

Explaining Credit Default Swap Spreads by Means of Realized Jumps and Volatilities in the Energy Market

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Abstract

This paper studies the relationship between credit default swap spreads (CDS) for the Energy sector and oil futures dynamics. Using data on light sweet crude oil futures from 2004 to 2013, which contains crisis period, we examine the importance of volatility and jumps extracted from the futures in explaining CDS spread changes. The analysis is performed at an index level and by rating group; as well as for the pre-crisis, crisis and post-crisis periods. Our findings are consistent with Merton's theoretical framework. At an index level, futures' jumps are important when explaining CDS spread changes, with negative jumps having higher impact during the crisis. The continuous volatility part is significant and positive indicating that futures volatility conveys relevant information for the CDS market. Negative jumps have an increasing importance as the credit rating deteriorates while futures volatility becomes more important for higher rating categories. For the highest rating category the CDS spread depends very weakly on both, futures' jumps and volatility. The relation between the CDS market and the futures market is stronger during volatile periods and strengthens after the Global Financial Crisis.

JEL Classification G12, G13, C14

Keywords: Oil futures, CDS spread, realized jumps, realized volatility

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1 Introduction

The seminal work of Merton (1974) investigates the intrinsic relationship between credit risk, equity volatility and equity returns. It underpins negative correlation between stock movements and credit risk as well as negative correlation between stock movements and volatility. Following this work many empirical studies have analyzed interactions between these three quantities. Credit risk was initially measured by bond yield spread while equity volatility was obtained by using mean squared log-returns. For example, Collin-Dufresne et al. (2001) finds that an important determinant of corporate credit spread changes is given by the volatility index VIX for the U.S market. Campbell and Taksler (2003) documents a strong empirical connection between rising idiosyncratic equity risk and increasing yields on corporate bonds relative to treasury bonds. The evolution of financial markets lead to reexamination of this intrinsic relationship between credit risk, equity volatility and equity returns. In particular, the rise of credit default swap (CDS) market has enabled the establishment of an alternative way of computation of credit risk whereas options provided a more forward looking point of view of equity volatility. Benkert (2004) investigates the effects of historical and option-implied equity volatility on corporate CDS premia and; and documents that option-implied volatility is a more important factor in explaining variation in CDS premia than historical volatility. Ericsson et al. (2009) analyze the determinants of corporate default risk given by CDS spreads; they document that leverage, equity volatility (computed using mean squared log-returns) and the risk free rate are important determinants of CDS premia.

More recently, many theoretical studies on high-frequency data developed a variety of models for the description of price dynamics. Aït-Sahalia (2002) and Aït-Sahalia (2004) investigate the presence of jumps in discretely sampled data and devise an approach for disentangling the diffusion component of a stochastic asset return process into a continuous part and a jump part. Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006) devise a robust theoretical technique for detecting jumps in high frequency financial time series by establishing a relationship between realized power variation and bipower variation which provides a consistent estimator of integrated variance unaffected by jumps. The proposed frameworks allow to separately measure the continuous sample path variation and the discontinuous jump part. These results are appealing because the importance of jumps in asset dynamics has been

emphasized in a number of theoretical and empirical studies. Starting with the theoretical result of Merton (1976) the importance of jumps during crisis periods has been convincingly illustrated in Bates (1991) and Bates (2000). In light of these new econometric results and empirical evidences the reassessment of the fundamental relationship found by Merton appears to be the next crucial objective.

Combining the research strand on the importance of jumps and separation of the diffusion component into a continuous part and a jump part, with the objective to explain CDS spreads, Zhang et al. (2009) examine to which extent equity jumps and volatility explain CDS spreads. Performing a panel analysis on a sample ranging from January 2001 to December 2003 for 307 U.S. firms authors find that equity volatility given by the bipower variation is an important determinant of CDS spread changes. Surprisingly, authors find that equity jumps, either positive or negative, as well as jump intensity or jump volatility, are not significant when explaining CDS spread changes.¹

Our work contributes to the literature by introducing the first comprehensive analysis of the relationship between CDS spread for the Energy sector and oil futures jump and volatility activities. As opposed to previous studies performed for equity markets, our sample is large, ranging from January 2004 to December 2013, and contains both, low and high volatility periods allowing to determine how market conditions affect this relationship. We perform the analysis for the CDS spread at an index level and per rating group, thus assessing the impact of creditworthiness on this relationship. We also split the underlying period into pre-crisis, crisis and post-crisis period. At an index level, we find that futures' jumps are an important ingredient when explaining CDS spread changes, with negative jump components having higher impact during the crisis period. Furthermore, we find that futures volatility conveys relevant information for the CDS market. Credit rating affects the results as follows: (i) negative jumps have an increasing importance as the credit rating deteriorates; (ii) futures volatility play a dominant role for higher rating categories; (iii) for the highest rating available the CDS spread depends very weakly on both, futures' jumps and volatility; (iv) lastly, the connection between the CDS market and the futures market appears to be stronger during volatile market conditions and strengthens after the global financial crisis. All our results are consistent with Merton's theoretical framework.

¹In the following we will be also referring to Tauchen and Zhou (2011) and Wright and Zhou (2009) who investigate the explanatory power of jumps, either for stocks, bonds and foreign exchange rates, but do not consider the CDS market.

The paper is organized as follows. We present the key ingredients for the jump detection framework in Section 2. A description of the empirical data used in our analysis is provided in Section 3. Regression tests and analysis are performed in Section 4. Section 5 provides concluding remarks. All graphs and tables are relegated to the appendix.

2 Model Specification

Let $s_t = \ln(S_t)$ be the log-asset price whose dynamics evolve under the influence of a jump-diffusion process

$$ds_t = \mu_t dt + \sigma_t dW_t + J_t dq_t, \quad (1)$$

where μ_t and σ_t are the instantaneous drift and diffusion terms of the return process, respectively; J_t is the log jump size with mean μ_J and standard deviation σ_J , W_t is a standard Brownian motion and dq_t is a Poisson process with intensity λ_J . Time is measured in daily units and we define the intraday returns as

$$r_{t,i} = s_{t,i\cdot\Delta} - s_{t,(i-1)\cdot\Delta}, \quad (2)$$

where $r_{t,i}$ refers to the i^{th} within-day return on day t , with Δ being the sampling frequency within each day such that $m = 1/\Delta$ observations occur every day and as $\Delta \rightarrow 0$ we have that $m \rightarrow \infty$.

Barndorff-Nielsen and Shephard (2004) propose two measures for quadratic variation process namely, the realized variance (RV) and the realized bipower variation (BV) that converge uniformly as $\Delta \rightarrow 0$ to different quantities of the jump diffusion process such as

$$RV_t = \sum_{i=1}^m r_{t,i}^2 \rightarrow \int_{t-1}^t \sigma_s^2 ds + \int_{t-1}^t J_s^2 dq_s, \quad (3)$$

$$BV_t = \frac{\pi}{2} \frac{m}{m-1} \sum_{i=2}^m |r_{t,i}| |r_{t,i-1}| \rightarrow \int_{t-1}^t \sigma_s^2 ds. \quad (4)$$

As it is evident from Eq.(3) and Eq.(4), the difference between the realized variance and the realized bipower variation is zero when there is no jump and strictly positive when there is a jump. For detecting jumps, we adopt the ratio test, proposed in Huang and Tauchen (2005)

and Andersen et al. (2005), where the test statistic

$$RJ_t \equiv \frac{RV_t - BV_t}{RV_t}, \quad (5)$$

is an indicator for the contribution of jumps to the total within-day variance of the process. This test statistic converges in distribution to a standard normal distribution when using an appropriate scaling:

$$z = \frac{RJ_t}{\sqrt{\left\{ \left(\frac{\pi}{2}\right)^2 + \pi - 5 \right\} \Delta \max\left(1, \frac{TP_t}{BV_t^2}\right)}} \rightarrow N(0, 1). \quad (6)$$

In Eq.(6) TP_t is the tripower quarticity that is robust to jumps; it is defined in Barndorff-Nielsen and Shephard (2004) as

$$TP_t \equiv m\mu_{4/3}^{-3} \frac{m}{m-2} \sum_{i=3}^m |r_{t,i-2}|^{4/3} |r_{t,i-1}|^{4/3} |r_{t,i}|^{4/3} \rightarrow \int_{t-1}^t \sigma_s^4 ds, \quad (7)$$

where

$$\mu_k \equiv 2^{k/2} \frac{\Gamma((k+1)/2)}{\Gamma(1/2)}, \quad k > 0.$$

Assuming that there is at most one jump per day (Merton (1976)) and that jump size dominates the return when jump occurs (Andersen et al. (2005)), daily realized jumps sizes can be obtained as

$$\hat{J}_t = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times I_{(ZJ_t \geq \Phi_\alpha^{-1})}}, \quad (8)$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function with α being the level of significance and $I_{(ZJ_t \geq \Phi_\alpha^{-1})}$ is an indicator function which takes the value of one if there is a jump on a given day, and zero otherwise.

Once the realized jumps have been established, we can compute the jump mean $\hat{\mu}_J$, the variance $\hat{\sigma}_J$ and intensity $\hat{\lambda}_J$ as follows:

$$\hat{\mu}_J = \text{Mean of } \hat{J}_t, \quad (9)$$

$$\hat{\sigma}_J = \text{Standard deviation of } \hat{J}_t, \quad (10)$$

$$\hat{\lambda}_J = \frac{\text{Number of jump days}}{\text{Number of trading days}}. \quad (11)$$

It has been shown in Tauchen and Zhou (2011) that such an approach for estimation of realized jump parameters is robust with respect to drift and diffusion function specifications. It allows to

easily specify the jump arrival rate, avoiding elaborating estimation methods, and yields reliable results under various settings, for instance, when the sample size is either finite, increasing or shrinking.

3 Data Description

In this study we consider both, the CDS market and the commodity market during the period from January 2004 to December 2013. As the financial markets went through very different behaviors throughout this time, we found it instructive to split the selected period into three sub-samples; the first runs from January 2004 to December 2007 and will be qualified as the pre-crisis period; the second spreads from January 2008 to end of 2009, covering the Global Financial Crisis (GFC), and is named the crisis period²; the third and last sub-sample goes from January 2010 to December 2013 and is referred to as the post-crisis period. For each market we provide details that include the data source, data processing and descriptive statistics. We first focus on the energy market and then discuss the credit default swap market.

For the energy market we consider the light sweet crude oil futures (i.e., the futures with the shortest maturity) quoted on the Chicago Mercantile Exchange since it is one of the most traded futures in the energy sector. Because of its liquidity and importance it has been used in many previous empirical studies (see Askara and Krichene (2008), Soucek (2013) and Chevallier and Sevi (2012) among others). High frequency data sampled at 5 minute-intervals from January 2004 to December 2013 is obtained from SIRCA³. We restrict the computations to quotes from 9:30 am to 15:30 pm as it is well known that this sampling frequency avoids micro structure noise effects that can cause biases in the estimation of the realized volatility. We compute daily realized volatility (RV_t) and realized bipower variation (BV_t) using Eq.(3) and (4), respectively. From these two quantities we extract daily jumps using Eq.(8) and split the resulting time series into positive and negative parts that will be denoted as J_t^+ and J_t^- , respectively. Following the literature, see Zhang et al. (2009) and Tauchen and Zhou (2011) among others, we average the

²It is usually agreed that the start of the GFC takes place end of July 2007 but for the Energy CDS market the surge of CDS spread occurred slightly later. Selecting December 2009 for the "end" of the GFC might be very surprising at first sight. This choice is mainly motivated by two facts. First, around that date the CDS spread corresponded to 150 bps that is well below the 400 bps observed during the GFC. Second, for the futures around that date the market was in a bull market configuration for at least 4 months.

³<http://www.sirca.org.au/>

data fortnightly from Thursday to Wednesday (i.e., Wednesday two weeks later) and keep only these two-week spaced averaged values. For simplicity, we use the same notation for BV_t , BV_t , J_t^+ and J_t^- for these fortnightly observations. Thus, our sample ranges from 21 January 2004⁴ to 18 December 2013, with fortnightly spaced observations. Figure 1 shows daily annualized realized volatility time series for the futures with the resulting BV_t and J_t series presented in Figure 2 and 3, respectively. From Figure 3 we note that both, negative and positive jumps are prevalent throughout the entire period under consideration.

[Insert Figure 1 here]

[Insert Figure 2 here]

[Insert Figure 3 here]

We also compute the mean jump intensities for both, the positive and negative jumps and denote these quantities as λ^+ and λ^- , respectively. We report in Table 1 the descriptive statistics for the differences in bipower variation ΔBV_t , the jumps J_t^+ and J_t^- as they will be used in the regression analysis performed in the next section, as well as the level of BV_t as it allows for more intuitive interpretation.

[Insert Table 1 here]

Over the entire sample the mean value for the realized bipower variation (BV_t) corresponds to 3.6×10^{-4} and when computed over the three different sub-samples, pre-crisis, crisis and post-crisis periods, it gives 3.22×10^{-4} , 7.86×10^{-4} and 1.89×10^{-4} , respectively. The discrepancies between these values and the peak reached during the crisis period illustrate the impact of the GFC that started on the futures market through a substantial increase of futures' volatility. The GFC impact is also well pronounced when looking at the standard deviation for the BV_t , which is higher during the crisis period compared to either pre- or post-crisis. Note also that the post-crisis value is smaller than the pre-crisis value, which is related to the influence of jumps as discussed below. Regarding the bipower variation change, given by ΔBV_t , it is large and positive

⁴Averaging from Thursday, 8 January to Wednesday, 21 January.

during the crisis period, implying an increase of CDS spreads during that critical period, while for the two other sub-samples the mean values are negative. Regarding the standard deviations, whether we consider the level BV_t or the change ΔBV_t , the values corresponding to the crisis period are substantially larger by a multiple factor of approximately 3 for the changes and 5 for the level. The discrepancies observed for the statistics computed over the different sub-samples justify the partitioning of the sample.

As for the jumps, positive and negative parts have approximately the same magnitude (means are close in absolute values) and standard deviation when computed over the entire sample period. For the three sub-samples, positive and negative jumps display large variability. During the crisis period the mean values (in absolute value terms) and standard deviations are at least twice as high as those obtained for the other two sub-samples. Furthermore, post-crisis mean values are larger than their pre-crisis counterparts, which is well pronounced for negative jumps (with an increase of 63%) while for positive jumps, an increase of 13% is observed. This fact should be put in perspective with the remark made for BV_t , the continuous component of the realized volatility. These findings suggest that the explanatory power of jumps has increased after the crisis period. Lastly, the mean jump intensities, which represent the average number of jumps per time unit⁵, characterize the jump activity and complete the statistics already presented. For the negative jumps the highest intensity is achieved during the post-crisis period, a result surprising at first as we would expect that the largest value should be observed during the crisis period. However, let us consider this value in perspective with other properties of negative jumps. Compared to the post-crisis period, negative jumps occurring during the crisis period are slightly less frequent (22% against 28% per year) but are 1.65 times larger in magnitude and also exhibit a greater variability (the standard deviation is at least twice as high as that during the post-crisis period). In contrast, post-crisis jumps are more frequent but of a smaller size. As for the pre-crisis period, the intensity and the jump size are 50% and 38% smaller compared to those for the post-crisis period, respectively. For the standard deviations, the results are as expected; they are at least twice as large during the crisis period compared to the pre- or post-crisis periods. If we restrict our consideration to the first two moments and the jump intensity, positive jumps exhibit similar qualitative statistical properties to those of negative jumps as all the remarks made for the latter apply to the positive jumps as well.

⁵In our case these values are the mean number of jumps per year.

A CDS is a credit derivative contract between two counterparties that essentially provides insurance against the default of an underlying. In a CDS, the protection buyer makes periodic payments to the protection seller until the occurrence of a credit event or the maturity date of the contract, whichever comes first. The premium paid by the buyer is denoted as an annualized spread, measured in basis points (bps), and referred to as the CDS spread. If a credit event (default) occurs on the underlying financial instrument, the buyer is compensated for the loss incurred as a result of the credit event, receiving the difference between the par-value of the bond and its market value after default.

Our dataset uses CDSs on corporate bonds collected on a daily basis from Markit. We restrict our analysis to the 5-year maturities, which is considered to be the most liquid, from January 2004 to December 2013. We take non-sovereign entities from the Energy sector (previously named Oil & Gas). The CDSs are written on senior unsecured debt (RED tier code: SNRFOR) and denominated in USD. We average the individual CDS spreads to produce a 5-year CDS index value for this sector. In order to be consistent with the volatility data the CDS time series are sampled fortnightly with the first and last observations corresponding to 21 January 2004 and 18 December 2013, respectively.⁶

Table 1 reports means and standard deviations for the CDS level and change for the entire sample as well as for the three sub-samples. For the entire sample, the mean CDS level is 139.73 and the corresponding standard deviation is 61.9. The mean CDS values for the pre-crisis, crisis and post-crisis periods are 105.00, 221.38 and 133.76, respectively. The historical high CDS spread levels observed during the GFC is consistent with the tremendous uncertainty taking place in the financial markets at that time. Furthermore, the standard deviation during that period is 79.81 which is four times higher than the standard deviation during the pre-crisis period and twice as high as that for the post-crisis period. Note also that the CDS market shows a greater level of uncertainty for the CDS spreads during the post-crisis period, compared to the pre-crisis period. It is mainly due to fact that the GFC lead to a general reassessment of risks understood in a very broad sense (the recent works on contagion, liquidity and counterparty risks constitute a convincing illustration).

Similar conclusions can be drawn for the CDS spread changes. The mean value of CDS changes

⁶These are the days on which fortnightly averages are computed for energy futures.

is positive during the crisis period and the standard deviation is three times higher than the one obtained during the pre-crisis period, and twice as high as that observed during the post-crisis period. During this latter period the mean CDS changes are negative indicating a decline in the CDS spread and a posteriori justifying the name of post-crisis for the third sub-sample. As for the standard deviation, it is larger for the post-crisis period compared to the pre-crisis period, which also indicate a permanent increase in uncertainty in the credit market.

Although our study focuses on the relationship between volatility of the futures price and CDS spread, it is of interest to consider statistical properties of futures log-returns. Table 1 also reports the mean and standard deviation for futures log-returns. During the pre-crisis period the mean of the futures log-returns is positive corresponding to 0.0099, or 25.7% per annum, and the standard deviation is 0.0547. During the crisis period the mean is, as expected, negative and the corresponding standard deviation is twice as large as during the pre-crisis period. For the post-crisis sample the mean is positive corresponding to 0.0023, or 6% per annum, and the standard deviation is 0.0484 which is slightly smaller than the pre-crisis value and is consistent with the observations made for the bipower variation values. Figure 4 shows the evolution of the CDS spread and futures for the entire sample, many of the statistical properties described above can be deduced from this figure.

[Insert Figure 4 here]

We also consider the CDS spreads by rating class: *AA*, *A*, *BBB*, *BB*.⁷ More precisely, we group the CDS spreads used to compute the index by rating (also provided by Markit) and compute a CDS spread index value for each rating category. The results are presented in Table 2. Similarly to the statistics produced for the CDS index, we observe that the means and standard deviations of credit spreads increase substantially as credit quality deteriorates from *AA* to *BB* throughout the entire period. The *AA* rating class has the lowest mean and standard deviation of 27.66 and 6.65, respectively, during the pre-crisis period. Due to higher levels of uncertainty in the financial market system during the GFC, the means and standard deviations corresponding to this period increase. Both statistics increase when moving from the highest (*AA*) to the lowest (*BB*) rating class. The highest levels for the mean and standard deviation of 422.99 and 124.76, respectively, are achieved for the lowest (*BB*) rating class. As noted earlier for the CDS spread

⁷Other rating classes are not considered due to unavailability of the data for the entire sample period.

level, both statistics have lowered during the post-crisis period but the values remained higher compared to those for the pre-crisis period across all rating classes, which can be explained by strict funding procedures enforced after the GFC. We also present Figure 5 which compares CDS spreads for the different rating classes. We note that as the credit quality deteriorates, there is an increase in the CDS spread levels throughout the entire sample period. All rating classes evolve in a similar manner except for the lowest *BB* rating class which has been extremely volatile during the pre-crisis and crisis periods, becoming less volatile and moving with other rating classes during the post-crisis.

[Insert Figure 5 here]

The mean values for CDS changes during the crisis period across all rating classes are positive which is consistent with the substantial increase of credit spread levels over that period. As expected, the lowest mean value during the crisis period corresponds to the *AA* rating class while the highest is attributed to the *BB* case. Similar results can be drawn for the standard deviations, which are higher during the crisis period across all rating classes with pre-crisis values being lower than those for post-crisis.

4 Methodology and Empirical Results

4.1 Regression analysis

The importance of jumps as an explanatory variable has been convincingly illustrated in Tauchen and Zhou (2011), Wright and Zhou (2009) and Zhang et al. (2009). Tauchen and Zhou (2011) show that the jump volatility (that is, the volatility of \hat{J}_t defined in Eq.(8)) explains a large portion of bond spreads. They use regression to analyze the relevance of jump volatility computed from a two-year rolling window on monthly *AAA* and *BAA* bond spreads⁸. They obtain large R^2 , around 20% when performing simple regression, see Table 5 in Tauchen and Zhou (2011) for more details. However, this choice of a rolling window assumes that volatility observations are computed on overlapping intervals, which implies strong autocorrelation of the explanatory variable and might be problematic if the sample size is small. Say, if the sample size is close to

⁸These ratings are provided by Moody's and correspond to *AAA* and *BBB* of Markit.

two years, a large part of the sample is used to compute volatility. In addition, one should note that Tauchen and Zhou (2011) consider bond yield spread *levels* that assume stationarity of the yield. This assumption is clearly not satisfied during the GFC. Another interesting contribution is that of Wright and Zhou (2009) who illustrate the importance of the mean jump size of the 30-year Treasury bond futures to explain the excess return on holding of an n-month maturity bond (with $n \in \{24; 36; 48; 60\}$). The mean value is computed using a 24-month rolling window, thus, imposing a constraint on the sample size when applying this methodology. In addition, authors demonstrate that jump volatility and intensity are not significant (see Table 2 in Wright and Zhou (2009) for details). It remains unclear whether their conclusions remain valid for a rolling window of a smaller size, which is crucial if the underlying sample size is small.⁹ Of particular interest to us is a study by Zhang et al. (2009) who investigate CDS market at a firm level. The authors show that jump activity has a strong explanatory power for corporate CDS spreads for a sample ranging from 2001 to 2003. Positive and negative jumps denoted as J_t^+ and J_t^- , respectively, have significant coefficients and are able to explain CDS spread levels. Jumps' volatility, on the contrary, is not significant when considered jointly with continuous volatility given by the bipower realized volatility estimator BV_t , see Table 3 in Zhang et al. (2009). Implementation strategies for both aforementioned papers require one-year averaging over the variables¹⁰ (either jumps or realized volatility) and can be problematic if the sample of interest is too small. Authors consider both, credit spread levels and credit spread changes but note that the results are less satisfactory when dealing with CDS spread changes. It is well known (Collin-Dufresne et al. (2001)) that spread changes are much more difficult to explain and generally result in a smaller R^2 . Consistently with this remark Zhang et al. (2009) found an R^2 of around 4% and, interestingly, the jumps (either positive or negative, or their intensity) are all statistically insignificant (see regression 2 in Table 5 in Zhang et al. (2009)). Focusing on CDS spread changes instead can be necessary if the underlying time series exhibit a trend. This situation is especially pronounced during the GFC.

The objective of this paper is to analyze the role of jumps extracted from the futures light sweet crude oil in explaining CDS spread changes for the energy market. We focus on CDS changes rather than levels because our sample contains GFC and therefore, is not stationary. Contrary to

⁹Specifically in Wright and Zhou (2009) this aspect is not an issue since their sample ranges from July 1982 to September 2006.

¹⁰The problem of autocorrelation of explanatory variables in the computation of the t-statistics can be resolved by using Newey-West's result.

existing literature, we consider fortnightly observations where explanatory variables are averaged over two weeks. This procedure avoids overlapping in the averaging process and allows us to work with relatively small sample sizes. As an example, we are able to consider GFC which spreads from January 2008 to December 2009, comprising 50 fortnightly observations. From an analytical point of view, we consider the regression equation of the form

$$\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t. \quad (12)$$

We do not incorporate VIX, treasury yield curve or other explanatory variables since our purpose is to focus exclusively on the relationship between CDS spreads and futures' volatility and jumps. Consistent with Merton (1974), we would expect negative correlation between stock movements and credit risk. When stock price increases due to a positive jump, probability of default decreases, CDS drops and therefore, one would expect $c_2 < 0$. On the other hand, if the stock price decreases due to a negative jump, probability of default increases, which leads to an increase in CDS and subsequently, $c_3 < 0$. Finally, with decreasing stock price (and thus, increasing CDS) volatility will increase due to a leverage effect, and thus, one would expect $c_1 > 0$.

4.2 Analysis at the index level

We perform regression analysis based on Eq.(12) first using the entire sample and then for each sub-sample. To filter out jumps we used $\alpha = 0.999$ in Eq.(8). The results are reported in Table 3. Whenever we refer to the regression coefficient as “significant”, we mean significance at 5% level, and will specify otherwise. For the entire sample we observe R^2 corresponding to 11%, which is an encouraging result given that CDS spread changes are difficult to explain, in contrast to CDS spread levels. The coefficient for the continuous volatility part, given by the bipower realized volatility estimator BV_t , is significant and positive (t-statistic of 2.31), which is consistent with the result in Merton (1974). Positive jumps are significant and the coefficient is negative (t-statistic of -2.58). Negative jumps have strong negative and significant coefficient (t-statistic of -4.15), which underpins the importance of this jump type. Both results are again in line with Merton (1974). Comparing positive and negative jump sizes along with the estimated coefficients we conclude that negative jumps, although occurring slightly less frequent, have a larger price impact on CDS spreads, compared to positive jumps. Summarizing, all the estimates

are consistent with Merton (1974)'s work and negative jumps are an important ingredient in the dynamics of futures prices. We now perform the regression on the different sub-samples to understand how market conditions affect the results.

[Insert Table 3 here]

During the pre-crisis period, ranging from January 2004 to December 2007, the regression analysis leads to insignificant coefficients for both, BV_t and J_t^+ . Only negative jumps appear to be significant and have a correct (negative) sign for the regression coefficient (t-statistic of -2.10). Furthermore, the R^2 is 5% which is rather low but is in line with the results reported in Zhang et al. (2009). Note that this weak relationship between futures' volatility and CDS spreads echoes the low correlation between futures log-returns and CDS spread changes reported in Table 4. This is again consistent with Merton (1974)'s theoretical framework.

[Insert Table 4 here]

For the crisis period the results are surprisingly satisfactory as R^2 reaches 16.4%. The continuous part of the volatility, BV_t , is not significant although it has a correct (positive) sign. Both, negative and positive jump coefficients have a correct (negative) sign, with negative jumps being significant (t-statistic of -2.23) and positive jumps being nearly significant (t-statistic of -1.93) suggesting that during financial turmoils stock price dynamics have more discontinuities, and information is impounded in the price abruptly. Indeed, from the descriptive statistics in Table 1 we note that jumps are substantially larger (in absolute value terms) during the crisis period and display larger variability. From Table 4 we observe that correlation between futures log-returns and CDS spread changes become strong and negative. To summarize, jump modeling is essential if the sample under consideration contains volatile periods.

For the last sub-sample, the post-crisis period, the regression analysis also leads to interesting results. Firstly, R^2 is high at 14% which implies that volatility, understood in a broad sense (that is, continuous and discontinuous), is able to explain CDS spread changes, which contrasts with the results for the pre-crisis period where R^2 only reaches 5%. Higher R^2 during the post-crisis period compared to pre-crisis, also implies that the relationship between the CDS market and the volatility market has been reinforced after the GFC, that is, the markets became more

connected. This aspect can be further confirmed when looking at the correlation level between futures log-returns and CDS spread changes that is nearly twice the pre-crisis value, see Table 4. In this particular case the significant variables are the bipower realized volatility (t-statistic of 2.73) and the negative jumps (t-statistic of -2.15), both with correct signs. The results provide useful guidance to build a parametric model.

To illustrate the results described above in a time-varying setting, we plot time-varying t-test statistics estimated using a moving window of 78 fortnightly observations (corresponding to approximately 3 years of data) in Figure 6. The first t-statistic is computed using 78 fortnightly observation from Wednesday, 21 January 2004 to Wednesday, 15 January 2005; the second test statistic is computed using observations from Wednesday, 4 February 2005 to Wednesday, 29 January 2006, etc. The results are consistent with those observed in Table 3; R^2 ranges from 5% for the pre-crisis period to above 20% during the crisis period. BV_t is mostly insignificant but has a correct (positive) sign. Positive jumps are mostly negative (with an exception of the post-crisis period) and significant during the crisis period. Negative jumps are negative and significant, with significance increasing during the crisis period.

[Insert Figure 6 here]

Putting the results obtained for the three sub-samples in perspective with those for the entire sample suggests strengthening of the relationship between the credit and volatility markets that has certainly been triggered by the GFC. This also explains a relatively low R^2 in Zhang et al. (2009) as their sample ranges from 2001 to 2003 which does not include the GFC. Due to a small sample size, aggregation of data along the time axis in this case can jeopardize the quality of the results.

So far we have worked at an index level which averages the CDS spreads for all U.S. energy companies. It is instructive to disaggregate the data and to perform this analysis for different rating groups that constitute the index to understand how creditworthiness affects the relation between CDS spread changes and futures' volatility.

4.3 Analysis by rating group

We report in Table 5 the regression results for different samples and rating groups (*AA*, *A*, *BBB*, *BB*). For the entire sample, across all rating groups all coefficients have correct signs. Namely, positive sign is observed for the bipower realized volatility and negative for both jump components. We also note the discrepancies between the coefficients across different rating groups that suggest dependency of CDS-volatility relationship on creditworthiness. This implies that a panel analysis which imposes the same relation across rating groups, is likely to perform poorly, which leads to low R^2 in regression 2, Table 5 of Zhang et al. (2009). Negative jump activity is extremely important for all rating groups as the coefficients are all significant (ranging from -2.18 for *AA* to -3.78 for *BBB*). For the positive jumps they are significant for all but the *A* rating¹¹. Rather surprising is the result for the bipower realized volatility that is significant for the *A* rating only. We now focus on regression analysis for the different sub-samples but we first raise some remarks on their validity. Whenever coefficient is significant, its sign is consistent with Merton (1974)'s model. For a given sub-sample the regression coefficients across all rating groups vary considerably. Similarly to the results reported for the entire sample, we observe discrepancies between the coefficients across different rating groups, suggesting that panel analysis might be inappropriate even for the sub-sample periods.

Considering the pre-crisis period, negative jumps are significant only for the *A* rating class whereas positive jumps are significant only for the *AA* rating class. These two groups also have the highest R^2 corresponding to 6.9% and 10%, respectively. However, the overall relationship between volatility and CDS spread changes appears to be weak.

For the crisis sample period, the *A*, *BBB* and *BB* rating classes display high R^2 values ranging from 13.4% to 17.8%. Negative jumps appear to be significant for lower rating categories (i.e., *BBB* and *BB*) whereas positive jumps are significant for the lowest rating *BB* only. For the *A* rating class, connection between CDS spread changes and realized volatility is explained through the bipower realized variation BV_t . Summing up, jumps are important for lower rating categories whereas the continuous volatility part is important for higher rating classes. Interestingly, for the highest rating group *AA* none of the coefficients are significant and R^2 is extremely low (3%). This latter result seems consistent with the “flight to quality” effect observed during the

¹¹T-statistic for *BBB* rating is nearly significant taking the value of -1.93. Note that if we assume lower significance level, our results will become significant for all rating classes, including *A*.

crisis. During the post-crisis period negative jumps are significant for all but the lowest rating group (*BB*) while the bipower realized volatility is significant for the *BB* rating class only. R^2 is slightly higher for the post-crisis period compared to the pre-crisis period.

When comparing correlations reported in Table 4 between futures log-returns and CDS spread changes by rating class and across sub-samples, the results appear consistent with those observed for the entire sample: the lowest correlations (in absolute terms) are observed for the pre-crisis period for all rating groups, with a subsequent increase during the crisis period and a minor drop afterwards during the post-crisis period. We note that correlations become stronger after the crisis compared to pre-crisis for all rating groups except for the highest rating *AA*.

Overall, our results underline the importance of negative jumps and the bipower realized volatility (i.e., the continuous part of the volatility) that are required for accurate modelling of the relationship between the CDS market and the volatility market. This relationship depends on global market conditions and appears to be strong during a bear market configuration. Our findings seem reasonable if we take into account the fact that the futures is a contract that can be short easily to provide hedge in the bear market conditions. Our conclusions are valid at an index level as well as per rating group.

5 Conclusion

This paper presents a comprehensive study on the relationship between CDS spreads for the Energy sector and oil futures dynamics. Motivated by Zhang et al. (2009) who examine to which extent equity jumps and volatility are determinant of CDS spreads, we analyze the importance of volatility and jumps extracted from the futures light sweet crude oil in explaining CDS spread changes for the energy market. Our sample is large, ranging from January 2004 to December 2013, and covers the Global Financial Crisis, hence the fundamental relationship between credit risk and futures' volatility and jumps can be analyzed under very different market conditions. The analysis is performed at an index level and per rating group thus assessing the impact of creditworthiness on this relationship; as well as for different sub-samples (pre-crisis, crisis and post-crisis periods).

Our findings are consistent with Merton's theoretical framework. First of all, at an index level, futures' jumps are important when explaining CDS spread changes, with negative jump components having higher impact during the crisis period. Furthermore, the continuous volatility part, given by the bipower realized volatility, is significant and positive indicating that futures volatility conveys relevant information for the CDS market. In terms of the results per rating group, we find that negative jumps have an increasing importance as the credit rating deteriorates while futures volatility becomes more important for higher rating categories. For the highest rating category the CDS spread depends very weakly on both, futures' jumps and volatility. Finally, the relation between the CDS market and the futures market appears to be stronger during volatile periods and strengthens after the Global Financial Crisis.

Our work suggests several extensions. First, our analysis has been performed for the U.S. market, a worldwide extension is definitively of interest. Second, we restrict our study to the pair CDS for the energy sector, constituting mainly of oil companies, and oil futures, but other commodities are worth considering. Metal, gas, agriculture and electricity futures jointly analyzed with CSD spreads for companies depending on these commodities could lead to a general overview of the credit risk and futures relationship for commodity markets. Regarding the electricity market, it is well known that jumps (spikes) play a very particular role, it would be of interest to understand how this affects the results. Third, in the present work the realized volatility and jump activities are examined under the historical probability. Alternatively, as mentioned in the introduction, one could extract volatility through options, or more generally through derivatives, which will result in dynamics under the risk neutral probability measure. These two dynamics and more precisely their difference, that is, the variance risk premium, is known to be of crucial importance and quantifying its impact on the credit default swap market is essential. Lastly, our performed analysis at an index level as well as per rating group is a first stage of index disaggregation, and working at a firm level will provide a complete picture of the fundamental relation found by Merton. It occurs to us that all these important questions that extend naturally our results, have not been yet considered in the literature, they are left for future research.

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A Appendix

A.1 Tables

Table 1: Descriptive Statistics

	Entire period	Pre-crisis	Crisis	Post-crisis
<i>CDS_t level</i>				
Mean	139.73	105.00	221.38	133.76
Std. dev.	61.97	20.98	79.82	38.58
<i>ΔCDS_t</i>				
Mean	-0.1224	0.0468	0.9407	-0.8164
Std. dev.	15.02	8.24	25.98	12.82
<i>Futures log-returns</i>				
Mean	4.02×10^{-3}	9.92×10^{-3}	-4.36×10^{-3}	2.31×10^{-3}
Std. dev.	6.61×10^{-2}	5.47×10^{-2}	10.64×10^{-2}	4.84×10^{-2}
<i>BV_t level</i>				
Mean	3.60×10^{-4}	3.22×10^{-4}	7.86×10^{-4}	1.89×10^{-4}
Std. dev.	4.16×10^{-4}	1.37×10^{-4}	7.50×10^{-4}	1.32×10^{-4}
<i>ΔBV_t</i>				
Mean	-2.23×10^{-6}	-4.01×10^{-6}	6.05×10^{-7}	-1.87×10^{-6}
Std. dev.	1.81×10^{-4}	1.17×10^{-4}	3.37×10^{-4}	1.13×10^{-4}
<i>J_t⁺</i>				
Mean	3.15×10^{-3}	2.52×10^{-3}	4.96×10^{-3}	2.87×10^{-3}
Std. dev.	2.43×10^{-3}	1.73×10^{-3}	3.77×10^{-3}	1.65×10^{-3}
λ ⁺	0.2408	0.1913	0.2658	0.2781
<i>J_t⁻</i>				
Mean	-3.05×10^{-3}	-1.91×10^{-3}	-5.15×10^{-3}	-3.12×10^{-3}
Std. dev.	2.96×10^{-3}	1.64×10^{-3}	5.04×10^{-3}	1.84×10^{-3}
λ ⁻	0.2185	0.1442	0.2275	0.2888

Note. Descriptive statistics computed using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods for CDS level and changes, futures log-returns, bipower variation (BV_t) level and changes, positive and negative jumps (J_t^+ and J_t^-). Jump intensities are denoted by λ^+ and λ^- for positive and negative jumps, respectively. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$.

Table 2: Descriptive Statistics for CDS by Rating Group

	Entire period	Pre-crisis	Crisis	Post-crisis
<i>CDS_t level</i>				
<i>AA</i>				
Mean	43.43	27.66	68.31	46.71
Std. dev.	23.59	6.65	31.03	17.57
<i>A</i>				
Mean	81.19	51.66	125.21	88.57
Std. dev.	42.65	18.45	51.27	32.15
<i>BBB</i>				
Mean	122.26	81.86	194.28	126.57
Std. dev.	58.26	12.20	82.48	28.01
<i>BB</i>				
Mean	267.63	224.32	422.99	233.93
Std. dev.	09.84	55.10	124.76	67.07
 <i>ΔCDS_t</i>				
<i>AA</i>				
Mean	-0.0459	-0.0466	0.3211	-0.2151
Std. dev.	6.80	2.37	11.83	6.53
<i>A</i>				
Mean	-0.0550	-0.0931	0.5454	-0.2001
Std. dev.	11.37	5.98	16.13	12.65
<i>BBB</i>				
Mean	0.0879	0.2647	1.2969	-0.4192
Std. dev.	13.92	8.85	25.06	9.87
<i>BB</i>				
Mean	-0.4564	-0.3886	2.7209	-1.4977
Std. dev.	31.69	22.38	49.61	28.04

Note. Descriptive statistics computed using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods for CDS level and changes by rating group.

Table 3: Regression Analysis

ΔBV_t	J_t^+	J_t^-	R^2
<i>Entire period: Jan 2004 - Dec 2013</i>			
1.218×10^4	-0.102×10^4	-0.139×10^4	0.109
(2.31)	(-2.58)	(-4.15)	
<i>Pre-crisis: Jan 2004 - Dec 2007</i>			
6.810×10^3	-0.224×10^3	-1.072×10^3	0.052
(0.97)	(-0.47)	(-2.10)	
<i>Crisis: Jan 2008 - Dec 2009</i>			
1.114×10^4	-0.200×10^4	-0.180×10^4	0.164
(0.91)	(-1.93)	(-2.23)	
<i>Post-crisis: Jan 2010 - Dec 2013</i>			
3.024×10^4	0.054×10^4	-0.143×10^4	0.140
(2.73)	(0.75)	(-2.15)	

Note. Regression results for the equation $\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t$ obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$. For a given sample and rating we report the coefficient estimates and corresponding t-statistics in parenthesis

Table 4: Futures and CDS Correlations

	Entire period	Pre-crisis	Crisis	Post-crisis
Index	-0.2639	-0.1411	-0.3341	-0.2207
AA	-0.1524	-0.0515	-0.1682	-0.1908
A	-0.2049	-0.0967	-0.2407	-0.2290
BBB	-0.3158	-0.1844	-0.3970	-0.2163
BB	-0.2313	-0.1129	-0.2888	-0.2046

Note. Correlations between fortnightly futures log-returns and CDS differences for the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods.

Table 5: Regression Analysis by Rating Group

	ΔBV_t	J_t^+	J_t^-	R^2
<i>Entire period: Jan 2004 - Dec 2013</i>				
AA	8.045×10^2 (0.32)	-3.664×10^2 (-1.96)	-3.470×10^2 (-2.18)	0.027
A	1.116×10^4 (2.77)	-0.056×10^4 (-1.87)	-0.0869×10^4 (-3.40)	0.096
BBB	5.313×10^3 (1.06)	-0.723×10^3 (-1.93)	-1.200×10^3 (-3.78)	0.071
BB	1.478×10^4 (1.29)	-0.244×10^4 (-2.86)	-0.216×10^4 (-2.98)	0.061
<i>Pre-crisis: Jan 2004 - Dec 2007</i>				
AA	3.099×10^3 (1.57)	-0.316×10^3 (-2.37)	-0.042×10^3 (-0.29)	0.069
A	7.896×10^3 (1.61)	-0.536×10^3 (-1.62)	-0.984×10^3 (-2.74)	0.108
BBB	-1.177×10^4 (-1.57)	0.003×10^4 (-0.07)	-0.058×10^4 (-1.05)	0.035
BB	3.259×10^4 (1.74)	-0.092×10^4 (-0.73)	-0.236×10^4 (-1.71)	0.059
<i>Crisis: Jan 2008 - Dec 2009</i>				
AA	1.390×10^3 (0.23)	-0.584×10^3 (-1.16)	-0.313×10^3 (-0.79)	0.034
A	1.514×10^4 (2.02)	-0.103×10^4 (-1.61)	-0.074×10^4 (-1.50)	0.178
BBB	1.089×10^4 (0.91)	-0.163×10^4 (-1.59)	-0.155×10^4 (-1.95)	0.134
BB	2.472×10^3 (0.10)	-4.963×10^3 (-2.46)	-3.337×10^3 (-2.12)	0.154
<i>Post-crisis: Jan 2010 - Dec 2013</i>				
AA	-3.589×10^3 (-0.06)	0.297×10^3 (0.07)	-8.572×10^3 (-2.40)	0.058
A	9.407×10^3 (0.83)	0.746×10^3 (1.00)	-1.737×10^3 (-2.56)	0.088
BBB	6.509×10^3 (0.73)	0.344×10^3 (0.59)	-1.251×10^3 (-2.34)	0.071
BB	7.253×10^4 (2.93)	0.168×10^4 (1.03)	-0.090×10^4 (-0.60)	0.099

Note. Regression results (per rating group) for the equation $\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t$ obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$. For a given sample and rating we report the coefficient estimates and corresponding t-statistics in parenthesis.

A.2 Figures

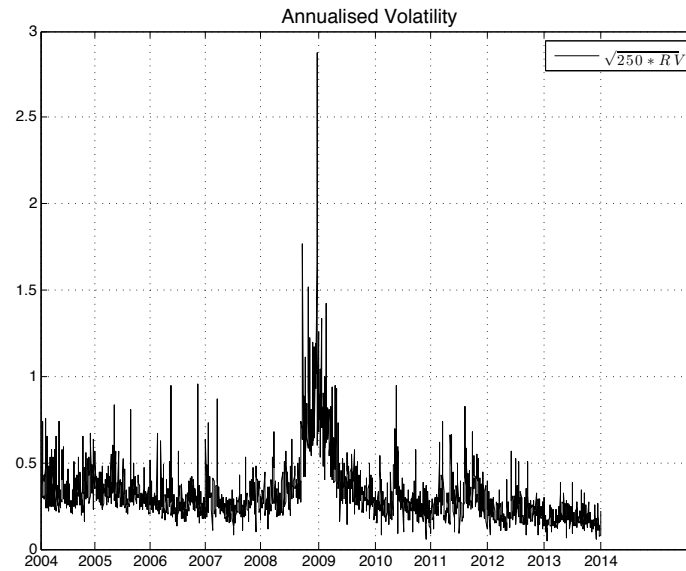


Figure 1: Annualized daily volatility $\sqrt{250 \times RV_t}$ for the period from January 2004 to December 2013.

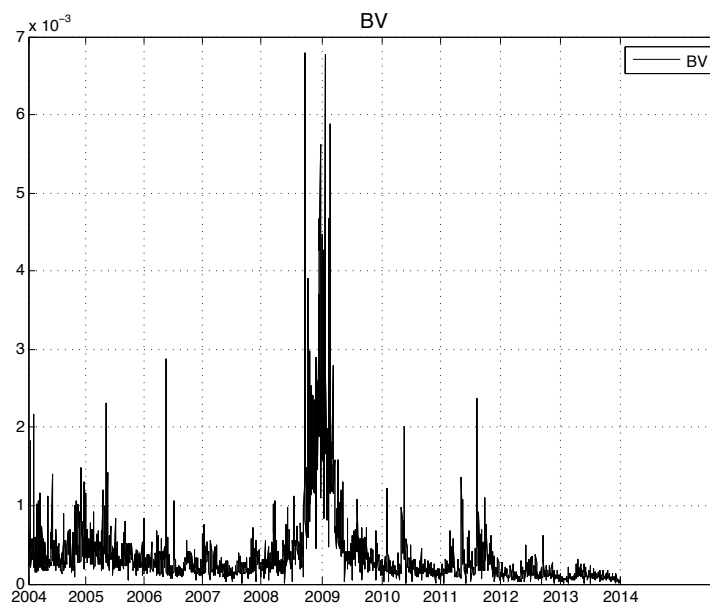


Figure 2: Bipower variation BV_t (daily observations) for the period from January 2004 to December 2013.

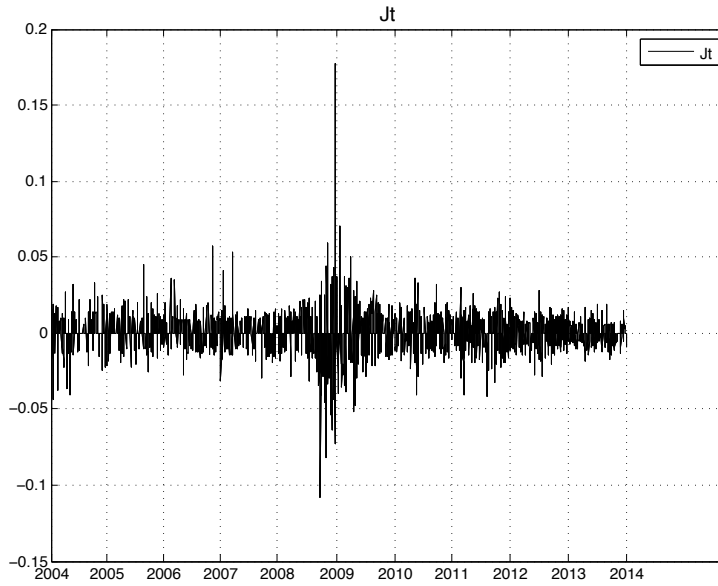


Figure 3: Jump components J_t (daily observations) for the period from January 2004 to December 2013.

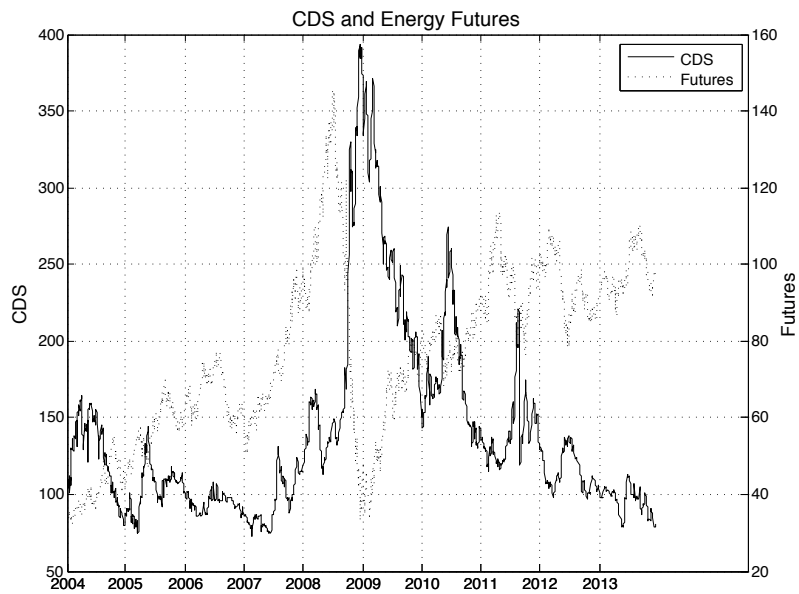


Figure 4: Energy sector CDS (5-year maturity) and front futures, daily data from January 2004 to December 2013.

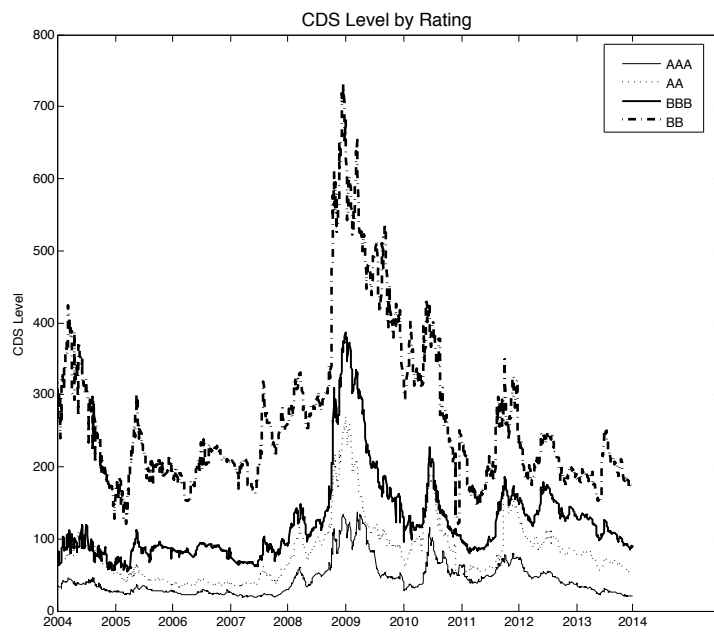


Figure 5: Energy sector CDS spread levels for different rating classes, daily data from January 2004 to December 2013.

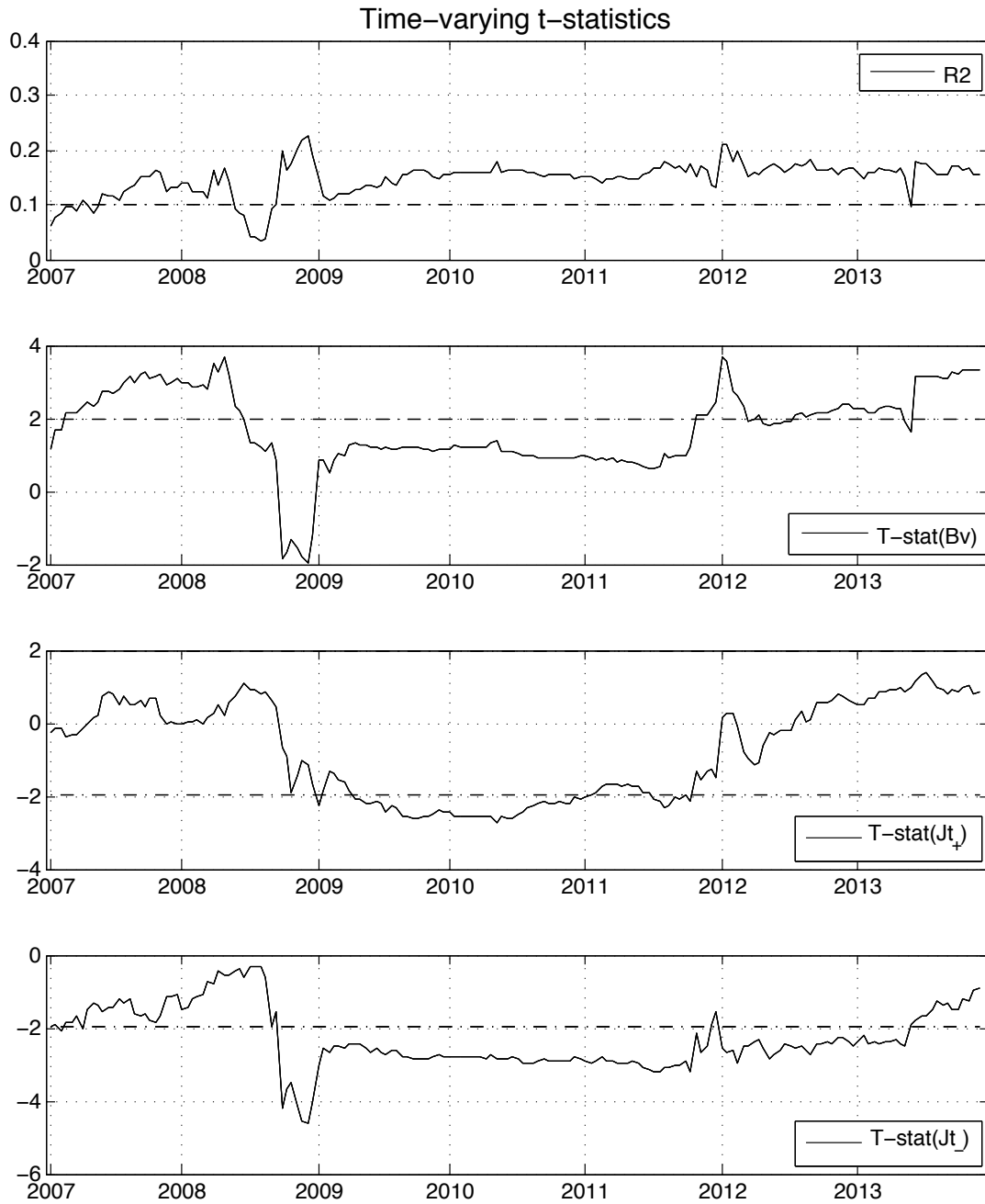


Figure 6: Time-varying t-statistics computed from the regression equation $\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t$ using fortnightly observations from January 2004 to December 2013. Moving window of 72 fortnightly observations (3 years) is used. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$.