Trading Costs and Market Microstructure Invariance: Identifying Bet Activity

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Abstract

Market microstructure invariance (MMI) stipulates that trading costs of financial assets are driven by the volume and volatility of bets, but these variables are inherently difficult to identify. With futures transactions data, we estimate bet volume as the trading volume of brokerage firms that trade on behalf of their clients and bet volatility as the trade-related component of futures volatility. We find that the futures bid-ask spread lines up with bet volume and bet volatility as predicted by MMI, and that intermediation by high frequency traders does not interfere with the MMI relation.

Key words: Market microstructure invariance, bet volume, bet volatility, transactions costs

1. Introduction

Kyle and Obizhaeva (2016a) propose the market microstructure invariance (MMI) theory. It stipulates that the distributions of risk transfers and transactions costs are constant across assets and over time, if trades are converted into bets, trading time into business time, and return volatility into bet volatility (Kyle, Obizhaeva and Kritzman, 2016). Bets are transactions intended to produce idiosyncratic gains based on investors' beliefs and can be thought of as portfolio managers' parent orders that are typically split into several trades.[1] Business time refers to the calendar time between bets, and bet volatility is the part of return volatility that is attributable to order flow imbalances, and not driven by public information.[2] The execution of bets affects transactions costs by generating market impact due to adverse selection.

In real financial markets, bets are difficult to distill from trades, which makes it hard to test MMI empirically. Our solution is to estimate bet volume as the trading volume of brokerage firms that trade on behalf of their clients and bet volatility as the trade-related component of return volatility. Regulatory data on futures transactions with trading firm IDs allow us to separate proprietary trading by high-frequency trading (HFT) firms from client-induced trading, and we assume that the latter constitutes bet volume. We use the decomposition technique from Hasbrouck (1991) to isolate bet volatility with tick-by-tick trade and quote futures data.

According to Kyle and Obizhaeva (2016a), the MMI implies a specific relation between the percentage bid-ask spread and the dollar bet activity of an asset, where dollar bet activity equals

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[1] Kyle and Obizhaeva (2016) define a bet as a "metaorder", i.e., a collection of trades that belong to the same trading decision of a single trader.

[2] Mandelbrot and Taylor (1967), Clark (1973), Jones, Kaul and Lipson (1994), and Ane and German (2000), among others, study the relationships between the number of trades, trading volume, and return variance, in business time.

the product of bet volatility and bet dollar volume.[3] We use our measures of bet volatility and

bet volume to construct a measure of dollar bet activity, and to empirically test the MMI by

regressing the percentage futures bid-ask spread on the dollar bet activity measure. Our findings

show that the futures bid-ask spread lines up with dollar bet activity, and, thus, bet volume and

bet volatility, as predicted by the MMI.

We perform the regressions by aggregating our measures of bid-ask spreads, bet activity, and

bet volatility on a daily and intraday basis, following the methodology in Andersen, Bondarenko,

Kyle and Obizhaeva (2018). Our regression results support MMI in both aggregation settings.

However, we find that MMI does a relatively worse job in describing the intraday pattern of the

futures bid-ask spread than the one across days. The reason is that the minimum tick size regime

in the futures market creates a boundary restriction for the bid-ask spread. MMI predicts that

the futures bid-ask spread level is positively related to bet volatility and the futures price, and

negatively related to bet volume. Our results show that the futures spread becomes binding with

the tick size towards the end of the trading day, when bet activity levels are high. Thus, we claim

that the futures tick size is too large as it hinders the spread from reaching levels low enough to

adapt to the large bet activity as predicted by MMI.

Andersen et al. (2018) translate the MMI into the intraday trading invariance (ITI) theory, in

which they assume that the distributions of risk transfers and transactions costs are invariant

without converting trades into bets, trading time into business time, and return volatility into

bet volatility. The intuition is that traders trade more often, and in smaller lots, when the

volatility is high. Andersen et al. (2018) stress that the auxiliary assumptions for MMI to carry

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[3] Kyle and Obizhaeva (2016b) derive the invariance relationships from dimensional analysis, leverage neutrality, and the market invariance hypothesis, while Kyle and Obizhaeva (2020) use a theoretical model of informed trading with different beliefs for the same purpose.

over to the ITI setting are very strict. Kyle and Obizhaeva (2016a) introduce a volatility multiplier that measures the fraction of return volatility attributed to bet volatility, and a volume multiplier that determines the ratio between bet volume and total trading volume. When the multipliers are assumed to be constants, MMI translates into ITI in the sense that MMI can be tested by regressing the percentage bid-ask spread on trading activity rather than bet activity.

We estimate the volatility multiplier and the volume multiplier from our measures of bet volatility and bet volume. Our results show that the multipliers vary somewhat both on a daily and intra-daily basis. Moreover, the multipliers are correlated with futures return volatility and trading volume. However, when we evoke the assumption of constant multipliers, and test the ITI version of MMI, we find that the percentage futures bid-ask spread lines up with trading activity in an almost identical way as to bet activity.

We contribute to previous research in several ways. First, by estimating bet activity and bet volatility we directly test if the relationship between transactions costs, bet activity, and bet volatility, implied by MMI, holds. We focus on the invariance of transactions costs while most previous studies study the invariance of the distributions of risk transfers. One notable exception is Rizopoulos (2018), who studies the invariance of transactions costs in the ITI version, without estimating bet activity and bet volatility, in the UK equity market. Papers that study the invariance of the distributions of risk transfers approximate bets with trades (Andersen et al., 2018), portfolio transitions related to rebalancing decisions made by institutional investors and executed by brokers (Kyle and Obizhaeva, 2016a), prints from the US equity TAQ data set (Kyle, Obizhaeva and Tuzun, 2020), and counting the time when traders change their trading direction from buying to selling or selling to buying (Bae, Kyle, Lee and Obizhaeva, 2016).

4

Second, we estimate the volatility multiplier and the volume multiplier and analyze their characteristics on a daily and intraday basis. According to Kyle and Obizhaeva (2016a), the assumptions that the multipliers are constants or time varying are important for testing the MMI predictions empirically. Also, if the multipliers are correlated with volatility and volume, empirical estimates of parameters predicted by invariance may be biased. We contribute by studying the validity of the assumption that the multipliers are constant in the test of MMI.

Third, we contribute to the literature of the determinants of futures transactions costs. Several studies investigate the relationship between transactions costs, trading volume and volatility on futures markets. Wang and Yau (2000) study the relations between trading volume, the bid-ask spread, and price volatility for financial and metal futures, and find a positive relationship between trading volume and price volatility and a negative relationship between trading volume and the bid-ask spread. Ding (1999) analyzes the determinants of bid-ask spreads in the foreign exchange futures market and reports that the number of transactions is negatively related to the bid-ask spread, whereas volatility in general is positively related to it. Xu (2014) examines the intertwined dynamics between trading patience, order flows, and liquidity in the OMXS 30 index futures and finds that a higher proportion of patient traders and a higher order arrival rate leads to a smaller spread. Our paper adds to these papers by documenting that the bid-ask spread is positively related to bet volatility and the futures price, and negatively related to bet activity, in a particular functional form suggested by MMI.

Another strand of literature discusses the relationship between the tick size and futures transactions costs. Kurov and Zabotina (2005) document that the minimum tick sizes of the E-mini S&P 500 and E-mini Nasdaq-100 futures contracts act as binding constraints on the bid-ask spreads by not allowing the spreads to decline to competitive levels. The authors suggest that

the Chicago Mercantile Exchange (CME) should decrease the minimum tick sizes of the S&P 500 and Nasdaq-100 E-mini futures contracts, and they claim that a tick size reduction is likely to result in lower futures trading costs. Indeed, in 2006, the CME reduced the minimum tick size of the floor-traded and the E-mini Nasdaq-100 futures from 0.5 to 0.25 index points. In a follow-up article, Kurov (2008) investigates this event and finds a significant reduction in the effective spreads in the E-mini futures. He also finds that the tick size reduction improves price discovery and informational efficiency. Similar results of improved liquidity and lowered transaction costs are found in Alampieski and Lepone (2008), for the tick size reduction in treasury bond futures traded on Sydney Futures Exchange, and in Martineza and Tse (2019), for the tick size reduction in foreign currency futures contracts on the Chicago Mercantile Exchange.

Complementing previous studies, we find that the current minimum tick size regime in the OMXS 30 index futures creates a boundary restriction for the bid-ask spread, and the futures spread becomes binding with the tick size especially towards the end of the trading day. Our main contribution to this strand of the literature is to analyze the tick size boundary conditions within the MMI framework, where bet activity is the key determinant of bid-ask spreads. Not only can we draw the, somewhat trivial, conclusion that the bid-ask spread often is binding with the tick-size, but by obtaining the implied bid-ask spread from MMI, we also learn how large the spread would be in a frictionless setting, given the level of bet activity in the market.

Our results offer important policy implications for exchange decision makers and regulators in financial markets. In the equity market, various tick size regimes are implemented. For example, in the US, the tick size equals one penny for all stocks with a price above one dollar, while in European equity markets, tick sizes vary with the price level and trading activity of stocks. The current debate, in both the US and in Europe, concerns how the "optimal" (Graziani and Rindi,

2022), or even the "intelligent" (Nasdaq, 2019), tick size should be determined. Our suggestion is to use the MMI framework and determine the asset tick size so that its bid-ask spread is allowed to line up with bet volatility, bet activity, and price level for the asset in question. If bet variables are difficult to obtain, we suggest to instead use the ITI framework, where return volatility, trading activity, and price level determine bid-ask spreads, and thus, also tick sizes. Roughly speaking, we suggest adapting the European equity market model by adding return volatility as a third reference point to the first two, namely, trading activity and price level, when determining tick sizes. Our suggested model (ITICKS) implies a relatively wide tick size for an asset with high return volatility, low trading activity, and high price level.

2. Theory and empirical framework

In this section, we briefly present the MMI theory according to Kyle and Obizhaeva (2016a) for the purpose of achieving an empirical testing framework. Although we perform tests with data from the index futures market, the following presentation applies for any financial asset. In the following, we first go through MMI and derive a testable regression equation for the futures bid-ask spread as a function of bet activity. Second, we show that the ITI concept of Andersen et al. (2018) is a special case of MMI. Finally, we present our measures for bet volatility and bet volume that enable us to measure bet activity, and we provide the multipliers necessary for converting MMI to ITI, or vice versa.

2.1 Market microstructure invariance

MMI stipulates two invariance principles that (i) the distributions of risk transfers, and (ii) transactions costs, are constant over business time. Business time is defined as the expected time

between the arrival of bets to the asset market, and bet activity is assumed to drive both the risk and transaction costs of an asset.

In the first principle of MMI, the volatility per bet is proportional to the expected dollar bet size. Kyle and Obizhaeva (2016a) formulate the first principle as that the random variable $\tilde{I}_\tau$ has an invariant distribution across all intervals τ:

$$\tilde{I}_\tau = P_\tau \tilde{Q}_\tau^B \tilde{\sigma}_\tau^B / (\tilde{N}_\tau^B)^{1/2}. \tag{1}$$

Using the notation in Andersen et al. (2018), for each interval τ, the random realization of a variable during the interval is indicated by the tilde on top of the variable. Likewise, by omitting the tilde, we denote the expected value of the variable in question in interval τ, conditional on the information available from interval $\tau - 1$. Thus, P_τ denotes the average futures price (dollar per contract) in interval τ, $\tilde{Q}_\tau^B$ is the average bet size (number of contracts per bet) in interval τ, $\tilde{\sigma}_\tau^B$ is the average percentage bet volatility per unit of time, and $\tilde{N}_\tau^B$ is the bet arrival rate per unit of time. In addition, bet volume (number of contracts per unit of time) equals $\tilde{V}_\tau^B = \tilde{N}_\tau^B \tilde{Q}_\tau^B$. The invariance of $\tilde{I}_\tau$ in Eq. (1) implies that betting agents adjust their bet sizes and bet arrival rates to control the risk (volatility) of their futures bets.

The second principle of the MMI theory according to Kyle and Obizhaeva (2016a) relates to the invariance of transactions costs. MMI implies that the percentage bid-ask spread is proportional to the product of bet volatility and bet activity to the power of $-1/3$ as[4]

$$\frac{\widetilde{SPR}_\tau}{P_\tau} = \widehat{RSPR}_\tau = K^B \tilde{\sigma}_\tau^B (\tilde{W}_\tau^B)^{-1/3}, \tag{2}$$

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[4] See Eq. (19) in Kyle and Obizhaeva (2016a).

where $\widehat{SPR}_\tau$ is the dollar bid-ask spread in interval τ, $\widehat{RSPR}_\tau$ is the percentage bid-ask spread, K^B is a constant, and $\widetilde{W}_\tau^B = \tilde{\sigma}_\tau^B P_\tau \tilde{V}_\tau^B$ is the bet activity as defined by Kyle and Obizhaeva (2016a).

Taking the natural logarithm of both sides in Eq. (2) yields

$$\log\big(\widehat{RSPR}_\tau\big) = \log(K^B) + \log(\tilde{\sigma}_\tau^B) - \frac{1}{3}\log\big(\widetilde{W}_\tau^B\big), \tag{3}$$

which can be rewritten as

$$\widehat{rspr}_\tau - \frac{1}{2}\tilde{s}_\tau^B = k^B - \frac{1}{3}\widetilde{w}_\tau^B, \tag{4}$$

where $\widehat{rspr}_\tau = \log\big(\widehat{RSPR}_\tau\big)$, $\tilde{s}_\tau^B = 2\log(\tilde{\sigma}_\tau^B)$, $k^B = \log(K^B)$ and $\widetilde{w}_\tau^B = \log\big(\widetilde{W}_\tau^B\big)$. By taking the expectation in Eq. (4), conditional on information at time $\tau - 1$, and adding an error term, we arrive at the following equation:

$$rspr_\tau - \frac{1}{2}s_\tau^B = k^B - \frac{1}{3}w_\tau^B + \varepsilon_\tau^B, \tag{5}$$

where ε_τ is a zero-mean residual. Testing the transactions costs part of the MMI theory boils down to regressing the bet variance-adjusted relative bid-ask spread on the left-hand side of Eq. (5) on the bet activity variable w_τ^B. Accordingly, the MMI theory can be rejected if the associated regression coefficient is significantly different from $-1/3$.

In the spirit of Andersen et al. (2018), it is possible to back out the implied average bid-ask spread from the left-hand side of Eq. (5) as a function of the average bet activity w_τ^B, the average bet variance s_τ^B, and the average futures price p_τ as

$$spr_\tau^* = k^* - \frac{1}{3}w_\tau^B + \frac{1}{2}s_\tau^B + p_\tau, \tag{6}$$

where the constant k^* is identified through the moment condition $E(\varepsilon_\tau - k) = 0$ from Eq. (5). When compared with the actual bid-ask spread, the implied spread from Eq. (6) could be seen as a metric of how well MMI works. Eq. (6) also showcases the intuition of MMI when it comes to the determinants of transactions costs. Accordingly, the bid-ask spread is positively related to bet volatility and the futures price, and negatively related to bet activity.

2.2 Intraday trading invariance

Andersen et al. (2018) introduce ITI with a focus on the first principle of MMI. By extending their framework to the second principle, which also Rizopoulos (2018) does, it follows that the percentage bid-ask spread is proportional to the product of return volatility and trading activity, rather than bet volatility and bet activity as in Eq. (2), to the power of $-1/3$ as

$$\frac{\widetilde{SPR}_\tau}{P_\tau} = \widehat{RSPR}_\tau = K\tilde{\sigma}_\tau \widetilde{W}_\tau^{-1/3}, \tag{7}$$

where K is a constant, $\tilde{\sigma}_\tau$ is the average percentage return volatility per unit of time, $\widetilde{W}_\tau = \tilde{\sigma}_\tau P_\tau \tilde{V}_\tau$ is trading activity, and $\tilde{V}_\tau$ is the trading volume (number of contracts per unit of time).

The expression in Eq. (7) can be transformed into a regression equation by taking the natural logarithm of both sides, taking the expectation, conditional on information at time $\tau - 1$, and adding an error term ε_τ as

$$rspr_\tau - \frac{1}{2}s_\tau = k - \frac{1}{3}w_\tau + \varepsilon_\tau. \tag{8}$$

10

where $s_\tau = 2\log(\tilde{\sigma}_\tau)$, $k = \log(K)$ and $w_\tau = \log(\widetilde{W}_\tau)$.

Thus, we can test the ITI version of the second principle of the MMI theory by regressing the return variance-adjusted relative bid-ask spread on the trading activity variable w_τ. If the ITI version of the MMI theory holds, the associated regression coefficient should not be significantly different from $-1/3$.

2.3 Multipliers

The variables bet variance and bet activity are not easily observed in data, which makes it difficult to directly test MMI. For ITI to correspond to MMI, we need restrictions on the trading variables s_τ and w_τ relative the bet variables s_τ^B and w_τ^B. Here, we present measures of the bet variables and of time-varying multipliers that relate the bet variables to the corresponding trading variables.

Bet volatility

Kyle and Obizhaeva (2016a) and Kyle et al. (2016) define bet volatility $\tilde{\sigma}_\tau^B$ as the fraction of return volatility $\tilde{\sigma}_\tau$ that is due to order flow imbalances caused by bet activity, and not driven by public information that is incorporated into prices without trading, in the following way: $\tilde{\sigma}_\tau^B = \tilde{\psi}_\tau \tilde{\sigma}_\tau$, where $0 \leq \tilde{\psi}_\tau \leq 1$ is the volatility multiplier measured over the interval τ. Hence, the natural logarithm of (expected) bet variance is a function of the corresponding return variance according to $s_\tau^B = 2\log\psi_\tau + s_\tau$.

We estimate ψ_τ from observed futures trades and quotes using the decomposition technique from Hasbrouck (1991). Let the time scale (s) be the transaction sequence so that a futures trade at time s during the interval τ is represented by the indicator variable $x_s = +1$ for a buyer-

initiated trade, $x_s = -1$ for a seller-initiated trade, and $x_s = 0$ for an unclassified trade. Denote the logarithm of the futures midpoint quote prior to the trade at time s as p_s, and the corresponding change in the logarithm of the midpoint quote since prior to the previous trade at time $s-1$ as r_s. Following Hasbrouck (1991), and using the notation in Barclay, Hendershott and McCormick (2003), we estimate the following vector autoregression (VAR) model:

$$r_s = \sum_{i=1}^{p} \alpha_i r_{s-i} + \sum_{i=0}^{p} \beta_i x_{s-i} + \varepsilon_{1,s}, \tag{9}$$

$$x_s = \sum_{i=1}^{p} \gamma_i r_{s-i} + \sum_{i=1}^{p} \delta_i x_{s-i} + \varepsilon_{2,s}, \tag{10}$$

where $\varepsilon_{1,s} \sim N(0, \sigma_{\varepsilon_1}^2)$, $\varepsilon_{2,s} \sim N(0, \sigma_{\varepsilon_2}^2)$, and $E\varepsilon_{1,r}\varepsilon_{1,s} = E\varepsilon_{2,r}\varepsilon_{2,s} = E\varepsilon_{1,r}\varepsilon_{2,s} = 0$ for lags $r < s$.

We invert the estimated VAR system into the vector moving average (VMA) model:

$$\begin{pmatrix} r_s \\ x_s \end{pmatrix} = \begin{pmatrix} a(L) & b(L) \\ c(L) & d(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{1,s} \\ \varepsilon_{2,s} \end{pmatrix}, \tag{11}$$

where $a(L)$, $b(L)$, $c(L)$, and $d(L)$ are lag polynomial operators.

Consistent with Hasbrouck (1991), we decompose the logarithm of the futures midpoint p_s into a permanent random-walk component m_s and a transitory stationary component s_s:

$$p_s = m_s + s_s, \tag{12}$$

where $m_s = m_{s-1} + v_s$, $v_s \sim N(0, \sigma_v^2)$, and $Ev_r v_s = 0$ for lags $r \neq s$. From the definition that $r_s = p_s - p_{s-1}$, and denoting $\Delta s_s = s_s - s_{s-1}$, we obtain $r_s = v_s + \Delta s_s$. Defining $\sigma_s^2 = E\Delta s_s^2$, we write the futures return variance as the sum of a permanent component and a transitory component:

$$\sigma_p^2 = \sigma_v^2 + \sigma_s^2. \tag{13}$$

Accordingly, the futures return variance is decomposed into an efficient part due to changes in the "true" futures value, and an inefficient part caused by microstructure imperfections. Using the VMA model above, we further decompose the efficient part of the variance into a trade-unrelated component and a trade-related component:

$$\sigma_v^2 = \left(\sum_{i=0}^{\infty} a_i\right)^2 \sigma_{\varepsilon_1}^2 + \left(\sum_{i=0}^{\infty} b_i\right)^2 \sigma_{\varepsilon_2}^2. \tag{14}$$

As stated by Hasbrouck (1991), the first component of the efficient variance in Eq (11) reflects the arrival of public information that is incorporated in futures prices through quote updates, without trading. The second component is caused by the arrival of private information through futures trading. The first component is not contributing to bet volatility. Thus, we measure ψ_τ as

$$\psi_\tau = \sqrt{1 - \frac{\sigma_{\varepsilon_1}^2}{\sigma_p^2}\left(\sum_{i=0}^{\infty} a_i\right)^2}, \tag{15}$$

and use it to convert s_τ into s_τ^B.

Bet volume

Following Kyle and Obizhaeva (2016a) and Kyle et al. (2016), bet volume relates to trading volume through the volume multiplier ζ_τ as $\tilde{V}_\tau^B = 2\tilde{V}_\tau/\tilde{\zeta}_\tau$, or in natural logarithms and expected form as $v_\tau^B = \log 2 - \log \zeta_\tau + v_\tau$. If the volume multiplier is equal to one in interval τ, each trade

matches two bets, where one side is buying, and the other one is selling. The volume multiplier exceeds one if each bet generates trades among intermediaries other than the betting sides.

If bet volume is known, ζ_τ could be obtained from the ratio between bet volume and trading volume. In our futures data, we know the trading firm behind each trade. Thus, we have information about the initiators of proprietary trades that are carried out by trading firms for their own accounts. However, if the trading firms trade on behalf of clients, we are not able to discern the client IDs. As a result, it is difficult to figure out individual client bets and which trades, carried out by each trading firm, that belong to the individual client bets.

As a proxy for bet volume V_τ^B, we use the *broker volume*, which is the trading volume by brokerage firms that trade on behalf of clients. On the one hand, this proxy will overestimate the actual bet volume if the brokerage firms engage in market making that generates intermediary trades other than the bet trades. On the other hand, the proxy might underestimate the actual bet activity if proprietary trading firms engage in bet activity. However, with our data it is straightforward to estimate ζ_τ from the ratio of total futures trading volume to broker volume during the interval τ.

With the estimated volatility multiplier and volume multiplier we can relate bet activity to trading activity according to: $W_\tau^B = \sigma_\tau^B P_\tau V_\tau^B = \psi_\tau \sigma_\tau P_\tau 2 V_\tau / \zeta_\tau = 2\psi_\tau/\zeta_\tau W_\tau$. Hence, by converting s_τ into s_τ^B and w_τ into w_τ^B, we can run the regression for the bid-ask spread in Eq. (5). Note that when $\psi_\tau = 1$ and $\zeta_\tau = 2$, we have that $s_\tau^B = s_\tau$ and $w_\tau^B = w_\tau$. In this case, the MMI and ITI specifications for transactions costs coincide. In the more general case, when ψ_τ and ζ_τ are constants, but not necessarily equal to 1 and 2 respectively, the regressions in Eq. (5) and Eq. (8)

differ only with respect to the constant terms k^B and k. Hence, when the multipliers are constant, both regressions yield a slope coefficient equal to $-1/3$ under both MMI and ITI.

Kyle and Obizhaeva (2016a) note that the assumptions that ψ_τ and ζ_τ are constants are important for testing the predictions of MMI empirically, and that these assumptions should be tested empirically. Moreover, Kyle and Obizhaeva (2016a) state that if ψ_τ and ζ_τ are correlated with V_τ and σ_τ in the ITI setting, empirical estimates of parameters predicted by invariance may be biased. The authors call for an empirical examination of these correlations, so that necessary adjustments can be made in tests of the invariance hypotheses. In a response to their call, we perform a correlation analysis of the multipliers.

3. Institutional setting and data

3.1 Description of the OMXS futures market

The OMXS 30 index futures are traded at Nasdaq Stockholm (henceforth Nasdaq). The OMXS 30 index consists of the 30 most actively traded and largest stocks on the same exchange, which is revisited every six months. The futures trading is consolidated to Nasdaq. It is, however, possible to trade the OMXS 30 index futures at other venues, but this rarely happens during our sample period. We focus on the transactions at Nasdaq and exclude other transactions (0.44%).

The trading environment for the OMXS 30 index futures constitutes an electronic limit order book. Traders can choose to submit market orders or limit orders, which are executed and stored according to the price-visibility-time priority rule. Limit orders posted on the same price level are executed according to the time of submission, except that visible orders have higher priority than hidden orders. Only members of the exchange, either dealers, brokers, or proprietary

trading firms, can trade directly through the Nasdaq. Some trading firms are designated market makers in the sense that they have contractual agreements with Nasdaq to provide liquidity.

Trading in the OMXS 30 index futures starts at 8:45 AM with an opening call auction which uncrosses at 9:00 AM. Continuous trading through the limit order book is possible during trading hours from 9:00 AM to 5:25 PM, when the closing call auction starts. If the day before a Swedish bank holiday is a trading day, the closing auction starts at 12:55 PM. The contract size equals 100 times the underlying index. The tick size for the OMXS 30 index futures contract is SEK 0.25. During our sample period, the futures price fluctuates in the range 1,238 and 1,679 (see Table 1). Hence, the relative tick size ranges between roughly 1.5 and 2.0 basis points (bps).

The OMXS 30 index futures contracts have different maturities. At any time, trading is possible in at least three futures contract series, with up to one, two, and three months left to maturity, respectively. On the third Friday of the expiration month, if it is a Swedish bank day, one contract series expires. If the day in question is not a Swedish bank day, or a half trading day, the contract series expires on the preceding bank day. A new expiration month series is listed four days prior to the expiration of the previous series. The futures contracts are settled in cash at maturity.

In addition to trading in the regular futures contracts, it is possible to trade calendar spreads in the OMXS 30 index futures. The OMXS 30 Roll contract is a standardized combination of trades in OMXS 30 futures. The OMXS 30 Roll technically implements a calendar spread strategy by selling the nearby contract and buying the second nearby contract simultaneously. Nasdaq automatically creates the combinations. The tick size for the OMXS 30 Rolls is 0.05 SEK compared to 0.25 SEK for the individual regular futures contracts. Calendar spreads trading is only active during the expiration weeks.

3.2 Data

We obtain tick-by-tick quote and trade records for the OMXS 30 index futures from Refinitiv Tick History (RTH), time-stamped at a microsecond granularity. The quote data include quote updates on the best bid and ask prices and the number of futures contracts available at the best bid and ask prices (i.e., the bid and ask size). The trade data include the execution prices and volumes. We follow Lee and Ready (1991) to classify whether a trade is buyer- or seller-initiated. In addition, we retrieve the intraday five-second data for the OMXS 30 index futures from RTH, which includes the best bid price, the best ask price, and the corresponding sizes at these prices.

To identify which trading firms are responsible for the OMXS 30 index futures trades, we use data from the Transaction Reporting System (TRS). All financial institutions supervised by one of the national financial supervisory authorities in the European Union must report their transactions to TRS, in accordance with the Markets in Financial Instruments Directive (MiFID). We get access to the TRS transaction data through the Swedish financial supervisory authority, Finansinspektionen. The TRS data include information on prices and volumes with a buy/sell indicator. The data also include identifiers for the trading firms responsible for their trades.

We use the identifiers to separate proprietary trading firms from brokerage firms. We follow Baron, Brogaard, Hagströmer and Kirilenko (2019) and identify trading firms that are members of the Futures Industry Association's European Principal Traders Association (FIA EPTA), which is an industry organization for principal trading firms, as high-frequency trading firms. These firms are known to be proprietary traders (see Aramian and Nordén, 2021, who perform a

similar classification of trading firms).[5] Moreover, trading firms that are not high-frequency traders are classified as brokerage firms.

In the TRS database, transactions are double reported with a time stamp down to the nearest second. For example, if trading firm X places a buy order of 100 contracts in OMXS 30 index futures and the order is executed against two sell orders, 40 contracts from trading firm Y and 60 contracts from trading firm Z, then this trade results in three entries, one from each trading firm X, Y and Z, respectively, in the TRS database.

In our analyses, we use all trades and quotes during continuous trading between 9:05 AM and 5:20 PM from RTH and TRS. We exclude the data during the first five and last five minutes of the continuous trading session to avoid possible issues due to the opening and closing auctions.

Our sample period is from January 4, 2016, to December 29, 2017, and half trading days are excluded. Following Andersen et al. (2018), we focus on the nearby futures contract (the one closest to maturity), which is the most actively traded contract. Each month, when the next nearby futures contract becomes more actively traded than the current one, we "roll over" to the next nearby contract. This rolling over always happens one or two trading days before the current nearby contract expires. We further exclude the trading days during the expiration weeks (with less than seven days to maturity) to avoid any calendar spread trades in the nearby contract and the next contract. The final sample period consists of 413 trading days.

3.3 Variables

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[5] Following Aramian and Nordén (2021), we also classify trading firms as high-frequency traders if they describe themselves as such on their websites.

MMI implies that the percentage bid-ask spread, adjusted for bet volatility, is proportional to bet activity to the power of $-1/3$. In this section, we explain how we measure the bid-ask spread, volatility, and volume that are relevant for our empirical investigations. Following the setting in Andersen et al. (2018), we measure all the variables in each interval for a given day. Specifically, the sample begins at time 0 and contains D trading days, each comprising T intraday intervals of length $\Delta t = 1/T$. Thus, the full sample includes the consecutive non-overlapping intervals $\tau = 1, \dots, D \times T$. To identify the specific trading day and the intraday period associated with a specific interval, we follow Andersen et al. (2018) and use the double-index notation (d, t), where $d \in D = \{1, \dots, D\}$ denotes the trading day, and $t \in T = \{1, \dots, T\}$ is the intraday interval.

We choose Δt to be five-minute intervals since only 0.26% of the intervals have a measured volatility equal to zero (no price changes), and thus, produce missing values of log volatility. Choosing, e.g., one-minute intervals instead produces 9.76% missing values of log volatility.

Quoted bid-ask spread and effective spread

Our first measure of the relative bid-ask spread $\widehat{RSPR}_\tau$ in Eq. (2) is the quoted bid-ask spread that we denote $\widehat{RSPR}_\tau^Q$. We measure $\widehat{RSPR}_\tau^Q$ as the average five-second quoted bid-ask spread for each τ, or alternatively, for each five-minute interval t and day d, according to:

$$\widehat{RSPR}_\tau^Q = \widehat{RSPR}_{d,t}^Q = \sum_{i=1}^{60} \frac{\left(ask_{i,d,t} - bid_{i,d,t}\right)}{60 \times mid_{i,d,t}} \tag{16}$$

where $bid_{i,t,d}$ and $ask_{i,t,d}$ are the best buy and sell prices, and $mid_{i,d,t}$ is the corresponding midpoint of these prices, which is our proxy of the futures price P_τ in Eq. (2), prevailing at the end of the five-second interval i, within the five-minute interval t, on day d.

As our second measure of the relative bid-ask spread, we obtain the effective bid-ask spread, denoted $\widehat{RSPR}^E_\tau$, for each τ as:

$$\widehat{RSPR}^E_\tau = \widehat{RSPR}^E_{d,t} = \sum_{s=1}^{S_{d,t}} \frac{2q_{s,d,t}(P_{s,d,t} - mid_{s,d,t})}{S_{d,t} \times mid_{s,d,t}} \tag{17}$$

where $P_{s,d,t}$ is the sth trade price in the trade sequence $s = 1, \ldots, S_{d,t}$, $mid_{s,d,t}$ is the midpoint price prevailing at the time of the trade, and $q_{s,d,t}$ is the trade side indicator, which is equal to +1 for a buyer-initiated trade and -1 for a seller-initiated trade, following Lee and Ready (1991), for the sth trade, during the five-minute interval t, on day d.

Bet volatility and volatility multiplier

We measure the return volatility from five-second squared midpoint returns for each τ, or alternatively, for each five-minute interval t on each day d, according to:

$$\tilde{\sigma}_\tau = \tilde{\sigma}_{d,t} = \sqrt{\sum_{i=1}^{60} \left[\ln(mid_{i,d,t}) - \ln(mid_{i-1,d,t})\right]^2}. \tag{18}$$

To estimate the bet volatility, we first need to estimate the volatility multiplier $\tilde{\psi}_\tau$. In the first estimation setting, we allow the volatility multiplier to vary across trading days but to be constant within the days. Hence, for each trading day d, we estimate $\tilde{\psi}_d$ according to Eq. (15) based on futures trades and quotes using the decomposition technique from Hasbrouck (1991) described in Eq. (9) to (14). In the second setting, we let the volatility multiplier vary on an intraday basis, but to be constant across trading days. Thus, for each five-minute interval t from 9:05 AM to 05:20 PM, we estimate $\tilde{\psi}_t$ according to Eq. (15) based on futures trades and quotes

20

to decompose the variance into bet volatility and non-bet volatility. We use five lags in the VAR

model and truncate the VMA at thirty lags following Hasbrouck (1991).[6]

Then, we follow Kyle and Obizhaeva (2016a) and Kyle et al. (2016), and obtain bet volatility as

$\tilde{\sigma}_d^B = \tilde{\psi}_d \tilde{\sigma}_d$ in the daily aggregation setting, and as $\tilde{\sigma}_t^B = \tilde{\psi}_t \tilde{\sigma}_t$ in the intraday aggregation setting.

Bet volume, volume multiplier, and bet activity

In the TRS data, we have information on the trading firm behind each entry with a buy/sell

indicator. For each interval τ, we obtain the bet volume, $\tilde{V}_\tau^B$ or $\tilde{V}_{d,t}^B$, as the trading volume by

brokerage firms. We also obtain the trading volume, $\tilde{V}_\tau$ or $\tilde{V}_{d,t}$, as the half of the total sum of

volume from all trading firms recorded in TRS, for each interval. Thus, we first calculate the

volume multiplier as $\tilde{\zeta}_\tau = 2\tilde{V}_\tau/\tilde{V}_\tau^B$ following Kyle and Obizhaeva (2016a) and Kyle et al. (2016)

for each interval τ. Then, we aggregate the volume multiplier in the daily setting to obtain $\tilde{\zeta}_d$ by

averaging across intraday periods on day d, and in the intraday setting to get $\tilde{\zeta}_t$ by averaging

across days in interval t. we

Finally, obtain trading activity $\widetilde{W}_\tau$ as $\tilde{\sigma}_\tau P_\tau \tilde{V}_\tau$ for each τ, where $P_\tau \tilde{V}_\tau$ equals half of the total sum of

dollar trading volume from all trading firms in the interval. We use both multipliers to get bet

activity $\widetilde{W}_\tau^B$ as $\tilde{\sigma}_\tau^B P_\tau \tilde{V}_\tau^B = \tilde{\psi}_\tau \tilde{\sigma}_\tau P_\tau 2\tilde{V}_\tau/\tilde{\zeta}_\tau = 2\tilde{\psi}_\tau/\tilde{\zeta}_\tau \widetilde{W}_\tau$, where $\tilde{\psi}_\tau$ and $\tilde{\zeta}_\tau$ are either $\tilde{\psi}_d$ and $\tilde{\zeta}_d$ in the

daily setting or $\tilde{\psi}_t$ and $\tilde{\zeta}_t$ in the intraday setting.

3.4 Descriptive statistics

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[6] We also estimate the VAR and VMA system using varying numbers of lagged trades and quote changes. Our results are not sensitive to the choice of the number of lags.

Table 1 presents descriptive statistics for our measures of futures return volatility, trading volume, trading activity, and different measures of liquidity. Each mean statistic is obtained as an average across all five-minute intervals during the sample period. The mean futures return volatility is 0.12 on an annual basis, and the mean double-counted trading volume equals more than 2,000 contracts per five-minute period. Double counting means that we measure volume from both the long and the short side of each futures trade to facilitate comparisons between bet and non-bet volume in subsequent analyses. Our measure of futures trading activity, which equals the product of return volatility, futures price, and trading volume amounts to an average of more than SEK 2,600 per five-minute period. In addition, a five-minute interval contains an average of 116 trades with an average size of six contracts per trade.

Insert Table 1 here

The average futures depth at the best quotes on both sides of the limit order book equals 46 contracts. Thus, the order book is liquid enough to accommodate an average trade of six contracts. Also, on average, the quoted futures bid-ask spread is just above the minimum quoted spread level 0.25, which is implied by the minimum tick size restriction. Thus, most of the time, the tick size puts a binding restriction on the quoted bid-ask spread. Relative to the midpoint of the bid and ask quotes, the quoted bid-ask spread equals 1.8 basis points on average. Numbers for the effective bid-ask spread are similar.

Andersen et al. (2018) report that, during US trading hours, the E-mini S&P 500 index futures is much more heavily traded, with an average one-minute trading volume of almost 5,000 contracts (not double counted), and more liquid, with an average depth of almost 1,000

contracts, than the OMXS 30 index futures. Both futures markets share the common feature of an average bid-ask spread being very close to the respective minimum tick size boundary.

4. Results

This section presents our main results. First, we show statistics for our estimates of bet volatility, bet volume, the volatility multiplier, and the volume multiplier, and we illustrate how the variables evolve on both a daily and an intraday basis. We test whether the multipliers are correlated with other variables. Then, we turn to the main results from the regressions of quoted and effective bid-ask spread and test the transaction costs principle of MMI.

4.1 Bet volatility and volatility multiplier

Table 2 reports descriptive statistics for the estimated bet volatility and volatility multiplier, obtained from the variance decompositions according to sections 2.3 and 3.3, for both the daily (Panel A) and the intraday (Panel B) frequency. The mean bet volatility is 0.115 across days, and 0.114 across intraday intervals, on an annual basis, and the corresponding volatility multiplier is 0.861 and 0.836, respectively. This implies that bets generate 86% and 84% of the return volatility on daily basis and during the five-minute intervals, respectively. We note that the standard deviation of the volatility multiplier is about twice as large when estimated across days than intraday. Across days, the volatility multiplier fluctuates in the range between 0.64 and 0.95, while the corresponding range, 0.75 to 0.91, is tighter on an intraday basis.

<center>Insert Table 2 here</center>

Figure 1 plots the daily time series of bet volatility, futures return volatility, and the volatility multiplier. Evidently, bet volatility and return volatility follow each other closely over time, and

<center>23</center>

both variables are much higher during the beginning of the sample than towards the end. Nevertheless, the volatility multiplier appears to oscillate around its daily mean 0.86 during the entire sample period.[7]

Insert Figure 1 here

In Figure 2, we illustrate the intraday time series of each variable bet volatility, return volatility, and the volatility multiplier. There is a clear commonality in the intraday variation in both the bet volatility and the return volatility in that they are at their respective highest level in the morning and diminish throughout the trading day with common jumps at 2:30 PM, when the US equity markets open, and at 3:30 PM, when several key news announcements occur. The volatility multiplier seems to fluctuate around the intraday mean 0.84 throughout the day, with, perhaps, more variability in the afternoon than in the morning.

Insert Figure 2 here

4.2 Bet volume and volume multiplier

We estimate bet volume as the trading volume by brokerage firms that trade on behalf of clients. Table 2 reports descriptive statistics for the estimated bet volume and volume multiplier for both the daily (Panel A) and the intraday (Panel B) frequency.

The average bet volume is 1,367 futures contracts, and the corresponding average bet multiplier is 1.68, both per day and per five-minute interval. From MMI, if the volume multiplier equals one, there are no intermediaries, and all trades are executed directly between the betting sides. If the

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[7] We do not conduct formal tests of whether the multipliers are constants. Instead, we perform subsequent bid-ask spread regressions under different assumptions about fluctuations of the multipliers. That way, we can evaluate how important the assumptions of constant multipliers are directly in the regression framework.

multiplier equals two, an intermediary is executing each bet. The value 1.68 implies that intermediaries are needed for some bets, but not for all. Sometimes, the betting sides manage to find each other directly in the futures limit order book.

Average bet volume is highly variable and ranges from a minimum of 312 contracts to a maximum of 3,568 contracts daily, and between from a minimum of 738 contracts and 4,144 contracts intraday. As for the volatility multiplier, the volume multiplier has a higher variability across days than across intraday intervals. The volume multiplier ranges between 1.03 and 2.35 across days, and between 1.43 and 1.79 across intraday intervals.

Insert Figure 3 here

Figure 3 shows the average bet volume, the average non-bet volume, and the average volume multiplier for each trading day in the sample. Both bet volume and non-bet volume (by intermediaries) exhibit large variability across trading days. The daily volume multiplier fluctuates around the average level 1.68.

We plot the intraday average bet volume and non-bet volume in Figure 4. Bet volume shows a "U-shaped" intraday pattern, with a relatively higher volume in the morning and late afternoon than during the midday. Non-bet volume has a similar "U-shaped" pattern, but less accentuated towards the end of the trading day. Interestingly, bet volume increases sharply and reaches levels of about twice as large as non-bet volume in the last half hour of trading. As a result, the volume multiplier diminishes towards the end of the trading day.

Insert Figure 4 here

4.3 Correlation analysis

Table 3 presents pair-wise correlations for our measures of the volatility multiplier, volume multiplier, return volatility, and dollar trading volume. We obtain the correlations for both daily (Panel A) and intraday observations (Panel B). The volatility multiplier is significantly negatively correlated with return volatility (at the 1% level) daily, but not intraday, and with dollar volume (at the 10% level) both daily and intraday. Hence, on days with low volatility and/or low dollar volume, bet volatility makes up a larger proportion of return volatility.

Insert Table 3 here

The volume multiplier is significantly positively correlated with dollar volume (at the 1% level) and return volatility (at the 1% level) daily. One interpretation of these daily positive correlations is that on days with high return volatility and dollar volume, bets generate more trades among intermediaries than on days when volatility and volume are low. However, intraday, the correlation between the volume multiplier and volume is significantly negative (at the 1% level). This stems from the observation in Figure 4 that the volume multiplier is relatively constant, while volume is distinctly U-shaped, throughout the trading day. Also, the two multipliers are significantly positively correlated both daily (at the 5% level) and intraday (at the 1% level).

4.4 Main regression results of quoted and effective spread

We now turn to the regression analysis that encompasses tests of MMI. We run two sets of regressions according to Eq. (5), in which we aggregate the five-minute observations during either days across intraday time periods, or during intraday time periods across days. In the following, we first present results from the daily regressions and then from the intraday ones.

Daily regression results

26

Table 4 presents the empirical results from the daily time series regressions with the following aggregated version of Eq. (5): $rspr_d - 0.5s_d^B = k^B + \beta w_d^B + \varepsilon_d^B$. The estimation is done by Ordinary Least Squares (OLS) with Newey and West (1987) standard errors with ten lags. The first line in Table 4 shows the results using the natural logarithm of the average percentage quoted bid-ask spread, as $rspr_d$. The main parameter of interest β equals -0.335, which is very close to $-1/3$, the value according to MMI, and we cannot reject the null hypothesis that $\beta = -1/3$ at any conventional significance level. The second line in Table 4 reports the results using the effective spread as a measure of transaction cost, with an estimated β of -0.347 which is still quite close to, and not significantly different from, $-1/3$. The results are very similar to the ones for the quoted bid-ask spread. Thus, we cannot reject MMI even at the 10% significance level based on the daily time series regression analysis.

Insert Table 4 here

Intraday regression results

Next, we aggregate the five-minute observations during intraday time periods across days and estimate intraday regressions using the following aggregated version of Eq. (5): $rspr_t - 0.5s_t^B = k^B + \beta w_t^B + \varepsilon_t^B$. The estimation is performed by OLS with Newey and West (1987) standard errors with six lags. The results are presented in Table 5, where the first line holds the results for the quoted bid-ask spread whereas the second line is for the effective spread. The main parameter of interest β is -0.301 (-0.285) for the quoted spread (effective spread) which deviates more from $-1/3$ than the β in the daily regressions. Nevertheless, we cannot reject the null hypothesis that $\beta = -1/3$ at any conventional significance level. Thus, we fail to reject MMI even at the 10% significance level based on either the daily or the intraday regression analysis.

Insert Table 5 here

Although we cannot reject MMI based on the results in Table 4 and Table 5, we do observe deviations of estimated βs from $-1/3$, especially in the intraday setting. In addition, it is well documented that liquidity measured as the quoted spread, or the effective spread, has an intraday pattern.[8] Thus, we zoom in on how transaction costs, bet volatility, and bet activity are related during the continuous trading hours and investigate how large the actual deviations between the predicted and observed quoted spread (effective spread) are.

Given the MMI specification in Eq. (5), we express the implied average spread as a function of the expected average bet activity and bet volatility and then back out the implied average spread according to Eq. (6) using the generalized method of moments. Figure 5 depicts the actual average quoted futures bid-ask spread (bold line) and the implied average quoted bid-ask spread (dotted line) for each five-minute interval from 9:05 AM to 5:20 PM for all trading days in the sample. Both the actual and the implied quoted bid-ask spread are relatively higher in the beginning of the trading day than towards the end. However, the variation in the implied spread is somewhat larger than in the actual spread. Towards the end of the trading day, at about 3:30 PM, the implied spread starts deviate substantially from the actual spread. In Figure 5, the implied spread drops below 0.2 after 5:00 PM while the actual spread remains above 0.25, which is the minimum level for the quoted spread given the tick size restriction.

Figure 6 plots the actual average effective spread and the implied effective spread for each five-minute interval from 9:05 AM to 5:20 PM for all trading days in the sample, which tells us the same story as for the average quoted spread. Clearly, the current tick size of SEK 0.25 for the

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[8] See, among others, the early work by Chan, Christie and Schultz (1995) and McInish and Wood (1992) for evidence from the equity market, and Xu (2014) for evidence from the index futures market.

OMXS 30 index futures is too large according to MMI. With the sizable increase in bet activity towards the end of the trading day, the spread should decrease accordingly. However, the actual spread is binding at the minimum tick of SEK 0.25.

Insert Figure 5 and Figure 6 here

4.5 Robustness check and ITI regression results

Kyle and Obizhaeva (2016a) note that the assumptions that the volume and volatility multiplier are constants are important for testing the predictions of MMI empirically, and that these assumptions should be tested empirically. One way to test whether the two multipliers are constants is the correlation analysis as in section 4.3 suggested by Kyle and Obizhaeva (2016a), where we see some evidence showing that the volume and volatility multipliers are correlated with volume and volatility. Here, we adopt a more practical approach, by running a series of regressions for testing MMI under different assumptions for the two multipliers.

In the main regression analysis presented in section 4.4, we allow the volatility multiplier and the volume multiplier to vary either across trading days, but to be constant within days, or to vary on an intraday basis, but to be constant across days. We conclude that we cannot reject MMI under either the daily or the intraday setting. In this section, we check if MMI still holds with more restrictive assumptions for the multipliers. In the spirit of section 4.4, we first present results from the daily regressions and then from the intraday ones.

Daily regressions with restrictive assumptions for the multipliers

In the daily regression setting, we allow the volatility multiplier and volume multiplier to vary across trading days but to be constant within the days. Here, we first assume that the two

multipliers are constants ($\tilde{\psi}$ and $\tilde{\zeta}$), and then we estimate these two constants based on the RTH and TRS data for the whole sample period. More specifically, we obtain $\tilde{\psi}$ by estimating a new VAR and VMA model using all trades and quotes data in the sample, and $\tilde{\zeta}$ by taking the average volume multipliers across all 5-min intervals. The estimated $\tilde{\psi}$ and $\tilde{\zeta}$ are 0.829 and 1.677, which means that bet volatility accounts for 82.9% of return volatility, and that one unit of bet volume generates 1.677 units of trading volume.

We use the two constants to transform s_τ into s_τ^B and w_τ into w_τ^B, and we estimate daily time series regressions with the following aggregated version of Eq. (5): $rspr_d - 0.5s_d^B = k^B + \beta w_d^B + \varepsilon_d^B$. Panel A of Table 6 holds the results. The main parameter of interest β is still very close to $-1/3$, the value according to MMI, and we cannot reject the null hypothesis that $\beta = -1/3$ at any conventional significance level for either the quoted bid-ask spread or the effective spread. We also set $\tilde{\psi} = 1$ and $\tilde{\zeta} = 2$, which means that $s_\tau^B = s_\tau$ and $w_\tau^B = w_\tau$, and re-estimate the regressions. Panel B of Table 6 shows the results. The estimated β parameters are identical to the ones in Panel A. The only difference between MMI with constant multipliers and ITI is that the constant terms are different.

<center>Insert Table 6 here</center>

Intraday regressions with restrictive assumptions for the multipliers

In the intraday regression setting, we allow the volatility multiplier and volume multiplier to vary on an intraday five-minute basis, but to be constant across trading days. Here, we also allow the multipliers to vary intraday, every ten minutes and every hour. In addition, we assume constant multipliers ($\tilde{\psi} = 0.829$ and $\tilde{\zeta} = 1.667$), and that the multipliers take on the values according to ITI ($\tilde{\psi} = 1$ and $\tilde{\zeta} = 2$). Thus, for each ten-minute (one hour) interval from 9:05 AM

to 05:20 PM, we estimate $\tilde{\psi}_t$ based on futures trades and quotes, and we obtain $\tilde{\tilde{\zeta}}_t$ as the average across days for each ten-minute (one hour) interval.

We estimate intraday regressions using the following aggregated version of Eq. (5): $rspr_t - 0.5 s_t^B = k^B + \beta w_t^B + \varepsilon_t^B$, where the ten-minute, one hour, or constant multipliers are used to calculate s_t^B and w_t^B. The regression results are presented in Table 7, where Panel A holds the results for the quoted bid-ask spread and Panel B for the effective spread. Irrespective of the specification, we cannot reject the null hypothesis that $\beta = -1/3$ at any conventional significance level. Thus, we fail to reject MMI and ITI even at the 10% significance level.

<div align="center">Insert Table 7 here</div>

We find some evidence that the volatility and volume multipliers vary across trading days and intraday empirically. However, these variations do not disrupt the MMI. We find that the futures bid-ask spread lines up with bet volume and bet volatility as predicted by MMI, and that the relationship is not sensitive to whether we allow the multipliers to vary or if we restrict them to be constant. Our findings imply that ITI works well and is indistinguishable from MMI in our empirical setting. Apparently, bet volatility is the most important part of return volatility, so that volatility caused by intermediaries' quote updates does not affect the MMI relation. Moreover, bet volume is the driving force behind the development of trading volume, and intermediation by high frequency traders does not interfere with the MMI relation.

5. Concluding remarks

Kyle and Obizhaeva (2016a) propose the market microstructure invariance (MMI) theory. It stipulates two invariance principles, i.e., the distributions of risk transferred by bets and the transactions costs of executing bets, are constant over business time and across assets. Given the

importance of the invariance theory, many studies test it empirically in different markets focusing on the invariance of the distributions of risk transferred by bets. However, bets - transactions intended to produce idiosyncratic gains based on investors' beliefs - are difficult to distill from trades, and we propose a solution to this difficulty.

Our main contribution to the existing literature is to test the invariance of transaction cost and to approximate bet volume as the trading volume of brokerage firms that trade on behalf of their clients and bet volatility as the trade-related component of return volatility. Our regulatory data on futures transactions with trading firm IDs allow us to separate proprietary trading by high-frequency trading (HFT) firms from client-induced trading, and we assume that the latter constitutes bet volume.

MMI stipulates a specific relationship between the bid-ask spread, bet volatility, and bet volume. Our empirical analysis finds that this relationship holds up well in the index futures market.

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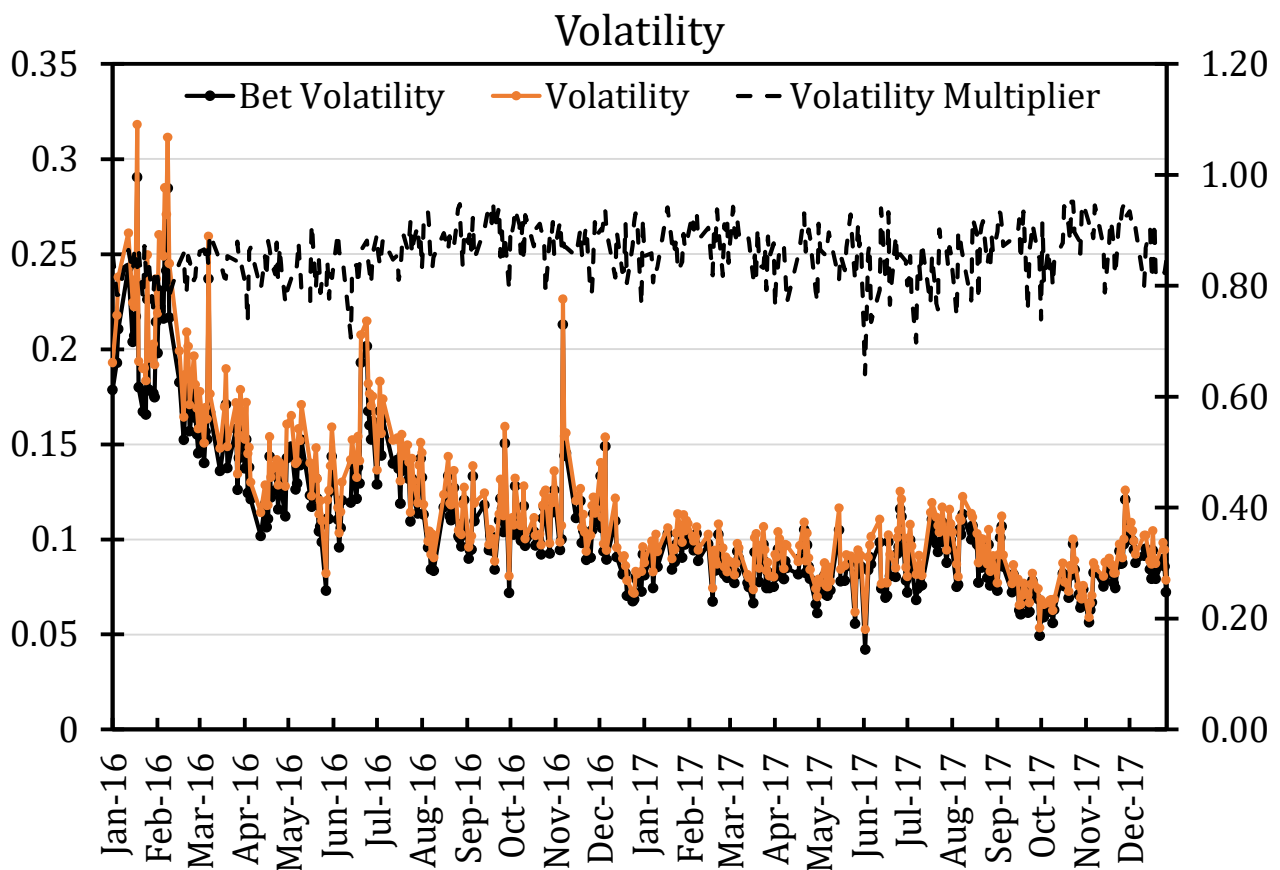
Figure 1: Daily bet volatility.

The figure shows the average bet volatility, the average futures return volatility, and the average volatility multiplier (right scale), for each trading day in the sample.
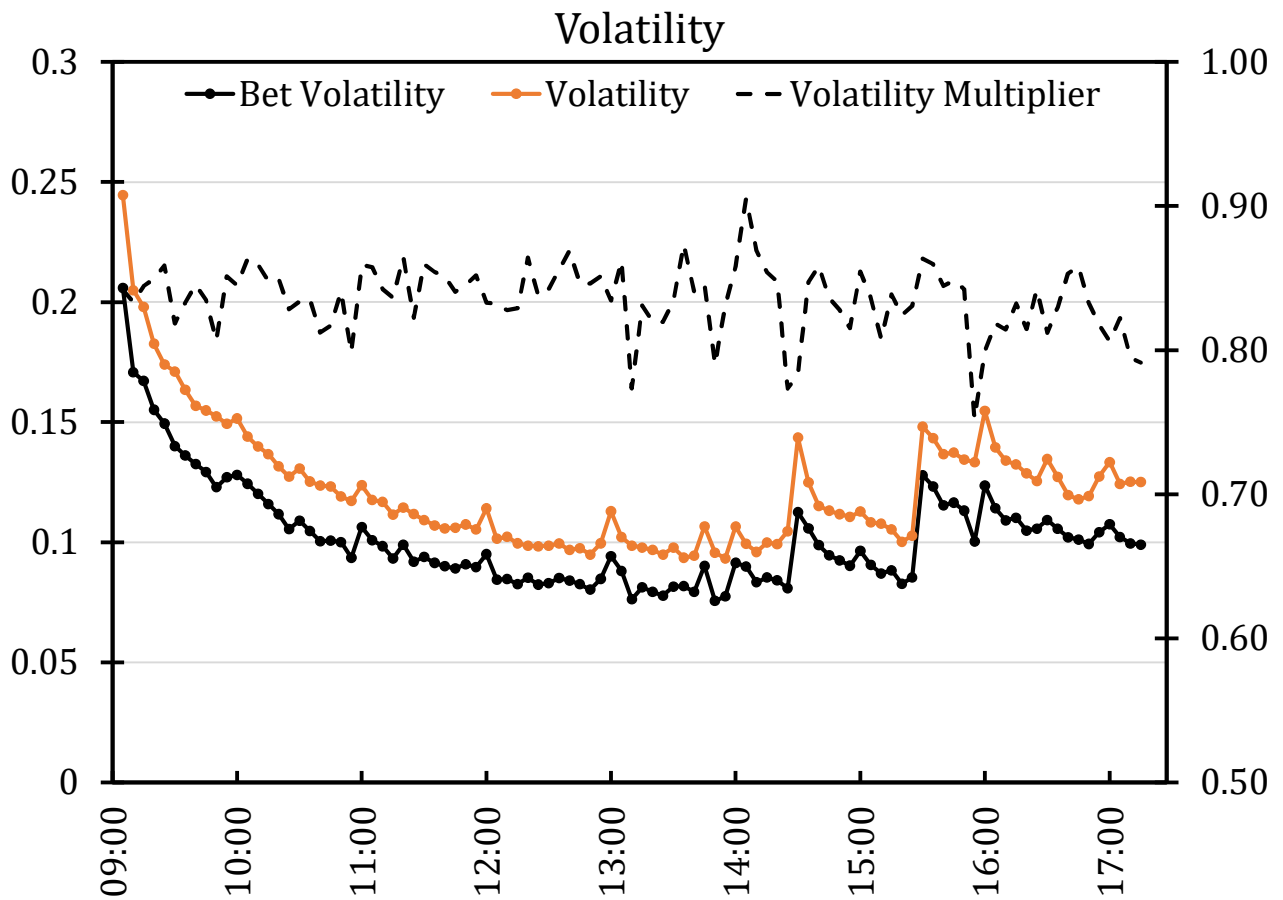
Figure 2: Intraday bet volatility.

The figure shows the average bet volatility, the average futures return volatility, and the average volatility multiplier (right scale), for each five-minute interval across all trading days in the sample.

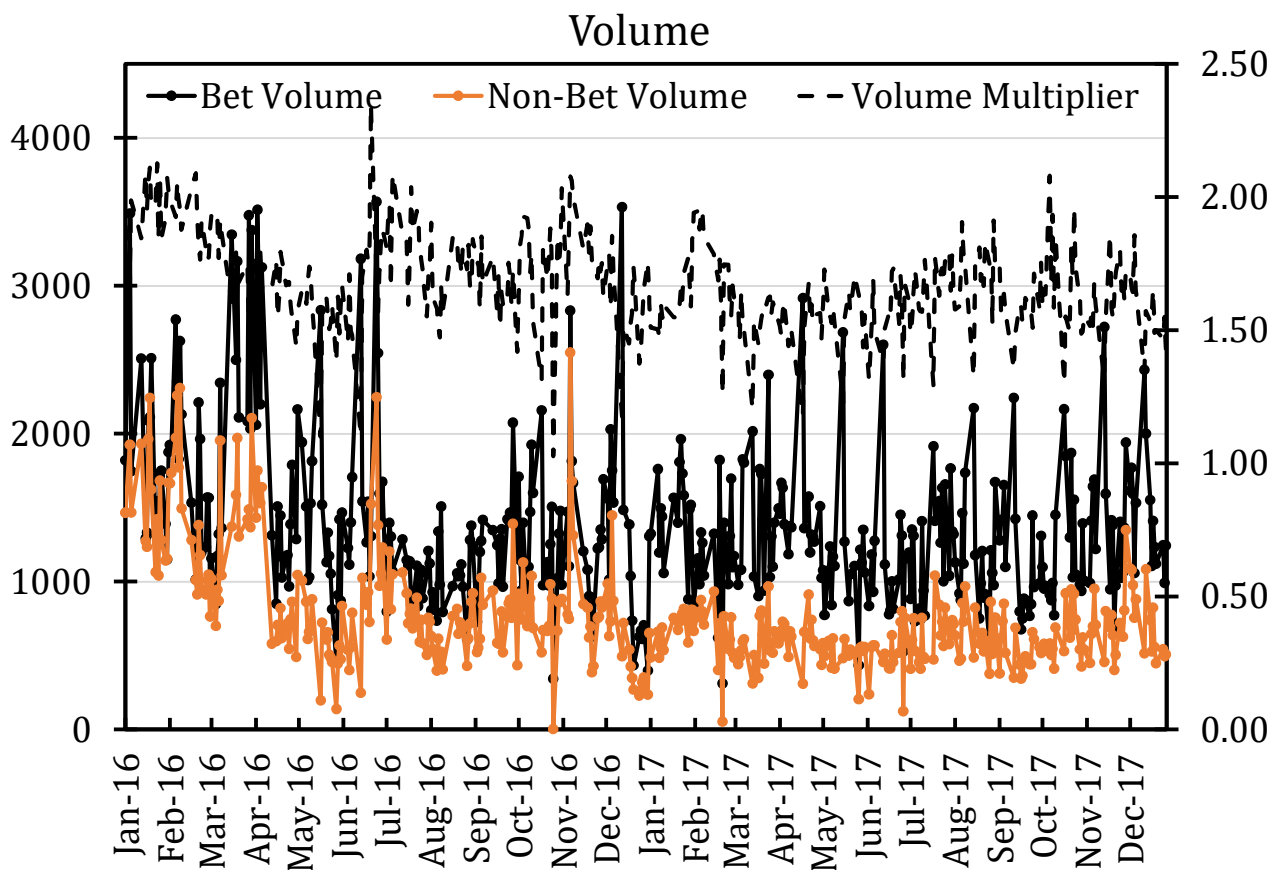**Figure 3**: Daily bet volume.

The figure shows the average bet volume, the average non-bet volume, and the average volume multiplier (right scale), for each trading day in the sample.
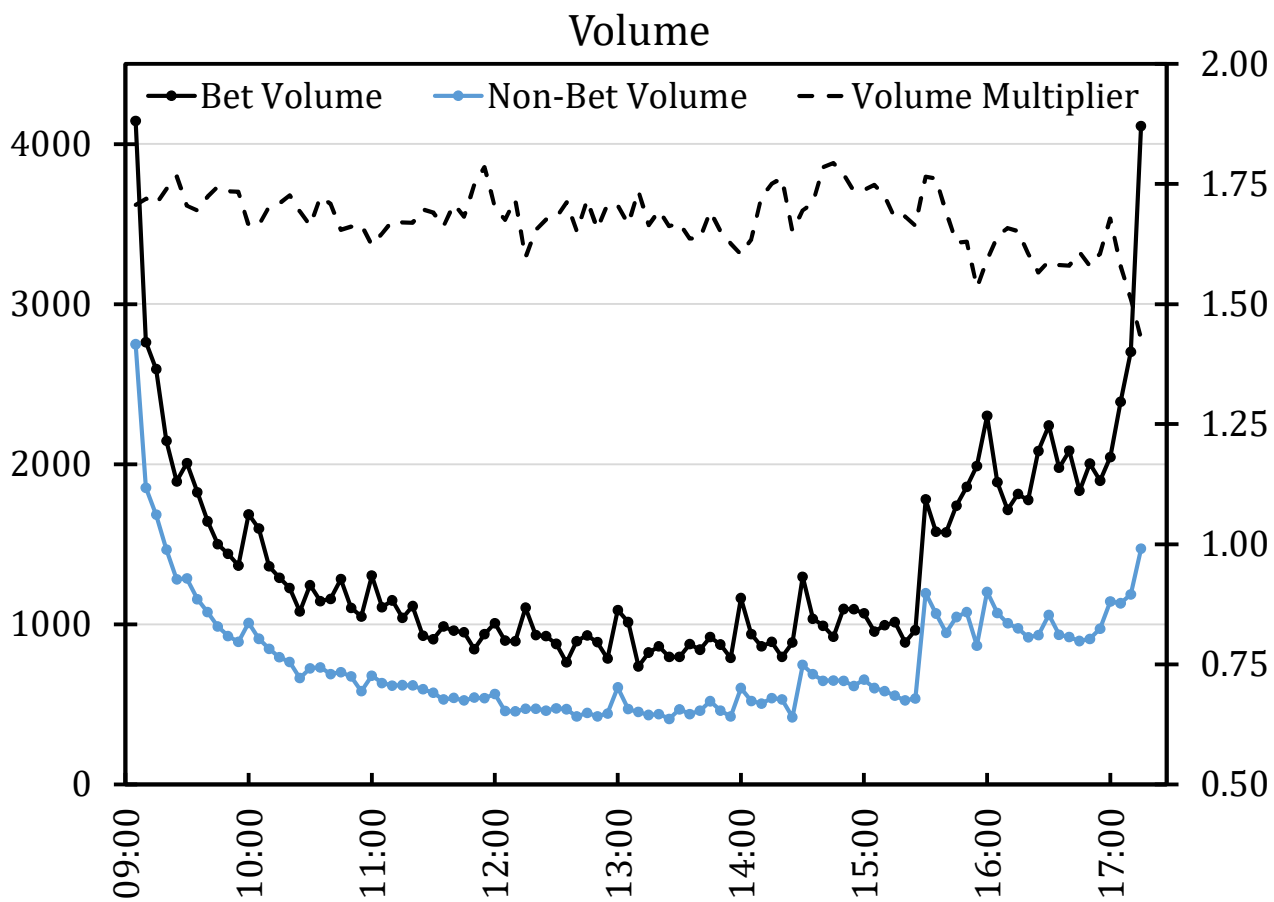
Figure 4: Intraday bet volume.

The figure shows the average bet volume, the average non-bet volume, and the average volume multiplier (right scale), for each five-minute interval across all trading days in the sample.

Quoted Bid-Ask Spread
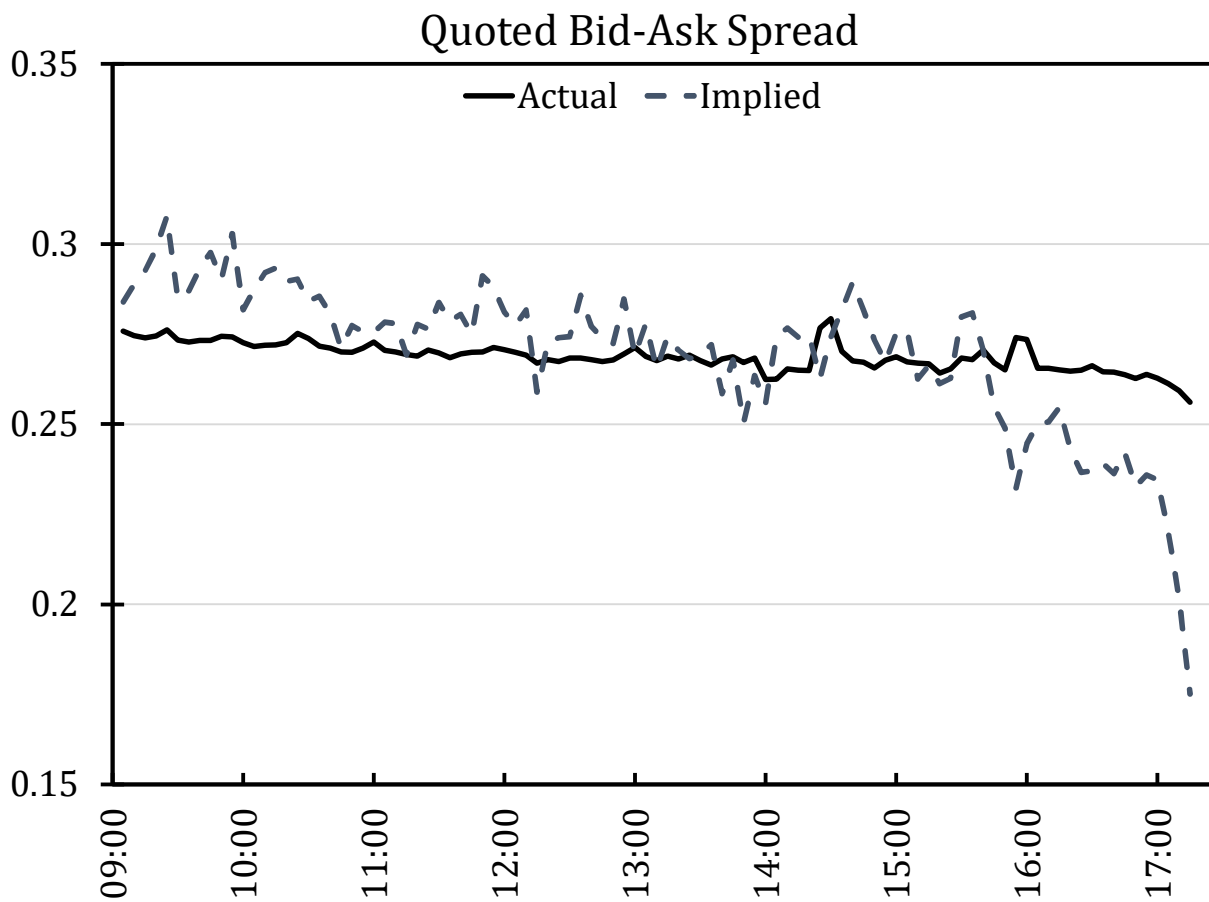
Figure 5: Intraday implied quoted bid-ask spread.

The figure shows the actual average quoted futures bid-ask spread and the implied average quoted bid-ask spread for each five-minute interval for all trading days in the sample.

Figure 6: Intraday implied effective spread.

The figure shows the actual average effective futures spread and the implied average effective spread for each five-minute interval for all trading days in the sample.

Table 1: Descriptive statistics

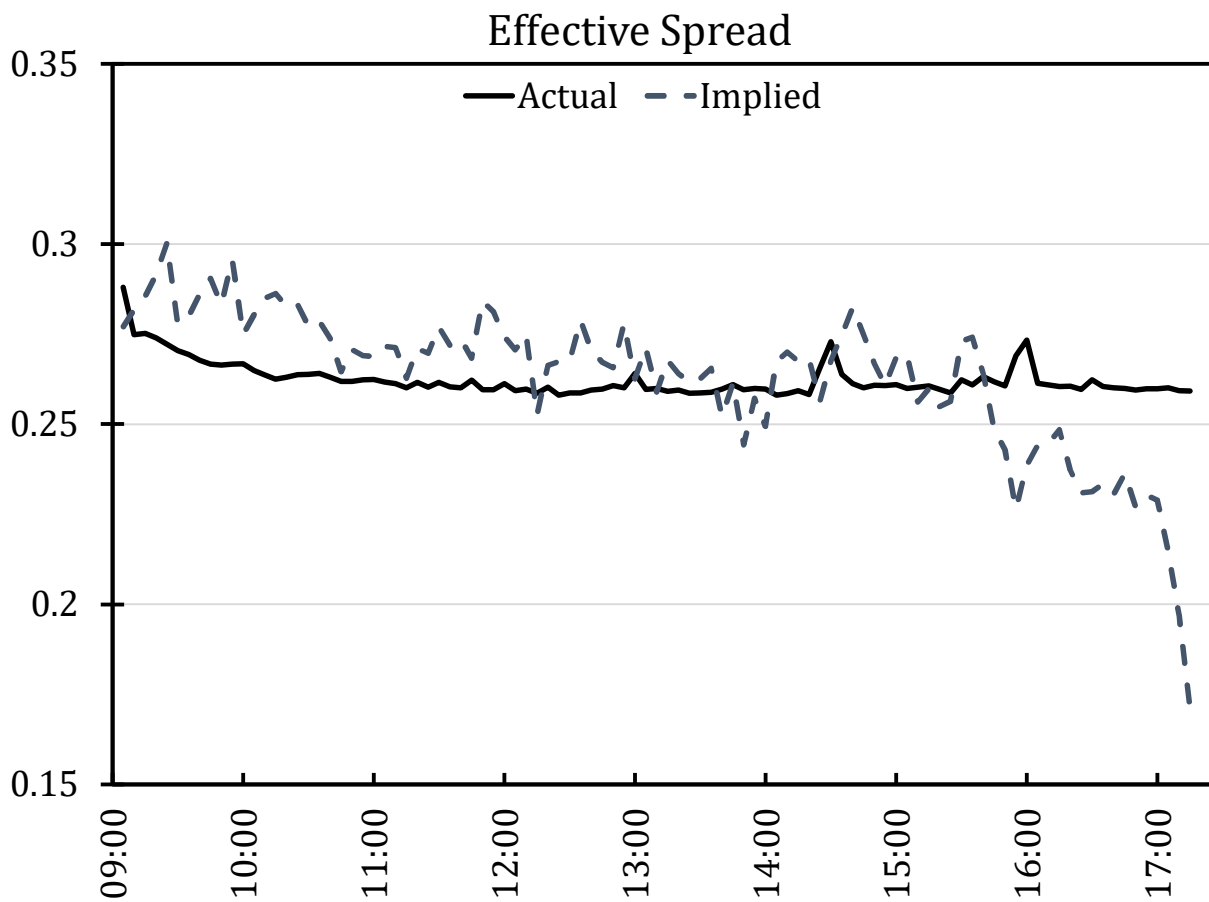|	Mean	Min	Max	St. Dev.
Volatility	0.125	0.000	0.826	0.142
Volume	2,137	21	59,624	2,145
Trading activity	2,694	0	106,434	4,164
No. of Trades	117	2	1,187	84
Trade size	6	1	26	2
Futures price	1,493	1,238	1,679	115
Depth	46	10	179	14
Absolute quoted spread	0.269	0.250	0.536	0.020
Relative quoted spread	1.820	1.489	4.032	0.236
Absolute effective spread	0.263	0.250	0.557	0.018
Relative effective spread	1.775	1.489	3.976	0.218

The table presents descriptive statistics for the nearby futures contract, which is defined as the most actively traded regular futures contract on a specific day. The data include 413 days and 99 five-minute intervals per day, from 9:05 to 17:20. Half trading days are excluded. Contract maturity spans from 36 days to 7 days before the expiration day. Volatility is the realized volatility, calculated from the sum of five-second squared midpoint returns during each five-minute interval, averaged across all observations, and reported in annualized terms. Volume is the number of contracts traded per five-minute interval (double counted). Trading activity equals the product of volatility, futures price, and volume. Futures price is the average midpoint between the best ask price and the best bid price, observed every five seconds, per five-minute interval. No. of Trades is the number of trades per five-minute interval. Trade Size equals Volume divided by No. of Trades per five-minute interval. Depth is the average number of futures contracts at the best bid and ask observed every five seconds, per five-minute interval. Absolute quoted spread is the average quoted bid-ask spread, i.e., best ask price minus best bid price, observed every five seconds, per five-minute interval. Relative quoted spread is the absolute quoted spread divided by the midpoint between the best ask price and the best bid price, observed every five seconds, per five-minute interval, reported in basis points. Absolute effective spread is twice the average signed difference between trade price and the midpoint between the best ask price and the best bid price at the time of the trade, per five-minute interval. Relative effective spread is the absolute effective spread divided by the midpoint between the best ask price and the best bid price at the time of the trade, per five-minute interval, reported in basis points.

Table 2: Bet volatility, bet volume, and bet activity: Daily and intraday averages

	Panel A: Daily				Panel B: Intraday			
	Mean	Min	Max	St. Dev.	Mean	Min	Max	St. Dev.
Volatility	0.125	0.053	0.318	0.117	0.125	0.093	0.245	0.088
Bet volatility	0.115	0.042	0.291	0.106	0.114	0.085	0.224	0.081
Volatility multiplier	0.861	0.641	0.952	0.049	0.836	0.752	0.905	0.024
Volume	2,137	348	5,816	852	2,137	1,191	6,893	983
Bet volume	1,367	312	3,568	552	1,367	738	4144	639
Volume multiplier	1.677	1.028	2.347	0.182	1.677	1.428	1.794	0.062
Trading activity	2,694	336	12,900	1,969	2,694	1,086	15,439	1,975
Bet activity	2,696	409	11,597	1,728	2,692	999	15,233	1,969

The table presents descriptive statistics for measures of average bet volatility, bet volume, and bet activity across 99 five-minute intervals for each trading day (Panel A), and across 413 trading days for each five-minute interval (Panel B). The variables volatility, volume, futures price, and trading activity are described in Table 1. Bet volatility is the part of volatility driven by bet activity. Volatility multiplier measures the fraction of bet volatility to volatility. Bet volume is the average five-minute trading volume by brokerage firms that trade on behalf of clients for each trading day. Volume multiplier equals the double counted volume divided by the bet volume. Bet activity equals the product of bet volatility, futures price, and bet volume.

Table 3: Correlation analysis

	Panel A: Daily			Panel B: Intraday		
	Volatility multiplier	Volume multiplier	Volatility	Volatility multiplier	Volume multiplier	Volatility
Volume multiplier	0.1076**			0.2891***		
Volatility	-0.2425***	0.5567***		-0.0583	0.0930	
Dollar volume	-0.0885*	0.1386***	0.5392***	-0.1757*	-0.3299***	0.8334***

The table presents pair-wise correlations for measures of volatility multiplier, volume multiplier, volatility, and dollar volume based on daily observations on the 413 trading days (Panel A) and intraday observations for the 99 five-minute intervals (Panel B). Dollar volume equals the product of futures price and total volume. Other variables are described in Table 1. *, ** and *** indicate significance at the 10%, 5% and 1% levels.

Table 4: Daily regression results

	N. Obs	k	β	se(k)	se(β)	$\bar{R}^2$	$t\,(\beta = -1/3)$	p-value
Quoted spread	413	1.125	-0.335	0.139	0.021	0.645	-0.079	0.937
Effective spread	413	1.183	-0.347	0.134	0.020	0.672	-1.135	0.256

The table presents results from daily regressions according to the following version of Eq. (5): $rspr_d - 0.5s_d^B = k^B + \beta w_d^B + \varepsilon_d^B$, where $rspr_d$ is the natural logarithm of the average percentage quoted bid-ask spread, or the average percentage effective spread, on day d, s_d^B denotes the natural logarithm of the average percentage bet variance per day d, w_d^B is the natural logarithm of the bet activity per day d, k^B is a constant, and ε_d^B is a residual with zero mean. Both spread measures are defined in Table 1. The data include 413 trading days, and each daily variable observation is obtained as an average across 99 five-minute intervals per day, from 9:05 to 17:20. Half trading days are excluded. Regressions are performed for the nearby regular futures contract using days in the sample with 7-36 days until the expiration day. The nearby contract is defined as the most actively traded regular futures contract on a specific day. Standard errors (se(c) and se(β)) are corrected for heteroskedasticity and autocorrelation in the residuals (10 lags) according to Newey and West (1987).

Table 5: Intraday regression results

	N. Obs	k	β	se(k)	se(β)	$\bar{R}^2$	$t\,(\beta=-1/3)$	p-value
Quoted spread	99	0.921	-0.301	0.202	0.032	0.871	1.010	0.312
Effective spread	99	0.791	-0.285	0.202	0.033	0.846	1.465	0.143

The table presents results from intraday regressions according to the following version of Eq. (5): $rspr_t - 0.5s_t^B = k^B + \beta w_t^B + \varepsilon_t^B$, where $rspr_t$ is the natural logarithm of the average percentage quoted bid-ask spread, or the average percentage effective spread, per intraday interval t, s_t^B denotes the natural logarithm of the average percentage bet variance per intraday interval t, w_t^B is the natural logarithm of the bet activity per interval t, k^B is a constant, and ε_t^B is a residual with zero mean. Both spread measures are defined in Table 1. The data include 99 five-minute intervals per day, from 9:05 to 17:20, and each intraday variable observation is obtained as an average across 413 trading days. Half trading days are excluded. Regressions are performed for the nearby regular futures contract using days in the sample with 7-36 days until the expiration day. The nearby contract is defined as the most actively traded regular futures contract on a specific day. Standard errors (se(c) and se(β)) are corrected for heteroskedasticity and autocorrelation in the residuals (6 lags) according to Newey and West (1987).

Table 6: Daily regression results for constant multipliers

	N. Obs	k	β	se(k)	se(β)	$\bar{R}^2$	$t\,(\beta = -1/3)$	p-value
Panel A: MMI with constant multipliers								
Quoted spread	413	1.192	-0.342	0.121	0.018	0.768	-0.471	0.638
Effective spread	413	1.235	-0.352	0.119	0.018	0.781	-1.037	0.300
Panel B: ITI with constant multipliers								
Quoted spread	413	1.008	-0.342	0.121	0.018	0.768	-0.471	0.638
Effective spread	413	1.052	-0.352	0.120	0.018	0.781	-1.037	0.300

The table presents results from daily regressions as in Table 4. $s_d^B = 2\log\psi_d + s_d$, where s_d equals the natural logarithm of the average percentage return variance per day d, and ψ_d is the volatility multiplier on day d. $w_d^B = \log 2 + \log\psi_d - \log\zeta_d + w_d$, where w_d the natural logarithm of the trading activity per day d, and ζ_d is the volume multiplier on day d. Panel A holds results for constant multipliers, estimated over the entire sample period, and Panel B contains results for specific assumptions of constant multipliers: $\psi_d = 1$, and $\zeta_d = 2$. Spread measures are defined in Table 1. Data and sample description is in Table 4.

Table 7: Intraday regression results for different multiplier aggregations

	N. Obs	k	β	se(k)	se(β)	$\bar{R}^2$	$t\,(\beta = -1/3)$	p-value
Panel A: Quoted Spread								
MMI (5 min)	99	0.921	-0.301	0.202	0.032	0.871	1.010	0.312
MMI (10 min)	99	0.937	-0.304	0.196	0.032	0.882	0.917	0.359
MMI (Hourly)	99	0.960	-0.307	0.189	0.031	0.894	0.849	0.396
MMI (Constant)	99	0.994	-0.312	0.152	0.025	0.925	0.853	0.394
ITI	99	0.810	-0.312	0.152	0.025	0.925	0.853	0.394
Panel B: Effective Spread								
MMI (5 min)	99	0.791	-0.285	0.202	0.033	0.846	1.465	0.312
MMI (10 min)	99	0.808	-0.288	0.196	0.032	0.859	1.417	0.359
MMI (Hourly)	99	0.831	-0.291	0.215	0.031	0.872	1.366	0.396
MMI (Constant)	99	0.865	-0.296	0.153	0.025	0.906	1.493	0.394
ITI	99	0.681	-0.296	0.153	0.025	0.906	1.493	0.394

The table presents results from intraday regressions as in Table 5. $s_t^B = 2\log\psi_t + s_t$, where s_t equals the natural logarithm of the average percentage return variance per intraday period t, and ψ_t is the corresponding volatility multiplier. $w_t^B = \log 2 + \log\psi_t - \log\zeta_t + w_t$, where w_t the natural logarithm of the trading activity per intraday period t, and ζ_t is the corresponding. Panel A holds results for the average percentage quoted bid-ask spread, and Panel B contains results for the average percentage effective spread. ITI refers to specific assumptions of constant multipliers: $\psi_d = 1$, and $\zeta_d = 2$. Spread measures are defined in Table 1. Data and sample description is in Table 5.