Equity Option Implied Probability of Default and Equity Recovery Rate

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Abstract

There is a close link between prices of equity options and the probability of default of a firm. We show that in the presence of positive expected equity recovery, the standard methods that assume zero equity recovery at default misestimate the probability of default implicit in option prices. We introduce a simple method to detect stocks with positive expected equity recovery by examining option prices, and propose a method to extract the probability of default from option prices in the presence of positive expected equity recovery. Our empirical results based on six large financial institutions in the US during the 2007-2009 crisis show that assuming zero recovery leads to significant mispricing of options and misestimation of implied probability of default.

Keywords: option, default, probability of default, arbitrage bounds

JEL Classification: G13

1 Introduction

There is a long line of literature linking option prices with the probability of default of a firm, following the seminal work by Merton (1974). Most of the studies in this literature (e.g., Bayraktar and Yang (2011), Carr and Linetsky (2006), Carr and Madan (2010), Carr and Wu (2009), and Linetsky (2006)) assume that there is no residual asset left to pay equity investors in the event of a firm's default. That is, the stock price goes to zero when a default occurs. In most cases, the assumption of zero equity recovery at default is valid. However, in some instances, there is significant residual value to equity investors even after

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a firm defaults.[1] Ignoring a positive expected equity recovery at default has important implications for pricing options and also for estimating probability of default from observed option prices.

The present study makes two important contributions to the literature. First, we propose a simple method to detect stocks with positive expected equity recovery by examining prices of equity options. Second, we propose a simple closed-form equity call option pricing formula that takes positive equity recovery into account.

Our methodology is based on new static arbitrage-free lower bounds for European call and put option prices when the expected default probability and equity recovery are known. We derive these lower bounds by generalizing the results of Orosi (2014).

We use the prices of options on General Motors (GM) stock close to its bankruptcy filing in June 2009 to illustrate how the violation of the proposed lower bounds can be used to detect positive expected recovery for GM. We also apply our method to six large U. S. financial institutions that were under tremendous stress during the financial crisis of 2007-2009. We find that lower bound violations occurred frequently for options on these stocks, indicating that the expected equity recovery rates of these stocks were often positive during the crisis. The positive equity recovery rates for these stocks are consistent with what happened to these firms eventually. Some were forced to be acquired by other institutions at very low prices (Bear Stearns and Merrill Lynch) while one was bailed out by the government (AIG). The positive expected recovery rates reflect the investors' expectations of what the stocks of these firms would be worth after going through near-default events such as forced mergers and government bail-out.

The new lower bounds with positive equity recovery can be also used to extend the closed-form equity option pricing formula of Orosi (2015a). The model allows us to estimate the equity recovery rate and the probability of default. We calibrate two versions of the model: the new formula that allows for positive equity recovery and the formula in Orosi (2015a) that assumes zero recovery. We then compare the probability of default estimated from the two versions of the model with the probability of default estimated from credit default swaps (CDS) for MGM Resorts International in 2009.

We find that the probabilities of default estimated with positive recovery are comparable to the CDS-implied probabilities of default whereas the probabilities estimated under zero recovery assumption are significantly lower than those implied from CDS. Our example illustrates that ignoring a positive expected equity recovery at default has important implications for estimating probability of default from observed option prices. Therefore, other methods of estimating probability of default from option prices such as Carr and Wu (2011) and Taylor et al. (2014) must be used with caution when the equity recovery is expected to be positive.

[1] See Leland (1994), Leland and Toft (1996), Fan and Sundaresan (2000), Broadie, Chernov, and Sundaresan (2007), Hackbarth, Hennessy, and Leland (2007), and Davydenko (2012).

The rest of paper is organized as follows. In Section 2, we show how the violation of lower bounds for option prices can be explained by positive equity recovery at default. Section 3 describes the data. In Section 4, we present examples of the application of the proposed methodology for detecting positive expected equity recovery from option prices. In Section 5, we propose an interpolation-based closed-form formula that can be used to infer the probability of default from equity options when expected recovery is positive. Section 6 concludes the paper.

2 Violation of Lower Bounds for Options on a Defaultable Asset

The Merton's lower bounds for European call and put prices are based on the assumption that the underlying asset price follows a strictly positive price process, which is not the case when there is a positive probability of default. Orosi (2014) proposes improved lower bounds under the assumption that the underlying asset can default and the price of the asset at default is zero. In the case of equity options, this means a zero equity recovery at default.

$$P(K,T) \geq \max\left(e^{-rT} \cdot K \cdot PD, 0\right), \tag{1}$$

$$C(K,T) \geq \max\left(S_0 e^{-dT} - e^{-rT}(1-PD) \cdot K, 0\right). \tag{2}$$

In practice, however, Orosi's lower bounds are often violated for equity options. In this section, we first present an alternative derivation of the lower bounds proposed in Orosi (2014), then show that relaxing the assumption of zero equity recovery can explain the presence of violations of Orosi's bounds in practice.

2.1 Lower bounds with zero equity recovery

Following Merton (1973), we assume: (i) perfect capital markets; (ii) there are no arbitrage opportunities; (iii) investors have positive marginal utility of wealth; and (iv) current and future interest rates are strictly positive. Let $C(K,T)$ and $P(K,T)$ be the current prices of European call and put options on the stock with strike K and maturity T. r and d denote the interest rate and the continuous dividend yield, respectively.

Consider a stock that has a current price of S_0 with a positive risk-neutral default probability of PD prior to some time T. Then, since the stock is worthless in the case of default

$$\begin{aligned} PD &= P(S_T = 0), \\ P(S_T &> 0) = 1 - PD. \end{aligned}$$

Moreover, as De Marco et al. (2013) show, $P(S_T > 0)$ can be calculated from call options using the identity of Breeden and Litzenberger (1978) and is given by

$$P(S_T > 0) = -e^{-rT} \left. \frac{\partial^+ C(K,T)}{\partial K} \right|_{K=0} = -e^{-rT} \left(\lim_{\Delta K \to 0} \frac{C(\Delta K, T) - C(0,T)}{\Delta K} \right).$$

Then, a digital contract that pays a unit currency at time T if default happens prior to time T and pays zero otherwise is given by

$$D(T) = e^{-rT} \cdot PD,$$

and can be replicated in terms of call options and cash as follows:

$$\begin{aligned} D(T) &= e^{-rT} \cdot PD = e^{-rT} - e^{-rT} P(S_T > 0) = \\ e^{-rT} + \left. \frac{\partial^+ C(K,T)}{\partial K} \right|_{K=0} &= e^{-rT} + \lim_{\Delta K \to 0} \frac{C(\Delta K, T) - C(0,T)}{\Delta K}. \end{aligned} \tag{3}$$

Proposition 1 *The lower bound of a European put option written on a defaultable asset is*

$$P(K,T) \geq e^{-rT} \cdot K \cdot PD. \tag{4}$$

Proof. *Assuming otherwise, one can assume (4) does not hold, and form the following zero value portfolio at time zero:*

$$\Pi = P(K,T) - K \cdot D(T) + B,$$

where B represent the amounts invested in bonds. In the case of default, the value of the portfolio at the time of expiry is given by:

$$\Pi = K - K + Be^{rT} = Be^{rT} > 0,$$

because the payoff of $D(T) = 1$. If the asset does not default prior to expiry and the option does not finish in the money (or $S_T > K$ equivalently), then the value of the portfolio at the time of expiry is given by

$$\Pi = Be^{rT} > 0$$

because the put option and $D(T)$ become worthless. Finally, if the asset does not default prior to expiry and the option finishes in the money (or $S_T \leqslant K$ equivalently), then the value of the portfolio at the time of expiry is given by

$$\Pi = K - S_T + Be^{rT} > 0.$$

■

Proposition 2 *The lower bound of a European call option written on a defaultable asset is*

$$C(K,T) \geq S_0 e^{-dT} - e^{-rT}(1 - PD) \cdot K. \tag{5}$$

Proof. *Using equation (4) and the put-call parity relationship:*

$$\begin{aligned} C(K,T) &= S_0 e^{-dT} - K \cdot e^{-rT} + P(K,T) \\ &\geq S_0 e^{-dT} - K \cdot e^{-rT} + e^{-rT} \cdot K \cdot PD \\ &\geq S_0 e^{-dT} - e^{-rT}(1 - PD) \cdot K. \end{aligned}$$

■

2.2 Lower bounds with positive equity recovery

We now assume that if a company's stock price falls below a default barrier, db, then the company defaults. Moreover, when a firm defaults, the stock price is worth $R \geq 0$, hereafter referred to as equity recovery.

2.2.1 Random equity recovery

We assume that equity recovery at expiry T is a continuously distributed random variable with a probability density function $f_R(\rho)$ at a known recovery value ρ. Then, we observe that the price of a put option can be written as follows:

$$\begin{aligned} P(K,T) &= e^{-rT}E\left[(K-S_T)_+\right] = \\ &\quad e^{-rT}E\left[(K-S_T)_+ \middle| D\right]\cdot PD + \\ &\quad e^{-rT}E\left[(K-S_T)_+ \middle| ND\right]\cdot(1-PD) \end{aligned} \tag{6}$$

where $E\left[(K-S_T)_+ \middle| D\right]$ is the conditional expectation if default occurs, $E\left[(K-S_T)_+ \middle| ND\right]$ is the conditional expectation if default does not occur, and $PD = P(S_T \leq db)$. Let $E(R)$, $\min(R)$, and $\max(R)$ be the expected value, minimum, and maximum values of the recovery at time T, respectively. Then, the value of a put option is

$$\begin{aligned} P(K,T) &= e^{-rT}E\left[(K-S_T)_+ \middle| D\right] = 0 \text{ if } K \leq \min(R), \\ P(K,T) &= e^{-rT}E\left[(K-S_T)_+ \middle| D\right]\cdot PD \\ &= e^{-rT}\left(\int_{\min(R)}^{K}(K-r)\cdot f_R(\rho)d\rho\right)\cdot PD \text{ if } \min(R)\leq K\leq\max(R), \\ P(K,T) &= e^{-rT}E\left[(K-S_T)_+ \middle| D\right] = e^{-rT}(K-E(R))\cdot PD \text{ if } \max(R)\leq K\leq db, \end{aligned} \tag{7}$$

The value of a call option follows from the put-call parity as

$$\begin{aligned} C(K,T) &= S_0e^{-dT} - Ke^{-rT} \text{ if } K \leq \min(R), \\ C(K,T) &= e^{-rT}\left(\int_{\min(R)}^{K}(K-r)\cdot f_R(\rho)d\rho\right)\cdot PD + S_0e^{-dT} - K\cdot e^{-rT} \\ \text{if } \min(R) &\leq K \leq \max(R), \\ C(K,T) &= e^{-rT}(K-E(R))\cdot PD + S_0e^{-dT} - K\cdot e^{-rT} \text{ if } \max(R)\leq K\leq db. \end{aligned} \tag{8}$$

Hence, for $\max(R) \leq K \leq db$, the values of the options are

$$P(K,T) = \max(K-E(R),0)e^{-rT}\cdot PD, \tag{9}$$

$$C(K,T) = S_0e^{-dT} - K\cdot e^{-rT} + \max(K-E(R),0)e^{-rT}\cdot PD. \tag{10}$$

Moreover, since option prices are convex functions of K, the lower bounds of calls and puts with $K \geq \max(R)$ are

$$\begin{aligned} P(K,T) &\geq \max(K - E(R), 0)e^{-rT} \cdot PD, & (11) \\ C(K,T) &\geq S_0 e^{-dT} - K \cdot e^{-rT} + \max(K - E(R), 0)e^{-rT} \cdot PD. & (12) \end{aligned}$$

Note that both calls and puts equal the lower bounds of Merton for $K \leq \min(R)$. Therefore, the lower bounds of the options in the presence of non-zero recovery is significantly lower than the lower bounds of options with zero recovery that have the same probability of default. Moreover, in the presence of recovery, the options equal their lower bounds for strikes $K \geq \max(R)$, and these values are significantly lower than the lower bounds of the options with zero recovery.

2.2.2 Constant equity recovery

Note that if one only deals with options for which $\max(R) \leq K \leq db$, option prices are given by

$$\begin{aligned} P(K,T) &= \max(K - E(R), 0)e^{-rT} \cdot PD, \\ C(K,T) &= S_0 e^{-dT} - K \cdot e^{-rT} + \max(K - E(R), 0)e^{-rT} \cdot PD. \end{aligned}$$

As a consequence of convexity, for options with $K \geq \max(R)$, the lower bounds of calls and puts are given by

$$\begin{aligned} P(K,T) &\geq \max(K - E(R), 0)e^{-rT} \cdot PD, \\ C(K,T) &\geq S_0 e^{-dT} - K \cdot e^{-rT} + \max(K - E(R), 0)e^{-rT} \cdot PD. \end{aligned}$$

The above equations indicate that if one deals with options for which $K \geq \max(R)$, then it is reasonable to replace $E(R)$ by a constant parameter R when examining lower bounds of European calls and puts. Furthermore, if $\min(R)$ and $\max(R)$ are close to each other, then a constant recovery assumption at expiry is also reasonable. Therefore, the lower option bounds for a constant R are given by

$$\begin{aligned} P(K,T) &\geq \max(K - R, 0)e^{-rT} \cdot PD, & (13) \\ C(K,T) &\geq \max(S_0 e^{-dT} - K \cdot e^{-rT} + \max(K - R, 0)e^{-rT} \cdot PD, 0). & (14) \end{aligned}$$

Although in our subsequent analysis we will frequently use (13) and (14), our conclusions can be easily generalized for non-constant recovery.

2.2.3 Comparison of lower bounds

Figure 1 (left panel) shows an example of what the proposed call price lower bounds look like compared to the Merton's lower bound,

$$C(K,T) \geq \max\left(Se^{-dT} - Ke^{-rT}, 0\right). \quad (15)$$

If we assume that the recovery is zero, then a positive PD simply makes the sloped portion of the bound steeper. But if we assume a positive recovery and PD, then the bound coincides with the Merton's bound up to the expected recovery R and starts decreasing linearly from R at the same slope as the bound with positive PD and zero recovery.

Figure 1 (right panel) shows what can happen if the recovery is indeed positive, but we apply the lower bound with zero recovery. In this case, we will find that some of the observed call prices violate the lower bound. This is an indication that the zero recovery assumption does not hold for the underlying stock.

< **Figure 1: Lower Bound for Call Prices** >

Figure 2 (left panel) shows an example of what the proposed put price lower bounds look like compared to the Merton's lower bound,

$$P(K,T) \geq \max\left(Ke^{-rT} - Se^{-dT}, 0\right). \tag{16}$$

Figure 2 (right panel) shows how we can identify the presence of positive recovery by looking at the observed put prices with respect to the lower bound with positive PD and zero recovery. If the recovery is positive, then some of the observed put prices will violate the lower bound with positive PD and zero recovery.

< **Figure 2: Lower Bound for Put Prices** >

3 Data

We obtain data on options on eight stocks from IVolatility[2]. The data includes daily end-of-day option settlement prices, dividend rates, and interest rates. IVolatility uses LIBOR rates for terms up to one year, and ISDA(R) interest rate swaps par mid rates for longer terms. Our sample includes options on:

- General Motors on 15 April 2009,
- MGM Resorts International in 2009,
- Six U.S. financial institutions considered in Taylor et al. (2014) between 2008 and 2009: American International Group (AIG), Bank of America, Bear Stearns, JP Morgan, Lehman Brothers, and Merrill Lynch.

[2] See http://www.ivolatility.com

4 Detecting Positive Equity Recovery with Lower Bound Violations

An important implication of (4) and (5) is that these static arbitrage lower bounds are based on minimal assumptions. If one observes a violation of these bounds (referred to as lower bound violation hereafter), then the non-zero recovery assumption can be readily rejected.

4.1 Illustrative example: General Motors

General Motors (GM) filed for bankruptcy on June 1, 2009, and the 'old' GM stock started trading over the counter (pink sheets) on June 2. Its stock traded at 75 cents the day before the bankruptcy filing, but shot up to about \$1.20 a share by June 12, less than two weeks after the bankruptcy filing. By April 15, 2009, the bankruptcy of GM was highly anticipated and the price of GM stock had plunged to \$1.89 from around \$40 in 2007. With this background in mind, we look at the prices of options on GM stock on April 15, 2009.

The bid and ask prices of call options are shown on Figure 3 (left panel), together with the Merton's lower bound and our new proposed lower bound with $PD = 40\%$ and $R = 0$. A conservative estimate of the PD of 40% (adjusted to the option maturity of 0.431 years) was implied from 1-year credit default swaps.

We observe that the observed prices satisfy the Merton's lower bound, but the prices of some of the lowest strike call options violate our proposed lower bound. The violations imply that investors could have made arbitrage profits if PD was sufficiently high, which was certainly the case in our example. How can we explain the presence of these arbitrage opportunities?

One assumption we made in the lower bound plotted in Figure 3 (left panel) is that the expected equity recovery is zero. We now relax this assumption and suppose that the expected recovery is \$1. The assumption seems reasonable since the price of GM stock went from 75 cents to \$1.20 in two weeks following the bankruptcy. We plot the lower bound with $PD = 40\%$ and $R = \$1$ in Figure 3 (right panel). Since the lower bound under the assumption of positive expected recovery, R, coincides with the Merton's lower bound when the strike is between zero and R, none of the observed call prices violate this new lower bound. Therefore, there are no arbitrage opportunities arising from lower bound violations if investors believed that when GM eventually defaults, the price of its stock would have dropped to \$1 or higher, but not to zero.

< Figure 3: Lower Bound Violations - General Motors >

This example shows that if the probability of default is positive and we find low strike options violating the lower bound with positive PD, but zero R, then this can be an indication that the expected equity recovery at default is greater than zero.

4.2 U.S. financial institutions during the 2007-2009 financial crisis

We apply our proposed methodology to identify stocks that are expected to have a positive equity recovery at default. We select six large financial institutions in the U.S. during the crisis for several reasons. These financial institutions were under tremendous stress during the financial crisis. As a result, Lehman Brothers declared bankruptcy while Bear Stearns and Merrill Lynch were forced to be acquired by JP Morgan Chase and Bank of America, respectively. AIG was bailed out by the government. The circumstances leading up to the failure of these firms provide an interesting case study because a government bail-out is a form of near-default that often leaves equity holders of the firm in distress with small but positive residual value. Moreover, the same set of firms and time period was used in a related study by Taylor et al. (2014) to demonstrate their methodology for extracting the probability of bankruptcy from stock and option prices.

The first step in identifying lower bound violations is to obtain an estimate of the PD. We estimate PDs by using the methodology in Carr and Wu (2011) described in Appendix A. One assumption behind Carr and Wu (2011) methodology is that the equity recovery is zero. As a result, for stocks with positive equity recovery, the methodology results in underestimating PD. Therefore, a Carr and Wu (2011) PD provides a downward biased estimate of the actual PD, which is sufficient for our purpose of detecting lower bound violations.

We then plug in the Carr-Wu PDs into the Orosi's call price lower bound, $C(K,T) \geq \max\left(S_0 e^{-dT} - e^{-rT}(1-PD)\cdot K, 0\right)$, to determine whether a call price violates this lower bound. We report the frequency of lower bound violations in Table 1. Note that the price of an American call is greater than or equal to the price of the corresponding European call. Hence, if the above procedure yields an arbitrage violation for an American call, it also implies an arbitrage violation for the corresponding European call. For the same reason, we use ask quotes rather than bid quotes.

$<$ **Table 1. Lower Bound Violations - U.S. Financial Institutions** $>$

The results show that lower bound violations occur frequently, indicating rejection of the zero recovery assumption for these firms. The frequency of lower bound violations is around 80% for three stocks (AIG, JP Morgan, and Lehman Brothers). Even for the stock with the smallest number of violations, the frequency of violations is 27%. The high frequency of lower bound violations indicates that the expected equity recovery of these stocks during the crisis is likely to have been often positive. In fact, for Bear Stearns, a positive expected recovery is consistent with the fact that the firm was eventually sold to JP Morgan Chase for $10 per share when it failed.

5 Extracting Probability of Default from Option Prices when Equity Recovery is Positive

Assuming zero recovery when the expected recovery is positive can lead to mispricing of options and misestimation of probability of default from equity options. There are several ways of estimating the probability of default from options, namely, Carr and Wu (2011), Taylor et al. (2014), and Orosi (2015a). In this Section, we focus on the method proposed in Orosi (2015a). Since the formula in Orosi (2015a) does not allow for positive recovery, we first derive an extension of the formula in Orosi (2015a) for the case of positive recovery. Using (11) and the results of Orosi (2015a), an analytic expression for European call options can be derived as:

$$C(K,T) = \begin{cases} S_0 e^{-dT} - Ke^{-rT} & \text{if } K \leq R \\ S_0 e^{-dT} - Ke^{-rT} + e^{-rT}(K-R)\cdot PD & \text{if } R \leq K \leq db \\ \left(S_0 e^{-dT} - e^{-rT}\cdot R\cdot PD\right)\cdot c & \text{if } db < K \end{cases}, \tag{17}$$

where

$$\begin{aligned} c &= \frac{-B_2 + \sqrt{(B_2)^2 - 4A_2C_2}}{2A_2} \\ A_2 &= 1 - G(1-PD) \\ B_2 &= x(1-PD) - 1 + 2G\cdot DB(1-PD), \\ C_2 &= -G\cdot DB^2(1-PD), \\ DB &= S_0e^{-dT} - e^{-rT}\cdot R\cdot PD - e^{-rT}(1-PD)\cdot db, \\ x &= \frac{e^{-rT}\cdot K}{S_0e^{-dT} - e^{-rT}\cdot R\cdot PD}. \end{aligned}$$

To derive (17), we extend the methodology in Orosi (2015a) by allowing non-zero equity recovery at default. We start by observing that equation (11) gives the following equations for call option prices when $K \leq db$:

$$C(K,T) = \begin{cases} S_0 e^{-dT} - Ke^{-rT} & \text{if } K \leq R \\ S_0 e^{-dT} - Ke^{-rT} + e^{-rT}(K-R)\cdot PD & \text{if } R \leq K \leq db \end{cases}.$$

To obtain analytic call option prices for $K > db$, apply the following transformations to the strikes and call option prices:

$$\begin{aligned} c &= \frac{C(K,T)}{S_0e^{-dT} - e^{-rT}\cdot R\cdot PD} \\ x &= \frac{Ke^{-rT}}{S_0e^{-dT} - e^{-rT}\cdot R\cdot PD}. \end{aligned}$$

Note that if $R \leq K \leq db$, the relation between c and x is given by

$$c = 1 - (1 - PD) \cdot x,$$

or equivalently

$$x = \frac{1 - c}{1 - PD}.$$

To ensure that the transformed call option prices, c, are decreasing and convex functions of x when $K > db$, we use the following expression to model the dependence between these two variables

$$x = \frac{1 - c}{1 - PD} + G\frac{(1 - c)^2}{c}, \tag{18}$$

where G is a positive constant. Note that the above relation also guarantees the continuity of call prices at $K = db$ and $c \to 0$ as $x \to 0$.

The equation (18) can be rearranged as following:

$$c^2(1 - G(1 - PD)) + c(x(1 - PD) - 1 + 2G(1 - PD)) - G(1 - PD) = 0.$$

Finally, the equation (17) is obtained as the positive root of the above quadratic equation.

5.1 Illustrative example: MGM Resorts International

We calibrate the call option formula in (17) to call options on MGM stock in 2009. We choose MGM because we have access to data on the credit default swaps and also because MGM did not pay any dividend after 2000. This simplifies our analysis since American-style call options have no early exercise premia when dividend is zero, and thus can be treated as European options.

We fit two versions of the formula. In Model 1, R, PD, and db are parameter that are extracted from option prices whereas Model 2 assumes that R is zero. For each trading day in the sample, (17) is calibrated by minimizing the root mean squared percent pricing errors:

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{C_{ij}\left(\theta_j\right) - C_{ij}}{C_{ij}}\right)^2}, \tag{19}$$

where θ_j represents the parameters for the j^{th} given trading day, the $\{C_{ij}\}_{i=1}^{n}$ are the observed option prices on the j^{th} trading day (for all strikes and expiries), and the $C_{ij}\left(\theta_j\right)$-s are the option prices based on the model.

We fit the model to in-the-money call options because Orosi (2015b) points out that the model provides poor fit to out-of-the-money calls. On many days, we cannot find enough in-the-money call options to calibrate the parameters, so we use simulated prices instead. First, for each trading day in the sample, a cubic Hermite spline-based interpolant is fitted to all available call options with the second longest maturity. The average maturity of the options used is around

0.74 years, which is approximately 9 months. Then, using this interpolant, 15 uniformly distributed call option prices with the same maturity are generated so that the minimum of the generated strike prices corresponded to 70% of the minimum values of all available strike prices, and the maximum of the generated strike prices corresponds to the stock price.

< Table 2. Probability of Default and Equity Recovery Rate - MGM Resorts International (2009) >

The results of the model calibration are reported in Table 2, and the 5-day moving averages of the estimated PDs are plotted in Figure 4. We also plot the PDs computed from American put option prices using the methodology in Car and Wu (2011) for comparison. Moreover, to assess whether the equity implied probabilities agree with those extracted from credit markets, we use CDS with a maturity of 5 years. We first estimate the constant hazard rate, λ_{CDS}, from CDS spreads by using the relation:

$$\lambda_{CDS} = \frac{SP}{1 - R_{Bond}},$$

where SP is the CDS spread and R_{Bond} is the bond recovery rate that is assumed to be 40%.

To make the PDs implied from options and those implied from CDS comparable in terms of time horizon, we use the relation in equation (21) to calculate 1-year PDs by adjusting PDs estimated from options of different maturities and λ_{CDS} estimated from 5-year CDS spreads.

We report the average estimates of the model parameters for both Model 1 and Model 2 in Table 2. The times series plot of the PDs computed from Model 1 and Model 2 show that Model 2 significantly underestimates PDs (Figure 4A). The PDs from Model 1 are comparable to the PDs implied from American puts or CDS. However, the PDs from Model 2 are significantly lower than PDs implied from American puts or CDS.

We also plot the time series of expected equity recovery rate in Figure 4B. The estimated equity recovery rate for MGM ranged from zero to 50% during this period.

< Figure 4: Probability of Default and Equity Recovery - MGM >

Our example illustrates that ignoring a positive expected equity recovery at default has important implications for estimating probability of default from observed option prices. Therefore, other methods of estimating probability of default from option prices such as Carr and Wu (2011) and Taylor et al. (2014) must be used with caution when the equity recovery is expected to be positive.

6 Conclusion

In the presence of positive expected equity recovery, the standard methods that assume zero equity recovery at default misestimate the probability of default implicit in option prices. We introduce a simple method to detect stocks with positive expected equity recovery by examining option prices, and propose a method to extract the probability of default from option prices in the presence of positive expected equity recovery. Our methodology is based on new lower bounds for European call and put option prices when the expected default probability and equity recovery are known. Our empirical results show that assuming zero recovery leads to significant mispricing of options and misestimation of implied probability of default.

The possibility of positive equity recovery at default has implications for other applications such as pricing convertible bonds (Ayache, Forsyth, and Vetzal (2003)) and exotic options. In our future research, we will investigate how our method can be applied to other related problems.

Appendix A: Probability of Default Implied from Puts - Carr and Wu (2011)

Carr and Wu (2011) defines a unit recovery claim (URC) as a contract that pays one dollar at default whenever the company defaults prior to the option expiry and pays zero otherwise. Under the assumption of a constant default arrival rate, λ, the value of a URC is given by

$$U(T) = \int_0^T \lambda e^{-rs} e^{-\lambda s} ds = \lambda \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}. \tag{20}$$

The probability of default, PD, can be computed from the default arrival rate, λ, using the relation,

$$PD = 1 - e^{-\lambda T}. \tag{21}$$

Carr and Wu (2011) show that for low strike prices, the URC can be replicated by an American put, $P_{Am}(K, T)$, the following way:

$$U(T) = \frac{P_{Am}(K, T)}{K}. \tag{22}$$

PD can be easily calculated by first calculating the value of $U(T)$ using (22), then determining the value of λ from (20), and finally plugging λ into the equation (21).

To compute $U(T)$, Carr and Wu (2011) use only options that satisfy the following criteria: (i) bid price is greater than zero; (ii) time-to-maturity is greater than 360 days; (iii) strike price is \$5 or less; and (iv) absolute value of the put's delta is not larger than 0.15.

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< **Table 1. Lower Bound Violations - U.S. Financial Institutions** >

	Violations (%)	Carr-Wu Prob. of Default mean (std)	# Options filtered
AIG	80	0.12 (0.14)	186
Bear Sterns	27	0.17 (0.17)	51
Bank of America	64	0.12 (0.14)	347
JP Morgan	81	0.08 (0.14)	354
Lehman Brothers	81	0.08 (0.08)	123
Merrill Lynch	53	0.12 (0.14)	73

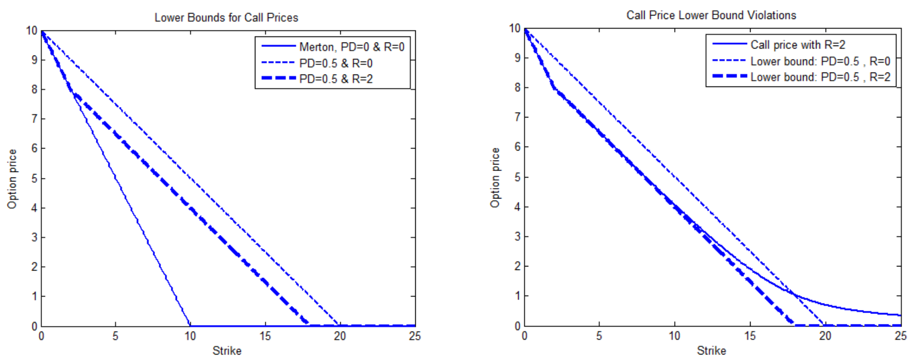
Note: We report the frequency of violation of the Orosi's lower bound for call options, $C(K,T) \geq \max\left(S_0 e^{-dT} - e^{-rT}(1-PD)\cdot K, 0\right)$, using all call options on the six U.S. stocks between 2008 and 2009. To do this, we first compute Carr and Wu (2011) probabilities of default, PD, by using the methodology described in Appendix A. We then plug in the estimated PDs into the lower bound formula to determine whether an option price violates it. We also report the number of options used in estimating Carr-Wu PDs. The filtered options satisfy: (i) bid price is greater than zero; (ii) time-to-maturity is greater than 360 days; (iii) strike price is $5 or less; and (iv) absolute value of the put's delta is not larger than 0.15.

< Table 2. Probability of Default and Equity Recovery Rate - MGM Resorts International (2009) >

	Root Mean Squared (%Error)	G	R (%)	PD (%)	db (\$)
Model 1: R>0					
Mean	1.02	0.3869	19.11	23.13	2.7330
Standard Deviation	n/a	0.3037	17.43	21.93	1.6211
Model 2: R=0					
Mean	4.82	0.1302	0	3.18	0
Standard Deviation	n/a	0.1140	n/a	7.16	n/a

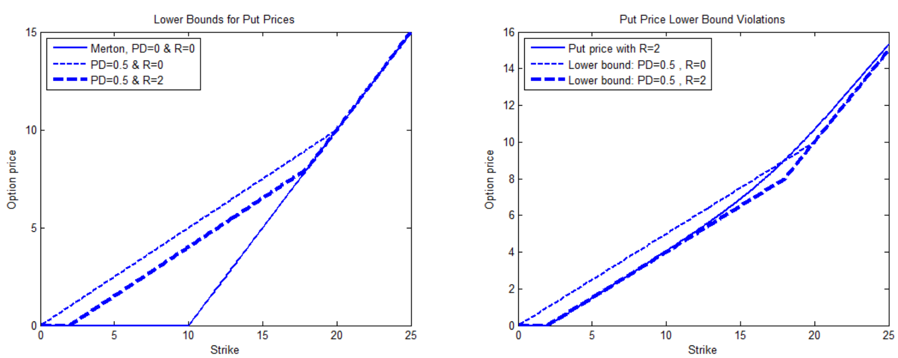
Note: We calibrate the call option price formula in (17) to call options on MGM Resorts International stock in 2009. Model 1 assumes positive equity recovery, thus estimates R by fitting the observed call prices to the pricing formula. Model 2 assumes zero recovery, thus R is simply fixed at zero. For each trading day in the sample, the model is calibrated by minimizing the root mean squared percent pricing errors of generated in-the-money options. To generated in-the-money call option prices, a cubic Hermite spline-based interpolant is fitted to all available call options with the second longest maturity. Then, using this interpolant, 15 uniformly distributed call option prices with the same maturity are generated so that the minimum of the generated strike prices corresponded to 70% of the minimum values of all available strike prices, and the maximum of the generated strike prices corresponds to the stock price.

< Figure 1: Lower Bound for Call Prices >



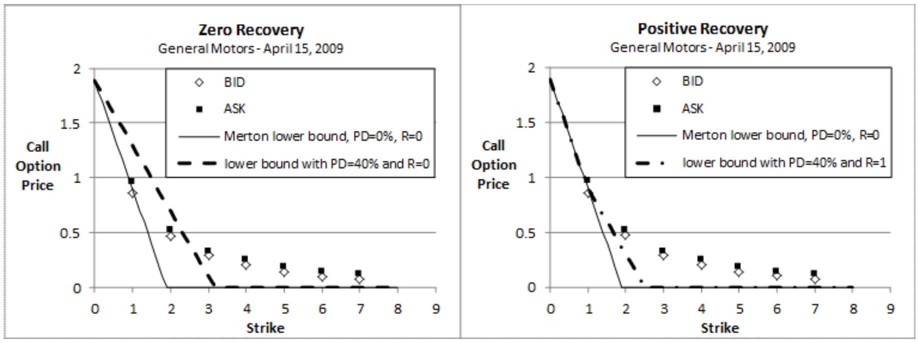
Note: The parameters used are: stock price(S)=\$10, riskfree rate(r)=0%, dividend yield(d)=0%, time-to-maturity(T)=1 year, default barrier(db)=\$4. Merton's lower bound for call prices, C, is $C(K) \geq \max\left(Se^{-dT} - Ke^{-rT}, 0\right)$ where K is the strike price. The lower bound for call prices with a positive probability of default, PD, is $C(K,T) \geq S_0 e^{-dT} - K \cdot e^{-rT} + \max(K-R,0)e^{-rT} \cdot PD$. The call prices on the right panel are generated using (17) with $G = 0.12$.

< Figure 2: Lower Bound for Put Prices >



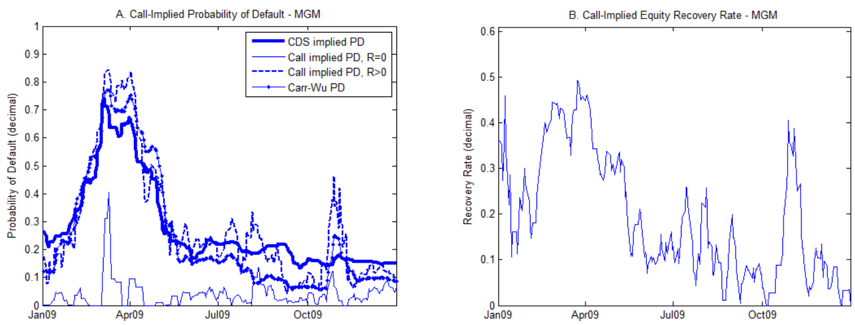
Note: The parameters used are: stock price(S)=\$10, riskfree rate(r)=0%, dividend yield(d)=0%, time-to-maturity(T)=1 year, default barrier(db)=\$4. Merton's lower bound for put prices, P, is $P(K) \geq \max\left(Ke^{-rT} - Se^{-dT}, 0\right)$ where K is the strike price. The lower bound for put prices with a positive probability of default, PD, is $P(K,T) \geq \max(K-R,0)e^{-rT} \cdot PD$. The put prices on the right panel are generated using put-call parity and (17) with $G = 0.12$.

< Figure 3: Lower Bound Violations - General Motors >



Note: We plot the bid and ask prices of call options on General Motors' stock on April 15, 2009. The parameters used are: stock price(S)=\$1.89, riskfree rate(r)=0.93%, dividend yield(d)=0%, time-to-maturity(T)=0.431 year, and probability of default (PD) = 40%. The probability of default was estimated from credit default swaps on General Motors. Merton's lower bound for call prices, C, is $C(K) \geq \max\left(Se^{-dT} - Ke^{-rT}, 0\right)$ where K is the strike price. The lower bound for call prices with a positive probability of default, PD, is $C(K,T) \geq S_0 e^{-dT} - K \cdot e^{-rT} + \max(K - R, 0)e^{-rT} \cdot PD$.

< **Figure 4: Probability of Default and Equity Recovery - MGM** >



Note: We calibrate the call option price formula in (17) to call options on MGM Resorts International stock in 2009. Model 1 assumes positive equity recovery, thus estimates R by fitting the observed call prices to the pricing formula. Model 2 assumes zero recovery, thus R is simply fixed at zero. For each trading day in the sample, the model is calibrated by minimizing the root mean squared percent pricing errors of generated in-the-money options. To generated in-the-money call option prices, a cubic Hermite spline-based interpolant is fitted to all available call options with the second longest maturity. Then, using this interpolant, 15 uniformly distributed call option prices with the same maturity are generated so that the minimum of the generated strike prices corresponded to 70% of the minimum values of all available strike prices, and the maximum of the generated strike prices corresponds to the stock price.