Higher Moment Risk Premiums for the Crude Oil Market: A Downside and Upside Conditional Decomposition

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Abstract

Relying on options written on the USO, an exchange traded fund tracking the daily price changes of the WTI light sweet crude oil, we extract variance and skew risk premiums in a model-free way. We further decompose these risk premiums into downside and upside conditional components and show that they are time varying; that they can be partially explained by USO excess returns and, more importantly, these decomposed risk premiums enable a much better prediction of USO excess returns than the standard, or undecomposed, variance and skew risk premiums.

JEL Classification: G11; G12; G13

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1 Introduction

Energy commodities have become a major part of financial markets as a result of the rapid growth in trading volume and the variety of derivative products, among which the crude oil futures and options have taken a significant proportion. Specifically, the trading volume of crude oil futures and options accounts for over 50% of the total trading volume of energy contracts on the NYMEX in 2015. As for the equity (index) option market, the commodity option market enables the study of variance risk premium, that is, the premium asked by market participants to invest/trade volatility risk. For the extensive literature on variance risk premium we refer, without being exhaustive, to Bakshi et al. (2003), Carr and Wu (2009), Trolle and Schwartz (2010) and Prokopczuk and Wese Simen (2014).

The fact that financial markets react differently to positive and negative shocks has been widely acknowledged in previous literature. Consequently, semivariance measures, considered in Barndorff-Nielsen et al. (2008) or Patton and Sheppard (2013), were found to carry more information than unconditional measures. For the specific case of crude oil market and the relevance of semivariance measures see Chevallier and Sévi (2012) or Sévi (2014). Following that line of research it is therefore natural to assess whether tail risk premium or conditional variance risk premium carries more information than standard (i.e. unconditional) variance risk premium. In Bollerslev et al. (2015), Lettau et al. (2014) and Kilic and Shaliastovich (2015), it was confirmed that conditional variance risk premium has higher forecasting power for equity index excess returns.

Beyond variance risk premium, skew risk premium has recently attracted a strong interest among academics. In Kozhan et al. (2013), see also the important and related work of Neuberger (2012), the authors found that skew risk premium naturally completes variance risk premium for the equity index option market. In Da Fonseca and Xu (2016), the authors analyze the specifics of the volatility index option market (i.e. VIX options) with respect to variance and skew risk premiums and show the consistency of the results with the shapes of the smile observed in the equity and volatility index option markets.

Based on these works we contribute to the literature by performing a conditional decomposition of variance and skew risk premiums extracted from options written on the USO (an exchange traded fund tracking the daily price changes of the WTI light sweet crude oil). We assess the time variation property of these risk premiums and analyze the relations between these decomposed higher moment risk premiums and the USO excess returns. Lastly, we show how these decomposed higher moment risk premiums enable a much better prediction of USO excess returns.

The paper is organized as follows. Section 2 presents a formal definition of the key quantities used in this work. Section 3 provides a description of the data used in the empirical analysis. We discuss the empirical implementation and the results in Section 4 while Section 5 concludes.

2 Pricing formulas

In this section we describe the variance and skew swaps that are the main financial products used in this work. A variance swap is a derivative contract by which two counterparties agree to exchange some cash flows at some prespecified dates. One counterparty will pay an amount at the initiation of the contract, denoted by t, and will receive an amount at the maturity of the contract, denoted by T, equals to the realized variance of the underlying asset computed between t and T. As the amount paid at time t is known during the life of the contract, it is called the fixed leg of the swap while the amount received at time T is only determined at date, which is unknown during the life of the contract, and is therefore called the floating leg of the swap. If between t and T the realized volatility increases to the point that it offsets the premium paid at time t, the net position will be positive for that counterparty who is often qualified as the protection buyer (against an increase of volatility). The counterparty holding the other side of the deal is called the protection seller.

A skew swap has the same characteristics as a variance swap except that the premium paid at time t is related to the risk-neutral third moment of the underlying asset return while at time T the realized skew of the underlying asset is paid. This product allows a market participant to hedge against a change in the skewness of the asset.

Computing the values of such contracts involves the evaluation of the realized variance and skewness of the underlying asset as well as the determination of the premiums paid at initiation of the contracts, that is to say, the computation of the fixed legs of the variance and skew swaps. This task can be achieved thanks to the results of Carr and Madan (1998) and Kozhan et al. (2013). Once these quantities are evaluated, by averaging over time we deduce the variance and skew risk premiums that are of fundamental importance in finance as they quantify the compensations asked by market participants to invest or bear those risks.

A closer look at the formulas suggests to decompose the variance and skew risk premiums conditionally on the evolution of the underlying asset and, therefore, to define conditional versions of such quantities and to expect them to have higher information content than unconditional ones. These analytical considerations are further justified by the well known empirical fact that an asset behaves differently depending on whether its return is positive or negative. This latter property has been extensively used in the finance literature, either in asset pricing papers such as Ang et al. (2006) which shows that investors ask for additional compensation when market goes down and the downside premium is reflected by the cross section of stock returns, or Lettau et al. (2014) which demonstrates that in the currency market the differential between high and low interest rate currencies is higher under bad market conditions compared to good market conditions, or Smith (2007) which proves that investors demand different premiums conditional on the sign and magnitude of market skewness. As a result, it seems judicious to perform such conditional decomposition for the variance and skew risk premiums and assess whether these decomposed quantities carry more information. Empirical results will confirm this intuition.

We first present the variance swap contract and related quantities such as variance risk premium, then conditional decompositions as well as excess returns of investments made on such contracts. It will allow us to specify the notations used throughout this work. We pursue with the skew swap and related quantities that are important for this work and constitute a contribution to the literature.

2.1 Variance risk premiums

The valuation of a variance swap contract of maturity T requires the computation of the fixed leg that is paid at time t, the initiation date of the contract. Let S_t be the underlying asset price at time t, then the log return from t to T is $r_{t,T} = \ln S_T - \ln S_t$. In this work the asset S_t will be the USO, an exchange traded fund tracking the daily price changes of the WTI light sweet crude oil. Firstly, we define the USO excess return from t to T as

$$xm_{t,T}^{USO} = r_{t,T} - r_{t,T}^{f}, \tag{1}$$

where $r_{t,T}^{f}$ is the risk-free rate.

Following the literature, to value the fixed leg of the swap, Kozhan et al. (2013) proposes the following formula

$$iv_{t,T} = E_t^{Q}[g^{v}(r_{t,T})], \tag{2}$$

where $E_t^{Q}[.]$ denotes the risk-neutral expectation conditional on time t and $g^{v}(r) = 2\left(e^{r} - 1 - r\right)$. A Taylor expansion of this function around zero shows that it is equal to r^2, thus its choice for this product.

It was shown in the literature that any twice-continuously differentiable payoff function can be spanned by a continuum of out-of-the-money (OTM) European calls and puts. Specifically, Kozhan et al. (2013) show that based on the payoff function g^{v} the risk-neutral variance $iv_{t,T}$ can be expressed as follows

$$\begin{aligned} iv_{t,T} &= 2\int_{S_t}^{+\infty} \frac{C_{t,T}(K)}{B_{t,T}K^2}dK + 2\int_{0}^{S_t} \frac{P_{t,T}(K)}{B_{t,T}K^2}dK \\ &= iv_{t,T}^{u} + iv_{t,T}^{d}, \end{aligned} \tag{3}$$

where $C_{t,T}(K)$ and $P_{t,T}(K)$ denote the prices at time t of calls and puts with expiry date T and strike price K and $B_{t,T}$ is the zero-coupon bond at time t with maturity T. In fact, $iv_{t,T}^{u}$ and $iv_{t,T}^{d}$ can be spanned by a continuum of OTM calls and puts, respectively, which correspond to the first and second integrals in Eq.(3). A detailed proof is presented in Appendix A. The decomposition is quite intuitive, the upside risk neutral variance $iv^u_{t,T}$ is constructed upon a set of call options that will pay only when the underlying asset return from t to T, that is to say $r_{t,T} = \ln S_T - \ln S_t$, is positive. In fact, it captures the second moment of the upper tail distribution. Likewise, the downside risk neutral variance $iv^d_{t,T}$ is constructed upon a set of put options that will pay only when the underlying asset return from t to T is negative and, in that case, it captures the second moment of the lower tail distribution. As we have

$$g^v(r_{t,T}) = g^v(r_{t,T})\mathbf{1}_{\{r_{t,T}>0\}} + g^v(r_{t,T})\mathbf{1}_{\{r_{t,T}\leq 0\}}, \tag{4}$$

it is natural to also name $iv^u_{t,T}$ and $iv^d_{t,T}$ the upside and downside risk-neutral variances and state

$$\begin{aligned} iv^u_{t,T} &= E^Q_t[g^v(r_{t,T})\mathbf{1}_{\{r_{t,T}>0\}}], \\ iv^d_{t,T} &= E^Q_t[g^v(r_{t,T})\mathbf{1}_{\{r_{t,T}\leq 0\}}]. \end{aligned} \tag{5}$$

As there are only a finite number of options available in the market, iv^u_t and iv^d_t can be approximated in practice by the following sums:

$$\begin{aligned} iv^u_{t,T} &= 2\sum_{S_t\leq K_i} \frac{C_{t,T}(K_i)}{B_{t,T}K_i^2}\Delta I(K_i), \\ iv^d_{t,T} &= 2\sum_{K_i\leq S_t} \frac{P_{t,T}(K_i)}{B_{t,T}K_i^2}\Delta I(K_i), \end{aligned} \tag{6}$$

with the weight function $\Delta I(K_i)$ defined as

$$\Delta I(K_i) = \begin{cases} \frac{K_{i+1}-K_{i-1}}{2}, 0 \leq i \leq N \quad (\text{with } K_{-1} = 2K_0 - K_1, K_{N+1} = 2K_N - K_{N-1}) \\ 0, \quad \text{otherwise.} \end{cases}$$

By definition of the variance swap contract, the floating leg is given by

$$rv_{t,T} = E^P_t[g^v(r_{t,T})] = \sum_{i=t}^{T-1} g^v(r_{i,i+1}), \tag{7}$$

where $r_{i,i+1}$ is the daily log return of the underlying asset (so we split the interval $[t\ T]$, which will be one-month long in this work, into daily sub-intervals), and $E^P_t[.]$ denotes the historical expectation conditional on time t. Following Kilic and Shaliastovich (2015), we decompose the realized variance $rv_{t,T}$ into two parts that are related to the two opposite sides of the asset return distribution. More precisely, we will write

$$\begin{aligned} rv_{t,T} &= \sum_{i=t}^{T-1} g^v(r_{i,i+1})\mathbf{1}_{\{r_{i,i+1}>0\}} + \sum_{i=t}^{T-1} g^v(r_{i,i+1})\mathbf{1}_{\{r_{i,i+1}\leq 0\}} \\ &= rv^u_{t,T} + rv^d_{t,T}, \end{aligned} \tag{8}$$

where $rv^u_{t,T}$ and $rv^d_{t,T}$ denote the upside and downside realized variances, respectively.

These quantities being defined, the payoff of a variance swap (payer of the fixed leg and receiver of the floating leg) is given by $rv_{t,T} - iv_{t,T}$ and after averaging under the historical probability measure the variance risk premium is obtained. As explained in Kozhan et al. (2013), it is convenient to define the excess return of an investment in the variance swap and it is given by

$$vp_{t,T} = \frac{rv_{t,T}}{iv_{t,T}} - 1. \tag{9}$$

The decomposition performed on the variance swap allows us to define the upside and downside variance swaps as $rv^u_{t,T} - iv^u_{t,T}$ and $rv^d_{t,T} - iv^d_{t,T}$, respectively, as well as the corresponding risk premiums. Here also, it is convenient to define excess returns associated with these swaps and it leads to

$$vp^u_{t,T} = \frac{rv^u_{t,T}}{iv^u_{t,T}} - 1, \quad vp^d_{t,T} = \frac{rv^d_{t,T}}{iv^d_{t,T}} - 1. \tag{10}$$

Remark 2.1 *The decomposition of the variance risk premium into two components follows Kilic and Shaliastovich (2015) where the authors qualified them as "good" (for the upside) and "bad" (for the downside) risk premiums, a naming justified by the fact that they analyze an equity index (the S&P500) for which a positive (negative) return is often favorably (unfavorably) considered. In the case of the crude oil, such naming is inappropriate as a too high oil price leads to a decrease of consumption and a weakening of the economy.*

Remark 2.2 *In Kilic and Shaliastovich (2015), the decomposition of Eq.*(8) *is computed using high frequency data and as explained by these authors it is known that, thanks to Barndorff-Nielsen et al. (2008), under the hypothesis that $(r_t)_{t\geq 0}$ satisfies the dynamic $r_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dw_s + J_t$ with $(w_t)_{t\geq 0}$ a Brownian motion and J_t a pure jump process, then the following convergences in probability hold*

$$rv^u_{t,T} \to \frac{1}{2}\int_t^T \sigma_s^2 ds + \sum_{t\leq s\leq T} (\Delta r_s)^2 \mathbf{1}_{\{\Delta r_s \geq 0\}},$$
$$rv^d_{t,T} \to \frac{1}{2}\int_t^T \sigma_s^2 ds + \sum_{t\leq s\leq T} (\Delta r_s)^2 \mathbf{1}_{\{\Delta r_s \leq 0\}},$$

with $\Delta r_s = r_s - r_{s-}$.

For the risk neutral part of the variance swap, they perform the decomposition in Eq.(3). Notice that while the upside risk neutral variance depends on positive evolutions of the underlying asset over the interval $[t\ T]$, the upside realized variance does not and similar remark applies to the downside decomposition. As a result, it does not lead exactly to risk premiums as, by definition, a risk premium requires the same quantity to be computed under the risk neutral and historical probabilities. Still, we will follow these authors and qualify the average value of $rv^u_{t,T} - iv^u_{t,T}$ and $rv^d_{t,T} - iv^d_{t,T}$ as risk premiums.

2.2 Skew risk premiums

For the skew swap we will closely follow Kozhan et al. (2013) and we refer to that work for further details. These authors propose to compute the fixed leg of the swap at time t with maturity T as

$$is_{t,T} = E^Q_t[g^s(r_{t,T})], \tag{11}$$

where $g^s(r) = 6\,(2 + r - 2e^r + re^r)$. A Taylor expansion of g^s shows that it behaves like r^3 and it justifies its use to compute the skewness. [1]The expectation in Eq.(12) can be expressed as a function of a continuum of OTM options as it can be written as

$$is_{t,T} = 3(v^E_{t,T} - iv_{t,T}), \tag{12}$$

where the quantity $v^E_{t,T}$ is defined as

$$\begin{aligned} v^E_{t,T} &= \frac{2}{B_{t,T}} \int_{S_t}^{+\infty} \frac{C_{t,T}(K)}{KF_{t,T}} dK + \frac{2}{B_{t,T}} \int_0^{S_t} \frac{P_{t,T}(K)}{KF_{t,T}} dK \\ &= v^{u,E}_{t,T} + v^{d,E}_{t,T}. \end{aligned}$$

Thanks to the decomposition of $iv_{t,T}$ into upside and downside parts and similar decomposition that can also be performed on $v^E_{t,T}$, still denoted by $v^{u,E}_{t,T}$ and by $v^{d,E}_{t,T}$, we conclude that $is_{t,T}$ can be decomposed as

$$\begin{aligned} is_{t,T} &= 3(v^{u,E}_{t,T} - iv^u_{t,T}) - 3(v^{d,E}_{t,T} - iv^d_{t,T}) \\ &= is^u_{t,T} - is^d_{t,T}. \end{aligned} \tag{13}$$

[1]We refer to Kozhan et al. (2013) for an explanation why the function g^s is is used instead of r^3 as well as g^v given in the variance swap section instead of r^2.

As $is^u_{t,T}$ only involves OTM calls, it depends on the third moment of the underlying asset return conditional on this return to be positive and, as such, it depends on the asset's right or upper tail distribution. A similar remark applies to $is^d_{t,T}$, this time OTM puts are involved and with the difference that it is conditional on the asset return to be negative, so it depends on the asset's left or lower tail distribution. As a result, we can write

$$g^s(r_{t,T}) = g^s(r_{t,T})\mathbf{1}_{\{r_{t,T}>0\}} - g^s(|r_{t,T}|)\mathbf{1}_{\{r_{t,T}\leq 0\}}, \tag{14}$$

from which we deduce

$$is^u_{t,T} = E^Q_t\left[g^s(r_{t,T})\mathbf{1}_{\{r_{t,T}>0\}}\right], \quad is^d_{t,T} = E^Q_t\left[g^s(|r_{t,T}|)\mathbf{1}_{\{r_{t,T}\leq 0\}}\right]. \tag{15}$$

In practice, there are only a finite number of options available in the market, so $v^{u,E}_{t,T}$ and $v^{d,E}_{t,T}$ can be approximated by the following sums:

$$\begin{aligned} v^{u,E}_{t,T} &= 2\sum_{S_t\leq K_i}\frac{C_{t,T}(K_i)}{B_{t,T}K_iF_{t,T}}\Delta I(K_i), \\ v^{d,E}_{t,T} &= 2\sum_{K_i\leq S_t}\frac{P_{t,T}(K_i)}{B_{t,T}K_iF_{t,T}}\Delta I(K_i), \end{aligned} \tag{16}$$

with weight function $\Delta I(K_i)$ as previously defined.

Regarding the floating leg of the skew swap, it is in fact the expectation of the payoff function $g^s(r)$ under the physical measure P, and it is given by

$$rs_{t,T} = E^P_t[g^s(r_{t,T})] = \sum_{i=t}^{T-1} g^s(r_{i,i+1}). \tag{17}$$

The decomposition of Eq.(14) leads to define the upside and downside realized skew, namely, $rs^u_{t,T}$ and $rs^d_{t,T}$, and these quantities are

$$\begin{aligned} rs^u_{t,T} &= \sum_{i=t}^{T-1} g^s(r_{i,i+1})\mathbf{1}_{\{r_{i,i+1}>0\}}, \\ rs^d_{t,T} &= \sum_{i=t}^{T-1} g^s(|r_{i,i+1}|)\mathbf{1}_{\{r_{i,i+1}\leq 0\}}. \end{aligned} \tag{18}$$

The skew swap contract value, payer of the fixed leg and receiver of the floating leg, can be written as $rs_{t,T} - is_{t,T}$ and after averaging under the historical probability measure the skew risk premium is obtained. As for the variance swap, it is convenient to define the excess return of an investment on a skew swap contract as

$$sp_{t,T} = \frac{rs_{t,T}}{is_{t,T}} - 1. \tag{19}$$

Lastly, the decompositions performed on the risk-neutral and realized skews lead to define the upside and downside skew swaps and after averaging under the historical probability measure to obtain the upside and downside skew risk premiums. Again, it is convenient to compute the excess returns associated with these upside and downside skew swaps, they are given by

$$\begin{aligned} sp^u_{t,T} &= \frac{rs^u_{t,T}}{is^u_{t,T}} - 1, \\ sp^d_{t,T} &= \frac{rs^d_{t,T}}{is^d_{t,T}} - 1. \end{aligned} \tag{20}$$

To implement these quantities we will use options on crude oil and only monthly maturities are available. As a consequence, the risk-neutral expectations can only be evaluated twelves times a year and t will run through the first days following the option maturity dates. Therefore, all the quantities are on a monthly basis. Also, to lighten notations we will drop the dependency with respect to T, that is to say, we will use iv_t instead of $iv_{t,T}$ in the following parts and the same rule applies to all the other quantities.

3 Data and descriptive statistics

The empirical analysis spans the period from January 2010 to June 2016. To compute the variance and skew risk premiums as well as their decompositions, we obtain both European call and put options written on the USO from Thomson Reuters Ticker History (TRTH) of SIRCA.[2] Option information such as Ticker, date, last price, close bid, close ask, expiration date, strike price and option type is extracted and consistently with the pricing formulas presented in the previous section only OTM options are used. As previously mentioned, the empirical study is carried out at monthly frequency, so only one-month maturity options will be used here, and the computation will run through the first days following the option maturity dates. Also, because the moneyness range of options varies a lot across time, especially for the puts, we restrict it

[2] http://www.sirca.org.au/

from 0.5 to 2.0 to avoid illiquidity issues caused by deep OTM options. We use Libor rates to proxy the risk-free rates, all of them provided by Bloomberg.

Figure 1 contains the evolution of the USO for the period considered while Figure 2 illustrates the distribution of its daily log returns as well as the normal distribution having the same mean and standard deviation as the data sample. Compared to the normally distributed curve, USO density curve exhibits a slightly negative skewness, fatter tails, and a higher peak, it highlights the importance of higher moment risks such as skewness and kurtosis.

[Insert Figure 1 here]

[Insert Figure 2 here]

Figure 3 exhibits the time series of total, upside and downside risk-neutral variances, namely, iv, iv^u and iv^d, from January 2010 to June 2016. The comovement of the three variables demonstrates their positive correlations. In general, the curve of iv^d is above that of iv^u, so we can expect the average value of iv^d to be larger, and it implies that the variance of the left tail is larger than the variance of the right tail. We also notice that compared to iv^d, the curve of iv^u exhibits more spikes.

[Insert Figure 3 here]

Figure 4 shows the time series of total, upside and downside realized variances, namely, rv, rv^u and rv^d, over the same period. All the three quantities are positively correlated as the curves move together, and the moving trend is similar to that of their risk-neutral counterparts. The magnitude of rv^d is slightly larger than that of rv^u but their difference is smaller compared to iv^u and iv^d. Moreover, there are more spikes on the downside realized variance curve compared to the upside realized variance curve.

[Insert Figure 4 here]

Figure 5 displays the time series of total, upside and downside variance risk premiums, namely, vp, vp^u and vp^d. Generally, vp, vp^u and vp^d show similar evolution patterns over time, it suggests positive correlations among the variables. On average, vp, vp^u and vp^d are negative, indicating that positive premiums are paid to hedge against the total, upside and downside volatility of the underlying asset. Moreover, the curve of vp^d shows more larger spikes than that of vp^u. There are mainly two concentrated periods of spikes revealed by the curves of vp^u and vp^d, namely, the period from 2010 to 2012 and the period from 2014 to 2016 during which the crude oil price dropped dramatically. The curves of vp^u and vp^d exhibit spikes at different times indicating that the decomposed variance premiums are driven by different underlying state variables.

[Insert Figure 5 here]

Figure 6 shows the time series of total, upside and downside risk-neutral skews, namely, is, is^u and is^d. The comovement of is^u and is^d suggests a positive correlation between these variables. Comparing the upper and lower figures, and consistently with the decomposition of is, is^u captures the positive spikes of is and is^d captures the negative spikes of is. Also of interest is to notice that we can have is that remains unchanged from one observation date to the next while is^u and is^d vary substantially. As a result, the disaggregation of is into is^u and is^d can provide additional information. A similar remark applies to iv although here iv^u and iv^d add up to give iv. Both the average value and volatility of is^d are greater than those of is^u, as the curve of is^d is above that of is^u for most part and it displays more larger spikes as well.

[Insert Figure 6 here]

Figure 7 shows the time series of total, upside and downside realized skews, namely, rs, rs^u and rs^d. It depicts similar moving trend to their risk-neutral counterparts. A positive correlation between rs^u and rs^d can be observed as they comove together. As for the risk-neutral variances, rs^u captures the positive spikes of rs while rs^d captures the negative spikes of rs, as shown by Figure 7. Lastly, rs^d reveals higher values on average compared to rs^u.

[Insert Figure 7 here]

Figure 8 exhibits the time series of total, upside and downside skew risk premiums, namely, sp, sp^u and sp^d. The three variables display distinct extreme values from those of the series in Figure 6 and Figure 7 as spikes in risk premiums, decomposed or not, are due to strong differences between realized and risk-neutral quantities. It suggests that the risk premium components contain different information. Also, during the two crisis periods, namely, years 2010 to 2012 and years 2014 to 2016, many spikes are present in sp^u and sp^d curves while for the sp curve there is only one extreme value around 2011. It suggests that the sp curve, which aggregates sp^u and sp^d, is less informative than its constituents considered separately. Lastly, the curve of sp^d exhibits larger spikes than the one of sp^u.

[Insert Figure 8 here]

Table I reports descriptive statistics such as mean and standard deviation for the realized and risk-neutral variance and skewness. Regarding the variance, either realized or risk-neutral, the downside component is larger than the upside component. A similar remark applies to the skewness, it results in negative skews (realized and risk-neutral). For both the variance and the skew, the realized values are smaller than the risk-neutral values, it implies that investors are willing to pay in order to hedge variance and skew risks. Lastly, the asymmetry between downside and upside risk-neutral quantities (variance and skew) explains the downward slope of the volatility smile observed in the USO option market and is similar to what is known for the equity index options (S&P500).

[Insert Table I here]

Table II reports descriptive statistics for the key quantities vp, vp^u, vp^d, sp, sp^u and sp^d for the period under study. On average, the variance risk premiums are negative, with the downside variance risk premium as the lowest. Note that in Kilic and Shaliastovich (2015), which investigates the upside and downside variance risk premiums for the S&P500 market, iv^u is positive while iv^d is negative[3], highlighting a difference between equity index and commodity markets. In other words, in the equity market, a downward market movement is bad news but an upward market movement is good news. In sharp contrast, in the commodity market, both upward and downward market shifts are bad news. Regarding the skew risk premiums, which were not analyzed in Kilic and Shaliastovich (2015), all of them are negative. Moreover, we notice that

[3]Kilic and Shaliastovich (2015) defines moment risk premium as the difference between risk-neutral and realized moments, it will result in risk a premium of opposite sign than ours.

upside and downside skew risk premiums are quite close. For the standard deviations, both downside variance and skew risk premiums are higher than their upside counterparts and the difference is even larger for the skew. It suggests that downside skew risk premium is the most sensitive variable to left tail market crashes.

[Insert Table II here]

Table III provides a correlation matrix for those variables. Both vp^u and vp^d have strong correlations with vp, as high as 0.487 and 0.885, respectively. As expected, vp^u and vp^d are weakly correlated, only 0.051. In contrast, sp^d has a much higher correlation with sp than sp^u with sp as we find -0.601 for the former while for the latter we find -0.195. Notice that in both cases, downside decompositions carry more information (i.e. higher correlations) with respect to aggregated or unconditional risk premiums than upside decompositions.

[Insert Table III here]

4 Empirical analysis

In order to deepen our understanding of the information content of the variables constructed in the previous part, namely, the upside and downside variance and skew risk premiums, a thorough empirical analysis of these quantities is performed. The first part aims to test time-varying properties of the total and decomposed risk premiums. Time variation of variance and skew risk premiums for the U.S. equity index market (i.e. S&P500 index options) has been documented in the literature such as Kozhan et al. (2013) while similar conclusion was obtained for the volatility index market (i.e. VIX index options) in Da Fonseca and Xu (2016). The second part proposes several factor models for the total as well as decomposed risk premiums using the USO excess return as explanatory variable. The third part is about predictability of USO excess returns by these quantities and to show that upside and downside risk premiums jointly have higher forecasting power than the (unconditional) variance and skew risk premiums. As shown by Bollerslev et al. (2009), variance risk premium contains significant predictive information for equity index excess returns within a forecast horizon of 6 months. From the construction of the quantities, it is intuitive that upside and downside variance and skew risk premiums jointly contain more information than total variance and skew risk premiums. The recent work of Kilic and Shaliastovich (2015) decomposed the variance premium into "good" and "bad" variance premiums and further demonstrated that the two components jointly have a stronger predictive power for equity index excess returns over a longer horizon (they study the same data as Bollerslev et al. (2009)). Our objective is to analyze the predictability of USO excess returns by the upside and downside variance and skew risk premiums over forecast horizons spanning from 1 week to 9 months. Our results extend existing results in both directions. First, along with variance risk premiums (unconditional and conditional) it also considers skew risk premiums (unconditional and conditional), thus it extends the study of Chevallier and Sévi (2013) that analyzes the predictability of crude oil futures returns using the (unconditional) variance risk premium.[4] Second, it underlines the specifics, compared with the equity index option market and volatility index option market, of the crude oil option market.

4.1 Time variation of risk premiums

Time variation of variance risk premiums: Following Kozhan et al. (2013), we test the time variation of the total variance risk premiums by performing the univariate regression of realized variance on risk-neutral variance, as well as the univariate regressions of upside and downside realized variances on upside and downside risk-neutral variances

$$rv_t = \alpha_0 + \alpha_1 iv_t + \epsilon_t^{\alpha}, \tag{21}$$

$$rv_t^u = \beta_0 + \beta_1 iv_t^u + \epsilon_t^{\beta}, \tag{22}$$

$$rv_t^d = \gamma_0 + \gamma_1 iv_t^d + \epsilon_t^{\gamma}, \tag{23}$$

and the results are reported in Table IV.

[Insert Table IV here]

Under the null hypothesis that the variance risk premium is constant over time, the slope should be one and the intercept should be zero. Regarding Eq.(21), the slope of total risk neutral variance (iv) is 0.798, thus significantly smaller than 1 (and different from 0), it indicates that

[4]The point of view of Chevallier and Sévi (2013) is somewhat different than ours as they consider along the variance risk premium other explanatory variables such as Han Index, Killian Index and the De RoonS Index, among others.

vp is time varying. Similar conclusion applies to iv^d as the slope coefficient is 0.528 and highly significant. It contrasts with iv^u as the estimated coefficient in that case is 1.042 and highly significant from which we deduce that the upside variance risk premium is not time varying if we follow Kozhan et al. (2013)'s interpretation. The empirical results, at least for iv and iv^d, are consistent with Figure 5 that shows the time-varying evolutions of total and downside variance risk premiums. Regarding the R^2, the values are as high as 61.29%, 55.28% and 32.08% and all the intercepts are not significantly different from zero.

Time variation of skew risk premiums: To test the dynamics of skew risk premiums, either total, upside or downside skew risk premiums, we run the following univariate regressions

$$rs_t = \alpha_0 + \alpha_1 is_t + \epsilon_t^{\alpha}, \tag{24}$$

$$rs_t^u = \beta_0 + \beta_1 is_t^u + \epsilon_t^{\beta}, \tag{25}$$

$$rs_t^d = \gamma_0 + \gamma_1 is_t^d + \epsilon_t^{\gamma}, \tag{26}$$

and report the results in Table V.

[Insert Table V here]

Regarding Eq.(24) about the total skew risk premium, the slope is significantly different from zero at 5%, while both slopes of upside and downside skew risk premiums are highly significantly different from one. It indicates that all the skew risk premiums are time varying. Moreover, the R^2 for Eq.(24) is as low as 10.15%, while for Eq.(25) and Eq.(26) they increase to 49.75% and 26.11%, respectively. It demonstrates that after decomposition, the upside and downside risk neutral skew provides more information on their realized counterparts.

4.2 Factor models for risk premiums

In this part, to better understand the source of risk premiums, we analyze to which extent they are related to USO excess returns. As previously stated, we adopt the ratio expressions given by Eqs.(9), (10), (19) and (20), so that the risk premiums can be interpreted as the excess returns of investments made on the corresponding moment swap contracts. For example, vp^d is actually the excess return from an investment made on the downside variance swap contract, for which the value of the floating leg is rv^d and the value of the fixed leg is iv^d. Therefore, the synthetic downside variance swap vp^d enables the buyer of the contract to hedge against an increase of the downside variance. Moreover, for simplicity, we name the underlying of vp^d the downside USO, it is related to negative USO returns. Similarly, the underlying of vp^u is the upside USO, it is related to positive USO returns. Same interpretation also applies to sp^u and sp^d.

Factor models for total variance and skew risk premiums: Regarding the total variance and skew risk premiums, we consider the regressions

$$vp_t = \alpha_0 + \alpha_1 xm_t^{USO} + \epsilon_t^{\alpha}, \tag{27}$$

$$sp_t = \beta_0 + \beta_1 xm_t^{USO} + \epsilon_t^{\beta}, \tag{28}$$

where xm^{USO} denotes the USO monthly excess return starting on day t as defined in Eq.(1). Results for Eq.(27) and Eq.(28) are reported in Table VI.

[Insert Table VI here]

The first regression leads to a highly significant and negative coefficient for xm^{USO} and R^2 of 10.44%, and the coefficient's sign is consistent with the leverage effect implied by the negative slope of the volatility smile observed on USO options. If the market goes down, that is, a negative value for xm^{USO}, it will lead to an increase of market volatility and thus an increase of vp. Furthermore, as the market volatility increases, the left tail of USO distribution grows larger and it will result in a volatility smile with a steeper slope. The coefficient of xm^{USO} in the second regression is not significantly different from zero, so the relationship between xm^{USO} and sp cannot be confirmed here. Note that xm^{USO} explains more vp than sp.

Factor models for upside variance and skew risk premiums: We consider whether the upside variance and skew risk premiums, which can be interpreted as the excess return of investments made on those swap contracts, can be explained by market excess returns. We run the following regressions

$$vp_t^u = \alpha_0 + \alpha_1 xm_t^{USO} + \epsilon_t^{\alpha}, \tag{29}$$

$$sp_t^u = \beta_0 + \beta_1 xm_t^{USO} + \epsilon_t^{\beta}, \tag{30}$$

with the estimation results reported in Table VI. Regarding the regression for the upside variance risk premium, the slope estimate is positive and highly significant and indicates that a positive relationship exists between the USO excess return and upside variance risk premium. Similarly, Eq.(30) also leads to a positive and significant coefficient for xm^{USO}, thus a positive relationship also exists between the USO excess return and upside skew risk premium. Also, the R^2 for Eq.(29) is 22.82% while it is 11.66% for Eq.(30), indicating that xm^{USO} explains more the variable vp^u than sp^u. Moreover, before decomposition, xm^{USO} contributes only to 10.44% of vp (the regression Eq.(27)), while after decomposition, xm^{USO} contributes to 22.82% of vp^u. Similar remark applies to sp and sp^u, as xm^{USO} is not correlated to the former while it explains a considerable part of the latter.

Factor models for downside variance and skew risk premiums: We preform univariate regressions of downside variance and skew risk premiums on the USO excess return

$$vp_t^d = \alpha_0 + \alpha_1 xm_t^{USO} + \epsilon_t^{\alpha}, \tag{31}$$

$$sp_t^d = \beta_0 + \beta_1 xm_t^{USO} + \epsilon_t^{\beta}, \tag{32}$$

and results are reported in Table VI. The coefficients for vp^d and sp^d are both negative and significant, the R^2 are equal to 38.16% and 19.22% for Eq.(31) and Eq.(32), respectively. Similar to the previous case, xm^{USO} explains more of vp^d than sp^d. In conclusion, the higher the risk premium moment order is, the less xm^{USO} can explain. Interestingly, xm^{USO} explains more vp^d than vp or vp^u. Likewise, among sp, sp^u and sp^d, xm^{USO} explains more of sp^d.

4.3 Predictability

Predictability by upside and downside variance risk premiums: In this part, we will focus on the role of upside and downside variance risk premiums in predicting USO excess returns. We will consider the following regressions

$$xm_{t,h}^{USO} = \alpha_{0,h} + \alpha_{1,h} vp_t + \epsilon_t^{\alpha}, \tag{33}$$

$$xm_{t,h}^{USO} = \beta_{0,h} + \beta_{1,h} vp_t^u + \epsilon_t^{\beta}, \tag{34}$$

$$xm_{t,h}^{USO} = \gamma_{0,h} + \gamma_{1,h} vp_t^d + \epsilon_t^{\gamma}, \tag{35}$$

$$xm_{t,h}^{USO} = \delta_{0,h} + \delta_{1,h} vp_t^u + \delta_{2,h} vp_t^d + \epsilon_t^{\delta}, \tag{36}$$

where h denotes the horizon of prediction and $xm_{t,h}^{USO}$ denotes the future USO excess return over the horizon h that is computed as

$$xm_{t,h}^{USO} = \frac{1}{h} \sum_{i=0}^{h} r_{t+i,T+i} - r_{t+h,T+h}^f, \tag{37}$$

with $r_{t,T}$ and $r_{t,T}^f$ representing the monthly USO return, as previously defined, and the monthly risk-free rate starting at day t and ending at time T, respectively. The results for Eqs.(33) - (36) are presented in Table VII.

[Insert Table VII here]

Eq.(33) analyzes the predictability of USO excess returns by the variance risk premium over various time horizons ranging from 1 week to 9 months. The regression results show that vp remains a significant predictor variable only over a short horizon of 2 weeks, with a low R^2 of 6.70%.[5] In contrast, Bollerslev et al. (2009) demonstrate that variance risk premium serves as a significant predictor for equity index returns over a forecasting horizon of 6 months, which is much longer than the 2-week horizon in the crude oil market, and illustrates a first difference between the equity index market and the crude oil market. Also, the coefficient of vp in Eq.(33) is negative, it indicates that investors are willing to pay a premium to hedge against the volatility of the underlying asset (i.e. the USO) regardless of the moving direction.

For comparison, Eq.(34) investigates the predictability of USO excess returns by the upside variance risk premium over various forecasting horizons. Compared to vp, the predictive information of vp^u remains significant over the much longer horizon of 3 months. For the 3-month ahead USO excess return regression, vp^u is only moderately significant and leads to a low R^2 of 5.28%.

[5] In fact it is an adjusted R-square but we will omit the term adjusted hereafter.

Considering the forecasting horizon of 2 months, the coefficient of vp^u is highly significant with a R^2 of 15.93% and suggests that vp^u contains more predictive information than vp.

Eq.(35) investigates the predictive information of vp^d for the USO excess return xm^{USO}. For the 2-week forecasting horizon, vp^d is highly significant with a R^2 of 26.73%, thus vp^d is the most informative variable among vp, vp^u and vp^d. Moreover, the longest forecastable horizon for vp^d is 3 months, even though vp^d is lowly significant in that case (i.e. the t-statistic is at a significance level of 5%).

The results of univariate regressions from the previous parts show that among the total and decomposed variance risk premiums, the latter, and especially vp^d, work better as predictor variables, in terms of forecasting horizons and level of significance, and generally vp^d contributes a bit more to explain the future USO excess returns than vp^u. We further analyze the joint predictive information of vp^u and vp^d for USO excess returns in Eq.(36). Compared to the univariate regressions, the R^2 increases for all forecasting horizons, it underlines the complementary contributions of vp^u and vp^d. Naturally, the R^2 decreases from 58.67% to 8.18% when the forecasting horizon increases from 1 week to 3 months, where for the 3-month horizon vp^u is only moderately significant while vp^d is lowly significant. For the 2-week ahead USO excess return, vp^u and vp^d jointly contribute to explain 45.16% of xm^{USO}, with both coefficients highly significant. In summary, it is statistically important to include upside and downside variance premiums to better predict future USO returns.

Predictability by upside and downside skew risk premiums: In this part, similar analysis is carried out for the predictability of USO excess returns by upside and downside skew risk premiums and a comparison is performed when the total skew risk premium is used. We will run the following regressions

$$xm_{t,h}^{USO} = \alpha_{0,h} + \alpha_{1,h} sp_t + \epsilon_t^{\alpha}, \tag{38}$$

$$xm_{t,h}^{USO} = \beta_{0,h} + \beta_{1,h} sp_t^u + \epsilon_t^{\beta}, \tag{39}$$

$$xm_{t,h}^{USO} = \gamma_{0,h} + \gamma_{1,h} sp_t^d + \epsilon_t^{\gamma}, \tag{40}$$

$$xm_{t,h}^{USO} = \delta_{0,h} + \delta_{1,h} sp_t^u + \delta_{2,h} sp_t^d + \epsilon_t^{\delta}, \tag{41}$$

and report the results in Table VIII.

[Insert Table VIII here]

Eq.(38) focuses on the predictability of USO excess returns by the total skew risk premium (sp) over horizons ranging from 1 week to 9 months. The results show that sp does not contain any predictive information about xm^{USO}, as the coefficients are insignificant and the R^2 are low for all horizons. In contrast, as previously shown, the total variance risk premium (vp) contains significant predictive information regarding xm^{USO} for a forecasting horizon of up to 2 weeks.

Eq.(39) investigates the predictive information for xm^{USO} contained in the upside skew risk premium (sp^u) over the same forecasting horizons. The coefficient of sp^u remains significant up to an horizon of 3 months, even though the significance level at 3 months is only at 5%. Unlike sp, sp^u contains predictive information for xm^{USO} as suggested by both the significant coefficients and the decent R^2. Moreover, sp^u is positively correlated with future USO excess returns as the coefficients of sp^u remain positive for all horizons. Note that the intercept term is also significant for up to 3 months.

For comparison, Eq.(40) analyzes the predictive information for xm^{USO} contained in the downside skew risk premium (sp^d). Similar to the case of sp^u, both the intercept and slope of sp^d remain significant for up to 3 months but notice that the R^2 only remain decent, that is to say above 10%, for horizons up to 1 month. Here also, the constant terms remain significant and of constant sign for forecasting horizons less than or equal to 3 months. The negative sign of sp^d shows that sp^d is negatively correlated to xm^{USO}.

Eq.(41) further analyzes the joint predictive information of sp^u and sp^d for xm^{USO}. Firstly, both sp^u and sp^d remain significant up to 3 months, with a high degree of significance for shorter horizons. Compared to the univariate regressions on sp^u and sp^d, for all the horizons, the R^2 is much higher and larger than the sum of the R^2 of the univariate regressions. It suggests that these variables not only do not have redundant information but, indeed, have complementary information. The constant terms that were significant in the univariate regressions are no longer significant (except for the 2-month regression). Lastly, the coefficient signs are consistent with those of the univariate regressions. Again, decomposed skew risk premiums have a much stronger predictive power for USO excess returns than the (undecomposed) skew risk premium.

Predictability by combining upside and downside risk premiums: The previous two parts demonstrate the advantage of decomposing variance and skew risk premiums. In this part, we will further explore the impact of this decomposition by considering the predictability of USO excess returns by combining upside variance and skew risk premiums as explanatory variables on one hand and downside variance and skew risk premiums as explanatory variables on the other hand. Lastly, we will also consider the combination of upside and downside variance and skew risk premiums. For simplicity, we use the total higher moment risk premiums to refer to the total variance risk premium and the total skew risk premium. Similarly, we use the upside (downside) higher moment risk premiums to refer to the upside (downside) variance risk premium and the upside (downside) skew risk premium. We run the following regressions

$$xm_{t,h}^{USO} = \alpha_{0,h} + \alpha_{1,h} vp_t + \alpha_{1,h} sp_t + \epsilon_t^{\alpha}, \tag{42}$$

$$xm_{t,h}^{USO} = \beta_{0,h} + \beta_{1,h} vp_t^u + \beta_{2,h} sp_t^u + \epsilon_t^{\beta}, \tag{43}$$

$$xm_{t,h}^{USO} = \gamma_{0,h} + \gamma_{1,h} vp_t^d + \gamma_{2,h} sp_t^d + \epsilon_t^{\gamma}, \tag{44}$$

$$xm_{t,h}^{USO} = \delta_{0,h} + \delta_{1,h} vp_t^u + \delta_{1,h} vp_t^d + \delta_{1,h} sp_t^u + \delta_{1,h} sp_t^d + \epsilon_t^{\delta}, \tag{45}$$

and report the results in Table IX.

[Insert Table IX here]

The Eq.(42) shows that total higher moment risk premiums can forecast USO excess returns only for an horizon of 2 weeks as beyond that horizon the R^2 is close to zero and only the variance variable is significant. In sharp contrast, upside high moment risk premiums, given by Eq.(43), and downside high moment risk premiums, given by Eq.(44), lead to decent forecast of USO excess returns for up to 2 months, thus confirming the interest of decomposition for forecasting. For the upside higher moment risk premiums the variance seems to contain all the information as it is the only significant variable and, as a result, the R^2 obtained for these regressions are close to those obtained when only the upside variance variable is used. For the downside higher moment risk premiums and for short horizons, both the variance and the skew are significant, and in that case the R^2 is higher than those obtained when regressing on the downside variance alone (i.e. Eq.(35)) or the downside skew alone (i.e. Eq.(40)), whereas for longer horizons the variance is the only significant variable with the natural consequence that the R^2 in those cases are close to those obtained when regressing on the variance alone. Lastly, in Eq.(45), all the variables are considered, it leads to regressions with very large R^2 for up to 2 months and among all the variables vp^d seems to be the most important one. The coefficients' signs are consistent with those obtained in the previous regressions. Notice also that there is a complementary effect between upside and downside variables as the R^2 in a given regression involving these variables largely dominates those obtained when only upside or downside variables are used and further confirm, if needed be, the interest of the decomposition proposed in this work.

5 Conclusion

In this work we provide a comprehensive analysis of the total and decomposed variance and skew risk premiums for the USO, an exchange traded fund tracking the daily price changes of the WTI light sweet crude oil. So far, most of the literature mainly discusses the use of decomposed variance risk premiums for the S&P 500 option market. We contribute to the literature by extending the analysis of decomposed variance risk premiums to the crude oil market, but also extend the discussion to skew risk premiums. To build these quantities we rely on two key works, the decomposition proposed by Kilic and Shaliastovich (2015) for the variance risk premium and the computation methodology for variance and skew risk premiums developed by Kozhan et al. (2013).

We obtain three main findings. Firstly, we find that all the risk premiums, no matter decomposed or not, are time varying. Secondly, if we apply one factor models to the total, upside and downside variance and skew risk premiums with the USO excess returns as explanatory variable, we find that it better explains the decomposed higher moment risk premiums (both variance and skew) than their total counterparts. Thirdly, by analyzing the predictability of crude oil market excess returns by decomposed variance and skew risk premiums, we found that the decomposed high moment risk premiums contain much more predictive information than their undecomposed counterparts. The downside higher moment risk premiums, the variance and to a lesser extent the skewness, are especially informative about future evolutions of the crude oil market excess return.

It would be interesting to fully explore how the decomposed risk premiums combine with observable economic variables commonly used in the literature, see for example Chevallier and Sévi

(2013), for analyzing the crude oil market. Also, other commodity markets such as gas and gold option markets could be considered along with commodity volatility option markets. We leave these open questions for further research.

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A Decomposition of risk-neutral variance and skew

Following Bakshi et al. (2003), any twice-continuously differentiable function $H(S)$ where S is spot price of the underlying can be spanned by a position in bonds, stocks and out-of-money options

$$H(S) = H(\bar{S}) + (S - \bar{S})H_S(\bar{S}) + \int_{\bar{S}}^{\infty} H_{SS}(K)(S-K)^+ dK + \int_0^{\bar{S}} H_{SS}(K)(K-S)^+ dK.$$

Under risk-neutral measure Q, the arbitrage-free price of the contingent claim with payoff $H(S)$ is

$$\begin{aligned} E^Q[e^{-r_f(T-t)}H(S)] &= \left(H(\bar{S}) - \bar{S}H_S(\bar{S})\right) e^{-r_f(T-t)} + H_S(\bar{S})S_t \\ &+ \int_{\bar{S}}^{\infty} H_{SS}(K)C(t,T;K)dK + \int_0^{\bar{S}} H_{SS}(K)P(t,T;K)dK. \end{aligned} \tag{46}$$

Specifically, Kozhan et al. (2013) define the payoff function for the variance swap contract as $g^v(r(S)) = 2\left(e^r - 1 - r\right)$, with $r(S) = \ln \frac{S}{S_t}$. Referring to Eq.(46), we set $\bar{S} = S(t)$. Under the risk-neutral measure Q, value of the payoff function is

$$E^Q[H(S)] = \frac{1}{B_{t,T}} \int_{S_t}^{+\infty} \frac{2}{K^2} C_{t,T}(K)dK + \frac{1}{B_{t,T}} \int_0^{S_t} \frac{2}{K^2} P_{t,T}(K)dK. \tag{47}$$

where $B_{t,T}$ is the time-t price of zero-coupon bond with maturity T.

Based on the previous work, we define the payoff function for the upside variance swap contract as

$$H^u(S) = \begin{cases} g^v(r(S)), & \text{if } S > S_t, \\ 0, & \text{otherwise.} \end{cases} \tag{48}$$

The first order derivative of $H^u(S)$ is

$$H^u_S(S) = \begin{cases} 2\left(\frac{1}{S_t} - \frac{1}{S}\right), & \text{if } S > S_t, \\ 0, & \text{otherwise,} \end{cases}$$

where $H^u_S(S)$ is continuous but not differentiable at $S = S_t$.

The second order derivative of $H^u(S)$ is

$$H^u_{SS}(S) = \begin{cases} \frac{2}{S^2}, & \text{if } S > S_t, \\ 0, & \text{otherwise,} \end{cases}$$

where $H^u_{SS}(S)$ is not continuous at $\bar{S} = S_t$, but it is continuous on $(-\infty, S_t)$ and (S_t, ∞) separately.

Even though $H^u(S)$ is not twice-continuously differentiable at $S = S_t$, it is well defined and the discontinuity will not result in an infinite integral. Therefore, for the upside variance swap, the expected value of the payoff function under risk-neutral measure Q is

$$E^Q[H^u(S)] = \frac{1}{B_{t,T}} \int_{S_t}^{+\infty} \frac{2}{K^2} C_{t,T}(K)dK. \tag{49}$$

Likewise, if we define the payoff function for the downside variance swap contract as

$$H^d(S) = \begin{cases} 0, & \text{if } S > S_t, \\ g^v(r(S)), & \text{otherwise,} \end{cases} \tag{50}$$

by taking the second derivative of $H^d(S)$, we get the expected value of the payoff function under risk-neutral measure Q

$$E^Q[H^d(S)] = \frac{1}{B_{t,T}} \int_0^{S_t} \frac{2}{K^2} P_{t,T}(K)dK. \tag{51}$$

Considering Eq.(47), we get

$$E^Q[H(S)] = E^Q[H^u(S)] + E^Q[H^d(S)]. \tag{52}$$

Eq.(52) demonstrates that the risk-neutral variance can be decomposed into upside and downside risk-neutral variance, respectively, with the former constructed upon a continuum of out-of-money calls and the latter constructed upon a continuum of out-of-money puts.

As to the decomposition of risk-neutral skew, the same methodology applies. We define the payoff function for the upside and downside skew swap as

$$H^u(S) = \begin{cases} g^s(r(S)), & \text{if } S > S_t, \\ 0, & \text{otherwise.} \end{cases} \tag{53}$$

and

$$H^d(S) = \begin{cases} 0, & \text{if } S > S_t, \\ -g^s(r(S)), & \text{otherwise.} \end{cases} \tag{54}$$

By utilizing Eq.(46), under risk-neutral measure Q, the expected value for $H^u(S)$ and $H^d(S)$ can be expressed by a continuum of out-of-money calls and puts, respectively

$$\begin{aligned} E^Q[H^u(S)] &= \frac{6}{B_{t,T}} \int_{S_t}^{\infty} \frac{K - S_t}{K^2 S_t} C_{t,T}(K) dK, \\ E^Q[H^d(S)] &= \frac{6}{B_{t,T}} \int_{0}^{S_t} \frac{S_t - K}{K^2 S_t} P_{t,T}(K) dK. \end{aligned} \tag{55}$$

Therefore, for skew swap contract, we have

$$E^Q[H(S)] = E^Q[H^u(S)] - E^Q[H^d(S)]. \tag{56}$$

B Tables

Table I: Descriptive statistics of variances and skews

	Mean	Std. dev.		Mean	Std. dev.
rv	7.64e-03	7.48e-03	rs	-2.526e-05	2.422e-04
rv^u	3.54e-03	4.32e-03	rs^u	1.417e-04	2.779e-04
rv^d	4.10e-03	4.09e-03	rs^d	1.670e-04	2.426e-04
iv	10.65e-03	7.36e-03	is	-8.446e-04	8.754e-04
iv^u	4.27e-03	3.10e-03	is^u	1.335e-03	1.257e-03
iv^d	6.38e-03	4.45e-03	is^d	2.179e-03	1.919e-03

Note: Descriptive statistics such as mean, standard deviation for the variables: the realized skew (rs, given by Eq.(17)), the upside realized skew (rs^u, given by Eq.(18)) and the downside realized skew (rs^d, given by Eq.(18)), the risk neutral skew (is, given by Eq.(12)), the upside risk neutral skew (is^u, given by Eq.(15)) and the downside risk neutral skew (is^d, given by Eq.(15)). Sample with monthly frequency ranging from January 2010 to June 2016.

Table II: Descriptive statistics of risk premiums

	Mean	Std. dev.	Q1	Median	Q3
vp	-0.304	0.395	-0.565	-0.408	-0.207
vp^u	-0.239	0.474	-0.531	-0.315	-0.020
vp^d	-0.323	0.605	-0.676	-0.435	-0.178
sp	-1.096	0.808	-1.071	-1.000	-0.919
sp^u	-0.915	0.094	-0.975	-0.938	-0.899
sp^d	-0.912	0.153	-0.981	-0.956	-0.921

Note: Descriptive statistics such as mean, standard deviation, the 25th percentile, median, and 75th percentile for the variables: the variance risk premium (vp, given by Eq.(9)), the upside variance risk premium (vp^u, given by Eq.(10)), the downside variance risk premium (vp^d, given by Eq.(10)), the skew risk premium (sp, given by Eq.(19)), the upside skew risk premium (sp^u, given by Eq.(20)) and the downside skew risk premium (sp^d, given by Eq.(20)). Sample with monthly frequency ranging from January 2010 to June 2016.

Table III: Correlations

	vp	vp^u	vp^d	sp	sp^u	sp^d
vp	1.000	0.487	0.885	-0.339	0.536	0.807
vp^u		1.000	0.051	-0.154	0.899	0.088
vp^d			1.000	-0.418	0.153	0.923
sp				1.000	-0.195	-0.601
sp^u					1.000	0.172
sp^d						1.000

Note: Correlation between the variables: the variance risk premium (vp, given by Eq.(9)), the upside variance risk premium (vp^u, given by Eq.(10)) and the downside variance risk premium (vp^d, given by Eq.(10)), the skew risk premium (sp, given by Eq.(19)), the upside skew risk premium (sp^u, given by Eq.(20)) and the downside skew risk premium (sp^d, given by Eq.(20)). Sample with monthly frequency ranging from January 2010 to June 2016.

Table IV: Time variation of upside and downside variance premiums

	Const.	iv	iv^u	iv^d	Adj. R^2(%)
rv	-0.0008	0.798^{***}			61.29
	(-0.91)	(8.82)			
rv^u	-0.0009		1.042^{***}		55.28
	(-1.46)		(6.49)		
rv^d	0.0008			0.528^{***}	32.08
	(1.31)			(6.34)	

Note: Regressions of the realized variance (rv, given by Eq.(7)) on the risk neutral variance (iv, given by Eq.(3)), the upside realized variance (rv^u, given by Eq.(8)) on the upside risk neutral variance (iv^u, given by Eq.(3)) and the downside realized variance (rv^d, given by Eq.(8)) on the downside risk neutral variance (iv^d, given by Eq.(3)). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. Sample with monthly frequency ranging from January 2010 to June 2016.

Table V: Time variation of upside and downside skew premiums

	Const.	is	is^u	is^d	Adj. R^2(%)
rs	-0.0001^{**}	-0.093^{*}			10.15
	(-2.96)	(-2.50)			
rs^u	-0.00006^{*}		0.157^{***}		49.75
	(-2.04)		(5.81)		
rs^d	0.00002			0.066^{***}	26.11
	(0.63)			(4.73)	

Note: Regressions of the realized skew (rs, given by Eq.(17)) on the risk neutral variance (is, given by Eq.(12)), the upside realized variance (rs^u, given by Eq.(18)) on the upside risk neutral variance (is^u, given by Eq.(15)) and the downside realized variance (rs^d, given by Eq.(18)) on the downside risk neutral variance (is^d, given by Eq.(15)). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. Sample with monthly frequency ranging from January 2010 to June 2016.

Table VI: Market excess returns and risk premiums

	Const.	xm^{USO}	Adj. R^2(%)
vp	-0.314***	-1.450*	10.46
	(-8.01)	(-2.15)	
sp	-1.098***	-0.225	-1.24
	(-15.84)	(-0.22)	
vp^u	-0.198***	2.554***	22.82
	(-3.79)	(4.56)	
sp^u	-0.908***	0.377**	11.66
	(-84.20)	(2.95)	
vp^d	-0.366***	-4.045***	38.16
	(-7.66)	(-4.57)	
sp^d	-0.920***	-0.726**	19.22
	(-66.13)	-3.23	

Note: The table shows to what extent the risk premiums, namely, the variance premium (vp, given by Eq.(9)), the skew premium (sp, given by Eq.(19)), the upside variance premium (vp^u, given by Eq.(10)), the upside skew premium (sp^u, given by Eq.(20)), the downside variance premium (vp^d, given by Eq.(10)) and the downside skew premium (sp^d, given by Eq.(20)), can be explained by the USO excess return (xm^{USO}, given by Eq.(1)). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. The monthly observations range from January 2010 to June 2016.

Table VII: Market return prediction using upside and downside variance premiums

		xm^{USO}						
		1w	2w	1m	2m	3m	6m	9m
1	Const.	-0.035*	-0.030	-0.022	-0.017	-0.015	-0.013	-0.013
		(-2.49)	(-1.91)	(-1.29)	(-0.89)	(-0.71)	(-0.59)	(-0.83)
	vp	-0.074***	-0.057*	-0.032	-0.013	-0.008	0.002	0.005
		(-3.33)	(-2.44)	(-1.34)	(-0.75)	(-0.42)	(0.18)	(0.49)
	Adj. R^2(%)	9.84	6.70	1.64	-0.65	-1.05	-1.38	-1.07
2	Const.	0.006	0.002	0.0007	-0.003	-0.006	-0.010	-0.012
		(0.57)	(0.21)	(0.06)	(-0.23)	(-0.40)	(-0.55)	(-0.87)
	vp^u	0.083***	0.068***	0.060***	0.043***	0.029**	0.014	0.010
		(5.21)	(5.30)	(4.84)	(4.08)	(2.81)	(1.77)	(1.24)
	Adj. R^2(%)	19.91	15.93	14.27	9.29	5.28	2.29	1.35
3	Const.	-0.041**	-0.035**	-0.029*	-0.022	-0.018	-0.015	-0.014
		(-3.29)	(-2.85)	(-2.23)	(-1.61)	(-1.16)	(-0.67)	(-0.92)
	vp^d	-0.089***	-0.070***	-0.049***	-0.028***	-0.018*	-0.004	0.0002
		(-4.84)	(-4.33)	(-3.57)	(-3.88)	(-2.06)	(-0.75)	(0.030)
	Adj. R^2(%)	35.56	26.73	14.53	5.46	2.48	-0.94	-1.47
4	Const.	-0.023	-0.020	-0.015	-0.012	-0.012	-0.011	-0.012
		(-1.90)	(-1.77)	(-1.38)	(-1.11)	(-0.87)	(-0.66)	(-0.96)
	vp^u	0.088***	0.071***	0.063***	0.045***	0.030**	0.015	0.010
		(6.78)	(6.06)	(5.21)	(4.00)	(2.73)	(1.82)	(1.30)
	vp^d	-0.092***	-0.072***	-0.051***	-0.029***	-0.019*	-0.005	-0.0001
		(-5.41)	(-4.77)	(-3.97)	(-4.24)	(-2.21)	(-0.95)	(-0.03)
	Adj. R^2(%)	58.67	45.16	30.56	15.62	8.18	1.47	-0.12

Note: The table shows the predictability of future USO excess returns (xm^{USO}) which is defined as Eq.(37), by using the variance premium (vp, given by Eq.(9)) alone, and using the upside variance premium (vp^u, given by Eq.(10)) and downside variance premium (vp^d, given by Eq.(10)) jointly. The forecasting horizon h can be 1 week (1w), 2 weeks (2w), 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m) and 9 months (9m). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. The monthly observations range from January 2010 to June 2016.

Table VIII: Market return prediction using upside and downside skew premiums

		xm^{USO}						
		1w	2w	1m	2m	3m	6m	9m
1	Const.	-0.010	-0.009	-0.007	-0.010	-0.007	-0.011	-0.013
		(-0.56)	(-0.57)	(-0.48)	(-0.67)	(-0.40)	(-0.45)	(-0.85)
	sp	0.002	0.003	0.005	0.002	0.005	0.002	0.001
		(0.15)	(0.22)	(0.54)	(0.42)	(1.55)	(0.79)	(0.39)
	Adj. R^2(%)	-1.28	-1.24	-1.02	-1.25	-0.82	-1.22	-1.43
2	Const.	0.281^{***}	0.257^{***}	0.251^{***}	0.192^{***}	0.127^{*}	0.053	0.043
		(4.27)	(4.75)	(4.71)	(3.98)	(2.42)	(1.47)	(1.23)
	sp^u	0.321^{***}	0.296^{***}	0.290^{***}	0.224^{***}	0.153^{*}	0.072	0.063
		(4.13)	(4.59)	(4.56)	(3.87)	(2.46)	(1.38)	(1.24)
	Adj. R^2(%)	11.54	12.09	13.45	10.35	6.06	2.50	3.07
3	Const.	-0.265^{**}	-0.222^{**}	-0.164^{**}	-0.091^{***}	-0.060^{*}	-0.015	-0.007
		(-2.94)	(-2.79)	(-2.61)	(-3.33)	(-2.34)	(-0.74)	(-0.48)
	sp^d	-0.277^{**}	-0.229^{**}	-0.165^{**}	-0.086^{***}	-0.051^{*}	-0.002	0.008
		(-2.97)	(-2.81)	(-2.63)	(-3.46)	(-2.40)	(-0.10)	(0.48)
	Adj. R^2(%)	20.95	17.46	9.91	2.67	0.61	-1.40	-1.29
4	Const.	0.063	0.074	0.114	0.115^{*}	0.079	0.046	0.044
		(0.65)	(0.86)	(1.49)	(2.16)	(1.53)	(1.14)	(1.22)
	sp^u	0.403^{***}	0.365^{***}	0.341^{***}	0.253^{***}	0.171^{*}	0.075	0.062
		(5.14)	(5.12)	(4.79)	(3.98)	(2.52)	(1.50)	(1.43)
	sp^d	-0.321^{***}	-0.269^{**}	-0.203^{**}	-0.114^{***}	-0.070^{**}	-0.010	0.002
		(-3.32)	(-3.21)	(-3.19)	(-4.16)	(-3.08)	(-0.66)	0.11
	Adj. R^2(%)	39.83	36.41	28.91	16.03	8.32	1.27	1.63

Note: The table shows the predictability of future USO excess returns (xm^{USO}) which is defined as Eq.(37), by using the variance premium (sp, given by Eq.(19)) alone, and using the upside variance premium (sp^u, given by Eq.(20)) and downside variance premium (sp^d, given by Eq.(20)) jointly. The forecasting horizon h can be 1 week (1w), 2 weeks (2w), 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m) and 9 months (9m). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. The monthly observations range from January 2010 to June 2016.

Table IX: Market return prediction using upside and downside variance and skew premiums

		xm^{USO}						
		1w	2w	1m	2m	3m	6m	9m
1	Const.	-0.050**	-0.040*	-0.023	-0.017	-0.009	-0.010	-0.010
		(-2.84)	(-2.08)	(-1.19)	(-0.79)	(-0.37)	(-0.39)	(-0.78)
	vp	-0.082***	-0.062**	-0.032	-0.013	-0.005	0.003	0.006
		(-3.73)	(-2.63)	(-1.37)	(-0.73)	(-0.24)	(0.35)	(0.53)
	sp	-0.012	-0.008	-0.0004	0.00005	0.004	0.003	0.002
		(-1.17)	(-0.82)	(-0.05)	(0.01)	(0.87)	(0.85)	(0.70)
	Adj. R^2(%)	9.64	5.98	0.32	2.01	-2.10	-2.55	-2.37
2	Const.	-0.226	-0.026	0.100	0.144	0.099	0.032	0.074
		(-1.60)	(-0.19)	(0.70)	(0.97)	(0.64)	(0.26)	(0.82)
	vp^u	0.132***	0.074*	0.039	0.012	0.007	0.005	-0.008
		(3.70)	(2.18)	(1.12)	(0.35)	(0.20)	(0.19)	(-0.40)
	sp^u	-0.267	-0.033	0.114	0.169	0.121	0.048	0.098
		(-1.64)	(0.84)	(0.68)	(0.94)	(0.63)	(0.30)	(0.84)
	Adj. R^2(%)	20.54	14.84	13.57	9.31	4.85	1.20	1.96
3	Const.	0.251**	0.129	0.061	0.077	0.066	0.062	0.029
		(2.88)	(1.45)	(0.68)	(0.78)	(0.58)	(0.70)	(0.63)
	vp^d	-0.169***	-0.115***	-0.073*	-0.055	-0.041	-0.025	-0.012
		(-6.06)	(-4.12)	(-2.42)	(-1.64)	(-1.03)	(-0.83)	(-0.76)
	sp^d	0.349***	0.196	0.107	0.118	0.101	0.091	0.052
		(3.30)	(1.79)	(0.94)	(0.92)	(0.68)	(0.75)	(0.96)
	Adj. R^2(%)	40.03	27.83	14.09	5.31	2.28	-0.21	-1.87
4	Const.	0.270	0.306	0.307	0.324	0.239	0.120	0.119
		(1.65)	(1.76)	(1.67)	(1.51)	(0.89)	(0.70)	(1.42)
	vp^u	0.070*	0.025	0.004	-0.012	-0.010	-0.0003	-0.010
		(2.43)	(0.83)	(0.13)	(-0.32)	(-0.25)	(-0.01)	(-0.59)
	vp^d	-0.155***	-0.108***	-0.070*	-0.055	-0.041	-0.024	-0.013
		(-5.30)	(-3.62)	(-2.25)	(-1.72)	(-0.99)	(-0.94)	(-0.84)
	sp^u	0.077	0.245	0.317	0.304	0.213	0.075	0.107
		(0.57)	(1.59)	(1.76)	(1.58)	(1.03)	(0.56)	(1.23)
	sp^d	0.270*	0.137	0.057	0.087	0.081	0.080	0.046
		(2.18)	(1.11)	(0.49)	(0.75)	(0.56)	(0.80)	(0.91)
	Adj. R^2(%)	60.85	46.26	32.07	17.60	8.74	0.91	-0.02

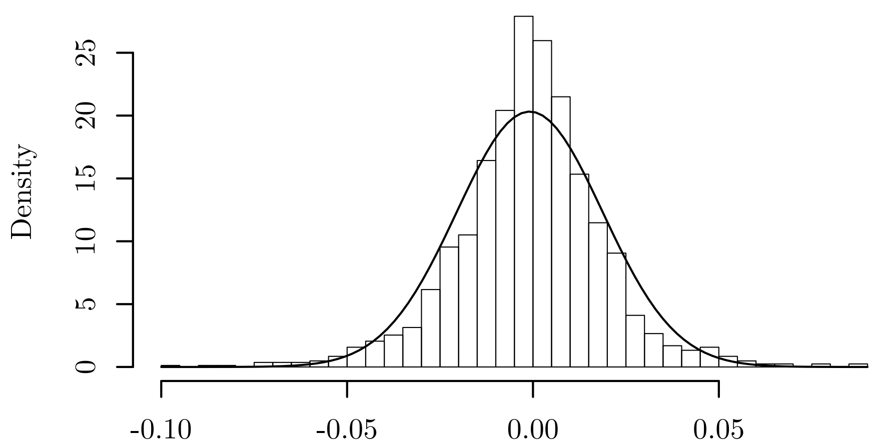
Note: The table compares the predictability of future USO excess returns (xm^{USO}) which is defined as Eq.(37), by dividing the risk premiums into two groups: the upside variance and skew premiums and the downside variance and skew premiums. The forecasting horizon h can be 1 week (1w), 2 weeks (2w), 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m) and 9 months (9m). The t-statistics are computed according to Newey and West (1987). We use $*$, $**$ and $***$ to denote the significance level of 5%, 1% and 0.1% respectively. The monthly observations range from January 2010 to June 2016.

C Figures

Figure 1: Evolution of USO price

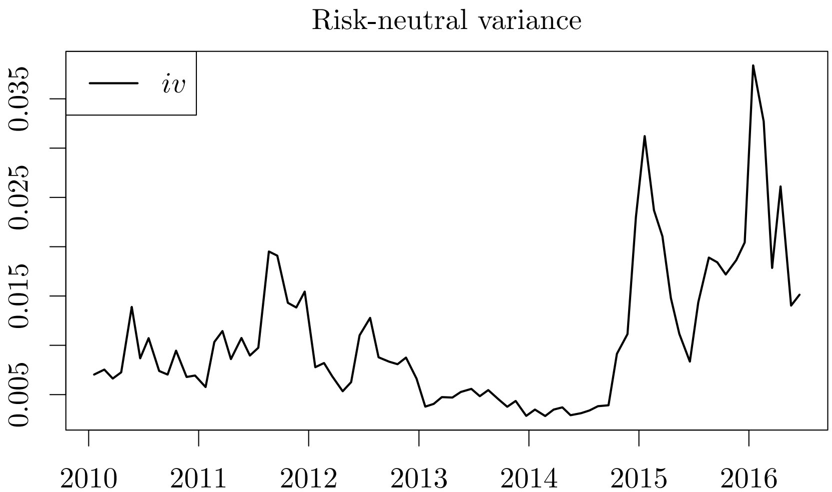
The curve shows the time series of USO price from January 2010 to July 2016. The market went through turmoil in 2015 and 2016.

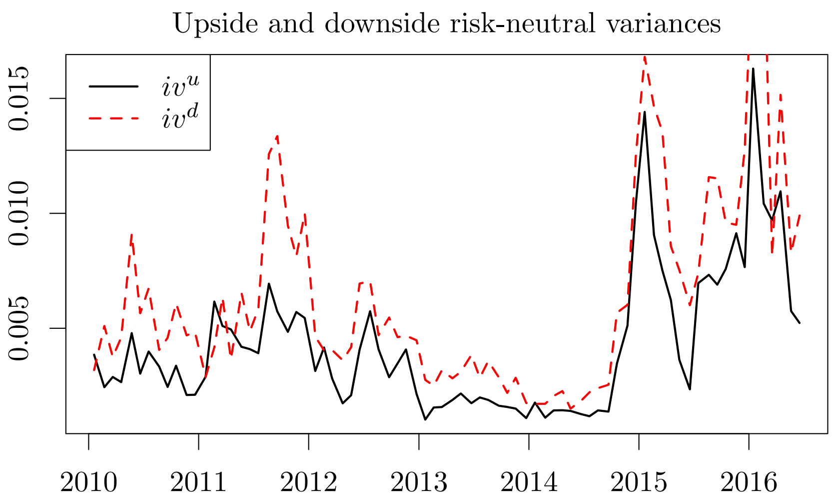
Figure 2: Density curve of USO returns



The histogram shows the empirical density of the daily log returns of USO from January 2010 to July 2016. The curve stands for the normal distribution with the same mean and standard deviation of the sample data.

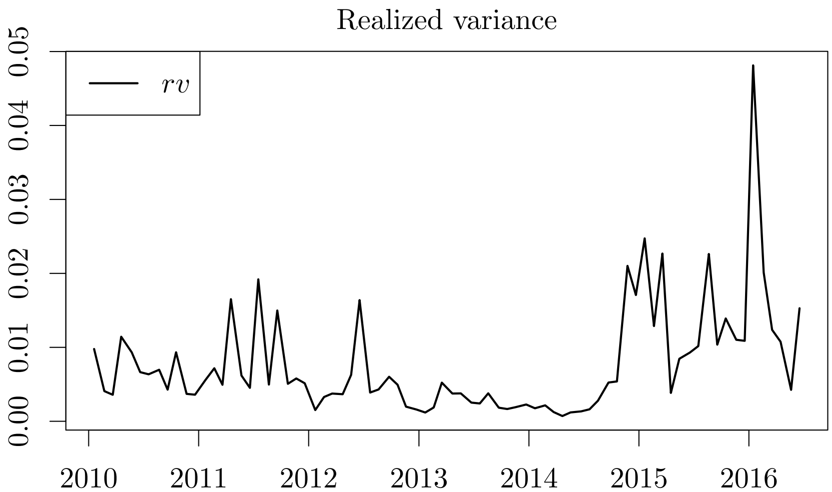
Figure 3: Decomposition of risk-neutral variance

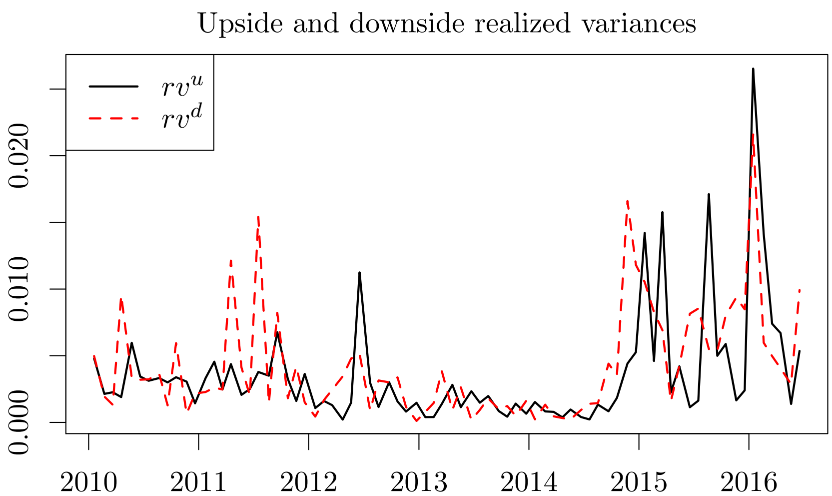




The upper figure shows the evolution of risk-neutral variance (iv, given by Eq.(3)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside risk-neutral variance (iv^u, given by Eq.(6), black solid line) and downside risk-neutral variance (iv^d, given by Eq.(6), red dashed line) of the same period, also based on monthly observations. In general, the downside risk-neutral variance is greater and more volatile than upside risk-neutral variance, and the two sum up to the total risk-neutral variance.

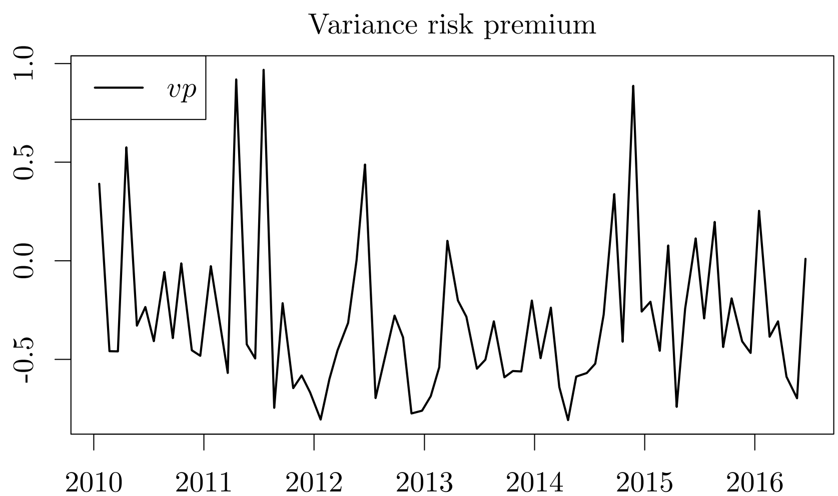
Figure 4: Decomposition of realized variance

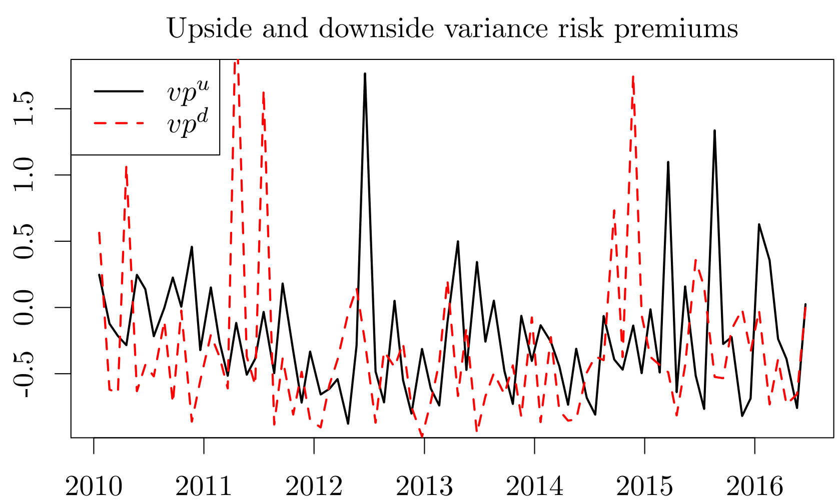




The upper figure shows the evolution of realized variance (rv, given by Eq.(7)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside realized variance (rv^u, given by Eq.(8), black solid line) and downside realized variance (rv^d, given by Eq.(8), red dashed line) of the same period, also based on monthly observations. In general, the volatility of the downside realized variance is greater than upside realized variance, and the two sum up to the total realized variance.

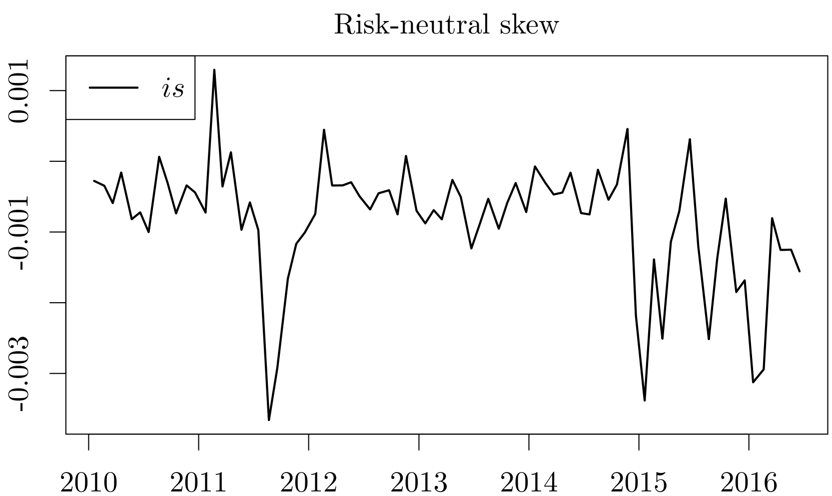
Figure 5: Decomposition of variance risk premium

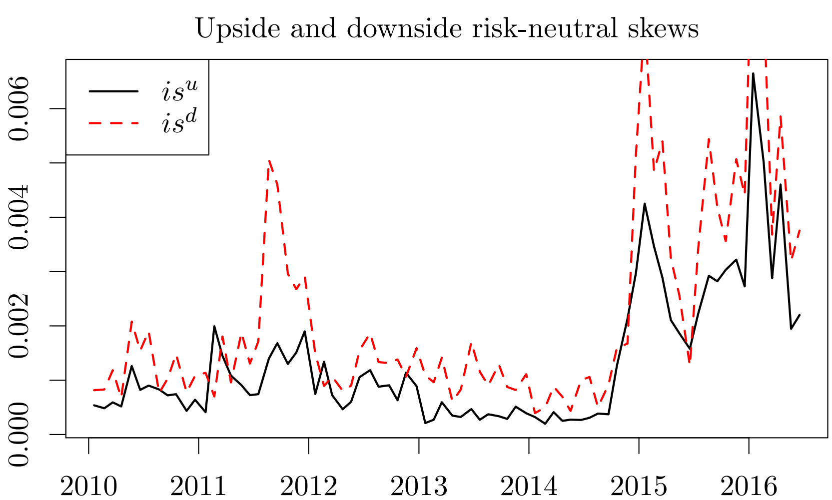




The upper figure shows the evolution of variance risk premium (vp, given by Eq.(9)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside variance risk premium (vp^u, given by Eq.(10), black solid line) and downside variance risk premium (vp^d, given by Eq.(10), red dashed line) of the same period, also based on monthly observations. In general, the volatility of the downside variance risk premium is greater.

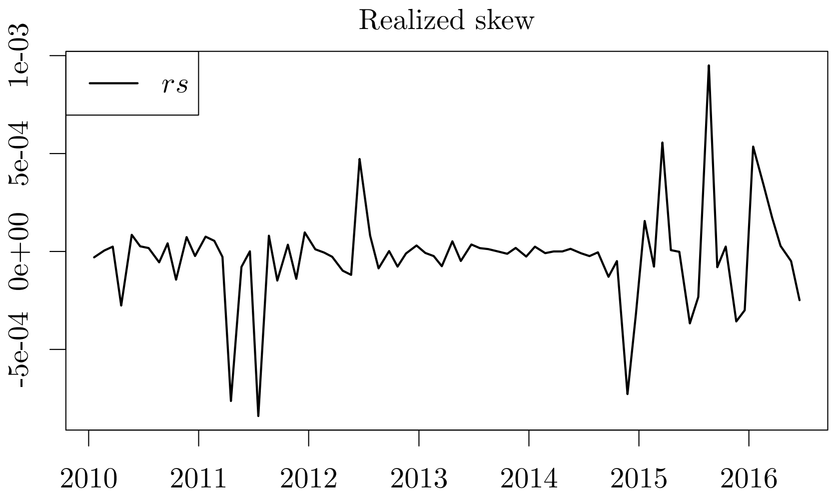
Figure 6: Decomposition of risk-neutral skew

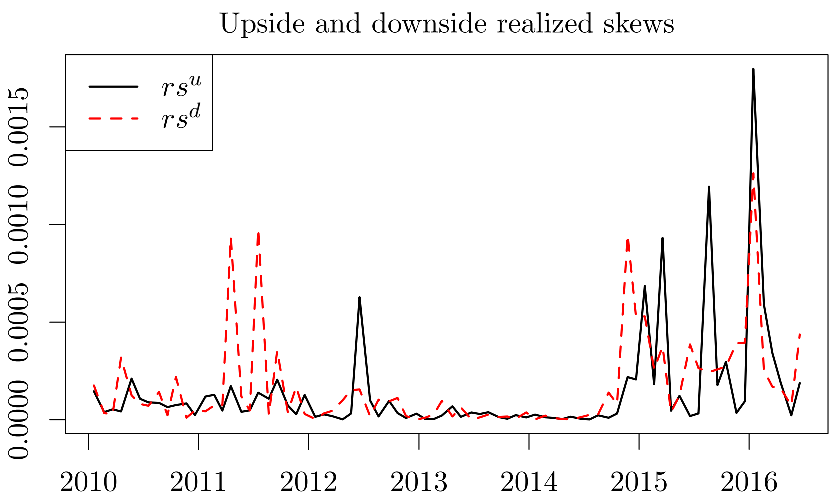




The upper figure shows the evolution of risk-neutral skew (is, given by Eq.(12)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside risk-neutral skew (is^u, given by Eq.(13), black solid line) and downside risk-neutral skew (is^d, given by Eq.(13), red dashed line) of the same period, also based on monthly observations. In general, the volatility of the downside risk-neutral skew is greater than upside risk-neutral skew, and the two sum up to the total risk-neutral skew.

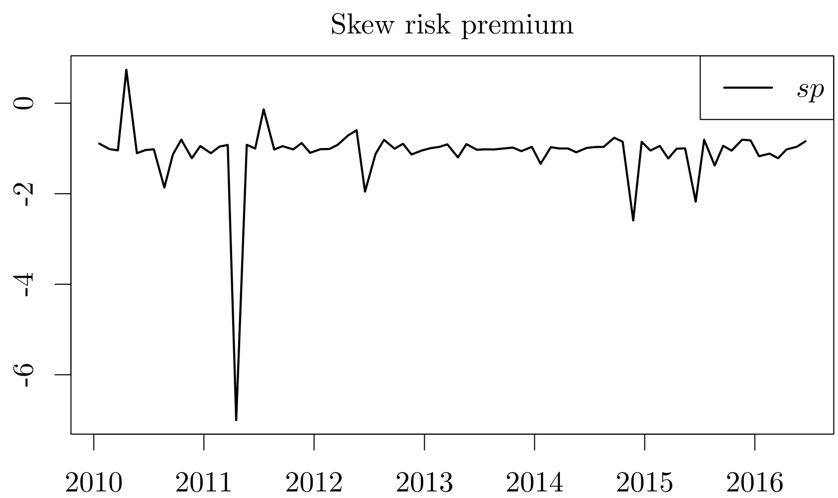
Figure 7: Decomposition of realized skew

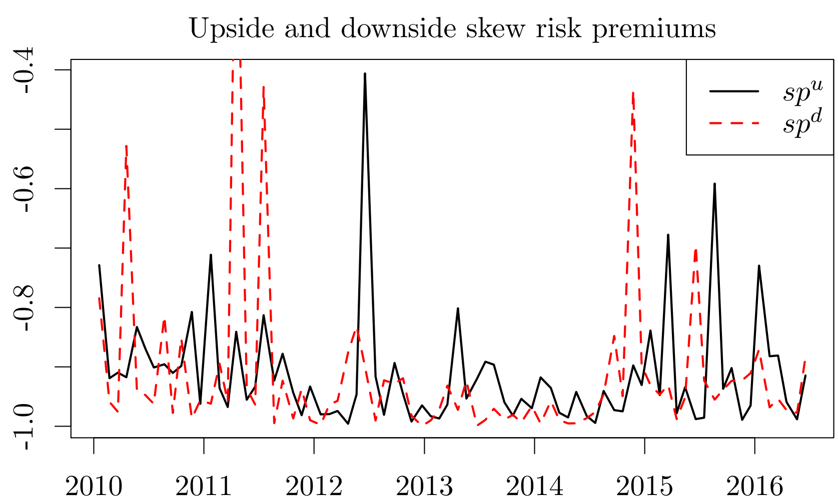




The upper figure shows the evolution of realized skew (rs, given by Eq.(17)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside realized skew (rs^u, given by Eq.(18), black solid line) and downside realized skew (rs^d, given by Eq.(18), red dashed line) of the same period, also based on monthly observations. In general, the volatility of the downside realized skew is greater than upside realized skew, and the two sum up to the total realized skew.

Figure 8: Decomposition of skew risk premium





The upper figure shows the evolution of skew risk premium (sp, given by Eq.(19)) from January 2010 to June 2016, based on monthly observations. The lower figure shows the evolutions of upside skew risk premium (sp^u, given by Eq.(20), black solid line) and downside skew risk premium (sp^d, given by Eq.(20), red dashed line) of the same period, also based on monthly observations. In general, the volatility of the downside skew risk premium is greater than upside skew risk premium.