FORECASTING THE PRICE OF CRUDE OIL WITH A STRUCTURAL MODEL

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ABSTRACT. The price of oil (WTI) has been extremely volatile recently due to many factors, such as shifts in geopolitical power, supply of major producers such as Saudi Arabia, war, demand from China, and the COVID-19 pandemic, to name a few. In this paper, I investigate fundamental sources of crude oil price movements, identify factors that impact the long-term trend and short-term fluctuations, respectively, and propose a structural two-equation model to forecast the price of oil. The 10-year out-of-sample price forecasts shows the strong efficacy of this structural approach in terms of both the model alpha accuracy and the portfolio performance.

1. Introduction

Crude oil is the lifeblood of the global economy. The world derives over a third of its total energy production from crude oil, more than any other source by far. In commodity investing, a deep understanding of the oil pricing mechanism and causal factors is very important for a profitable strategy in the futures market.

In this paper, I focus on the price forecast of the West Texas Intermediate (WTI) at both monthly and daily level with three main parts: (1) Fundamental sources of crude oil price movements. The price of oil exhibits a long-term trend determined by geopolitical factors, production, and demand. Meanwhile, there are many factors that contribute to price movements on a daily basis. I identify the long- and short-term factors that impact the trend and daily fluctuations of crude oil prices, respectively. (2) A structural two-equation model to forecast the daily price of oil. To capture these structural features, I construct a structural two-equation model: one equation for the long-term trend (monthly frequency, long model), and the other equation for short-term volatilities (daily frequency, short model). (3) Evaluating efficacy of this structural model via a 10-year-period out-of-sample analysis.¹

There has been a large volume of literature seeking to identify significant causal factors to forecast the price of oil. For example, Fattouh (2007) investigates factors

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¹We first check the accuracy of the structural model forecasts by econometric measurements. We then form a real strategy by investing in WTI futures and investigate the portfolio performance.

in the framework of non-structural, supply-demand and the informal approach, respectively, and find that those factors are very limited if used for long-term price projections. Alquist, et al. (2013) explore different measures for the price of oil and focus on econometric techniques, particularly time series analysis, such as Granger causality, to measure the forecasting power of oil prices and a few other macro indicators, such as GDP and world industrial production. They also discuss a structural model accounting for the shocks from demand and supply using a VAR framework. Baumeister and Kilian (2012) conduct time-series forecast for the real spot price of WTI using a vector autoregression framework, which is extended Baumeister and Kilian (2015) that combine different methods of oil price forecasting and find that the combination outperforms any single method. Given the importance of the price of Brent oil outside of the US, Manescu and Robays (2014) provide an excellent summary of methodologies and factors used in the literature and then apply those methods to the price of Brent oil: the factors comprise future contract prices, time series analysis, and industrial production. Most recently, several studies have applied machine learning and neural networks to forecast crude oil prices (Sehgal and Pandey (2015), Chen et al. (2017), Zhang and Hamori (2020)).

It should be noted that much of the literature on crude oil pricing focuses on econometric techniques, which is very important given the asset's times series characteristics.² Typically, these studies don't intend to address and capture the fundamental drivers and signals of crude oil price movements. Technically, there may be identification issues. For example, some studies use quarterly GDP data alone to identify monthly oil price variations. Some use macroeconomic indicators, such as interest or inflation rates, as causal factors in price movements. There are several problems with these approaches. First, these rates are not significant underlying causes of crude oil price movements. Second, their values do not change much, especially in the past 20 years, and therefore cannot explain the dramatic changes in oil prices. Third, some studies explore fundamental sources of price movements, such as industry activities but fail to capture nonlinear effects, as these underlying forces exhibit strong tail behaviors, but not over the whole distribution. In this study, unlike the oil price forecasting literature that focuses on econometric techniques such as VAR, ARMA, GARCH which are of secondary significance, I focus more on the fundamental drivers of crude oil price movements and a structural method to combine this information.

Other sources of crude oil price forecasts include government agencies and industry. For example, the energy information administration (EIA), a US government agency, publishes price forecasts for WTI on a monthly basis. Many industry firms and organizations also publish forecasts regularly though they are usually descriptive listing a few factors and then a conclusion about the price of oil either a single value,

²Kilian and Zhou (2020) provide an excellent survey on the VAR models for the oil pricing with special attention to the identifying assumptions and methods of inference.

or a range for a given period.³. These public industry and government studies can be used as references but are not very useful for real-world for investment strategies.

The contribution of this study is twofold. First, to my knowledge, this is the first paper to identify a suite of factors that capture the long-term trend and short-term fluctuations of oil price movements using a system of equation models. Second, the paper explores the efficacy of the out-of-sample forecasts which is helpful for practitioners. For example, the factors and analysis described in this paper would be helpful in hedging for risk, such as for airline companies or alpha for the oil commodity in multi-asset investment strategies.

The rest of the paper is organized as follows. Section 2 proposes a structural pricing model with identified factors to forecast both the long-term trend and short-term fluctuations in the price of oil. Section 3 presents the in-sample analysis of the structural model. Section 4 performs an out-of-sample study to forecast the price of oil. Section 5 summarizes.

2. Structural model: Long-term trend and short-term fluctuations

In this section, I explore the underlying fundamental causes of crude oil price movements, with a focus on forecasting the long-term trend and short-term fluctuations. I first investigate the price of WTI and its special characteristics, then identify the drivers and signals that impact the long-term trend and short-term fluctuations, respectively. I propose a two-equation structural model is proposed with one equation for the long-term trend (long model) and another for the short term price fluctuations (short model). This system of multi-factor equations captures the unique features of the dynamic price movements of crude oil.

- 2.1. Crude Oil Pricing: long-term trend and drivers. The price of crude oil is not decided exclusively by the market via supply and demand, at least not in the linear equilibrium sense, for the following reasons:
 - Demand is rigid.
 - Supply is concentrated in a few countries.
 - Crude oil transactions are in USD.
 - Geopolitical power plays a significant role in the price of crude oil.

Rigid demand establishes a floor for prices; the price may decrease only if demand drops to a very low level. Compared with demand which impacts price only in extreme cases, supply has a greater impact on price in general via controls on both price and production. Geopolitical power plays a significant role in oil production and price and is usually intertwined with supply. Based on fundamental insights about the crude oil market, I identify significant themes (forces) that decide the market direction of crude oil prices:

³This is partially due to the non-disclosure compliance, methods, factors and process of industry forecasts are not disclosed in full.

- (1) World: the balance between supply and demand in extreme cases
- (2) Saudi Arabia: oil production and price policy in the name of OPEC
- (3) US: production and demand, geopolitical power
- (4) China: large and increasing demand that supports the price floor
- (5) Trend: technical price momentum

I discuss each theme and its associated factors for the long-term WTI price trend below.

2.1.1. World theme. Given the strategic importance of crude oil, oil price is not impacted as much by world supply and demand as other goods. There is a nonlinear equilibrium relationship between the price and supply/demand at the overall global level. Under extreme conditions when demand is very weak (recession) and producers fight for market share, there will be oversupply (a so-called glut), which creates downward pressure on the price of crude oil. On the other hand, if demand increases or the supply is tight for some reason, the price will increase.

I use monthly data from OECD countries on the industrial production index (IPI) to measure demand from industry.⁴ This index is based on a reference period that expresses change in the volume of production output. I use world production of crude oil as supply. In algebraic terms, the nonlinear equilibrium can be modeled by changes in the IPI and world oil production as follows. Let ΔS be the change in world crude oil production and ΔI be the change in the IPI for OECD countries. There is an equilibrium overtime:

$$\Delta S = b_0 + b_1 \Delta I + \nu,$$

where $E(\nu) = 0$ over a long period, and $b = (b_0, b_1)$ is the equilibrium parameters. Now define supply over demand as

$$\nu = \Delta S - b_0 - b_1 \Delta I,$$

which is usually not equal to zero. I employ the tail values of ν to capture the nonlinear equilibrium. Under extreme conditions, the supply-demand imbalance will impact the price. In this study, the world imbalance factor (WORLDimb) is defined as follows:

WORLDimb =
$$\nu$$
, over supply: $\Delta I < -0.25 \& \Delta S > 0.25$
WORLDimb = ν , over demand: $\Delta I > 0.25 \& \Delta S < -0.50$

The other supply-demand scenarios have non-significant effects, so WORLDimb = 0 for non-extreme cases.

This nonlinear imbalance hypothesis is supported by the data. I run an OLS regression to estimate ν . As expected, the value of $\hat{\nu}$ has no significant impacts on the price and returns of WTI overall. However, when we create a WORLDimb factor

⁴According to OECD: the industrial production index refers to the output of industrial establishments and covers sectors such as mining, manufacturing, electricity, gas and steam, and airconditioning.

to capture the nonlinearity, it becomes clear that this new factor has significant impacts on WTI forward returns. The correlation between WORLDimb and one-month forward returns on WTI is presented in Table 3.2 and regression results in Table 3.4.

- 2.1.2. Saudi Arabia theme. There are two mechanisms that Saudi Arabia uses to influence the price of crude oil: production and official selling price (OSP). I discuss each below.
- 1. Production control. To conduct quantitative analysis, I obtain data on Saudi Arabia's monthly crude oil production from Jan 1973 to May 2020. The change of monthly production (SAUDIprod) is employed as a factor to characterize the country's production control mechanism. A production increase corresponds to a higher price (returns) in previous periods but predicts a lower price in the future. It should be noted that a change in Saudi Arabia's oil production is usually followed by other OPEC members. Sometimes this is the result of a collective bargain either within OPEC or between OPEC and other major crude oil producers, such as Russia. In particular, a production cut requires agreement among producers to avoid cheating and is therefore more difficult to implement. However, once it is executed, the effect of a production cut is more dramatic than a production increase.
- 2. Price control. In addition to production and capacity controls, Saudi Arabia also directly impacts the crude oil market via official selling price (OSP). In this study, I use Aramco's OSP for the Arab Light grade to the US, Asia and EU regions. Arab Light accounts for more than 60% of all Saudi crude oil exports. Usually, Aramco announces the OSP on the first Monday of each month for delivery in the next month. Note that the OSP is based on the deviation (discount or premium) of a benchmark. For example, the OSP to the US is relative to WTI before Nov 2009 and the ASCI index after Nov 2009. The bar plot in Fig. 2.1 presents the autocorrelation values of OSP of Arab Light crude to the US, Asia, and Northwestern Europe. The OSP for the US is consistent over time, with an autocorrelation of 80% ten months later, while the pricing for both the Asia and NW EU regions changes faster, with an autocorrelation of 30-40% ten months later. In this study, I use the average of monthly change in the OSPs for the three regions:

SAUDIosp = (ArabLightUSchg + ArabLightASIAchg + ArabLightEUchg)/3 where ArabLightUSchg is defined as the monthly change in the OSP of Arab Light to the US.

2.1.3. The US theme. The US has impacts on the price of crude oil through changes in production via the market and geopolitical power.

⁵Aramco is the state owned company of Saudi Arabia, its IPO went on Dec 11, 2019 with 1.5 values of the shares traded on Saudi's stock exchange.

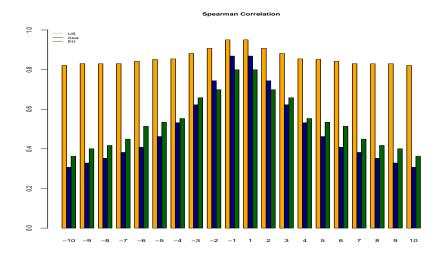


FIGURE 2.1. Autocorrelation values of Saudi Arabia OSP for the Arab Light grade to the US, Asia, and Northwestern Europe, with back and forward periods of 1 to 10 months. Data source: Bloomberg

- 1. US crude oil production. The US crude oil production has significant impacts on the price of WTI. I acquired monthly production data and we use the monthly change in production as a factor. It should be noted that the US oil production increases are usually the result of technological innovation and/or deregulation on environments. However, US crude oil production decreases are always the result of low pricy that are too low to be profitable. Low prices result from either oversupply (e.g., a Saudi Arabia production increase) or a significant drop in demand, such as during the COVID-19 pandemic in 2020.
- 2. Geopolitical factor. I list below major historical events that have impacted the price of oil from 1973 to 2020. In terms of asset pricing, we can quantify such events as a social and economic crisis factor. Because of the persistence of such events over time, we can use such a factor to help forecast the price of oil. It should be noted that we are not forecasting the events themselves but simply modeling the events in terms of their impacts on the price of crude oil. Moreover, since the US is the most powerful country in the world in terms of its economy and military power, its impacts in all geopolitical events are dominant, particularly for the strategic commodity of crude oil. I construct a factor called *USgeo*, whose values depend on three considerations: seriousness of the event, whether the entity imports or exports oil, and the entity's relationship with the US. Depending on the latter two, the impact of an event on oil prices is either negative or positive. For example, if the entity involved in an event exports oil and has a negative relationship with the US, the score is negative; if the

entity imports oil, the score is positive. I assign a score from -3 to 3 to indicate the direction and magnitude of impacts.

• Oil Embargo: Oct 16, 1973 - March 17, 1974

• Iranian Revolution: November 1978 - December 1979

• Collapse of the USSR: 1986 - 1991

• Iraq-Kuwait War: 2 August 1990 - 28 February 1991

• 2008 US Financial Crisis

• Arab Spring: 7 December 2010 - mid-2012.

• Crimean Crisis: 2014, and Sanctions on Russia, 2014 - present

• Shale Boom: 2015 - 2019

• COVID-19 Pandemic: 2019 - present

- 2.1.4. China theme. China accounts for approximately 40% of global oil demand growth between 2009 and 2020. Chinese crude oil demand has grown in response to the country's increasing economic size and industrial scale. Moreover, since the country's oil production did not keep up with its industrial expansion, most of its increasing oil demand resulted in increased imports, which underpin the price of oil. To better understand the size of China's oil demand growth during this period, consider that China's crude oil demand grew by 0.28 MBD in 2009 roughly the daily consumption of the Philippines, and by 1.16 mbd in 2010 roughly the daily consumption of Taiwan.⁶
- 2.1.5. Trend: technical theme. I use time series price data to capture the trend in price movements. This can be regarded as price momentum, a typical technical factor in security investing. The momentum factor is PMm, measured by the most recent monthly return:

$$PMm_t = \frac{P_t}{P_{t-1}},$$

where P_t is the average of daily prices of WTI for the month t.

- 2.2. Crude Oil Pricing: short-term fluctuations and signals. While the long-term themes set the trend for oil price movements, there are factors that impact the price movements of WTI on a daily basis, which may follow or deviate from the trend. These short-term sources can be categorized into five themes:
 - (1) Transaction: daily values of the petrodollar
 - (2) US: weekly crude oil production and commercial stock
 - (3) China: news of manufacturing activities
 - (4) World: short term disruptions
 - (5) Technical: short-term technical effects

 $^{^6 \}mbox{Source:} \mbox{ https://www.bakerinstitute.org/media/files/files/e0b5a496/WorkingPaper-ChinaOil-093016.pdf}$

Note that investors and speculators monitor and use these short-term themes in their trading activities on a daily basis. I discuss each theme and its associated factors for short-term WTI price movements below.

- 2.2.1. Transaction theme: Petrodollar. Because crude oil transactions are conducted in USD, any change in the value of the USD will impact the demand and supply for oil and hence daily oil prices. For example, lower values of the USD will trigger higher demand, so it is expected that USD depreciation will have positive impacts on the price of oil. In this study, the US dollar index (USDX) is used as a proxy for the strength of the U.S. dollar. The USDX measures the performance of the US dollar against a basket of currencies: the Euro, Canadian dollar, Japanese yen, British pound, Swiss franc, Australian dollar, and Swedish krona. These seven foreign currencies trade widely in currency markets outside their respective home areas. In this study, I use daily changes in the US dollar index (USDXchg) to measure changes in the strength of the petrodollar.
- 2.2.2. The US theme: weekly commercial stock and production. The US theme includes two factors: one is the existing commercial stock of crude oil, and the other is the produciton of new crude oil.
- 1. Weekly commercial stock. Changes in the US commercial crude oil stock stand for crude oil supply/demand in the US on a weekly basis. A significant increase indicates less demand over a short period and hence downward pressure on daily oil prices. I use data from the weekly EIA report on US commercial crude oil stock. EIA reports stock changes every Wednesday at 10:30 am EST. This is closely watched by both investors and speculators in crude oil markets. A significant increase or decrease can cause dramatic changes in oil prices during the day and the days after.
- 2. Weekly production. The US weekly crude oil production data can be obtained from EIA, though it is rare at a weekly frequency for other countries. Notably, US production responds more to the price of WTI in the short run, though it has more significant impacts on the price of oil in the long term. Nevertheless, I use weekly changes in production (USprodw) to indicate the direction of daily price movements.
- 2.2.3. China theme: PMI. I use monthly Purchasing Managers' Index (PMI) data to measure short-term changes in oil demand from China. Chinese PMI data provides an early indication each month of economic activities in the Chinese manufacturing sector. It is reported at the end of each month. I use the previous month's values to avoid look-ahead bias.
- 2.2.4. World theme: disruptions. Because oil production is concentrated in a few countries, any disruptions to oil production, delivery, or stock can impact the price immediately. Supply disruptions in the short-term are measured by the agreed cut or increase in production between major producers or events that interrupt normal oil production, such as a fire in a major oil field. I construct the oil production disruption

factor (DISR) based on events involved major oil-producing countries, such as the US, Russia, and Saudi Arabia.

2.2.5. Technical theme: momentum. There are constantly over- and underreactions to events in the crude oil market in the short term, so price momentum is expected for short-term price deviations. In this paper, we define the short-term price momentum (PMd) as 2-week daily price changes:

$$PMd_t = \sum_{j} (P_t - P_{t-j}), \quad j = 4, 7, 10, 12, 13,.$$

where P_{t-j} is the total price of WTI on the j^{th} business day before day t.

2.3. Crude oil pricing: A structural model. Based on the discussion in the previous section, we know that the long- and short-term themes have different mechanisms and impacts on the price of crude oil. Accordingly, for the asset pricing of crude oil, it makes sense to construct separate long and short models, separately.

In this study, we decompose the total daily price of WTI into the long-term monthly average and the short-term daily deviation:

$$P = P_l + P_s$$

where P is the total daily price of WTI, P_l is the monthly average, and P_s is the daily deviation (Fig. 2.2). I run unit root tests using both the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) methods for the price of oil from 1973 to 2020. Table 2.1 shows that P_l has a unit root while the daily price deviation does not. To avoid the spurious issue when modeling monthly prices, we use R_l , returns on the monthly average prices, as the dependent variable. We specify the following linear structural model to forecast the price of crude oil:

$$(2.1) R_{l,t+1} = X_t \beta + u_{t+1}$$

$$(2.2) P_{s,t+1} = Z_t \gamma + e_{t+1}$$

$$(2.3) P_{t+1} = (1 + R_{l,t+1}) \times P_{l,t} + P_{s,t+1}$$

where $R_{l,t+1} = \frac{P_{l,t+1}}{P_{l,t}} - 1$, X is the set of long-term factors, and Z is the set of short-term factors. Thus, we have constructed a structural model of two equations which jointly models the price movements of WTI.

Based on the discussion above, we employ the following factors (X) in the long-term model (2.1):

- WORLDimb: the world oil supply/demand imbalance
- SAUDIprod: change in Saudi Arabia's crude oil production
- SAUDIosp: change in Saudi Arabia's OSP
- USprod: change in US crude oil production
- USgeo: geopolitical events associated with the US and the crude oil market

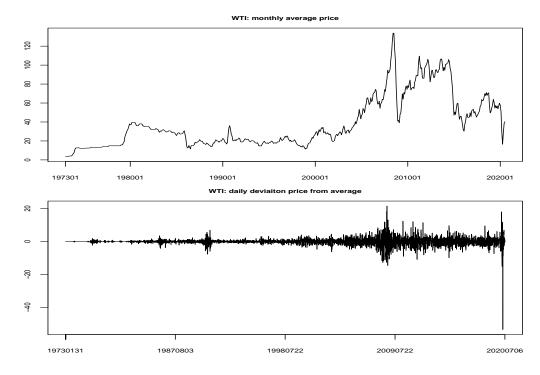


FIGURE 2.2. Monthly average and daily deviation of the price movements of crude oil (WTI) from Jan 31, 1973 to July 6, 2020.

Unitroot Test	Р	P_l	P_s
ADF	0.27	0.41	0.01
PP	0.36	0.10	0.01

TABLE 2.1. Unit root tests of the price of WTI (P) and its decompositions, the monthly average (P_l) and deviation (P_s) , from 1973 to 2020.

- CHINAimp: China's crude oil imports
- PMm: monthly price momentum of crude oil

We employ the following factors (Z) in the short-term model: (2.2)

- USDXchg: daily change in the US dollar index
- USstock: weekly change in US commercial crude oil stock
- USprodw: weekly US crude oil production change
- DISR: disruptions to the oil market
- PMI: China's Purchasing Manager's Index
- PMd: daily price momentum of crude oil

The long- and short-term equations can thus be specified as follows:

$$R_{l,t+1} = \beta_0 + \beta_1 \text{WORLDimb} + \beta_2 \text{SAUDIprod} + \beta_3 \text{SAUDIosp}$$

$$(2.4) + \beta_4 \text{USprod} + \beta_5 \text{USgeo} + \beta_6 \text{CHINAimp} + \beta_7 \text{PMm} + u_{t+1}$$

$$P_{s,t+1} = \gamma_0 + \gamma_1 \text{USDXchg} + \gamma_2 \text{USstock} + \gamma_3 \text{USprodw}$$

$$(2.5) + \gamma_4 \text{DISR} + \gamma_5 \text{PMI} + \gamma_6 \text{PMd} + e_{t+1}$$

These two equations, (2.4) and (2.5), constitute the structural model for the total price forecast. Given the linear decomposition of the total price, the expected total price is the sum of the expected long-term trend plus the expected short-term deviation. Thus, we can use the least squares method to forecast the monthly average using the long-term factors X and daily deviations using the short-term factors Z. The sum is a forecast of the total oil price. In the following sections, we first perform an in-sample analysis of the structural model and then conduct out-of-sample forecasts based on the formula above.

3. Empirical study: in-sample results

We have identified fundamental factors for the long-term price trend and short-term price deviations and proposed a structural model to capture each component. To better understand these factors and how the long and short model perform, we conduct an in-sample analysis in this section. The long-model factors have a monthly frequency, while the short-model factors are daily. The in-sample study will prepare us for the out-of-sample forecasting in the next section.

- 3.1. **In-sample analysis: long model.** First, we focus on the long model, progressing from univariate analysis to bivariate analysis and then the multi-factor model. For univariate analysis, we describe statistical values for each variable. For bivariate analysis, we investigate relationships between variables and between each variable and returns. To evaluate the fit of the multi-factor model, we explore OLS estimates and their significance.
- 3.1.1. Univariate Analysis. We summarize each variable and forward returns (WTImfRet) in Table 3.1, where Q1 and Q3 are for the first and third quartiles. It is interesting to see that many factors are skewed, indicating the volatility of the crude oil market. For example, the OSP of Arab Light can go down by \$7 but up by only \$3.15. The monthly change in US crude oil production ranges from -2.06 MBD to 0.76 while Saudi Arabia's range is -2 to 2.13, confirming Saudi Arabia's practice of production control. China's oil imports reach the peak of 11.16 MBD maximum recently.
- 3.1.2. Bivariate Analysis. We first investigate the correlation between each factor and the one-month forward return variable (WTImfRet) for the study period from Jan 1973 to May 2020. The results of Pearson and Spearman correlations are presented in Table 3.2. For each factor, the Pearson and rank correlation values have the same sign,

	Mean	Std	Median	Q1	Q3	Min	Max
WTImfRet (%)	0.82	9.04	0.16	-3.20	4.78	-43.35	72.61
SAUDIprod	0.01	0.48	0	-0.16	0.2	-2.00	2.13
SAUDIosp	-0.01	0.73	0.02	-0.30	0.27	-7.00	3.15
USprod	0	0.17	0	-0.07	0.07	-2.06	0.76
USgeo	-0.10	1.62	0	0	0	-3	3
CHINAimp	2.39	3.12	1.22	-0.17	4.57	-0.53	11.16
WORLDimb	-0.65	1.53	-0.70	-1.10	0.34	-5.70	1.94
PMm (%)	0.76	8.93	0.16	-3.20	4.66	-43.35	72.61

TABLE 3.1. Summary statistics for each variable. Data are monthly from Jan 1973 to May 2020.

implying the strength of the forecasting power of each factor. On the other hand, the rank correlation and Pearson correlation values differ greatly for some factors, indicating the existence of outliers in those factors, such as SAUDIosp and USprod. This is in keeping with the univariate analysis described above.

	Pearson	Spearman
SAUDIprod	-0.08	-0.03
SAUDIosp	0.30	0.13
USprod	-0.17	-0.03
USgeo	0.12	0.10
CHINAImp	0.01	0.08
WORLDimb	-0.30	-0.23
PMm	0.28	0.19

TABLE 3.2. Correlation between each factor and one-month forward returns from Jan 1973 to May 2020.

We now explore correlations between factors. Table 3.3 presents both Pearson (upper triangle) and rank correlations (lower triangle) between factors for the study period. Except for the 24% Pearson correlation between SAUDIprod and WORLDimb and 27% Pearson correlation between SAUDIosp and PMm, all the correlation values are below 10%. The low correlation values between factors confirm the diversification of alpha (price change) sources and ensure non-significance of the multicollinearity issue in multi-factor modeling.

3.1.3. Multi-factor model. We perform multi-factor analysis using OLS for the pooled monthly data from Jan 1973 to May 2020. Table 3.4 lists the estimates for coefficients with t-values and R^2 (%) in parentheses. Columns 2-8 are for the single-factor models,

	SAUDIprod	SAUDIosp	USprod	USgeo	CHINAimp	WORLDimb	PMm
SAUDIprod	1	0.04	-0.09	0	0.02	0.24	0.05
SAUDIosp	-0.01	1	-0.07	0.01	-0.03	0.02	0.27
USprod	-0.03	0.06	1	0.05	0.08	-0.03	-0.19
USgeo	-0.04	0	0.01	1	-0.2	-0.01	0.15
CHINAimp	-0.01	0.08	0.17	-0.05	1	0.07	-0.04
WORLDimb	0.09	-0.01	-0.02	0	0.07	1	-0.03
PMm	0.04	0.06	-0.02	0.11	0.07	-0.05	1

Table 3.3. Correlations between factors, with Pearson in the upper triangle and Spearman in the lower triangle. Data are monthly from Jan 1973 to May 2020.

and the last column is for the multi-factor model (2.4). First, for single factor models, each factor is significant at the 10% level except for the CHINAimp factor. For the Saudi Arabia theme, a 1 mbd increase in crude oil production will cause the price of WTI to decrease by 1.55% in the following month, and a \$1 increase in OSP will cause a 4.54% price increase in the following month. For the US theme, a 1 MBD increase in crude oil production will cause price decrease by 9.11%, while one unit of significant geopolitical events will be associated with a price increase of 0.65%. Chinese crude oil imports do not exhibit much effect in the model, this is mainly because the monthly change in price varies too much to be identified by the smooth long-term uptrending imports data. Regarding the world imbalance between crude oil demand and supply, one unit of such an imbalance will cause the price of WTI to decrease by 1.74%. The technical factor, PMm, has positive effects: a 1% price increase in a given month will continue into the next month with a 0.29% increase in price.

After pooling all the factors together in a linear model (2.4), the OLS coefficients and t-values remain about the same, while the joint explainatory power increases to 14% measured by R^2 . From an investment perspective, this is a very significant level, such that information, if used properly, can result in a profitable strategy. Overall, the results of both single- and multi-factor models confirm the fundamental analysis presented in previous sections. The efficacy of these identified factors should give us confidence to employ them to capture WTI price movements during the study period from 1973 to 2020. However, it is important not to overstate the significance of the in-sample results. Also, the results are for the pooled data of the entire period, and there may be different dynamics at different times within the study period.

3.2. **In-sample analysis: short model.** We perform in-sample analysis of the short model in this section. Similar to the data analysis for the long model, we first perform

⁷We expect that a longer horizon, such as annual or quarterly frequency, may be more significant. This could be explored in future research.

	Single	Multi-factor
SAUDIprod	-1.55 (-1.98, 1)	-1.78 (-2.34, 14)
SAUDIosp	4.54 (4.68, 9)	3.35 (4.07, 14)
USprod	-9.11 (-4.02, 3)	-7.29 (-3.30, 14)
USgeo	0.65(2.79, 1)	0.53 (2.36, 14)
CHINAimp	0.03 (0.19, 0.4)	$0.08 \ (0.65, 14)$
WORLDimb	-1.74 (-2.10, 9)	-1.25 (-1.59, 14)
PMm	0.29 (6.98, 8)	0.20 (4.74, 14)

TABLE 3.4. OLS estimates for single factor models and the multifactor model. The t-values and R^2 (%) are included in parentheses. Data are monthly from Jan 1973 to May 2020.

univariate and bivariate analysis, then apply OLS to estimate the linear multi-factor model. The data are daily from Jan 1, 1973 to May 31, 2020.

3.2.1. Univariate Analysis. Following the same pattern as the long model, we present summary statistics for each variable of the short model in Table 3.5. We can see that one-day-forward daily price deviations (WTIdf) are very volatile, with the minimum value of \$-53 on April 21, 2020, and maximum value of \$21.64 on April 22, 2020. The standard deviation of daily price deviations is \$1.91 per barrel. The weekly change in US crude oil production (USprodw) ranges from -1.07 to 1.1 MBD, which is quite dramatic for a weekly frequency. The daily change in the US dollar index (USDXchg) has a mean of zero with a range from -4.87 to 3.61, while the weekly change in the US commercial stock (USstock) averages 0.11 mbd but with a wide range from -14.98 to 19.25. PMI, the factor that measures Chinese manufacturing activities, has values from 35.7 to 59.2 during the study period.

The variations in the daily and weekly factor values provide strong identification capability for the short model, which focuses on forecasting the daily price deviation from the monthly average of WTI prices.

3.2.2. Bivariate Analysis. We now investigate correlations between factors and between each factor and the one-day-forward value of daily price deviation (Table 3.6). First, factors have low correlations with each other except DISR and PMI. Second, in terms of forecasting power, the short-term momentum factor (PMd) has the highest correlation of 43% (Pearson) and 39%(rank), indicating that the impacts of short-term events during the preceding 10 business days persist strongly for several days. The commercial stock change factor, which is reported every Wednesday, also has significant negative effects on the next business day's price movements, as practiced by investors. The change in the USDX has marginally negative effects on the price on the following business day, which may be because the new daily value of the USDX

	Mean	Std	Median	Q1	Q3	Min	Max
WTIdf	0	1.91	0	-0.53	0.55	-53.53	21.64
USDXchg	0	0.52	0	-0.27	0.27	-4.87	3.61
USstock	0.11	4.44	0.19	-2.78	2.95	-14.98	19.25
USprodw	0	0.11	0	-0.02	0.02	-1.07	1.10
DISR	2.25	0.97	2.24	1.75	2.92	0.07	4.42
PMI	51.67	1.71	51.66	51.66	51.66	35.70	59.20
PMd	0.02	2.79	0	-0.75	1.02	-59.83	19.86

Table 3.5. Summary statistics for each variable of the short model from Jan 1, 1973 to May 31, 2020.

will mostly impact the price on the same day. Both DISR and weekly US oil production barely have any effect.⁸ Note that of course, all the short-term factors have more significant effects on the price of WTI on the same day than on the next business day. For forecasting purposes, we employ the factors that continue to impact the price on the following business day.

	WTIdf	USDXchg	USstockw	USprodw	DISR	PMI	PMd
WTIdf	1	-0.02	-0.05	0.01	0.02	-0.01	0.43
USDXchg	-0.02	1	0.02	0.01	0.01	-0.01	-0.06
USstock	-0.06	0.02	1	0.08	0.08	-0.04	-0.1
USprodw	0	0	0.05	1	-0.06	-0.01	-0.03
DISR	0	0.02	0.04	-0.12	1	-0.6	-0.08
PMI	0.01	-0.02	-0.05	-0.04	-0.63	1	0.2
PMd	0.39	-0.05	-0.1	-0.03	-0.08	0.06	1

TABLE 3.6. Correlation between factors and between each factor and one-day-forward price deviation. Pearson correlation is in the upper triangle and Spearman in the lower triangle. Data is daily from Jan 1, 1973 to May 31, 2020.

3.2.3. Multi-factor model. We present the OLS estimates for the single-factor and multi-factor models (2.5) in Table 3.7 with t-values and R^2 included in parentheses. For the single-factor models, in desceding order of magnitude, PMd, USstock, and USDXchg have significant effects on the next-day's price deviation. Note that except for PMd, all other factors have close to zero R^2 (rounded to the nearest 1%). The multi-factor model has an R^2 of 16%, with 15% contributed from PMd and the remaining 1% from other factors. In the multi-factor model, PMd has a coefficient of 0.28 with a t-value of 41.51. The next is the factor USstock, which has a coefficient of

⁸Due to its high correlation with PMI, DISR is removed from the multi-factor model in the out-of-sample study.

-0.01 with a t-value of -1.97. Due to the dominant effect of PMd, the effects of a factor with even a small correlation with PMd can be distorted in the multi-factor model. For example, the t-value for USDXchg dropped from -1.89 in the single-factor model to 0.61 in the multi-factor model; the coefficient for PMI changes sign. However, because of this dominant effect of PMd, these small distortions will not impact the forecast significantly. In an out-of-sample study, we use an expanding window such that data are added on a daily basis, so these coefficients will change over time, and we use all these factors except DISR to forecast the next-day WTI price deviation.

	Single	Multi-factor
USDXchg	-0.07 (-1.89, 0)	0.02 (0.61, 16)
UStock	-0.03 (-5.48, 0)	-0.01 (-1.97, 16)
USprodw	0.03 (0.16, 0)	0.3 (1.73, 16)
DISR	0.004 (0.12, 0)	-0.01 (-0.70, 16)
PMI	0.01 (0.87, 0)	-0.09 (-7.41, 16)
PMd	0.27 (44.03, 15)	0.28 (41.51, 16)

TABLE 3.7. OLS estimates for single-factor and multi-factor models (2.5). The t-values and R^2 (%) are in parentheses. Data are daily from Jan 1, 1973 to May 31, 2020.

Having conducted in-sample analysis for both the long and short models, we show that the fundamentally identified factors have forecasting power for the monthly average and daily deviations of the WTI prices. In the next section, we perform an out-of-sample study to forecast the total daily prices of WTI and verify the efficacy of forecasts based on portfolio performance.

4. Out-of-sample forecasts

Following the structural model of (2.1) - (2.3), we conduct out-of-sample forecasts for both the monthly average and daily deviation of WTI prices from the period of Jan 1, 2010 to May 31, 2020. For this 10-year out-of-sample period, we forecast monthly returns using the long model and then forecast the daily price deviation using the short model. We then combine the monthly average forecast and daily deviation forecast to obtain the total price forecast for each business day during the out-of-sample period.

We first check the accuracy of the structural model forecasts by econometric measurements. We then form a real strategy by investing in WTI futures and investigate the portfolio performance.

Long Model Note that we use returns instead of prices as the dependent variable in the long model because the latter is nonstationary. We use one-month forward returns and factors specified in model (2.1). The returns are based on the monthly

average price of WTI. At time T (month end), we use the factor values known as of T-1 and forward returns from T-1 to T to obtain estimates for the coefficients, then use the most recent available information on factor values available at T to derive the forecast of the monthly average price for the next month, T+1.

Estimate:
$$R_{l,t+1} = X_t \beta_{t+1} + u_{t+1}, \quad t = T - 1, \dots, T - N$$

Forecast: $\hat{R}_{l,T+1} = X_T \hat{\beta}_{T+1}$
 $\hat{P}_{l,T+1} = (1 + \hat{R}_{l,T+1}) \times P_{l,T}$

The long model data are monthly from Jan 1973 to May 2020. We set the period from Jan 1973 to Dec 2009 as in-sample and the period from 2010 to 2020 as out-of-sample. The out-of-sample forecast and portfolio are constructed as follows: Using the example of the date of Dec 31, 2009, we estimate the model parameters using the in-sample factor values from Jan 1973 to Nov 2009, then use the most recent month's factor value on Dec 31, 2009 to forecast the monthly return for the next month from Dec 31, 2009 to Jan 31, 2010.

Short Model For the daily deviation forecasts, note that we do not know the value of P_s until the end of the month because we simply do not know the true value of P_l until then. To make sure we have information available at time T for the out-of-sample forecasts, we use the data of T-30 (30 business days prior to the time T) as the in-sample data to derive the estimates for factor weights. We then use the information (factor values) at time T to forecast the daily price deviation for T+1.

Estimate:
$$P_{s,t+1} = Z_t \gamma_{t+1} + e_{t+1}, \quad t = T - 30, \dots, T - 30 - N$$

Forecast: $\hat{P}_{s,T+1} = Z_T \hat{\gamma}_{T+1}$

With the forecasts in hands for both the monthly average and daily deviation, we obtain the forecast for the daily total price at time T:

$$\hat{P}_{T+1} = \hat{P}_{l,T+1,l} + \hat{P}_{s,T+1}.$$

We discuss the out-of-sample forecasting accuracy by conventional econometric measures.

4.1. Alpha Model Efficacy. We measure the efficacy of the forecasts from the long and short models according to both standard econometric norms and the performance of the derived portfolios. Regarding econometric norms, we follow the convention of using measures such as correlation and mean square percent error to measure the accuracy of the price forecasts. We also use the hit ratio (HIT) to evaluate the forecast accuracy by direction (up or down), which is very important for investment decisions.

We first obtain the values of R^2 for the long and short model, respectively:

$$Actual = \beta_0 + \beta_1 Forecast + \nu$$

We also calculate the correlations between actual values and forecasts for the long and short forecasts (Table 4.1). First, the forecasts from both the long and short models perform well in terms of correlation and R^2 . Second, the long model has much better forecasting power than the short model. There are two reasons for this: on one hand, crude oil market trend is usually obvious, dictated by a few major underlying forces; on the other hand, daily price fluctuations arise from numerous sources, many of which are difficult to characterize in a quantitative model. Nevertheless, the total price forecast has a very high correlation with actual prices, indicating the strong efficacy of the structural model.

Clearly, the accuracy of the daily total price forecast is based on the monthly forecasts. If the monthly forecasts are very far from the true values, the daily forecasts will add little value even if they are accurate. On the other hand, even if the monthly forecasts are highly accurate, the accuracy of the total price forecasts can still be low if the daily deviation forecasts are inaccurate.

	Pearson Corr.	Rank Corr.	OLS R^2
Long model	0.97	0.97	0.95
Short model	0.37	0.44	0.14
Total price	0.99	0.99	0.98

TABLE 4.1. Correlations between forecasts and true values and OLS R^2 (true values \sim forecasts) for the out-of-sample forecasts.

Because the daily deviation can be large or small, the percentage for the daily price deviation does not make much sense from either a price forecasting or investment perspective. We measure the accuracy of the monthly average price forecast and the total price forecast, from which we can derive the forecasting power of the daily deviation forecasts. For notational convenience, we drop the time subscript.

(4.1) Monthly average:
$$P_l = \hat{P}_l + \hat{u}_l$$

(4.2) Total Price:
$$P = (\hat{P}_l + \hat{u}_l) + (\hat{P}_s + \hat{e}_s) = \hat{P} + \hat{\nu}$$

where \hat{P}_l is the monthly forecast of P_l and \hat{P}_s is the daily forecast of P_s , while \hat{u}_l and \hat{e}_s are the error terms for the monthly and daily forecasts, respectively. \hat{P} is the forecast for the total daily price.

We measure the forecasting accuracy using conventional metrics: mean absolute percentage errors (MAPE), mean of squared errors (MSE), and mean absolute error (MAE). We also use the nonparametric measure of hit ratio, which is based on the

directional count of forecast and the actual values.

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|P - \hat{P}|}{P}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (P - \hat{P})^{2}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |P - \hat{P}|$$

$$HIT = \frac{1}{n} \sum_{i=1}^{n} I(R \times \hat{R})$$

where I() is an indicator function with the value of 1 for a positive input and zero otherwise. We list the results in Table 4.2 for monthly average price forecasts and total price forecasts. We also present results for the return version. The contribution of daily deviation forecasts can be derived from the accuracy of the total price and the long-term price. The MAPE values indicate that on average, the monthly average price forecast can be as accurate as 93%, and the total price forecast can be as accurate as 96%, which agree with the correlation and R^2 values. Since MSE and MAE measure magnitude instead of percentage, it is more meaningful to compare them with different models. In terms of the hit ratio, the value for the monthly average return forecasts is 59.52%, indicating that the long model correctly predicts the direction of WTI price movements 60% of the time; the value for the total daily price return forecasts is 58.19%, which is quite high. From an investment perspective, a hit ratio for return forecasts above 55% implies significant forecasting power. Such forecasts are called alpha and can usually yield a profitable portfolio. This is confirmed by the portfolio performance results in the next section.

	MAPE	MSE	MAE	HIT
Long Model: forecast				
\hat{P}_l	0.07	29.09	4.14	NA
\hat{R}_l	NA	0.01	0.07	59.52
Total Price: forecast				
\hat{P}	0.04	14.36	2.59	NA
\hat{R}	NA	0.01	0.04	58.19

Table 4.2. Accuracy measurements (MAPE, MSE, MAE, and HIT) for the out-of-sample forecasts, Jan 1, 2010 to May 30, 2020.

The results above are for the entire period. Fig. 4.1 presents results in greater detail within the out-of-sample study period, including time-series differences between predicted and and actual values for monthly average (top plot) and total daily prices (bottom plot). The plots show that first, for both monthly average and total prices, the differences between the forecast and actual values are around zero with a similar range over time, indicating the robustness of the model forecasts over time. Second, the symmetry around zero shows no systematic over- or undervaluations from the

model. Third, for the most recent period in 2020, forecasts can differ greatly from actual values, for example, there is an outlier difference of \$-50 during the COVID-19 pandemic.

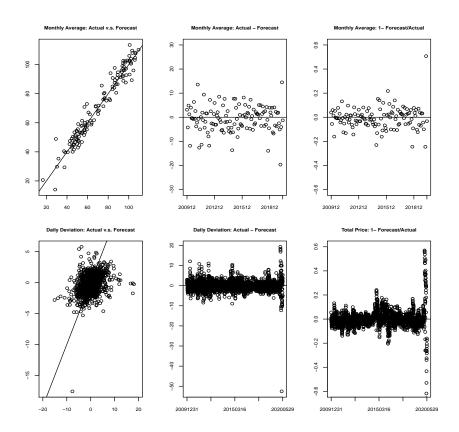


FIGURE 4.1. Comparing forecasts and actual values over time for monthly average and total daily prices: out-of-sample results from Jan 1, 2001 to May 31, 2020.

To be more precise about the forecast accuracy over time, Fig. 4.2 presents the monthly forecasts for the long model with a 90% confidence interval and actual values from Jan 2001 to May 2020. Overall, the forecasts follow actual monthly average prices very closely over time, though much more closely during certain periods. The narrow band indicates a high level of accuracy. However, the 90% band does not cover actual values for some periods, indicating that the model performs poorly there. These will all be reflected in the portfolio performance if we use forecasts as alpha and invest in this commodity.

4.2. Portfolio Performance by Investing in WTI Futures. We explore the efficacy of the structural model by investing in WTI via futures. Since this is leveraged, we use just one contract for each trading period. We specify the same investment

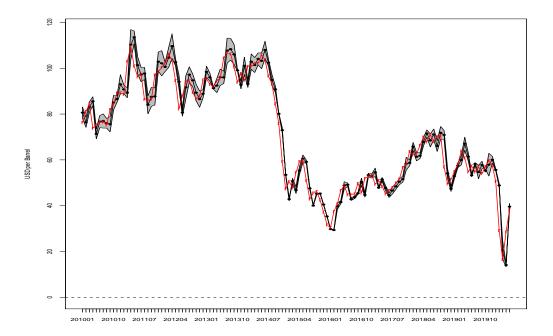


FIGURE 4.2. Out-of-sample forecasts for monthly average WTI prices from Jan 2001 to May 2020. The gray area is the 90% confidence band. The red line is the actual monthly average price.

decision criteria as before, with a lambda value of 0.005 for the monthly portfolio and 0.05 for the daily portfolio.

In the ETF or spot market, we calculate portfolio performance based on the geometric return. In the futures market, we calculate profit and loss (PL) and the returns are arithmetic. The margin for one NYMEX WTI contract is around \$5000 most of the time during the out-of-sample period; a broker may require extra margin in a derivatives trading account.⁹ To be conservative and avoid margin calls when loss occurs, we maintain principal of \$10,000 and carry out transactions for only one contract each period (month or day). Since the unit per WTI contract is 1000 barrels, the PL for each transaction is as follows:

Long buy:
$$PL = (P_{t+1} - P_t) \times 1000$$

Short sell: $PL = (P_t - P_{t+1}) \times 1000$

Therefore, the return for each and the entire trading period is

each period:
$$FR = PL/10000$$

entire period: $CR = \sum_{t} FR_{t}$

⁹https://www.cmegroup.com/trading/energy/.

where FR is the future return for each period, and CR is the cumulative return (arithmetic) for the entire period. It should be emphasized that because futures are leveraged with a leverage ratio of $\frac{P_t \times 1000}{\text{margin}}$, the PL will enlarge with the number of contracts, but return will stay the same.

Based on the above specifications, we carry out investments via futures of the front month (CL). We summarize the overall performance of the futures portfolio for the entire out-of-sample period in Table 4.3. The monthly portfolio is based on the monthly average price forecast of WTI, assuming such an average price exists each month. The daily portfolio is based on the total price forecast (monthly average plus daily deviation), which is more realistic from an execution perspective. Table 4.3 shows that both the monthly and daily portfolios are profitable. For the monthly futures portfolio, over the out-of-sample period, the average profit is \$1,444 per month, and the total profit is \$181,989, resulting in an annualized return of 1.73 and Sharpe ratio of 0.98. For the daily futures portfolio, over the out-of-sample period, the average profit is \$199 per day, and the total profit is \$517,200, resulting in an annualized return of 4.78 and Sharpe ratio of 1.88. Note that we do not trade for every period, and the results are based on only one contract. These performance numbers confirm the strong efficacy of the structural alpha.

Portfolio	meanPL	totalPL	stdPL	AnnuRet	AnnuStd	Ratio
Monthly	1,444	181,989	5,087	1.73	1.76	0.98
Daily	199	517,200	1,641	4.78	2.54	1.88

TABLE 4.3. Performance of the futures portfolio over the entire out-of-sample period from Jan 1, 2010 to May 31, 2020.

To show portfolio performance over time, we present the cumulative PL in Fig. 4.3. There are smooth upward curves for both the monthly and daily portfolios, indicating their robust performance over time.

We now analyze portfolio characteristics based on PL and bets. The former describes some details about profit and loss, while the latter enables us to see how PLs are generated from the short sell and long buy transactions.

Table 4.4 describes portfolio statistics for trading profits and losses for the monthly portfolio in the top section and daily portfolio in the bottom section. Consider the monthly portfolio first: Of 108 trades, 40 of them incurre losses with an average loss of \$3,407, and 68 generate profits with an average profit of \$4,681. The profitable performance of the monthly portfolio arises from not only the high number profitable trades but also the higher profits per trade than losses. Now consider the daily portfolio. There are 640 trades in total, with 204 trades with losses and 436 trades with profits. The average loss is \$928 and the median loss is \$670, while the average profit is \$1,651 and the median profit is \$1,140. This shows that the portfolio performance is robust, which confirms the predictive power of the structural model.

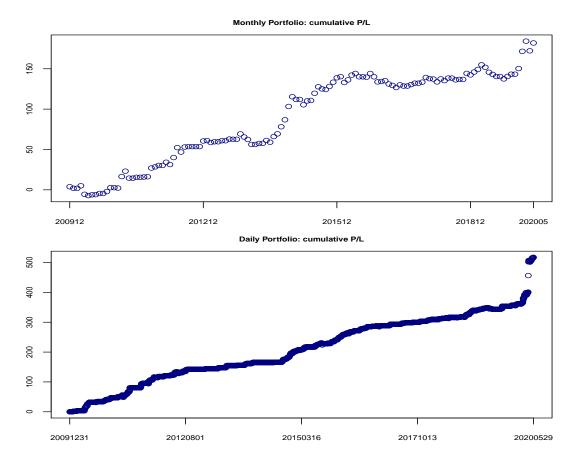


FIGURE 4.3. Cumulative sum of the profits and loss (PL) for the monthly and daily portfolios over the out-of-sample period from Jan 1, 2010 to May 31, 2020.

by PL	Number	meanPL	stdPL	medianPL	minPL	maxPL
Monthly						
Loss	40	-3,407	2,771	-2,656	-12,015	-13
Profit	68	4,681	4,308	3,452	25	21,335
Daily						
Loss	204	-928	927	-670	-4,900	-10
Profit	436	1,651	3,613	1,140	10	55,290

Table 4.4. Profit and loss characteristics for the monthly and daily portfolios. Data period: Jan 1, 2010 to May 31, 2020

Table 4.5 summarizes PL statistics across transactions: short sell, no trade, and long buy. The top section is for the monthly portfolio, and the bottom section is

for the daily portfolio. The "Number" column shows how many trades are performed for each category. The other columns present the summary PL statistics for each category. We see that for the monthly portfolio, over the 126-month out-of-sample period, 43 months have a short sell, 65 months have a long buy, and 18 months have no trade. For the monthly portfolio, both the short and long positions are profitable: the short positions generate \$2,342 profit on average and a median of \$918, while the long positions generate \$1,250 profit on average and a median of \$1,243. The PL results of the transactions for the monthly portfolio indicate strong forecasting power of alphas derived from the long model. Now consider the daily portfolio, where among 2,596 business days over the out-of-sample period, there are 306 days with a short sell, 334 days with a long buy, and 1,956 days with no trade. Both the short and long positions are profitable: the short positions generate \$863 on average and a median of \$605, while the long positions generate \$759 on average and a median of \$560. The performance of the daily portfolio trades is based on the total price forecast for each day, which implies strong alpha efficacy of the structural model. The profits are generated by the forecasts of both price decreases and increases over the entire period, not concentrated within a few trades or a few days, which confirms the robustness of the alpha.

by Bets	Number	meanPL	stdPL	medianPL	minPL	maxPL
Monthly						
Short	43	2,343	6,964	917	-12,015	21,335
No Trade	18	0	0	0	0	0
Long	65	1,250	4,188	1,243	-10,551	14,278
Daily						
Short	306	863	3,577	605	-4,900	55,290
No Trade	1,956	0	0	0	0	0
Long	334	758	2,883	560	-4,530	45,890

TABLE 4.5. Summary statistics for the monthly and daily portfolios based on bets: short sell, no trade, and long buy. Data period: Jan 1, 2010 to May 31, 2020.

5. Conclusion

In this paper, we forecast the price of WTI using a structural equation model, which comprises a long model for monthly average price and a short model for daily fluctuations. The structural model is built in such a way that it reflects the real world sources of the crude oil price movements: the long-term trend is impacted by the pricing and production policy of Saudi Arabia, US technological innovation (such as shale oil), geopolitical power, Chinese demand, and world supply/demand

imbalance; while the short-term price movements are impacted by the value of the petrodollar, US commercial stock, short-term production, and other news such as oil market disruptions and Chinese manufacturing activity.

We conduct both in-sample analysis and out-of-sample forecasts. The results indicate strong efficacy of the structural model based on both econometric measures. The structural model performs well not only during a normal market but also during extreme volatile periods, such as for March and April in 2020.

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