The imprecision of volatility indexes

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Abstract

Concerns about imprecision arise in a VIX estimator computed by aggregating several imprecise implied volatility estimates (IVs). Imprecision in the estimation of VIX, when discounted in its applications, may have adverse consequences ranging from modest to material. In this paper, we propose a bootstrap strategy to capture this imprecision for a model based VIX estimator. We use the confidence band and the standard deviation of the bootstrap estimates of VIX as imprecision indicators. We find that the imprecision of VIX is considerable and has important consequences for real-world applications of volatility indexes. In addition, we use this measure of imprecision as a model selection criterion by choosing the volatility index which has the lowest imprecision.

Keywords: Implied Volatility; Volatility index; Imprecision
JEL Classification: G12; G13; G17

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Contents

1 Introduction

The volatility index (VIX) is a measure of market's expectation of volatility. The forecasting superiority of VIX over historical volatility has been explored extensively in the literature (Blair, Poon, and Taylor, 2001; Jiang and Tian, 2005; Corrado and Miller, 2005; Giot, 2005). This has lent a predominant role to the VIXs in applications such as option pricing and value-at-risk calculation. The negative correlation of VIX with the market index has also been employed as a market timing device for trading strategies that result in positive returns by switching between two portfolios based on whether VIX rises or falls (Copeland and Copeland, 1999; Arak and Mijid, 2006; Cipollini and Manzini, 2007). Derivatives on the new model free VIX exploit this negative correlation to allow investors to obtain direct exposure to the market's volatility and diversify portfolios.

The CBOE introduced VIX futures and options on March 26, 2004 and February 24, 2006 respectively. Since then trading on VIX derivatives has increased substantially. In 2012, the open interest was at 326,066 contracts and 6.3 million contracts for VIX futures and options respectively.[1]

The matter of precision in VIX assumes importance as volatility indexes themselves become useful. Concerns about imprecision arise in a VIX estimator computed by aggregating several imprecise implied volatility estimates (IVs). Imprecision in the estimation of VIX, when discounted in its applications cited above, may have adverse consequences ranging from modest to material. This issue has been given very little attention in the literature. To address this issue, the paper presents a bootstrap confidence interval approach to capture this uncertainty in the estimation of a model based VIX – the vega weighted VIX (VVIX).

The width of the confidence interval and the standard deviation of the sampling distribution of a VIX estimate are motivated as measures of the estimate's precision. Various model based VIX estimators proposed in the literature (Grover and Thomas, 2012) are then compared in terms of their precision using these performance measures.

The data used in this paper is based on the NSE-50 (Nifty) index options traded at the National Stock Exchange of India Ltd (NSE). NSE is the third largest derivatives exchange with approximately two billion contracts traded for the period January - December, 2012.[2] The period of analysis used in the

[1]Source: CBOE, http://www.cboe.com/data/AnnualMarketStatistics.aspx
[2]Source: FIA, http://www.futuresindustry.org/volume-.asp

paper: February 2009 - September 2010.

The paper finds for a given point in time in the sample, using a cross-section of Nifty index options with 29 and 57 days to expiry, the point estimate of the VVIX is 17.82% with 95% confidence limits of 16.03% to 19.91%.

Pairwise comparisons of VIX estimators for the period of analysis show that the vega weighted VIX is the best estimator with the smallest confidence interval width and standard deviation, followed by the liquidity weighted VIXs, and the elasticity weighted VIX.

Imprecision in a VIX estimator translates into imprecise option prices when used as an estimate of future volatility in an option pricing formula. For example, a 6100 out-of-money call option on the Nifty index at 5464.75 with 29 days to expiry, would have been priced at Rs. 1.92 — using the VVIX point estimate of 17.82 %. However, when considering the uncertainty in the estimate the price may have been anywhere between Rs. 0.89 and Rs. 3.86.

Moreover, such uncertainty may be exacerbated due to use of a more imprecise estimate. For example, in our sample, we find the VVIX to be much more precise than say an elasticity weighted VIX (EVIX). While the median confidence interval width for the VVIX is 2.9 percentage points in our sample, it is 7.4 percentage points for the EVIX. Clearly, the price band for the option under such uncertainty is too wide to be useful.

The importance of gauging the precision in the VIX estimators cannot be understated. However, a large part of the literature focusses on identifying/reducing potential sources of error in the estimation of VIXs with little or no emphasis on explicitly estimating the precision in these estimates (Latane and Rendleman, 1976; Chiras and Manaster, 1978; Jiang and Tian, 2007). Hentschel (2003) is only study which measures the level of accuracy in IV and the old CBOE VIX by estimating confidence intervals derived from the Black-Scholes formula. We contribute to the existing literature by constructing bootstrapped confidence intervals to measure the precision in VIX estimators. Unlike the previous study, our methodology does not rely on any distributional assumptions for the errors and hence is widely applicable.

This paper also relates to the literature that develops empirical proxies to measure ambiguity among investors. Under the expected utility paradigm, investors know or act as if they know the probabilities of all states of nature. In other words, the investor only faces risk in his decision making process. In reality investors do not know their precise probabilities and hence are exposed not only to risk but also to ambiguity (Knightian uncertainty). While an extensive literature exists on decision making under uncertainty, limited studies exist on developing empirical proxies to capture ambiguity. For instance, Brenner and Izhakian (2011) use variance of probability of gain or loss, Baltussen, Van Bekkum, and Van der Grient (2013) use volatility of implied volatility, and Ehsani, Krause, and Lien (2013) use the mean divergence between implied probability distributions to proxy ambiguity. Since VIX is an implied volatility estimate derived from option prices that are forward looking, it captures risks perceived by investors about expected stock returns. Measurement errors in the estimation of VIX render it imprecise resulting in a fuzzy estimator of volatility. Uncertainty in VIX could be another way of capturing ambiguity. The imprecision indicators could be used to compute ambiguity for a wide range of stocks and at higher frequencies.

The paper is presented as follows: Section 2 discusses the sources of errors that may hamper the estimation of a volatility index. Section 3 discusses the model-free strategy to measure the precision of a volatility index. Section 4 describes the data. Section 5 shows the working of the methodology through an example. Section 6 discusses the results for the entire period of analysis. Section 7 discusses model selection using the precision criterion. Section 9 concludes.

2 Concerns about measurement

Volatility indexes are computed using a chain of option prices at different strikes and maturities. There are two broad ways to compute a volatility index. The first involves summarising implied volatilities of each observed option price, using an option pricing formula.[3] The second involves deriving a formula for VIX from the concept of fair value of a variance swap. This is referred to the *model free* method and is currently employed by CBOE to compute VIX every 15 seconds.[4]

The approach of this paper is to see alternative methods for computing volatility indexes as statistical estimators. Microstructure noise and the lim-

[3]The most common example of an index that employs this procedure is the old CBOE VIX (VXO) (Whaley, 1993). It is computed from eight near-the-money options for the two nearest maturities. Other examples of the model based approach include aggregating IVs weighted by vega, liquidity, and volatility elasticity (Grover and Thomas, 2012).

[4]The concept of a model-free implied variance was first introduced by Dupire (1993) and Neuberger (1994) and further developed by Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999) and Britten-Jones and Neuberger (2000). In contrast to the VXO, VIX uses a wide range of out-of-money (OTM) options to estimate the expected volatility of the market.

its of arbitrage introduce imprecision into each observed option price. Every observed option price suffers from a certain degree of measurement error. The method through which the volatility index is computed should thus be seen as a statistical estimator. The imprecision of measurement of option prices can introduce economically significant imprecision into the resulting volatility index. We briefly discuss the sources of errors that may hamper both estimation strategies.

2.1 Model based approach

Errors in option prices can result in considerable errors in the implied volatilities estimated by inverting the Black-Scholes option pricing formula. For example, Hentschel (2003) discuss measurement errors that result from finite-quote precision, bid-ask spreads and non-synchronous prices of options and underlying. The errors due to finite quote precision exist, since unlike the ideal market conditions assumed in the Black-Scholes model, markets in the real world record prices in discrete increments. The bid-ask spread is another important source of error that may obfuscate the true prices. The non-synchronous alignment of the underlying prices with the option prices may also render inefficiencies in the IV estimation.

Small errors in option prices result in large errors in IVs, and this effect is more pronounced for OTM options (Hentschel, 2003). For measuring the precision of IV estimates, Hentschel (2003) estimates confidence bands derived from the Black-Scholes formula.

2.2 Model free approach

Jiang and Tian (2007) discuss methodological errors that may induce inefficiencies in the model free estimates of implied variance. They discuss four types of errors: truncation, discretisation, expansion, and maturity interpolation:

1. Only a finite range of strikes prices are available. This induces truncation error in VIX and causes a downward bias in the estimate.
2. The absence of a continuum of strike prices as a result of availability of strikes only in discrete increments leads to discretisation error. This mainly stems out from using a numerical integration procedure to approximate the integrals in the VIX formula.

3. The Taylor series expansion of the log function in the VIX formula induces a negative error in the expansion.
4. The last error arises from the linear maturity interpolation scheme used to compute VIX. The evidence on the term structures of the implied variances reveals that they may not be linear or monotonic in option maturity.

Building on these ideas, Andersen, Bondarenko, and Gonzalez-Perez (2011) show that varying ranges of strike prices induce considerable error in intraday estimates of VIX.

3 Measuring the imprecision of a volatility index

Our approach to the problem is non-parametric in comparison with Hentschel (2003). We make no assumptions about the distribution of the error. Our approach is a statistical one. We treat each option price as an imprecise transformation of the true implied volatility index. In this perspective, the computational method for the volatility index is a statistical estimator. We then propose a bootstrap strategy for estimating the imprecision of the estimator.

While this approach is fully general, for the purposes of this paper, we analyse one specific method for obtaining the volatility index: the vega adjusted volatility index (VVIX). We focus on a model-based estimator of a volatility index as opposed to a model-free estimator. The former is applicable to a wider class of assets since illiquidity in options market for several assets renders the estimation of a model-free estimator difficult. For example, the availability of a very limited range of strikes with larger strike intervals limit the applicability of the later estimator. The CBOE uses the later estimator to calculate VIX from S&P 500 index options available for a wide range of strikes and at shorter intervals as presented in Appendix A.1. In contrast, the options on Nifty index are available with limited strikes and at larger intervals as presented in Appendix B.1. This limits the applicability of the later estimator for emerging economies and for assets with a less developed options market in comparison to the US options market.

The VVIX is computed from all option prices as follows:

1. Estimation of IVs using the Black-Scholes model for the two nearest maturities.

2. Computation of the average weighted IV for each maturity i:

$$IV_i = \frac{\sum_{j=1}^{n} w_{ij} IV_{ij}}{\sum_{j=1}^{n} w_{ij}}$$

where, IV_{ij} refers to a vector of IVs for $j = \{1 \dots n\}$ and two nearest maturities, $i = \{near, next\}$, w_{ij} refers to the vega weight for the corresponding IV_{ij}.

3. The vega weighted average IVs are interpolated to compute the 30 day expected volatility, VVIX.

3.1 A bootstrap inference strategy

Each option provides an imprecise IV number, several of which are aggregated into a volatility index. A similar problem is faced in estimating LIBOR, where the situation faced is as follows. There is a true unobserved price on an OTC market. Each dealer who is polled reports a noisy estimate of this true price. The estimator for LIBOR is a robust estimator that utilises these pieces of information to construct a more precise estimate (Cita and Lien, 1992; Berkowitz, 1999; Shah, 2000).

As with the LIBOR question, we propose a bootstrap estimator through which we construct the distribution of VVIX in the following steps:

1. At each maturity, we sample with replacement among all observed option prices to construct a bootstrap replicate.

2. Thousands of times, we obtain an estimate of VVIX at each maturity and thus an overall VVIX estimate.

3. The standard deviation (σ) and confidence bands are computed from the bootstrapped sampling distribution of VVIX using the adjusted bootstrap percentile method (Efron, 1987).

4 Data description

4.1 S&P 500 index (SPX)

The end-of-day data on SPX index options at the CBOE obtained from Bloomberg is used to estimate the volatility index values. The data is available for the months of September, October and November 2010. This includes: transaction date, expiry date of the options contract, strike price, type of the option i.e. call or put, price of the underlying index, best buy and ask price of options. The risk-free rates used are the one and three month US Treasury bill rates provided by the US department of the Treasury.

Options with zero bid/ask prices, and negative bid-ask spreads are discarded from the dataset. Further, we also check whether the market price of options lie within the no-arbitrage band of the Black-Scholes model. Options that violate this condition are also discarded from the set.

4.2 S&P CNX Nifty index

Data on the NSE-50 (Nifty) index options at the National Stock Exchange (NSE) of India Ltd is used to estimate the volatility index values. NSE is the fourth largest derivative exchange in the world in terms of number of contracts traded (Table 1). It is also the second largest exchange in terms of number of contracts traded in equity index (Table 2). NSE is a source of high quality data on exchange-traded derivatives with an extremely active derivatives segment.

Table 1 Global exchanges: number of contracts traded and/or cleared

Rank	Exchanges	Jan-Dec 2012	Annual % change
1	CME Group	2,890,036,506	-14.7%
2	Eurex	2,291,465,606	-18.8%
3	National Stock Exchange of India	2,010,493,487	-8.6%
4	NYSE Euronext	1,951,376,420	-14.5%
5	Korea Exchange	1,835,617,727	-53.3%

Source: FIA, http://www.futuresindustry.org/volume-.asp

Table 2 Ranked by the number of contracts traded in equity index

1	Kospi 200 Options, KRX	3,671,662,258	1,575,394,249	-51.7%
2	S&P CNX Nifty Options, NSE India	868,684,582	803,086,926	-7.6%
3	SPDR S&P 500 ETF Options, CME	729,478,419	585,945,819	-19.7%
4	E-mini S&P 500 Futures, CME	620,368,790	474,278,939	-23.5%
5	RTS Futures, Moscow Exchange	377,845,640	321,031,540	-15.0%

Source: FIA, http://www.futuresindustry.org/volume-.asp

The options data is available at the tick-by-tick frequency. This includes: transaction date, expiry date of the options contract, strike price, type of the option i.e. call or put, price of the underlying index, best buy and ask price of options. The risk-free rates used are the one and three month MIBOR rates provided by NSE.

A volatility index number is estimated for a given trading interval. For example, the CBOE disseminates real-time values of VIX updated every fifteen second. The VIX may be contaminated with market microstructure noise if calculated at a very high frequency. Thus, we also conduct all estimations with data sampled at a frequency of fifteen seconds. For a robustness check, sampling frequencies of thirty and sixty seconds are also used.

The tick-by-tick data on options with approximately 200,000 observations per day are sampled at a frequency of fifteen seconds using the procedure outlined in Andersen, Bondarenko, and Gonzalez-Perez (2011) to estimate a volatility index. The sampling procedure involves constructing fifteen seconds series for each individual option using the *previous tick method*. The last available quote is retained at the end of each fifteen second interval throughout the trading day. If no new quote arrives in a fifteen second interval, the last available quote prior to the interval is retained. If no quote is available in the previous interval, the last available quote from the last five minutes is retained.

In addition to the above, options with zero traded volume, zero bid/ask prices, and negative bid-ask spreads are discarded from the dataset. Further, we also check whether the market price of options lie within the no-arbitrage band of the Black-Scholes model. Options that violate this condition are also discarded from the set.

5 Two empirical examples

As an example of the working of this methodology, we estimate the distribution of VVIX at one point in time for a sample of SPX and Nifty options respectively.

5.1 Estimating confidence interval using SPX options

As the first example, we show the estimation of a confidence interval for VVIX from a particular sample of end-of-day SPX options, on 2010-09-17. The numerical values in this sample are shown in Appendix A.1.

Figure 1 The distribution of VVIX: One example, SPX

The kernel density plot captures the distributions of the weighted average IVs for the two nearest maturities referred to as IVNear and IVNext respectively and the corresponding VVIX. The distributions are estimated by drawing 1000 bootstrap replicates from a particular sample of SPX options, end-of-day on 2010-09-17, given in Appendix A.1. The dashed lines indicate the point estimate of VVIX and its 95% confidence bounds (LB, UB). The distribution of VVIX overlaps with the distribution of IVNear. The interpolation scheme used in the computation of VVIX assigns higher weight to IVNear when first month options have higher liquidity. This results in the distribution of VVIX being biased towards IVNear. This pattern reverses when first month options near expiration and IV estimates from these options are more noisy.

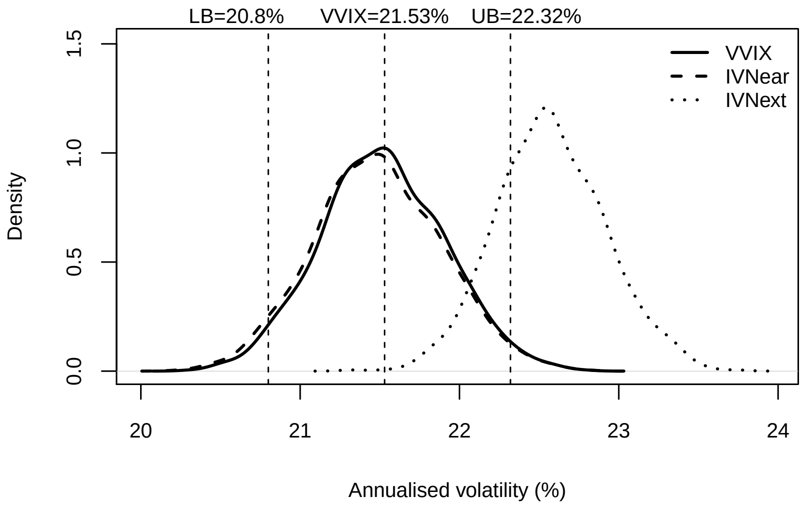


Table 3 Near-the-money SPX options for the first month maturity

The underlying is at 1125.59, the number of days to expiry is 29 and the risk-free rate is 0.12%.

Strike	Type	Mid-Quote	IVol (%)	Strike	Type	Mid-Quote	IVol (%)
1095	c	42.50	19.31	1095	p	14.00	21.29
1100	c	38.00	18.30	1100	p	15.25	20.87
1105	c	35.25	18.71	1105	p	16.65	20.48
1110	c	31.75	18.35	1110	p	18.05	19.99
1115	c	28.60	18.14	1115	p	19.70	19.59
1120	c	25.75	18.05	1120	p	21.55	19.24
1125	c	22.75	17.70	1125	p	24.55	19.68
1130	c	19.35	16.90	1130	p	26.30	19.00
1135	c	16.85	16.66	1135	p	28.10	18.22
1140	c	14.00	15.98	1140	p	30.85	18.05
1145	c	12.35	16.12	1145	p	33.65	17.79
1150	c	10.50	15.94	1150	p	36.45	17.37
1155	c	8.55	15.49	1155	p	39.75	17.22

Note: We define near-the-money-options as call and put options with strike-to-spot ratio between 0.97 and 1.03 (Pan and Poteshman, 2006).

In order to fix intuition, Table 3 shows the near-the-money SPX options for the near month only. As we see, there is considerable variation in the IV. If a sample mean of these were computed it would have a 95% confidence interval from 17.65 to 18.84 which shows a wide band of width 1.19 percentage points.

In order to illustrate the working of the method, one bootstrap replicate of the SPX sample shown in Appendix A.1 is presented at Appendix A.2. The VVIX estimated from the original sample is 21.53%. Bootstrap replicates are drawn from the original sample and the distribution of VVIX is estimated as described in Figure 1. The lower and upper 95% confidence bounds estimated from the bootstrap distributions of VVIX are 20.8% and 22.32% respectively.

This measure of imprecision, of roughly 1.5 percentage points, is an economically significant one. For a sense of scale, the one-day change in VVIX is smaller than 1.5 percentage points on 62% of the days.[5] This suggests that on 62% of the days, we know little about whether VVIX went up or down when compared with the previous day.

The above example demonstrates that the level of imprecision found in a VIX estimate computed from SPX options may not be trivial. The level of imprecision in VVIX captured by its wide confidence band of width 1.5

[5] Out of a total of 60 days.

percentage points is economically significant and cannot be ignored.

5.2 Estimating confidence interval using Nifty options

As the second example, we show the estimation of a confidence interval for VVIX from a particular sample of Nifty options, from 14:59:45 to 15:00:00 on 2010-09-01. The numerical values in this sample are shown in Appendix B.1.

Table 4 Near-the-money NIFTY options for the first month maturity

Strike	Type	Underlying	Mid-Quote	Maturity (Days)	Risk-free (%)	IVol (%)
5500	c	5464.75	72.57	29	6.29	12.43
5600	c	5464.75	30.55	29	6.29	11.57
5400	c	5464.75	133.95	29	6.29	13.11
5600	p	5464.75	160.65	29	6.29	15.75
5400	p	5464.75	68.75	29	6.29	17.80
5500	p	5464.75	105.30	29	6.29	16.48

Note: We define near-the-money-options as call and put options with strike-to-spot ratio between 0.97 and 1.03 (Pan and Poteshman, 2006).

Table 4 shows the near-the-money Nifty options for the near month only. As we see, there is considerable variation in the IV. If a sample mean of these were computed it would have a 95% confidence interval from 12.53 to 16.52 which shows a wide band of width 4 percentage points.

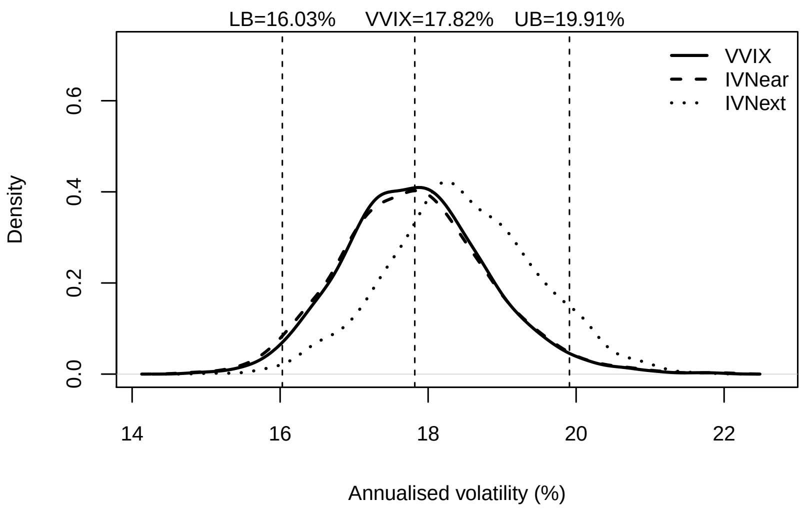
One bootstrap replicate of the Nifty shown in Appendix B.1 is presented at Appendix B.2. The VVIX estimated from the original sample is 17.82%. Bootstrap replicates are drawn from the original sample and the distribution of VVIX is estimated as described in Figure 3. The lower and upper 95% confidence bounds estimated from the bootstrap distributions of VVIX are 16.03% and 19.91% respectively.

This measure of imprecision, of roughly 4 percentage points, is an economically significant one. For a sense of scale, the one-day change in VVIX is smaller than 4 percentage points on 92% of the days.[6] This suggests that on 92% of the days, we know little about whether VVIX went up or down when compared with the previous day. This level of imprecision is higher in comparison to the imprecision number estimated using SPX options.

[6] Out of a total of 379 days.

Figure 2 The distribution of VVIX: One example, NIFTY

The kernel density plot captures the distributions of the weighted average IVs for the two nearest maturities referred to as IVNear and IVNext respectively and the corresponding VVIX. The distributions are estimated by drawing 1000 bootstrap replicates from a particular sample of NIFTY options, from 14:59:45 to 15:00:00 on 2010-09-01, given in Appendix B.1. The dashed lines indicate the point estimate of VVIX and its 95% confidence bounds (LB, UB). The distribution of VVIX overlaps with the distribution of IVNear. The interpolation scheme used in the computation of VVIX assigns higher weight to IVNear when first month options have higher liquidity. This results in the distribution of VVIX being biased towards IVNear. This pattern reverses when first month options near expiration and IV estimates from these options are more noisy.



6 How large is the imprecision of VVIX

The subsection above was about only one example. We now present evidence about imprecision that runs over a bigger dataset for Nifty options. We also discuss the differences of our methodology against the earlier study and the resulting difference in inferences.

6.1 Imprecision of VVIX

For the period February 2009 - September 2010, the median imprecision observed in VVIX measured in terms of confidence interval width is 2.9 percentage points (Table 5), which is a large number when compared with the one-day change in VVIX which has a median value of 1.18 percentage points. In addition, the median standard deviation of the bootstrap estimates for the period is 0.739 percentage points (Table 5).

Table 5 Measures of imprecision for VVIX

	Width of CI	Standard Dev.
No. of Obs.	1516	1516
Min	1.03	0.25
1st Qu	2.27	0.58
Median	**2.92**	**0.74**
Mean	3.91	0.94
3rd Qu	4.06	1.02
Max	50.49	11.78
Std Dev	3.64	0.75

Note: Both in percentage points.

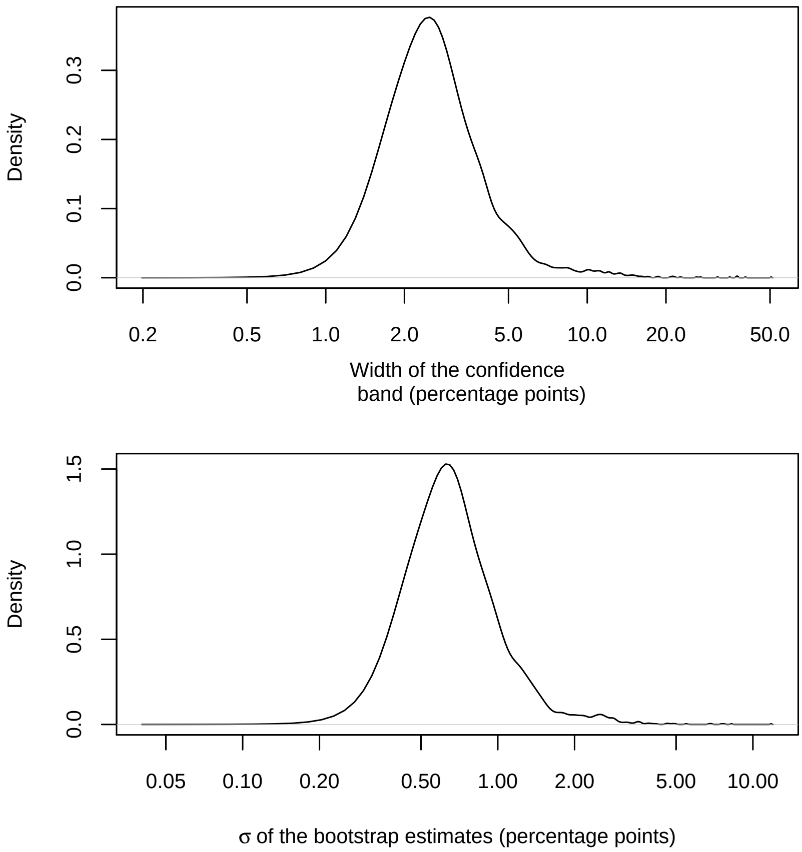
Figure 3 captures the distributions of the measures of imprecision: the width of the confidence band and the standard deviation of the bootstrap estimates for the VVIX estimator for the period February 2009 - September 2010.

6.2 Comparing against earlier study

Based on several assumptions about the distribution of the errors, Hentschel (2003) shows that for an ATM stock option with twenty days to expiry, the ninety-five percent confidence intervals are of the order of ±6 percentage points. This is an economically significant scale of imprecision. While individual option prices are observed with low precision, Hentschel (2003) argues

Figure 3 Measures of imprecision for VVIX

The figure plots the kernel density for the width of the confidence band and the standard deviation of the bootstrap estimates measured in percentage points. The 95% confidence interval and the standard deviation are computed four times a day, for the period February 2009 - September 2010. For each point in time, the confidence interval and the standard deviation are computed from the bootstrapped sampling distribution of VVIX.



that the old CBOE VIX is fairly efficient, with 95% confidence intervals of the order of ±25 basis points. This is because the VXO is estimated from near-the-money options that tend to be less contaminated with errors, low weights are attached to options near expiry, and errors in underlying prices cancel when averaging implied volatilities from puts and calls. By this argument, even though each option price is measured with imprecision, volatility indexes are fairly precisely measured. We on the contrary find a substantially higher level of imprecision in the volatility indexes. This may be attributed to the following reasons.

First, we employ a model-free strategy to gauge the level of imprecision in the implied volatility index. On the other hand, Hentschel (2003) derives confidence bands from the Black-Scholes model. Second, our methodology does not require any assumptions on the distribution of errors. In contrast, Hentschel (2003) assumes that the errors stemming from bid-ask quotes are normally distributed with zero mean and standard deviation proportional to the magnitude of the bid-ask spread. Third, Hentschel (2003) finds VXO to be an efficient estimator with 95% confidence intervals of the order ±25 basis points based on a given price level and a set of measurement error assumptions, computed for a pair of strike prices and maturities sampled from a range of strike prices and maturities. We on the contrary find VVIX to be observed with significant imprecision. Our analysis is based on a large set of data which includes 1,516 samples of the Nifty index options. Each sample consists of options for a range of strike prices, and two maturities.

In addition to the above, there is a difference in the choice of options used for the estimation of VXO and VVIX. Our non-parametric methodology applies to a large set of model based volatility indexes estimated from all options that give a market price. The VXO however is estimated from a fixed choice of eight options corresponding to two strikes and two maturities with limited information content.

6.3 Implications

In recent years, numerous applications of volatility indexes have sprung up, which are predicated on observing the volatility index precisely. Our model-free strategy for understanding the imprecision of VVIX suggests that we observe this volatility index with substantial imprecision. This evidence of the imprecision of volatility indexes may suggest greater caution in the use of these indexes.

7 Using this measure of imprecision for model selection

Weighting options by their vega to estimate a volatility index is one of the several approaches available to compute a volatility index. The alternatives include weighting options by volatility elasticity, liquidity etc (Grover and Thomas, 2012). Traditionally, volatility indexes have been judged on the extent to which they forecast future realised volatility.

However, another approach to model selection can be rooted in the problem of statistical imprecision. Just as model selection for LIBOR has been done by identifying the estimator which has the lowest bootstrap standard deviation, we could do model selection for the volatility index by choosing the index which has the lowest imprecision.

7.1 Methodology

We benchmark the performance of the four volatility indexes computed in Grover and Thomas (2012). They are: vega weighted VIX (VVIX), liquidity weighted VIX (SVIX, TVVIX), and volatility elasticity weighted VIX (EVIX). The liquidity weighted VIXs incorporate the spread and traded volume weights in the computation of SVIX and TVVIX respectively. The VIXs are computed four times a day, for the period February 2009 - September 2010. The performance indicators utilised are the standard deviation of the bootstrap estimates and the width of the confidence band. The performance of the volatility index estimators are compared pairwise by employing the Wilcoxon signed rank test. The test is based on the null hypothesis that the median difference between a pair of volatility indexes is zero.

We present the results for the pairwise comparisons of VIXs in the next section. The results are robust to the size of sampling frequency adopted, thus we present results only for the fifteen seconds frequency.

7.2 Pairwise comparisons of VIXs

Table 6 reports the summary statistics for the width of the 95% confidence interval for the four VIXs. The EVIX has the widest 95% band of 7.37 percentage points and largest σ of 1.87 percentage points in the median case. Thus, implying lowest precision among the volatility indexes. On the other

Figure 4 Width of the confidence band

The kernel density plot captures the distributions of the width of the confidence band for four VIXs. The 95% confidence band for a VIX is estimated estimated four times a day, for the period February 2009 - September 2010. For each point in time, the confidence band is computed from the bootstrapped sampling distribution of the VIX

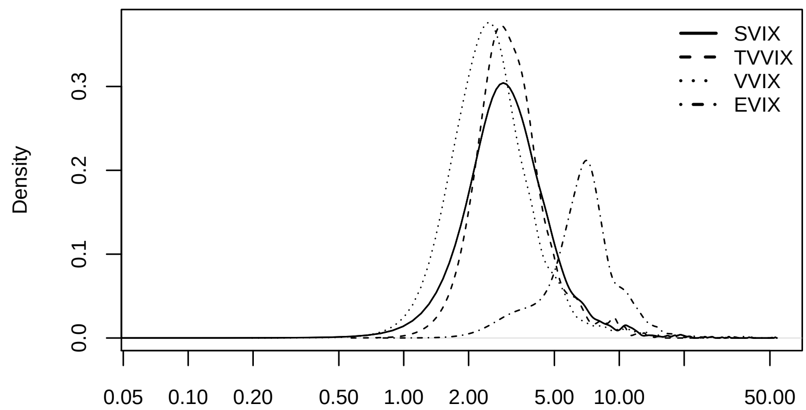


Figure 5 Standard deviation of the bootstrap estimates

The kernel density plot captures the distributions of the standard deviation of the bootstrap estimates for four VIXs. The standard deviation for a VIX is computed four times a day, for the period February 2009 - September 2010. For each point in time, the standard deviation is calculated from the bootstrapped sampling distribution of the VIX.

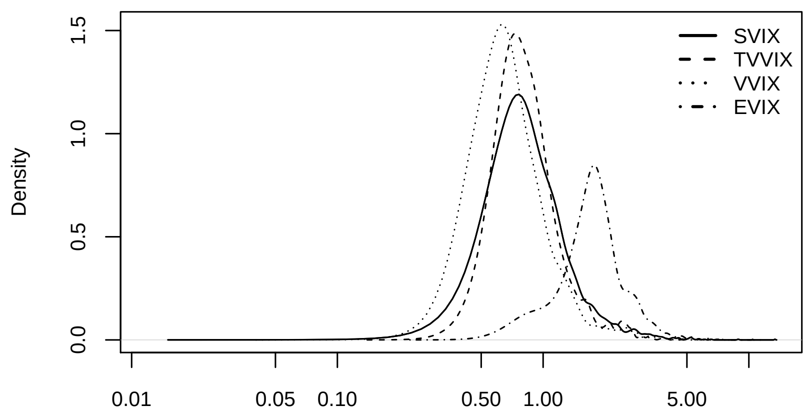


Table 6 Measures of imprecision

Width of the confidence band (percentage points)				
	SVIX	TVVIX	VVIX	EVIX
No. of Obs	1516	1516	1516	1516
Min	0.93	1.36	1.03	2.18
1st Qu	2.71	2.74	2.27	6.20
Median	3.55	3.42	2.92	7.37
Mean	4.54	4.02	3.91	8.24
3rd Qu	4.84	4.44	4.06	9.26
Max	52.94	23.79	50.49	51.08
Std Dev	3.80	2.11	3.64	3.93
σ of the bootstrap estimates (percentage points)				
No. of Obs	1516	1516	1516	1516
Min	0.24	0.34	0.25	0.57
1st Qu	0.71	0.71	0.58	1.58
Median	0.91	0.88	0.74	1.87
Mean	1.14	1.03	0.94	2.05
3rd Qu	1.25	1.14	1.02	2.32
Max	13.39	4.77	11.78	10.58
Std Dev	0.82	0.53	0.75	0.88

hand, VVIX has the highest precision with a confidence interval width of 2.92 percentage points and a σ of 0.74 percentage points in the median case.

Figure 4 represents the kernel density plot for the width of the 95% confidence interval for the four VIXs, estimated four times a day, for the period February 2009 - September 2010. The distributions of the VVIX, SVIX, and TVVIX are similar, while the EVIX is shifted heavily towards the right of the plot implying larger confidence bands than the rest. A similar pattern is seen for the distributions of the standard deviation of the bootstrap estimates across the VIXs (Figure 5).

Table 7 reports the median differences and the p-values for each pair of VIXs. The pairwise comparisons imply that VVIX outperforms the other three VIXs in terms of precision. It has the highest precision, whereas EVIX has the lowest precision. The difference in precision for the liquidity VIXs is statistically insignificant. Thus, ranking the VIXs based on their performance, we get: VVIX, SVIX and TVVIX, and EVIX.

Table 7 Wilcoxon signed rank test

	Width of the confidence band		σ of the bootstrap estimates	
	Median Diff	Pval	Median Diff	Pval
EVIX - SVIX	3.745	0.000	0.923	0.000
EVIX - TVVIX	3.846	0.000	0.962	0.000
EVIX - VVIX	4.326	0.000	1.097	0.000
SVIX - TVVIX	-0.004	1.000	0.013	0.341
SVIX - VVIX	0.641	0.000	0.183	0.000
TVVIX - VVIX	0.618	0.000	0.165	0.000

8 Reproducible research

An R package named ifrogs has been released into the public domain, with an open source implementation of the methods of this paper. An example shown in this package replicates all the calculations of this paper for a sample of index options for two series: S&P 500 and Nifty presented in Appendix A.1 and B.1 respectively.

9 Conclusion

Hentschel (2003) has argued that the imprecision of volatility indexes is small. Our result disagrees substantially with his. These differences may be attributed to the following reasons: we employ a non-parametric strategy, free from any distributional assumptions on errors, and for a large number of samples on index options. In addition to the above, we compute a set of volatility indexes estimated from implied volatilities for a wide range of strike prices while VXO is computed from implied volatilities of eight fixed near-the-money options with limited information content.

This paper also builds on Grover and Thomas (2012) which has worked on model selection of volatility indexes. The approach of this paper was based on the extent to which a given volatility index is effective at forecasting future realised volatility. While this is an important perspective, it has certain constraints. The statistical thinking of this paper offers an additional mechanism through which this model selection can be done. They find the liquidity adjusted VIX (SVIX) to be the best model. We find, that our model selection technique chooses VVIX to be the most precise estimator of VIX followed by the liquidity adjusted VIXs.

This paper also relates to the literature that develops empirical proxies to measure ambiguity. Uncertainty in VIX may be another way of capturing ambiguity. Imprecision indicators proposed in the paper can be computed to measure ambiguity for a wide range of stocks and at higher frequencies. This measure of ambiguity can further be used to test the relationship between ambiguity and expected stock returns. Traditional pricing factors discussed in the literature can also be included in the analysis to check whether uncertainty in VIX captures information beyond that found in these factors.[7]

The most important finding of this paper is that the imprecision of VIX is considerable. In recent years, a large number of applications of VIX have arisen. These applications largely treat VIX as a precisely observed number. The key finding of this paper is that the imprecision of VIX is substantial. The median value of the width of the 95% confidence interval for VVIX, which is the most precise volatility index, is 2.9 percentage points, which is a large number when compared with the one-day change in VVIX which has a median value of 1.18 percentage points.

For a sample of SPX options, we find that the confidence interval of VVIX estimated has a width of 1.5 percentage points which is an economically significant number when compared with the one-day change in VVIX which is smaller than this number for 62% of the days in our sample.

This finding has important consequences for real-world applications of volatility indexes. As an example, exchanges such as CBOE may find it useful to disseminate a confidence interval for VIX instead of only showing a point estimator.

The research of this paper was centred around volatility index estimation that is based on the Black-Scholes option pricing formula. Future extensions of this work are required in order to obtain inference procedures for model-free estimators such as the CBOE VIX. The bootstrap strategy proposed in this paper does not directly transport to that setting.

[7] Similar analysis has been conducted in Brenner and Izhakian (2011), Baltussen, Van Bekkum, and Van der Grient (2013), and Ehsani, Krause, and Lien (2013).

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A SPX options: A sample and replicate

We present a sample of end-of-day SPX options and one bootstrap replicate for the sample, on 2010-09-17. The sample consists of options for a range of strikes, and two nearest maturities. The example confidence intervals presented in Section 5 are estimated for this sample of options by generating thousand bootstrap replicates of this sample. A single such replicate is presented in the subsection of the Appendix.

A.1 Sample of SPX options

	Strike	Type	Underlying	Mid-Quote	Maturity	Risk-free	IVol
1	965	c	1125.59	160.70	29	0.12	18.02
2	970	c	1125.59	155.85	29	0.12	21.87
3	975	c	1125.59	150.85	29	0.12	21.19
4	980	c	1125.59	146.00	29	0.12	22.29
5	985	c	1125.59	141.00	29	0.12	21.57
6	990	c	1125.59	135.95	29	0.12	20.36
7	995	c	1125.59	131.70	29	0.12	24.24
8	1000	c	1125.59	126.75	29	0.12	23.62
9	1005	c	1125.59	121.95	29	0.12	23.55
10	1010	c	1125.59	116.95	29	0.12	22.70
11	1015	c	1125.59	112.40	29	0.12	23.27
12	1020	c	1125.59	107.45	29	0.12	22.52
13	1025	c	1125.59	102.85	29	0.12	22.65
14	1030	c	1125.59	98.35	29	0.12	22.85
15	1035	c	1125.59	93.50	29	0.12	22.21
16	1040	c	1125.59	89.00	29	0.12	22.19
17	1045	c	1125.59	84.65	29	0.12	22.31
18	1050	c	1125.59	80.30	29	0.12	22.29
19	1055	c	1125.59	75.55	29	0.12	21.58
20	1060	c	1125.59	71.25	29	0.12	21.45
21	1065	c	1125.59	67.05	29	0.12	21.34
22	1070	c	1125.59	62.70	29	0.12	20.95
23	1075	c	1125.59	58.65	29	0.12	20.82
24	1080	c	1125.59	54.45	29	0.12	20.42
25	1085	c	1125.59	50.40	29	0.12	20.08
26	1090	c	1125.59	46.30	29	0.12	19.60
27	1095	c	1125.59	42.50	29	0.12	19.31
28	1100	c	1125.59	38.00	29	0.12	18.30
29	1105	c	1125.59	35.25	29	0.12	18.71
30	1110	c	1125.59	31.75	29	0.12	18.35
31	1115	c	1125.59	28.60	29	0.12	18.14
32	1120	c	1125.59	25.75	29	0.12	18.05
33	1125	c	1125.59	22.75	29	0.12	17.70

34	1130	c	1125.59	19.35	29	0.12	16.90
35	1135	c	1125.59	16.85	29	0.12	16.66
36	1140	c	1125.59	14.00	29	0.12	15.98
37	1145	c	1125.59	12.35	29	0.12	16.12
38	1150	c	1125.59	10.50	29	0.12	15.94
39	1155	c	1125.59	8.55	29	0.12	15.49
40	1160	c	1125.59	6.80	29	0.12	15.01
41	1165	c	1125.59	5.60	29	0.12	14.89
42	1170	c	1125.59	4.65	29	0.12	14.87
43	1175	c	1125.59	3.55	29	0.12	14.48
44	1180	c	1125.59	2.58	29	0.12	13.99
45	1185	c	1125.59	2.08	29	0.12	14.01
46	1190	c	1125.59	1.82	29	0.12	14.36
47	1195	c	1125.59	1.30	29	0.12	14.00
48	1200	c	1125.59	0.95	29	0.12	13.81
49	1205	c	1125.59	0.68	29	0.12	13.59
50	1210	c	1125.59	0.50	29	0.12	13.53
51	1215	c	1125.59	0.45	29	0.12	13.92
52	1220	c	1125.59	0.35	29	0.12	13.98
53	1225	c	1125.59	0.28	29	0.12	14.06
54	1230	c	1125.59	0.25	29	0.12	14.44
55	1235	c	1125.59	0.30	29	0.12	15.40
56	1240	c	1125.59	0.25	29	0.12	15.56
57	1245	c	1125.59	0.22	29	0.12	15.89
58	1250	c	1125.59	0.25	29	0.12	16.66
59	1255	c	1125.59	0.22	29	0.12	16.97
60	1260	c	1125.59	0.22	29	0.12	17.50
61	1265	c	1125.59	0.10	29	0.12	16.35
62	1270	c	1125.59	0.08	29	0.12	16.32
63	1275	c	1125.59	0.10	29	0.12	17.32
64	1280	c	1125.59	0.08	29	0.12	17.26
65	1285	c	1125.59	0.08	29	0.12	17.72
66	1290	c	1125.59	0.08	29	0.12	18.18
67	1295	c	1125.59	0.08	29	0.12	18.64
68	1300	c	1125.59	0.08	29	0.12	19.10
69	675	p	1125.59	0.08	29	0.12	62.00
70	680	p	1125.59	0.08	29	0.12	61.16
71	690	p	1125.59	0.08	29	0.12	59.51
72	700	p	1125.59	0.12	29	0.12	60.72
73	710	p	1125.59	0.12	29	0.12	59.04
74	715	p	1125.59	0.12	29	0.12	58.21
75	720	p	1125.59	0.12	29	0.12	57.38
76	725	p	1125.59	0.12	29	0.12	56.56
77	730	p	1125.59	0.12	29	0.12	55.74
78	740	p	1125.59	0.15	29	0.12	55.13
79	750	p	1125.59	0.18	29	0.12	54.36
80	760	p	1125.59	0.18	29	0.12	52.73
81	770	p	1125.59	0.22	29	0.12	52.53
82	775	p	1125.59	0.22	29	0.12	51.71

83	780	p	1125.59	0.25	29	0.12	51.50
84	790	p	1125.59	0.28	29	0.12	50.42
85	800	p	1125.59	0.32	29	0.12	49.76
86	810	p	1125.59	0.35	29	0.12	48.56
87	820	p	1125.59	0.38	29	0.12	47.33
88	825	p	1125.59	0.40	29	0.12	46.89
89	830	p	1125.59	0.45	29	0.12	46.77
90	840	p	1125.59	0.42	29	0.12	44.80
91	850	p	1125.59	0.48	29	0.12	43.81
92	860	p	1125.59	0.52	29	0.12	42.75
93	870	p	1125.59	0.65	29	0.12	42.37
94	875	p	1125.59	0.73	29	0.12	42.20
95	880	p	1125.59	0.65	29	0.12	40.72
96	890	p	1125.59	0.80	29	0.12	40.29
97	900	p	1125.59	0.78	29	0.12	38.44
98	910	p	1125.59	0.95	29	0.12	37.97
99	920	p	1125.59	1.02	29	0.12	36.74
100	925	p	1125.59	1.08	29	0.12	36.18
101	930	p	1125.59	1.07	29	0.12	35.33
102	935	p	1125.59	1.23	29	0.12	35.26
103	940	p	1125.59	1.20	29	0.12	34.28
104	945	p	1125.59	1.30	29	0.12	33.90
105	950	p	1125.59	1.40	29	0.12	33.49
106	955	p	1125.59	1.48	29	0.12	32.94
107	960	p	1125.59	1.57	29	0.12	32.46
108	965	p	1125.59	1.73	29	0.12	32.14
109	970	p	1125.59	1.82	29	0.12	31.61
110	975	p	1125.59	2.03	29	0.12	31.37
111	980	p	1125.59	2.10	29	0.12	30.69
112	985	p	1125.59	2.42	29	0.12	30.72
113	990	p	1125.59	2.45	29	0.12	29.86
114	995	p	1125.59	2.75	29	0.12	29.70
115	1000	p	1125.59	2.88	29	0.12	29.05
116	1005	p	1125.59	3.17	29	0.12	28.78
117	1010	p	1125.59	3.25	29	0.12	27.97
118	1015	p	1125.59	3.70	29	0.12	27.91
119	1020	p	1125.59	3.75	29	0.12	27.01
120	1025	p	1125.59	4.15	29	0.12	26.74
121	1030	p	1125.59	4.60	29	0.12	26.49
122	1035	p	1125.59	5.15	29	0.12	26.32
123	1040	p	1125.59	5.35	29	0.12	25.55
124	1045	p	1125.59	5.95	29	0.12	25.33
125	1050	p	1125.59	6.30	29	0.12	24.69
126	1055	p	1125.59	7.00	29	0.12	24.47
127	1060	p	1125.59	7.70	29	0.12	24.16
128	1065	p	1125.59	8.30	29	0.12	23.66
129	1070	p	1125.59	8.85	29	0.12	23.04
130	1075	p	1125.59	9.65	29	0.12	22.63
131	1080	p	1125.59	10.90	29	0.12	22.60

132	1085	p	1125.59	11.75	29	0.12	22.07
133	1090	p	1125.59	12.80	29	0.12	21.66
134	1095	p	1125.59	14.00	29	0.12	21.29
135	1100	p	1125.59	15.25	29	0.12	20.87
136	1105	p	1125.59	16.65	29	0.12	20.48
137	1110	p	1125.59	18.05	29	0.12	19.99
138	1115	p	1125.59	19.70	29	0.12	19.59
139	1120	p	1125.59	21.55	29	0.12	19.24
140	1125	p	1125.59	24.55	29	0.12	19.68
141	1130	p	1125.59	26.30	29	0.12	19.00
142	1135	p	1125.59	28.10	29	0.12	18.22
143	1140	p	1125.59	30.85	29	0.12	18.05
144	1145	p	1125.59	33.65	29	0.12	17.79
145	1150	p	1125.59	36.45	29	0.12	17.37
146	1155	p	1125.59	39.75	29	0.12	17.22
147	1160	p	1125.59	43.40	29	0.12	17.24
148	1165	p	1125.59	46.80	29	0.12	16.87
149	1170	p	1125.59	50.65	29	0.12	16.79
150	1175	p	1125.59	54.95	29	0.12	17.07
151	1180	p	1125.59	59.20	29	0.12	17.19
152	1185	p	1125.59	63.45	29	0.12	17.19
153	1190	p	1125.59	67.60	29	0.12	16.89
154	1195	p	1125.59	72.20	29	0.12	17.12
155	1200	p	1125.59	77.75	29	0.12	18.87
156	1205	p	1125.59	81.60	29	0.12	17.71
157	1210	p	1125.59	86.45	29	0.12	18.20
158	1215	p	1125.59	91.30	29	0.12	18.65
159	1220	p	1125.59	96.20	29	0.12	19.18
160	1225	p	1125.59	101.15	29	0.12	19.80
161	1230	p	1125.59	106.10	29	0.12	20.41
162	1235	p	1125.59	111.05	29	0.12	21.00
163	1240	p	1125.59	116.10	29	0.12	21.84
164	1245	p	1125.59	121.10	29	0.12	22.54
165	1250	p	1125.59	126.10	29	0.12	23.24
166	1255	p	1125.59	131.00	29	0.12	23.65
167	1260	p	1125.59	135.95	29	0.12	24.18
168	1265	p	1125.59	141.10	29	0.12	25.29
169	1270	p	1125.59	146.10	29	0.12	25.96
170	1275	p	1125.59	151.10	29	0.12	26.62
171	1280	p	1125.59	155.95	29	0.12	26.82
172	1285	p	1125.59	161.10	29	0.12	27.93
173	1290	p	1125.59	166.10	29	0.12	28.58
174	1295	p	1125.59	171.10	29	0.12	29.22
175	1300	p	1125.59	176.10	29	0.12	29.86
176	1305	p	1125.59	181.10	29	0.12	30.49
177	1310	p	1125.59	186.10	29	0.12	31.12
178	1315	p	1125.59	190.95	29	0.12	31.23
179	1320	p	1125.59	195.95	29	0.12	31.84
180	1325	p	1125.59	200.95	29	0.12	32.45

181	1350	p	1125.59	225.95	29	0.12	35.42
182	1375	p	1125.59	250.95	29	0.12	38.29
183	1400	p	1125.59	275.90	29	0.12	40.86
184	1450	p	1125.59	325.90	29	0.12	46.16
185	1500	p	1125.59	375.90	29	0.12	51.19
186	1550	p	1125.59	425.90	29	0.12	55.96
187	920	c	1125.59	206.10	64	0.16	20.29
188	925	c	1125.59	201.25	64	0.16	21.03
189	930	c	1125.59	196.65	64	0.16	22.69
190	935	c	1125.59	192.00	64	0.16	23.49
191	940	c	1125.59	187.15	64	0.16	23.41
192	945	c	1125.59	182.45	64	0.16	23.70
193	950	c	1125.59	177.75	64	0.16	23.88
194	955	c	1125.59	172.95	64	0.16	23.74
195	960	c	1125.59	168.40	64	0.16	24.07
196	965	c	1125.59	163.85	64	0.16	24.29
197	970	c	1125.59	159.15	64	0.16	24.16
198	975	c	1125.59	154.55	64	0.16	24.15
199	980	c	1125.59	150.15	64	0.16	24.37
200	985	c	1125.59	145.25	64	0.16	23.82
201	990	c	1125.59	140.95	64	0.16	24.06
202	995	c	1125.59	136.45	64	0.16	23.97
203	1000	c	1125.59	131.80	64	0.16	23.66
204	1005	c	1125.59	127.55	64	0.16	23.76
205	1010	c	1125.59	123.15	64	0.16	23.64
206	1015	c	1125.59	118.65	64	0.16	23.38
207	1020	c	1125.59	114.25	64	0.16	23.18
208	1025	c	1125.59	110.05	64	0.16	23.13
209	1030	c	1125.59	105.55	64	0.16	22.76
210	1035	c	1125.59	101.55	64	0.16	22.78
211	1040	c	1125.59	97.15	64	0.16	22.43
212	1045	c	1125.59	93.30	64	0.16	22.47
213	1050	c	1125.59	89.20	64	0.16	22.26
214	1055	c	1125.59	85.10	64	0.16	22.02
215	1060	c	1125.59	80.95	64	0.16	21.70
216	1065	c	1125.59	76.95	64	0.16	21.45
217	1070	c	1125.59	73.40	64	0.16	21.44
218	1075	c	1125.59	69.45	64	0.16	21.13
219	1080	c	1125.59	66.10	64	0.16	21.14
220	1085	c	1125.59	62.15	64	0.16	20.74
221	1090	c	1125.59	58.50	64	0.16	20.48
222	1095	c	1125.59	55.25	64	0.16	20.40
223	1100	c	1125.59	51.55	64	0.16	20.01
224	1105	c	1125.59	48.55	64	0.16	19.96
225	1110	c	1125.59	45.30	64	0.16	19.72
226	1115	c	1125.59	42.35	64	0.16	19.59
227	1120	c	1125.59	38.95	64	0.16	19.16
228	1125	c	1125.59	35.05	64	0.16	18.41
229	1130	c	1125.59	32.95	64	0.16	18.56

230	1135	c	1125.59	30.50	64	0.16	18.47
231	1140	c	1125.59	27.75	64	0.16	18.15
232	1145	c	1125.59	25.25	64	0.16	17.90
233	1150	c	1125.59	22.15	64	0.16	17.26
234	1155	c	1125.59	20.80	64	0.16	17.51
235	1160	c	1125.59	18.75	64	0.16	17.31
236	1165	c	1125.59	16.65	64	0.16	17.01
237	1170	c	1125.59	14.80	64	0.16	16.77
238	1175	c	1125.59	12.65	64	0.16	16.27
239	1180	c	1125.59	11.50	64	0.16	16.32
240	1185	c	1125.59	10.05	64	0.16	16.10
241	1190	c	1125.59	8.80	64	0.16	15.93
242	1195	c	1125.59	7.55	64	0.16	15.68
243	1200	c	1125.59	6.40	64	0.16	15.42
244	1205	c	1125.59	5.55	64	0.16	15.31
245	1210	c	1125.59	4.95	64	0.16	15.36
246	1215	c	1125.59	4.00	64	0.16	14.98
247	1220	c	1125.59	3.40	64	0.16	14.87
248	1225	c	1125.59	2.83	64	0.16	14.70
249	1230	c	1125.59	2.42	64	0.16	14.67
250	1235	c	1125.59	1.90	64	0.16	14.35
251	1240	c	1125.59	1.60	64	0.16	14.30
252	1245	c	1125.59	1.25	64	0.16	14.06
253	1250	c	1125.59	1.15	64	0.16	14.28
254	1255	c	1125.59	1.07	64	0.16	14.53
255	1260	c	1125.59	0.65	64	0.16	13.73
256	1265	c	1125.59	0.50	64	0.16	13.56
257	1270	c	1125.59	0.62	64	0.16	14.43
258	1275	c	1125.59	0.40	64	0.16	13.86
259	1280	c	1125.59	0.32	64	0.16	13.83
260	1285	c	1125.59	0.30	64	0.16	14.03
261	1290	c	1125.59	0.28	64	0.16	14.22
262	1295	c	1125.59	0.35	64	0.16	15.05
263	600	p	1125.59	0.12	64	0.16	53.11
264	620	p	1125.59	0.18	64	0.16	52.21
265	625	p	1125.59	0.18	64	0.16	51.55
266	630	p	1125.59	0.22	64	0.16	52.23
267	640	p	1125.59	0.25	64	0.16	51.49
268	650	p	1125.59	0.25	64	0.16	50.18
269	660	p	1125.59	0.28	64	0.16	49.40
270	670	p	1125.59	0.30	64	0.16	48.58
271	675	p	1125.59	0.35	64	0.16	48.78
272	680	p	1125.59	0.30	64	0.16	47.31
273	690	p	1125.59	0.30	64	0.16	46.05
274	700	p	1125.59	0.57	64	0.16	48.41
275	710	p	1125.59	0.55	64	0.16	46.84
276	720	p	1125.59	0.62	64	0.16	46.30
277	725	p	1125.59	0.65	64	0.16	45.88
278	730	p	1125.59	0.60	64	0.16	44.77

279	740	p	1125.59	0.68	64	0.16	44.18
280	750	p	1125.59	0.95	64	0.16	44.97
281	760	p	1125.59	1.00	64	0.16	43.99
282	770	p	1125.59	0.92	64	0.16	42.22
283	775	p	1125.59	0.98	64	0.16	41.90
284	780	p	1125.59	1.02	64	0.16	41.56
285	790	p	1125.59	1.23	64	0.16	41.38
286	800	p	1125.59	1.35	64	0.16	40.70
287	810	p	1125.59	1.43	64	0.16	39.75
288	820	p	1125.59	1.60	64	0.16	39.19
289	825	p	1125.59	1.72	64	0.16	39.03
290	830	p	1125.59	1.82	64	0.16	38.75
291	840	p	1125.59	2.00	64	0.16	38.05
292	850	p	1125.59	2.30	64	0.16	37.68
293	860	p	1125.59	2.45	64	0.16	36.78
294	870	p	1125.59	2.70	64	0.16	36.12
295	875	p	1125.59	2.95	64	0.16	36.08
296	880	p	1125.59	3.00	64	0.16	35.52
297	890	p	1125.59	3.42	64	0.16	35.12
298	900	p	1125.59	3.80	64	0.16	34.52
299	910	p	1125.59	4.20	64	0.16	33.89
300	920	p	1125.59	4.75	64	0.16	33.44
301	925	p	1125.59	4.90	64	0.16	32.98
302	930	p	1125.59	5.00	64	0.16	32.43
303	935	p	1125.59	5.45	64	0.16	32.40
304	940	p	1125.59	5.70	64	0.16	32.05
305	945	p	1125.59	6.00	64	0.16	31.74
306	950	p	1125.59	6.40	64	0.16	31.55
307	955	p	1125.59	6.60	64	0.16	31.07
308	960	p	1125.59	7.00	64	0.16	30.83
309	965	p	1125.59	7.45	64	0.16	30.63
310	970	p	1125.59	7.80	64	0.16	30.28
311	975	p	1125.59	8.35	64	0.16	30.13
312	980	p	1125.59	8.70	64	0.16	29.73
313	985	p	1125.59	9.15	64	0.16	29.42
314	990	p	1125.59	9.55	64	0.16	29.04
315	995	p	1125.59	10.05	64	0.16	28.73
316	1000	p	1125.59	10.80	64	0.16	28.63
317	1005	p	1125.59	11.05	64	0.16	28.04
318	1010	p	1125.59	11.70	64	0.16	27.79
319	1015	p	1125.59	12.35	64	0.16	27.51
320	1020	p	1125.59	13.10	64	0.16	27.29
321	1025	p	1125.59	13.85	64	0.16	27.03
322	1030	p	1125.59	14.50	64	0.16	26.66
323	1035	p	1125.59	15.30	64	0.16	26.38
324	1040	p	1125.59	16.10	64	0.16	26.07
325	1045	p	1125.59	17.00	64	0.16	25.80
326	1050	p	1125.59	18.00	64	0.16	25.56
327	1055	p	1125.59	18.95	64	0.16	25.26

328	1060	p	1125.59	20.00	64	0.16	24.98
329	1065	p	1125.59	21.05	64	0.16	24.67
330	1070	p	1125.59	22.00	64	0.16	24.27
331	1075	p	1125.59	23.70	64	0.16	24.28
332	1080	p	1125.59	24.40	64	0.16	23.66
333	1085	p	1125.59	25.85	64	0.16	23.44
334	1090	p	1125.59	27.15	64	0.16	23.10
335	1095	p	1125.59	28.80	64	0.16	22.91
336	1100	p	1125.59	30.40	64	0.16	22.65
337	1105	p	1125.59	32.00	64	0.16	22.35
338	1110	p	1125.59	33.80	64	0.16	22.11
339	1115	p	1125.59	35.60	64	0.16	21.82
340	1120	p	1125.59	37.40	64	0.16	21.49
341	1125	p	1125.59	39.70	64	0.16	21.37
342	1130	p	1125.59	41.55	64	0.16	20.96
343	1135	p	1125.59	44.05	64	0.16	20.84
344	1140	p	1125.59	46.70	64	0.16	20.75
345	1145	p	1125.59	48.70	64	0.16	20.26
346	1150	p	1125.59	51.65	64	0.16	20.22
347	1155	p	1125.59	54.65	64	0.16	20.15
348	1160	p	1125.59	57.30	64	0.16	19.83
349	1165	p	1125.59	60.40	64	0.16	19.70
350	1170	p	1125.59	63.55	64	0.16	19.54
351	1175	p	1125.59	66.85	64	0.16	19.41
352	1180	p	1125.59	70.25	64	0.16	19.28
353	1185	p	1125.59	73.45	64	0.16	18.96
354	1190	p	1125.59	77.25	64	0.16	18.96
355	1195	p	1125.59	81.25	64	0.16	19.03
356	1200	p	1125.59	85.20	64	0.16	19.03
357	1205	p	1125.59	88.75	64	0.16	18.67
358	1210	p	1125.59	93.45	64	0.16	19.12
359	1215	p	1125.59	97.15	64	0.16	18.74
360	1220	p	1125.59	101.75	64	0.16	19.04
361	1225	p	1125.59	106.25	64	0.16	19.23
362	1230	p	1125.59	110.70	64	0.16	19.34
363	1235	p	1125.59	115.05	64	0.16	19.31
364	1240	p	1125.59	120.05	64	0.16	19.88
365	1245	p	1125.59	124.60	64	0.16	19.99
366	1250	p	1125.59	129.35	64	0.16	20.28
367	1255	p	1125.59	133.95	64	0.16	20.38
368	1260	p	1125.59	139.00	64	0.16	20.96
369	1265	p	1125.59	143.65	64	0.16	21.08
370	1270	p	1125.59	148.55	64	0.16	21.47
371	1275	p	1125.59	153.55	64	0.16	21.98
372	1280	p	1125.59	158.85	64	0.16	22.86
373	1285	p	1125.59	163.65	64	0.16	23.12
374	1290	p	1125.59	168.30	64	0.16	23.16
375	1295	p	1125.59	173.35	64	0.16	23.71
376	1300	p	1125.59	178.45	64	0.16	24.33

377	1305	p	1125.59	183.20	64	0.16	24.47
378	1310	p	1125.59	188.20	64	0.16	24.94
379	1315	p	1125.59	193.65	64	0.16	26.04
380	1320	p	1125.59	198.15	64	0.16	25.81
381	1340	p	1125.59	218.40	64	0.16	28.00
382	1350	p	1125.59	228.35	64	0.16	28.83
383	1400	p	1125.59	278.35	64	0.16	33.14
384	1450	p	1125.59	328.40	64	0.16	37.27
385	1500	p	1125.59	378.15	64	0.16	40.61
386	1550	p	1125.59	428.25	64	0.16	44.41
387	1600	p	1125.59	478.10	64	0.16	47.53

A.2 A single bootstrap replicate

	Strike	Type	Underlying	Mid-Quote	Maturity	Risk-free	IVol
1	680	p	1125.59	0.08	29	0.12	61.16
2	1055	c	1125.59	75.55	29	0.12	21.58
3	1070	p	1125.59	8.85	29	0.12	23.04
4	900	p	1125.59	0.78	29	0.12	38.44
5	1245	p	1125.59	121.10	29	0.12	22.54
6	1050	p	1125.59	6.30	29	0.12	24.69
7	995	p	1125.59	2.75	29	0.12	29.70
8	1105	c	1125.59	35.25	29	0.12	18.71
9	1105	c	1125.59	35.25	29	0.12	18.71
10	1240	c	1125.59	0.25	29	0.12	15.56
11	1210	p	1125.59	86.45	29	0.12	18.20
12	1195	c	1125.59	1.30	29	0.12	14.00
13	875	p	1125.59	0.73	29	0.12	42.20
14	1350	p	1125.59	225.95	29	0.12	35.42
15	1210	p	1125.59	86.45	29	0.12	18.20
16	1305	p	1125.59	181.10	29	0.12	30.49
17	1220	p	1125.59	96.20	29	0.12	19.18
18	1050	c	1125.59	80.30	29	0.12	22.29
19	1030	p	1125.59	4.60	29	0.12	26.49
20	1095	c	1125.59	42.50	29	0.12	19.31
21	1015	c	1125.59	112.40	29	0.12	23.27
22	1155	c	1125.59	8.55	29	0.12	15.49
23	1225	p	1125.59	101.15	29	0.12	19.80
24	1450	p	1125.59	325.90	29	0.12	46.16
25	810	p	1125.59	0.35	29	0.12	48.56
26	675	p	1125.59	0.08	29	0.12	62.00
27	990	p	1125.59	2.45	29	0.12	29.86
28	1105	c	1125.59	35.25	29	0.12	18.71
29	1235	c	1125.59	0.30	29	0.12	15.40
30	975	c	1125.59	150.85	29	0.12	21.19
31	750	p	1125.59	0.18	29	0.12	54.36
32	1045	c	1125.59	84.65	29	0.12	22.31

33	1055	c	1125.59	75.55	29	0.12	21.58
34	920	p	1125.59	1.02	29	0.12	36.74
35	970	c	1125.59	155.85	29	0.12	21.87
36	1295	c	1125.59	0.08	29	0.12	18.64
37	1310	p	1125.59	186.10	29	0.12	31.12
38	1185	p	1125.59	63.45	29	0.12	17.19
39	965	c	1125.59	160.70	29	0.12	18.02
40	1205	c	1125.59	0.68	29	0.12	13.59
41	1100	p	1125.59	15.25	29	0.12	20.87
42	790	p	1125.59	0.28	29	0.12	50.42
43	985	p	1125.59	2.42	29	0.12	30.72
44	1225	c	1125.59	0.28	29	0.12	14.06
45	1120	c	1125.59	25.75	29	0.12	18.05
46	710	p	1125.59	0.12	29	0.12	59.04
47	1270	p	1125.59	146.10	29	0.12	25.96
48	1185	p	1125.59	63.45	29	0.12	17.19
49	1005	p	1125.59	3.17	29	0.12	28.78
50	925	p	1125.59	1.08	29	0.12	36.18
51	775	p	1125.59	0.22	29	0.12	51.71
52	1040	p	1125.59	5.35	29	0.12	25.55
53	1230	p	1125.59	106.10	29	0.12	20.41
54	1090	p	1125.59	12.80	29	0.12	21.66
55	1165	c	1125.59	5.60	29	0.12	14.89
56	1270	p	1125.59	146.10	29	0.12	25.96
57	1255	c	1125.59	0.22	29	0.12	16.97
58	1160	c	1125.59	6.80	29	0.12	15.01
59	1095	p	1125.59	14.00	29	0.12	21.29
60	1055	c	1125.59	75.55	29	0.12	21.58
61	1225	p	1125.59	101.15	29	0.12	19.80
62	1225	p	1125.59	101.15	29	0.12	19.80
63	990	c	1125.59	135.95	29	0.12	20.36
64	1120	p	1125.59	21.55	29	0.12	19.24
65	945	p	1125.59	1.30	29	0.12	33.90
66	935	p	1125.59	1.23	29	0.12	35.26
67	1255	c	1125.59	0.22	29	0.12	16.97
68	1130	p	1125.59	26.30	29	0.12	19.00
69	995	p	1125.59	2.75	29	0.12	29.70
70	1220	p	1125.59	96.20	29	0.12	19.18
71	1215	c	1125.59	0.45	29	0.12	13.92
72	1080	p	1125.59	10.90	29	0.12	22.60
73	1300	c	1125.59	0.08	29	0.12	19.10
74	1090	c	1125.59	46.30	29	0.12	19.60
75	1165	p	1125.59	46.80	29	0.12	16.87
76	1290	p	1125.59	166.10	29	0.12	28.58
77	1165	p	1125.59	46.80	29	0.12	16.87
78	1105	c	1125.59	35.25	29	0.12	18.71
79	1300	c	1125.59	0.08	29	0.12	19.10
80	1280	c	1125.59	0.08	29	0.12	17.26
81	1295	p	1125.59	171.10	29	0.12	29.22

82	1010	p	1125.59	3.25	29	0.12	27.97
83	1250	p	1125.59	126.10	29	0.12	23.24
84	810	p	1125.59	0.35	29	0.12	48.56
85	860	p	1125.59	0.52	29	0.12	42.75
86	1050	c	1125.59	80.30	29	0.12	22.29
87	1300	c	1125.59	0.08	29	0.12	19.10
88	990	p	1125.59	2.45	29	0.12	29.86
89	1105	c	1125.59	35.25	29	0.12	18.71
90	1075	c	1125.59	58.65	29	0.12	20.82
91	1300	p	1125.59	176.10	29	0.12	29.86
92	990	c	1125.59	135.95	29	0.12	20.36
93	1285	c	1125.59	0.08	29	0.12	17.72
94	990	p	1125.59	2.45	29	0.12	29.86
95	715	p	1125.59	0.12	29	0.12	58.21
96	1235	c	1125.59	0.30	29	0.12	15.40
97	850	p	1125.59	0.48	29	0.12	43.81
98	1120	c	1125.59	25.75	29	0.12	18.05
99	985	c	1125.59	141.00	29	0.12	21.57
100	1030	c	1125.59	98.35	29	0.12	22.85
101	1035	p	1125.59	5.15	29	0.12	26.32
102	1030	p	1125.59	4.60	29	0.12	26.49
103	1170	c	1125.59	4.65	29	0.12	14.87
104	1285	c	1125.59	0.08	29	0.12	17.72
105	1210	p	1125.59	86.45	29	0.12	18.20
106	1300	c	1125.59	0.08	29	0.12	19.10
107	890	p	1125.59	0.80	29	0.12	40.29
108	970	c	1125.59	155.85	29	0.12	21.87
109	1315	p	1125.59	190.95	29	0.12	31.23
110	930	p	1125.59	1.07	29	0.12	35.33
111	1220	c	1125.59	0.35	29	0.12	13.98
112	1280	c	1125.59	0.08	29	0.12	17.26
113	930	p	1125.59	1.07	29	0.12	35.33
114	1090	c	1125.59	46.30	29	0.12	19.60
115	1165	c	1125.59	5.60	29	0.12	14.89
116	1260	c	1125.59	0.22	29	0.12	17.50
117	1145	p	1125.59	33.65	29	0.12	17.79
118	1060	c	1125.59	71.25	29	0.12	21.45
119	1215	p	1125.59	91.30	29	0.12	18.65
120	1265	c	1125.59	0.10	29	0.12	16.35
121	740	p	1125.59	0.15	29	0.12	55.13
122	1270	c	1125.59	0.08	29	0.12	16.32
123	1055	c	1125.59	75.55	29	0.12	21.58
124	1120	c	1125.59	25.75	29	0.12	18.05
125	690	p	1125.59	0.08	29	0.12	59.51
126	770	p	1125.59	0.22	29	0.12	52.53
127	1115	c	1125.59	28.60	29	0.12	18.14
128	1150	p	1125.59	36.45	29	0.12	17.37
129	710	p	1125.59	0.12	29	0.12	59.04
130	1400	p	1125.59	275.90	29	0.12	40.86

131	1095	p	1125.59	14.00	29	0.12	21.29
132	1095	c	1125.59	42.50	29	0.12	19.31
133	1290	c	1125.59	0.08	29	0.12	18.18
134	1160	c	1125.59	6.80	29	0.12	15.01
135	715	p	1125.59	0.12	29	0.12	58.21
136	930	p	1125.59	1.07	29	0.12	35.33
137	965	c	1125.59	160.70	29	0.12	18.02
138	1175	p	1125.59	54.95	29	0.12	17.07
139	850	p	1125.59	0.48	29	0.12	43.81
140	1155	p	1125.59	39.75	29	0.12	17.22
141	1030	c	1125.59	98.35	29	0.12	22.85
142	1280	c	1125.59	0.08	29	0.12	17.26
143	900	p	1125.59	0.78	29	0.12	38.44
144	740	p	1125.59	0.15	29	0.12	55.13
145	1110	c	1125.59	31.75	29	0.12	18.35
146	1025	p	1125.59	4.15	29	0.12	26.74
147	1120	p	1125.59	21.55	29	0.12	19.24
148	1450	p	1125.59	325.90	29	0.12	46.16
149	1080	c	1125.59	54.45	29	0.12	20.42
150	995	c	1125.59	131.70	29	0.12	24.24
151	1005	c	1125.59	121.95	29	0.12	23.55
152	860	p	1125.59	0.52	29	0.12	42.75
153	875	p	1125.59	0.73	29	0.12	42.20
154	1120	p	1125.59	21.55	29	0.12	19.24
155	1160	p	1125.59	43.40	29	0.12	17.24
156	990	c	1125.59	135.95	29	0.12	20.36
157	1030	c	1125.59	98.35	29	0.12	22.85
158	1220	c	1125.59	0.35	29	0.12	13.98
159	1220	p	1125.59	96.20	29	0.12	19.18
160	950	p	1125.59	1.40	29	0.12	33.49
161	1085	c	1125.59	50.40	29	0.12	20.08
162	1090	p	1125.59	12.80	29	0.12	21.66
163	1200	c	1125.59	0.95	29	0.12	13.81
164	1100	p	1125.59	15.25	29	0.12	20.87
165	820	p	1125.59	0.38	29	0.12	47.33
166	810	p	1125.59	0.35	29	0.12	48.56
167	1290	c	1125.59	0.08	29	0.12	18.18
168	875	p	1125.59	0.73	29	0.12	42.20
169	725	p	1125.59	0.12	29	0.12	56.56
170	950	p	1125.59	1.40	29	0.12	33.49
171	1145	c	1125.59	12.35	29	0.12	16.12
172	1175	p	1125.59	54.95	29	0.12	17.07
173	740	p	1125.59	0.15	29	0.12	55.13
174	1215	p	1125.59	91.30	29	0.12	18.65
175	1180	p	1125.59	59.20	29	0.12	17.19
176	1130	c	1125.59	19.35	29	0.12	16.90
177	1080	p	1125.59	10.90	29	0.12	22.60
178	1300	c	1125.59	0.08	29	0.12	19.10
179	1220	p	1125.59	96.20	29	0.12	19.18

180	1065	c	1125.59	67.05	29	0.12	21.34
181	1075	p	1125.59	9.65	29	0.12	22.63
182	1115	p	1125.59	19.70	29	0.12	19.59
183	950	p	1125.59	1.40	29	0.12	33.49
184	1240	p	1125.59	116.10	29	0.12	21.84
185	1165	c	1125.59	5.60	29	0.12	14.89
186	1285	c	1125.59	0.08	29	0.12	17.72
187	1080	c	1125.59	66.10	64	0.16	21.14
188	985	c	1125.59	145.25	64	0.16	23.82
189	1180	p	1125.59	70.25	64	0.16	19.28
190	1315	p	1125.59	193.65	64	0.16	26.04
191	955	p	1125.59	6.60	64	0.16	31.07
192	965	p	1125.59	7.45	64	0.16	30.63
193	930	p	1125.59	5.00	64	0.16	32.43
194	1245	p	1125.59	124.60	64	0.16	19.99
195	910	p	1125.59	4.20	64	0.16	33.89
196	1025	c	1125.59	110.05	64	0.16	23.13
197	710	p	1125.59	0.55	64	0.16	46.84
198	1240	p	1125.59	120.05	64	0.16	19.88
199	1095	p	1125.59	28.80	64	0.16	22.91
200	1045	c	1125.59	93.30	64	0.16	22.47
201	1290	p	1125.59	168.30	64	0.16	23.16
202	1300	p	1125.59	178.45	64	0.16	24.33
203	630	p	1125.59	0.22	64	0.16	52.23
204	1115	c	1125.59	42.35	64	0.16	19.59
205	1095	c	1125.59	55.25	64	0.16	20.40
206	1145	p	1125.59	48.70	64	0.16	20.26
207	1070	p	1125.59	22.00	64	0.16	24.27
208	1115	c	1125.59	42.35	64	0.16	19.59
209	670	p	1125.59	0.30	64	0.16	48.58
210	1310	p	1125.59	188.20	64	0.16	24.94
211	1135	p	1125.59	44.05	64	0.16	20.84
212	880	p	1125.59	3.00	64	0.16	35.52
213	1195	p	1125.59	81.25	64	0.16	19.03
214	1195	p	1125.59	81.25	64	0.16	19.03
215	1125	c	1125.59	35.05	64	0.16	18.41
216	820	p	1125.59	1.60	64	0.16	39.19
217	1080	c	1125.59	66.10	64	0.16	21.14
218	790	p	1125.59	1.23	64	0.16	41.38
219	1055	p	1125.59	18.95	64	0.16	25.26
220	975	p	1125.59	8.35	64	0.16	30.13
221	860	p	1125.59	2.45	64	0.16	36.78
222	1090	c	1125.59	58.50	64	0.16	20.48
223	1550	p	1125.59	428.25	64	0.16	44.41
224	920	c	1125.59	206.10	64	0.16	20.29
225	1085	p	1125.59	25.85	64	0.16	23.44
226	1125	p	1125.59	39.70	64	0.16	21.37
227	1170	p	1125.59	63.55	64	0.16	19.54
228	1190	p	1125.59	77.25	64	0.16	18.96

229	1290	c	1125.59	0.28	64	0.16	14.22
230	1350	p	1125.59	228.35	64	0.16	28.83
231	1000	c	1125.59	131.80	64	0.16	23.66
232	1290	p	1125.59	168.30	64	0.16	23.16
233	1240	p	1125.59	120.05	64	0.16	19.88
234	1000	c	1125.59	131.80	64	0.16	23.66
235	925	p	1125.59	4.90	64	0.16	32.98
236	1255	c	1125.59	1.07	64	0.16	14.53
237	925	p	1125.59	4.90	64	0.16	32.98
238	740	p	1125.59	0.68	64	0.16	44.18
239	1240	p	1125.59	120.05	64	0.16	19.88
240	1290	c	1125.59	0.28	64	0.16	14.22
241	1000	p	1125.59	10.80	64	0.16	28.63
242	1210	c	1125.59	4.95	64	0.16	15.36
243	1195	c	1125.59	7.55	64	0.16	15.68
244	670	p	1125.59	0.30	64	0.16	48.58
245	1295	p	1125.59	173.35	64	0.16	23.71
246	775	p	1125.59	0.98	64	0.16	41.90
247	1060	c	1125.59	80.95	64	0.16	21.70
248	940	c	1125.59	187.15	64	0.16	23.41
249	650	p	1125.59	0.25	64	0.16	50.18
250	1145	p	1125.59	48.70	64	0.16	20.26
251	640	p	1125.59	0.25	64	0.16	51.49
252	600	p	1125.59	0.12	64	0.16	53.11
253	1145	p	1125.59	48.70	64	0.16	20.26
254	1075	c	1125.59	69.45	64	0.16	21.13
255	725	p	1125.59	0.65	64	0.16	45.88
256	1225	c	1125.59	2.83	64	0.16	14.70
257	1225	c	1125.59	2.83	64	0.16	14.70
258	1175	p	1125.59	66.85	64	0.16	19.41
259	995	c	1125.59	136.45	64	0.16	23.97
260	1225	p	1125.59	106.25	64	0.16	19.23
261	1100	c	1125.59	51.55	64	0.16	20.01
262	1055	p	1125.59	18.95	64	0.16	25.26
263	1150	c	1125.59	22.15	64	0.16	17.26
264	1215	c	1125.59	4.00	64	0.16	14.98
265	1170	c	1125.59	14.80	64	0.16	16.77
266	950	c	1125.59	177.75	64	0.16	23.88
267	1310	p	1125.59	188.20	64	0.16	24.94
268	840	p	1125.59	2.00	64	0.16	38.05
269	1115	c	1125.59	42.35	64	0.16	19.59
270	1295	p	1125.59	173.35	64	0.16	23.71
271	1045	p	1125.59	17.00	64	0.16	25.80
272	1010	c	1125.59	123.15	64	0.16	23.64
273	1135	p	1125.59	44.05	64	0.16	20.84
274	1000	c	1125.59	131.80	64	0.16	23.66
275	1190	p	1125.59	77.25	64	0.16	18.96
276	1245	c	1125.59	1.25	64	0.16	14.06
277	820	p	1125.59	1.60	64	0.16	39.19

278	990	p	1125.59	9.55	64	0.16	29.04
279	1285	c	1125.59	0.30	64	0.16	14.03
280	600	p	1125.59	0.12	64	0.16	53.11
281	1020	p	1125.59	13.10	64	0.16	27.29
282	985	c	1125.59	145.25	64	0.16	23.82
283	1265	c	1125.59	0.50	64	0.16	13.56
284	1140	p	1125.59	46.70	64	0.16	20.75
285	1160	c	1125.59	18.75	64	0.16	17.31
286	935	c	1125.59	192.00	64	0.16	23.49
287	1235	p	1125.59	115.05	64	0.16	19.31
288	1400	p	1125.59	278.35	64	0.16	33.14
289	1060	p	1125.59	20.00	64	0.16	24.98
290	975	c	1125.59	154.55	64	0.16	24.15
291	940	p	1125.59	5.70	64	0.16	32.05
292	940	c	1125.59	187.15	64	0.16	23.41
293	1115	p	1125.59	35.60	64	0.16	21.82
294	1245	c	1125.59	1.25	64	0.16	14.06
295	1315	p	1125.59	193.65	64	0.16	26.04
296	1135	c	1125.59	30.50	64	0.16	18.47
297	1450	p	1125.59	328.40	64	0.16	37.27
298	800	p	1125.59	1.35	64	0.16	40.70
299	1215	c	1125.59	4.00	64	0.16	14.98
300	990	c	1125.59	140.95	64	0.16	24.06
301	1260	c	1125.59	0.65	64	0.16	13.73
302	1045	p	1125.59	17.00	64	0.16	25.80
303	1000	p	1125.59	10.80	64	0.16	28.63
304	840	p	1125.59	2.00	64	0.16	38.05
305	1500	p	1125.59	378.15	64	0.16	40.61
306	1265	p	1125.59	143.65	64	0.16	21.08
307	1280	c	1125.59	0.32	64	0.16	13.83
308	1055	p	1125.59	18.95	64	0.16	25.26
309	910	p	1125.59	4.20	64	0.16	33.89
310	630	p	1125.59	0.22	64	0.16	52.23
311	945	p	1125.59	6.00	64	0.16	31.74
312	1035	c	1125.59	101.55	64	0.16	22.78
313	1135	p	1125.59	44.05	64	0.16	20.84
314	1295	c	1125.59	0.35	64	0.16	15.05
315	920	p	1125.59	4.75	64	0.16	33.44
316	1070	c	1125.59	73.40	64	0.16	21.44
317	1120	p	1125.59	37.40	64	0.16	21.49
318	1040	c	1125.59	97.15	64	0.16	22.43
319	600	p	1125.59	0.12	64	0.16	53.11
320	1270	p	1125.59	148.55	64	0.16	21.47
321	1240	c	1125.59	1.60	64	0.16	14.30
322	1250	c	1125.59	1.15	64	0.16	14.28
323	1265	c	1125.59	0.50	64	0.16	13.56
324	860	p	1125.59	2.45	64	0.16	36.78
325	1225	p	1125.59	106.25	64	0.16	19.23
326	1095	c	1125.59	55.25	64	0.16	20.40

327	1065	c	1125.59	76.95	64	0.16	21.45
328	1200	p	1125.59	85.20	64	0.16	19.03
329	1185	p	1125.59	73.45	64	0.16	18.96
330	940	c	1125.59	187.15	64	0.16	23.41
331	1090	p	1125.59	27.15	64	0.16	23.10
332	1205	p	1125.59	88.75	64	0.16	18.67
333	1500	p	1125.59	378.15	64	0.16	40.61
334	1135	p	1125.59	44.05	64	0.16	20.84
335	990	c	1125.59	140.95	64	0.16	24.06
336	875	p	1125.59	2.95	64	0.16	36.08
337	1230	p	1125.59	110.70	64	0.16	19.34
338	1160	c	1125.59	18.75	64	0.16	17.31
339	890	p	1125.59	3.42	64	0.16	35.12
340	940	p	1125.59	5.70	64	0.16	32.05
341	935	p	1125.59	5.45	64	0.16	32.40
342	730	p	1125.59	0.60	64	0.16	44.77
343	950	p	1125.59	6.40	64	0.16	31.55
344	1180	p	1125.59	70.25	64	0.16	19.28
345	940	p	1125.59	5.70	64	0.16	32.05
346	650	p	1125.59	0.25	64	0.16	50.18
347	920	c	1125.59	206.10	64	0.16	20.29
348	1315	p	1125.59	193.65	64	0.16	26.04
349	965	c	1125.59	163.85	64	0.16	24.29
350	860	p	1125.59	2.45	64	0.16	36.78
351	1080	c	1125.59	66.10	64	0.16	21.14
352	1290	c	1125.59	0.28	64	0.16	14.22
353	950	c	1125.59	177.75	64	0.16	23.88
354	1275	p	1125.59	153.55	64	0.16	21.98
355	1195	c	1125.59	7.55	64	0.16	15.68
356	995	p	1125.59	10.05	64	0.16	28.73
357	1180	p	1125.59	70.25	64	0.16	19.28
358	1110	p	1125.59	33.80	64	0.16	22.11
359	1115	p	1125.59	35.60	64	0.16	21.82
360	1315	p	1125.59	193.65	64	0.16	26.04
361	925	p	1125.59	4.90	64	0.16	32.98
362	650	p	1125.59	0.25	64	0.16	50.18
363	1085	c	1125.59	62.15	64	0.16	20.74
364	770	p	1125.59	0.92	64	0.16	42.22
365	955	p	1125.59	6.60	64	0.16	31.07
366	810	p	1125.59	1.43	64	0.16	39.75
367	1285	p	1125.59	163.65	64	0.16	23.12
368	780	p	1125.59	1.02	64	0.16	41.56
369	1145	p	1125.59	48.70	64	0.16	20.26
370	1000	p	1125.59	10.80	64	0.16	28.63
371	680	p	1125.59	0.30	64	0.16	47.31
372	670	p	1125.59	0.30	64	0.16	48.58
373	1235	c	1125.59	1.90	64	0.16	14.35
374	710	p	1125.59	0.55	64	0.16	46.84
375	1210	p	1125.59	93.45	64	0.16	19.12

376	630	p	1125.59	0.22	64	0.16	52.23
377	995	c	1125.59	136.45	64	0.16	23.97
378	1040	c	1125.59	97.15	64	0.16	22.43
379	1125	c	1125.59	35.05	64	0.16	18.41
380	1040	c	1125.59	97.15	64	0.16	22.43
381	1180	p	1125.59	70.25	64	0.16	19.28
382	1085	p	1125.59	25.85	64	0.16	23.44
383	1050	p	1125.59	18.00	64	0.16	25.56
384	1005	c	1125.59	127.55	64	0.16	23.76
385	1110	c	1125.59	45.30	64	0.16	19.72
386	955	c	1125.59	172.95	64	0.16	23.74
387	880	p	1125.59	3.00	64	0.16	35.52

B Nifty options: A sample and replicate

We present a sample of Nifty options and one bootstrap replicate for the sample, from 14:59:45 to 15:000:00 on 2010-09-01. The sample consists of options for a range of strikes, and two nearest maturities. The example confidence intervals presented in Section 5 are estimated for this sample of options by generating thousand bootstrap replicates of this sample. A single such replicate is presented in the subsection of the Appendix.

B.1 Sample of Nifty options

	Strike	Type	Underlying	Mid-Quote	Maturity	Risk-free	IVol
1	6100	c	5464.75	1.25	29	6.29	16.76
2	6000	c	5464.75	1.85	29	6.29	15.33
3	5900	c	5464.75	2.20	29	6.29	13.22
4	5800	c	5464.75	4.53	29	6.29	12.14
5	5500	c	5464.75	72.57	29	6.29	12.43
6	5700	c	5464.75	9.82	29	6.29	11.03
7	5600	c	5464.75	30.55	29	6.29	11.57
8	5400	c	5464.75	133.95	29	6.29	13.11
9	5300	c	5464.75	208.50	29	6.29	13.01
10	4400	p	5464.75	2.12	29	6.29	35.15
11	4300	p	5464.75	2.05	29	6.29	38.17
12	4600	p	5464.75	2.67	29	6.29	29.86
13	4500	p	5464.75	2.12	29	6.29	32.00
14	4000	p	5464.75	1.67	29	6.29	46.87
15	4700	p	5464.75	3.05	29	6.29	27.24
16	5000	p	5464.75	9.88	29	6.29	22.37
17	4900	p	5464.75	5.18	29	6.29	22.91
18	6100	p	5464.75	627.58	29	6.29	29.20

19	5600	p	5464.75	160.65	29	6.29	15.75
20	6000	p	5464.75	526.62	29	6.29	25.39
21	5700	p	5464.75	240.82	29	6.29	17.11
22	5900	p	5464.75	426.85	29	6.29	21.83
23	5800	p	5464.75	330.77	29	6.29	19.07
24	5200	p	5464.75	28.85	29	6.29	20.63
25	5400	p	5464.75	68.75	29	6.29	17.80
26	5100	p	5464.75	17.52	29	6.29	21.65
27	5500	p	5464.75	105.30	29	6.29	16.48
28	5300	p	5464.75	45.27	29	6.29	19.35
29	4800	p	5464.75	3.38	29	6.29	24.45
30	6100	c	5464.75	3.27	57	6.95	13.18
31	6000	c	5464.75	4.57	57	6.95	12.06
32	5900	c	5464.75	9.38	57	6.95	11.78
33	5800	c	5464.75	18.95	57	6.95	11.60
34	5700	c	5464.75	40.12	57	6.95	11.99
35	5500	c	5464.75	125.58	57	6.95	13.16
36	5400	c	5464.75	186.50	57	6.95	13.46
37	5600	c	5464.75	75.62	57	6.95	12.56
38	5200	c	5464.75	337.20	57	6.95	13.08
39	5300	c	5464.75	260.00	57	6.95	13.87
40	4900	p	5464.75	29.25	57	6.95	25.40
41	5200	p	5464.75	75.22	57	6.95	23.35
42	5300	p	5464.75	95.10	57	6.95	21.97
43	5400	p	5464.75	120.35	57	6.95	20.55
44	5500	p	5464.75	157.75	57	6.95	19.73
45	5000	p	5464.75	42.12	57	6.95	25.03
46	4800	p	5464.75	20.12	57	6.95	25.86
47	5100	p	5464.75	57.15	57	6.95	24.29
48	5700	p	5464.75	266.45	57	6.95	18.89
49	5900	p	5464.75	428.80	57	6.95	21.03
50	6100	p	5464.75	621.80	57	6.95	25.98
51	5800	p	5464.75	342.58	57	6.95	19.62
52	6000	p	5464.75	524.80	57	6.95	23.53
53	5600	p	5464.75	204.60	57	6.95	18.97

B.2 A single bootstrap replicate

	Strike	Type	Underlying	Mid-Quote	Maturity	Risk-free	IVol
1	4300	p	5464.75	2.05	0.08	6.29	38.17
2	5900	c	5464.75	2.20	0.08	6.29	13.22
3	5700	p	5464.75	240.82	0.08	6.29	17.11
4	5000	p	5464.75	9.88	0.08	6.29	22.37
5	5100	p	5464.75	17.52	0.08	6.29	21.65
6	6000	p	5464.75	526.62	0.08	6.29	25.39
7	6100	p	5464.75	627.58	0.08	6.29	29.20
8	5500	c	5464.75	72.57	0.08	6.29	12.43

9	5500	c	5464.75	72.57	0.08	6.29	12.43
10	5300	c	5464.75	208.50	0.08	6.29	13.01
11	5400	p	5464.75	68.75	0.08	6.29	17.80
12	5400	c	5464.75	133.95	0.08	6.29	13.11
13	4700	p	5464.75	3.05	0.08	6.29	27.24
14	4800	p	5464.75	3.38	0.08	6.29	24.45
15	5400	p	5464.75	68.75	0.08	6.29	17.80
16	5300	p	5464.75	45.27	0.08	6.29	19.35
17	5400	p	5464.75	68.75	0.08	6.29	17.80
18	5900	c	5464.75	2.20	0.08	6.29	13.22
19	5600	p	5464.75	160.65	0.08	6.29	15.75
20	5500	c	5464.75	72.57	0.08	6.29	12.43
21	6000	c	5464.75	1.85	0.08	6.29	15.33
22	5600	c	5464.75	30.55	0.08	6.29	11.57
23	5400	p	5464.75	68.75	0.08	6.29	17.80
24	4800	p	5464.75	3.38	0.08	6.29	24.45
25	4000	p	5464.75	1.67	0.08	6.29	46.87
26	4300	p	5464.75	2.05	0.08	6.29	38.17
27	6100	p	5464.75	627.58	0.08	6.29	29.20
28	5500	c	5464.75	72.57	0.08	6.29	12.43
29	5300	c	5464.75	208.50	0.08	6.29	13.01
30	6100	c	5464.75	3.27	0.16	6.95	13.18
31	4900	p	5464.75	29.25	0.16	6.95	25.40
32	5900	c	5464.75	9.38	0.16	6.95	11.78
33	5900	c	5464.75	9.38	0.16	6.95	11.78
34	5300	p	5464.75	95.10	0.16	6.95	21.97
35	6100	c	5464.75	3.27	0.16	6.95	13.18
36	5200	c	5464.75	337.20	0.16	6.95	13.08
37	6000	p	5464.75	524.80	0.16	6.95	23.53
38	5900	p	5464.75	428.80	0.16	6.95	21.03
39	6100	c	5464.75	3.27	0.16	6.95	13.18
40	5400	c	5464.75	186.50	0.16	6.95	13.46
41	5100	p	5464.75	57.15	0.16	6.95	24.29
42	4900	p	5464.75	29.25	0.16	6.95	25.40
43	5500	p	5464.75	157.75	0.16	6.95	19.73
44	5400	c	5464.75	186.50	0.16	6.95	13.46
45	5700	c	5464.75	40.12	0.16	6.95	11.99
46	5300	c	5464.75	260.00	0.16	6.95	13.87
47	5800	p	5464.75	342.58	0.16	6.95	19.62
48	5900	p	5464.75	428.80	0.16	6.95	21.03
49	5500	p	5464.75	157.75	0.16	6.95	19.73
50	5300	p	5464.75	95.10	0.16	6.95	21.97
51	4900	p	5464.75	29.25	0.16	6.95	25.40
52	5000	p	5464.75	42.12	0.16	6.95	25.03
53	6100	p	5464.75	621.80	0.16	6.95	25.98