The Determinants of Liquidity Risk *

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Abstract

This paper examines the large liquidity risk premium documented in Acharya and Pedersen (2005). Using a standard return decomposition, I show that the liquidity premium has two components: covariation of liquidity costs with (i) market risk premium shocks and (ii) macroeconomic shocks. In 1964–2017 US stock market data, both components are priced but the expected return premium associated with the latter is approximately three times larger than that for the former. Liquidity volatility is primarily incorporated in stock prices via its common variation with macroeconomic shocks.

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1 Introduction

Liquidity matters. Many studies show that a lack of liquidity affects asset prices not only as a direct cost (Amihud and Mendelson, 1986; Brennan and Subrahmanyam, 1996; Jones, 2002) but also as a systematic risk factor (Pástor and Stambaugh, 2003; Acharya and Pedersen, 2005; Amihud, 2014). Amihud et al. (2006) argue that liquidity costs and risks have implications for the allocation of economy's real resources as they affect firms' costs of capital. Nevertheless, relatively little is known about the precise links between liquidity risk and asset prices.

Acharya and Pedersen (2005) derive a liquidity-adjusted capital asset pricing model (LCAPM hereafter) in which systematic liquidity risks determine asset prices. In addition to the standard beta, three liquidity betas also appear in the LCAPM: (i) β_2 – the covariation between firm liquidity costs (c_i) and market liquidity costs (c_M), (ii) β_3 – the (negative) covariation between stock returns (r_i) and market liquidity costs (c_M), and (iii) β_4 – the (negative) covariation between liquidity costs and market returns (r_M). Using 1964-99 US stock market data, Acharya and Pedersen estimate that the market price associated with these risks is 0.16%, 0.08%, and 0.82% respectively, from which they conclude that β_4 is the most important source of liquidity risk.

Somewhat surprisingly however, this result has received little attention in the literature. In this paper, I therefore examine more closely the nature and source of β_4 risk. First, I make use of the Campbell and Shiller's (1988) decomposition to show that β_4 can be written as the sum of two sub-betas, representing the covariation of liquidity costs with macroeconomic shocks (shocks to interest rates and aggregate expected dividends) and financial shocks (shocks to the market risk premium) respectively. Second, aided by the Merton's (1980) risk-return relationship, I estimate the market price of each of these sub-betas using 1963–2017 US stock market data. I find that the return premium associated with the macroeconomic shock beta (0.84%) is three times as large as that associated with the financial shock beta (0.29%). Investor concerns about illiquidity apparently relate more to its covariation with adverse macroeconomic shocks than with shocks to risk premia. By investigating relative importance of cash-flow and risk premium shocks for firm-level stock returns, Vuolteenaho (2002) document that firm-level stock returns are predominantly determined by information about future cash-flow.

The difference in return premium associated with the macroeconomic shock beta and the financial shock beta can be partly explained by the nature of systematic shocks that stock illiquidity covaries with. Adverse financial shocks and macroeconomic shocks reduce value of the market portfolio (and hence investor's wealth), but only financial shocks improve future investment opportunities (Campbell and Vuolteenaho, 2004). Therefore, investor is more sensitive to macroeconomic shocks, and requires greater return premium for stocks whose liquidity costs strongly covary with macroeconomic shocks.

I also find that both macroeconomic and financial shock betas cross-sectionally differ by firm size (the market capitalization of equity) and firm illiquidity. Both the negative macroeconomic shock beta and the financial shock beta decrease (increase) as a function of firm size (firm illiquidity). This finding can be understood as follows. Adverse macroeconomic shocks and/or financial shocks reduce the demand for risky assets (flight to quality), making small and illiquid stocks even more illiquid (illiquidity spiral). Therefore, liquidity costs of small and/or illiquid firms show greater covariation with macroeconomic shocks and financial shocks.

In the next section, I outline the underlying theoretical relationships linking liquidity and expected returns. Section 3 then presents the estimation results, while section 4 considers some robustness issues. Finally, Section 5 contains some concluding remarks.

2 The Determinants of Liquidity Risk

In this section, I first describe how liquidity betas affect asset prices in the Acharya and Pedersen's LCAPM. Then, applying the Campbell and Shiller's return decomposition to the market returns in β_4, I show that β_4 has two sub-betas: the macroeconomic shock beta and financial shock beta.

2.1 Liquidity-Adjusted CAPM (LCAPM)

Acharya and Pedersen (2005) derive the LCAPM from an overlapping generations model, in which investor maximizes his expected utility over time by allocating its endowment into a portfolio of assets. More importantly, investor is required to pay transaction costs when selling (liquidating) risky assets. Hence, investor cares about net returns (e.g., after transaction costs returns). The sale of riskless assets does not incur transaction costs and short-selling of risky assets is not allowed. Under these conditions, Acharya and Pedersen derive the LCAPM in equation (1).

$$
\begin{aligned}
E\left[r_{i,t} - r_{f,t}\right] &= E\left[c_{i,t}\right] + \beta_i E\left[r_{M,t} - c_{M,t} - r_{f,t}\right] \\
&= E\left[c_{i,t}\right] + \frac{\mathrm{Cov}(r_{i,t} - c_{i,t}, r_{M,t} - c_{M,t})}{\mathrm{Var}(r_{M,t} - c_{M,t})} E\left[r_{M,t} - c_{M,t} - r_{f,t}\right] \\
&= E\left[c_{i,t}\right] + \lambda\Bigg(\beta_{1,i} + \beta_{2,i} - \beta_{3,i} - \beta_{4,i}\Bigg) \qquad (1)
\end{aligned}
$$

where

$$
\begin{aligned}
&\lambda = E\left[\lambda_t\right] = E\left[r_{M,t} - c_{M,t} - r_{f,t}\right] \\
&\beta_{1,i} = \frac{\mathrm{Cov}\left(r_{i,t}, r_{M,t} - E_{t-1}\left[r_{M,t}\right]\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\
&\beta_{2,i} = \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], c_{M,t} - E_{t-1}\left[c_{M,t}\right]\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\
&\beta_{3,i} = \frac{\mathrm{Cov}\left(r_{i,t}, c_{M,t} - E_{t-1}\left[c_{M,t}\right]\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\
&\beta_{4,i} = \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], r_{M,t} - E_{t-1}\left[r_{M,t}\right]\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)}
\end{aligned}
$$

In the LCAPM in addition to the market beta ($\beta_{1,i}$), asset prices are determined by expected liquidity costs and three liquidity betas ($\beta_{2,i}$, $\beta_{3,i}$, and $\beta_{4,i}$). Equation (1) is the unconditional LCAPM where risk premium is expected to be constant. r_i and r_M are firm i and the market returns respectively. r_f is the riskless interest rate. $c_{i,t}$ and $c_{M,t}$ are relative liquidity costs, liquidity costs per dollar invested, for stock i and the market respectively. λ is the market risk premium.

$\beta_{2,i}$ is liquidity commonality beta, the covariation between stock i's liquidity costs and market liquidity costs. It implies that investor requires greater compensation for a stock that becomes particularly illiquid during the times when the market as a whole is illiquid (Chordia et al., 2000; Hasbrouck and Seppi, 2001; Huberman and Halka, 2001; Coughenour and Saad, 2004).

$\beta_{3,i}$ is the covariation between stock i returns and market liquidity costs. Stocks with high $\beta_{3,i}$ provides a hedge against a drop in market-wide liquidity, and therefore, investor is willing to pay premium for stocks with greater exposures to $\beta_{3,i}$.

Finally, $\beta_{4,i}$, the main interest of the present paper, is the covariation between stock liquidity costs and the market returns. Using US 1964-99 US stock market data, Acharya and Pedersen document that return premium for $\beta_{4,i}$ is much greater than that for other liquidity betas: the return premium associated with $\beta_{4,i}$ (0.82%) is about 5 and 10 times greater than that for $\beta_{2,i}$ and $\beta_{3,i}$ respectively. The intuition for $\beta_{4,i}$ is that investor requires return premium for a stock if the stock becomes particularly illiquid during the times when investor needs the most (economic downturns).

Given its strong economic significance, I examine $\beta_{4,i}$ more closely. In the next section, I show that $\beta_{4,i}$ can be decomposed into two sub-betas and the large return premium associated with $\beta_{4,i}$ is the combined pricing effect of the two sub-betas.

2.2 The Determinants of β_4

Campbell and Vuolteenaho (2004) use the return decomposition approximation of Campbell and Shiller and show that the standard market beta can be decomposed into two sub-betas: the cash-flow beta and the discount-rate beta. In this section, applying the market return decomposition of Campbell and Shiller to $\beta_{4,i}$, I show that the $\beta_{4,i}$ can be decomposed into the macroeconomic shock beta and the financial shock beta.

Equation (2) presents the stock market return decomposition.

$$\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right] \approx \eta_{Mac,t} - \eta_{\hat{\pi},t} \tag{2}$$

where

$$\eta_{Mac,t} = \Delta E_t\left[\sum_{j=0}^{\infty}\rho^j \Delta\hat{d}_{t+j}\right] - \Delta E_t\left[\sum_{j=1}^{\infty}\rho^j \hat{r}_{f,t+j}\right] \quad \text{and}$$
$$\eta_{\hat{\pi},t} = \Delta E_t\left[\sum_{j=1}^{\infty}\rho^j \hat{\pi}_{t+j}\right]$$

ΔE_t denotes the change in expectations from $t-1$ to t. $\hat{r}$ and $\hat{d}$ are logged stock returns and dividends respectively. $\hat{\pi}$ is logged stock market excess returns. ρ is average ratio of the stock price to the sum of the stock price and the dividend. Equation (2) implies that unexpected logged stock market returns, $\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$, are due to either macroeconomic shocks ($\eta_{Mac,t}$, shocks to interest rates and aggregate expected dividends), financial shocks ($\eta_{\hat{\pi},t}$, shocks to the market risk premium) or combination of the two.

Before I make use of the return decomposition in equation (2), I assume that r_M (stock market returns) is approximately equals to $\hat{r}_M = ln(r_M + 1)$ and that stock market returns are log-normally distributed (e.g., $ln\left(E[r_M + 1]\right) = E\left[\hat{r}_M\right] + 0.5\sigma^2\left(\hat{r}_M\right)$). These assumptions are needed because returns in the Campbell and Shiller's decomposition are logged ($\hat{r}_{M,t}$) whereas the market returns in $\beta_{4,i}$ are simple returns ($r_{M,t}$). These assumptions are also acceptable for monthly market returns. Combining these two assumptions yields:

$$r_{M,t} - E_{t-1}[r_{M,t}] \approx \eta_{Mac,t} - \eta_{\hat{\pi},t} - \frac{1}{2}\sigma^2_{t-1}(\hat{r}_M)$$

Then, by substituting the return decomposition approximation above into β_4, I now show that β_4 can be written as the sum of two sub-betas, the macroeconomic shock beta

$(\beta_{4a,i})$ and the financial shock beta $(\beta_{4b,i})$.

$$\begin{aligned}\beta_{4,i} &\approx \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], \eta_{Mac,t} - \eta_{\hat{\pi},t} - \frac{1}{2}\sigma^2_{t-1}(\hat{r}_M)\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\ &= \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], \eta_{Mac,t} - \eta_{\hat{\pi},t}\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\ &= \beta_{4a,i} + \beta_{4b,i} \end{aligned} \tag{3}$$

where

$$\begin{aligned}\beta_{4a,i} &= \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], \eta_{Mac,t}\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)} \\ \beta_{4b,i} &= \frac{\mathrm{Cov}\left(c_{i,t} - E_{t-1}\left[c_{i,t}\right], -\eta_{\hat{\pi},t}\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)}\end{aligned}$$

The economic intuition behind $\beta_{4a,i}$ is that when adverse macroeconomic shocks reduce investor's wealth, even a small increase in liquidity costs will exert a larger impact on investor's marginal utility. Hence, investor would require much greater premium for stocks whose liquidity costs are strongly (negatively) correlated to macroeconomic shocks. Furthermore, pricing effect of $\beta_{4a,i}$ suggests that macroeconomic shocks affect asset prices not only through its co-variation with stock returns (Campbell and Vuolteenaho, 2004) but also through its co-variation with stock liquidity costs, shedding light on the role of macroeconomic shocks in the asset pricing literature.

The financial shock beta, $\beta_{4b,i}$, implies that stocks whose illiquidity costs are negatively correlated with financial shocks should provide premium. An increase in the expected market return premium (increase in discount rates) implies immediate price reductions and a simultaneous increase in liquidity costs will accelerate the price reduction process. Therefore, stocks whose liquidity costs are strongly correlated with financial shocks should have greater expected returns.

In order to examine how the macroeconomic shock beta and the financial beta affect asset prices, in the next section using 1963-2017 US stock market data I estimate $\beta_{4a,i}$ and $\beta_{4b,i}$ and their pricing effects.

3 Estimation

In this section, using 1963–2017 US stock market data (common stocks listed on the New York Stock Exchange, and American Stock Exchange), I separately estimate the macroeconomic shock beta and financial shock beta as well as their pricing effects. In preparation for estimation of these two sub-betas, I first construct 25 illiquidity portfolios and the market portfolio to use as test assets (Section 3.1). Then, I estimate market and portfolio liquidity costs shocks (Section 3.2), and macroeconomic shocks and financial shocks (Section 3.3). Lastly, using the estimated series of portfolio liquidity cost shocks, and macroeconomic and financial shocks, I present estimation results for the two sub-betas in Section 3.4 and their pricing effects in Section 3.5.

3.1 Portfolio Formation

I construct a market portfolio as well as 25 illiquidity portfolios to use as test assets. In the beginning of each month t, I form a market portfolio. For the market portfolio, I exclude stocks with prices less than \$5. Harris (1994) shows that liquidity and trading volume of low-price stocks are affected by changes in market micro-structures, such as changes in tick-size. Angel (1997) argues that stocks with a large relative tick size provides an incentive for liquidity providers to trade those stocks more frequently which in turn increases the pool of investors who know about the companies. Therefore, in order to minimize micro-structure effect captured by low-price stocks, I exclude penny stocks.[1] I also exclude stocks that do not have more than 15 days of return and volume data in month t. I form 25 illiquidity portfolios for year y by sorting stocks by $y-1$ average illiquidity. I exclude stocks whose beginning of the year, y, prices less than \$5 and stocks that do not have more than 100 days of return and volume data in year $y-1$.

The model requires value-weighted returns and value-weighted illiquidity for the market portfolio. However, many prior liquidity studies focus on equal-weighted return and illiquidity measures (Chordia et al., 2000; Amihud, 2002; Acharya and Pedersen, 2005;

[1] According to the U.S. Securities and Exchange Commission (SEC), *penny stock* refers to a security whose trading price is less than five dollars per share (`https://www.sec.gov/fast-answers/answerspennyhtm.html`).

Lee, 2011; Acharya et al., 2013; Vu et al., 2015). Constructing the equal-weighted market return and market illiquidity is better dealing with over-representation of large and liquid stocks in my sample especially when my sample does not include illiquid assets (i.e., corporate bonds, real estate and may small stocks) that constitute a significant fraction of aggregate wealth (Acharya and Pedersen, 2005).[2] Therefore, I focus on equal-weighted market returns and equal-weighted market illiquidity in this section. Using equal-weighted market returns and illiquidity also makes it easier to compare my estimation results with prior studies that examine the LCAPM. However, as a robustness check, I also estimate the model with value-weighted market returns and value-weighted market illiquidity.

3.2 Measuring Illiquidity Shocks

Liquidity is not an observable variable, but fortunately, several proxies are available. In the present paper, I use Amihud's illiquidity measure. Amihud and Noh (2018) document that unlike volume based measures, Amihud illiquidity correctly depicts several liquidity crises (e.g., the October 1987 stock market crash and the great financial crisis of 2007-2009). Moreover, Amihud illiquidity measures overall costs of selling stocks while bid-ask spread based measures only measure cost of selling small number of stocks.

The monthly Amihud illiquidity for stock i is defined as:

$$ILLIQ_{i,t} = \frac{1}{Days_{i,t}} \sum_{d=1}^{Days_{i,t}} \frac{|r_{i,t,d}|}{V_{i,t,d}} \tag{4}$$

$r_{i,t,d}$ and $V_{i,t,d}$ are stock i returns and dollar trading volume (in million \$) on day d in month t. $Days_{i,t}$ is the number of valid trading days in month t for stock i. The absolute return and volume ratio in equation (4) measures the relative size of price reactions to trading volume on day d. A stock with greater price reactions (high $|r_{i,t,d}|$) for a given trading size ($V_{i,t,d}$) will have higher $ILLIQ_{i,t}$. Therefore, $ILLIQ_{i,t}$ measures average monthly *illiquidity* (rather than liquidity) for stock i.

[2] Less than 15 big stocks account for more than 20% of total market equity capitalization in the sample market portfolio at the end of 2017.

The illiquidity proxy in equation (4) has two issues. First, $ILLIQ_{i,t}$ measures average price reactions (in %) relative to trading volumes (in \$), and therefore, $ILLIQ_{i,t}$ is not stationary. Second, $ILLIQ_{i,t}$ measures price impact whereas the liquidity costs in equation (1) is 'dollar costs per dollar invested'. Hence, I use normalized illiquidity, $c_{i,t+1}$, in equation (5), instead of $ILLIQ_{i,t}$.

$$c_{i,t} = \min\left\{0.25 + 0.30 ILLIQ_{i,t} P_{M,t-1}, 30.00\right\} \tag{5}$$

The normalization process in equation (5) is proposed by Acharya and Pedersen and it solves the two problems mentioned above. $P_{M,t-1}$ is the ratio of market equity capitalizations at the end of $t-1$ and market equity capitalizations at the end of July 1962. This adjustment makes illiquidity series stationary. The coefficients 0.25 and 0.30 are chosen to match the cross-sectional distribution of $c_{i,t}$ to the distribution of the effective half spread. Lastly, in order to avoid estimated liquidity costs being biased by extremely illiquid stocks, normalized illiquidity is capped at 30%.

Portfolio illiquidity is a weighted sum of stock i's illiquidity costs and it is defined as:

$$c_{p,t} = \sum_{i \in p} w_{i,t} c_{i,t} \tag{6}$$

where $c_{p,t}$ is a weighted illiquidity costs for a portfolio p, and $w_{i,t}$ is either equal or value-based weights for stock i in portfolio p. If a portfolio p includes all sample stocks, $c_{p,t}$ becomes market weighted illiquidity costs.

Lastly, I estimate portfolio p illiquidity innovations, $c_{p,t} - E_{t-1}\left[c_{p,t}\right]$. The estimation of the macroeconomic shock beta and the financial shock beta in equation (3) requires illiquidity innovations rather than illiquidity. More importantly, I find that the first order autocorrelation in the market illiquidity over the sample period is near 0.90 at monthly frequency, and therefore, using illiquidity innovations also resolves issues related to non-stationarity. Similar to Acharya and Pedersen, I estimate illiquidity innovations by AR(3) from equation (7).

$$
\begin{aligned}
\left(0.25 + 0.30\overline{ILLIQ}_{p,t}P_{M,t-1}\right) &= \alpha_0 + \alpha_1\left(0.25 + 0.30\overline{ILLIQ}_{p,t-1}P_{M,t-1}\right) \\
&\quad + \alpha_2\left(0.25 + 0.30\overline{ILLIQ}_{p,t-2}P_{M,t-1}\right) \\
&\quad + \alpha_3\left(0.25 + 0.30\overline{ILLIQ}_{p,t-3}P_{M,t-1}\right) + u_{p,t} \qquad (7) \\
\text{where} \quad \overline{ILLIQ}_{p,t} &= \sum_{i\in p} w_{i,t}\min\left\{ILLIQ_{i,t}, \frac{30.00-0.25}{0.30P_{M,t-1}}\right\}
\end{aligned}
$$

I consider the residuals, $u_{p,t}$, in equation (7) as portfolio p illiquidity cost shocks. The equation (7) is different from the standard AR(3) model. Changes in $c_{p,t}$, the normalized portfolio illiquidity costs, arise when there is a change in portfolio illiquidity ($ILLIQ_{p,t}$), a change in market equity capitalization ($P_{M,t-1}$), or a combination of the two. Therefore, residuals from the standard AR(3) would capture both illiquidity shocks as well as unanticipated changes in market equity capitalizations. In order to capture portfolio liquidity shocks only, in equation (7), I use the same market index, $P_{M,t-1}$, for all liquidity costs terms ($c_{p,t}$, $c_{p,t-1}$, $c_{p,t-2}$ and $c_{p,t-3}$).

The first order autocorrelations and standard deviation of standardized market liquidity costs innovations ($u_{p,t}/\sigma(u_{p,t})$) estimated by equation (7) is 0.03 and 0.14% respectively.[3]

In addition to liquidity costs shock, I also need macroeconomic shocks ($\eta_{Mac,t}$) and financial shocks ($-\eta_{\hat{\pi},t}$) to estimate the macroeconomic shock beta and the financial shock beta. In the next section, I describe how I estimate these systematic shocks.

3.3 Measuring Market Risk and Macroeconomic Shocks

In this section, I estimate macroeconomic shocks and financial shocks. I do this in two steps. I first present the regression model derived from applying Merton's (1980) theoretical relationship between the market risk premium and market return variance (market risk) to the return decomposition in equation (2). Then, I estimate the corresponding

[3]Acharya and Pedersen (2005) find that the standard deviation of market illiquidity innovation is 0.17% and the first order autocorrelation is -0.03 for 1964-1999 period.

shocks from the regression model.

Merton (1980) shows that if investors have homogeneous beliefs about expected returns and if the investment opportunity set is constant (e.g., uncorrelated with news about future investment opportunities), expected market risk premium is linearly related to its conditional variance.

$$E_t\left[\pi_{t+1}\right] = \gamma\sigma_t^2(\pi) \tag{8}$$

$$\text{where} \quad \pi_{t+1} = r_{M,t+1} - r_f$$

Equation (8) implies that the market risk premium is proportional to the market risk premium variance, $\sigma_t^2(\pi)$. γ is interpreted as relative risk aversion of representative investor. Other studies have shown similar risk-return relationships. Assuming quadratic utility or normally distributed returns, Huang and Litzenberger (1988) show that market risk premium is a product of aggregate relative risk aversion and market return variance. Incorporating risk-free asset market clearing condition, Boyle (2005) shows that market risk premium is a product of average risk aversion of all investors and market return variance.

Unfortunately, the systematic return-risk relationship in equation (8) cannot be directly applied to the return decomposition in equation (2). In order to make the use of the return-risk relationship, I assume that $\pi_{M,t+1} \approx \hat{\pi}_{M,t+1}$ where $\hat{\pi}_{M,t+1} = ln(\pi_{M,t+1} + 1)$ and that log-normally distributed market risk premium (e.g., $ln\left(E[\pi_M + 1]\right) = E[\hat{\pi}_M] + 0.5\sigma_t^2(\hat{\pi})$), and then, the equation (8) can be rewritten as:

$$E_t\left[\hat{\pi}_{t+1}\right] \approx \gamma\sigma_t^2(\pi) - \frac{1}{2}\sigma_t^2(\hat{\pi}) \approx \left(\gamma - \frac{1}{2}\right)\sigma_t^2(\hat{\pi})$$

The latter approximation is due to close relationship between logged market excess return variance and simple market excess return variance. I find that the correlation between $\sigma_t^2(\pi)$ and $\sigma_t^2(\hat{\pi})$ during the sample period is 0.998. In other words, market excess return variance and logged market excess return variance are pretty much identical in practice.

Then, assuming constant expectation of risk premium and applying the law of iterated expectations to the above expected logged market excess return expectation equation, I show that

$$\begin{aligned}
\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right] &= \eta_{Mac,t} - \left(\sum_{j=1}^{\infty} \rho^j \bar{E}_t\left[\hat{\pi}_{t+j}\right] - \sum_{j=1}^{\infty} \rho^j \bar{E}_{t-1}\left[\pi_{t+j}\right]\right) \\
&= \eta_{Mac,t} - \frac{\rho}{1-\rho}\left(\bar{E}_t\left[\hat{\pi}_{t+j}\right] - \bar{E}_{t-1}\left[\hat{\pi}_{t+j}\right]\right) \\
&= \eta_{Mac,t} + \phi\left(\sigma_t^2(\hat{\pi}) - E_{t-1}\left[\sigma_t^2(\hat{\pi})\right]\right) \qquad (9)
\end{aligned}$$

where $\phi = -\frac{\rho}{1-\rho}\left(\gamma - \frac{1}{2}\right)$, and $-\eta_{\hat{\pi},t} \approx \phi\left(\sigma_t^2(\hat{\pi}) - E_{t-1}\left[\sigma_t^2(\hat{\pi})\right]\right)$

Equation (9) suggests that both macroeconomic and financial shocks can be estimated by regressing unexpected changes in logged market excess returns on unexpected changes in logged market excess return variance. In this study, I consider the part of the unexpected logged market excess returns that are explained by unexpected changes in logged market excess return variance as financial shocks ($-\eta_{\hat{\pi},t}$), and the remaining components as macroeconomic shocks. As $0 < \rho < 1$, if $\gamma > \frac{1}{2}$, the relationship between unexpected logged market excess returns and variance, captured by $\phi = -\frac{\rho}{1-\rho}\left(\gamma - \frac{1}{2}\right)$, is negative. French et al. (1987) argue that the negative relationship between unexpected changes in market excess returns and market excess return variance is indirect evidence of a positive relationship between conditional market return and risk.[4]

However, the estimation of macroeconomic shocks and financial shocks in equation (9) requires an additional step – estimation of unexpected changes in logged market excess returns and variance. I estimate both variables from ARMA(m,n). m and n in ARMA(m,n) represent order of the autoregressive and the the moving average part respectively. Unexpected changes in logged market excess returns, $\epsilon_{\hat{r}_M,t}$, and unexpected changes in logged market excess return variance, $\epsilon_{\sigma^2(\hat{\pi}),t}$, are estimated by ARMA(m,n)

[4]When investors receive positive market risk shocks, due to autocorrelations in market risk, investors will make upward adjustments in their market risk forecasts. If market risk premium is positively related to ex-ante market risk, investors' discount rates for future cash flows will be revised upward. The increase in discount rates will bring the current stock prices down, and therefore, there should be negative contemporaneous relationship between market return and market risk shocks.

Table 1: ARMA (m,n) Estimation For Market Return and Market Risk

This table presents the ARMA estimation results for the logged market excess returns ($\hat{r}_{M,t}$) and variance ($\sigma_t^2(\hat{\pi})$). Sample period for the estimation is from January 1964 to December 2017 (648 months). Monthly logged equal-weighted market excess return is the logged average monthly stock returns for all stocks in the market portfolio minus the logged riskless interest rate. Monthly logged market excess return variance is computed from daily logged market excess returns for each month adjusted for numbers of trading days in the month. The first four columns present ARMA estimation results for the logged market excess returns and the latter four columns present ARMA estimation results for logged market excess return variance. Standard errors are presented in parentheses. AIC (Akaike information criterion) and BIC (Bayesian information criterion) measures estimation fitness adjusting for the numbers of estimated parameters. ACF(1) is first-order autocorrelation coefficient for estimated residuals. LB (6) and LB (12) are Ljung-Box test statistics with 6 lags (six month) and with 12 lags (one year), testing for independency in the estimated residuals from the ARMA model. P-value for Ljung-Box test are in square brackets.

	Market Return, $\hat{r}_{M,t}$				Market Excess Return Variance, $\sigma_t^2(\hat{\pi})$			
	ARMA (1,0)	ARMA (1,1)	ARMA (2,0)	ARMA (2,1)	ARMA (1,0)	ARMA (1,1)	ARMA (2,0)	ARMA (2,1)
Const.	0.011 (0.003)	0.011 (0.003)	0.011 (0.003)	0.012 (0.003)	0.003 (0.000)	0.003 (0.000)	0.003 (0.000)	0.003 (0.001)
ψ_1	0.165 (0.039)	-0.165 (0.214)	0.176 (0.039)	0.088 (0.538)	0.558 (0.033)	0.730 (0.049)	0.488 (0.039)	1.070 (0.228)
ψ_2			-0.064 (0.039)	-0.050 (0.099)			0.127 (0.039)	-0.205 (0.146)
θ_1		0.342 (0.204)		0.088 (0.538)		-0.256 (0.071)		-0.589 (0.216)
AIC	-1811.28	-1811.85	-1811.96	-1809.99	-5264.00	-5274.18	-5272.47	-5273.53
BIC	-1797.85	-1793.96	-1794.06	-1787.62	-5250.58	-5256.29	-5254.58	-5251.16
ACF(1)	0.011	-0.000	0.001	0.000	-0.070	0.007	-0.005	-0.003
LB(6)	5.861 [0.439]	3.436 [0.753]	3.305 [0.770]	3.290 [0.772]	16.177 [0.013]	4.419 [0.620]	5.806 [0.445]	3.125 [0.793]
LB(12)	10.526 [0.570]	8.011 [0.784]	7.814 [0.800]	7.824 [0.799]	17.851 [0.120]	4.723 [0.967]	6.393 [0.895]	3.442 [0.992]

equations in (10) and (11).

$$\hat{r}_{M,t} = \text{Constant.} + \epsilon_{\hat{r}_M,t} + \sum_{j=1}^{m} \psi_j \hat{r}_{M,t-j} + \sum_{k=1}^{n} \theta_k \epsilon_{\hat{r}_M,t-k} \tag{10}$$

$$\sigma_t^2(\hat{\pi}) = \text{Constant.} + \epsilon_{\sigma^2(\hat{\pi}),t} + \sum_{j=1}^{m} \psi_j \sigma_{t-j}^2(\hat{\pi}) + \sum_{k=1}^{n} \theta_k \epsilon_{\sigma^2(\hat{\pi}),t-k} \tag{11}$$

Table 1 presents ARMA estimation results for the logged market excess returns and variance for January 1964 - December 2017 (648 months). First of all, looking at AIC (Akaike information criterion), BIC (Bayesian information criterion), and ACF(1) (the first order autocorrelation), ARMA (2,0) seems the best specification for logged market excess returns and ARMA (1,1) for logged market excess return variance. The first order autocorrelation in the estimated unexpected changes in the logged market excess returns,

Table 2: Estimation of Macroeconomic and Financial shocks

The first two columns in this table present estimation results for equation (9) with and without constant. The left hand side variable is the unexpected logged equal-weighted market excess returns estimated from ARMA (2,0) and the right hand side variable is unexpected logged equal-weighted market excess return variance estimated from ARMA (1,1). The sample period for the estimated is Jan 1964 - Dec 2017 (648 months). Newey and West (1994) heteroskedasticity and autocorrelation consistent (HAC) standard errors are reported in parenthesis. The last two columns present descriptive statistics for the estimated macroeconomic shocks (residuals, $\eta_{Mac,t}$) and the estimated financial shocks (fitted values, $-\eta_{\hat{\pi},t} = \hat{\phi}\left(\sigma_t^2(\hat{\pi}) - E_{t-1}\left[\sigma_t^2(\hat{\pi})\right]\right)$). 'Std. Dev.' is standard deviation of the estimated macroeconomic shocks and financial shocks. 'Autocorr.' is the first-order autocorrelation. 'Corr. with $\epsilon_{\hat{r}_M,t}$' is the correlation between unexpected logged market excess returns ($\epsilon_{\hat{r}_M,t} = \hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$) with the estimated macroeconomic and financial shocks.

	(1)	(2)		$\eta_{Mac,t}$	$-\eta_{\hat{\pi},t}$
Const.	0.000		Std. Dev.	0.055	0.022
	(0.002)		Min	-0.229	-0.341
ϕ	-5.463	-5.463	Max	0.229	0.143
	(0.764)	(0.765)	Autocorr.	-0.087	0.007
adj. R^2	0.141	0.141	Corr. with $\epsilon_{r_M,t}$	0.926	0.378

$\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$, and market excess return variance, $\sigma_t^2(\hat{\pi}) - E_{t-1}\left[\sigma_t^2(\hat{\pi})\right]$, are near zero under these specifications. The last two rows report Ljung-Box test statistics and their p-values for the estimated unexpected changes in the logged market excess returns and variances with six lags (six month) and with twelve lags (one year). Ljung-Box test statistics are small and have p-value greater than 5%, indicating that the estimated $\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$ and $\sigma_t^2(\hat{\pi}) - E_{t-1}\left[\sigma_t^2(\hat{\pi})\right]$ are time-independent.

Using the estimated unexpected logged market excess returns and variance from ARMA models, I now estimate macroeconomic and financial shocks. Table 2 presents estimation results of the time series regression in equation (9). The left hand side variable is unexpected logged market excess returns estimated by ARMA(2,0) and the right hand side variable is unexpected logged market excess return variance estimated by ARMA(1,1). I estimate the regression equation (9) without a constant (model restriction) and with a constant as a free parameter.

I find that the estimated coefficients on unanticipated changes in logged market excess return variance ($\hat{\phi}$) is negative and statistically significant. The adjusted R^2 is 0.141, implying that only 14 percent of the variations in the unexpected changes in logged market returns are explained by unexpected changes in the variances. I consider the residuals in the restricted model (column (2) in Table 2) as macroeconomic shocks ($\eta_{Mac,t}$) and the fitted values as negative of financial shocks ($-\eta_{\hat{\pi},t}$).

The last two columns in Table 2 present summary statistics for the estimated macroeconomic and financial shocks. The first order autocorrelation in macroeconomic and financial shocks is low. The correlation between unexpected logged market excess returns, $\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$ and $\eta_{Mac,t}$ is 0.926 and the correlation between $\hat{r}_{M,t} - E_{t-1}\left[\hat{r}_{M,t}\right]$ and $-\eta_{\hat{\pi},t}$ is 0.378. This indicates that unexpected logged stock market excess returns are much more closely related to macroeconomic shocks, $\eta_{Mac,t}$ than with financial shocks, $-\eta_{\pi,t}$. With the estimated series of $\eta_{Mac,t}$ and $-\eta_{\pi,t}$, I now move onto the estimation of the macroeconomic shock beta ($\beta_{4,a}$) and financial shock beta ($\beta_{4,b}$).

3.4 Measuring Macroeconomic and Financial Shock Betas

In Section 2.2, I have shown that $\beta_{4,p}$, the covariation between firm liquidity costs and the market returns, has two components - the macroeconomic shock beta, $\beta_{4a,p}$, and the financial shock beta, $\beta_{4b,p}$. In this section, I measure the two sub-liquidity betas separately. The two sub-liquidity betas are:

$$\beta_{4a,p} = \frac{\mathrm{Cov}\left(c_{p,t} - E_{t-1}\left[c_{p,t}\right], \eta_{Mac,t}\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)}$$

$$\beta_{4b,p} = \frac{\mathrm{Cov}\left(c_{p,t} - E_{t-1}\left[c_{p,t}\right], -\eta_{\hat{\pi},t}\right)}{\mathrm{Var}\left(r_{M,t} - E_{t-1}[r_{M,t}] - (c_{M,t} - E_{t-1}[c_{M,t}])\right)}$$

And, the sum of the two sub-liquidity betas must be approximately equal to $\beta_{4,p}$.

$$\beta_{4,p} = \beta_{4a,p} + \beta_{4b,p}$$

Panel A in Table 3 presents $\beta_{4a,p}$ and $\beta_{4b,p}$ for 25 illiquidity portfolios for the sample period, January 1964 to December 2017 (648 months). I find that exposures to both betas substantially vary cross-sectionally. Relatively more illiquid portfolios have stronger liquidity costs covariation with both macroeconomic shocks and financial shocks than liquid portfolios. The cross-sectional difference can be partly explained by flight to liquidity effect. Adverse systematic shocks reduce the demand for illiquid stocks, making illiq-

Table 3: The macroeconomic shock and financial shock betas estimation

This table presents estimated macroeconomic shock betas ($100 \cdot \beta_{4a,p}$) and financial shock betas ($100 \cdot \beta_{4b,p}$) for 25 illiquidity sorted portfolios and size (market equity capitalization) portfolios. $t - statistics$, ratios of estimated betas and bootstrap standard error, are reported in parenthesis. Bootstrap standard errors are computed from 10,000 simulated realizations. Estimates are for the January 1964 - December 2017 period (648 months). To save space, this table presents $\beta_{4a,p}$ and $\beta_{4b,p}$ only for odd-numbered portfolios.

Portfolio	1	3	5	7	9	11	13	15	17	19	21	23	25
Panel A: Illiquidity Sorted Portfolios													
$100{\cdot}\beta_{4a,p}$	-0.00	-0.02	-0.04	-0.07	-0.10	-0.20	-0.30	-0.52	-0.71	-1.13	-1.99	-2.83	-5.87
	(-2.54)	(-7.08)	(-7.49)	(-7.47)	(-7.85)	(-7.60)	(-5.84)	(-7.15)	(-6.45)	(-6.76)	(-7.86)	(-6.45)	(-5.23)
$100{\cdot}\beta_{4b,p}$	-0.00	-0.00	-0.01	-0.01	-0.01	-0.03	-0.04	-0.05	-0.08	-0.14	-0.14	-0.39	-2.01
	(-1.51)	(-3.43)	(-3.28)	(-3.13)	(-3.60)	(-2.94)	(-3.38)	(-3.06)	(-2.80)	(-3.00)	(-1.47)	(-1.70)	(-2.31)
Panel B: Market Equity Capital (Size) Sorted Portfolios													
$100{\cdot}\beta_{4a,p}$	-0.00	-0.02	-0.05	-0.09	-0.17	-0.22	-0.43	-0.63	-0.93	-1.43	-1.99	-3.62	-7.42
	(-5.28)	(-5.31)	(-6.92)	(-5.00)	(-6.44)	(-5.79)	(-7.12)	(-6.84)	(-6.21)	(-6.13)	(-6.66)	(-6.08)	(-6.66)
$100{\cdot}\beta_{4b,p}$	-0.00	-0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.08	-0.15	-0.17	-0.21	-0.53	-1.56
	(-2.79)	(-2.83)	(-3.34)	(-2.75)	(-2.96)	(-3.14)	(-2.93)	(-2.90)	(-2.80)	(-2.09)	(-1.93)	(-1.97)	(-2.09)

uid stocks even more illiquid. Therefore, illiquid portfolios have much greater illiquidity sensitivity to macroeconomic and financial shocks (e.g., $\beta_{4a,p}$ and $\beta_{4b,p}$).

Looking at the relative size of the two sub-betas, stock liquidity costs co-vary much more strongly with macroeconomic shocks than with financial shocks. For illiquid portfolio (Portfolio 25), risk exposures to $\beta_{4a,p}$ is -5.87 whereas risk exposures to $\beta_{4b,p}$ is only -2.01. This large difference can be partly explained by the nature of the systematic shocks, macroeconomic shocks and financial shocks, that liquidity costs covary with. Financial shocks reduce stock prices (and hence investors' wealth), but it improves future investment opportunities as discount rates go up. On the other hand, macroeconomic shocks affect investors' wealth but have limited impact on future investment opportunity sets, so investor's demand for risky assets would be more sensitive to macroeconomic shocks, making stocks' illiquidity much more strongly correlated with macroeconomic shocks.

Panel B in Table 3 presents the estimated $\beta_{4a,p}$ and $\beta_{4b,p}$ for 25 size portfolios. Similar to illiquidity portfolios, small size portfolios have greater risk exposures to both $\beta_{4a,p}$ and $\beta_{4b,p}$. The greater risk exposures to $\beta_{4a,p}$ and $\beta_{4b,p}$ may, in part, explain the large return premium associated with small stocks (size premium). Looking at the ratio of the two betas, I find that the exposures to the macroeconomic shock beta is about 5 times greater than the exposures to the financial shock beta for portfolios of small stocks.

So far, I have examined the macroeconomic shock and financial shock betas associated with illiquidity and size portfolios and find that small and illiquid portfolios have greater exposures to both sub-liquidity betas than big and liquid portfolios, and that portfolios have greater exposures to $\beta_{4a,p}$ than to $\beta_{4b,p}$. Motivated by this finding, I examine return premium associated with the two sub-liquidity betas in the following section.

3.5 Pricing Macroeconomic Shock and Financial Shock Betas

In this section, I estimate return premium associated with the macroeconomic and financial shock betas in the context of Acharya and Pedersen's LCAPM. The playground for this section is 25 value weighted illiquidity portfolios.

The expected return premium associated with the macroeconomic and financial beta is a product of the risk exposures to each beta and price of risk. Therefore, in addition to the estimation of $\beta_{4a,p}$ and $\beta_{4b,p}$ in Section 3.4, I also need to estimate the *price of risk*, λ, in equation (1). This requires me to estimate the other two liquidity betas ($\beta_{2,p}$ and $\beta_{3,p}$) as well as the market beta ($\beta_{1,p}$). So, I first present estimated betas along with descriptive statistics for each illiquidity portfolio. Then, I estimate the price of risk in the context of the LCAPM in equation (1), and finally I present the expected return premium associated with each beta.

3.5.1 Illiquidity Portfolio Beta Estimation

Table 4 presents estimated portfolio betas and descriptive statistics for 25 illiquidity portfolios. Average portfolio liquidity costs, $E\left[c_{p,t}\right]$, monotonically increases from portfolio 1 (most liquid) to portfolio 25 (most illiquid). Average illiquidity portfolio returns increase with portfolio illiquidity. Size and illiquidity are negatively related.

$\beta_{1,p}$ measures portfolio exposures to market risk. Illiquid portfolios tend to have greater exposures to market risk. $\beta_{1,p} \times 100$ (non-monotonically) increases from 54.37 for the most liquid portfolio to 74.98 for the most illiquid portfolio. On the other hand, the overall liquidity beta (e.g., ($\beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$) $\times 100$) monotonically increases from 0.38 for the most liquid portfolio to 8.85 for the most illiquid portfolio. This suggests

Table 4: Portfolio Betas and Characteristics for 25 Illiquidity Portfolios

This table reports estimated portfolio betas for 25 illiquidity portfolios, updated every January based on previous years' stock illiquidity during 1964-2017 (54 years). The market beta, $\beta_{1,p}$, and liquidity betas, $\beta_{2,p}$, $\beta_{3,p}$, $\beta_{4a,p}$, and $\beta_{4b,p}$, are computed by value weighted monthly illiquidity portfolio returns and illiquidity innovations, and equally weighted monthly market portfolio returns and illiquidity innovations. $t-statistics$, a ratio of estimated β and bootstrap standard error, are reported in parentheses. Bootstrap standard errors are computed from 10,000 simulated realizations. Column 6-11 in this table presents portfolio characteristics. $E[c_p]$ column reports average illiquidity of portfolio p and $\sigma(\Delta c_p)$ column reports standard deviation of portfolio p's illiquidity innovations. $E[r_{e,p}]$ and $\sigma(r_p)$ are average and standard deviation of value weighted monthly portfolio excess returns for portfolio p. Portfolio turnover (trn) and market capitalization (Size) are reported in the last two columns. To save space, I only report properties of odd numbered portfolios.

	Estimated Portfolio Betas					Portfolio Characteristics					
	$\beta_{1,p}$ (·100)	$\beta_{2,p}$ (·100)	$\beta_{3,p}$ (·100)	$\beta_{4a,p}$ (·100)	$\beta_{4b,p}$ (·100)	$E[c_p]$ (%)	$\sigma(\Delta c_p)$ (%)	$E[r_{e,p}]$ (%)	$\sigma(r_p)$ (%)	trn (%)	Size (bl$)
1	54.37 (29.31)	0.00 (1.95)	-0.38 (-4.93)	-0.00 (-2.54)	-0.00 (-1.51)	0.25	0.00	0.47	1.53	5.68	24.16
3	66.09 (41.23)	0.00 (6.74)	-0.51 (-5.35)	-0.02 (-7.08)	-0.00 (-3.43)	0.26	0.00	0.55	1.76	7.90	4.48
5	69.32 (48.79)	0.00 (5.81)	-0.54 (-5.94)	-0.04 (-7.49)	-0.01 (-3.28)	0.27	0.01	0.65	1.82	8.57	2.41
7	76.40 (44.64)	0.00 (6.15)	-0.66 (-5.47)	-0.07 (-7.47)	-0.01 (-3.13)	0.28	0.01	0.73	1.95	9.56	1.37
9	78.50 (54.32)	0.00 (6.33)	-0.69 (-6.20)	-0.10 (-7.85)	-0.01 (-3.60)	0.30	0.02	0.74	1.99	9.28	0.92
11	76.99 (53.31)	0.00 (6.30)	-0.68 (-6.43)	-0.20 (-7.60)	-0.03 (-2.94)	0.34	0.03	0.70	2.04	8.64	0.66
13	81.06 (49.27)	0.01 (6.28)	-0.71 (-5.86)	-0.30 (-5.84)	-0.04 (-3.38)	0.38	0.05	0.74	2.11	8.45	0.48
15	82.73 (58.67)	0.01 (6.22)	-0.74 (-7.09)	-0.52 (-7.15)	-0.05 (-3.06)	0.45	0.07	0.82	2.22	7.82	0.36
17	85.09 (56.39)	0.01 (6.42)	-0.82 (-7.17)	-0.71 (-6.45)	-0.08 (-2.80)	0.59	0.11	0.78	2.32	6.88	0.27
19	87.38 (43.66)	0.03 (6.60)	-0.80 (-7.07)	-1.13 (-6.76)	-0.14 (-3.00)	0.79	0.17	0.89	2.43	6.41	0.18
21	88.04 (46.70)	0.04 (6.07)	-0.86 (-6.56)	-1.99 (-7.86)	-0.14 (-1.47)	1.27	0.31	0.90	2.58	5.23	0.12
23	81.82 (32.94)	0.08 (6.93)	-0.80 (-6.70)	-2.83 (-6.45)	-0.39 (-1.70)	2.45	0.59	0.95	2.72	4.53	0.08
25	74.98 (26.13)	0.22 (6.13)	-0.75 (-7.36)	-5.87 (-5.23)	-2.01 (-2.31)	8.29	1.47	0.95	3.05	2.96	0.03

that portfolios with greater illiquidity also have greater exposures to illiquidity beta risks.

Similar to Acharya and Pedersen, I find negative relationship between exposures to liquidity beta risks and average liquidity costs and size. If investors apply higher discount rates to stocks with greater liquidity beta and liquidity costs, the illiquid stocks would have lower market equity capitalization. The converse is also true. Small firms tend to have reduced ability to absorb negative economic shocks, so they are less attractive to investors, which in turn make these stocks illiquid. Therefore, I interpret the negative relationship as reinforcement effect, rather than causal relationship.

3.5.2 LCAPM Estimation

In this section, using the estimated βs and portfolio excess returns, I estimate price of risk (λ) from the cross-sectional regression of the LCAPM. The LCAPM estimation results are presented in Table 5.

The first three rows in Table 5 report estimation results of the following regression model:

$$E\left[r_{p,t} - r_{f,t}\right] = \alpha + \mathcal{K}E\left[c_{p,t}\right] + \lambda_{net}\beta_{net,p} \tag{12}$$

$$\text{where } \mathcal{K} \in \{\bar{\kappa}, \kappa, 0\} \quad \text{and} \quad \beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$$

In equation (12), I do not restrictions the intercept, α, to be equal zero, but leave it as a free parameter. Equation (12) implies that price of risk (λ_{net}) is the same for different beta risks. For example, price of one unit of market risk, $\beta_{1,p}$, is the same as the price of one unit of liquidity commonality risk, $\beta_{2,p}$, and so on.

The second term in equation (12), $\mathcal{K}E\left[c_{p,t}\right]$, is costs of holding a portfolio p. In the LCAPM, every investor rebalances his/her portfolio every period, but I estimate liquidity costs at monthly frequency. Therefore, average monthly liquidity costs would overestimate liquidity costs for some portfolios if the portfolios are, on average, held more than a month. Similarly, average monthly liquidity costs would underestimate liquidity costs for some other portfolios if the portfolios are held less than one month. This imbalance in holding periods and estimation frequency is adjusted by the parameter, $\mathcal{K}$ in equation (12). For example, if average investment holding period for portfolio p is one and half year, then the average monthly liquidity costs should be $E\left[c_{p,t}\right]/18$ where $\mathcal{K} = 1/18$, rather than $E\left[c_{p,t}\right]$.

I estimate equation (12) with three different restrictions on $\mathcal{K} \in \{\bar{\kappa}, \kappa, 0\}$. I estimate the equation (1) with $\mathcal{K} = \bar{\kappa}$ (calibrated by turnover), with $\mathcal{K} = \kappa$ (a free parameter), and with $\mathcal{K} = 0$. The average monthly turnover of sample stocks is 0.073 (7.3%), implying that investors' average stock holding period is 13.66 months ($\approx 1/0.073$) for 1964-2017, so I use $\bar{\kappa} = 0.073$.

Table 5: LCAPM Estimation for 25 Illiquidity Portfolios

This table reports cross-sectional liquidity-adjusted CAPM estimation results. The estimation is based on the 25 illiquidity portfolio monthly value weighted returns for Jan 1964 - Dec 2017 (648 months). The regression model is $E[r_{p,t} - r_{f,t}] = \alpha + \mathcal{K}E[c_{p,t}] + \lambda_{net}\beta_{net}$ where $\mathcal{K} \in \{\bar{\kappa}, \kappa, 0\}$. $\bar{\kappa}$ is average monthly turnover for all stocks (0.073) and κ indicate a free parameter. $\beta_{net,p}$ is the overall LCAPM beta of portfolio p, $\beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$. t-statistics in parentheses are estimated by GMM framework. The last column reports adjusted R^2s obtained from OLS.

	Constant	$E[c_p]$	$\beta_{1,p}$	$\beta_{2,p}$	$\beta_{3,p}$	$\beta_{4a,p}$	$\beta_{4b,p}$	$\beta_{net,p}$	R^2
1	-0.005	0.073						0.867	0.38
	(-0.027)	(—)						(4.097)	
2	-0.210	0.021						1.191	0.89
	(-2.668)	(4.136)						(11.944)	
3	-0.295							1.324	0.82
	(-2.986)							(10.814)	
4	-0.335	0.073	5.069					-3.676	0.84
	(-3.576)	(—)	(8.636)					(-6.849)	
5	-0.068	-0.031	-5.396					6.357	0.89
	(-0.580)	(-0.941)	(-1.598)					(1.966)	
6	-0.148		-2.253					3.344	0.89
	(-1.831)		(-4.452)					(7.226)	
7	-0.314	0.073	1.647	-36.419	39.070	19.046	-3.177		0.85
	(-1.754)	(—)	(2.937)	(-0.282)	(0.944)	(2.249)	(-0.844)		

The first row in Table 5 presents estimation results of equation (12) with $\mathcal{K}$=0.073. The estimated intercept is economically small and statistically not different from zero. The estimated price of risk, λ_{net}, is 0.867 and statistically significant. However, the adjusted R^2 is only 0.38, indicating that β_{net} poorly explains cross-sectional variations in average illiquidity portfolio net returns, $E[r_{p,t} - \bar{\kappa}c_{p,t} - r_{f,t}]$.

The second row in Table 5 reports estimation results of equation (12) with $\mathcal{K} = \kappa$ where κ is a free parameter. The estimated price of risk, λ_{net}, is 1.191, slightly greater than what is estimated in the first row. The estimated κ is 0.021, smaller than the average stock turnover ratio, and the adjusted R^2 is 0.89. When κ is estimated as a free parameter, there is a substantial improvement in the model fit. The third row presents estimation results of equation (12) with $\kappa = 0$. I find that the estimated price of risk is not much different from what is estimated in the second row.

The next three rows report estimation results of the following regression model:

$$E\left[r_{p,t} - r_{f,t}\right] = \alpha + \mathcal{K}E\left[c_{p,t}\right] + \lambda_1\beta_{1,p} + \lambda_{net}\beta_{net,p} \tag{13}$$

$$\text{where } \mathcal{K} \in \{\bar{\kappa}, \kappa, 0\} \quad \text{and} \quad \beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$$

In equation (13), I assume that price of market risk is different from price of liquidity risk. The interpretation of estimated coefficients requires caveat. λ_1 does not estimate a

Table 6: Correlations Between Portfolio Betas

This table reports correlations between the market and liquidity betas for 25 value-weighted illiquidity portfolios reported in Table 4.

	$\beta_{1,p}$	$\beta_{2,p}$	$\beta_{3,p}$	$\beta_{4a,p}$	$\beta_{4b,p}$
$\beta_{1,p}$	1.000	0.200	-0.948	-0.313	-0.075
$\beta_{2,p}$		1.000	-0.448	-0.980	-0.957
$\beta_{3,p}$			1.000	0.547	0.306
$\beta_{4a,p}$				1.000	0.920
$\beta_{4b,p}$					1.000

risk premium on market risk as $\beta_{1,p}$ is also contained in $\beta_{net,p}$. The price of market risk is the sum of λ_1 and λ_{net} and price of liquidity risk is λ_{net}.

Depending on the restriction on $\mathcal{K}$, price of liquidity risk varies from -3.676 to 6.357. Acharya and Pedersen also find similar amount of variations in the estimated liquidity risk premium. The negative price of liquidity risk in the fourth row, however, is puzzling. The estimated market price of risk ranges from 0.961 to 1.393.

The last row in Table 5 reports the estimation results of the following LCAPM regression model.

$$E[r_{p,t} - r_{f,t}] = \alpha + 0.073E\left[c_{p,t}\right] + \lambda_1\beta_{1,p} + \lambda_2\beta_{2,p} + \lambda_3\beta_{3,p} + \lambda_{4a}\beta_{4a,p} + \lambda_{4b}\beta_{4b,p} \qquad (14)$$

The regression model (14) estimates prices for each beta risk separately. Estimated price of market risk is 1.647 and is statistically significant. However, estimated prices of liquidity risks seem unrealistic. This is a typical result of (severe) multicollinearity problem. Under the multicollinearity problem, the estimated coefficients are very sensitive to minor changes in the data. Therefore, the estimated coefficients in the last row in Table 5 are difficult to interpret. Table 6 presents correlation between the estimated betas in Table 4. The estimated absolute correlation is as high as 0.980 between $\beta_{2,p}$ and $\beta_{4a,p}$.

In the following section, using the estimated price of risks in Table 5, I estimate return premium associated with the macroeconomic shock beta and the financial shock beta, as well as return premium associated with other betas in the LCAPM.

3.5.3 Economic Significance

Table 4 suggests that the most illiquid portfolio on average generates about 0.48%p (5.76%p annually) more returns than the most liquid portfolios. The LCAPM and the decomposition in Section 2.2 suggest that this large illiquidity return premium of 5.76% can be explained by expected liquidity costs, $E[c_p]$, the market beta ($\beta_{1,p}$), the covariation between firm liquidity costs and market liquidity costs ($\beta_{2,p}$), and the covariation between stock returns and market liquidity costs ($\beta_{3,p}$), and more importantly, by the macroeconomic shock beta ($\beta_{4a,p}$) as well as by the financial shock beta ($\beta_{4b,p}$).

In this section, I estimate the return premium associated with each beta. The required returns associated with each beta are estimated by the product of market price of risk and the excess risk exposures to corresponding betas. For market price of risk, I use λ_{net} from the second row of Table 5 where I assume the price of risk is the same for all betas and where κ is estimated as a free parameter. λ_{net} in the second row is relatively more precisely estimated (smaller standard error).

Table 4 suggests that the monthly turnover adjusted liquidity costs of holding illiquid portfolio is 0.245% ($\approx 8.29 * 0.0296$) and the costs for holding liquid portfolio is 0.014% ($\approx 0.25 * 0.0568$). Therefore, the annualized return premium due to liquidity costs is 2.772% for 1964-2017.

Illiquid portfolios have greater exposure to market risk. The return premium due to the market beta ($\beta_{1,p}$) is:

$$12 \cdot \lambda_{net}\big(\beta_{1,25} - \beta_{1,1}\big) = 2.946\% \tag{15}$$

It implies that about half ($\approx$ 51%) of the illiquidity premium comes form the excess exposures to the market risk. Put it differently, about half of the illiquidity premium is coming from liquidity costs and exposures to liquidity beta risks.

$\beta_{2,p}$ is liquidity commonality beta (the covariation with firm liquidity costs and market liquidity costs). The return premium due to $\beta_{2,p}$ is:

$$12 \cdot \lambda_{net}\big(\beta_{2,25} - \beta_{2,1}\big) = 0.031\% \tag{16}$$

The illiquidity return premium due to $\beta_{2,p}$ is positive but is not economically significant. The 95% confidence interval for the return premium associated with $\beta_{2,p}$ is [0.018, 0.054].[5]

Similarly, the return premium due to the covariation between stock returns and market liquidity costs, $\beta_{3,p}$, is

$$-12 \cdot \lambda_{net}\big(\beta_{3,25} - \beta_{3,1}\big) = 0.054\% \tag{17}$$

and 95% confidence interval is [-0.0016, 0.1354].

Finally, the return premium associated with the macroeconomic shock beta ($\beta_{4a,p}$) and the financial shock beta ($\beta_{4b,p}$) is:

$$-12 \cdot \lambda_{net}\big(\beta_{4a,25} - \beta_{4a,1}\big) = 0.839\% \tag{18}$$

$$-12 \cdot \lambda_{net}\big(\beta_{4b,25} - \beta_{4b,1}\big) = 0.288\% \tag{19}$$

The estimated return premium due to the macroeconomic shock beta ($\beta_{4a,p}$) is 0.839% annually and its 95% confidence interval is [0.421%, 1.483%]. The estimated return premium due to the financial shock beta ($\beta_{4b,p}$) is 0.288% annually and its 95% confidence interval is [0.059%, 0.772%]. This implies that the return premium due to the covariation between firm liquidity costs and market returns, $\beta_{4,p} \approx \beta_{4a,p} + \beta_{4a,p}$, is approximately 1.127% annually.

The return premium due to $\beta_{4a,p}$ is approximately three times greater than that for $\beta_{4b,p}$. Greater return premium for $\beta_{4a,p}$ is rather an expected result as illiquid portfolios have much greater exposures to $\beta_{4a,p}$ than to $\beta_{4b,p}$. Campbell and Vuolteenaho (2004)

[5]Risk premium associated with liquidity commonality beta is estimated by the product of price of risk, λ_{net} from row 2 in Table 5, and difference in $\beta_{2,p}$ (i.e., $\beta_{2,25} - \beta_{2,1}$). The 97.5% confidence interval of λ_{net} is [0.9676, 1.4146] and the 97.5% bias-corrected bootstrapped confidence interval for $100 \cdot \beta_{2,25}$ and $100 \cdot \beta_{2,1}$ are [0.0015, 0.0032] and [0.0000, 0.0000] respectively. Therefore, 97.5% confidence interval for $100 \cdot (\beta_{2,25} - \beta_{2,1})$ is [0.0015, 0.0032]. 95% confidence interval for $100 \cdot \lambda_{net} (\beta_{2,25} - \beta_{2,1})$ is [0.0176, 0.0537]. In this calculation, I assume that $\beta_{2,1}$, $\beta_{2,25}$ and λ_{net} are independent.

document that large return premium associated with small stocks is in part due to their greater exposures to return sensitivity to real economic shocks (e.g., macroeconomic shocks). They find that pricing effect of the covariation between stock returns and cash flow news shocks (e.g., macroeconomic shocks) is much greater than the pricing effect of the covariation between stock returns and discount rate news shocks (e.g., financial shocks). Therefore, strong return premium associated with the macroeconomic shock beta implies that that macroeconomic shocks strongly affect asset prices not only through their covariation with stock returns but also through their covariation with stock liquidity.

The return premium associated with the financial shock beta is 0.288% annually. This finding is particularly interesting not only because it is the first study that documents pricing effect of the systematic covariation between liquidity costs and financial shocks, but it also explains about a quarter ($\approx 26\%$) of the large return premium associated with liquidity sensitivity to market returns, $\beta_{4,p}$.

4 Robustness Tests

I have shown that $\beta_{4,p}$, the covariation between firm liquidity costs and market returns, has two components: the macroeconomic shock beta ($\beta_{4a,p}$) and the financial shock beta ($\beta_{4b,p}$). Then, using portfolios sorted on illiquidity, I find that both sub-liquidity betas are strongly priced. In this section, I repeat the same exercise, but with different playgrounds to examine whether both $\beta_{4a,p}$ and $\beta_{4a,p}$ are consistently priced.

4.1 Value-Weighted Market Portfolio

In this section, to examine whether my results are robust to the choice of value-weighted versus equal weighted market portfolio returns and illiquidity. Estimated betas and portfolio characteristics and the LCAPM results are reported in Table 7 and Table 8.

Consistent with prior estimation results, I find that exposures to the macroeconomic shock beta and the financial shock beta are substantially vary cross-sectionally and that illiquid portfolios have much greater exposures to the macroeconomic shock beta than to

Table 7: Portfolio Betas and Characteristics for 25 Illiquidity Portfolios (Value weighted Market Portfolio)

This table reports estimated portfolio betas for 25 illiquidity portfolios, updated every January based on previous years' stock illiquidity during 1964-2017 (54 years). The market beta, $\beta_{1,p}$, and liquidity betas, $\beta_{2,p}$, $\beta_{3,p}$, $\beta_{4a,p}$, and $\beta_{4b,p}$, are computed by value weighted monthly portfolio returns and illiquidity innovations, and macroeconomic and financial shocks estimated from value-weighted market returns. $t-statistics$, a ratio of estimated β and bootstrap standard error, are reported in parentheses. Bootstrap standard errors are computed from 10,000 simulated realizations. Column 6-11 in this table presents portfolio characteristics. $E\left[c_p\right]$ column reports average illiquidity of portfolio p and $\sigma\left(\Delta c_p\right)$ column reports standard deviation of portfolio p's illiquidity innovations. $E\left[r_{e,p}\right]$ and $\sigma\left(r_p\right)$ are average and standard deviation of value weighted monthly portfolio excess returns for portfolio p. Portfolio turnover (trn) and market capitalization (Size) are reported in the last two columns. To save space, I only report properties of odd numbered portfolios.

	Estimated Portfolio Betas					Portfolio Characteristics					
	$\beta_{1,p}$ (·100)	$\beta_{2,p}$ (·100)	$\beta_{3,p}$ (·100)	$\beta_{4a,p}$ (·100)	$\beta_{4b,p}$ (·100)	$E\left[c_p\right]$ (%)	$\sigma\left(\Delta c_p\right)$ (%)	$E\left[r_{e,p}\right]$ (%)	$\sigma\left(r_p\right)$ (%)	trn (%)	Size (bl$)
1	87.43 (67.50)	0.00 (1.94)	-0.05 (-7.76)	-0.00 (-3.32)	-0.00 (-1.24)	0.25	0.00	0.47	1.53	5.68	24.16
3	94.82 (68.03)	0.00 (6.35)	-0.06 (-8.66)	-0.02 (-6.73)	-0.00 (-3.01)	0.26	0.00	0.55	1.76	7.90	4.48
5	95.68 (52.27)	0.00 (5.72)	-0.07 (-8.75)	-0.05 (-7.39)	-0.01 (-2.86)	0.27	0.01	0.65	1.82	8.57	2.41
7	102.39 (41.00)	0.00 (5.61)	-0.07 (-8.91)	-0.08 (-7.31)	-0.01 (-2.95)	0.28	0.01	0.73	1.95	9.56	1.37
9	102.86 (36.09)	0.00 (6.39)	-0.08 (-9.34)	-0.12 (-7.78)	-0.01 (-3.23)	0.30	0.02	0.74	1.99	9.28	0.92
11	97.42 (31.09)	0.00 (6.24)	-0.08 (-9.21)	-0.25 (-7.55)	-0.03 (-2.76)	0.34	0.03	0.70	2.04	8.64	0.66
13	102.86 (31.21)	0.00 (5.83)	-0.08 (-8.67)	-0.38 (-6.15)	-0.04 (-2.76)	0.38	0.05	0.74	2.11	8.45	0.48
15	102.30 (29.18)	0.00 (6.20)	-0.08 (-8.31)	-0.62 (-6.85)	-0.05 (-2.80)	0.45	0.07	0.82	2.22	7.82	0.36
17	103.01 (27.61)	0.00 (6.13)	-0.09 (-8.89)	-0.84 (-6.49)	-0.08 (-2.59)	0.59	0.11	0.78	2.32	6.88	0.27
19	104.85 (23.77)	0.00 (6.34)	-0.09 (-8.96)	-1.32 (-6.34)	-0.15 (-2.89)	0.79	0.17	0.89	2.43	6.41	0.18
21	103.74 (22.73)	0.01 (6.19)	-0.09 (-8.72)	-2.50 (-7.57)	-0.12 (-1.21)	1.27	0.31	0.90	2.58	5.23	0.12
23	94.88 (18.44)	0.01 (7.70)	-0.09 (-7.78)	-3.26 (-5.11)	-0.47 (-1.68)	2.45	0.59	0.95	2.72	4.53	0.08
25	84.88 (17.10)	0.02 (8.07)	-0.08 (-7.02)	-8.32 (-5.48)	-2.14 (-1.90)	8.29	1.47	0.95	3.05	2.96	0.03

the financial shock beta.

The covariation between portfolio illiquidity shock and macroeconomic shocks ($100 \cdot \beta_{4a,p}$) estimated from equal-weight market excess returns and value-weighted market excess return for the most illiquidity portfolio is -5.84 and -8.32 respectively. This maybe in part due to that the systematic macroeconomic shocks (e.g, aggregate cash-flow shocks), which firm liquidity costs covary with, is better captured when using value-weight market returns. On the other hand, when using equal-weight market returns, adverse cash-flow shocks coming from big size firms, which affect many investors, can easily be offset by positive cash-flow shocks arising from small firms. Vuolteenaho (2002) find that when firm level cash-flow shocks is largely diversified away when they are aggregated into an equal weighted portfolio.

Table 8: LCAPM Estimation for 25 Illiquidity Portfolios (Value Weighed Portfolio Returns)

This table reports cross-sectional liquidity-adjusted CAPM estimation results. The estimation is based on the 25 illiquidity portfolio monthly value weighted returns for Jan 1964 - Dec 2017 (648 months). The regression model is $E[r_{p,t} - r_{f,t}] = \alpha + \mathcal{K}E\left[c_{p,t}\right] + \lambda_{net}\beta_{net}$ where $\mathcal{K} \in \{\bar{\kappa}, \kappa, 0\}$. $\bar{\kappa}$ is average monthly turnover for all stocks (0.087) and κ indicate a free parameter. $\beta_{net,p}$ is the overall LCAPM beta of portfolio p, $\beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$. t-statistics in parentheses are estimated by GMM framework. The last column reports adjusted R^2s obtained from OLS.

	Constant	$E\left[c_p\right]$	$\beta_{1,p}$	$\beta_{2,p}$	$\beta_{3,p}$	$\beta_{4a,p}$	$\beta_{4b,p}$	$\beta_{net,p}$	R^2
1	-1.492	0.073						2.161	0.61
	(-4.452)	(—)						(6.517)	
2	-1.319	0.049						2.014	0.75
	(-4.564)	(6.742)						(7.074)	
3	-0.948							1.697	0.31
	(-1.990)							(3.601)	
4	-1.414	0.073	1.550					0.555	0.68
	(-4.752)	(—)	(2.653)					(0.826)	
5	-0.833	-0.085	-10.354					11.883	0.80
	(-2.783)	(-1.837)	(-2.933)					(3.523)	
6	-1.146		-3.932					5.772	0.78
	(-4.375)		(-7.645)					(9.748)	
7	-0.444	0.073	0.594	1244.200	-711.202	2.633	16.848		0.79
	(-1.156)	(—)	(1.022)	(0.578)	(-2.166)	(0.434)	(1.603)		

The covariation between portfolio illiquidity shock and financial shocks ($100 \cdot \beta_{4b,p}$) estimated from equal-weight market excess returns and value-weighted market excess return for the most illiquidity portfolio is -2.01 and -2.14 respectively.

Looking at the LCAPM estimation results in Table 8, the market price of risk, λ_{net}, is also priced with value-weighted market portfolio returns. The return premium associated with the macroeconomic shock beta and the financial shock beta is 2.01% and 0.52% respectively.

4.2 Size Portfolio

In this section, I examine size portfolios. I form 25 portfolios sorted by market equity capitalization. Estimated betas and portfolio characteristics and the LCAPM estimation results are reported in Table 9 and Table 10.

Consistent with prior studies (Banz, 1981; Fama and French, 1992; Fama and French, 1993), I find that small-sized stocks have greater average returns. Small stocks also have greater average illiquidity, illiquidity variance, and exposures to liquidity betas. I find that exposures to the market beta increases from 53.10 for portfolios with large company stocks to 82.21 for portfolios of small company stocks. Similarly, The overall risk exposures to liquidity betas, $100 \cdot (\beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p})$, increase from 0.36 to

Table 9: Portfolio Betas and Characteristics for 25 Size Portfolios

This table reports estimated portfolio betas for 25 size portfolios updated every January based on previous years' stocks market equity capitalization during 1964-2017 (54 years). The market beta, $\beta_{1,p}$, and liquidity betas, $\beta_{2,p}, \beta_{3,p}, \beta_{4a,p}$, and $\beta_{4b,p}$, are computed by monthly portfolio value weighted average returns and portfolio illiquidity innovations. $t - statistics$, ratio of estimated β and bootstrap standard error, are reported in parentheses. Bootstrap standard errors are computed from 10,000 simulated realizations. Column 6-11 in this table presents portfolio characteristics. $E[c_p]$ column reports average illiquidity of portfolio p and $\sigma(\Delta c_p)$ column reports standard deviation of portfolio p's illiquidity innovations. $E[r_{e,p}]$ and $\sigma(r_p)$ are average and standard deviation of value weighted monthly portfolio excess returns for portfolio p. Portfolio turnover (trn) and market capitalization (Size) are reported in the last two columns. To save space, I only report properties of odd numbered portfolios.

	Estimated Portfolio Betas					Portfolio Characteristics					
	$\beta_{1,p}$ (·100)	$\beta_{2,p}$ (·100)	$\beta_{3,p}$ (·100)	$\beta_{4a,p}$ (·100)	$\beta_{4b,p}$ (·100)	$E[c_p]$ (%)	$\sigma(\Delta c_p)$ (%)	$E[\pi_p]$ (%)	$\sigma(r_p)$ (%)	trn (%)	Size (bl$)
1	53.10	0.00	-0.36	-0.00	-0.00	0.25	0.00	0.48	1.52	5.21	25.28
	(28.24)	(7.27)	(-4.80)	(-5.28)	(-2.79)						
3	67.24	0.00	-0.54	-0.02	-0.00	0.26	0.00	0.61	1.77	8.17	4.42
	(47.77)	(5.89)	(-5.97)	(-5.31)	(-2.83)						
5	74.42	0.00	-0.59	-0.05	-0.01	0.27	0.01	0.64	1.90	9.68	2.19
	(55.22)	(6.23)	(-6.02)	(-6.92)	(-3.34)						
7	81.28	0.00	-0.70	-0.09	-0.02	0.30	0.02	0.69	1.98	10.11	1.30
	(59.11)	(5.46)	(-5.92)	(-5.00)	(-2.75)						
9	82.60	0.00	-0.71	-0.17	-0.03	0.33	0.03	0.77	2.06	9.80	0.87
	(79.51)	(5.85)	(-6.74)	(-6.44)	(-2.96)						
11	84.02	0.01	-0.70	-0.22	-0.04	0.38	0.05	0.79	2.17	10.25	0.62
	(73.69)	(5.05)	(-6.26)	(-5.79)	(-3.14)						
13	91.52	0.01	-0.79	-0.43	-0.05	0.43	0.07	0.78	2.29	9.94	0.45
	(68.27)	(5.61)	(-6.96)	(-7.12)	(-2.93)						
15	91.45	0.01	-0.80	-0.63	-0.08	0.53	0.10	0.82	2.39	9.93	0.32
	(67.19)	(6.42)	(-7.21)	(-6.84)	(-2.90)						
17	93.13	0.02	-0.85	-0.93	-0.15	0.69	0.15	0.81	2.57	9.47	0.23
	(55.73)	(6.23)	(-6.98)	(-6.21)	(-2.80)						
19	96.39	0.04	-0.93	-1.43	-0.17	0.96	0.25	0.78	2.70	9.08	0.15
	(47.30)	(6.05)	(-6.89)	(-6.13)	(-2.09)						
21	97.08	0.05	-0.89	-1.99	-0.21	1.53	0.38	0.87	2.85	8.08	0.10
	(46.03)	(5.90)	(-6.27)	(-6.66)	(-1.93)						
23	94.66	0.10	-0.88	-3.62	-0.53	2.89	0.71	0.91	3.00	6.95	0.05
	(40.61)	(6.48)	(-7.13)	(-6.08)	(-1.97)						
25	82.21	0.26	-0.85	-7.42	-1.56	8.14	1.40	0.93	3.17	5.09	0.01
	(29.39)	(7.92)	(-8.53)	(-6.66)	(-2.09)						

10.09. Therefore, larger return premium in small size company stocks is not only due to exposures to market beta, but also due to exposure to liquidity betas.

I find that the portfolio of small company stocks, Portfolio 25 in Table 9, have much greater exposures to liquidity sensitivity to macroeconomic shocks compared to the risk exposures to liquidity sensitivity to market risk, indicating the size premium is more strongly associated with $\beta_{4b,p}$. Campbell and Vuolteenaho (2004) document that small stocks have considerably higher *cash-flow beta*, the covariation between stock return and cash-flow shocks, than larger stocks and the pricing effect of cash-flow beta is greater than the pricing effect of *discount-rate beta*, the covariation between stock return and discount rate shocks.

In terms of the LCAPM estimation, similar to Fama and French (1993) and Acharya

Table 10: LCAPM Estimation for 25 Size Portfolios

This table reports cross-sectional liquidity-adjusted CAPM estimation results. The estimation is based on the 25 size portfolio monthly value weighted returns for Jan 1964 - Dec 2017 (648 months). The regression model is $E[r_{p,t} - r_{f,t}] = \alpha + \mathcal{K}E\left[c_{p,t}\right] + \lambda_{net}\beta_{net}$ where $\mathcal{K} \in \{\bar{\kappa}, \kappa, 0\}$. $\bar{\kappa}$ is average monthly turnover for all stocks (0.087) and κ indicate a free parameter. $\beta_{net,p}$ is the overall LCAPM beta of portfolio p, $\beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$. t-statistics in parentheses are estimated by GMM framework. The last column reports adjusted R^2s obtained from OLS.

	Constant	$E[c_p]$	$\beta_{1,p}$	$\beta_{2,p}$	$\beta_{3,p}$	$\beta_{4a,p}$	$\beta_{4b,p}$	$\beta_{net,p}$	R^2
1	0.299	0.087						0.433	0.14
	(1.794)	(—)						(2.282)	
2	0.077	0.018						0.779	0.87
	(1.277)	(3.384)						(10.905)	
3	0.021							0.867	0.82
	(0.304)							(10.779)	
4	-0.033	0.087	5.739					-4.782	0.88
	(-0.498)	(—)	(12.500)					(-11.301)	
5	0.067	0.024	0.530					0.266	0.86
	(0.782)	(0.664)	(0.177)					(0.092)	
6	0.105		-1.440					2.176	0.87
	(1.636)		(-3.296)					(5.402)	
7	0.064	0.087	0.575	-251.412	-25.171	0.162	-8.941		0.89
	(0.658)	(—)	(1.530)	(-0.597)	(-0.783)	(0.029)	(-0.203)		

and Pedersen (2005), I find that the fit of the LCAPM is relatively poor (R^2=87%, Row 2 in Table 10) and the estimated coefficient on the net beta is small, $\lambda_{net} = 0.78$, suggesting that the LCAPM does not explain much of the cross-sectional average returns on size portfolios. The return premium associated with the macroeconomic shock beta and the financial shock beta is 0.69% and 0.15% respectively. Consistent with illiquidity portfolios, return premium associated with the macroeconomic shock beta is greater than that associated with the financial shock beta.

4.3 Including NASDAQ Stocks

In this section, I include NASDAQ stocks in the sample. One of the reasons that I exclude NASDAQ stocks from the main analysis is that NASDAQ joins the CRSP in December, 1972, limiting period of analysis. Moreover, different trading mechanisms in NASDAQ and NYSE may have their own micro-structure effects on required return on the listed stocks. And, trading volumes recorded in NASDAQ is inflated due to double counting (Atkins and Dyl, 1997; Amihud, 2002; Ben-Rephael et al., 2015), so estimated illiquidity for NASDAQ stocks could be underestimated.

I form 25 illiquidity portfolios with NASDAQ stocks and re-estimate return premium associated with the two sub-liquidity betas. Therefore, in this section, the dataset in-

Table 11: Portfolio Betas and Characteristics for 25 Illiquidity Portfolios

This table reports estimated portfolio betas for 25 illiquidity portfolios that are updated every January based on previous years' stock illiquidity during 1974-2017 (44 years). The sample includes NYSE/AMEX/NASDAQ stocks. The market beta, $\beta_{1,p}$, and liquidity betas, $\beta_{2,p}, \beta_{3,p}, \beta_{4a,p}$, and $\beta_{4b,p}$, are computed by monthly portfolio value weighted average returns and portfolio illiquidity innovations. $t-statistics$, ratio of estimated β and bootstrap standard error, are reported in parentheses. Bootstrap standard errors are computed from 10,000 simulated realizations. Column 6-11 in this table presents portfolio characteristics. $E\left[c_p\right]$ column reports average illiquidity of portfolio p and $\sigma\left(\Delta c_p\right)$ column reports standard deviation of portfolio p's illiquidity innovations. $E\left[r_{e,p}\right]$ and $\sigma\left(r_p\right)$ are average and standard deviation of value weighted monthly portfolio excess returns for portfolio p. Portfolio turnover (trn) and market capitalization (Size) are reported in the last two columns. To save space, I only report properties of odd numbered portfolios.

	Estimated Portfolio Betas					Portfolio Characteristics					
	$\beta_{1,p}$ $(\cdot 100)$	$\beta_{2,p}$ $(\cdot 100)$	$\beta_{3,p}$ $(\cdot 100)$	$\beta_{4a,p}$ $(\cdot 100)$	$\beta_{4b,p}$ $(\cdot 100)$	$E\left[c_p\right]$ (%)	$\sigma\left(\Delta c_p\right)$ (%)	$E\left[\pi_p\right]$ (%)	$\sigma\left(r_p\right)$ (%)	trn (%)	Size (bl$)
1	48.03	0.01	-1.67	-0.01	-0.00	0.25	0.05	0.67	1.66	8.82	45.94
	(20.10)	(0.98)	(-2.96)	(-0.98)	(0.43)						
3	57.15	0.00	-1.98	-0.00	-0.00	0.25	0.00	0.83	1.92	11.96	6.42
	(21.81)	(3.90)	(-2.67)	(-4.48)	(-2.59)						
5	64.34	0.00	-2.29	-0.01	-0.00	0.26	0.00	0.87	2.06	13.42	2.77
	(24.52)	(0.32)	(-2.67)	(-2.44)	(-2.66)						
7	67.04	0.00	-2.07	-0.01	-0.01	0.27	0.01	0.85	2.18	12.71	1.66
	(25.20)	(0.59)	(-2.19)	(-3.19)	(-2.74)						
9	69.34	0.00	-2.04	-0.03	-0.01	0.28	0.01	1.01	2.29	12.26	1.09
	(23.86)	(0.56)	(-2.31)	(-4.39)	(-3.12)						
11	72.91	0.00	-2.00	-0.07	-0.02	0.30	0.02	0.90	2.40	10.92	0.77
	(24.32)	(0.25)	(-2.01)	(-4.39)	(-2.61)						
13	72.50	0.00	-2.56	-0.10	-0.03	0.35	0.03	0.83	2.53	10.29	0.55
	(24.02)	(0.71)	(-2.63)	(-4.79)	(-2.49)						
15	74.10	0.00	-2.47	-0.15	-0.04	0.42	0.05	1.01	2.60	9.35	0.41
	(24.44)	(0.89)	(-1.97)	(-4.27)	(-2.37)						
17	75.05	0.01	-3.43	-0.32	-0.06	0.55	0.10	1.09	2.71	8.27	0.30
	(22.66)	(1.06)	(-3.81)	(-4.36)	(-1.54)						
19	72.26	0.02	-2.53	-0.55	-0.13	0.82	0.18	1.07	2.79	7.74	0.21
	(22.45)	(1.41)	(-2.36)	(-4.41)	(-2.13)						
21	68.96	0.08	-2.30	-0.63	-0.59	1.42	0.39	1.08	2.91	6.32	0.15
	(22.18)	(1.64)	(-2.28)	(-2.20)	(-2.49)						
23	59.40	0.23	-3.09	-1.51	-0.85	3.25	0.72	1.00	3.02	5.25	0.10
	(17.88)	(1.58)	(-4.48)	(-3.03)	(-2.05)						
25	61.51	1.69	-2.12	-4.19	-1.55	9.75	1.92	0.79	3.79	5.22	0.05
	(14.93)	(3.15)	(-2.97)	(-3.93)	(-2.30)						

cludes NYSE/AMEX and NASDAQ stocks from 1973 to 2017. The estimated betas and portfolio characteristics and the LCAPM estimation results are reported in Table 11 and Table 12.

Consistent with portfolios with NYSE/AMEX stocks, portfolios with NYSE/AMEX and NASDAQ stocks on average show that more illiquid portfolios tend to have greater exposures to liquidity betas. Total liquidity risk exposures, $100\cdot(\beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p})$, increase from 1.69 for the most liquid portfolio to 9.55 for the most illiquid portfolio. In terms of the LCAPM estimation, the fit of the model is low, R^2=0.33. However, the estimated overall risk premium, λ^{net} is similar to the price of risk estimated in Section 3.5.2. The estimated annualized return premium associated with the macroeconomic shock beta and the financial shock beta is 0.50% and 0.19% respectively.

Table 12: LCAPM Estimation for 25 Illiquidity Portfolios

This table reports cross-sectional liquidity-adjusted CAPM estimation results. The estimation is based on the 25 size portfolio monthly value weighted returns for 1974-2017 (44 years). The sample includes NYSE/AMEX/NASDAQ stocks. The regression model is $E[r_{p,t} - r_{f,t}] = \alpha + \mathcal{K}E\left[c_{p,t}\right] + \lambda_{net}\beta_{net}$ where $\mathcal{K} \in \{\bar{\kappa}, \kappa, 0\}$. $\bar{\kappa}$ is average monthly turnover for all stocks (0.095) and κ indicate a free parameter. $\beta_{net,p}$ is the overall LCAPM beta of portfolio p, $\beta_{net,p} \equiv \beta_{1,p} + \beta_{2,p} - \beta_{3,p} - \beta_{4a,p} - \beta_{4b,p}$. t-statistics in parentheses are estimated by GMM framework. The last column reports adjusted R^2s obtained from OLS.

	Constant	$E\left[c_p\right]$	$\beta_{1,p}$	$\beta_{2,p}$	$\beta_{3,p}$	$\beta_{4a,p}$	$\beta_{4b,p}$	$\beta_{net,p}$	R^2
1	0.017	0.095						1.123	0.07
	(0.039)	(—)						(1.781)	
2	0.212	0.003						1.006	0.33
	(1.182)	(0.324)						(3.967)	
3	0.218							1.003	0.36
	(1.216)							(3.949)	
4	0.190	0.095	10.752					-9.386	0.74
	(0.817)	(—)	(8.175)					(-7.073)	
5	0.217	-0.090	-11.867					12.487	0.47
	(1.391)	(-2.709)	(-2.852)					(3.098)	
6	0.204		-0.850					1.834	0.35
	(1.151)		(-0.850)					(1.817)	
7	0.153	0.095	1.034	-65.723	-0.468	4.474	-26.544		0.91
	(1.115)	(—)	(4.576)	(-3.761)	(-0.113)	(0.428)	(-2.005)		

5 Conclusion

Acharya and Pedersen (2005) derive the LCAPM and find that the covariation between firm liquidity costs and market returns (β_4) is the most important source of liquidity risk. In this study, making the use of the return decomposition of Campbell and Shiller (1988), I show that β_4 has two sub-components: the macroeconomic shock beta and the financial shock beta.

I first show that the return on the market portfolio can be decomposed into two components: macroeconomic shocks (shocks to interest rates and aggregate expected dividends) and financial shocks (shocks to the market risk premium). The macroeconomic shock beta measures firm liquidity costs covariation with macroeconomic shocks and the financial shock beta measures firm liquidity costs covariation with financial shocks.

Using 1964-2017 US stock market data, I find that both sub-betas are priced, but the expected return premium associated with firm liquidity costs covariation with macroeconomic shocks is approximately three times larger than that for the covariation between firm liquidity costs and financial shocks. The threefold difference in expected return premium can be explained by the nature of the systematic shocks that covary with firm liquidity costs.

Both adverse financial shocks and macroeconomic shocks reduce value of the market portfolio (and hence investor's wealth), but only financial shocks improve future investment opportunities (Campbell and Vuolteenaho, 2004). Therefore, investor is more sensitive to macroeconomic shocks, and requires greater return premium for stocks whose liquidity costs strongly covary with macroeconomic shocks.

The strong pricing effect of the macroeconomic shock beta also implies that macroeconomic shocks affect asset prices not only through its covariation with stock returns (Campbell and Vuolteenaho, 2004), but also through its covariation with firm liquidity costs.

References

Acharya, V. V., Amihud, Y., and Bharath, S. T. (2013). Liquidity risk of corporate bond returns: A conditional approach. *Journal of Financial Eocnomics*, 110 (2):358–386.

Acharya, V. V. and Pedersen, L. H. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics*, 77:375–410.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of financial markets*, 5 (1):31–56.

Amihud, Y. (2014). The pricing of the illiquidity factor's systematic risk. Available at SSRN http://ssrn.com/abstract=2411856.

Amihud, Y. and Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of financial Economics*, 17 (2):223–249.

Amihud, Y., Mendelson, H., and Pedersen, L. H. (2006). Liquidity and asset prices. *Foundations and Trends in Finance*, 1 (4):269–364.

Amihud, Y. and Noh, J. (2018). Illiquidity and stock returns – II: Cross-section and time-series effects. Available at SSRN: https://ssrn.com/abstract=3139180.

Angel, J. J. (1997). Tick size, share prices and stock splits. *Journal of Finance*, 2:655–681.

Atkins, A. B. and Dyl, E. A. (1997). Market structure and reported trading volume: Nasdaq versus the nyse. *Journal of Financial Research*, 20 (3):291–304.

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9:3–18.

Ben-Rephael, A., Kadan, O., and Wohl, A. (2015). The diminishing liquidity premium. *Journal of Finanical and Quantitative Analysis*, 50 (1):197–229.

Boyle, G. (2005). Risk, expected return, and the cost of equity capital. *New Zealand Economic Papers*, 39 (2):181–194.

Brennan, M. J. and Subrahmanyam, A. (1996). Market microstructure and asset pricing: On the compensation for illiquidity in stock returns. *Journal of financial economics*, 41 (3):441–464.

Campbell, J. Y. and Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*, 1 (3):195–228.

Campbell, J. Y. and Vuolteenaho, T. (2004). Bad beta, good beta. *American Economic Review*, 94 (5):1249–1275.

Chordia, T., Roll, R., and Subrahmanyam, A. (2000). Commonality in liquidity. *Journal of Financial Economics*, 56 (1):3–28.

Coughenour, J. and Saad, M. (2004). Common market makers and commonality in liquidity. *Journal of Financial Economics*, 73:37–69.

Fama, E. F. and French, K. R. (1992). The cross-section of expected stock returns. *The Journal of Finance*, 47:427–465.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33:3–56.

French, K. R., Schwert, G. W., and Stambaugh, R. F. (1987). Expected stock returns and volatility. *Journal of financial Economics*, 19 (1):3–29.

Harris, L. E. (1994). Minimum price variations, discrete bid–ask spreads, and quotation sizes. *Review of Financial Studies*, 7:149–178.

Hasbrouck, J. and Seppi, D. J. (2001). Common factors in prices, order flows and liquidity. *Journal of Financial Economics*, 59 (3):383–411.

Huang, C. F. and Litzenberger, R. H. (1988). *Foundations for Financial Economics*. Prentice Hall.

Huberman, G. and Halka, D. (2001). Systematic liquidity. *Journal of Financial Research*, 24 (2):161–178.

Jones, C. M. (2002). A century of stock market liquidity and trading costs. Available at SSRN https://ssrn.com/abstract=313681.

Lee, K. (2011). The world price of liquidity risk. *Journal of Financial Economics*, 99:136–161.

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory analysis. *Journal of Financial Economics*, 8:323–361.

Newey, W. K. and West, K. D. (1994). Automatic lag selection in covariance matrix estimation. *Review of Economic Studies*, 61:631–653.

Pástor, L. and Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. *Journal of Political economy*, 111 (3):642–685.

Vu, V., Chai, D., and Do, V. (2015). Empirical tests on the liquidity-adjusted capital asset pricing model. *Pacific-Basin Finance Journal*, 35:73–89.

Vuolteenaho, T. (2002). What drives firm-level stock returns? *The Journal of Finance*, 57 (1):233–264.