Disaster Recovery, Jump Propagation and the Multihorizon UIP Pattern

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Abstract

Forward premium puzzle and the exchange rate level puzzle are two violations of interest parity. The former implies high interest rate currency is riskier while the latter implies the opposite. We propose a consumption-based general equilibrium models with disaster recovery and jump propagation to explain the two puzzles and hence address the paradox. The model reproduces patterns of multihorizon UIP regression slopes in Engel (2016), which is the key to resolve the currency puzzles. It also matches the term structure of real yields, as well as some basic moments of exchange rate growth and interest rate.

JEL Classification: F31, G15

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1 Introduction

The forward premium puzzle is a well-documented puzzle that concerns the relationship between exchange rates and interests. According to the uncovered interest rate parity, the country whose interest rate is relative low should experiences a appreciation in its currency, so that risk-free assets across countries have same returns when evaluated in the same currency. However, empirical works, including Fama et al. (1984), Bilson et al. (1981) and Hansen and Hodrick (1980), reveals that when the interest in one country is high relative to another, the short term deposits of that currency tend to earn an excess return. That is, currency with high interest-rate tend to appreciate. These findings violate the UIP condition and therefore inspire new models to explain it. There are a rich number of models in the literature that dissolve the forward premium puzzle.

However, the forward premium puzzle, if examined together with the exchange rate level puzzle, generates another paradoxical implication. The exchange rate level puzzle also describes the relationship between exchange rates and interest rates, but concerns the level instead of the growth rate of exchange rate. Classic exchange models, such as Dornbusch (1976) and Frankel (1979), assume UIP condition holds. In these model, the level of exchange is determined by the weighted average of the expectation of future interest rate differentials. Their models predict that, the country whose interest rate is higher than average should have a lower-than-average price of foreign currency. In other words, high interest rate relates to strength home currency. However, according to Engel (2016), though this relationship is borne out in the data, home currency tends to be stronger than is warranted by the models.

The two puzzles seem contradict to each other in the aspect of risk. To explain the forward premium puzzle, the high interest rate currency should be riskier, so that the compensation for risk-bearing is positive. On the contrary, the exchange rate level puzzle requires the high interest rate currency to be less risky, as lower expected future risk premiums can explain the strength of exchange rate. This contradiction is firstly documented in Engel (2016).

To reconcile Engel puzzle, the key is to reproduce the multiperiod currency risk premium regression slopes. Engel (2016) runs the following regression for different horizon j

$$E_t[\rho_{t+j}] = \alpha_j + \beta_j(r_t^* - r_t) + u_t^j,$$

where ρ_{t+j} is the excess return on the foreign deposit held by domestic investor from time $t+j$ to $t+j+1$, r_t^* and r_t are the foreign and domestic risk rate respectively. He finds that for the first few j, β_j is positive, but it eventually turns negative at long horizons, and finally converges to 0. What's more, the overall average of β_j is negative. The positive β_j in the short term corresponds to the forward premium puzzle, and the negative sum of β_j reflects the exchange rate level puzzle.

Chernov and Creal (2021) also investigate the multihorizon currency risk premium. They regress future depreciation rate, instead of expected future currency risk premium, on the interest rate differential, and find similar pattern as Engel (2016), especially the reversal slope sign and the convergence to 0 in the long end. The implication of this finding is that the shape of slopes of multihorizon currency risk premium regressions is inherited from expected depreciation, instead of expected future interest rate differentials.

The literature documents many theories that are able to explain the forward premium puzzle. Among them are external formation model by Verdelhan (2010), long-run risk model by Bansal and Shaliastovich (2013), rare disaster models by Du (2013) and Farhi and Gabaix (2016), model of overconfidence by Burnside, Han, Hirshleifer and Wang (2011). However, there are few models aims to explain the Engel puzzle or multihorizon UIP puzzle. Engel (2016) checks two basic models of exchange risk premium and "delayed overshooting", and concludes that both models can not account for the exchange level puzzle. He therefore proposes a model that introduces a non-pecuniary liquidity return on assets to account for the puzzle. Bacchetta and Van Wincoop (2021) shows that a model featured with delayed portfolio adjustment can address Engel puzzle. In these models, markets is with fraction of either liquidity or portfolio management cost. There are also some models with exogenously specified SDF that attempts to explain the multihorizon currency risk premium. Examples are a no-arbitrage term structure model that includes exchange rate as state variable in Chernov and Creal (2021), a term structure model with unspanned macroeconomic risks in Chen, Du and Zhu (2016).

To the best of our knowledge, there are not consumption-based general equilibrium models with frictionless markets in the literature that can address Engel puzzle. In this paper we propose a consumption-based model to fill the gap. In the model, rational investors in two countries are with recursive preference. In the economy of each country there are disasters followed by recoveries. The risk of disaster is highly but imperfectly shared between countries. The key feature is jump propagation for the unshared disasters. After the occurrence of a disaster, recovery and jump propagation have opposite implications of risk and therefore determines expectation of future currency risk premium that coincides with Engel's.

Specifically, the unshared disaster and diffusion risks together determine the spot currency risk premium while the future risk premium is shaped by recovery and jump propagation. When the domestic disaster probability is higher than foreign, the interest rate at home is lower as domestic investor will save more for precautionary reason. At the same time, higher disasters risk derives higher price of risk at home, and hence domestic shocks dominates exchange rate dynamics. As a consequence, the foreign risk-free asset is riskier to home investor than domestic one to foreign investor, and domestic investor require risk premium from foreign risk-free asset for risk-bearing. This channel explains the forward premium puzzle. The diffusion risk have similar effect.

For future currency risk premium, jump propagation plays an important role. Suppose the foreign country is under strong recovery, which implies that its consumption growth rate will be high in the near future, hence there are less precautionary savings, and the foreign interest rate is high. However, as recovery is accompanied with jump propagation, In the near future, the foreign disaster risk will ramp up, and the currency risk premium will decrease. With appropriate strength of recovery and jump propagation, the risk premium can turn into negative. In other words, future foreign exchange risk for foreign investor will increase more than for domestic investor, and foreign currency is less riskier for domestic investor. As in the long run, both recovery and jump propagation will fade away, β_j will gradually converge to 0 as in Engel (2016).

To show that the multihorizon currency risk premia are inherited from expected depreciation, rather than the expected future interest rate differentials, we also calculate the theoretical slopes with ex ante depreciation rate as dependent variable. The resulting pattern of slopes coincides with Chernov and Creal (2021).

The model also generate pro-cyclical interest rate and counter-cyclical term structure of real bond yields. In good times, when interest rate is high, the term structure is downward sloping and vise verse.

Our model builds on the rare disasters literature. Since proposed by Rietz (1988), rare disasters explain a lot of puzzles about asset returns. Recent studies show that incorporating time-varying risk of disasters enriches its explanation power (e.g. Gabaix (2012), Wachter (2013) and Seo and Wachter (2018)). Pioneered by Gourio (2008), the recoveries after disasters start to attract attention. Hasler and Marfe (2016) show that recovery explains the downward sloping term structure of equity premium. Our setup of recovery follows Hasler and Marfe (2016) but in a slightly different way. This model contribute to the disaster literature by showing that, combined with jump propagation, disaster recovery can explain the multihorizon UIP puzzle.

Our setup of jump propagation is new to the literature. Previous in the literature, jump propagation is modelled based on Hawkes process (Hawkes (1971)), examples are Boswijk, Laeven and Lalu (2015) and Du and Luo (2019)). In their setup, Each jump only has an ephemeral effect on the intensity. While in our model, self-excitation of jump will last for some time. Besides, as recovery and jump propagation start at the same time, our model is the first to investigate how they interact with each other to determine the multihorizon currency risk premium.

We proceed as follows. Section 2 proposes an equilibrium model with disaster recovery and jump propagation. Section 3 presents the model solutions of stochastic discount factor, bonds and exchange rate. Section 4 describes the details of calibration. Section 5 presents the models implications, especially for multihorizon UIP regressions and the term structure of bond yields. Section 6 concludes.

2 The Model

The model assumes two endowment economics, namely the home country and the foreign country. The two economics are completely symmetry. In other words, the economies evolve under the same mechanism and with identical parameters; The representative agents in both countries are characterized by the same underlying structural parameters. The model features with disaster recoveries and jump propagation.

2.1 Disasters with recoveries

Throughout this paper, the variables in the foreign country is super-indexed by "*". Let $c_t = \log C_t$, where C_t is the aggregate consumption in the home country. The dynamics of c_t is assumed to be as follows

$$\begin{aligned} dc_t &= (\mu_c + \omega_c z_t)dt + \sigma_t dW_{c,t} + \xi_t^i dN_t^i + \xi_t^g dN_t^g, \\ d\sigma_t^2 &= \phi_\sigma(\bar{\sigma}^2 - \sigma_t^2)dt + \nu_\sigma \sqrt{\sigma_t^2} dW_{\sigma,t}, \\ dz_t &= -\phi_z z_t dt + \xi_t^i dN_t^i + \xi_t^g dN_t^g. \end{aligned} \tag{1}$$

where $W_{c,t}$ and $W_{\sigma,t}$ are two independent standard Brownian motions. There are two sets of rare disasters on consumption. One is a common global disaster N_t^g, and the other is a idiosyncratic disaster N_t^i. In other words, the disaster risk is imperfectly shared among countries. Both N_t^g and N_t^i are Poisson processes. ξ_t^g and ξ_t^i are the sizes of disasters respectively. The global jump size ξ_t^g follows a negative time invariant distribution with probability density function(PDF) given by

$$f(\xi_t^g;\eta_0^g,\eta_1^g)=\begin{cases}\eta_1^g e^{\eta_1^g(\xi_t^g+\eta_0^g)}, & \text{if } \xi_t^g\le -\eta_0^g;\\ 0, & \text{otherwise}.\end{cases}$$

where η_0^g and η_1^g are positive parameters. The distribution can be obtained from translation of a negative exponential distribution. η_1^g is the intensity of the exponential distribution and $-\eta_0^g$ is the upper bound of the distribution, or η_0^g is the lower bound of the global disaster size. The moment generating function (MGF) is hence

$$\varrho^g(u)=E_t(e^{u\xi_t^g})=\frac{1}{1+\frac{u}{\eta_1^g}}e^{-\eta_0^g u}.$$

The first two moments of ξ_t^g are therefore

$$E_t(\xi_t^g)=-\left(\frac{1}{\eta_1^g}+\eta_0^g\right),\quad E_t(\xi_t^{g2})=\frac{1}{\eta_1^{g2}}+\left(\frac{1}{\eta_1^g}+\eta_0^g\right)^2.$$

The distribution of ξ_t^i is of the same form as ξ_t^g, but the parameters are η_0^i and η_1^i respectively.

Similar to Hasler and Marfe (2016), the recovery factor z_t models the recovery after disaster. However, in Hasler and Marfe (2016), z_t is directly added to the level of consumption, while in our model, it only affects the expected consumption growth rate. When a disaster occurs, z_t turns into negative, but the mean-reverting property makes it revert to its long term average 0. $\omega_c<0$ captures the impact of recoveries factor on consumption growth. After disaster, a negative z_t will increase the expected growth rate of consumption, which represents that the consumption is recovering. However, as z_t reverts, the expected growth rate of consumption will decrease gradually towards its normal level.

Given the dynamics of z_t, ω_c determines the recovery rate of consumption, therefore, we term ω_c as 'recovery degree' for convenience.

2.2 Jump propagation

Let λ_t^g and λ_t^i denote the intensities of the Poisson processes N_t^g and N_t^i. The jump intensities are supposed to follow the processes below.

$$\begin{aligned} d\lambda_t^g &= \phi_g(\bar{\lambda}^g - \lambda_t^g)dt + \sigma_g\sqrt{\lambda_t^g}dW_{g,t}, \\ d\lambda_t^i &= \phi_i(\bar{\lambda}^i + \omega_i z_t - \lambda_t^i)dt + \sigma_i\sqrt{\lambda_t^i}dW_{i,t}, \end{aligned} \tag{2}$$

where $\omega_i < 0$. The standard Brownian motions $W_{g,t}$, $W_{i,t}$, $W_{c,t}$ and $W_{\sigma,t}$ are mutually independent.

The intensity of global disaster follows a CIR process (Cox, Ingersoll Jr and Ross (1985)), which is highly skewed. As discussed in Wachter (2013), with CIR intensity, there are times when rare disasters can occur with high probability, but these times are themselves unusual. If $\omega_i = 0$, the domestic disaster intensity also follows a CIR process. The recovery factor z_t in the drift of domestic disaster intensity λ_t^i models the jump propagation and ω_i measures the degree of jump propagation. With jump propagation, the probability of domestic disaster is expected to be higher than usual after the occurrence of a disaster (either global or domestic). The non-negative property of $\omega_i z_t$ promises that jump intensity is well-defined, if the Feller condition $\sigma_i^2 < 2\phi_i\bar{\lambda}^i$ (Feller (1951)) is satisfied. Intuitively, without z_t term, the drift is already strong enough to pull the process back to its long-term mean when its value is approaching 0. The z_t term in drift will only strengthen the pull-back effect, therefore the process will always stay non-negative.

Previous studies in the literature usually model jump propagation based on

Hawkes process (Hawkes (1971)) and directly add jump component to the dynamics of intensity as $d\lambda_t = \phi(\bar{\lambda} - \lambda_t)dt + \beta dN_t$ (e.g. Boswijk et al. (2015); Du and Luo (2019)). In their setup, the occurrence of one jump will cause the jump intensity to ramp up instantly. Each jump only has an ephemeral effect on the intensity. However, we model the self-excitation of jump in a different way, where the occurrence of one jump will increase the long-term mean of the intensity. Note that z_t will gradually revert to its long-term mean zero, so the effect of one jump on intensity will also disappear gradually.

In Seo and Wachter (2018), the disaster intensity also mean-reverts to a time-varying long-term mean. In their model, the long-term mean itself is a diffusion CIR process without jumps. In our model, however, the long-term mean of each λ_t is $\bar{\lambda} + \omega_i z_t$, where z_t will jump at the events of consumption disasters. The simultaneousness of consumption disaster and z_t, and the higher disaster probabilities in the subsequent years, enable the model to capture the characteristic of jump propagation in the real world, e.g. the 2008 financial crisis.

Note that λ_t^g in both countries are identical and has not direct influence on the exchange rate. However, it affects both consumption growth and the recovery factor z_t, which have real impact on the exchange rate. We do not assume jump propagation for global disasters, just to make sure that $\lambda_t^g = \lambda_t^{g,*}$ for all t.

2.3 The recursive preference

Assume the preference of the representative investor in the home country can be described by the recursive utility function, as developed by Epstein and Zin (1989). In continuous time, the recursive utility function is

$$U_t = \left[(1-\delta^{dt})C_t^{\frac{1-\gamma}{\theta}} + \delta^{dt} E_t \left(U_{t+dt}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}$$

where γ is the coefficient of risk aversion, δ the subjective discount rate, ψ is the elasticity of intertemporal substitution(EIS) and $\theta = \frac{1-\gamma}{1-1/\psi}$.

3 Model Solution

We follow the methodology developed by Duffie, Pan and Singleton (2000) and Eraker and Shaliastovich (2008) to solve the model. Eraker and Shaliastovich (2008) shows that asset prices are approximately exponential affine in state variables under the affine jump-diffusion structure when the representative investor's preference is recursive. The methodology is based on the log-linearization of returns, as in Campbell and Shiller (1988), which facilitates the analytical tractability of the model. For convenience and to put the affine jump-diffusion structure more clear, we stack the variables in the home economy as a vector Y_t. That is, $Y_t = (c_t, z_t, \lambda_t^g, \lambda_t^i, \sigma_t^2)^\top$. Then we have

$$\begin{aligned} dY_t &= \mu(Y_t)dt + \Sigma(Y_t)dW_t + J_t dN_t, \\ \mu(Y_t) &= M + KY_t, \\ \Sigma(Y_t)\Sigma(Y_t)^\top &= h + \sum_{j=1}^{5} H^j Y_t^j, \\ J_t &= \begin{pmatrix} J_g \xi_t^g & J_i \xi_t^i \end{pmatrix}, \\ l_t(Y_t) &= l_0 + l_1 Y_t. \end{aligned}$$

where $W_t = (W_{c,t}, W_{z,t}, W_{g,t}, W_{i,t}, W_{\sigma,t})^\top$, $N_t = (N_t^g, N_t^i)^\top$, H^j is the jth page of H, Y_t^j is the jth element in Y_t and $l(Y_t)$ is the vector of jump intensity. $M \in \mathbb{R}^{5\times1}$, $K \in \mathbb{R}^{5\times5}$, $h \in \mathbb{R}^{5\times5}$, $H \in \mathbb{R}^{5\times5\times5}$, J_g, $J_i \in \mathbb{R}^{5\times1}$, $l_0 \in \mathbb{R}^{2\times1}$ and $l_1 \in \mathbb{R}^{2\times5}$ are matrixes derived from the setup of variables in Y_t. The specification of these matrixes is in Appendix.

3.1 The Price-consumption ratio

Let V_t^c denote the aggregate wealth, or the price of a claim on all future consumption and $R_{c,t}$ denote the cumulative return till time t on the wealth portfolio that pays consumption as its dividend. Hence $\nu_{c,t} = \log \frac{V_t^c}{C_t}$ denotes the log of the price-consumption ratio. A log-linearization of R_t is

$$d \log R_{c,t} \approx k_0 dt + k_1 d\nu_{c,t} - (1-k_1)\nu_{c,t} dt + dc_t, \tag{3}$$

where

$$k_0 = -\log\left((1-k_1)^{1-k_1} k_1^{k_1}\right), \quad k_1 = \frac{e^{E(\nu_{c,t})}}{1+e^{E(\nu_{c,t})}}. \tag{4}$$

This is the standard log-linearization method in the literature that is adopted in Eraker and Shaliastovich (2008) and Hasler and Marfe (2016), among others. It is the continuous time version of the counterpart in Campbell and Shiller (1988). The approximation error is a second order Taylor term. Campbell, Lo and MacKinlay (2012) and Bansal, Kiku and Yaron (2007) show that, when shocks are normally distributed, the approximation error is quite small and does not have material effects. Hasler and Marfe (2016) shows that the approximation is still of high accuracy when there are jumps in the model.

Recursive utility generates the continuous time dynamics of the stochastic discount factor(SDF) M_t as

$$d \log M_t = \theta \log \delta dt - \frac{\theta}{\psi} dc_t - (1-\theta) d \log R_{c,t}. \tag{5}$$

The Euler equation applied on $R_{c,t}$, is

$$E_t\left[\exp\left(\int_t^{t+\tau} d\log M_s + \int_t^{t+\tau} d\log R_{c,s}\right)\right] = 1 \tag{6}$$

Assume that log of price-consumption ratio is affine on Y_t,

$$\nu_{c,t} \equiv \log \frac{V_t^c}{C_t} = A + B^\top Y_t,$$

where $A \in \mathbb{R}$ and $B = (B_c, B_z, B_g, B_i, B_\sigma)^\top$. The Appendix shows that the solution coefficient A and B satisfy the following equations

$$\begin{aligned} 0 &= K^\top \chi - \theta(1-k_1)B + \frac{1}{2}\chi^\top H \chi + l_1^\top \big(\varrho(\chi) - \mathbf{1}_{2\times 1}\big), \\ 0 &= \theta\big(\log\delta + k_0 - (1-k_1)A\big) + M^\top \chi + \frac{1}{2}\chi^\top h \chi + l_0^\top \big(\varrho(\chi) - \mathbf{1}_{2\times 1}\big). \end{aligned} \tag{7}$$

with $\chi = \theta((1-1/\psi)\delta_c + k_1 B)$, $\delta_c = (1,0,0,0,0)^\top$. And $\varrho(\cdot)$ is defined as

$$\varrho(\alpha) = \left(\varrho^g(\alpha^\top J_g), \varrho^i(\alpha^\top J_i)\right)^\top, \tag{8}$$

where $\alpha \in \mathbb{R}^{5\times 1}$, $\varrho^g(\cdot)$ and $\varrho^i(\cdot)$ are the MGF of ξ_t^g and ξ_t^i respectively. Combining equation (4) and (7), we can solve k_0, k_1, A and B systematically.

3.2 The Stochastic Discount Factor

Plugging equation (3) and the dynamics of consumption into equation (5), applying the Ito-Doeblin Formula and using the fact that $\theta - \theta/\psi - 1 = -\gamma$, we have the SDF dynamics as

$$\frac{dM_t}{M_{t-}} = -r_t dt - \Lambda_t^{c\top} dW_t - \left(\Lambda_t^{d\top} dN_t - l_t(Y_t)^\top \left[\mathbf{1}_{2\times 1} - \varrho(-\Omega)\right] dt\right), \tag{9}$$

where the risk free rate r_t, the price of risk for Brownian shocks Λ_t^c and the price of risk for disasters Λ_t^d are defined by

$$\begin{aligned}
r_t &= \Phi_0 + \Phi_1^\top Y_t, \\
\Lambda_t^c &= \Sigma(Y_t)^\top \Omega, \\
\Lambda_t^d &= \left(1 - e^{-\Omega^\top J_g \xi_t^g}, 1 - e^{-\Omega^\top J_i \xi_t^i}\right)^\top, \\
\Omega &= \gamma\delta_c + (1-\theta)k_1 B,
\end{aligned}$$

with Φ_0 and Φ_1 given by

$$\begin{aligned}
\Phi_1 &= (1-\theta)(k_1 - 1)B + K^\top\Omega - \frac{1}{2}\Omega^\top H\Omega - l_1^\top\big(\varrho(-\Omega) - \mathbf{1}_{2\times 1}\big), \\
\Phi_0 &= -\theta\log\delta + (1-\theta)\left[k_0 + (k_1 - 1)A\right] + M^\top\Omega - \frac{1}{2}\Omega^\top h\Omega - l_0^\top\big(\varrho(-\Omega) - \mathbf{1}_{2\times 1}\big).
\end{aligned}$$

where $\varrho(\cdot)$ is defined by equation (8).

The second two last term in Φ_1 and Φ_0 can be interpreted as precautionary saving terms originating from the diffusion shocks and jump risk, respectively. r_t is a decreasing linear function of all of the four state variables (z_t, λ_t^g, λ_t^i and σ_t^2), irrespective to the value of EIS. A small z_t implies that the economy is recovering and the consumption is growing in a rate higher than average, which lowers the investor's willing to save and increases the interest rate. An increase on λ_t^g, λ_t^i or σ_t^2 means that the jump risk or diffusion risk increases, therefore investor requires more risk-free assets to hedge the increasing risks, which drives down the risk-free rate.

3.3 Bond prices

Let Q denotes the risk-neutral measure induced by the SDF. The Appendix shows that under measure Q, the state variables follow

$$dY_t = (M^Q + K^Q Y_t)dt + \Sigma(Y_t)dW_t^Q + J_t^Q dN_t^Q,$$

where

$$M^Q = M - h\Omega,$$

$$K^Q = K - H\Omega,$$

$$dW_t^Q = dW_t + \Lambda_t^c dt.$$

And the intensity of the jump is given by

$$l_t^Q = l_t \cdot Q(-\Omega).$$

The MGFs for jump sizes are

$$\varrho^{g,Q}(u) = \varrho^g(u - \Omega^\top J_g)/\varrho^g(-\Omega^\top J_g), \quad \varrho^{i,Q}(u) = \varrho^i(u - \Omega^\top J_i)/\varrho^i(-\Omega^\top J_i).$$

For convenience, we define

$$\varrho^Q(\alpha) = \left(\varrho^{g,Q}(\alpha^\top J_g), \varrho^{i,Q}(\alpha^\top J_i)\right)^\top = \varrho(u - \Omega)./\varrho(-\Omega), \tag{10}$$

with $./$ being the element-by-element division.

Let $P_t(\tau)$ denote the price at time t of zero-coupon bond of maturity τ with unit payment. Then

$$P_t(\tau) = E_t^Q\left(\exp\left(-\int_t^{t+\tau} r_u du\right)\right) = \exp\left[a_B(\tau) + b_B^\top(\tau)Y_t\right]$$

where $a_B(\tau)$ and $b_B(\tau)$ solve the ODEs below.

$$\dot{b}_B(\tau) = -\Phi_1 + K^{Q^\top} b_B(\tau) + \frac{1}{2} b_B^\top(\tau) H b_B(\tau) + l_1^{Q^\top}(\varrho^Q(b_B(\tau)) - 1),$$
$$\dot{a}_B(\tau) = -\Phi_0 + M^{Q^\top} b_B(\tau) + \frac{1}{2} b_B^\top(\tau) h b_B(\tau) + l_0^{Q^\top}(\varrho^Q(b_B(\tau)) - 1)$$

with initial conditions $a_B(0) = 0$ and $b_B(0) = \mathbf{0}_{5\times 1}$.

The n-period holding return of the zero-coupon bond is $R_{t+n}(\tau) = P_{t+n}(\tau - n)/P_t(\tau)$, for $n \leq \tau$.

3.4 The exchange rate and the currency risk premia

Recall that we assume complete symmetry on the foreign country. Both the diffusion risk and the jump risk on consumption between the two countries are partially correlated. Assume the correlation coefficient between $W_{c,t}$ and $W_{c,t}^*$ is ρ_c. They share the global jump risk N_t^g, and the sizes of the global disaster, ξ_t^g and $\xi_t^{g,*}$, are assume to be the same across countries.

Let $s_t = \log S_t$, where S_t denotes the real exchange rate between home country and foreign country, expressed in domestic goods per foreign good. An increasing S_t implies depreciation of the home currency. It is a standard result in the international finance literature (e.g., Backus, Foresi and Telmer (2001)) that

$$s_t = \log M_t^* - \log M_t. \tag{11}$$

Generally in no-arbitrage jump models, the market is not complete, especially when jump sizes follows continuous distribution. Therefore there are multiple SDFs in each economy, and appropriate M_t and M_t^* have to be chosen to satisfy equation (11) (see e.g. Brandt, Cochrane and Santa-Clara (2006)). However, the present model is a consumption-based general equilibrium model, and the SDF is determined endogenously.

We define the currency risk premium in continuous time as

$$\begin{aligned} rx_{t+\tau}^{FX} &= \int_t^{t+\tau} r_u^* du - \int_t^{t+\tau} r_u du + s_{t+\tau} - s_t \\ &= \int_t^{t+\tau} r_u^* du - \int_t^{t+\tau} r_u du + \int_t^{t+\tau} d\log M_u^* - \int_t^{t+\tau} d\log M_u. \end{aligned} \tag{12}$$

which can be interpreted as the excess return on the foreign deposit held by domestic investor from period t to $t+\tau$. The domestic investor borrows funds at home, and investor on the foreign risk-free asset at time t. During time t to $t+\tau$, she earns a return $\int_t^{t+\tau} r_u^* du$ in the foreign risk-free asset, and when she transfers to home currency, her gain/loss is $s_{t+\tau} - s_t$. And her cost is the domestic risk-free return $\int_t^{t+\tau} r_u du$.

According to the dynamics of SDF in equation (9), we have

$$\begin{aligned} d\log M_t &= \left(-r_t - l_t^\top \left[\varrho(-\Omega) - \mathbf{1}_{2\times 1}\right] - \frac{1}{2}\Lambda_t^{c\top}\Lambda_t^c\right) dt - \Lambda_t^{c\top} dW_t + \log(1 - \Lambda_t^{d\top}) dN_t, \\ &\equiv \left(M_0 + K_0 Y_t\right) dt - \Lambda_t^{c\top} dW_t - \Omega^\top J_t dN_t \end{aligned}$$

where

$$\begin{aligned} M_0 &= -\Phi_0 - l_0^\top \left[\varrho(-\Omega) - \mathbf{1}_{2\times 1}\right] - \frac{1}{2}\Omega^\top h\Omega, \\ K_0 &= -\Phi_1 - l_1^\top \left[\varrho(-\Omega) - \mathbf{1}_{2\times 1}\right] - \frac{1}{2}\Omega^\top H\Omega. \end{aligned}$$

And the dynamics of log real exchange rate is

$$\begin{aligned} ds_t =& d\log M_t^* - d\log M_t \\ =& K_0\left(Y_t^* - Y_t\right) dt - \Lambda_t^{c,*\top} dW_t^* + \Lambda_t^{c\top} dW_t \\ & - \Omega^\top J_i \xi_t^{i,*} dN_t^{i,*} + \Omega^\top J_i \xi_t^i dN_t^i. \end{aligned} \tag{13}$$

For the expected currency risk premium, we have the following proposition.

Proposition 1. *expected currency risk premium is given by*

$$E_t[rx^{FX}_{t+\tau}] = \int_t^{t+\tau} E_t[r^*_u]du - \int_t^{t+\tau} E_t[r_u]du + E_t[s_{t+\tau}] - s_t, \tag{14}$$

where

$$E_t[r^*_T] = A_r(\tau) + B_r^\top(\tau)Y^*_t,$$
$$E_t[r_T] = A_r(\tau) + B_r^\top(\tau)Y_t,$$
$$E_t[s_T] - s_t = B_m^\top(\tau)(Y^*_t - Y_t),$$

with $\tau = T - t$, and $A_r(\tau)$, $B_r(\tau)$ and $B_m(\tau)$ satisfied ODE systems given in the Appendix.

Let $B_x^\top(\tau)$ denote

$$B_x(\tau) = \int_0^\tau B_r(s)ds + B_m(\tau),$$

where the integration is evaluated for each entry in $B_r(s)$ individually, with a slight abuse of notation. Then

$$E_t[rx^{FX}_{t+\tau}] = B_x^\top(\tau)(Y^*_t - Y_t), \tag{15}$$

If we regress for the annualized expected currency excess return for holding foreign deposit from $t+\tau$ to $t+\tau+j$ on the interest rate difference, the slope coefficient is

$$\beta^{rx}_{\tau,\tau+j} = \frac{Cov\left(\frac{E_t[rx^{FX}_{t+\tau+j}]-E_t[rx^{FX}_{t+\tau}]}{j}, r^*_t - r_t\right)}{Var(r^*_t - r_t)} = \frac{[B_x^\top(\tau+j) - B_x^\top(\tau)]V_Y\Phi_1}{j\Phi_1^\top V_Y\Phi_1},$$

where V_Y is the covariance matrix of $Y^*_t - Y_t$. If j goes to 0, we have the slope for the instantaneous currency risk premium in $t+\tau$,

$$\beta_\tau^{rx} = \frac{[B_r(s) + \dot{B}_m(\tau)]^\top V_Y \Phi_1}{\Phi_1^\top V_Y \Phi_1}. \tag{16}$$

Here $\dot{B}_m(\tau)$ is is a vector of the derivatives of each entry in $B_m(\tau)$. If the dependent variable is the expected exchange rate growth, the slope coefficient is

$$\beta_\tau^{s} = \frac{\dot{B}_m^\top(\tau) V_Y \Phi_1}{\Phi_1^\top V_Y \Phi_1}. \tag{17}$$

4 Data and Calibration

In this section, we calibrate the model to the data of United States and United Kingdom.

4.1 Data

The consumption on United States is calculated as the sum of per capita personal consumption expenditure on nondurables and services. The data on consumption of nondurables and services are from Bureau of Economic Analysis (BEA). The U.S. population data comes from Factset. The consumption on United Kingdom is the per capita 'Total Final Consumption Expenditure', which is from Office for National Statistics (ONS). The UK population data is from Thomson Reuters Datastream. The risk-free interest rate is proxied by the yield on 3-month T-bills, which are obtained from Federal Reserve Economic Data (FRED) for U.S. and from Bank of England (BoE) for U.K.. The U.S. Consumer Price Index (CPI) inflation comes from CRSP, while the U.K. CPI inflation is from BoE. Both consumption and risk-free rate data on both countries are quarterly, and span from 1955Q1 to 2020Q4. The choice of the beginning date of 1955Q1 is because of the availability of U.K. consumption data.

The USD/GBP exchange rate data is also quarterly in order to in line with consumption and interest rate. The data is downloaded from Datastream and spans from 1975Q1 to 2020Q4, the period after dissolution of the Bretton Woods System.

4.2 Calibration

Table 1 presents the calibrated parameter values. The two countries share the same set of parameters.

We set the subjective discount rate at 0.975. The coefficient of risk aversion is set at 8, in the range between 1 and 10, as suggested by Mehra and Prescott (1985). It is also a reasonable level compared with the number commonly used in models with recursive utilities (e.g., Colacito and Croce (2013)). I set The elasticity of intertemporal substitution at 2, consistent with Colacito and Croce (2011) and Du (2013).

The expected consumption growth rate without disaster, μ_c is set at 0.0218. After taking disasters and recoveries into account, the average consumption growth is 0.0188, which falls in between the historical means of consumption growth rates in U.S. and U.K. in our sample (see Table 2). The standard deviation of consumption growth is 0.0095 for U.S. in Du (2013), which is based on a sample period between 1952Q1 and 2006Q4, when there are no economic disasters. Barro (2006) reports that the volatility of consumption growth in U.S. from 1890 to 2004 is 0.035, and the value is 0.045 for real per capita GDP. In our sample, the consumption volatilities is 0.0236 and 0.0329. All the evidence above considered, we let the volatility of consumption growth without disasters be 0.01. The unconditional consumption growth volatility, taking both diffusion risk and jump risk into consideration, is therefore 0.0381.

The literature documents low correlation between U.S. and U.K. consumption growth (e.g. 0.28 in Colacito and Croce (2011) and 0.31 in Brandt et al. (2006)). It is worth noting that their samples are before year 2000, and cover a period without economic disasters. Table 2 show that the consumption growth correlation on our full sample is 0.6372. However, if we focus on the period before 2008, the correlation becomes as low as 0.1630. And the value is 0.9125 for period between 2008Q1 and 2020Q4, when their are two disasters happened, i.e., the 2008 financial crisis and the pandemic of Covid-19. We therefore choose the value 0.2 for the correlation between domestic and foreign consumption diffusions.

We assume that home and global disaster sizes share the same distribution, i.e. $\eta_0^g = \eta_0^i$ and $\eta_1^g = \eta_1^i$. The average probability of disaster and the average size of disaster are crucial in asset pricing models, and they highly dependent on the definition of a "disaster". Based on Table 10 in Barro and Ursúa (2008), we set the lower bound of a disaster to be 0.10, considering that a smaller drop down of consumption could be the result of diffusion shocks instead of jump. We calibrate the total disaster probability to 0.023 ($\bar{\lambda}_i + \bar{\lambda}_g$), which is conservative compared with empirical data in Barro and Ursúa (2008). We set $\bar{\lambda}_g/(\bar{\lambda}_g + \bar{\lambda}_i) = 22/23$, which is close to the counterpart in Du (2013) and implies that more than 95% of the disaster risk is shared among countries. We set $\eta_1^g = 9$ and therefore the average size of disaster is $0.10 + 1/9 = 0.211$, which is close to the empirical result in Barro and Ursúa (2008).

The speed of recovery is equal to 0.30, which is close to the value 0.302 in

Hasler and Marfe (2016). We set $\omega_c = -0.12$, which means that, the instantaneous consumption growth rate right after an average disaster is 0.0471. The jump propagation degree is set at -0.047 artificially, implying that the conditional expectation of domestic jump intensities will increase by around 0.99% right after the occurrence of a disaster. However, as the disasters are themselves rare, ω_i only causes an increase of 0.079% on the domestic jump probability on average.

Other parameters are mean reversions and volatilities of consumption volatility and jump intensities. We set $\phi_g = \phi_i$, and calibrate these parameters to match the empirical mean and volatility of risk-free rate, as well as the volatility of exchange rate.

5 Model implications

5.1 Basic moments

Table 2 reports the moments from both empirical data from U.S. and U.k. and theoretical value. The model well matches the mean, volatility and autocorrelation of risk free rate and exchange rate. The correlation of stochastic discount factor is 0.8430 in our model, which promises the smooth movements of exchange rate. The theoretical consumption growth is close to the data in level, but the volatility is higher than the data, mainly because disasters are seldom in our sample period. The correlation of interest rate between countries is too high in our model, due to the highly shared disasters. We can lower the correlation by decreasing the proportion of global disasters. However, it will cause an increase in the volatility of exchange rate growth. For similar reason, the cross-country

Table 1: **Calibration Parameters**

Parameter description	Symbol	Value
Preference		
Subjective discount rate	δ	0.975
Elasticity of intertemporal substitution	ψ	2
Coefficient of risk aversion	γ	8
Consumption parameters not related to disasters		
Long-run expected growth	μ_c	0.0218
Volatility of consumption growth without disaster	$\bar{\sigma}$	0.01
Reversion speed of consumption growth volatility	ϕ_σ	0.8
Volatility of of consumption growth volatility	ν_σ	0.008
Correlation of consumption diffusions between countries	ρ_c	0.2
Disasters and recovery		
Long-run intensity of global disaster	$\bar{\lambda}_g$	0.022
Long-run intensity of idiosyncratic disaster	$\bar{\lambda}_i$	0.001
Reversion speed of global disaster intensity	ϕ_g	0.9
Reversion speed of idiosyncratic disaster intensity	ϕ_i	$=\phi_g$
Volatility of global disaster intensity	σ_g	0.16
Volatility of idiosyncratic disaster intensity	σ_i	0.006
Reversion speed of recovery factor	ϕ_z	0.3
Consumption recovery coefficient	ω_c	-0.12
Jump propagation degree	ω_λ	-0.047
Minimum size of global disaster	η_0^g	0.1
Minimum size of idiosyncratic disaster	η_0^i	$=\eta_0^g$
Parameter for distribution of global disaster size	η_1^g	9
Parameter for distribution of idiosyncratic disaster size	η_1^i	$=\eta_1^g$

Table 2: **Empirical and theoretical moments**

Moment	US data	UK data	Theoretical
Correlation of SDF, corr$(d\log M_t, d\log M_t^*)$	-		0.8430
Average consumption growth	0.0171	0.0203	0.0188
volatility of consumption growth	0.0236	0.0329	0.0381
Autocorrelation of of consumption growth	0.1461	0.1156	0
Correlation of consumption growth (full sample)	0.6372		0.8750^{*}
Correlation of consumption growth (1955Q1-2007Q4)	0.1630		0.2^{**}
Correlation of consumption growth (2008Q1-2020Q4)	0.9125		-
Average risk-free interest rate	0.0117	0.0133	0.0123
Volatility of risk-free interest rate	0.0163	0.0230	0.0180
Autocorrelation of of risk-free interest rate	0.7245	0.8032	0.6616
Correlation of risk-free interest rate	0.6551		0.9988
Average exchange rate growth	-0.0131		0
Volatility of exchange rate growth	0.1128		0.1202
Autocorrelation of exchange rate growth	0.07		0

The table displays moments both empirical and theoretical. The means and variance are calculated using quarterly data and the results are annualized. Correlations are using annual data. (*) is the unconditional correlation of consumption growth, and (**) is the correlation conditional on no disasters.

correlation of consumption growth in the model is a little bit higher than in the data. Overall, the model fits the basic moments regarding consumption, interest rate and exchange rate.

5.2 The forward premium puzzle

From equation (12) we have

$$d\,rx_t^{FX} = r_t^* dt - r_u dt + d\log M_t^* - d\log M_t.,$$

Therefore

$$E_t[\frac{d\,rx_t^{FX}}{dt}] = K_x[Y_t^* - Y_t],$$
$$K_x = -l_1^\top \left[\varrho(-\Omega) - \mathbf{1}_{2\times 1} - \Omega^\top E(J_t)\right] - \frac{1}{2}\Omega^\top H\Omega.$$

Note that since $e^x - 1 - x \geq 0$, we have $\varrho(-\Omega) - \mathbf{1}_{2\times 1} - \Omega^\top E(J_t) \geq 0$, and $K_x \leq 0$ with strict inequality for at least three of the elements.

The UIP regression slope coefficient can be repressed as

$$\beta_{UIP} = \frac{Cov(E_t[\frac{d\,rx_t^{FX}}{dt}], r_t^* - r_t)}{Var(r_t^* - r_t)} = \beta_0^{rx} \leq 0,$$

as $\Phi_1 \leq 0$. Here β_0^{rx} is defined in equation (16). In our model, β_{UIP} is 1.871 based on calibration (see Figure 1), which is close to the counterpart in empirical literature (e.g., in Engel (2016), β_{UIP} is 1.850 for USD/GBP; in Chernov and Creal (2021), the number is 1.770 averaged across 4 currencies, including GBP, against USD with nominal data.).

What is the underlying mechanism that enables our model to explain the UIP puzzle? In K_x, the loadings on z_t is 0, and on the other three variables are negative (see Figure 3 for values at $\tau = 0$). However, as the global disaster is shared among countries, λ_t^g does not have an impact on the currency risk premium. Suppose a shock increases the domestic disaster probability, $\lambda_t^i > \lambda_t^{i,*}$, and let other variables be equal across countries. The higher domestic disaster risk results in decrease on domestic risk-free rate, due to more precautionary savings. At the same time, the higher disaster intensity gives rise to higher price of risk in home country. In this case, domestic disaster shocks dominates the effect of foreign disaster shocks in exchange rate. As risk-free assets in their own currency are riskless, the riskiness for currency return comes from the movement of exchange rate. Since the domestic price of risk is higher, Equation (13) implies that, the riskiness of the foreign risk-free asset for home investor is higher than the riskiness of the domestic risk-free asset for foreign asset. Hence the domestic investor expects an excess return for risk-bearing. The analysis above concludes that the expected currency risk premium is higher at times when $r_t^* - r_t > 0$, which reflects a positive β_{UIP}. A larger impact of λ_t^i on the price of risk relates to a larger UIP regression slope.

The mechanism for σ_t^2 to contribute to the UIP regression slope is similar to λ_t^i (see Figure 3). In other words, both diffusion risk and jump risk play important roles in determination of UIP regression slope. As for z_t, $B_r^{(z_t)}(0)$ is negative. A small z_t means higher expected consumption growth and decreases investor's willing to save, and therefore increases the interest rate. However, z_t does not contribute to the expected currency risk premium, as z_t is not in the price of risk. Therefore, any change in the spot interest rate differential will be cancelled out by the expected depreciation/appreciation of spot exchange rate.

Figure 1 displays the impact of recovery degree ω_c and jump propagation degree ω_i on the UIP regression slope. The top panel shows that, a large recovery degree will lower the UIP regression of spot currency return. When $\omega_c = 0.15$, β_{UIP}

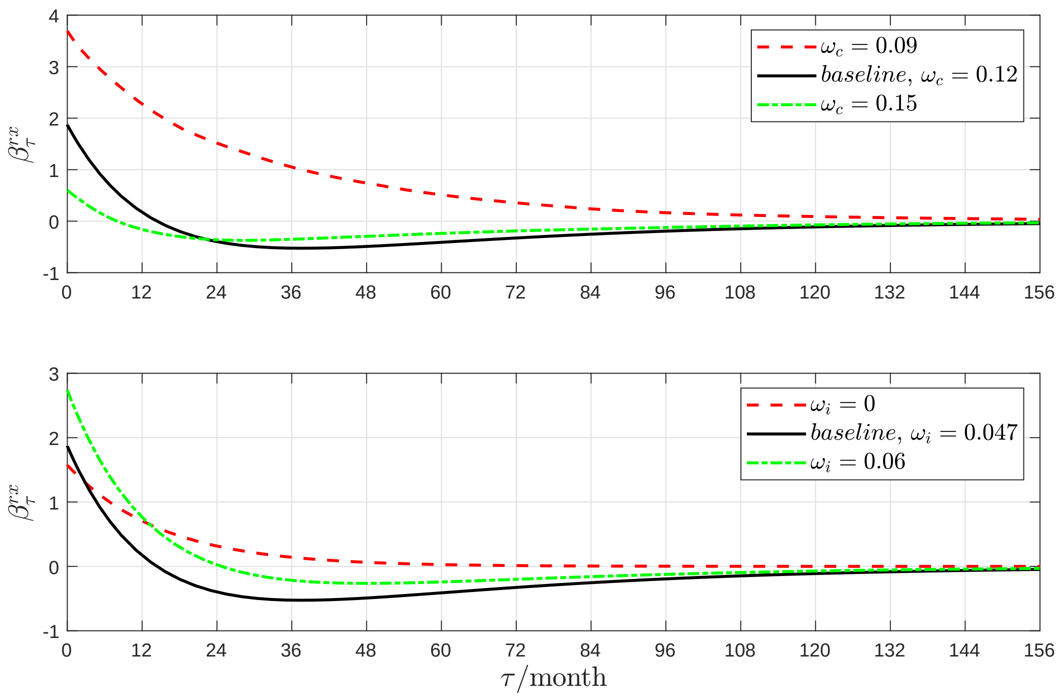

Figure 1: **Theoretical Slope of Ex ante Return Regression.** This figure plots the theoretical slope for regression of ex ante currency excess return on risk-free rate differential, as defined in equation (16). In the top panel, the solid line is the baseline calibration. The dashed line is with weaker recovery, and the dash-dotted line is with stronger recovery. In the bottom panel, the solid line is the baseline calibration. The dashed line is without jump propagation, and the dash-dotted line is with higher degree of jump propagation.

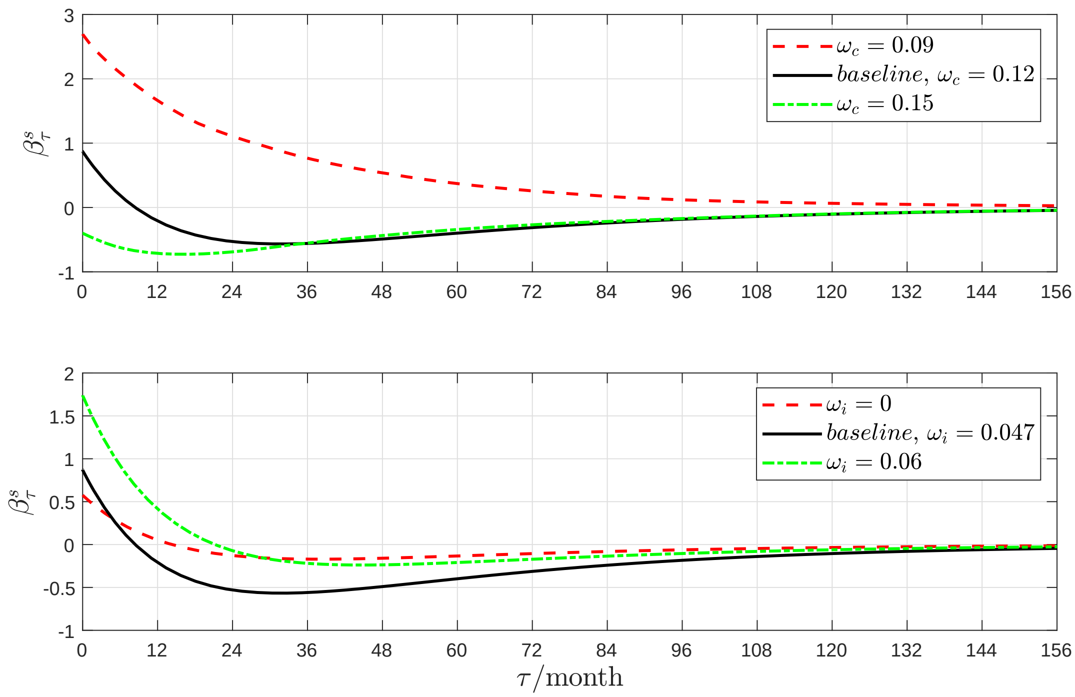

Figure 2: **Theoretical Slope of Ex ante Exchange Depreciation Regression.** This figure plots the theoretical slope for regression of ex ante exchange rate depreciation on risk-free rate differential, as defined in equation (17). In the top panel, the solid line is the baseline calibration. The dashed line is with weaker recovery, and the dash-dotted line is with stronger recovery. In the bottom panel, the solid line is the baseline calibration. The dashed line is without jump propagation, and the dash-dotted line is with higher degree of jump propagation.

is only 0.584. This result is intuitive, as recovery lessens jump risk. As showed in Figure 3, with larger recovery coefficient, the loading on λ_t^i for spot interest rate is smaller, which reflects less precautionary saving, as the investor expects stronger recoveries after possible disasters. It also shows that $\dot{B}_m^{(\lambda_t^i)}(0)$ becomes smaller for larger ω_c, which implies that, for a given $\lambda_t^i - \lambda_t^{i,*}$, the expected change in exchange rate is weaker, since λ_t^i contributes less to the price of risk.

The effects of jump propagation on β_{UIP} are twofold. First, it aggravates the impacts of a disaster, and therefore increase the loading on λ_t^i for both interest rate and exchange rate (Figure 4). Second, it increases the unconditional variance of λ_t^i, and hence its weight on determining β_{UIP}. Overall, a larger ω_i leads to larger β_{UIP} (see the bottom panel in Figure 1).

5.3 The multihorizon UIP regressions

In addition to the forward premium puzzle, can our model generates the pattern of multihorizon UIP regressions as in Engel (2016)? The solid curve in Figure 1 shows the theoretical multihorizon UIP regression slope coefficients define in Equation (16), which is the theoretical counterpart as in Engel's regression. The curve starts from a positive value (β_{UIP}) and then turns into negative at the horizon of around 13 months. In further horizons, it reaches the trough and then converges to 0 gradually. It catches main features with comparable values as in Engel (2016). If we run the following regression

$$E_t[\int_0^\infty d\,rx_\tau^{FX}] = \alpha_\infty + \beta_\infty(r_t^* - r_t) + u_t,$$

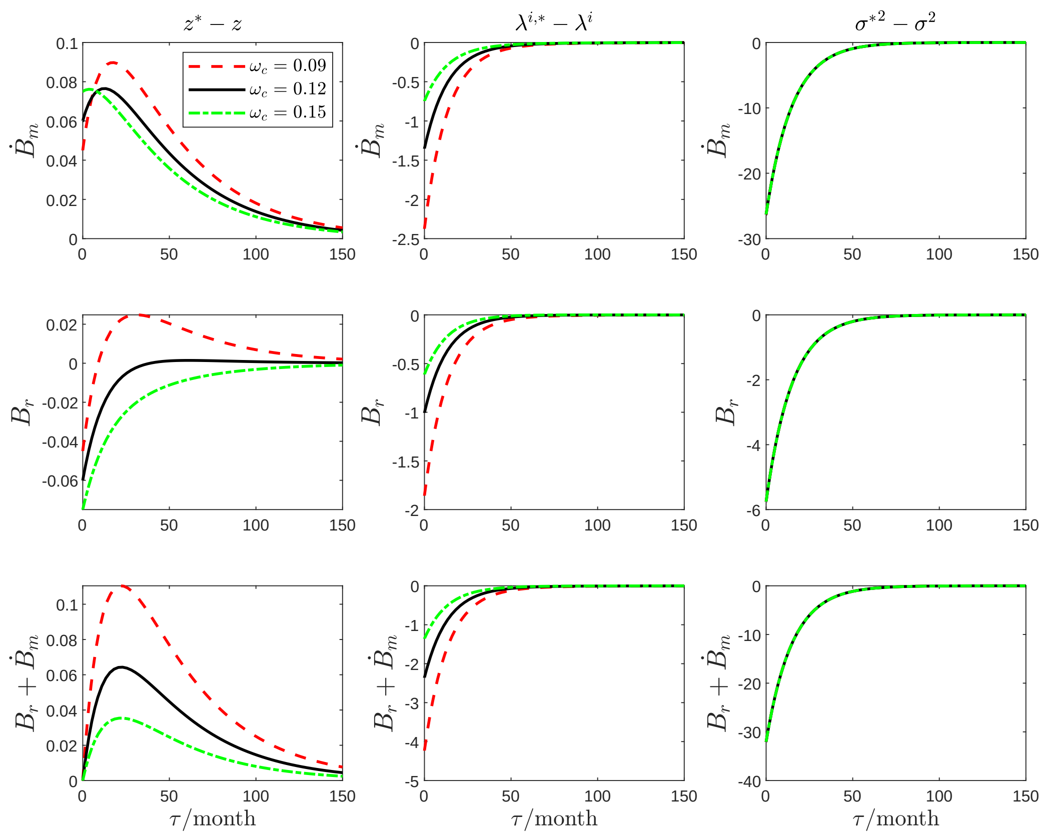

Figure 3: **Loadings with Different Recovery Degree.** The figure plots instantaneous exchange rate growth, interest rate differential, and currency risk premium loadings on the state variable differentials for different horizons and with different ω_c.

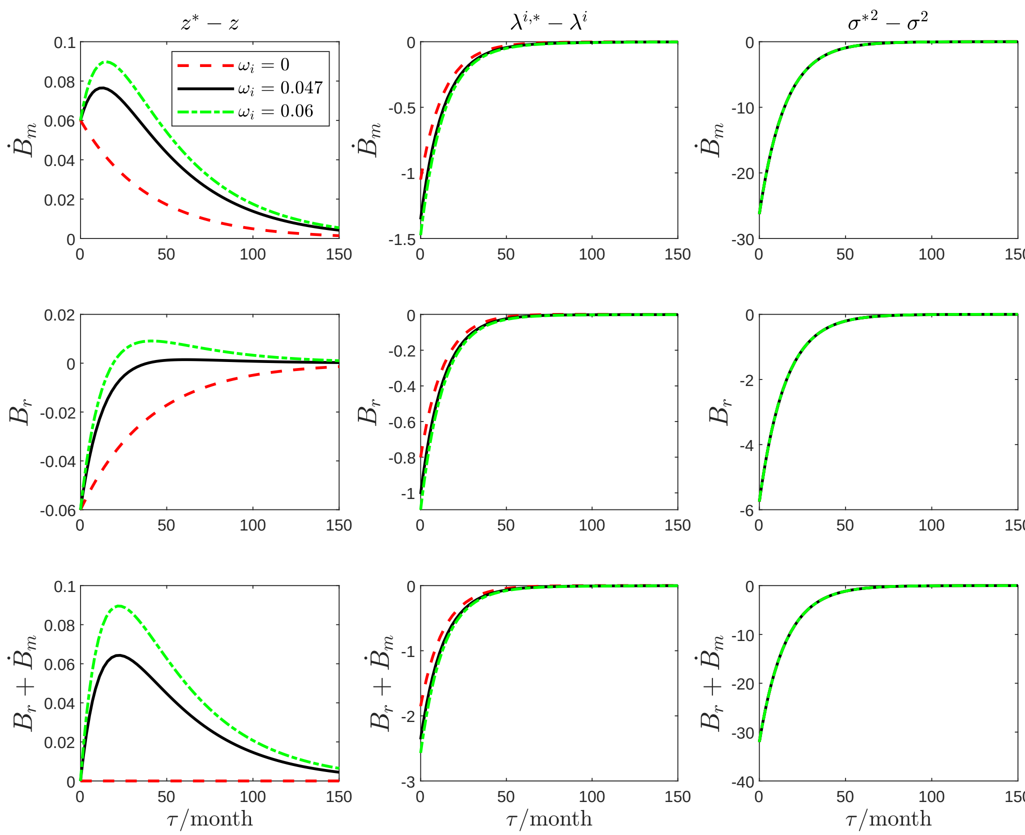

Figure 4: **Loadings with Different Degrees of Jump Propagation.** The figure plots instantaneous exchange rate growth, interest rate differential, and currency risk premium loadings on the state variable differentials for different horizons and with different ω_i.

the slope coefficient will be

$$\beta_\infty = \int_0^\infty \beta_\tau^{rx} d\tau.$$

In Engel (2016), $\beta_\infty = -30.890$ for G6, and -10.717 for United Kingdom with a 90% interval $(-27.130, 1.060)$. For individual countries in G6, the value ranges from -10.717 to -33.895. In our calibration, $\beta_\infty = -2.300$, which is transferred to -27.600 to compared with Engel (2016) (his dependent variable is constructed in monthly frequency). Therefore, our model not only catches the shape of the multihorizon UIP regression slope coefficient curve, but also matches its size. Negative β_∞ implies that high risk interest rate currency is less risky. Hence, our model provides a risk-based explanation for the puzzle proposed by Engel (2016).

What explains the model's ability to address Engel puzzle? The key is the feature of jump propagation. Even though z_t does not affect the spot currency risk premium, it determines expectation of future currency risk premium.

Suppose $z_t < z_t^*$, then $r_t > r_t^*$ as discussed before. The smaller z_t indicates that in the near future, the domestic disaster intensity is expected to be larger than foreign because of jump propagation. Based on the discussion in last section, a lower expectation of $\lambda_{t+\tau}^{i,*} - \lambda_{t+\tau}^i$ is related to higher expectation of exchange growth rate. In other words, $\dot{B}_m^{(z)}(\tau)$ is increasing for small τ, as showed in the top left panel of Figure 3. At the same time, the lower expectation $\lambda_{t+\tau}^{i,*} - \lambda_{t+\tau}^i$ increases the expected interest rate differential until even a reverse sign (see middle left panel in Figure 4. Though with $\omega_i = 0$, $B_r^{(z)}$ is already an increase function on τ due to mean-reversion of z_t, jump propagation accelerates the increase of $B_r^{(z)}$). The effects of $z_t^* - z_t$ on the expectation of exchange rate growth and interest rate differential add up to a hump-shape loading function on $z_t^* - z_t$ for currency risk premium (see bottom left panel in Figure 4).

Without jump propagation, the mean-reverting property of λ_t^i and σ_t^2 will lead to a monotone decreasing multihorizon UIP regression slopes, which never change signs, as the red dash curve in the bottom panel of Figure 1 shows. The slope coefficients therefore inherits both the monotone decreasing effects of λ_t^i and σ_t^2, and the (reversed) hump-shape effects of z_t.

Next we investigate how the degree of recovery and jump propagation affects the multihorizon currency risk premium. As mentioned before, recovery lessens the effect of λ_t^i. This is true for not only spot exchange rate. As a consequence, a larger ω_c generates a lower hump in the bottom left panel in Figure 3. As the currency risk premium loading on $\lambda_t^{i,*} - \lambda_t^i$ also getting smaller in level on lager ω_c, the resulting slope coefficient curve is flatter (see top panel in Figure 1). If the recovery degree is too small, however, the slope coefficient curve could become monotone, as the effect of $\lambda_t^{i,*} - \lambda_t^i$ is too strong and dominates the hump-shape effect of $z_t^* - z_t$. The effect of ω_i is not apparent. On the one hand, a larger ω_i results in a bigger hump of currency risk premium loading on $z_t^* - z_t$. On the other hand, it also increase both the loading on and the unconditional variance of $\lambda_t^{i,*} - \lambda_t^i$. The ultimate effect dependents on the relative strength of these two effects.

Figure 2 displays β_τ^s defined in Equation (17). Similar to Figure 1, the solid line starts at a positive value and then turns to negative. It coincides with the empirical results in Chernov and Creal (2021). It shows that the slope coefficient curve of multihorizon UIP regressions originates from the expectation of future exchange rate depreciations, instead of the expectation of future interest rate differentials.

5.4 Term Structure of Real Yields

As there are not perfect real bonds in real world, the empirical literature usually relies on inflation-indexed bonds or TIPS contracts to study the term structure of real yields. However, because of issues like sample size, liquidity or indexation lags, there are not consensus on the average term structure of the real yields. For example, the term structure is downward sloping using U.K. inflation-indexed bonds in Evans (1998), and it becomes flat when Verdelhan (2010) extends the results using Bank of England real yields and with a different sample period. Ang, Bekaert and Wei (2008) shows that the unconditional term structure of real yields is fairly flat, with a slight hump at 1-year maturity with U.S. data.

However, it is consensus that long term yields are less volatile than short term yields (Evans (1998), Evans (2003), Seppälä (2004) and Ang et al. (2008)). Besides, the real rate is pro-cyclical, and therefore, Ang et al. (2008) show that in regimes when interest rate is high, the yield term structure is downward-sloping, and when interest rate is low, it is upward sloping. In other words, the slope is counter-cyclical.

Table 3: **The Theoretical Bond Yields**

maturity	r	2 years	4 years	6 years	8 years	10 years
mean(%)	1.234	1.262	1.261	1.259	1.258	1.256
volatility(%)	1.799	0.825	0.482	0.330	0.248	0.199

Our model can reproduce these patterns. Table 3 presents the theoretical mean and volatility of bond yields for different maturities. It shows that the long term yields are highly stable, while the short term yields are volatile. This observation is also displayed in Figure 5. Besides, Figure 5 shows that the

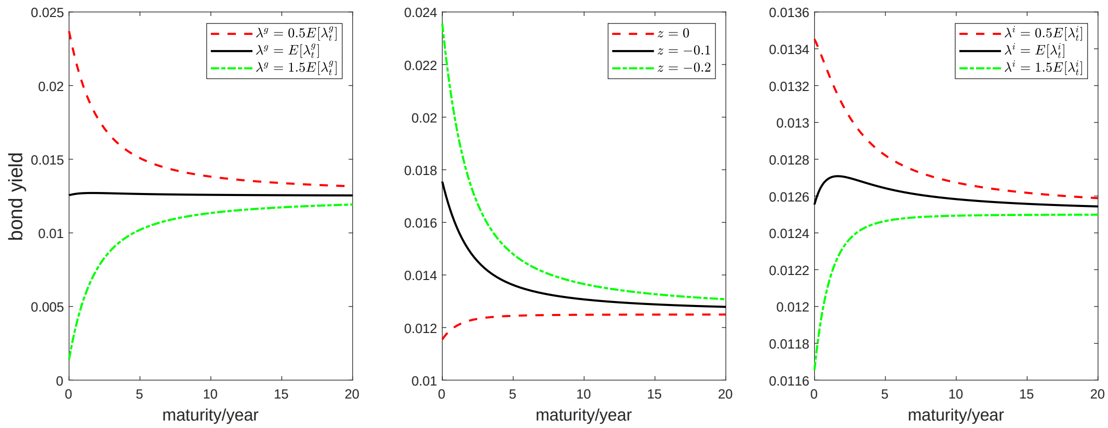

Figure 5: **The Term Structure of Bond Yields.** The figure plots the term structure of bond yields with state variables set at different levels.

unconditional term structure is flat with a slight hump (see the solid line in left and right panel) as in Ang et al. (2008). When z_t is small, which means that the economy is under recovery and can be interpreted as good times for the consumption growth rate is high, the term structure is downward sloping. When the disaster intensity is large, which can be interpreted as bad times, term structure is upward sloping. Hence the slope of real yields term structure is counter-cyclical in our model, consistent with Ang et al. (2008).

6 Conclusion remarks

Empirical research documents two puzzles that violates the UIP condition: the forward premium puzzle and the exchange rate level puzzle. The former implies that high interest rate currency is riskier and the latter implies the opposite. We propose a consumption-based general equilibrium models to reconcile the paradox.

The model features disaster recovery and jump propagation. The unshared part of disasters generates the forward premium puzzle, while jump propagation explains the multihorizon UIP pattern. Specifically, jump propagation produces negative correlation between interest rate differential and ex ante currency risk premium. When foreign country is under recovery, its interest rate is high. In the meanwhile, jump propagation implies that in the near future, foreign price of risk will dominate domestic in determining the exchange rate, and there foreign currency will be less risky to domestic investor. This channel drives the multihorizon UIP regression slope curve to reverse sign in the near future.

In addition to address the forward premium puzzle and Engel's puzzle, the model also accounts for the term structure of real bond yields, as well as some basic moments of the exchange rate growth and interest rate.

Appendix A The dynamics of Y_t

The matrixes which defines the dynamics of Y_t are given by

$$M = \begin{pmatrix} \mu_c \\ 0 \\ \phi_g \bar{\lambda}^g \\ \phi_i \bar{\lambda}^i \\ \phi_\sigma \bar{\sigma}^2 \end{pmatrix}, \quad K = \begin{pmatrix} 0 & \omega_c & 0 & 0 & 0 \\ 0 & -\phi_z & 0 & 0 & 0 \\ 0 & 0 & -\phi_g & 0 & 0 \\ 0 & \omega_i \phi_i & 0 & -\phi_i & 0 \\ 0 & 0 & 0 & 0 & -\phi_\sigma \end{pmatrix}, \quad H^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_i^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad H^5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu_\sigma^2 \end{pmatrix}, \quad H^1 = H^2 = h = \mathbf{0}_{5\times 5},$$

$$l_0 = \mathbf{0}_{2\times 1}, \quad l_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad J_g = J_i = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \end{pmatrix}^\top.$$

Appendix B Solve the price-consumption ratio

Define

$$Z_t = M_t R_{c,t}$$

Then

$$\begin{aligned}
d\log Z_t =& d\log M_t + d\log R_{c,t}\\
=&\theta\log\delta dt - \frac{\theta}{\psi}dc_t + \theta d\log R_{c,t}\\
=&\Big[\theta\big(\log\delta + k_0\big) - \theta(1-k_1)\nu_{c,t}\Big]dt + \left(\theta - \frac{\theta}{\psi}\right)dc_t + \theta k_1 B^\top dY_t\\
=&\Big[\theta\big(\log\delta + k_0\big) - \theta(1-k_1)(A + B^\top Y_t)\Big]dt + \chi^\top dY_t\\
=&\Big[\theta\big(\log\delta + k_0\big) - \theta(1-k_1)(A + B^\top Y_t) + \chi^\top(M + KY_t)\Big]dt\\
&+ \chi^\top\Sigma(Y_t)dW_t + \chi^\top J_t dN_t
\end{aligned}$$

where $\chi = \theta((1-1/\psi)\delta_c + k_1 B)$, $\delta_c = (1,0,0,0,0)^\top$. The first equality is by definition. The second equality uses equation(5). The third equality is based on substitution of $d\log R_{c,t}$ and rearrangement. The fourth equality uses the fact that $dc_t = \delta_c^\top dY_t$. The final line is straightforward.

An application of Ito-Doeblin Formula yields

$$\begin{aligned}
\frac{dZ_t}{Z_{t-}} =& \Big[\theta\big(\log\delta + k_0\big) - \theta(1-k_1)(A + B^\top Y_t) + \chi^\top(M + KY_t) + \frac{1}{2}\chi^\top\Sigma(Y_t)\Sigma(Y_t)^\top\chi\Big]dt\\
&+ \chi^\top\Sigma(Y_t)dW_t + \left(e^{\chi^\top J_g\xi_t^g} - 1\right)dN_t^g + \left(e^{\chi^\top J_i\xi_t^i} - 1\right)dN_t^i\\
=&\Big[\theta\big(\log\delta + k_0\big) - \theta(1-k_1)(A + B^\top Y_t) + \chi^\top(M + KY_t) + \frac{1}{2}\chi^\top\Sigma(Y_t)\Sigma(Y_t)^\top\chi\Big]dt\\
&+ \lambda_t^g\big[\varrho^g(\chi^\top J_g) - 1\big]dt + \lambda_t^i\big[\varrho^i(\chi^\top J_i) - 1\big]dt + \chi^\top\Sigma(Y_t)dW_t\\
&+ \left[\left(e^{\chi^\top J_g\xi_t^g} - 1\right)dN_t^g - \lambda_t^g\big[\varrho^g(\chi^\top J_g) - 1\big]dt\right]\\
&+ \left[\left(e^{\chi^\top J_i\xi_t^i} - 1\right)dN_t^i - \lambda_t^i\big[\varrho^i(\chi^\top J_i) - 1\big]dt\right]
\end{aligned}$$

The Euler equation (6) implies that Z_t is martingale, therefore its drift is 0, which yields equation (7) by collecting terms in drift. Combining equation (4) and (7), we can solve k_0, k_1, A and B either systematically or iteratively.

Appendix C Variables dynamics under the risk neutral measure

Define

$$\tilde{h}_t = (e^{-\Omega^\top J_g \xi_t^g}, e^{-\Omega^\top J_i \xi_t^i})^\top ./\varrho(-\Omega),$$

where $./$ denotes element-by-element division. $\varrho(\cdot)$ is defined by (8). Note that $E(\tilde{h}_t) = \mathbf{1}_{2\times 1}$ and $E(\varrho(-\Omega)\cdot\tilde{h}_t) = \varrho(-\Omega)$.

Let

$$\begin{aligned} L_t = & \exp\left\{\int_0^t \left[l_s^\top\left(\mathbf{1}_{2\times 1} - \varrho(-\Omega)\cdot E(\tilde{h}_s)\right) - \frac{1}{2}\Lambda_s^{c\top}\Lambda_s^c\right] ds - \int_0^t \Lambda_s^{c\top} dW_s\right\} \\ & \times \exp\left\{\int_0^t \log\left[\varrho(-\Omega)\cdot\tilde{h}_s\right] dN_s\right\} \end{aligned}$$

with $\cdot$ denoting element-by-element multiplication[1]. Then by Ito-Doeblin formula we have

$$\begin{aligned} \frac{dL_t}{L_{t-}} &= -\Lambda_t^{c\top} dW_t + \left[\varrho(-\Omega)\cdot\tilde{h}_t - \mathbf{1}_{2\times 1}\right] dN_t - l_t^\top\left[\varrho(-\Omega)\cdot E(\tilde{h}_t) - \mathbf{1}_{2\times 1}\right] dt, \\ &= \frac{dM_t}{M_{t-}} + r_t dt. \end{aligned}$$

Therefore L_t is a P-local martingale and it is non-negative by definition. Define an equivalent probability measure Q by $dQ/dP|\mathcal{F}_t = L_t/L_0$. We can easily show that, for any asset payment P_T on time T, its price conditional on the information

[1] The definition of L_t is in the sense of the more general Radon-Nikodym derivatives in Runggaldier (2003)

up to time t, with $t < T$, satisfies the following equation.

$$P_t = E_t\left[\frac{M_T}{M_t}P_T\right] = E_t^Q\left[\frac{M_T L_t}{M_t L_T}P_T\right] = E_t^Q\left[e^{-\int_t^T r_s ds}P_T\right].$$

In other words, the time t price is the expectation of the discounted time T price under measure Q. Therefore Q is actually the risk-neutral probability measure.

Let

$$W_t^Q = W_t + \int_0^t \Lambda_s^c ds,$$
$$\tilde{M}_t^Q = N_t - \int_0^t l_s \cdot \varrho(-\Omega) ds.$$

We can show that $L_t W_t^Q$ and $L_t \tilde{M}_t^Q$ are P-local martingales by mimicking the proof of Lemma 2 and Lemma 3 in Duffie et al. (2000). Hence we conclude that W_t^Q and $\tilde{M}_t^Q$ are Q-local martingales. It follows that N_t is a vector Piosson process with intensity $l_t^Q = l_t \cdot \varrho(-\Omega)$. Next we will solve the distribution of jump sizes under measure Q. Define

$$\tilde{M}_t^{c,Q} = \xi_t \cdot N_t - \int_0^t l_s^Q \cdot E^Q[\xi_s] ds,$$

where $\xi_t = (\xi_t^g, \xi_t^i)^\top$. It is clear that $\tilde{M}_t^{c,Q}$ is a 2×1 vector of compensated compound Poisson process under measure Q, so it is Q-local martingale (see, for example, Theorem 11.3.1 in Shreve (2004)). $L_t \tilde{M}_t^{c,Q}$ is thus P-local martingale. By Ito-

Doeblin formula, under measure P, we obtain the dynamics of $L_t\tilde{M}_t^{c,Q}$ as

$$
\begin{aligned}
d\left[L_t\tilde{M}_t^{c,Q}\right] =& \tilde{M}_{t-}^{c,Q} dL_t + L_{t-} d\tilde{M}_t^{c,Q} + L_{t-}\left[\varrho(-\Omega)\cdot\tilde{h}_t - \mathbf{1}_{2\times 1}\right]\cdot\xi_t\cdot dN_t \\
=& \tilde{M}_{t-}^{c,Q} dL_t + L_{t-}\xi_t\cdot dN_t - L_{t-}l_t^Q\cdot E^Q[\xi_t]dt + L_{t-}\left[\varrho(-\Omega)\cdot\tilde{h}_t - \mathbf{1}_{2\times 1}\right]\cdot\xi_t\cdot dN_t \\
=& \tilde{M}_{t-}^{c,Q} dL_t + L_{t-}\varrho(-\Omega)\cdot\tilde{h}_t\cdot\xi_t\cdot dN_t - L_{t-}l_t^Q\cdot E^Q[\xi_t]dt \\
=& \tilde{M}_{t-}^{c,Q} dL_t + L_{t-}\varrho(-\Omega)\cdot\tilde{h}_t\cdot\xi_t\cdot dN_t - L_{t-}l_t\cdot\varrho(-\Omega)\cdot E[\tilde{h}_t\cdot\xi_t]dt \\
& + L_{t-}l_t\cdot\varrho(-\Omega)\cdot E[\tilde{h}_t\cdot\xi_t]dt - L_{t-}l_t\cdot\varrho(-\Omega)\cdot E^Q[\xi_t]dt
\end{aligned}
$$

Ito-Doeblin formula yields the first equality. Substitution of $d\tilde{M}_t^{c,Q}$ generates the second. Collecting terms relative to dN_t results in the third. The last equality is constructing compensated process for dN_t term, and uses $l_t^Q = l_t\cdot\varrho(-\Omega)$. For $L_t\tilde{M}_t^{c,Q}$ to be P-local martingale, it should hold that

$$
L_{t-}l_t\cdot\varrho(-\Omega)\cdot E[\tilde{h}_t\cdot\xi_t]dt - L_{t-}l_t\cdot\varrho(-\Omega)\cdot E^Q[\xi_t]dt = 0.
$$

Therefore

$$
E^Q[\xi_t] = E[\tilde{h}_t\cdot\xi_t].
$$

We thus conclude that the PDF of ξ_t under measure Q is

$$
\begin{aligned}
f^Q(\xi_t^g;\eta_0^g,\eta_1^g) = \tilde{h}_t^{(1)} f(\xi_t^g;\eta_0^g,\eta_1^g) = \frac{e^{-\Omega^\top J_g\xi_t^g}}{E\left[e^{-\Omega^\top J_g\xi_t^g}\right]} f(\xi_t^g;\eta_0^g,\eta_1^g), \\
f^Q(\xi_t^i;\eta_0^i,\eta_1^i) = \tilde{h}_t^{(2)} f(\xi_t^i;\eta_0^i,\eta_1^i) = \frac{e^{-\Omega^\top J_i\xi_t^i}}{E\left[e^{-\Omega^\top J_i\xi_t^i}\right]} f(\xi_t^i;\eta_0^i,\eta_1^i).
\end{aligned}
$$

The MGFs of jump sizes under measure Q are hence

$$
\begin{aligned}
\varrho^{g,Q}(u) =& E_t^Q(e^{u\xi_t^g}) = \int e^{u\xi_s^g} \frac{e^{-\Omega^\top J_g \xi_s^g}}{E\left[e^{-\Omega^\top J_g \xi_s^g}\right]} f(\xi_s^g; \eta_0^g, \eta_1^g) ds \\
=& \frac{1}{E\left[e^{-\Omega^\top J_g \xi_s^g}\right]} \int e^{\left(u-\Omega^\top J_g\right)\xi_s^g} f(\xi_s^g; \eta_0^g, \eta_1^g) ds \\
=& \varrho^g(u - \Omega^\top J_g)/\varrho^g(-\Omega^\top J_g), \\
\varrho^{i,Q}(u) =& \varrho^i(u - \Omega^\top J_i)/\varrho^i(-\Omega^\top J_i).
\end{aligned}
$$

With the results above, we can immediately obtain the dynamics of Y_t under measure Q as in the main text.

Appendix D The expected currency risk premium

Note that $\log M_t$ has an affine structure on Y_t. Let $Y_t^e = (Y_t^\top, \log M_t)^\top$ be the extension of Y_t to include $\log M_t^\top$, then Y_t^e is a 6×1 vector.

$$
\begin{aligned}
dY_t^e &= \mu^e(Y_t^e)dt + \Sigma^e(Y_t^e)dW_t + J_t^e dN_t, \\
\mu^e(Y_t^e) &= M^e + K^e Y_t^e, \\
\Sigma^e(Y_t^e)\Sigma^e(Y_t^e)^\top &= h^e + \sum_{i=1}^{6} H^{e,i} Y_t^{e,i}, \\
J_t^e &= \begin{pmatrix} J_g^e \xi_t^g & J_i^e \xi_t^i \end{pmatrix}, \\
l^e(Y_t^e) \equiv \lambda_t &= l_0^e + l_1^e Y_t^e.
\end{aligned}
$$

where

$$
M^e = \begin{pmatrix} M \\ M_0 \end{pmatrix}, \quad K^e = \begin{pmatrix} K & \mathbf{0}_{5\times 1} \\ K_0 & 0_{1\times 1} \end{pmatrix}, \quad H^{e,i} = \begin{pmatrix} H^i & H^i\Omega \\ \Omega^\top {H^i}^\top & \Omega^\top H^i \Omega \end{pmatrix} \quad \forall i = 1, ..., 5,
$$

$$h^e = \begin{pmatrix} h & h\Omega \\ \Omega^\top h^\top & \Omega^\top h \Omega \end{pmatrix}, \quad H^{e,6} = \mathbf{0}_{6\times 6},$$

$$l_0^e = l_0, \quad l_1^e = \begin{pmatrix} l_1 & \mathbf{0}_{2\times 1} \end{pmatrix}, \quad J_g^e = \begin{pmatrix} J_g \\ -\Omega^\top J_g \end{pmatrix}, \quad J_i^e = \begin{pmatrix} J_i \\ -\Omega^\top J_i \end{pmatrix}.$$

Based on Duffie et al. (2000), we have

Lemma 1. *The expectation of* $\log M_T$*, conditional on time* t *information, is given by*

$$E_t(\log M_T) = A_m^e(\tau) + {B_m^e}^\top(\tau) Y_t^e, \tag{18}$$

where $\tau = T - t$*, and* $A_m^e(\tau)$ *and* $B_m^e(\tau)$ *solve the following ODEs.*

$$\begin{aligned} \frac{dB_m^e(\tau)}{d\tau} &= K^{e\top} B_m^e(\tau) + l_1^{e\top} E[J_t^{e\top} B_m^e(\tau)], \\ \frac{dA_m^e(\tau)}{d\tau} &= M^{e\top} B_m^e(\tau) + l_0^{e\top} E[J_t^{e\top} B_m^e(\tau)]. \end{aligned}$$

with boundary conditions $A_m^e(0) = 0$ *and* $B_m^e(0) = (0, 0, 0, 0, 0, 1)^\top$*.*

Proof: If we apply Ito-Deoblin formula on $E_t(\log M_T)$ in equation (18), from the resulting SDE we can see that $E_t(\log M_T)$ is a martingale when the above ODEs holds, which proves the Lemma. $\square$

Note that the last element of $B_m^e(\tau)$ is always 1. Let $A_m(\tau) = A_m^e(\tau)$, $B_m(\tau) = B_m^e(\tau)[1:5]$, where $B_m^e(\tau)[1:5]$ is the first five entries of $B_m^e(\tau)$, then

$$\begin{aligned} E_t[s_{t+\tau}] - s_t &= E_t(\log M_T^*) - E_t(\log M_T) - (\log M_t^* - \log M_t) \\ &= \left({B_m^e}^\top(\tau) - {B_m^e}^\top(0)\right)\left(Y_t^e - Y_t^{e,*}\right) = B_m^\top(\tau)(Y_t^* - Y_t). \end{aligned}$$

Similarly, the expectation of the risk-free rate is

$$E_t(r_T) = A_r(\tau) + B_r^\top(\tau) Y_t,$$

where $A_r(\tau)$ and $B_r(\tau)$ solve the following ODEs,

$$\begin{aligned}
\frac{dB_r(\tau)}{d\tau} &= K^\top B_r(\tau) + l_1{}^\top E^\xi[J_t^\top B_r(\tau)], \\
\frac{dA_r(\tau)}{d\tau} &= M^\top B_r(\tau) + l_0{}^\top E^\xi[J_t^\top B_r(\tau)].
\end{aligned}$$

with initial conditions $A_r(0) = \Phi_0$ and $B_r(0) = \Phi_1$.

Appendix E The stable state unconditional moments of the state variables

The unconditional mean and variance of the state variables crucial for the model to provide empirical implications. Some of the setup of the state variables in this model are trivial while some are not. Therefore here we provide the methodology to calculate the moments concerned. By 'stable state', we mean that the processes are assumed to start infinitely long time ago.

Both the volatility of consumption growth rate and the global jump intensity are standard CIR processes. Based on Cox et al. (1985), the stable state mean and variance are given by

$$E[\sigma_\infty^2] = \bar{\sigma}^2, \quad Var[\sigma_\infty^2] = \frac{\nu_\sigma^2 \bar{\sigma}^2}{2\phi_\sigma}, \quad E[\lambda_\infty^g] = \bar{\lambda}^g, \quad Var[\lambda_\infty^g] = \frac{\sigma_g^2 \bar{\sigma}^g}{2\phi_g}. \tag{19}$$

And as σ_t^2 and λ_t^g are uncorrelated to each other and any other variables, the correlation coefficients between them and with any other variables are 0.

Appendix E.1 The unconditional moments of z_t

Das (2002) calculate the moments of a jump-diffusion CIR process using characteristic function. In our model, the recovery factor z_t has time-varying jump intensities, and λ_t^i and z_t are inter-dependent, which makes it impossible to solve the characteristic function or moment generating function in closed-form. Even so, our method relies on moment generating function, but in a more circuitous way.

Recall that z_t follows the SDE in (1). The solution for z_t is

$$
\begin{aligned}
z_t &= z_0 e^{-\phi_z t} + \sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{-\phi_z (t-\tau_n)} + \sum_{m=1}^{N_t^g} \xi_{\tau_m}^g e^{-\phi_z (t-\tau_m)}, \\
&= e^{-\phi_z t} \left(z_0 + \sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{\phi_z \tau_n} + \sum_{m=1}^{N_t^g} \xi_{\tau_m}^g e^{\phi_z \tau_m} \right), \\
&= e^{-\phi_z t} \left(z_0 + \int_0^t \xi_s^i e^{\phi_z s} dN_s^i + \int_0^t \xi_s^g e^{\phi_z s} dN_t^g \right),
\end{aligned}
\tag{20}
$$

where τ_n is the arrival time of the nth jump. τ_n, $\tau_m \leq t$ as $n \leq N_t^i$ and $m \leq N_t^g$. The following lemma describes the characteristic function for the first summations in z_t. Note that the results for summations across N_t^g are similar.

Lemma 2. *Let*

$$
y_t = \sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{\phi_z \tau_n}
$$

The moment generating function (MGF) of y_t is given by

$$
M_{y,t}(u) \equiv E[e^{u y_t}] = E\left[\exp\left(\int_0^t \lambda_s^i \left[\varrho^i \left(u e^{\phi_z s} \right) - 1 \right] ds \right) \right],
$$

where $\varrho^i(\cdot)$ is the MGF of ξ_t^i.

Proof: The expression for MGF can be obtained by the method of conditioning. See, for example, Léveillé and Hamel (2018).

$\square$

Therefore the first and second moments of y_t can be calculated as

$$
\begin{aligned}
E[y_t] = \frac{dM_{y,t}(u)}{du}\Big|_{u=0} =& E\left[\int_0^t \lambda_s^i E[\xi^i] e^{\phi_z s} ds\right] = \int_0^t E[\lambda_s^i] E[\xi^i] e^{\phi_z s} ds, \\
E[y_t^2] = \frac{d^2M_{y,t}(u)}{du^2}\Big|_{u=0} =& E\left[\left(\int_0^t \lambda_s^i E[\xi^i] e^{\phi_z s} ds\right)^2\right] + E\left[\int_0^t \lambda_s^i E[{\xi^i}^2] e^{2\phi_z s} ds\right]
\end{aligned}
$$

where we use $\varrho^i(0) = 1$, $\frac{d\varrho^i(u)}{du}\Big|_{u=0} = E[\xi^i]$ and $\frac{d^2\varrho^i(u)}{du^2}\Big|_{u=0} = E[{\xi^i}^2]$. The first term in the second moment is

$$
\begin{aligned}
E\left[\left(\int_0^t \lambda_s^i E[\xi^i] e^{\phi_z s} ds\right)^2\right] =& E\left[\int_0^t \int_0^t \lambda_s^i \lambda_\tau^i E^2[\xi^i] e^{\phi_z (s+\tau)} ds d\tau\right] \\
=& \int_0^t \int_0^t E\left[\lambda_s^i \lambda_\tau^i\right] E^2[\xi^i] e^{\phi_z (s+\tau)} ds d\tau \\
=& E^2[\xi^i] \int_0^t \int_0^t \left(Cov[\lambda_s^i, \lambda_\tau^i] + E[\lambda_s^i] E[\lambda_\tau^i]\right) e^{\phi_z (s+\tau)} ds d\tau \\
=& E^2[\xi^i] \int_0^t \int_0^t Cov[\lambda_s^i, \lambda_\tau^i] e^{\phi_z (s+\tau)} ds d\tau + E^2[y_t]
\end{aligned}
$$

Hence, the variance of $e^{-\phi_z t} y_t$ is given by

$$
\begin{aligned}
Var[e^{-\phi_z t} y_t] =& e^{-2\phi_z t}\left(E[y_t^2] - E^2[y_t]\right) \\
=& e^{-2\phi_z t} E^2[\xi^i] \int_0^t \int_0^t Cov[\lambda_s^i, \lambda_\tau^i] e^{\phi_z (s+\tau)} ds d\tau + e^{-2\phi_z t} E\left[\int_0^t \lambda_s^i E[{\xi^i}^2] e^{2\phi_z s} ds\right] \\
=& 2e^{-2\phi_z t} E^2[\xi^i] \int_0^t \int_0^\tau Cov[\lambda_s^i, \lambda_\tau^i] e^{\phi_z (s+\tau)} ds d\tau + e^{-2\phi_z t} \int_0^t E[\lambda_s^i] E[{\xi^i}^2] e^{2\phi_z s} ds.
\end{aligned}
$$

Assuming the process starts infinitely long time ago, i.e., letting t goes to infinity, we obtain

$$\lim_{t\to\infty}\frac{2E^2[\xi^i]\int_0^t\int_0^\tau Cov[\lambda_s^i,\lambda_\tau^i]e^{\phi_z(s+\tau)}dsd\tau}{e^{2\phi_z t}}=\lim_{t\to\infty}\frac{2E^2[\xi^i]\int_0^t Cov[\lambda_s^i,\lambda_t^i]e^{\phi_z(s+t)}ds}{2\phi_z e^{2\phi_z t}}$$
$$=\lim_{t\to\infty}\frac{E^2[\xi^i]\int_0^t Cov[\lambda_s^i,\lambda_t^i]e^{\phi_z s}ds}{\phi_z e^{\phi_z t}}=\frac{E^2[\xi^i]Var[\lambda_\infty^i]}{\phi_z^2},$$

and

$$\lim_{t\to\infty}\frac{\int_0^t E[\lambda_s^i]E[{\xi^i}^2]e^{2\phi_z s}ds}{e^{2\phi_z t}}=\frac{E[\lambda_\infty^i]E[{\xi^i}^2]}{2\phi_z}$$

by L'Hopital's rule. So

$$\lim_{t\to\infty}Var[e^{-\phi_z t}y_t]=\frac{E^2[\xi^i]Var[\lambda_\infty^i]}{\phi_z^2}+\frac{E[\lambda_\infty^i]E[{\xi^i}^2]}{2\phi_z}.$$

In like fashion, we have

$$\lim_{t\to\infty}E[e^{-\phi_z t}y_t]=\frac{E[\lambda_\infty^i]E[\xi^i]}{\phi_z}.$$

Similar results can be obtain for the second summation across N_t^g in (20). Therefore, as N_t^i and N_t^g are independent, we have

$$E[z_\infty]=\frac{E[\lambda_\infty^i]E[\xi^i]}{\phi_z}+\frac{E[\lambda_\infty^g]E[\xi^g]}{\phi_z},\tag{21}$$

and

$$Var[z_\infty]=E^2[\xi^i]\frac{Var\,[\lambda_\infty^i]}{\phi_z^2}+E[{\xi^i}^2]\frac{E[\lambda_\infty^i]}{2\phi_z}+E^2[\xi^g]\frac{Var\,[\lambda_\infty^g]}{\phi_z^2}+E[{\xi^g}^2]\frac{E[\lambda_\infty^g]}{2\phi_z},\tag{22}$$

where the first two moments for ξ^i and ξ^g are given in the main text, $Var\,[\lambda_\infty^g]$ and $E[\lambda_\infty^g]$ are in equation (19). $Var\,[\lambda_\infty^i]$ and $E[\lambda_\infty^i]$ are left to be calculated in next section.

Appendix E.2 The stable state unconditional moments of λ_t^i

It can be easily verified that the SDE for λ_t^i in (2) is equivalent to the following equation

$$
\begin{aligned}
\lambda_t^i =& \lambda_0^i e^{-\phi_i t} + \bar{\lambda}^i \left(1 - e^{-\phi_i t}\right) + \int_0^t e^{-\phi_i (t-s)} \sigma_i \sqrt{\lambda_s^i}\, dW_{i,s} \\
&+ \frac{\phi_i \omega_\lambda}{\phi_i - \phi_z} e^{-\phi_z t} \left[z_0 + \sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{\phi_z \tau_n} + \sum_{m=1}^{N_t^g} \xi_{\tau_m}^g e^{\phi_z \tau_m} \right] \\
&- \frac{\phi_i \omega_\lambda}{\phi_i - \phi_z} e^{-\phi_i t} \left[\sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{\phi_i \tau_n} + \sum_{m=1}^{N_t^g} \xi_{\tau_m}^g e^{\phi_i \tau_m} \right] \\
=& \lambda_0^i e^{-\phi_i t} + \bar{\lambda}^i \left(1 - e^{-\phi_i t}\right) + \int_0^t e^{-\phi_i (t-s)} \sigma_i \sqrt{\lambda_s^i}\, dW_{i,s} \\
&+ \frac{\omega_\lambda \phi_i}{\phi_i - \phi_z} (z_t - w_t^i) + \frac{\phi_i \omega_\lambda}{\phi_i - \phi_z} e^{-\phi_i t} w_0^i.
\end{aligned}
\tag{23}
$$

Here w_t^i is is a stochastic process with the same SDE as z_t, but with a different mean-reverting rate ϕ_i,

$$dw_t^i = -\phi_i w_t^i dt + \xi_t^i dN_t^i + \xi_t^g dN_t^g.$$

So we calculate its stable state mean and variance by the same method for z_t.

$$E[w_\infty^i] = \frac{E[\lambda_\infty^i] E[\xi^i]}{\phi_i} + \frac{E[\lambda_\infty^g] E[\xi^g]}{\phi_i}, \tag{24}$$

$$Var[w_\infty^i] = E^2[\xi^i] \frac{Var\left[\lambda_\infty^i\right]}{\phi_i^2} + E[\xi^{i2}] \frac{E[\lambda_\infty^i]}{2\phi_i} + E^2[\xi^g] \frac{Var\left[\lambda_\infty^g\right]}{\phi_i^2} + E[\xi^{g2}] \frac{E[\lambda_\infty^g]}{2\phi_i}. \tag{25}$$

If we let

$$y_{w,t} = \sum_{n=1}^{N_t^i} \xi_{\tau_n}^i e^{\phi_i \tau_n},$$

the joint moment generating function (MGF) of y_t and $y_{w,t}$ is

$$M_{y,y_w,t}(u,v) \equiv E[e^{uy_t+vy_{w,t}}] = E\left[\exp\left(\int_0^t \lambda_s^i \left[\varrho^i\left(ue^{\phi_z s}+ve^{\phi_i s}\right)-1\right] ds\right)\right].$$

Following the same procedure as calculation for z_t, we obtain the stable state covariance from the joint MGF as

$$Cov[z_\infty, w_\infty^i] = E^2[\xi^i]\frac{Var\left[\lambda_\infty^i\right]}{\phi_z\phi_i} + E[{\xi^i}^2]\frac{E[\lambda_\infty^i]}{\phi_z+\phi_i} + E^2[\xi^g]\frac{Var\left[\lambda_\infty^g\right]}{\phi_z\phi_i} + E[{\xi^g}^2]\frac{E[\lambda_\infty^g]}{\phi_z+\phi_i}. \tag{26}$$

From Equation (23), we have

$$E[\lambda_\infty^i] = \bar{\lambda}^i + \frac{\omega_\lambda\phi_i}{\phi_i-\phi_z}(E[z_\infty]-E[w_\infty^i]), \tag{27}$$

and

$$Var[\lambda_\infty^i] = \frac{\sigma_i^2 E[\lambda_\infty^i]}{2\phi_i} + \left(\frac{\omega_\lambda\phi_i}{\phi_i-\phi_z}\right)^2\left(Var[z_\infty]+Var[w_\infty^i]-2Cov[z_\infty,w_\infty^i]\right). \tag{28}$$

Combining Equation (21), (24) and (27) solves

$$\begin{aligned} E[z_\infty] =& \frac{E[\xi^i]\bar{\lambda}^i + E[\xi^g]\bar{\lambda}^g}{\phi_z-\omega_\lambda E[\xi^i]}, \\ E[\lambda_\infty^i] =& \bar{\lambda}^i + \omega_\lambda E[z_\infty]. \end{aligned} \tag{29}$$

Note that in the model we assume $E[\xi^i] < 0$, $E[\xi^g] < 0$ and $\omega_\lambda < 0$. When $|\omega_\lambda|$ is a small number, so that

$$\omega_\lambda E[\xi^i] < \phi_z,$$

we have $E[\lambda_\infty^i] > \bar{\lambda}^i$.

Combining Equation (22), (25), (26) and (28) solves

$$\begin{aligned} Var[\lambda_\infty^i] =& A_i + \omega_\lambda^2 C_1 + C_2, \\ Var[z_\infty] =& a + C_1 + B_i C_2. \end{aligned} \tag{30}$$

where

$$\begin{aligned} a =& E[\xi^{i2}]\frac{E[\lambda_\infty^i]}{2\phi_z} + E[\xi^{g2}]\frac{E[\lambda_\infty^g]}{2\phi_z}, \\ A_i =& \frac{\sigma_i^2 E[\lambda_\infty^i]}{2\phi_i}, \quad A_g = \frac{\sigma_g^2 E[\lambda_\infty^g]}{2\phi_g}, \quad B_i = \frac{E^2[\xi^i]}{\phi_z^2}, \quad B_g = \frac{E^2[\xi^g]}{\phi_z^2}, \\ C_1 =& \frac{B_i A_i + B_g A_g}{1-\omega_\lambda^2 B_i}, \quad C_2 = \frac{\omega_i^2 \phi_i a}{(1-\omega_\lambda^2 B_1)(\phi_i+\phi_z)}. \end{aligned}$$

Appendix E.3 The unconditional covariance between z_t and jump intensities

From (23), we know that

$$Cov(z_\infty, \lambda_\infty^i) = \frac{\omega_\lambda \phi_i}{\phi_i - \phi_z}\left(Var[z_\infty] - Cov[z_\infty, w_\infty^i]\right).$$

Appendix F The stable state covariance matrix of $Y_t - Y_t^*$

As both the exchange rate and difference of risk-free rate are affine function on $Y_t - Y_t^*$, the covariance matrix of $Y_t - Y_t^*$ is crucial for the calculation of theoretical UIP regression coefficient. As the coefficient on c_t is 0 for both risk-free rate and exchange rate, here we only calculate the covariance matrix for the other variables in $Y_t - Y_t^*$, except $c_t - c_t^*$.

a. Variance and covariance of $\lambda_\infty^g - \lambda_\infty^{g,*}$

λ_t^g and $\lambda_t^{g,*}$ are identity, so the variance of $\lambda_t^g - \lambda_t^{g,*}$ and the covariance with other variables in $Y_t - Y_t^*$ are 0.

b. Variance and covariance of $\sigma_\infty^2 - {\sigma_\infty^*}^2$

As σ_t^2 is uncorrelated to other variables and independent across countries, we have $Var[\sigma_\infty^2 - {\sigma_\infty^*}^2] = 2Var[\sigma_\infty^2]$, and its covariance with the difference of other variables are 0.

b. Variance and covariance of $z_\infty - z_\infty^*$

z_t and z_t^* shares the same global jumps, and their jump sizes for each time t are i.i.d., therefore

$$Cov[z_\infty, z_\infty^*] = E^2[\xi^g]\frac{Var\left[\lambda_\infty^g\right]}{\phi_z^2} + E[{\xi^g}^2]\frac{E[\lambda_\infty^g]}{2\phi_z},$$

And hence by the assumption that countries are symmetry, we have

$$Var[z_\infty - z_\infty^*] = 2Var[z_\infty] - 2Cov[z_\infty, z_\infty^*].$$

In like fashion, we can obtain the following covariances.

$$\begin{aligned}
Cov[z_\infty, w_\infty^{i,*}] &= E^2[\xi^g]\frac{Var\left[\lambda_\infty^g\right]}{\phi_z\phi_i} + E[{\xi^g}^2]\frac{E[\lambda_\infty^g]}{\phi_z + \phi_i}, \\
Cov[w_\infty^i, w_\infty^{i,*}] &= E^2[\xi^g]\frac{Var\left[\lambda_\infty^g\right]}{\phi_i^2} + E[{\xi^g}^2]\frac{E[\lambda_\infty^g]}{2\phi_i} \\
Cov[z_\infty, \lambda_\infty^{i,*}] &= \frac{\omega_\lambda\phi_i}{\phi_i - \phi_z}(Cov[z_\infty, z_\infty^*] - Cov[z_\infty, w_\infty^{i,*}]), \\
Cov[z_\infty - z_\infty^*, \lambda_\infty^i - \lambda_\infty^{i,*}] &= 2Cov[z_\infty, \lambda_\infty^i] - 2Cov[z_\infty, \lambda_\infty^{i,*}].
\end{aligned}$$

d. Variance of $\lambda_\infty^i - \lambda_\infty^{i,*}$

Based on Equation (23), we have

$$Cov[\lambda_\infty^i, \lambda_\infty^{i,*}] = \left(\frac{\omega_\lambda \phi_i}{\phi_i - \phi_z}\right)^2 \left(Cov[z_\infty, z_\infty^*] + Cov[w_\infty^i, w_\infty^{i,*}] - 2Cov[z_\infty, w_\infty^{i,*}]\right).$$

Therefore

$$Var[\lambda_\infty^i - \lambda_\infty^{i,*}] = 2Var[\lambda_\infty^i] - 2Cov[\lambda_\infty^i, \lambda_\infty^{i,*}].$$

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