown in panels II and III. For low levels of A, the firm is more reliant on external finance. With limited borrowing capacity, the level of capital that can be fully financed with debt is relatively low. Therefore, it is worthwhile to issue equity to capture the relatively higher marginal benefits of capital. However, as A increases, the combined amount of A and B can fund a higher level of capital, such that it is no longer worthwhile to issue equity. Therefore, optimal K sharply drops as the firm switches to debt as the only source of external finance. Eventually, as A increases further, debt capacity no longer constrains optimal investment and the firm chooses a higher level of capital. When $\gamma_b > \gamma_e$, it takes a higher level of A for the firm to adopt a debt-only financing policy, since the amount it can borrow is limited by the steeper marginal cost curve of debt. Therefore, the firm uses equity financing over a greater range of A and, for a subset of this region, it will invest more relative to the scenario where $\gamma_b = \gamma_e$.

4. Dynamic Model

In this section, we present a partial equilibrium dynamic model in discrete time. Our framework shares many features with Gomes & Schmid (2010).

4.1 Firms

The economy consists of multiple firms, indexed by subscript j. At time t, a firm's after-tax operating profits $\Pi_{j,t}$ are given by:

$$\Pi_{j,t} = (1-\tau)(\exp\{x_t + z_{j,t}\}\, K_{j,t}^{\alpha} - f) \tag{D1}$$

Parameter τ represents the corporate tax rate, α is the capital share of output and f represents fixed costs of production. As for the state variables, $K_{j,t}$ is the firm's current level of capital stock, x_t represents aggregate productivity and $z_{j,t}$ represents firm-specific productivity. The productivity dynamics are modelled as follows:

$$x_t = (1-\rho_x)\mu_x + \rho_x x_{t-1} + \sigma_x \varepsilon_t^x \tag{D2}$$

$$z_{j,t} = (1-\rho_z)\mu_z + \rho_z z_{j,t-1} + \sigma_z \varepsilon_{j,t}^z \tag{D3}$$

The shocks ε_t^x and $\varepsilon_{j,t}^z$ are *i.i.d.* standard normal. Also $\forall j$, $\text{corr}(\varepsilon_t^x, \varepsilon_{j,t}^z) = 0$ and $\forall j \neq j'$, $\text{corr}\big(\varepsilon_{j',t}^z, \varepsilon_{j,t}^z\big) = 0$. Parameters μ_x and μ_z are the unconditional long-run means of each process, σ_x and σ_z govern the conditional volatility, whereas ρ_x and ρ_z govern the degree of persistence.

Every period t, firms choose the capital level for $t+1$. Investment $I_{j,t}$ follows the standard capital accumulation equation:

$$I_{j,t} = K_{j,t+1} - (1-\delta)K_{j,t} \tag{D4}$$

Here, δ is rate of depreciation. Following Gomes & Schmid (2010), we impose an investment irreversibility constraint i.e. $I_{j,t} \geq 0$.

As in the static model, firms can issue one period bonds valued at par. The face value outstanding is denoted $B_{j,t}$ and the coupon rate is denoted $c_{j,t}$. Net of interest tax shields, we can write a firm's total debt commitment in period t as:

$$\widehat{B}_{j,t} = (1 + (1-\tau)c_{j,t})B_{j,t} \tag{D5}$$

The firm's equity position is given by the variable $E_{j,t}$:

$$E_{j,t} = \Pi_{j,t} + \tau\delta K_{j,t} - I_{j,t} + B_{j,t+1} - \widehat{B}_{j,t} \tag{D6}$$

Note that we account for the tax shields of depreciation through $\tau\delta K_{j,t}$. $E_{j,t}$ can be either positive or negative, but it cannot be both. A positive value signifies a dividend payment, while a negative value indicates the firm is issuing equity, which incurs the following cost:

$$\Lambda(E_{j,t}) = (\lambda_0 - \lambda_1 E_{j,t})\mathbf{1}_{E_{j,t}<0} \tag{D7}$$

Therefore, the final distribution to shareholders is given by:

$$D_{j,t} = E_{j,t} - \Lambda(E_{j,t}) \tag{D8}$$

4.2 Investors

As in the static framework, our key innovation is that we model two separate investors, one for bonds (b) and one for equity (e). The pricing kernel of each investor $i \in \{e, b\}$ is given by:

$$M_{t,t+1}^{i} = \exp\left\{-r_t^f - \Gamma_{i,t}\sigma_x\varepsilon_{t+1}^x - \frac{1}{2}\Gamma_{i,t}^2\sigma_x^2\right\} \tag{D9}$$

Relative to the static setting, the key difference is that the risk-free rate is now dynamic and $\Gamma_{i,t}$ may also be dynamic. Note that both investors have the same conditional risk-free rate i.e. $\forall i$, $E_t[M_{t,t+1}^i] = \exp\{-r_t^f\}$. We specify an exogenous parsimonious process whereby the risk-free rate is a function of the aggregate productivity state:

$$r_t^f = r_0 + r_1(x_t - \mu_x) \tag{D10}$$

For the pricing of risk, we assume the equity holder has constant aversion to aggregate risk. On the other hand, the bondholder's aversion to aggregate risk can vary independently of the aggregate state:

$$\Gamma_{e,t} = \gamma_e \tag{D11}$$

$$\Gamma_{b,t} = \gamma_b \exp\left\{-w_t - \frac{1}{2}\sigma_w^2\right\} \tag{D12}$$

The pricing of risk in the bond market can vary for market-specific reasons, captured by the factor $w_t \sim N(0, \sigma_w^2)$; w_t is not serially correlated and is independent of both x_t and $z_{j,t}$. This reflects the observation of Collin-Dufresne et al. (2001) that credit spread changes appear to be driven by an aggregate market-specific factor. For $w_t > -\frac{1}{2}\sigma_w^2$, the term $\exp\{-w_t - \frac{1}{2}\sigma_w^2\}$ is lower than 1, such that $\Gamma_{b,t} < \gamma_b$. This scenario corresponds to a credit market boom. Conversely, if $w_t < -\frac{1}{2}\sigma_w^2$, then $\Gamma_{b,t} > \gamma_b$, which corresponds to credit tightening. Note that $E[\Gamma_{b,t}] = \gamma_b$, so that we can conveniently observe the unconditional difference in risk aversion between the equity and bond investor.

4.3 Valuation

4.3.1 Equity

For firm j at time t, the observable state space is defined as $S_{j,t} = \{K_{j,t}, \widehat{B}_{j,t}, x_t, w_t, z_{j,t}\}$. Conditional on S_t, the equity holders' goal is to choose $\{K_{j,t+1}, \widehat{B}_{j,t+1}\}$, such that equity value $V(S_{j,t})$ is maximised:

$$V(S_{j,t}) = \max\left\{0, \max_{K_{j,t+1}, \widehat{B}_{j,t+1}} \{D_{j,t} + E_t[M^e_{t,t+1}V(S_{j,t+1})]\}\right\} \tag{D13}$$

The equity holders also have the option to default when the following conditions arise:

1. $\forall\{K_{j,t+1}, \widehat{B}_{j,t+1}\},\ D_{j,t} < 0$ *(necessary condition)*

   For any choice of debt and capital, the firm requires external equity finance.

2. $\forall\{K_{j,t+1}, \widehat{B}_{j,t+1}\},\ |D_{j,t}| > E_t[M^e_{t,t+1}V(S_{j,t+1})]$ *(necessary and sufficient condition)*

   For any choice of debt and capital, the amount of required equity issuance (inclusive of underwriter costs) exceeds the expected discounted continuation value of the firm.

In the event of default, the equity holders receive a payoff of zero.

4.3.2 Corporate Bonds

Conditional upon the realisation of $x_{t+1}, w_{t+1}, z_{j,t+1}$, some choices of $K_{j,t+1}$ and $\widehat{B}_{j,t+1}$ may lead to default in the next period. The role of the bond investor is to offer the equity holder a coupon rate $c_{j,t+1}$ for any combination of $K_{j,t+1}$ and $\widehat{B}_{j,t+1}$, conditional on the current values of $x_t, w_t, z_{j,t}$[18]. This coupon rate will determine how much debt the firm can raise today i.e. since $B_{j,t+1} = \widehat{B}_{j,t+1}/(1 + (1-\tau)c_{j,t+1})$. As the coupon rate increases, there is a decrease in the amount of funds raised today per unit of total debt commitment tomorrow i.e. $B_{j,t+1}/\widehat{B}_{j,t+1}$ falls as $c_{j,t+1}$ increases.

As in the static model, the bond is priced at par and the following valuation must hold:

$$B_{j,t+1} = E_t\left[M^b_{t,t+1}\left\{(1 + c_{j,t+1})B_{j,t+1}\mathbf{1}_{V_{j,t+1}>0} + R_{j,t+1}\mathbf{1}_{V_{j,t+1}=0}\right\}\right] \tag{D14}$$

In (D14) we have an indicator function for default states as well as a recovery function $R_{j,t+1}$ for the payoff received by bond holders in the event of default:

[18] The transition probabilities to any future $x_{t+1}, z_{j,t+1}$ are conditional on the $x_t, z_{j,t}$ observed today. The bond market specific factor w_t affects the price of risk used to value the bond investor's contingent claims.

$$R_{j,t+1} = \Pi_{j,t+1} + \tau\delta K_{j,t+1} + \xi_1(1-\delta)K_{j,t+1} - \xi_0 \tag{D15}$$

The parameters ξ_0 and ξ_1 represent the fixed and proportional deadweight costs of default. We also impose a constraint that $R_{j,t+1} \leq B_{j,t+1}$ i.e. bond holders cannot recover more than the amount outstanding.

Finally, we rearrange (D14) and substitute $B_{j,t+1}$ using (D5) to obtain the following coupon rate function:

$$c_{j,t+1} = \frac{1 - E_t\left[M^b_{t,t+1}\left\{\mathbf{1}_{V_{j,t+1}>0} + \frac{R_{j,t+1}}{\widehat{B}_{j,t+1}}\mathbf{1}_{V_{j,t+1}=0}\right\}\right]}{E_t\left[M^b_{t,t+1}\left\{\mathbf{1}_{V_{j,t+1}>0} + (1-\tau)\frac{R_{j,t+1}}{\widehat{B}_{j,t+1}}\mathbf{1}_{V_{j,t+1}=0}\right\}\right]} \tag{D16}$$

Overall, this formulation highlights that the equity holder takes the coupon rate schedule as given. For any combination of $K_{j,t+1}$ and $\widehat{B}_{j,t+1}$ and across any realisation of $x_{t+1}, w_{t+1}, z_{j,t+1}$, the bond holder has full information on the future default decision of the equity holder. Regardless of what $K_{j,t+1}$ and $\widehat{B}_{j,t+1}$ is chosen, (D14) ensures that bond holders will break even in expectation (on a risk-adjusted basis).

4.4 First Order Conditions

The derivation of first order conditions for $K_{j,t+1}$ and $\widehat{B}_{j,t+1}$ is complicated by the existence of fixed equity issuance costs and investment irreversibility. We therefore adopt a simplified approach designed to provide the key intuition behind the marginal cost and benefit trade-off for debt and capital:

1. We drop the firm and time subscripts such that, for example, $K_{j,t+1}$ becomes K' and K_j becomes K.
2. To account for irreversibility, capital only has continuation value in states where $I' \geq 0$.
3. We assume that the equity cost function $\Lambda(E)$ is increasing in K' and decreasing in $\widehat{B}'$ (and vice versa for $\Lambda(E')$).

4. We denote the default state indicator variable as $\mathbf{1}_d$ and the complementary[19] solvent state indicator as $\mathbf{1}_s$. For an increase in $\widehat{B}'$ (K'), the number of default states captured by $\mathbf{1}_d$ increases (decreases).

First, we consider how the coupon rate varies with the choice of debt and capital. It can be shown that:

$$\frac{\partial c'}{\partial \widehat{B}'} > 0 \text{ and } \frac{\partial c'}{\partial K'} < 0$$

See Appendix A.1 for the proof.

4.4.1 Capital F.O.C.

With our simplifying assumptions, the marginal cost MC_K and marginal benefit MB_K functions for capital are given by:

$$MC_K = 1 + (1-\tau)\frac{\partial c'}{\partial K'}\frac{\widehat{B}'}{(1+(1-\tau)c')^2} + \frac{\partial \Lambda(E)}{\partial K'}$$

$$MB_K = E\left[M_e'\left\{\left((1-\tau)e^{x'+z'}\alpha K'^{\alpha-1} - (1-\tau)\delta + \mathbf{1}_{I'\geq 0} - \frac{\partial \Lambda(E')}{\partial K'}\right)\mathbf{1}_s + V(K')\frac{\partial \mathbf{1}_s}{\partial K'}\right\}\right]$$

Our focus is to understand how changes in the bond holder's price of risk are likely to affect these functions, relative to a scenario where the price of risk is the same for both investors. We therefore highlight the following components from the above conditions:

- If the bond holder's price of risk increases, the magnitude of $\frac{\partial c'}{\partial K'}$ will be greater and marginal cost will be reduced. Higher K' reduces default probability and improves the recovery amount. A more risk averse bond holder will place relatively more value to the reduction in risk and the improvement in recovery. At the same time, expected marginal benefit will increase since $\mathbf{1}_s$ increases in K'.
- However, since $\widehat{B}'$ and K' are determined jointly, we must consider that $\frac{\widehat{B}'}{(1+(1-\tau)c')^2}$ will also be affected as $\widehat{B}'$ and c' change with the bond holder's price of risk.

[19] $\mathbf{1}_s$ is complementary in the sense that, together, $\mathbf{1}_s + \mathbf{1}_d$ encompass the entire state space.

- We also recall the intuition from *Figure 4* with regard to equity issuance costs. The extent to which the firm is willing to incur the fixed component of $\Lambda(E)$ depends on the amount of K' that can be financed with debt as well as with current net worth. As optimal $\widehat{B}'$ changes with the bond holder's price of risk, the change in optimal K' is not always obvious.

Our analysis of the first order condition for capital suggests that the effect of a change in $\Gamma_{b,t}$ relative to $\Gamma_{e,t}$ on optimal K' will likely still be state-dependent, as was the case in the static model with equity issuance costs.

4.4.2 Debt F.O.C.

We define:

$$\zeta = \frac{1+c'}{1+(1-\tau)c'} \tag{D17}$$

If $c' > 0$, then $\zeta > 1$ and $\frac{\partial \zeta}{\partial c'} > 0$ (i.e. $(1-\tau) < 1$ implies that numerator increases more than denominator for higher c'). Since $\frac{\partial c'}{\partial \widehat{B}'} > 0$, it follows that $\frac{\partial \zeta}{\partial \widehat{B}'} > 0$. The variable ζ is the ratio of the pre-tax to the post-tax gross cost of borrowing and reflects the tax shield benefit of debt. It enables a more intuitive exposition of the marginal cost $MC_{\widehat{B}}$ and marginal benefit $MB_{\widehat{B}}$ functions for debt:

$$MB_{\widehat{B}} = E\left[M_b'\left\{\mathbf{1}_s + (\zeta-1)\mathbf{1}_s + \widehat{B}'\mathbf{1}_s\frac{\partial \zeta}{\partial \widehat{B}'} + \zeta\widehat{B}'\frac{\partial \mathbf{1}_s}{\partial \widehat{B}'} + R'\frac{\partial \mathbf{1}_d}{\partial \widehat{B}'}\right\}\right] - \frac{\partial \Lambda(E)}{\partial \widehat{B}'}$$

$$MC_{\widehat{B}} = E\left[M_e'\left\{\mathbf{1}_s + \frac{\partial \Lambda(E')}{\partial \widehat{B}'}\mathbf{1}_s - V\left(\widehat{B}'\right)\frac{\partial \mathbf{1}_s}{\partial \widehat{B}'}\right\}\right]$$

The trade-off components are similar to those in the static model:

- In the solvent states, one unit of debt is repaid. When the bond holder's price of risk differs, we have a valuation wedge for this claim: $E[(M_b' - M_e')\mathbf{1}_s]$. This wedge deepens when $\Gamma_{b,t}$ rises relative to $\Gamma_{e,t}$ and debt becomes costlier.

- The tax shields of the coupon payment are reflected in $E\left[M_b'\left\{(\zeta-1)\mathbf{1}_s + \widehat{B}'\mathbf{1}_s\frac{\partial\zeta}{\partial\widehat{B}'}\right\}\right]$.[20] Since the decision variable is $\widehat{B}'$, the bond holder offers the coupon schedule along this dimension. They are aware that the tax deductibility of interest eases the future debt burden for the firm and thus more debt outstanding B can be raised per unit of $\widehat{B}'$. A higher $\Gamma_{b,t}$ increases the tax-deductible coupon rate and $\frac{\partial\zeta}{\partial\widehat{B}'}$, but decreases the risk-adjusted value of the tax-shields.
- At the margin, an extra unit of $\widehat{B}'$ trades away solvent states for default states, such that there are fewer states were bondholders are repaid in full and instead incur bankruptcy costs. With a higher $\Gamma_{b,t}$, $E\left[M_b'\left\{\zeta\widehat{B}'\frac{\partial\mathbf{1}_s}{\partial\widehat{B}'} + R'\frac{\partial\mathbf{1}_d}{\partial\widehat{B}'}\right\}\right]$ falls, which reflects rising costs of distress.
- Debt does help lower equity issuance costs today. However, in the dynamic model, a high debt burden may need to be refinanced with equity, especially if credit conditions worsen in the subsequent period.

Although these first order conditions offer some insight into the potential consequences of market segmentation on optimal corporate policy, they do not speak for the magnitude of the effects. For this latter goal, the dynamic model must be solved numerically, calibrated and analysed to quantify these effects.

5. Results

5.1 Calibration

We use numerical techniques to solve the dynamic model on a monthly frequency and then use the optimal policy solutions to simulate a dynamic cross-section of firms. Our approach is very similar to that employed by Gomes & Schmid (2010) and we describe the procedure in the Appendix. For the most part, our parameter choices for the dynamic model are sourced directly from the literature. The rest are chosen such that our simulated cross-section of firm yields moments close to those observed in the data. We report our baseline parameter set in Table 1 and the sample moments in Table 2.

[20] Note there is an effect from tax shields at the current coupon rate $(\zeta-1)\mathbf{1}_s$ as well as from the increase in the coupon rate due to $\widehat{B}'$ i.e. $\widehat{B}'\mathbf{1}_s\frac{\partial\zeta}{\partial\widehat{B}'}$.

[Table 1]

Most of our parameter values are consistent with those used by Gomes & Schmid (2010) and Garlappi & Yan (2011), given their dynamic models are closest to ours.[21] As in both these studies, we set α and δ as 0.65 and 0.01 respectively, which is standard in the literature. The same applies to the values we choose for aggregate productivity persistence ρ_x and conditional volatility σ_x. The long-run aggregate productivity mean μ_x is reported as -3.1 in Garlappi & Yan (2011) and it has a scaling effect on production output relative to capital. We adjust it slightly to -3.2 to bring the profitability ratio closer to the data. Firm-specific productivity parameters μ_z, σ_z, ρ_z are the same as in Gomes & Schmid (2010). Fixed costs of production f are set to 0.034 as in Garlappi & Yan (2011).

For the corporate tax rate τ, we use an effective tax rate of 14% rather than the statutory rate, as done by Kuehn & Schmid (2014). The fixed cost of equity issuance λ_0 is set to 0.07, in between the value of 0.08 used by Garlappi & Yan (2011) and 0.06 used by Kuehn & Schmid (2014). We then adjust λ_1 to match the equity issuance rate. For recovery function parameters, we set ξ_0 to 0.1 as in Gomes & Schmid (2010), but set ξ_1 to 0.70 instead of 0.75 to more closely match the default rate.

For our investor parameters, we follow Zhang (2005) and set the stockholder's unconditional risk aversion γ_e to 50. As Zhang (2005) shows, this value pins down the equity holder's theoretical maximum Sharpe ratio, which can then be matched to the data.[22] Our risk-free rate parameters, r_0 and r_1, are set to match the mean and volatility of the risk-free rate. However, we find that r_1 also greatly affects equity volatility, hence we limit the magnitude of r_1 to prevent the volatility of the equity premium from becoming unreasonably high.

For our baseline set of parameters, γ_b and σ_w are set to 150 and 0.90 respectively. The goal is to match market leverage as closely as possible and, to a certain extent, book leverage. In the data, average book leverage is lower than average market leverage. This is driven by the fact that many high market-

[21] Note that Garlappi & Yan (2011) present their dynamic model in the Internet Appendix of their paper.

[22] In our case, the formula for the maximum Sharpe ratio simplifies to $\sqrt{\exp\{\gamma_e^2 \sigma_x^2\} - 1}$. Since Zhang (2005) uses the same value for σ_x as we do, he also obtains a value of 50 for γ_e. Zhang (2005) also specifies a time-varying component in the equity holder's risk premium, but we find it does not affect the value of the maximum Sharpe ratio. Rather, its purpose appears to be to control the dynamics of an endogenous risk-free rate, which is not applicable in our case.

to-book firms have very little or no leverage at all (Strebulaev & Yang, 2013). If firms with significant leverage have market-to-book closer to one, then the denominators of book and market leverage will be very similar. Therefore, average book and market leverage are very close in the data. Although our model generates relatively lower leverage for high market-to-book firms, we cannot achieve this to a sufficient degree to simultaneously match book leverage, market leverage and market-to-book.

[Table 2]

Table 2 displays our target and simulated moments, along with an outline of our empirical sources. The simulated moments are based on variables generated by our model. The risk-free rate is given by $\exp\{r_t^f\} - 1$ and the realised equity return for firm j with respect to time t is given by $V_{j,t+1}/(V_{j,t} - D_{j,t})$. The equity premium is the difference of the two. The investment ratio is defined as $I_{j,t}/K_{j,t}$, book leverage is $B_{j,t}/K_{j,t}$, market leverage $B_{j,t}/(B_{j,t} + V_{j,t})$ and market-to-book ratio $(B_{j,t} + V_{j,t})/K_{j,t}$. The profitability ratio is given by $\Pi_{j,t}/K_{j,t}$. Default rate is simply the frequency with which we observe default. All these are annualised to compare with the data. For the frequency of equity issuance, we group our monthly simulated observations into years and compute the fraction of years in which $D_{j,t} < 0$ for at least one of the months.[23]

5.2 Counterfactual Analysis

We conduct a counterfactual experiment to gauge the importance of parameters γ_b and σ_w and compile the results in Table 3. Parameters γ_b and σ_w differentiate how the bond holder prices risk relative to the equity holder. Therefore, our main counterfactual scenario is one where markets are integrated i.e. $\gamma_b = \gamma_e$ and $\sigma_w = 0$, such that $\forall t\ \Gamma_{b,t} = \Gamma_{e,t}$. This scenario corresponds to column IV of Table 3. We also consider the two interim steps which add up to our baseline model. First, in column II of Table 3, we increase γ_b to 150, while $\sigma_w = 0$. Second, in column III, we fix $\gamma_b = \gamma_e = 50$ and set $\sigma_w = 0.9$. In column I, we display the baseline model results where $\gamma_b = 150$ and $\sigma_w = 0.9$. All other parameters remain as shown in Table 1.

To ensure our conclusions are robust, the simulated paths for $\{x_t, w_t, z_{j,t}\}$ are equivalent across all specifications. Furthermore, for every individual equivalent simulation, we take the difference between

[23] We find this is consistent with how Hennessy & Whited (2007) define the variable in their study.

the moments generated by Model IV and Model I. [24] For each moment, we compute the standard deviation of the differences across all individual simulations. We then divide the overall difference by the standard deviation to create a t-stat as a measure of consistency. We also compute the frequency of simulations in which the single simulation difference is the same sign as the overall difference for all simulations.

[Table 3]

5.2.1 Optimal Leverage

The most consistent counterfactual difference we observe in Table 3 is the change in book and market leverage. Relative to the baseline in column I, the integrated markets scenario in column IV generates market (book) leverage which is 2.8 (3.3) percentage points higher. This difference is positive in 100% of simulations and has a relatively high t-stat. Both γ_b and σ_w contribute to the change in optimal leverage, but the effect of γ_b is quantitatively stronger.

The effect of γ_b largely follows the same logic as in the static model. For the same given level of default risk and policy choices, a higher γ_b increases the marginal cost of debt (relatively more than it increases tax shield benefits via a higher coupon rate). All else equal, optimal leverage is lower when γ_b is higher. This effect is strongest during economic downturns (i.e. where $x_t < \mu_x$), when default risk is higher. Conditional on $x_t < \mu_x$, optimal market (book) leverage is 3.0 (3.2) percentage points lower in column I relative to column IV. However, when $x_t > \mu_x$, average optimal market (book) leverage is 1.7 (2.4) percentage points lower.

To understand what drives the counterfactual leverage results conditional on x_t, first note that we generate counter-cyclical optimal book and market leverage, consistent with the recent empirical work

[24] As described in the Appendix, for each model specification, we simulate the economy 500 times and take the average of the moments produced in each of these simulations. Within each model specification, each simulation generates a different stochastic shock path for $\{x_t, w_t, z_{j,t}\}$. Across model specifications, the stochastic shock paths are the same. For example, simulation 29 under Model IV uses the same stochastic shock path as simulation 29 under Model I. Therefore, any changes in simulated moments are caused only by changes in optimal corporate policy. To continue our example, this means that the average market leverage ratio of simulation 29 under Model IV is comparable with the one of simulation 29 under Model I. We can thus obtain a series of differences in average market leverage between Models I and IV across all simulations (i.e. to compute t-stats or the proportion of differences with the same sign).

of Halling, Yu & Zechner (2016). In our model, this is primarily due to the counter-cyclicity of the risk-free component of coupon rates, whereby marginal interest tax shields are higher in downturns. To reap these relatively larger tax shield benefits, firms choose higher leverage during downturns as they are willing to trade off against a higher probability of default. On the other hand, the leverage choices during upturns are associated with a lower risk of default.[25] When there is more distress risk, the marginal cost of debt is more sensitive to γ_b, hence γ_b affects optimal leverage most when $x_t < \mu_x$.

The primary purpose of σ_w is to govern the volatility of $\Gamma_{b,t}$ (which depends on w_t). As $\Gamma_{b,t}$ varies through time, the credit market will undergo periods of tightening supply, where $\Gamma_{b,t} > \gamma_b$, and vice versa. As the marginal cost of debt is relatively high when $\Gamma_{b,t} > \gamma_b$, firms choose relatively lower optimal leverage. However, this effect is offset by states where the cost of debt is relatively lower (i.e. $\Gamma_{b,t} < \gamma_b$), which induces a higher optimal leverage choice. Overall, the magnitude of reduction in optimal leverage during bad credit market states seems to outweigh the increase in optimal leverage during good credit market states. Hence we observe a modest decrease in leverage when $\sigma_w > 0$. Another potential effect of σ_w is that it creates greater roll-over uncertainty. Although equity holders are risk-neutral with respect to future realisations of w_{t+1}, if the effect of $\Gamma_{b,t}$ is asymmetric as described above, the expected cost of refinancing debt will rise and result in lower optimal leverage.

The magnitude of the effect of σ_w depends on γ_b. As γ_b increases, the spread in $\Gamma_{b,t}$ for good and bad credit market states widens, holding σ_w constant. For example, when $\gamma_b = 50$, there is a 1.56% chance on the downside that $\Gamma_{b,t} = 305$ and a 1.56% chance on the upside that $\Gamma_{b,t} = 4$. When $\gamma_b = 150$, for the same probability, $\Gamma_{b,t} = 915$ on the downside and $\Gamma_{b,t} = 11$ on the upside. This amplifies the asymmetric effect of σ_w on optimal leverage, whereby reductions in borrowing during bad credit market states are greater in magnitude relative to increases in borrowing during good credit market states.

[25] Naturally, for the same level of leverage, default risk is higher during downturns, all else equal. This is due to the high auto-correlation of x_t, which means that profitability is persistently low and hence the expected discounted continuation value of equity is low (the latter also being reduced by a higher risk-free discount rate). However, firms are willing to bear more default risk during downturns because coupon rates are higher and they can secure more tax shield benefits.

Finally, we also note that both γ_b and σ_w have a modest, but consistent effect on equity issuance frequency. This suggests that some firms substitute towards equity financing as bond investors' price of risk becomes higher and more uncertain.

5.2.2 Investment

Across columns I and IV in Table 3, we observe only modest changes in the investment ratio mean and volatility, mostly driven by the change in γ_b. These aggregate results are quantitively weak because the investment rate equals the depreciation rate in the majority of our firm-year observation. This is due to the irreversible investment assumption, whereby firms are constrained to maintain to same capital level until they experience a sufficiently large positive stochastic shock[26] to incentivise them to expand.

Therefore, we focus on situations where firms invest to expand productive capacity (i.e. $I_{j,t}/K_{j,t} > \delta$) and ignore investment expenditure for depreciation purposes. We find that, when markets are integrated, firms expand their productive capacity in 6.4% of our simulated firm-year observations, with an average investment ratio of 51.3%.[27] In our baseline specification in column I, the frequency of firm-years in which firms increase productive capacity drops to 5.7%, with a slightly lower investment ratio of 50.7%. The reduction in the frequency of capacity expansion occurs in 97% of simulations, which suggests the effect is consistent.

We recall from Figure 4 that there are conditions under which, despite the higher price of risk in the bond market, investment could be higher. Our results suggest that, in our simulated economies, the forces which cause investment to drop are stronger. However, these differences could also be driven by the replacement of defaulting firms with small unlevered firms. When $\gamma_b = 50$, bond holders' required premium for distress risk is relatively lower and firms are willing to bear more default risk, which causes them to default more frequently. This also means that there is a higher entry frequency of small firms with growth options, such that we observe more instances of productive capacity expansion.

[26] Specifically, firms need to experience a more favourable combination of $\{x_t, w_t, z_{j,t}\}$ than they have ever had in their past. Therefore, firms that experience relatively favourable states of $\{x_t, w_t, z_{j,t}\}$ in the burn-in period are relatively less likely to expand productive capacity in the simulated sample.

[27] This suggests that our model generates rare, but relatively, large bursts of investment activity. These dynamics are qualitatively similar to those found empirically (e.g. Cooper & Haltiwanger, 2006).

A more thorough analysis could help clarify the exact channel by which changes in γ_b and σ_w affect investment. It could be that the overall investment results are not all that different, in that firms reach the same end point in terms of productive capacity, but undertake different journeys to get there. For example, take two identical firms, facing the same $\{x_t, w_t, z_{j,t}\}$, but different γ_b. The firm for which γ_b is higher initially invests less, as it is constrained by its debt capacity and does not find it worthwhile to issue equity to invest as much as the other firm. However, at some point in the future, as $\{x_t, w_t, z_{j,t}\}$ become more favourable, both firms have the capacity to source sufficient external finance to reach the same level of capital.

5.2.3 Other Results

With a higher unconditional price of risk in the bond market, the default rate does not necessarily rise. In fact, it is lower in 95% of simulations when we compare columns I and IV. This aligns with the behaviour of firms in the static model, whereby a rise in γ_b incentivises firms to use less debt to lower the risk of distress. This also explains why observed credit spreads do not necessarily rise with γ_b. Relative to column IV, credit spreads are lower in 99.2% of simulations. Another notable result is that changes in optimal leverage affect the volatility of equity returns which, in turn, affects the levered equity premium.

5.3 Sensitivity Analysis

We turn our attention to how results vary across different values of γ_b and σ_w. We show the outcome in Table 4. For γ_b, the direction of sensitivity appears to be consistent with the insights from Table 3 across all key moments. For σ_w, the magnitude of sensitivity is significantly weaker as we increase σ_w from 0.90 to 1.35. The purpose of σ_w is to make $\Gamma_{b,t}$ more volatile and simultaneously fix its unconditional mean to γ_b, but $\Gamma_{b,t}$ cannot drop below 0.[28] As σ_w increases, the values of $\Gamma_{b,t}$ in the lower tail of the distribution of w_t becomes more extreme, whereas for the remainder of the distribution, $\Gamma_{b,t}$ starts to approach 0. This indicates that, beyond a certain threshold for σ_w, firms' decisions are, on average, unaffected by the distribution of credit market shocks. It would be beneficial to conduct a more

[28] Note that $\Gamma_{b,t} = \gamma_b \exp\{-w_t - \frac{1}{2}\sigma_w^2\}$. We set $\gamma_b > 0$, hence $\forall w_t, \Gamma_{b,t} > 0$ since $\exp\{-w_t - \frac{1}{2}\sigma_w^2\} > 0$.

thorough analysis of the conditional effects of w_t on corporate policy to identify the precise channels by which we observe the diminishing effect of σ_w.

[Table 4]

Finally, Table 4 suggests that a similar set of benchmark results can be obtained if we were to set $\gamma_b = 200$ and $\sigma_w = 0$. However, it is important to consider that, after a certain point, the unconditional risk aversion for the bondholder can seem unreasonably high. Our calibrated value of 150 is already quite high, but it is nevertheless an improvement. A natural extension would be to carefully consider credit spread dynamics to pin down the values for γ_b and σ_w. However, our current model is not designed to address credit spreads, given one-month bonds do not capture the multi-period nature of default risk which is priced in more realistic long-term bonds (i.e. we only predict an annual credit spread of 7 basis point with our baseline model). Despite said limitations, our model still delivers important insights about corporate policy when bond and equity holders price risk differently.

6. Conclusion

Although equity and corporate bond holders are implicitly separated by agency conflicts throughout much of the corporate finance literature, little consideration has been given to how firms' policies, particularly the leverage choice, are affected by heterogeneity in investors' pricing of risk. The standard assumption is that risk is priced the same way in equity and corporate bond markets, despite evidence that suggests otherwise (Titman, 2002). Therefore, to address the under-leverage puzzle, we relax this assumption and construct a two-investor framework in which we embed key differences in the pricing of risk between equity and bond holders.

First, we incorporate evidence that bond investors' price of risk is higher relative to equity investors and show that it has a first order effect on optimal leverage. Not only does relatively higher bond holder risk aversion increase the risk-adjusted cost of financial distress, it also adds a new friction related to a wedge in the valuation of the bond holder's contingent claims. This latter effect further steepens the marginal cost curve of debt and thus drives optimal leverage even lower. Second, optimal leverage further decreases due to fluctuations in bond investors' price of risk, which arise independently of macroeconomic factors.

This study offers a new interesting angle to explore in the context of equity and bond holder conflicts of interest. So far, much of the literature focuses on the notion that equity holders maximise equity value, as opposed to firm value which is preferable for bond holders. We show that, beyond disagreement over the appropriate objective function of the firm, frictions can arise simply due to differences in the pricing of risk.

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Appendix

A.1 First order conditions for the coupon rate

We first consider the partial derivative of the coupon rate with respect to $\widehat{B}$:

$$c = \frac{\widehat{B} - E\left[M\left(\widehat{B}\mathbf{1}_s + R\mathbf{1}_d\right)\right]}{E\left[M\left(\widehat{B}\mathbf{1}_s + (1-\tau)R\mathbf{1}_d\right)\right]} = \frac{g(\widehat{B})}{h(\widehat{B})}$$

$$\frac{\partial c}{\partial \widehat{B}} = \frac{1}{\left[h\left(\widehat{B}\right)\right]^2}\left\{\left\{1 - E\left[M\left(\mathbf{1}_s + \widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} + R\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right)\right]\right\} h\left(\widehat{B}\right) - g(\widehat{B})E\left[M\left(\mathbf{1}_s + \widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} + (1-\tau)R\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right)\right]\right\}$$

$$\frac{\partial c}{\partial \widehat{B}} = \frac{1}{h\left(\widehat{B}\right)}\left\{1 - E\left[M\left(\mathbf{1}_s + \widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} + R\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right)\right] - cE\left[M\left(\mathbf{1}_s + \widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} + (1-\tau)R\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right)\right]\right\}$$

$$\frac{\partial c}{\partial \widehat{B}} = \frac{1}{h\left(\widehat{B}\right)}\left\{1 - (1+c)E[M\mathbf{1}_s] - (1+c)E\left[M\widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}}\right] - (1+(1-\tau)c)E\left[MR\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right]\right\}$$

Assuming that $h\left(\widehat{B}\right) > 0$,

$$\frac{\partial c}{\partial \widehat{B}} > 0 \Leftrightarrow (1+c)E[M\mathbf{1}_s] + (1+c)E\left[M\widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}}\right] + (1+(1-\tau)c)E\left[MR\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right] < 1$$

Note that $1 = E[M((1+c)\mathbf{1}_s + (R/B)\mathbf{1}_d)] \Rightarrow (1+c)E[M\mathbf{1}_s] < 1,$ *assuming* $E[M(R/B)\mathbf{1}_d] > 0$. *Therefore we just need to prove that:*

$$E\left[M\left((1+c)\widehat{B}\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} + (1+(1-\tau)c)R\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right)\right] < 0$$

As we increase $\widehat{B}$ *the amount of potential solvent states decrease i.e.* $\frac{\partial \mathbf{1}_s}{\partial \widehat{B}} < 0$ *and vice versa i.e.* $\frac{\partial \mathbf{1}_d}{\partial \widehat{B}} > 0$. *Since the number of total states* $\mathbf{1}_s + \mathbf{1}_d$ *is fixed, it follows that:*

$$\left|\frac{\partial \mathbf{1}_s}{\partial \widehat{B}}\right| = \left|\frac{\partial \mathbf{1}_d}{\partial \widehat{B}}\right|$$

Thus, to prove that $\frac{\partial c}{\partial \widehat{B}} > 0$, *all we have left is to impose that:*

$$(1+c)\widehat{B} > (1+(1-\tau)c)R \;\Leftrightarrow (1+c)B > R$$

Essentially, the total amount recovered in default must not exceed the total principal and interest owed to debtholders. We impose a stronger condition that $R \leq B$.

We now consider the partial derivative of the coupon rate with respect to K:

$$c = \frac{\widehat{B} - E\left[M\left(\widehat{B}\mathbf{1}_s + R(K)\mathbf{1}_d\right)\right]}{E\left[M\left(\widehat{B}\mathbf{1}_s + (1-\tau)R(K)\mathbf{1}_d\right)\right]} = \frac{g(K)}{h(K)}$$

$$\begin{aligned}\frac{\partial c}{\partial K} = \frac{1}{h(K)}\Bigg\{ & -E\left[M\left(\widehat{B}\frac{\partial \mathbf{1}_s}{\partial K} + R(K)\frac{\partial \mathbf{1}_d}{\partial K} + R'(K)\mathbf{1}_d\right)\right] \\ & - cE\left[M\left(\widehat{B}\frac{\partial \mathbf{1}_s}{\partial K} + (1-\tau)\left(R(K)\frac{\partial \mathbf{1}_d}{\partial K} + R'(K)\mathbf{1}_d\right)\right)\right]\Bigg\}\end{aligned}$$

Assuming that $h(K) > 0$,

$$\frac{\partial c}{\partial K} < 0 \Leftrightarrow (1+c)E\left[M\widehat{B}\frac{\partial \mathbf{1}_s}{\partial K}\right] + (1+(1-\tau)c)E\left[M\left(R(K)\frac{\partial \mathbf{1}_d}{\partial K} + R'(K)\mathbf{1}_d\right)\right] > 0$$

Since $R'(K) > 0$, *we can infer that* $(1+(1-\tau)c)[MR'(K)\mathbf{1}_d] > 0$. *Similar to before, we are left with:*

$$E\left[M\left(\widehat{B}(1+c)\frac{\partial \mathbf{1}_s}{\partial K} + (1+(1-\tau)c)R\frac{\partial \mathbf{1}_d}{\partial K}\right)\right] > 0$$

The key difference is that $\frac{\partial \mathbf{1}_s}{\partial K} > 0$ *and* $\frac{\partial \mathbf{1}_d}{\partial K} < 0$, *so in the end we still require:*

$$(1+c)\widehat{B} > (1+(1-\tau)c)R \ \Leftrightarrow (1+c)B > R$$

A.2 Dynamic Model: Numerical Solution Method

We first discretise the state space $S_{j,t} = \{K_{j,t}, \widehat{B}_{j,t}, x_t, w_t, z_{j,t}\}$. Given the persistent nature of x_t and $z_{j,t}$, we use the Rouwenhorst (1995) method to generate a 7-state discrete transition matrix for each variable. We also discretise w_t with 7 states. The grid for $K_{j,t}$ is set up such that the ratio between each point is equal. We find that $K_{j,t} \in [1,20]$ is an appropriate range for our parameter set. We define a new state variable $L_{j,t} = \widehat{B}_{j,t}/K_{j,t}$.[29] We create and equispaced grid for $L_{j,t}$ within the range [0,1]. The solution algorithm closely resembles that of Gomes & Schmid (2010). We start with a relatively coarse grid for $K_{j,t}$ and $L_{j,t}$ of 25 and 15 points respectively then perform the following steps:

1. We guess a starting point for the coupon schedule matrix $c(K_{j,t+1}, L_{j,t+1}, x_t, w_t, z_{j,t})$

[29] This way, the debt choice grid is reasonably fine for both low and high $K_{j,t}$ firms.

2. We use the coupon matrix to solve the equity maximisation problem (D13), which determines the default policy matrix of the equity holder
3. We then incorporate the default policy matrix to compute $c(K_{j,t+1}, L_{j,t+1}, x_t, w_t, z_{j,t})$ again
4. We use the updated coupon matrix to solve equity maximisation problem (D13) and check if the default policy matrix has changed. If the magnitude of the change is outside our tolerance level, we repeat steps 3 and 4 until the convergence criteria is met.

Once a solution is found we interpolate $c(K_{j,t+1}, L_{j,t+1}, x_t, w_t, z_{j,t})$ and $V(S_{j,t})$ over a finer grid for $K_{j,t}$ and $L_{j,t}$ of 65 and 35 points respectively and repeat steps 1-4, albeit with more relaxed convergence criteria to ensure computational feasibility (results are robust if we apply more stringent convergence criteria or increase grid size).

A.3 Dynamic Model: Simulation

We use the optimal policy matrices from our numerical optimisation routine to simulate a dynamic cross-section of firms. The discrete transition matrices for $x_t, w_t, z_{j,t}$ determine the path of our stochastic state variables. We note the following key features of our simulation:

- We initialise 5000 firms at $K_{j,t} = 1$ and uniformly sample their starting leverage from the grid of $L_{j,t+1}$.
- The firms make investment, financing and default decisions for a total of 1920 months of which the first 1200 months are discarded. This leaves a sample period of 720 months (60 years).
- Every time a firm defaults, it is replaced with an unlevered firm for which $K_{j,t} = 1$, as is done in Kuehn & Schmid (2014).
- The simulation is repeated 500 times. We use a set 'seed' to: (i) ensure results can be easily replicated and (ii) to ensure that we can make robust counterfactual experiments. For the latter point, note that parameter changes results in changes in firms' optimal policies. Therefore, when we run simulations for a new parameter set, we want to ensure that the initial cross-section and the transition path for $x_t, w_t, z_{j,t}$ remains unchanged, such that different parameter sets are comparable.

Figure 1. We plot the marginal cost and benefit of debt across different available choices of book leverage. The intersection represents the optimal book leverage ratio B^*/K under each scenario. In each panel, the faded dashed lines represent the cost/benefit curves from the other two panels.

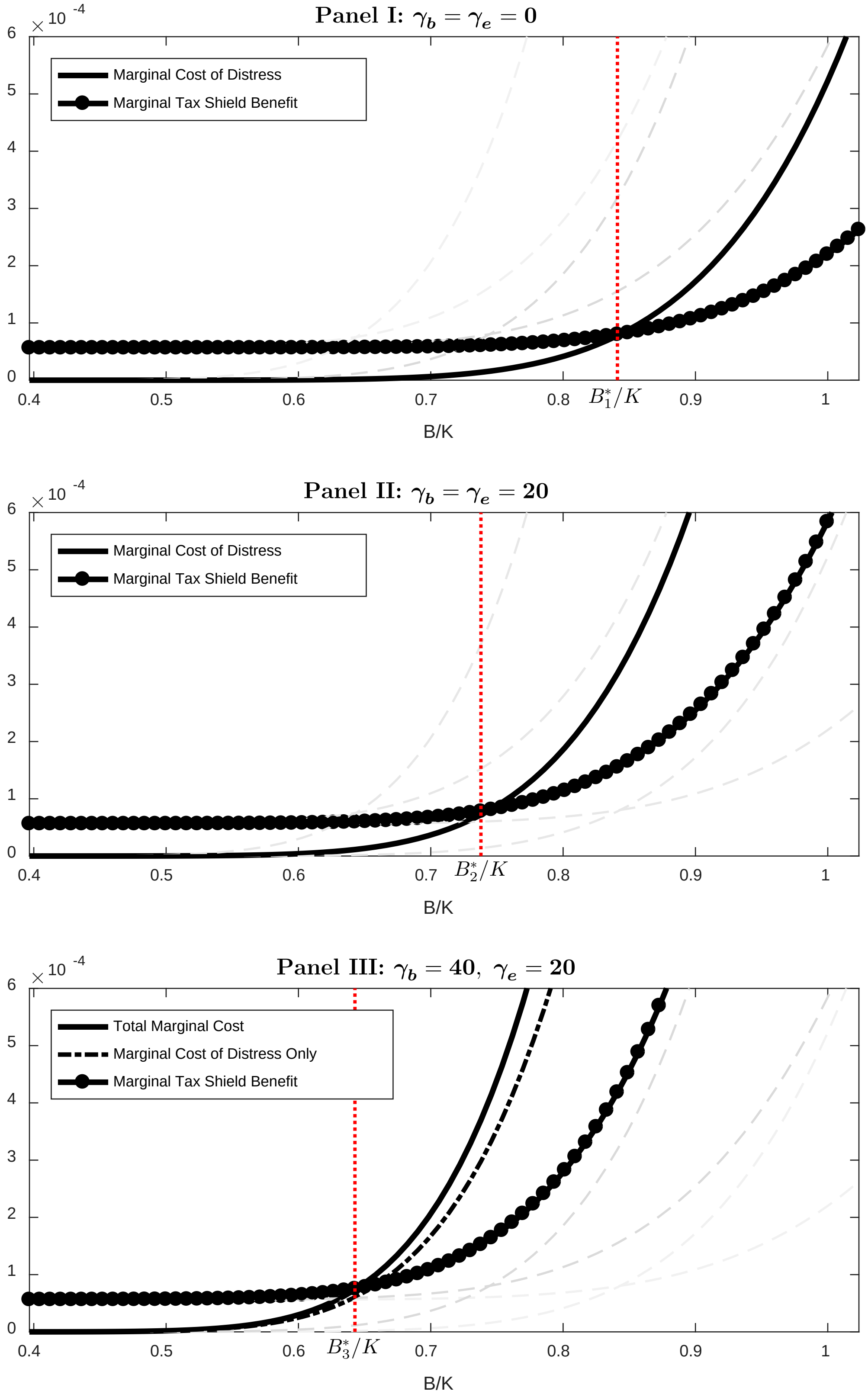

Figure 2. We plot variables of interest across different ratios of γ_b to γ_e. In panel I, we plot the optimal capital (K) and debt (B) choice of the firm. In panels II, III and IV we plot equity value, credit spreads and default probability respectively. In each of these three panels, the solid line shows the value associated with the optimal policies in the first panel. As a counterfactual, the dashed lines show what the value would be if the firm continues to pursue policies of K and B that are optimal when $\gamma_b = \gamma_e$. In panel II, we add a dotted line which represents the maximum value of equity attained when $\gamma_b = \gamma_e$.

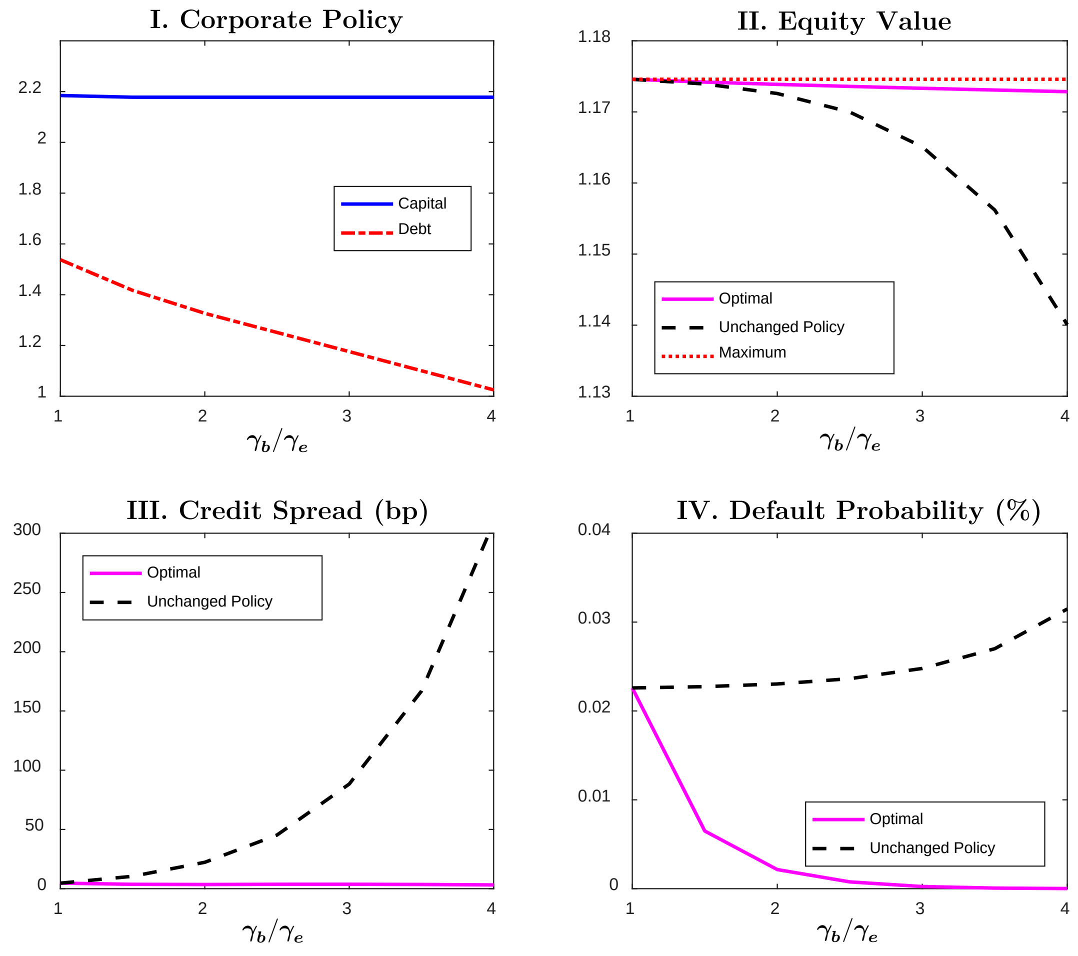

Figure 3. Refer to the description of Figure 2. Here we repeat the analysis shown in Figure 2, except that equity issuance is now costly.

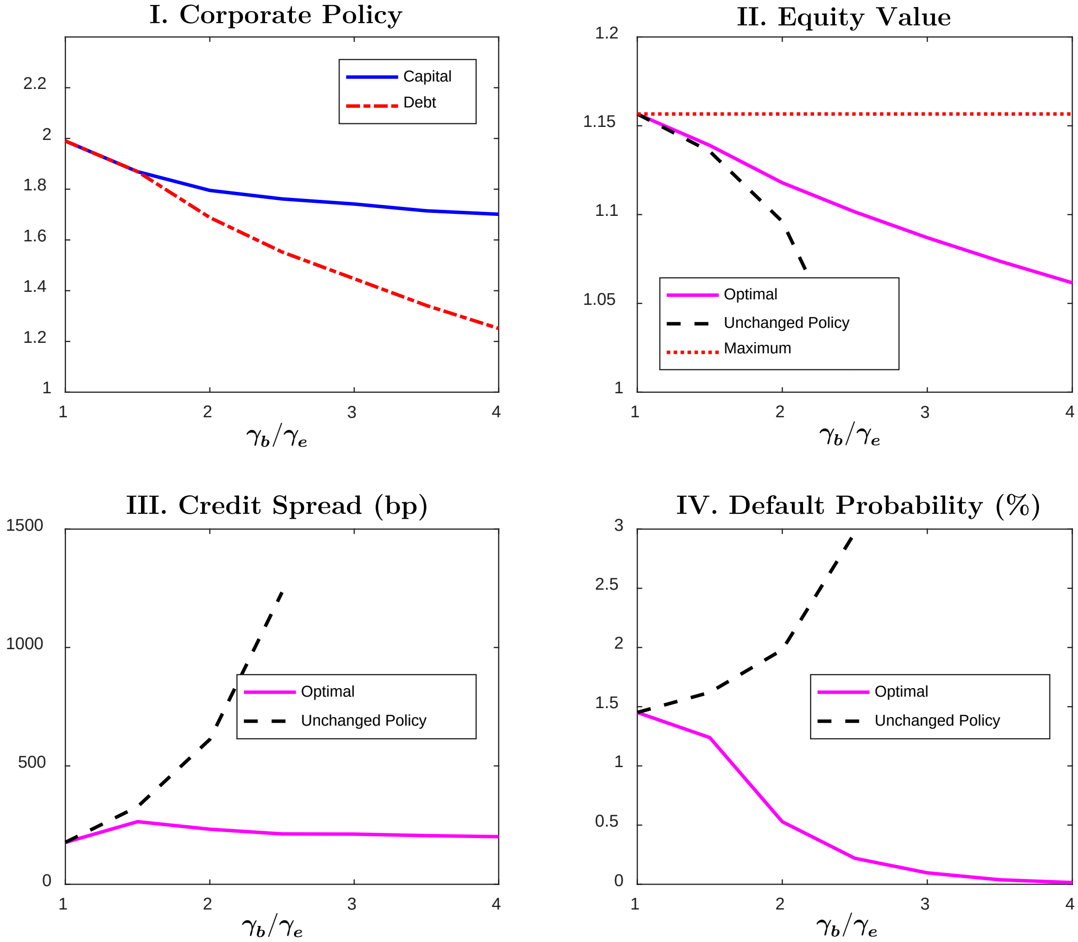

Figure 4. We plot optimal policy for K (panel I) and B (panel II) against different initial levels of net worth A. We compare the scenario where markets are integrated ($\gamma_b = \gamma_e$) to a scenario where they are not ($\gamma_b > \gamma_e$). Panel III shows the cumulative fraction of each finance source relative to total capital expenditure. A/K is the contribution of net worth, while $(A+B)/K$ shows the incremental contribution of debt finance. Although not explicitly plotted, $1-(A+B)/K$ is the contribution of equity issuance.

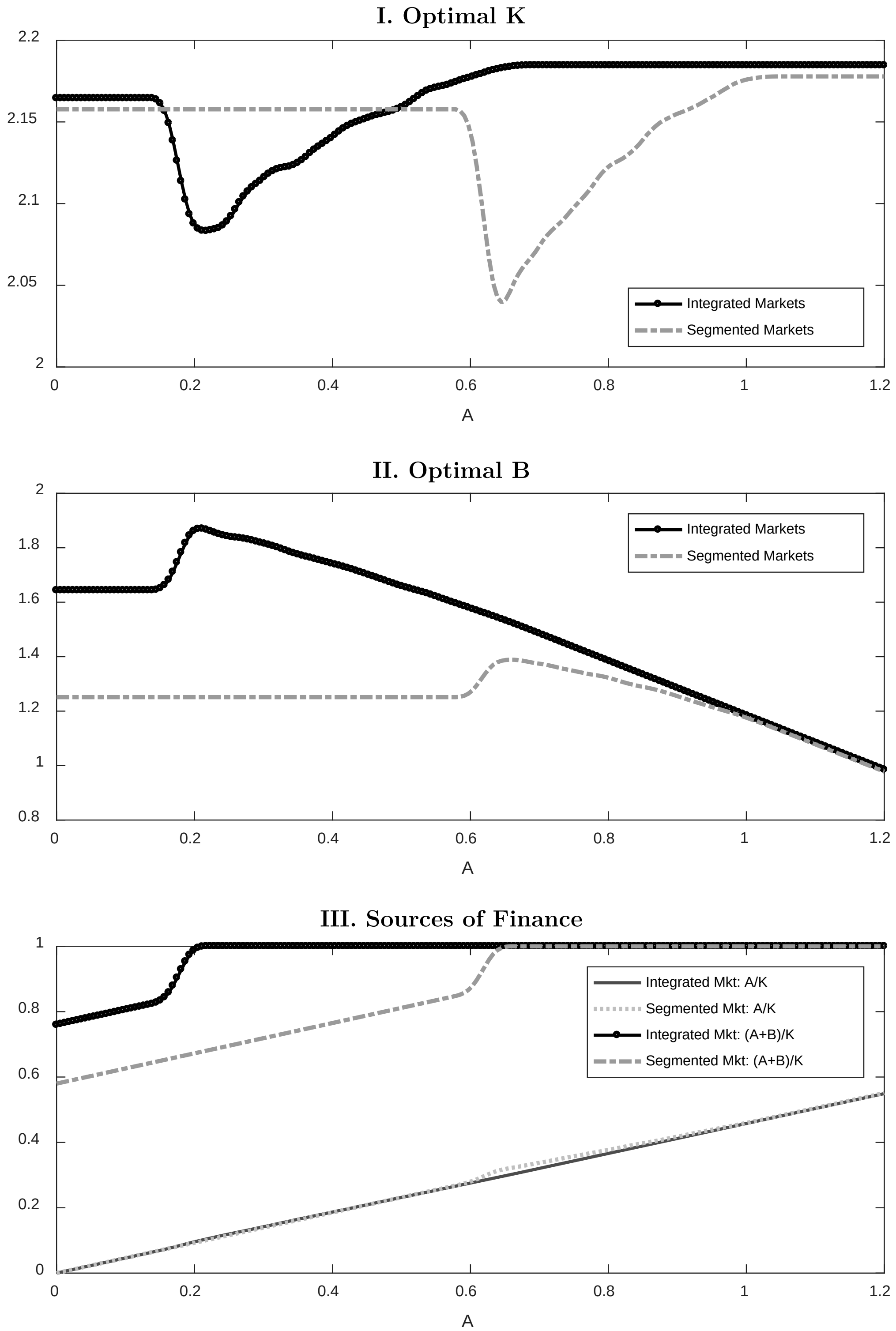

Table 1. Parameter Choices

Most of our parameters are sourced from a range of studies with similar dynamic frameworks, which include Gomes & Schmid (2010), Garlappi & Yan (2011) and Kuehn & Schmid (2014). The remainder of parameters are calibrated to match target moments. We discuss our parameter choices in more detail in section 5.1.

Parameter	Symbol	Value
Risk free rate constant	r_0	0.0012
Risk free rate exposure to aggregate state	r_1	-0.3850
Equity holder unconditional risk aversion	γ_e	50.0000
Bond holder unconditional risk aversion	γ_b	150.0000
Variability of bond holder risk aversion	σ_w	0.9000
Long-run aggregate productivity mean	μ_x	-3.2000
Conditional aggregate productivity volatility	σ_x	0.0023
Aggregate productivity persistence	ρ_x	0.9830
Long-run firm-specific productivity mean	μ_z	0.0000
Conditional firm-specific productivity volatility	σ_z	0.1500
Firm-specific productivity persistence	ρ_z	0.9200
Production function curvature	α	0.6500
Capital depreciation rate	δ	0.0100
Fixed costs of production	f	0.0340
Corporate tax rate	τ	0.1400
Fixed cost of equity issuance	λ_0	0.0700
Proportional cost of equity issuance	λ_1	0.0500
Fixed bankruptcy deadweight cost	ξ_0	0.1000
Proportion of capital recovered upon default	ξ_1	0.7000

Table 2. Sample Moments

We present the benchmark moments for our calibration and compare the moments generated from our model with those in the data. The risk-free rate and equity premium data moments are sourced from Barro (2006). Investment ratio moments are from Gomes (2001). Book leverage, market leverage and market-to-book are from Lemmon, Roberts & Zender (2008). The profitability ratio is from DeAngelo, DeAngelo & Whited (2011). Equity issuance frequency is from Hennessy & Whited (2007) and the one-year default rate is from Moody's Investors Service (Ou, Chiu & Metz, 2011). All moments are annualised.

Target Moment	Data	Model
Risk-free rate mean	0.014	0.014
Risk-free rate volatility	0.021	0.015
Equity premium mean	0.076	0.084
Equity premium volatility	0.175	0.220
Investment ratio mean	0.145	0.143
Investment ratio volatility	0.139	0.141
Book leverage mean	0.270	0.301
Market leverage mean	0.280	0.279
Market-to-book mean	1.590	1.211
Profitability ratio	0.192	0.203
Equity Issuance Frequency	0.175	0.165
Default Rate	0.011	0.009

Table 3. Counterfactual Analysis

We present the results of our counterfactual analysis across four different model specifications. Column I corresponds with our baseline model, while column IV corresponds with an integrated markets scenario where $\gamma_b = \gamma_e$ and $\sigma_w = 0$. Columns II and III show the results as we change γ_b and σ_w one at a time. For this analysis, we also measure the frequency of firm-years in which firms expand productive capacity (i.e. $I_{j,t}/K_{j,t} > \delta$) and compute the average investment ratio for these firms (which we denote as the mean productive capacity increase ratio). We also add measure for average book and market leverage conditional on the aggregate state. Economic expansions are when $x > \mu_x$ and contractions when $x < \mu_x$. We also report the value-weighted average of observed credit spreads. The t-stat column is simply the overall difference in the moment between IV and I, divided by the standard deviation of the difference across simulations. The 'freq.' column counts the fraction of simulations in which the difference between the moments of IV and I are the same sign as the overall difference.

γ_b	150	150	50	50		
σ_w	0.9	0.0	0.9	0.0	**IV - I**	
	I	**II**	**III**	**IV**	t-stat	freq.
Equity premium mean	0.084	0.085	0.088	0.089	1.063	0.972
Equity premium volatility	0.220	0.222	0.231	0.233	0.651	0.992
Investment ratio mean	0.143	0.144	0.146	0.146	1.542	0.984
Investment ratio volatility	0.141	0.143	0.154	0.155	1.542	0.974
Productive capacity increase frequency	0.057	0.057	0.064	0.064	1.484	0.968
Mean productive capacity increase ratio	0.507	0.511	0.509	0.513	0.339	0.688
Book leverage mean	**0.301**	**0.310**	**0.331**	**0.334**	**4.318**	**1.000**
Book leverage ($x < \mu_x$)	0.396	0.404	0.425	0.428	3.983	0.998
Book leverage ($x > \mu_x$)	0.203	0.209	0.225	0.227	2.689	0.998
Market leverage mean	**0.279**	**0.286**	**0.304**	**0.307**	**3.927**	**1.000**
Market leverage ($x < \mu_x$)	0.421	0.428	0.449	0.451	3.056	0.998
Market leverage ($x > \mu_x$)	0.147	0.151	0.163	0.164	2.789	0.998
Equity Issuance Frequency	0.165	0.162	0.159	0.158	-2.118	0.992
Default Rate	0.009	0.009	0.010	0.010	1.126	0.948
Credit Spread (bp)	6.985	6.390	9.365	9.847	1.629	0.992

Table 4. Sensitivity Analysis

We conduct sensitivity tests with respect to γ_b and σ_w. We first fix σ_w to 0 and show the results across different values of γ_b. We then fix γ_b and show the results across different values of σ_w.

	Sensitivity to γ_b			*Sensitivity to* σ_w		
γ_b	100	150	200	150	150	150
σ_w	0.0	0.0	0.0	0.45	0.90	1.35
Equity premium mean	0.086	0.085	0.084	0.085	0.084	0.084
Equity premium volatility	0.225	0.222	0.220	0.221	0.220	0.220
Investment ratio mean	0.145	0.144	0.143	0.143	0.143	0.144
Investment ratio volatility	0.147	0.143	0.142	0.143	0.141	0.142
Book leverage mean	**0.319**	**0.310**	**0.300**	**0.306**	**0.301**	**0.301**
Book leverage $(x < \mu_x)$	0.412	0.404	0.393	0.401	0.396	0.396
Book leverage $(x > \mu_x)$	0.216	0.209	0.205	0.207	0.203	0.202
Market leverage mean	**0.294**	**0.286**	**0.278**	**0.283**	**0.279**	**0.279**
Market leverage $(x < \mu_x)$	0.436	0.428	0.419	0.426	0.421	0.421
Market leverage $(x > \mu_x)$	0.156	0.151	0.148	0.149	0.147	0.147
Equity Issuance Frequency	0.160	0.162	0.162	0.163	0.165	0.165
Default Rate	0.010	0.009	0.009	0.009	0.009	0.009
Credit Spread (bp)	6.524	6.390	6.555	6.400	6.985	7.472