Modelling Intraday Correlations using Multivariate GARCH

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Abstract

The study of high frequency volatility and correlation dynamics is motivated by a large number of practical financial applications, with institutions such as banks and hedge funds requiring up-to-date risk profiles for their portfolios. To date, very little work exists on the dynamics of correlations over the trading day despite a growing literature on modelling intraday volatilities. This paper outlines important intraday features of pairwise correlations and presents a novel multivariate GARCH approach to estimate the intraday correlations of a portfolio of equities. Based on the Dynamic Conditional Correlation model, the framework captures the daily level fluctuations evident in pairwise correlations and takes into account the intraday inverted U-shape pattern seen in the intraday correlations between assets. An equicorrelated version of the approach based on the Dynamic Equicorrelation model is also provided. At the 5-minute frequency, modelling both persistence at the daily frequency and the intraday diurnality evident in the correlations provides promising results over the sample. The intraday pattern in the correlations is most evident between stocks that have a lower level of unconditional correlation, such as those from different industries. Stocks that are highly correlated display a pattern quite different to those with lower unconditional correlations.

Keywords

Intraday correlation modelling, volatility, multivariate GARCH, equicorrelation

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1 Introduction and Motivation

This paper aims to develop a modelling framework for the intraday correlation matrix, examining the correlation dynamics of a portfolio of equities at a high frequency. The idea of intraday correlations is distinct to the 'realized covariance' or *RCOV* literature, that is using intraday data sampled at high frequencies for the purposes of generating daily covariance or correlation matrices. The focus here is modelling intraday correlations using intraday data. The study of high frequency correlations is motivated by the requirement of institutions such as banks and hedge funds to have up-to-date risk profiles for their portfolios. Yet further incentives for understanding these processes as they evolve throughout the trading day include the numerous applications of such work, such as hedging (see Frey, 2000), temporal trading strategies and the impact of news arrival (see Goodhart and O'Hara, 1997).

The near-continuous flow of price and trade data presents researchers with opportunities, as well as unique challenges, to capture the dynamics of univariate and multivariate systems. A well documented complication of modelling intraday univariate volatilities is the diurnal or U-shaped pattern seen in volatility over the trading day, see Wood, McInish, and Ord (1985) for perhaps the earliest discussion of this phenomena. A successful univariate intraday volatility model needs to capture this diurnal pattern. Most recent work is based on the multiplicative component structure of Andersen and Bollerslev (1997), for example Engle and Sokalska (2012).

Along a similar line to the univariate work in this area, this paper identifies an important intraday pattern in the pairwise correlations for a portfolio of equities. In contrast to the volatility process of an individual asset, pairwise correlations of a portfolio of assets appear to display an inverted U-shaped pattern over the trading day. Existence of patterns in the intraday correlations leads to questions about how to model and subsequently forecast these dynamics. Commonalities between intraday correlations have been noted in the literature, see for example Allez and Bouchaud (2011) and Tilak, Széll, Chicheportiche, and Chakraborti (2013).[2] Allez and Bouchaud (2011) document the average correlation increased over the trading day, however they did not model these effects. This paper details an approach that is quite different to previous studies in this area, examining the correlation dynamics over the trading day with the specific aim of modelling these processes.

[2] Allez and Bouchaud (2011) and Tilak, Széll, Chicheportiche, and Chakraborti (2013) use eigenvector decompositions of the correlation matrix to study the dynamics of correlations over the trading day for U.S. equities.

The models presented are based on the consistent DCC (cDCC) model of Aielli (2013) and the DECO model of Engle and Kelly (2012), adapted to capture both the daily persistence and the intraday inverted U-shape pattern seen in the correlations between assets over the trading day. Estimation results indicate modelling the diurnal pattern in correlations over the trading day is potentially useful, in a similar way to the importance of accounting for diurnal patterns seen in volatilities. The analysis also highlights the relevance of daily persistence in correlations, with the models allowing for both the intraday pattern in correlations and daily level fluctuations in correlations providing promising results in terms of fit over the sample. A further examination of bivariate relationships and sub-portfolios based on industry reveals the intraday pattern in the correlations is most evident between stocks that have a lower level of unconditional correlation, such as those from different industries. Stocks that are highly correlated display a pattern quite different to the stock pairs with lower unconditional correlation.

The paper is organised as follows. Section 2 presents the modelling framework and the dataset studied is outlined in Section 3. Preliminary analysis of the pairwise correlations is given in Section 4. Empirical estimation results are provided in Section 5. Lastly, the paper concludes in Section 6 with outlining work still underway at time of writing with a view to expand on the study presented here. Suggestions for future applications and extensions are also provided.

2 Methodology

The decomposition of the conditional covariance matrix into univariate and multivariate components, popularised by Engle (2002), is extended to the intraday context as

$$\mathbf{H}_{t,i} = \mathbf{D}_{t,i}\mathbf{R}_{t,i}\mathbf{D}_{t,i} \ . \tag{1}$$

The intraday conditional correlation matrix is denoted $\mathbf{R}_{t,i}$ and the diagonal matrix of intraday conditional standard deviations of the returns on day t for intraday interval i is $\mathbf{D}_{t,i}$. As is the case at the daily frequency, $\mathbf{H}_{t,i}$ is estimated in two stages: firstly, the univariate standard deviations of $\mathbf{D}_{t,i}$ and, secondly the correlations between assets contained in $\mathbf{R}_{t,i}$. This section details the model used to estimate the univariate intraday volatility process of each asset in the portfolio, before describing the framework used to model the intraday correlations.

2.1 Intraday Univariate Volatility

The univariate framework used to estimate the individual volatility process of each stock is based on the multiplicative component GARCH model of Engle and Sokalska (2012). The volatility is decomposed into daily, diurnal and intraday variances as

$$r_{t,i} = \sqrt{h_t s_i q_{t,i}}\, \epsilon_{t,i} \qquad\qquad \epsilon_{t,i} \sim N(0,1)\ . \tag{2}$$

The daily variance component is denoted h_t, s_i is the diurnal pattern over the trading day, $q_{t,i}$ the intraday variance, and, $\epsilon_{t,i}$ an error term. The estimation procedure involves modelling the daily variance, h_t, in the first instance, and then conditioning the intraday returns in order to estimate the diurnal pattern, s_i. The returns are then conditioned by the diurnal component with an univariate GJR–GARCH model to capture the remaining intraday persistence.

Engle and Sokalska (2012) used commercially available volatility forecasts for h_t based on a risk factor model, however in this paper the daily variance is linked to the lagged volatility of the previous day. This approach allows for the use of the realized volatility, $RV_t = \sum_{i=0}^{I} r_{t,i}^2$, and does not require selection of any common risk factors (as in Engle and Sokalska, 2012). The AR(1) used here is

$$h_t = \mu + \varphi RV_{t-1}\ , \tag{3}$$

where RV_{t-1} is the realized volatility on day $t-1$, μ the unconditional volatility and φ a scaling parameter.

The intraday returns are scaled by the daily variances, allowing for the intraday diurnal pattern in the returns, s_i, to be modelled using

$$s_i = \frac{1}{T}\sum_{t=1}^{T} \frac{r_{t,i}^2}{h_t}\ . \tag{4}$$

The returns are then scaled by both the daily and diurnal variance components, denoted $z_{t,i}$,

$$z_{t,i} = \frac{r_{t,i}}{\sqrt{h_t s_i}} = \sqrt{q_{t,i}}\, \epsilon_{t,i}\ , \tag{5}$$

and the residual intraday variance modelled using a GJR–GARCH(1,ϕ,1) specification

$$q_{t,i} = \omega + \alpha z_{t,i-1}^2 + \phi z_{t,i-1}^2 I[z_{t,i-1} < 0] + \beta q_{t,i-1}\ . \tag{6}$$

Here, $\omega = (1 - \alpha - \beta - \phi/2)$ and $I[z_{t,i-1} < 0]$ is a dummy indicator variable that takes the value 1 if $z_{t,i-1}$ is negative and 0 otherwise. The usual constraints apply, that is $\omega > 0$, $\alpha + (\phi/2) \geq 0$, $\beta \geq 0$ and $\alpha + (\phi/2) + \beta < 1$. To summarise, the parameters estimated for the multiplicative component GARCH are $[\mu, \varphi, \alpha, \beta, \phi]$.

2.2 Intraday Dynamic Conditional Correlation

The cDCC specification for modelling intraday conditional correlations is defined

$$\mathbf{R}_{t,i} = \text{diag}(\mathbf{Q}_{t,i})^{-1/2} \, \mathbf{Q}_{t,i} \, \text{diag}(\mathbf{Q}_{t,i})^{-1/2} \, . \tag{7}$$

Several new forms of the pseudo-correlation matrix, $\mathbf{Q}_{t,i}$, are provided for modelling pairwise intraday correlations. The first is simply the original cDCC model,[3] applied at an intraday frequency rather than daily

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1 - a - b) + a \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t,i-1} \hat{\boldsymbol{\epsilon}}'_{t,i-1} \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b \, \mathbf{Q}_{t,i-1} \, , \tag{8}$$

where $\bar{\mathbf{Q}}$ is the unconditional sample correlation of volatility standardised returns, a and b are parameters subject to the positivity constraints $a > 0$, $b > 0$ and $a + b < 1$, and $\hat{\boldsymbol{\epsilon}}_{t,i-1}$ the vector of volatility standardised returns for day t, interval $i - 1$. As the parameters here are scalar values, the correlation dynamics are the same for all assets. For the purposes of this paper this model is referred to as cDCC and will represent a benchmark to which the following extensions are compared.

In equation 8, the pseudo-correlation is mean reverting to the unconditional correlation, $\bar{\mathbf{Q}}$. The approach taken here is in the spirit of how the intraday diurnal pattern is captured in the univariate case for the Engle and Sokalska (2012) method, described above. In the following DCC-Intraday model, the intention is to allow the intraday correlation to revert to the diurnal

[3] Recall the daily version of the cDCC (Engle, 2002) conditional correlation matrix, $\mathbf{R}_t$, as

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2} \, .$$

The pseudo-correlation matrix $\boldsymbol{Q}_t$ (Aielli, 2013) is

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - a - b) + a \, \text{diag}(\mathbf{Q}_{t-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t-1} \hat{\boldsymbol{\epsilon}}'_{t-1} \, \text{diag}(\boldsymbol{Q}_{t-1})^{1/2} + b \, \mathbf{Q}_{t-1}$$

where $\bar{\mathbf{Q}}$ is the unconditional sample correlation of volatility standardised returns, a and b are parameters subject to the positivity constraints $a > 0$, $b > 0$ and $a + b < 1$, and $\hat{\boldsymbol{\epsilon}}_{t-1}$ the vector of volatility standardised returns. As the parameters here are scalar values, the correlation dynamics are the same for all assets.

pattern seen in the pairwise correlations over the trading day, shown as

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}_i^{DI}(1-a-b) + a\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}'_{t,i-1}\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b\ \mathbf{Q}_{t,i-1}\ . \tag{9}$$

The parameters a and b are subject to the same constraints as in equation 8. The matrix $\bar{\mathbf{Q}}_i^{DI}$ is the outer product of standardised returns for each 5-minute interval i of the trading day, averaged over the T days and scaled to give a $N \times N$ correlation matrix for each of the I intervals,

$$\bar{\mathbf{Q}}_i^{DI} = \bar{\mathbf{Q}}_i^* \left(\frac{1}{T} \sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_{t,i}\hat{\boldsymbol{\epsilon}}'_{t,i} \right) \bar{\mathbf{Q}}_i^*\ . \tag{10}$$

Here, $\bar{\mathbf{Q}}_i^* = \mathrm{diag}\left(\frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_{t,i}\hat{\boldsymbol{\epsilon}}'_{t,i}\right)^{-1/2}$.

Allez and Bouchaud (2011, p. 11) find *"... average correlation between stocks increases throughout the day ... "* and this is later confirmed by Tilak, Széll, Chicheportiche, and Chakraborti (2013). Here, the suggestion of a diurnal pattern in correlations over the trading day provides further confirmation. Certainly, a model accounting for any intraday pattern in the pairwise conditional correlation processes is desirable. Further, evidence presented in Section 4 suggests the intraday pattern to be dependent to some degree on the level of correlation between the stocks. This empirical observation allows speculation regarding the magnitude of estimated values of the parameter b. If the value of b is high (a somewhat case-specific value, although usually this can be considered to be above 0.6), relative weighting will be given to the interval-to-interval persistence in forming the pseudo-correlation, $\mathbf{Q}_{t,i}$. The result will be a conditional correlation process similar to that seen at lower frequencies, for example daily. In contrast, should the value of b be low (less than 0.5) the emphasis is passed to the intraday diurnal pattern provided by $\bar{\mathbf{Q}}_i^{DI}$. In that case, the conditional correlations will have an obvious intraday shape for each trading day.

A similar technique can be used to account for correlation persistence at the daily level. The outer product of standardised returns is averaged over the I intervals of the trading day t and scaled to give an $N \times N$ correlation matrix for each of the T days,

$$\bar{\mathbf{Q}}_t^{DY} = \bar{\mathbf{Q}}_t^* \left(\frac{1}{I} \sum_{i=1}^{I} \hat{\boldsymbol{\epsilon}}_{t,i}\hat{\boldsymbol{\epsilon}}'_{t,i} \right) \bar{\mathbf{Q}}_t^*\ . \tag{11}$$

Here, $\bar{\mathbf{Q}}_t^* = \text{diag}\left(\frac{1}{I}\sum_{i=1}^{I}\hat{\boldsymbol{\epsilon}}_{t,i}\hat{\boldsymbol{\epsilon}}_{t,i}'\right)^{-1/2}$. This fluctuation in the dynamic correlations at a daily level can be incorporated into the cDCC framework in equation 8, additively. The previous day's daily level correlation, $\bar{\mathbf{Q}}_{t-1}^{DY}$, enters the model and replaces the lagged intraday pseudo-correlation $\mathbf{Q}_{t,i-1}$. Referred to as DCC-Daily I, $\mathbf{Q}_{t,i}$ becomes

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1-a-c) + a\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}_{t,i-1}'\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + c\ \bar{\mathbf{Q}}_{t-1}^{DY}\ . \qquad (12)$$

The correlation is mean reverting in the sense of the original cDCC, that is reverting to the unconditional $\bar{\mathbf{Q}}$. The scaling parameter c is constrained to be positive, $c > 0$, to ensure positive definiteness, and $a + c < 1$.

An unrestricted version of equation 12 can also be estimated, denoted DCC-Daily II. In this case both the previous interval's pseudo-correlation, $\mathbf{Q}_{t,i-1}$, as well as the additive term for the persistence in the daily correlations, $\bar{\mathbf{Q}}_{t-1}^{DY}$, are included

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1-a-b-c) + a\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}_{t,i-1}'\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b\ \mathbf{Q}_{t,i-1} + c\ \bar{\mathbf{Q}}_{t-1}^{DY}\ . \qquad (13)$$

Here, $a > 0$, $b > 0$, $c > 0$ and $a + b + c < 1$.

The final model is designed to account for both persistence in the daily correlations and the diurnal pattern evident over the trading day, in the spirit of the full univariate model of Engle and Sokalska (2012). Given the importance of capturing both the intraday diurnal pattern and daily-level variance in the univariate case, it is reasonable to expect the two effects will be important in the correlation context. DCC-Both includes the intraday correlation $\bar{\mathbf{Q}}_i^{DI}$ as the intercept, accounting for the daily level persistence additively with the term $c\ \bar{\mathbf{Q}}_{t-1}^{DY}$,

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}_i^{DI}(1-a-c) + a\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}_{t,i-1}'\ \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + c\ \bar{\mathbf{Q}}_{t-1}^{DY}\ . \qquad (14)$$

The parameters are constrained to be positive, $a > 0$ and $c > 0$, and $a + c < 1$. This allows the conditional correlations to revert to the intraday pattern, whilst capturing the daily level persistence of the correlations. The specification omits the $\mathbf{Q}_{t,i-1}$ term, representing the relationship of the previous interval's correlation to the current correlation. Preliminary experiments found that the addition of both intraday and daily level correlation terms rendered this variable redundant.

2.3 Intraday Dynamic Equicorrelation

All the cDCC-based models above readily extend to the equicorrelation context. The assumption of equicorrelation has been found to be useful in the context of modelling correlations at the daily frequency, see Engle and Kelly (2012) and Clements, Scott, and Silvennoinen (2015). It is reasonable to conjecture that similar benefits may exist at the intraday frequency and subsequently the equicorrelated models are included in the analysis.

The DECO framework using intraday data is shown as

$$\rho_{t,i} = \frac{1}{N(N-1)}\left(\mathbf{1}'\mathbf{R}_{t,i}^{DCC}\mathbf{1} - N\right) = \frac{2}{N(N-1)}\sum_{n>m}\frac{q_{n,m,t,i}}{\sqrt{q_{n,n,t,i}q_{m,m,t,i}}} \tag{15}$$

where $q_{n,m,t,i}$ is the n, mth element of the pseudo-correlation matrix $\mathbf{Q}_{t,i}$ using equation 8. Similarly, the intraday diurnal pattern in the correlations as well as a daily persistence variable can be included in the conditional pseudo-correlations as described above. Subsequently the equicorrelations are formed using equation 15, with the relevant specification of $\mathbf{Q}_{t,i}$. In keeping with the terminology used previously these models are referred to as DECO, DECO-Intraday, DECO-Daily I, DECO-Daily II, and DECO-Both.

3 Data

The dataset contains 5-minute returns of six stocks traded on the Australian Stock Exchange (ASX)[4] over the period 4 January 2011 to 30 December 2015. The companies are ANZ, BHP, NAB, RIO, WES and WOW representing two banks, two resource companies, a conglomerate and one retailer. The stock WES (Wesfarmers Ltd) is included to pair with WOW (Woolworths Ltd) as the two represent the largest supermarket operators in the Australian market. Part of the conglomerate Wesfarmers Ltd holdings is Coles Group, which together with Woolworths has an approximate 70% share of the market. Secondary interests of Wesfarmers Ltd include mining businesses, however it is contained in this dataset due to its supermarket (Coles) component.

There are 85,750 5-minute observations over 1,225 trading days, with 70 5-minute intervals per trading day. Trading begins at 10:10 AM and finishes at 4:00 PM, Monday to Friday. The market technically opens at 10:00 AM, however common practice is to discard the first 10

[4] ASX data, as opposed to the U.S. or European, is used in this paper as it was the most reliable source of high frequency data available.

minutes of the trading day when using ASX data. This avoids the opening auction period of the ASX used by the exchange to determine opening prices, see Hall and Hautsch (2006), among many others.

Intraday returns are generated as $r_{t,i} = \log(C_{t,i}/C_{t,i-1})$ where $C_{t,i-1}$ and $C_{t,i}$ are the closing prices of interval $i-1$ and i on day t. The exception is the first period of the day, when the price at the opening of the 10:10 AM - 10:15 AM interval is used to generate the return $r_{t,1}$, that is $C_{t,i-1} = O_{t,1}$. Figure 1 shows a snapshot of the intraday returns for each of the six stocks over a period of high volatility common to all. This particular sub-sample of market turbulence is observed from August 2015 to October 2015 and corresponds to global equity markets recording large losses during August 2015. These decreases were widely reported to be a response to investor concern regarding China and falling commodity prices.

A common feature in all measures of intraday trading is a diurnal pattern in the volatility process. This U-shape is documented by many researchers, see Andersen and Bollerslev (1997) and Engle and Sokalska (2012) among others. It is easily seen in the average squared intraday returns for each stock, $\bar{r}_i^2$, as in Figure 2. The squared returns series $r_{t,i}^2$ has been averaged across the t days for each i to generate $\bar{r}_i^2$. Evidence of this pattern in the volatility process of equity returns sampled at a high intraday frequency has complicated modelling of these processes.

Prior to any formal analysis of intraday correlations, it is useful to examine simple unconditional correlations (Table 1), using the raw returns $r_{t,i}$. ANZ and NAB are both banking stocks; BHP and RIO belong to the resource sector; WES is a conglomerate paired with WOW (retail), as discussed above. As would be expected, the correlations are higher for those stocks from the same industry. The pair of resource companies are more highly correlated with the banking pair than they are with WES and WOW. Also expected is that the resources pair is more highly correlated with WES than WOW, due to the mining interests of WES. Although the unconditional correlation of WES-WOW are not as high as those of the banking and resource pairs, it is noted that the two 'retail' stocks are more correlated with each other than any of the alternative stocks. In the analysis contained in the following section, these between- and within-industry differences are explored in terms of the effect (if any) on the behaviour of the correlation dynamics of the portfolio.

4 Preliminary Analysis

For the analysis, the returns are volatility standardised, denoted $\hat{\boldsymbol{\epsilon}}_{t,i}$, using the univariate multiplicative component GARCH model outlined in Section 2.1. These volatility adjusted returns are shown in Figure 3 for the same sub-sample as above (August 2015 to October 2015). It can be seen that the periods of turbulence and calm have normalised when compared to the raw returns of Figure 1. In essence, the intraday volatility adjusted returns are similar to what would be expected of volatility standardised returns at a lower frequency (for example, daily).

It is useful to again consider the unconditional correlations (this time of the volatility standardised returns) and Table 2 contains these values. In line with expectations, the unconditional correlations are similar to those in Table 1, leading to the same qualitative conclusions described above.

Figure 4 plots the pairwise intraday correlations contained in $\bar{\mathbf{Q}}_i^{DI}$ of equation 9. Recall this is the outer product of volatility standardised returns, averaged over the T days of the sample and scaled to be a true correlation matrix. A pattern over the trading day can be seen, as each of the pairwise relationships show an inverted U-shape. The notable exceptions to this common shape during trading hours is ANZ-NAB and BHP-RIO. Reasoning for these exceptions is provided below, following discussion of the intraday pattern itself.

The intraday inverted U-shape is common to all pairs exhibiting a relatively low, or weak, unconditional correlation. Those stock pairings displaying a moderate to high unconditional correlation over the sample behave differently, particularly over the morning and middle sessions. The distinguishing factor here appears to be whether or not the two stocks are related by industry. In the dataset analysed in this paper, the banks (ANZ-NAB) and resource companies (BHP-RIO) exhibit correlation dynamics over the trading day that are different to those where the equities are from different or diverse industries. Furthering the industry effect argument is the interesting case of WES-WOW. Recall that WES is a conglomerate, owner of the (retail) supermarket chain Coles. Coles is the major competitor of WOW (retail). The WES-WOW pairing is not a perfect industry match however, due to the mining interests of WES. It follows then that the intraday correlation behaviour of the pair falls somewhere between that of pairs from the same industry and the diverse-industry pairings, and Figure 4 displays this intuition.

The inverted U-shape is clearly shown when the trading day is broken into sessions, as in Table 3, which displays the mean of the pairwise intraday correlations in $\bar{\mathbf{Q}}_i^{DI}$ over three periods of trade. The three sessions are defined as *Morning*, 10:10AM to 11:30AM; *Middle* of the day, 11:30AM to 14:30PM; and, *Afternoon*, 14:30PM to 16:00PM. It is clear for each pair (bar the exceptions highlighted above) that the mean value is higher during the middle session, further illustrating the pattern evident in the intraday correlations of Figure 4.

Having outlined evidence of the intraday U-shape pattern evident in the correlations over the trading day, possible reasoning for these dynamics can be given. Perhaps the simplest explanation is variation in the timing of news arrival. That is, arrival of news overnight (before the opening session) may be different in nature to that arriving during active trading. News timing variation could manifest itself in increased firm level, or idiosyncratic, effects over the morning to middle sessions of trade, leading to correlations that begin low. Correlations then rise steeply during late morning and stay high until late afternoon. Industry-wide news appears to arrive before the market opens, leading to correlations that begin relatively high and remain high until later in the trading day.

Interestingly, it appears from Figure 4 that the diurnal pattern is strongest for those pairs that are otherwise weakly correlated in this context. For example, the between-industry pairing of BHP (resources) and WES (conglomerate) reveals a pronounced rise during the morning session of trade, between 10:10AM and 11:30AM. This pair exhibits a relatively weak (0.32) unconditional correlation over the full sample. Calculating the difference between the mean of the morning session and that of the middle of the day reveals a difference of 0.06 for the BHP-WES pairing. Similar stories could be told of any of the other diverse-industry pairings. In contrast, ANZ-NAB has a difference of -0.01. This pair of banking stocks display a level of correlations over the morning and midday trading sessions that only slightly deviates from their unconditional level of correlation of 0.55. The resources pairing of BHP-RIO also display a mean correlation that is unchanged over the morning session, with a difference of 0.00.

The apparent relationship between the unconditional level of correlations and the difference in means is not as pronounced in the afternoon. All mean correlations fall between the middle session and the afternoon trading period, with differences in the mean of the sessions ranging from -0.06 to -0.03. The implication of this result for the afternoon session lends further weight to the reasoning of timing with regard to industry-wide versus firm-specific news. The correlation dynamics over the trading day appear to depend not only on the pairwise unconditional correlations, but also time of day.

Figure 5 contains the daily level pairwise correlations contained in $\bar{\mathbf{Q}}_t^{DY}$ of equation 12. It is the outer products of the volatility standardised returns averaged over the I intervals for each of the T days and scaled to be a true correlation matrix. All pairs display similar trends over the sample, although the magnitude of changes in the correlations are larger for some than others.

5 Estimation Results

A full sample comparison of the conditional correlations outlined in Section 2.2 is provided in Tables 4 to 7. All models estimate easily and appear to fit the data well over the sample, given comparable log-likelihood values and information criterion (IC).[5] A likelihood ratio (LR) test restricting the parameter $b = 0$ is used to compare the nested models -Daily I and -Daily II of equations 12 to 13. The unrestricted model (-Daily II) is favoured in all cases, except $N = 6$ for DCC-Daily II, where $b = 0.00$ and not significant.

The previous section highlighted apparent differences in the inverted U-shape of the pairwise correlations over the trading day, providing motivation for bivariate comparison. Table 4 provides full sample parameter estimates for a within-industry pair (BHP-RIO) and diverse industry pair (ANZ-BHP). The resources pair, BHP-RIO, exhibit the highest unconditional correlation coefficient over the entire sample. The bank-resources pair, ANZ-BHP, have a higher unconditional correlation with each other than any other stock in the sample (excluding stocks from the same industry).[6] Figures 8 and 9 display the equicorrelation series for the DECO-Both model. The differences in the diurnal shape in Figure 4 are seen at the start of each trading day, as the three week snapshot shown in each plot illustrates the sudden jump in equicorrelation for the within-industry pair. Comparatively, the diverse industry pairing has a more gradual shift over the morning period of trade. Using the average intraday correlation matrix $\bar{\mathbf{Q}}_i^{DI}$ as the intercept allows for any differing shapes across the early session of trade, as the psuedo-correlation $\mathbf{Q}_{t,i-1}$ can revert readily to unique pairwise patterns.

In terms of differences in parameters between the pairs, the within-industry pairing exhibits higher values of a and lower values of b. Recall the parameter a scales the outer product of

[5] A range of possible starting values are used when estimating parameter values for each model specification.
[6] All bivariate pairings were estimated and the results are available upon request.

the volatility standardised returns from the previous interval; b governs the weight given to the lagged pseudo-correlation of the previous interval.[7] The implication here is that the relative importance between present (outer product of returns) and past (lagged pseudo-correlation) information in the correlation dynamics changes dependent on whether the pairing is highly unconditionally correlated or not. In terms of the impact of differences in parameter values, higher values of parameter b lead to smoother correlation dynamics for the diverse-industry pair (for example, ANZ-BHP). This point is illustrated in Figures 6 and 7. For ease of interpretation, two volatile trading periods of approximately three trading weeks each have been presented. The first begins 2 August 2011 to 18 August 2011 and corresponds to the downgrading of US credit. The second is 18 August 2015 to 4 September 2015, where large losses recorded by global equity markets led to increased market volatility. Unsurprisingly, the behaviour of the correlation dynamics between the 2011 and 2015 periods of relative market turmoil is similar: correlations decrease during the periods of volatility spiking, then increase again as the market normalises. Worth noting is the relative magnitude of the correlation fluctuations between 2011 and 2015, particularly in the ANZ-BHP case where the more recent turbulence led to larger movements in the correlation dynamics than 2011.

To examine the industry effect on the intraday diurnal pattern further, three portfolios are formed. The first contains 3 stocks of different industries; the second contains 4 stocks from two industries; and, the final portfolio contains all 6 equities.[8] Parameter estimates for each of the models outlined in Section 2 are contained in Tables 5 to 7. The first theme to arise from the portfolio combinations presented here is that parameter a is in general higher and b lower for the DECO class of model in comparison to the cDCC family. This is a well documented result for lower frequencies, see Engle and Kelly (2012) and Clements, Scott, and Silvennoinen (2015) among others. Similar trends in the high frequency, intraday setting provides confirmation of the relevance of a multivariate GARCH framework here. Highlighted for the bivariate case above, higher values of b lead to smoother conditional correlation dynamics for the equicorrelated models.

In the context of the unrestricted '-Daily II' models in particular, there is a large difference in the estimated b coefficient, the parameter weighting of the previous interval's pseudo-correlation, $\mathbf{Q}_{t,i-1}$. For the cDCC-based models, this parameter is much lower. For example, in $N = 6$ the

[7] In the bivariate case, $N = 2$, the DECO model is equivalent to a cDCC framework as no 'averaging' of pseudo-correlations needs to occur to form the equicorrelation ρ_t.

[8] All diverse and within-industry combinations were formed, results are available on request.

DCC-Daily II $b = 0.00$ compared to 0.92 for DECO-Daily. This leads to the overall persistence in the correlations, that is $a+b+c$, to be much lower for the DCC-Daily II model than DECO-Daily II. As above, the implication is that the contribution of present and past information in the correlation dynamics is very different. For $N = 3$ and $N = 4$, the DCC-Daily II parameter values of b are higher (0.09 and 0.14 respectively), although remain relatively low in comparison to the DECO-Daily II values. The impact of this difference in terms of the behaviour of correlations is most easily displayed visually, as in Figures 10 and 13. The two periods examined are the same high volatility snapshots used previously.

Recall that the '-Daily' models do not explicitly take the intraday correlation pattern into account. It is then interesting to note some regime-style transitions in the average correlation dynamics of DCC-Daily II, particularly for the August 2011 sub-period in Figure 10. More generally though, the impact of lower overall persistence in the correlations compared to the DECO-Daily II model is clear. The DECO-Daily II equicorrelation is much smoother than the DCC-based model, although all the key movements in correlation (for example, a relatively large decrease on 28 August 2015, among others) are similar between the two frameworks. These differences (and similarities) may be important in a practical sense and forecasting exercises would shed light on these dynamics out-of-sample.

The isolation of a three week trading period also provides a useful illustration of how the '-Both' models behave. Figures 14 through 17 show the dynamics of the correlations for the DCC-Both and DECO-Both models over two high volatility sub-periods. Unlike the differences evident in the '-Daily II' example, both the non-equicorrelated and equicorrelated versions display a clear pattern over the trading day. Recall that these models are designed to capture both the intraday pattern in correlations and correlation clustering at the daily frequency. In terms of parameters, the estimated coefficient a, which governs the input of new information into the pseudo-correlation, is similar to the other models. Recall that the '-Both' specification omits the lagged pseudo-correlation $\mathbf{Q}_{t,i-1}$, thus there is no b coefficient to report. Instead, each '-Both' model is designed to capture the daily level persistence in correlations through the additive term $\bar{\mathbf{Q}}_t^{DY}$, with the coefficient c. In general, the DCC-Both model has lower estimated values of c than the corresponding DECO-Both model. This effectively smooths the equicorrelated process over the day, as the daily level correlation is given a higher weighting.

It appears that capturing the intraday diurnality in the correlations may be as important as capturing temporal dependence at the daily frequency. This is reflected in the log-likelihood and IC values of the DCC-Both model in comparison to the DCC-Intraday and -Daily models, for all portfolio sizes. Of the DCC-Daily models, DCC-Daily II provides a reasonable fit and it is the same in the case of the equivalent DECO-Daily model group. This is presumably due to the fact that the Daily II models are an unrestricted version, aiming to account for daily level persistence additively whilst allowing for both present and past information explicitly. This seems to indicate that a complete picture of correlation persistence is helpful in the absence of directly capturing the diurnal intraday pattern.

6 Conclusion

The availability of high frequency intraday data has presented opportunities to model the intraday correlation dynamics of a portfolio of assets. Modelling of these processes is important for a range of practical applications over the trading day, including hedging, risk management, trade scheduling and setting limit orders. Traders and market makers require up-to-date information at increasingly small intervals, motivating studies such as that in this paper. Increased reliance on computerised trading depends on forecasts of volatility and as financial decisions are realistically made in terms of more than one asset, correlations are important. Given a thorough search of the intraday correlation literature, this is the first piece of research to explicitly consider modelling high frequency intraday correlation dynamics and the first to use the MGARCH framework in this context. Despite the interest in modelling intraday volatilities of individual assets, the area of intraday correlations is a relatively new literature.

This paper outlined key features of the behaviour of correlations over the trading day. In particular, evidence of an inverted U-shape pattern in the intraday correlations is found. Further, several models based on Dynamic Conditional Correlation and Dynamic Equicorrelation are presented to capture this intraday diurnal pattern and illustrate its apparent importance. Results of full sample estimation indicate that it may be worthwhile to incorporate both day-to-day correlation fluctuations and the intraday pattern in the correlations. In terms of the intraday pattern evident in the correlations, stocks that are highly correlated such as those from the same industry, seem to start highly correlated. This results in intraday correlation dynamics quite different to those pairs with lower unconditional correlations, where a steep rise in correlations is seen early in the day. In general, the cDCC framework appears to model the intraday correlation dynamics more effectively than DECO.

The insights into the behaviour of high intraday frequency correlations between equities, and the novel modelling approach suggested to capture these patterns, contribute to the intraday correlation literature. Future research could consider alternative models for capturing the intraday pattern in correlations. For example, a seasonal ARIMA (see Hamilton, 1994, for a thorough examination of this class of model), smooth transition GARCH framework of Silvennoinen and Teräsvirta (2015) or flexible Fourier form similar to that of Frijns and Margaritis (2008) may be appropriate.

As research continues in the area of intraday correlations and risk, forecasting promises to yield interesting results. A host of financial applications require intraday measures of risk and such techniques would benefit derivative traders and institutional investors, for example hedge funds. Future work in terms of forecasting exercises could be along these lines.

7 Tables and Figures

Intraday Returns: August 2015 to October 2015

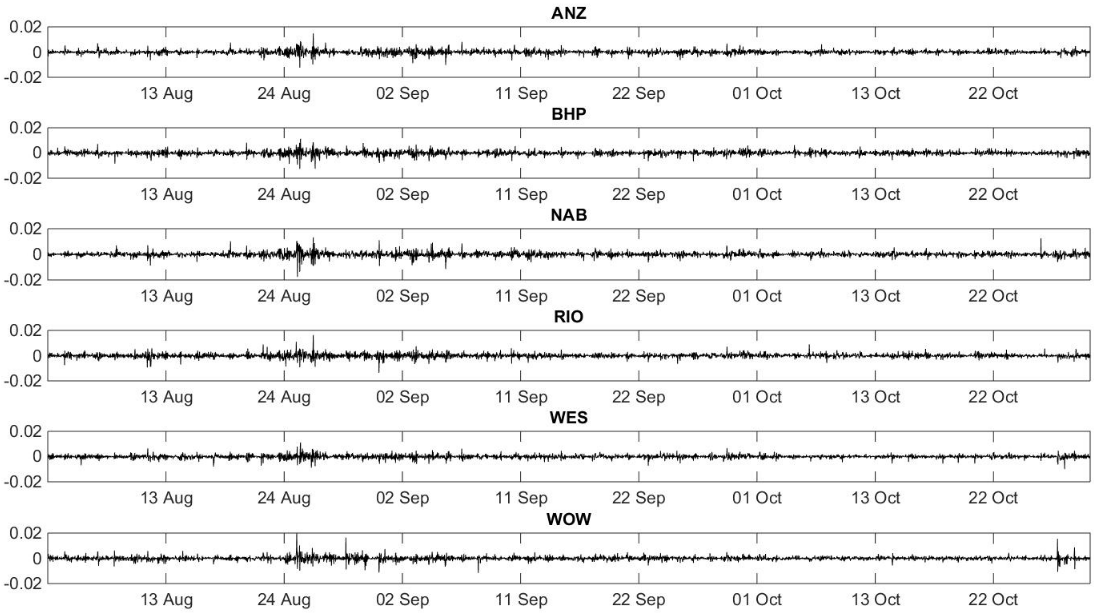

Figure 1: Sample of 5-minute intraday returns for each of the 6 Australian equities, sub-period spans 3 August 2015 to 30 October 2015.

Average Squared Returns

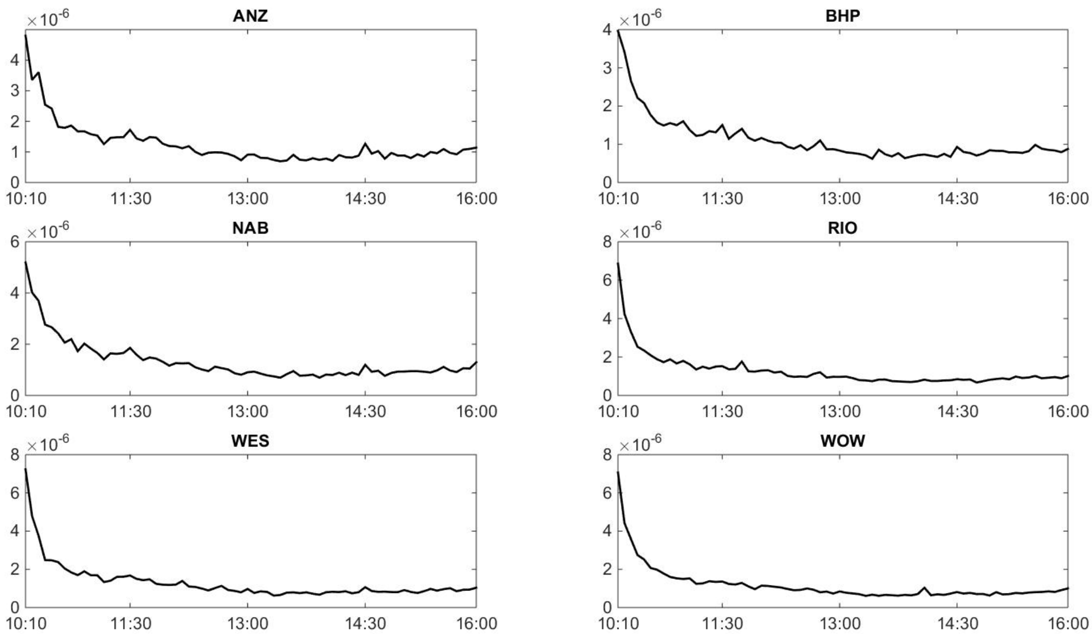

Figure 2: Average squared 5-minute intraday returns of each stock, $\bar{r}_i^2$, entire period spans 4 January 2011 to 30 December 2015.

Unconditional Correlations

Industry	Banking		Resources		Conglomerate	Retail
ρ	ANZ	NAB	BHP	RIO	WES	WOW
ANZ	–	0.6069	0.4193	0.3597	0.3356	0.2988
NAB		–	0.4072	0.3538	0.3355	0.2947
BHP			–	0.5969	0.3179	0.2860
RIO				–	0.2681	0.2367
WES					–	0.4566
WOW						–

Table 1: Unconditional correlations of 5-minute intraday returns for each pair of stocks, raw returns $r_{t,i}$ used, entire period spans 4 January 2011 to 30 December 2015.

Volatility Standardised Returns: August 2015 to October 2015

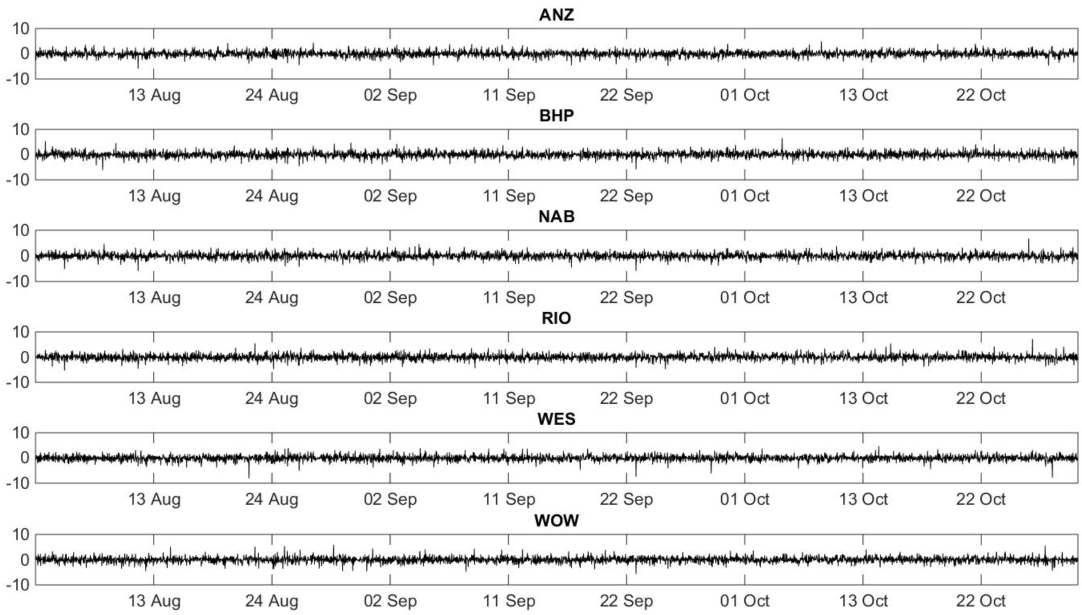

Figure 3: Sample of volatility standardised returns, $\hat{\boldsymbol{\epsilon}}_{t,i}$, for each of the 6 Australian equities. The sub-period spans 3 August 2015 to 30 October 2015.

Unconditional Correlations, volatility adjusted

Industry	Banking		Resources		Conglomerate	Retail
ρ	ANZ	NAB	BHP	RIO	WES	WOW
ANZ	–	0.5533	0.4086	0.3488	0.3378	0.3053
NAB		–	0.3930	0.3407	0.3336	0.2973
BHP			–	0.5683	0.3233	0.2944
RIO				–	0.2797	0.2553
WES					–	0.4557
WOW						–

Table 2: Unconditional correlations of 5-minute intraday returns for each pair of stocks, volatility adjusted returns $\hat{\boldsymbol{\epsilon}}_{t,i}$ used, entire period spans 4 January 2011 to 30 December 2015.

Average Intraday Correlations, volatility adjusted: $\bar{\mathbf{Q}}_i^{DI}$

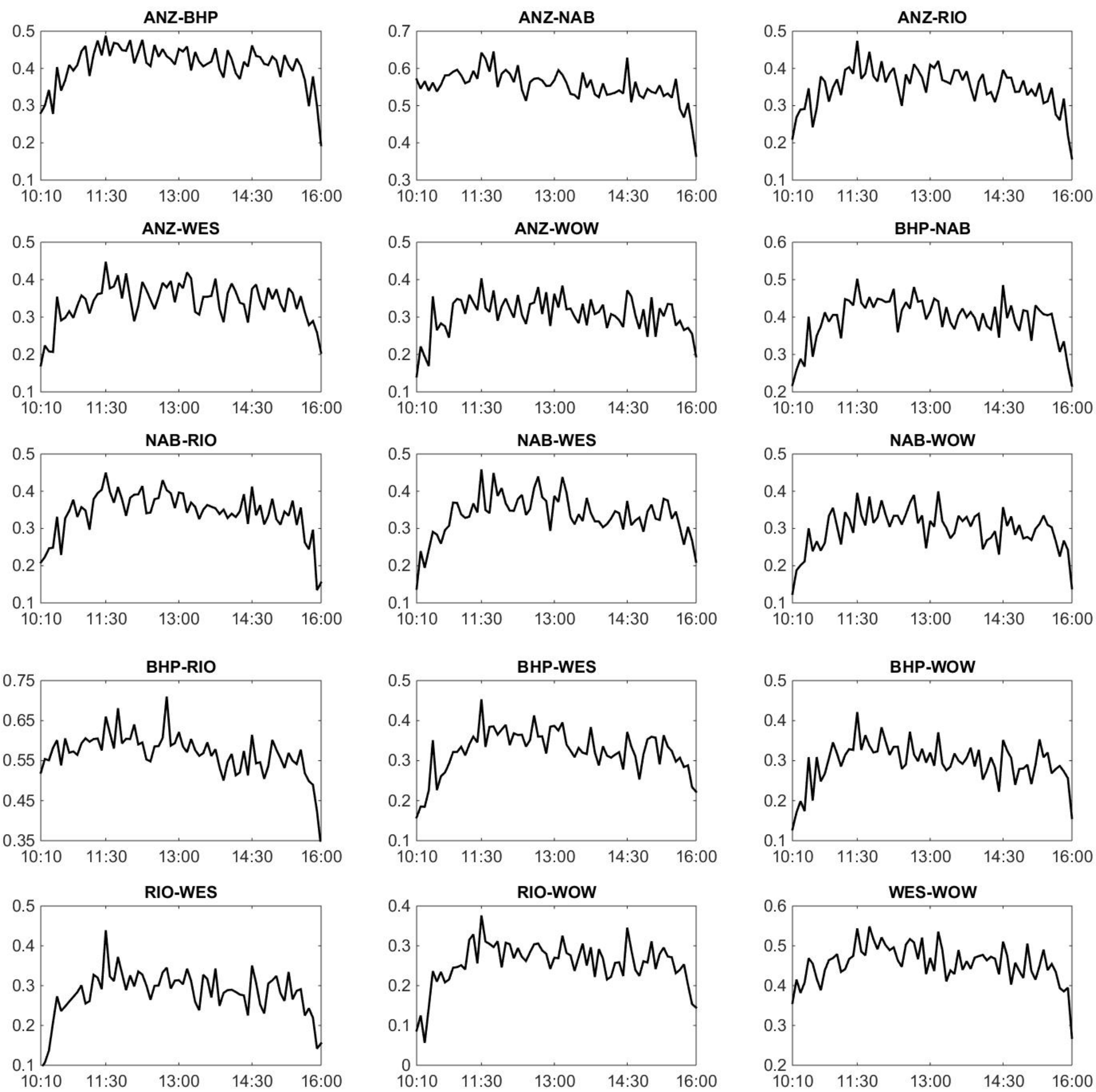

Figure 4: Plot of pairwise intraday correlations, $\bar{\mathbf{Q}}_i^{DI}$ of equation 9. Entire period spans 4 January 2011 to 30 December 2015.

$\bar{\mathbf{Q}}_i^{DI}$: Means over Trading Day

Morning - 10:10AM to 11:30AM						
Mean	ANZ	NAB	BHP	RIO	WES	WOW
ANZ	–	0.5731	0.3910	0.3356	0.3077	0.2888
NAB		–	0.3667	0.3235	0.3032	0.2727
BHP			–	0.5823	0.2910	0.2736
RIO				–	0.2535	0.2222
WES					–	0.4413
WOW						–
Middle - 11:30AM to 14:30PM						
Mean	ANZ	NAB	BHP	RIO	WES	WOW
ANZ	–	0.5637	0.4307	0.3731	0.3594	0.3216
NAB		–	0.4184	0.3683	0.3586	0.3193
BHP			–	0.5830	0.3480	0.3126
RIO				–	0.3030	0.2770
WES					–	0.4775
WOW						–
Afternoon - 14:30PM to 16:00PM						
Mean	ANZ	NAB	BHP	RIO	WES	WOW
ANZ	–	0.5126	0.3838	0.3148	0.3272	0.2909
NAB		–	0.3707	0.3041	0.3163	0.2793
BHP			–	0.5234	0.3078	0.2796
RIO				–	0.2610	0.2454
WES					–	0.4271
WOW						–

Table 3: The mean of pairwise average intraday correlations, $\bar{\mathbf{Q}}_i^{DI}$. Trading day split into three sessions, entire period spans 4 January 2011 to 30 December 2015.

Daily Correlations, volatility adjusted: $\bar{\mathbf{Q}}_t^{DY}$

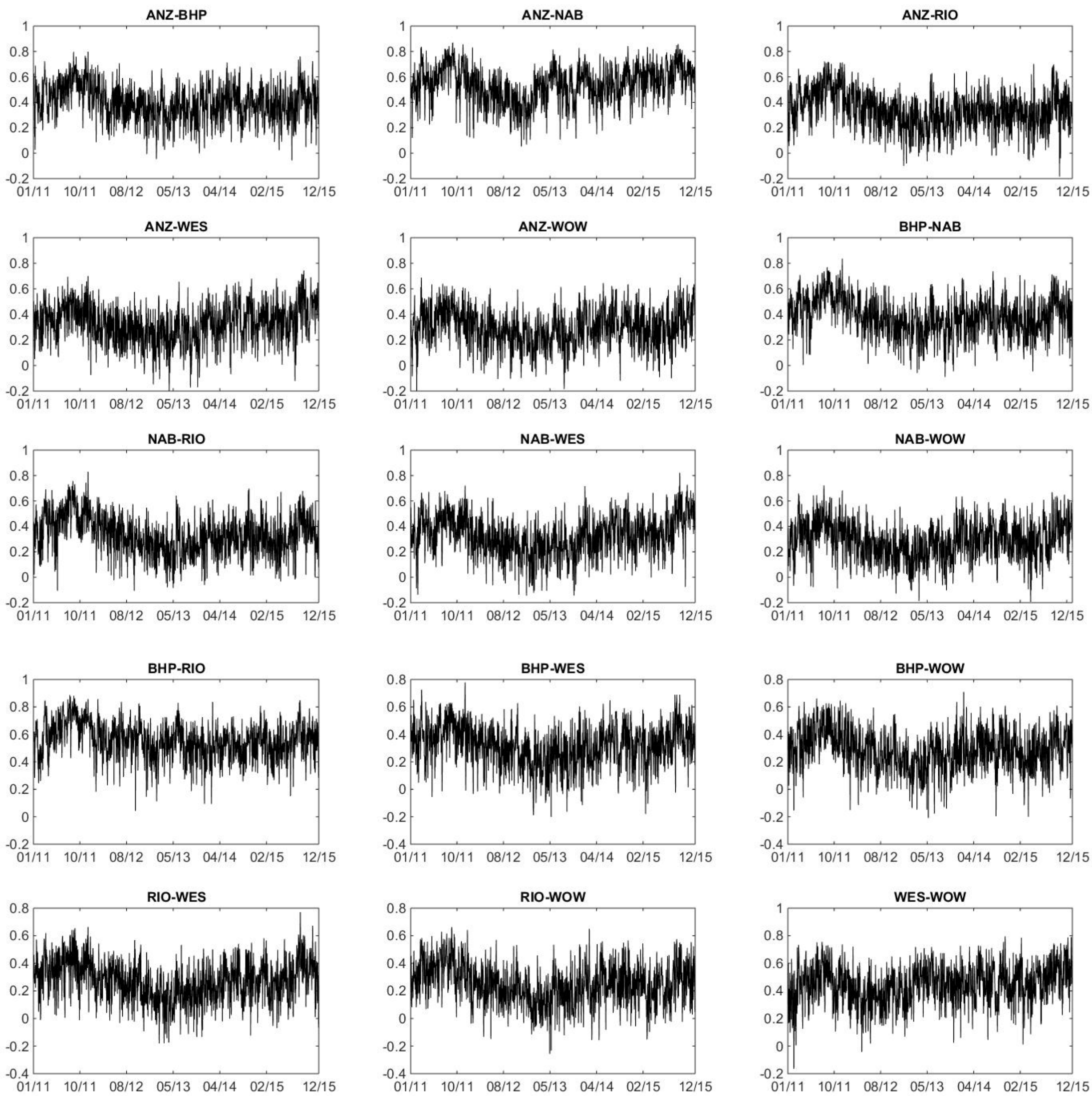

Figure 5: Plot of daily pairwise correlations contained in $\bar{\mathbf{Q}}_t^{DY}$ of equation 12. Entire period spans 4 January 2011 to 30 December 2015.

$N = 2$, Bivariate Relationships: Full Sample Results

Model	a	b	c	Log-Like	AIC	BIC	p-value
BHP-RIO							
cDCC	0.0131 (0.0022)	0.9780 (0.0044)		-68737	137478	137497	
DCC-Intraday	0.0144 (0.0025)	0.9750 (0.0054)		-68728	137460	137479	
DCC-Daily I	0.0265 (0.0022)		0.4109 (0.0184)	-69012	138028	138046	
DCC-Daily II	0.0224 (0.0032)	0.9381 (0.0136)	0.0153 (0.0050)	**-68702**	137409	137437	0.0000
DCC-Both	0.0240 (0.0019)		0.3559 (0.0174)	-68742	137487	137506	
DECO	0.0133 (0.0015)	0.9775 (0.0032)		-68737	137478	137497	
DECO-Intraday	0.0144 (0.0019)	0.9749 (0.0044)		-68728	137460	137479	
DECO-Daily I	0.0267 (0.0023)		0.4011 (0.0205)	-69012	138028	138047	
DECO-Daily II	0.0224 (0.0044)	0.9382 (0.0161)	0.0153 (0.0053)	**-68702**	137409	137437	0.0000
DECO-Both	0.0239 (0.0044)		0.3558 (0.0171)	-68742	137487	137506	
ANZ-BHP							
cDCC	0.0082 (0.0010)	0.9853 (0.0022)		-77680	155364	155383	
DCC-Intraday	0.0086 (0.0010)	0.9844 (0.0024)		-77679	155362	155381	
DCC-Daily I	0.0163 (0.0150)		0.3611 (0.0234)	-77816	155636	155655	
DCC-Daily II	0.0114 (0.0022)	0.9694 (0.0093)	0.0057 (0.0027)	-77675	155357	155385	0.0000
DCC-Both	0.0140 (0.0020)		0.3077 (0.0219)	**-77634**	155272	155291	
DECO	0.0082 (0.0009)	0.9853 (0.0020)		-77680	155364	155383	
DECO-Intraday	0.0085 (0.0011)	0.9844 (0.0027)		-77679	155362	155381	
DECO-Daily I	0.0161 (0.0027)		0.3605 (0.0224)	-77816	155636	155655	
DECO-Daily II	0.0116 (0.0012)	0.9684 (0.0057)	0.0059 (0.0019)	-77675	155357	155385	0.0000
DECO-Both	0.0140 (0.0037)		0.3013 (0.0293)	**-77634**	155272	155291	

Table 4: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. The p-value relates to the LR test of the restriction $b = 0$, applicable only in the case of the Daily models. Entire period spans 4 January 2011 to 29 December 2012. Selected $N = 2$ portfolios: Industry Pair (BHP, RIO) and Diverse Industry (ANZ, BHP).

Average DCC-Daily II Correlations

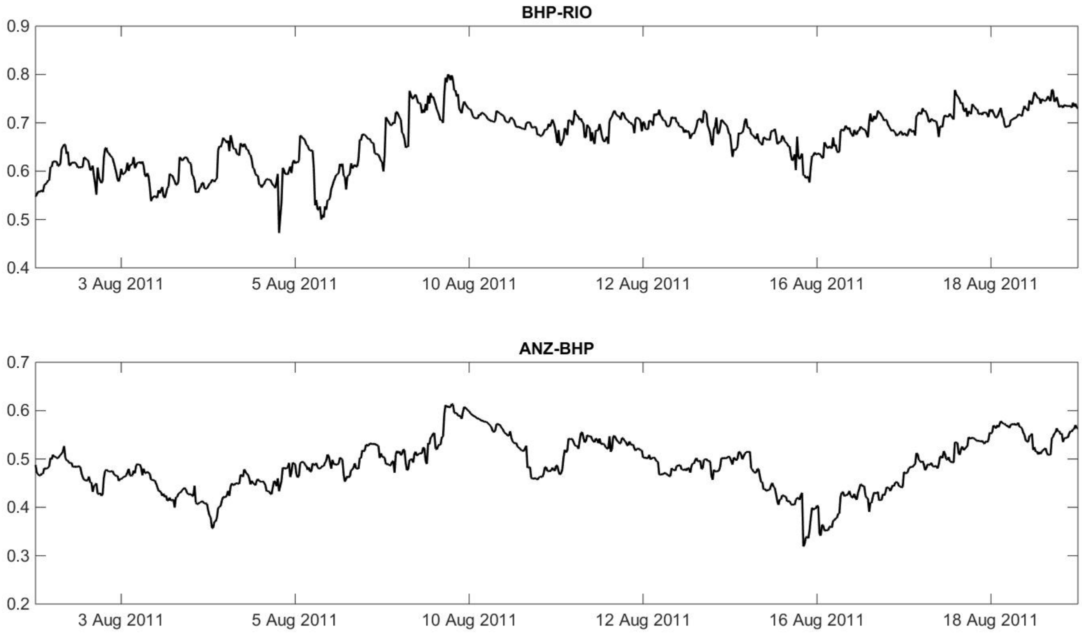

Figure 6: Average DCC-Daily II correlations over the period 2 August 2011 to 18 August 2011, for selected $N = 2$ portfolios: Industry Pair (BHP, RIO) and Diverse Industry (ANZ, BHP).

Average DCC-Daily II Correlations

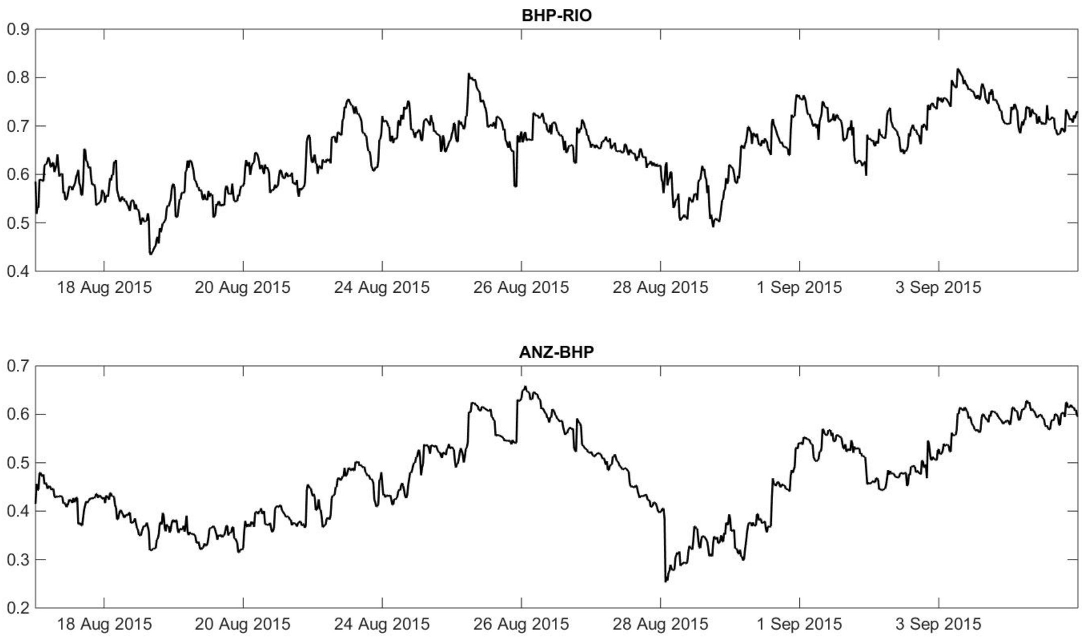

Figure 7: Average DCC-Daily II correlations over the period 17 August 2015 to 4 September 2015, for selected $N = 2$ portfolios: Industry Pair (BHP, RIO) and Diverse Industry (ANZ, BHP).

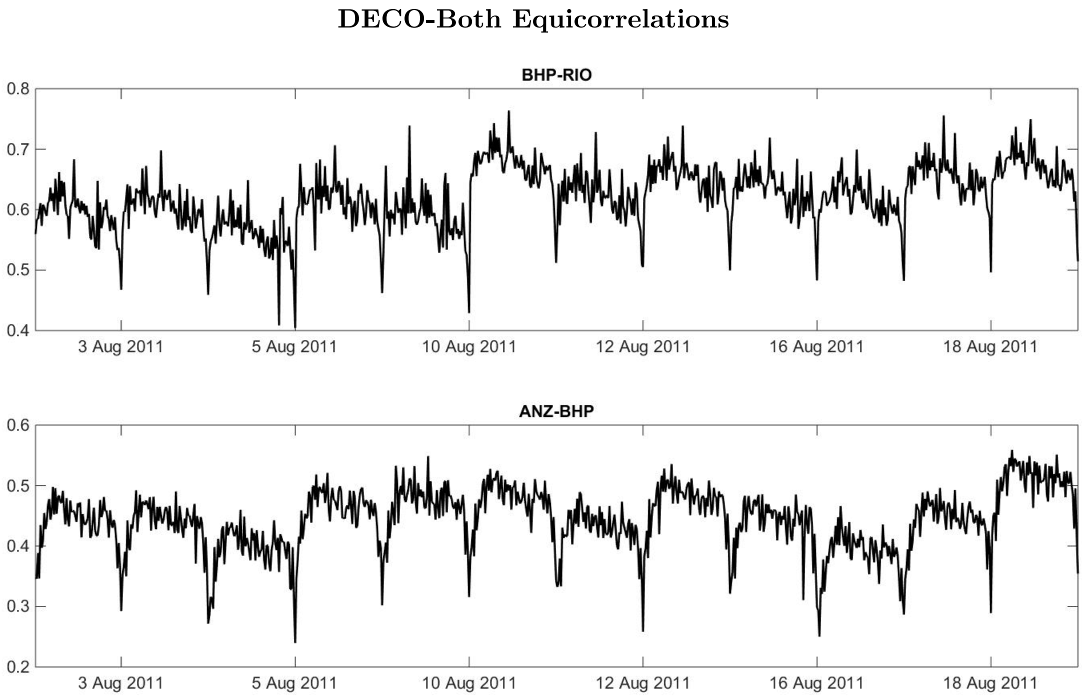

Figure 8: Equicorrelations of DECO-Both model over the period 2 August 2011 to 18 August 2011, for selected $N = 2$ portfolios: Industry Pair (BHP, RIO) and Diverse Industry (ANZ, BHP).

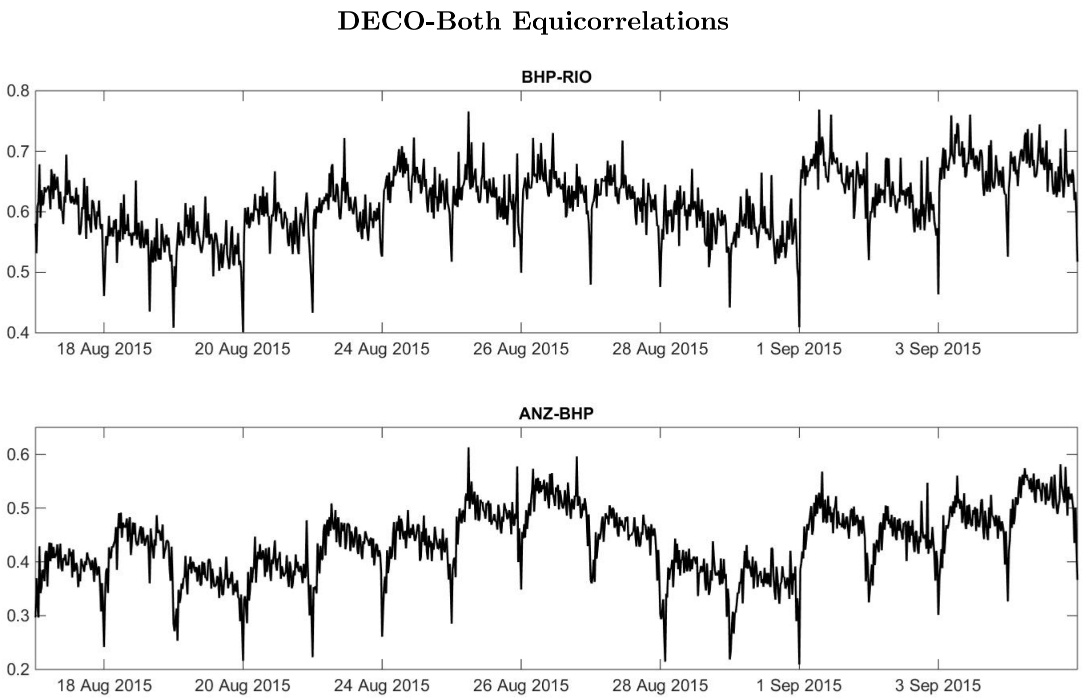

Figure 9: Equicorrelations of DECO-Both model over the period 17 August 2015 to 4 September 2015, for selected $N = 2$ portfolios: Industry Pair (BHP, RIO) and Diverse Industry (ANZ, BHP).

$N = 3$, Diverse Industry Portfolio (NAB, RIO, WES)

Model	a	b	c	Log-Like	AIC	BIC	p-value
cDCC	0.0032 (0.0005)	0.9952 (0.0008)		-116550	233104	233122	
DCC-Intraday	0.0033 (0.0006)	0.9949 (0.0010)		-116557	233119	233137	
DCC-Daily I	0.0166 (0.0018)		0.3012 (0.0123)	-116975	233955	233974	
DCC-Daily II	0.0178 (0.0019)	0.0921 (0.0120)	0.2741 (0.0106)	-116968	233942	233970	0.0001
DCC-Both	0.0151 (0.0095)		0.2593 (0.0113)	**-116511**	233026	233044	
DECO	0.0108 (0.0044)	0.9826 (0.0082)		-116662	233327	233346	
DECO-Intraday	0.0120 (0.0014)	0.9799 (0.0032)		-116676	233356	233375	
DECO-Daily I	0.0254 (0.0066)		0.4325 (0.0151)	-117059	234121	234140	
DECO-Daily II	0.0188 (0.0011)	0.9522 (0.0037)	0.0107 (0.0016)	-116717	233439	233467	0.0000
DECO-Both	0.0204 (0.0035)		0.3632 (0.0161)	-116563	233129	233148	

Table 5: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. The p-value relates to the LR test of the restriction $b = 0$, applicable only in the case of the Daily models. Entire period spans 4 January 2011 to 29 December 2012. $N = 3$, Diverse Industry Portfolio: NAB, RIO and WES.

$N = 4$, Industry Pairs Portfolios (ANZ, NAB, BHP, RIO)

Model	a	b	c	Log-Like	AIC	BIC	p-value
cDCC	0.0075 (0.0007)	0.9863 (0.0017)		-126542	253088	253107	
DCC-Intraday	0.0079 (0.0006)	0.9855 (0.0015)		-126533	253070	253089	
DCC-Daily I	0.0192 (0.0045)		0.3215 (0.0097)	-127316	254635	254654	
DCC-Daily II	0.0216 (0.0163)	0.1446 (0.5068)	0.2747 (0.1754)	-127279	254564	254592	0.0000
DCC-Both	0.0177 (0.0018)		0.2854 (0.0085)	**-126481**	252965	252984	
DECO	0.0161 (0.0018)	0.9730 (0.0034)		-126776	253556	253575	
DECO-Intraday	0.0177 (0.0017)	0.9688 (0.0038)		-126783	253570	253588	
DECO-Daily I	0.0241 (0.0025)		0.4760 (0.0144)	-127529	255062	255081	
DECO-Daily II	0.0216 (0.0013)	0.9487 (0.0046)	0.0107 (0.0020)	-126742	253489	253517	0.0000
DECO-Both	0.0198 (0.0044)		0.4007 (0.0148)	-126602	253208	253227	

Table 6: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. The p-value relates to the LR test of the restriction $b = 0$, applicable only in the case of the Daily models. Entire period spans 4 January 2011 to 29 December 2012. $N = 4$, Industry Pairs Portfolio: ANZ, NAB, BHP and RIO.

$N = 6$, All Stocks Portfolio

Model	a	b	c	Log-Like	AIC	BIC	p-value
cDCC	0.0040 (0.0004)	0.9928 (0.0009)		-191494	382992	383011	
DCC-Intraday	0.0045 (0.0006)	0.9912 (0.0015)		-191493	383604	383623	
DCC-Daily I	0.0161 (0.0010)		0.2370 (0.0067)	-192574	385152	385170	
DCC-Daily II	0.0161 (0.0012)	0.0003 (0.0007)	0.2369 (0.0075)	-192574	385153	385181	-
DCC-Both	0.0152 (0.0012)		0.2095 (0.0058)	**-191030**	382063	382082	
DECO	0.0211 (0.0068)	0.9625 (0.0137)		-193075	386154	386172	
DECO-Intraday	0.0301 (0.0067)	0.9379 (0.0200)		-193488	386981	386999	
DECO-Daily I	0.0314 (0.0040)		0.4731 (0.0144)	-193694	387392	387411	
DECO-Daily II	0.0310 (0.0017)	0.9163 (0.0055)	0.0212 (0.0020)	-193125	386255	386283	0.0000
DECO-Both	0.0246 (0.0769)		0.3883 (0.0243)	-191664	383331	383350	

Table 7: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. The p-value relates to the LR test of the restriction $b = 0$, applicable only in the case of the Daily models. Entire period spans 4 January 2011 to 29 December 2012. $N = 6$, All Stocks Portfolio: ANZ, BHP, NAB, RIO, WES and WOW.

Average DCC-Daily II Correlations

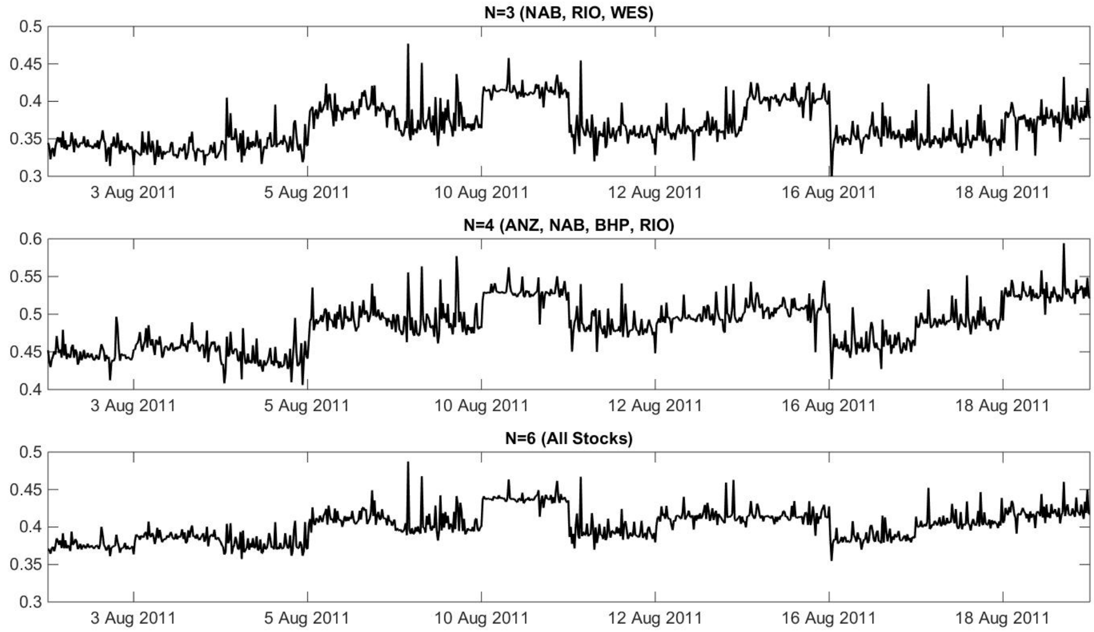

Figure 10: Average DCC-Daily II correlations over the period 2 August 2011 to 18 August 2011, for selected $N = 3, 4, 6$ portfolios.

Average DCC-Daily II Correlations

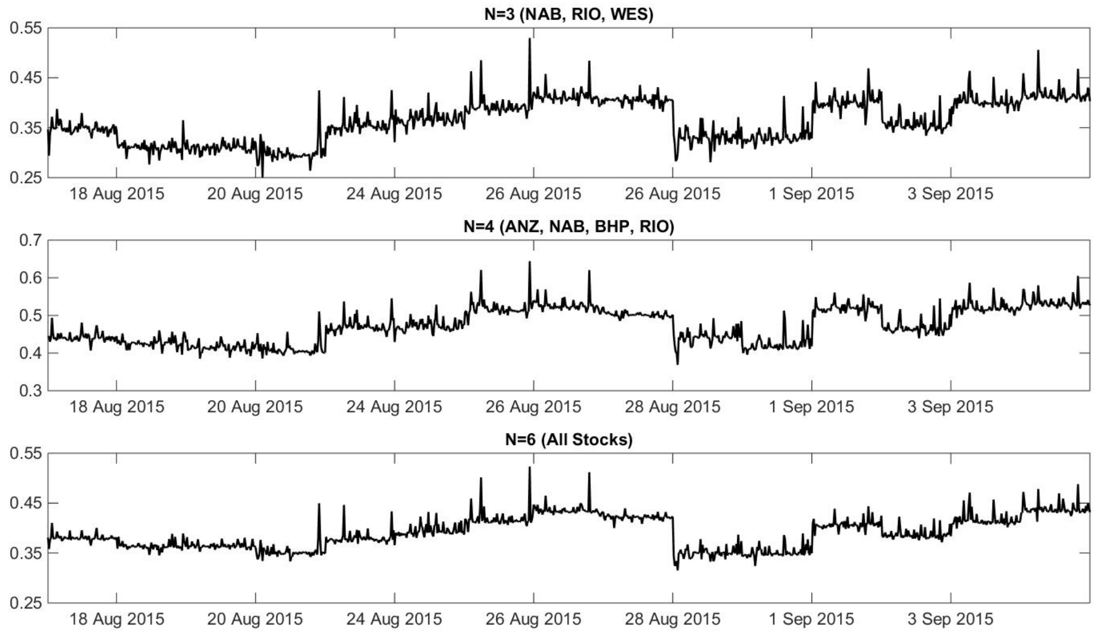

Figure 11: Average DCC-Daily II correlations over the period 18 August 2015 to 4 September 2015, for selected $N = 3, 4, 6$ portfolios.

DECO-Daily II Equicorrelations

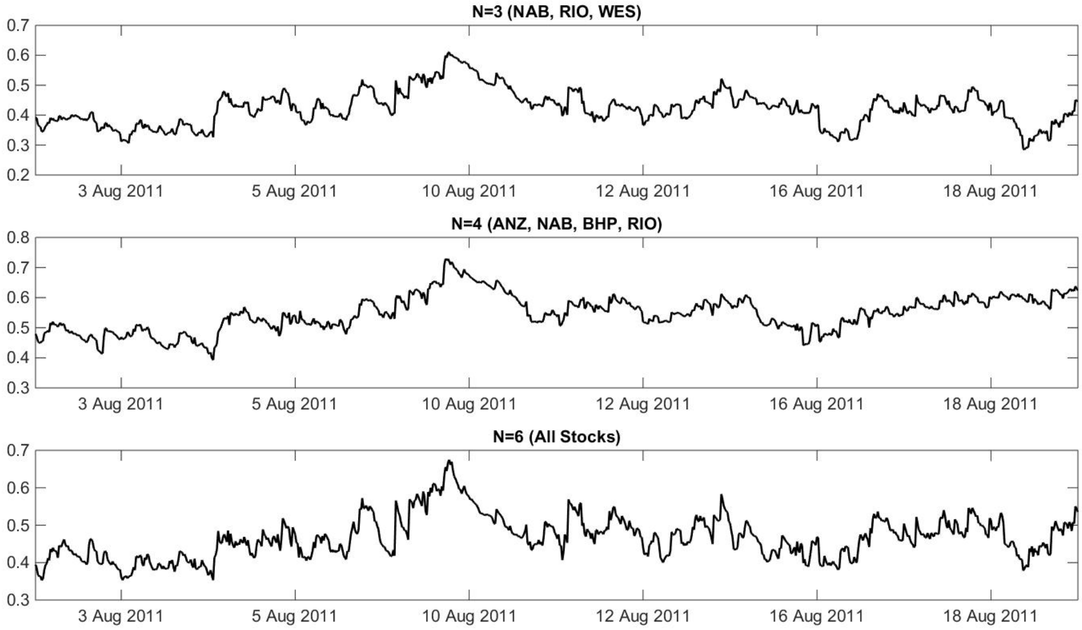

Figure 12: DECO-Daily II equicorrelations over the period 2 August 2011 to 18 August 2011, for selected $N = 3, 4, 6$ portfolios.

DECO-Daily II Equicorrelations

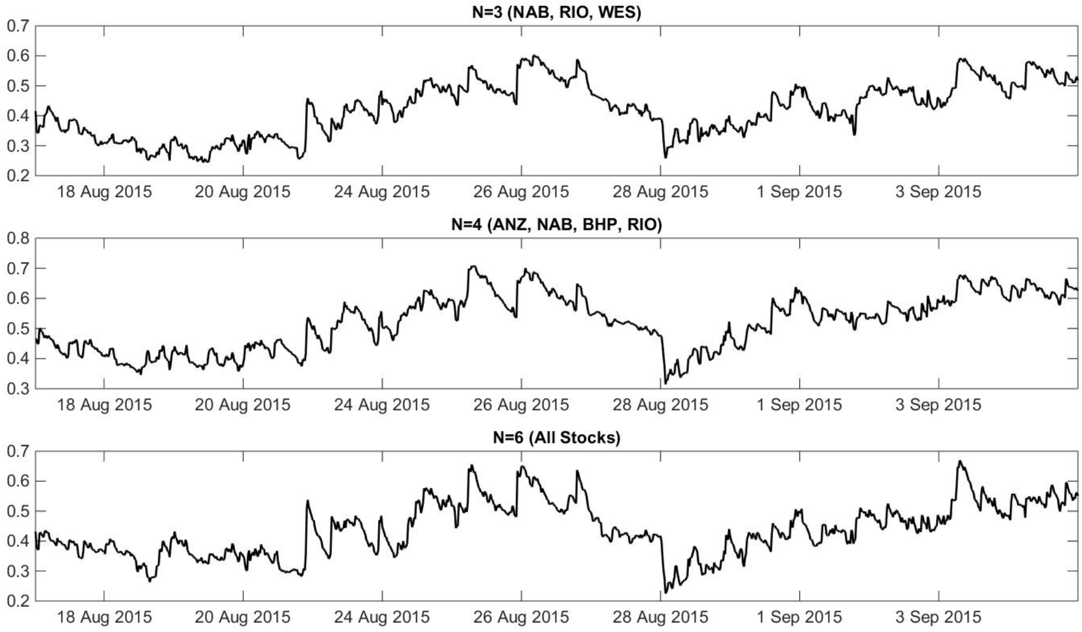

Figure 13: DECO-Daily II equicorrelations over the period 18 August 2015 to 4 September 2015, for selected $N = 3, 4, 6$ portfolios.

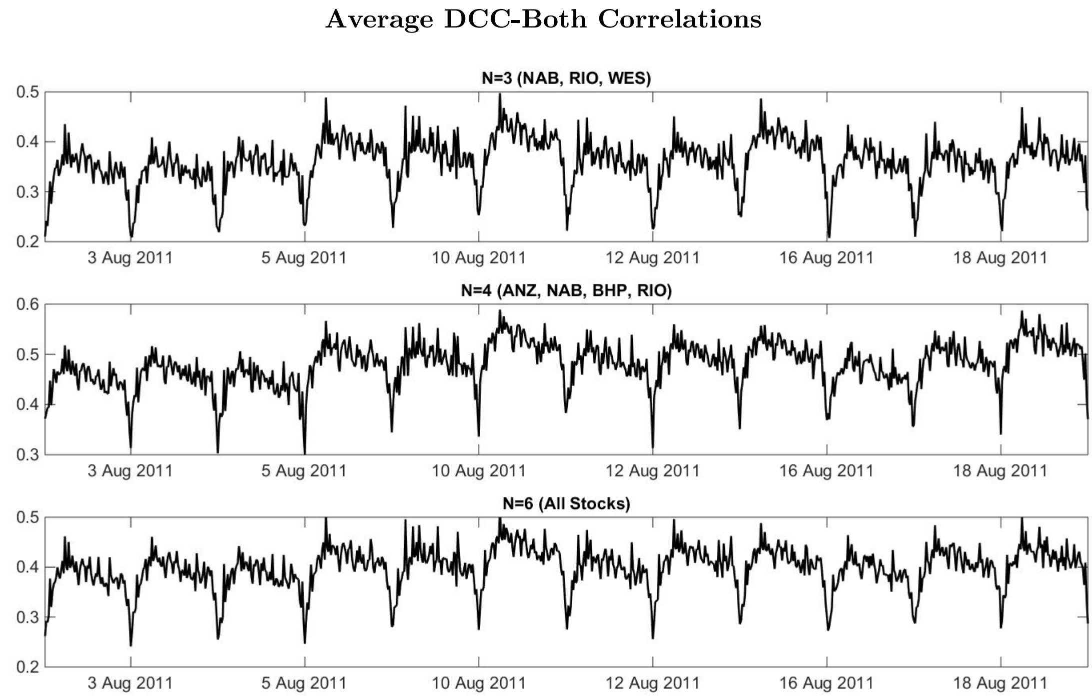

Figure 14: Average DCC-Both correlations over the period 2 August 2011 to 18 August 2011, for selected $N = 3, 4, 6$ portfolios.

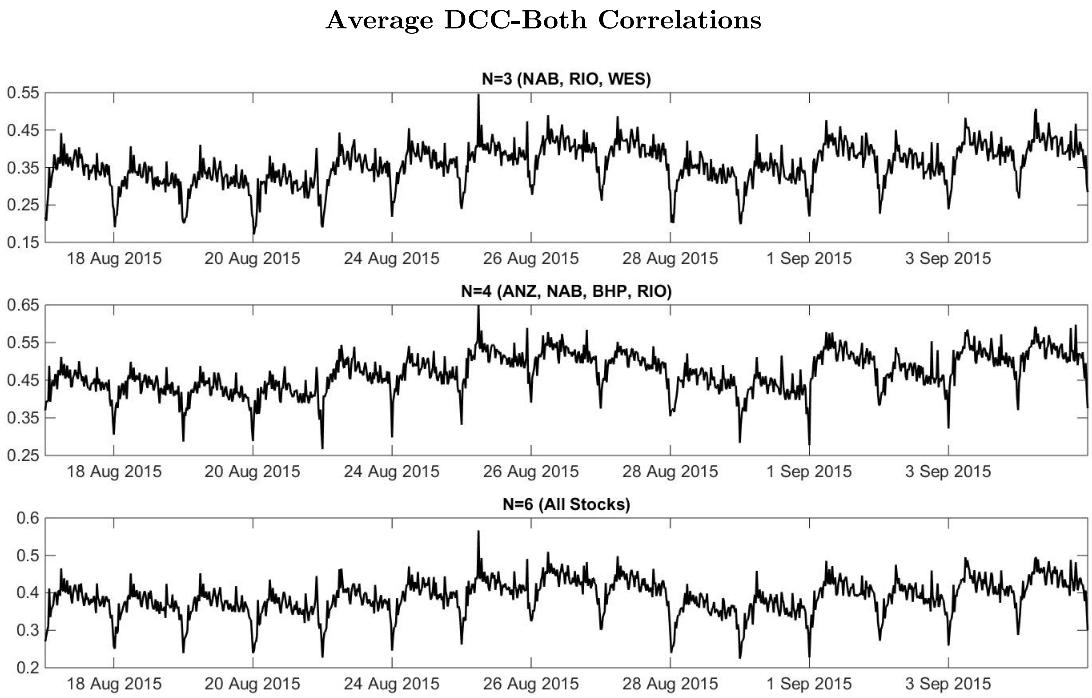

Figure 15: Average DCC-Both correlations over the period 18 August 2015 to 4 September 2015, for selected $N = 3, 4, 6$ portfolios.

DECO-Both Equicorrelations

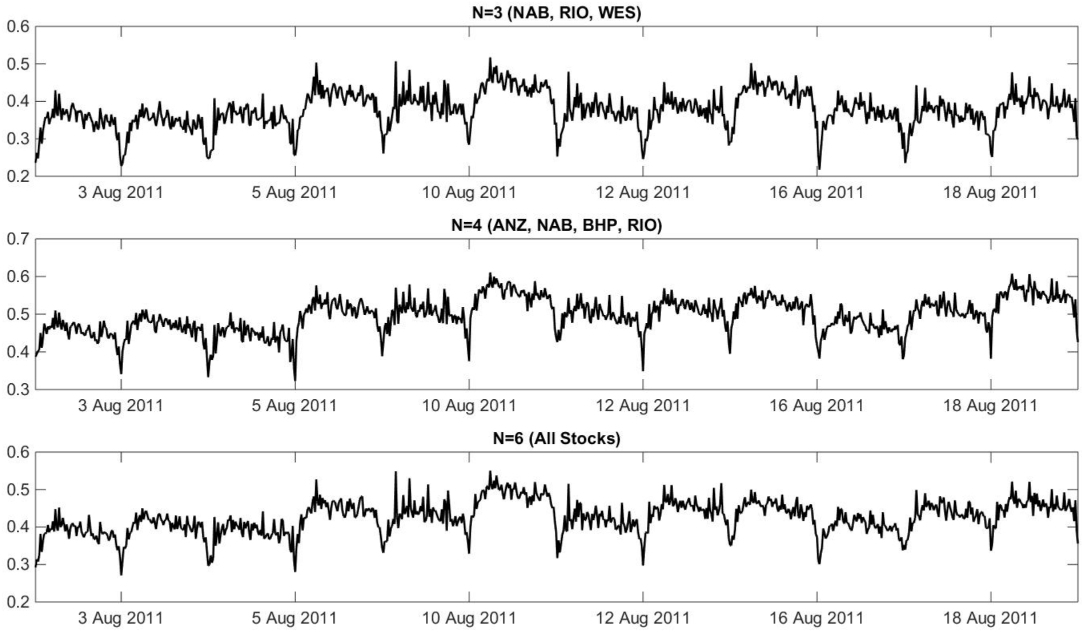

Figure 16: DECO-Both equicorrelations over the period 2 August 2011 to 18 August 2011, for selected $N = 3, 4, 6$ portfolios.

DECO-Both Equicorrelations

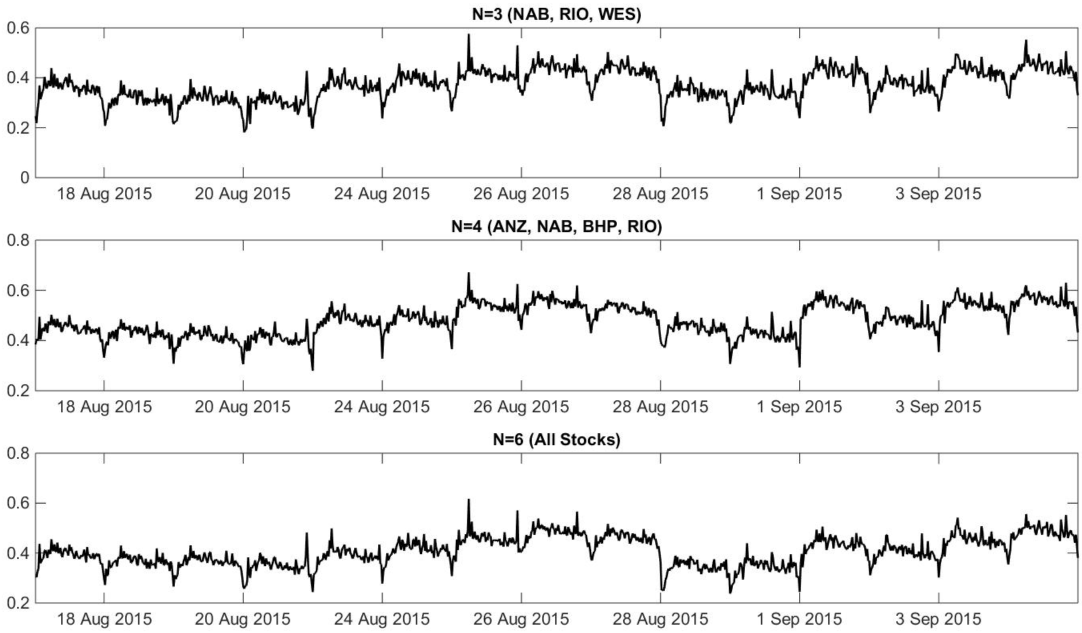

Figure 17: DECO-Both equicorrelations over the period 18 August 2015 to 4 September 2015, for selected $N = 3, 4, 6$ portfolios.

References

AIELLI, G. P. (2013): "Dynamic Conditional Correlations: On Properties and Estimation," *Journal of Business & Economic Statistics*, 31, 282–299.

ALLEZ, R., AND J.-P. BOUCHAUD (2011): "Individual and collective stock dynamics: intra-day seasonalities," *New Journal of Physics*, 13, 1–12.

ANDERSEN, T. G., AND T. BOLLERSLEV (1997): "Intraday periodicity and volatility persistence in financial markets," *Journal of Empirical Finance*, 4, 115–158.

CLEMENTS, A., A. SCOTT, AND A. SILVENNOINEN (2015): "On the benefits of equicorrelation for portfolio allocation," *Journal of Forecasting*, 34, 507–522.

ENGLE, R. F. (2002): "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business and Economic Statistics*, 20, 339–350.

ENGLE, R. F., AND B. KELLY (2012): "Dynamic Equicorrelation," *Journal of Business & Economic Statistics*, 30, 212–228.

ENGLE, R. F., AND M. E. SOKALSKA (2012): "Forecasting intraday volatility in the U.S. equity market: multiplicative component GARCH," *Journal of Financial Econometrics*, 10, 54–83.

FREY, R. (2000): "Risk minimization with incomplete information in a model for high-frequency data," *Mathematical Finance*, 10, 215–225.

FRIJNS, B., AND D. MARGARITIS (2008): "Forecasting daily volatility with intraday data," *The European Journal of Finance*, 14, 523–540.

GOODHART, C. A. E., AND M. O'HARA (1997): "High frequency data in financial markets: Issues and applications," *Journal of Empirical Finance*, 4, 73–114.

HALL, A. D., AND N. HAUTSCH (2006): "Order aggressiveness and order book dynamics," *Empirical Economics*, 30, 973–1005.

HAMILTON, J. D. (1994): *Time Series Analysis*. Princeton University Press.

SILVENNOINEN, A., AND T. TERÄSVIRTA (2015): "Modelling conditional correlations of asset returns: A smooth transition approach," *Econometric Reviews*, 34, 174–197.

TILAK, G., T. SZÉLL, R. CHICHEPORTICHE, AND A. CHAKRABORTI (2013): "Study of Statistical Correlations in Intraday and Daily Financial Return Time Series," in *Econophysics of Systemic Risk and Network Dynamics*, ed. by F. Abergel, B. K. Chakrabarti, A. Chakraborti, and A. Ghosh, pp. 77–104. Springer, Milan.

WOOD, R. A., T. H. MCINISH, AND J. K. ORD (1985): "An Investigation of Transactions Data for NYSE Stocks," *The Journal of Finance*, 40, 723–739.

Appendix A

Univariate parameter estimates

Stock	μ	φ	α	β	ϕ
ANZ	4.7400 (0.5624)	0.4684 (0.0382)	0.0745 (0.0038)	0.8727 (0.0089)	0.0157 (0.0023)
BHP	−5.2833 (1.8089)	0.3832 (0.0561)	0.0745 (0.0056)	0.8846 (0.0119)	0.0038 (0.0021)
NAB	−14.9047 (0.6343)	0.5581 (0.0312)	0.0824 (0.0042)	0.8614 (0.0084)	0.0119 (0.0011)
RIO	−8.2350 (2.1877)	0.4066 (0.0448)	0.0890 (0.0044)	0.8592 (0.0105)	0.0030 (0.0031)
WES	−7.1155 (1.2706)	0.3919 (0.0310)	0.0801 (0.0049)	0.8708 (0.0097)	0.0005 (0.0025)
WOW	−4.8943 (0.5443)	0.3831 (0.0378)	0.0766 (0.0061)	0.8759 (0.0131)	0.0054 (0.0009)

Table 8: Univariate intraday volatility model parameter estimates and robust standard errors for each stock, see Section 2.1. Entire period spans 4 January 2011 to 30 December 2015.

Details of dataset, including summary statistics

Stock	Min	Max	$\bar{x}$	s	Skewness	Kurtosis
ANZ	-0.0151	0.0149	0.0000	0.0011	0.0733	9.2677
BHP	-0.0126	0.0145	0.0000	0.0011	0.0432	9.1082
NAB	-0.0176	0.0133	0.0000	0.0012	0.0111	9.8609
RIO	-0.0136	0.0163	0.0000	0.0011	0.0497	9.4246
WES	-0.0135	0.0158	0.0000	0.0011	-0.0430	9.7997
WOW	-0.0127	0.0197	0.0000	0.0011	0.1537	12.9862

Table 9: List of 6 Australian stocks and summary statistics, period spans 4 January 2011 to 30 December 2015.