

# Assessing Systemic Risk Based on Interbank Exposures in the Japanese Banking System

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## Abstract

This paper contributes to the systemic risk literature by assessing the network structure of bilateral exposures in the Japanese interbank market. The Japanese interbank market is composed of call and bankers' acceptance markets, and the market participants are restricted to financial institutions domiciled in Japan. We analyze the systemic risk implied in Japanese interbank networks based on various network measures such as directed graphs, centrality measures, degree distributions, and susceptible-infected-removable (SIR) models. The main findings show that the degree distributions of the Japanese interbank network follows a power law, and three mega-bank groups currently designated as globally systemically important banks (G-SIBs) overwhelm others in terms of size, interconnectedness, and substitutability.

*Keywords:* systemic risk; network topology; centrality measure; degree distribution; susceptible-infected-removable (SIR) model; globally systemically important banks (G-SIBs)

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## 1. Introduction

Systemic risk has been a focus of financial literature long before the global financial crisis and the European debt crisis. The interbank network represents a significant component in the analysis of systemic risk. This paper contributes to the systemic risk literature by assessing systemic risk of bilateral exposures based on the interbank network.

Compared to Western banks, Japanese banks suffered little from these crises, with the exception of Norinchukin bank, which suffered a loss of 190

billion yen in fiscal year 2009, by investing in securitized products. Norinchukin bank is a cooperative bank in the agriculture, forestry, and fisheries industry and a major institutional investor. Also, other major Japanese banks suffered from the crises, although their losses in absolute amounts were smaller.

Some large Japanese financial institutions declared bankruptcy during the Heisei great recession of 1997–1998 prior to the global financial crisis. Hokkaido Takushoku Bank, Yamaichi Securities Company, Sanyo Securities Company, Long-Term Credit Bank of Japan, and Nippon Credit Bank all defaulted before the appearance of material systemic risk. Given this history, there is indeed a need in Japan for a system for monitoring and measuring systemic risk. It is mainly recently that banks have begun to contribute to systemic risk by holding securitized products or credit derivatives (see Kanno (2014)).

In the past several years, triggered by the global financial crisis, the banking supervisory authorities have focused on micro level prudential policies emphasizing traditional consumer protection and on macro level prudence by focusing on the soundness of the financial system as a whole. This latter approach must take into account the interconnectedness between the financial system and the world economy.

In 2009, the Financial Stability Board (FSB) was established as a successor to the Financial Stability Forum. As an international body representing central bankers and international financial bodies such as the Basel Committee on Banking Supervision (BCBS), it intends to promote financial stability.

In September 2009, the G20 leaders requested the FSB to designate “Global Systemically Important Financial Institutions” (G-SIFIs). As a result, the FSB, IMF, and BIS cooperatively adopted the three valuation points – size, interconnectedness, and substitutability – as the evaluation criterion for G-SIFIs (see IMF, BIS and FSB (2009)).

The BCBS issued a consultative paper for the evaluation method of Global Systemically Important Banks (see BCBS (2011)). The BCBS adopted a scoring-based valuation approach, and picked five evaluation criteria of wide-ranging categories such as size, interconnectedness, lack of readily available substitutes, global (cross-jurisdictional) activity, and complexity. In November 2011, BCBS published 29 bank names as G-SIFIs and announced the policy to revise and publish the G-SIFIs list officially every November. Three Japanese mega-bank groups were then selected.

## 2. Literature review

The finance or econophysics literature on financial networks has addressed systemic risk in recent years. This paper relates to various aspects of systemic risk related to financial networks.

Financial networks are composed of a complex financial system as a set of “nodes” connected by “edges.” In a financial network, nodes might represent financial institutions, sectors, regions, or countries. Edges represent the connections between the nodes, such as financial transactions or trades.

The analysis of financial networks provides supervisory authorities or individual institutions with implications concerning contagion risk from the channels through which shocks propagate. Hence, the resilience of a network can be tested, and systemically significant nodes can be identified. Network analysis also provides an empirical tool for testing the effectiveness of macroprudential policies.

The literature on financial networks includes two approaches. The first approach describes network structure using topological indicators. The literature often relates these indicators to model graphs based on network theory. This approach does not assume a mechanism by which shocks are transmitted within the network and hence is referred to as static network analysis (see Alves et al. (2013)). Boss et al. (2004), Puhf (2012), or Tirado (2012) are examples of this approach. For example, based on the Austrian central credit register, Boss et al. (2004) and Puhf (2012) find that the Austrian interbank market is tiered and that banks within sub-sectors tend to cluster together. The topological indicators discussed in section 4 of our paper are directed graph, centrality measures, degree distribution, and average path length.

The financial system is composed of various networks. However, a representative example of analytically tractable financial networks is the interbank network. Financial networks are characterized by bilateral exposures in the interbank market. Unfortunately, in many countries, bilateral exposure data are not published, and researchers are unable to use these data. Therefore, estimating the bilateral exposures matrix, which element is exposed from one bank to another, is a significant endeavor. Recently, some papers adopted a method that minimizes the relative entropy of the bilateral exposures matrix on the information theory, for example, Censor and Zenios (1998), Sheldon and Maurer (1998), Upper and Worms (2002), and Wells (2004).

The second approach observes the financial network structure response

to shocks to assess the strength of contagion channels and the resilience of the network. The introduction of a shock assumes a specific transmission mechanism, such as defaults by market participants, and is referred to as dynamic network analysis in Alves et al. (2013). Some papers contribute to the literature on systemic risk in interbank markets by focusing on the analyses of contagion effects, for example, Elsinger et al. (2006), Cocco (2009), Haldane and May (2011), or Duan and Zhang (2013). The Susceptible-Infected-Removable (SIR) model discussed in section 4 of our paper belongs to this approach.

The motivation for our research is the development of a practical systemic risk indicator based on interbank bilateral exposures and the network theory. Additionally, we empirically analyze its applicability to the Japanese banking sector.

### 3. Estimation of the bilateral exposure matrix

The stylized balance sheets of banks show the interbank assets and liabilities, non-interbank assets and liabilities, and net worth. The Japanese interbank market is composed of call and bankers' acceptance markets. The market participants include the Bank of Japan, city banks, trust banks, regional banks, second-tier regional banks, foreign banks' Japanese branches, Norinchukin Bank, Shinkin Central Bank and shinkin banks, security companies, and insurance companies. The Japanese banking system is, therefore, composed of city banks, trust banks, Norinchukin Bank, and regional banks.

The lending relationship in the interbank market can be represented by the  $N \times N$  matrix as follows:

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,N} \end{bmatrix} \begin{matrix} \sum_j \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_N \end{matrix} \quad (1)$$

$$\sum_i \begin{matrix} l_1 & \cdots & l_j & \cdots & l_N \end{matrix}$$

where  $x_{i,j}$  denotes outstanding loans and deposits of bank  $i$  to bank  $j$ .

Summing across row  $i$  gives the total value of bank  $i$ 's interbank assets, whereas summing down column  $j$  gives bank  $j$ 's total liabilities as follows:

$$a_i = \sum_j x_{i,j}, \quad l_j = \sum_i x_{i,j} \quad (2)$$

Each bank's total interbank claims and liabilities are typically only observable from the balance sheet; hence, it is not possible to estimate the matrix  $X$  without imposing further restrictions. In case of the absence of any additional information, one approach is to choose a distribution that minimizes the uncertainty, such as the entropy of the distribution for these exposures. Following a normalization such that  $\sum_i a_i = \sum_j l_j = 1$ , this yields the solution  $x_{i,j} = a_i * l_j$ , which implies the normalized amount credited by bank  $i$  to bank  $j$ . Therefore, the exposures reflect the relative importance of each bank in the interbank market.

Calculating the matrix  $X$ , we consider the fact that a bank cannot have an exposure to itself. Therefore, we populate the values into the initial values  $x_{i,j}^0$  as follows:

$$x_{i,j}^0 = \begin{cases} 0 & , \forall i = j \\ a_i l_j & , \text{otherwise} \end{cases} \quad (3)$$

The matrix  $X^0 = (x_{i,j}^0)$  violates the adding up constraints expressed in equations (2). Hence, we need to find a new matrix  $X$  that satisfies the constraints. Some methodologies are presented by Upper (2010), Elsinger et al. (2002), and Wells (2004). The solution can be provided by solving the optimization problem as follows:

$$\begin{aligned} & \min \sum_{i=1}^N \sum_{j=1}^N x_{i,j} \ln \left( \frac{x_{i,j}}{x_{i,j}^0} \right) \\ & \text{subject to } \sum_{j=1}^N x_{i,j} = a_i, \quad \sum_{i=1}^N x_{i,j} = l_j, \quad x_{i,j} \geq 0 \end{aligned} \quad (4)$$

We employ the RAS algorithm to solve this type of problem, which is outlined in Appendix A. For further details, refer to Censor and Zenios (1998).

The data of the interbank assets and liabilities on the balance sheets are

available at the web site of Japanese Bankers Association<sup>1</sup>. The data items are “call loan” on the asset side and “call money” on the liability side.

## 4. Analysis methodologies and results

### 4.1. Network topology

We describe the interbank network as a graph. The first approach is to treat the loans and deposits matrix  $X$  as a directed graph by using Gephi software (see Cherven (2013)).

Figures 1 and 2 denote the directed graphs as of March 2009 (during the global financial crisis) and March 2013 (after the crisis). The width of edge denotes its weight, and the color is the mixing of its source (start) node color and target (end) node one. The node size denotes its strength. The nodes in the graphs are all Japanese banks, and the number as of March 2009 (during the global financial crisis) is 127, including 19 major banks<sup>2</sup>, 64 regional banks, and 44 second-tier regional banks. Contrastingly, the total number of banks as of March 2013 decreased to 121, including 16 major banks, 64 regional banks, and 41 second-tier regional banks, because of mergers and other activities<sup>3</sup>.

The set for all source nodes is the banks with liabilities in the interbank market, and the set for all target nodes is the set of banks with claims in the interbank market. Hence, each bank with liabilities toward other banks in the interbank network is an initial node in the direct graph. Each bank acting as a counterparty is represented by an end node in the direct graph.

Figure 3 denotes the log-log histogram of the contract size. From Figure 3, the logarithms of contract sizes as of March 2009 and March 2013, follow a power law with a power exponent of 1.9554 and 1.7274 respectively<sup>4</sup>, in the regional external banks with large contract sizes. The regions in which the logarithms of contract sizes follow a power law are mainly composed of medium- and small-sized regional banks and second-tier regional banks. This distinction implies that the Japanese interbank market has two tiers;

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<sup>1</sup>see <http://www.zenginkyo.or.jp/en/>.

<sup>2</sup>Three mega-bank groups (Mizuho Financial Group, Mitsubishi UFJ Financial Group, and Sumitomo Mitsui Financial Group), Resona Group, Shinsei bank, Aozora bank, Citibank Japan, Japan Post bank, and so on are included.

<sup>3</sup>It is to be noted that no Japanese bank has defaulted during the period.

<sup>4</sup>Each figure is an absolute value of the slope  $-1.9554$  or  $-1.7272$  in Figure 3.

the first tier is composed of major banks and some top-tier regional banks, and the second tier is composed of medium- and small-sized regional banks and second-tier regional banks.

In addition, we analyze some centrality measures as follows:

*Betweenness centrality.* The first measure, betweenness centrality, is a centrality measure of a node within a graph. This measure quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. A node with high betweenness centrality can potentially influence the spread of information through the network. If this ratio is close to one, then  $i$  acts as a bridge along most of the shortest paths connecting banks  $j$  and  $k$ , whereas if it is close to zero, then bank  $i$  is less important to banks  $j$  and  $k$ . Betweenness centrality of bank  $i$  in the network is therefore calculated as follows:

$$B(b_i) = \sum_{j < k; i \notin \{j, k\}} \frac{g_{j,k}(b_i)}{g_{j,k}} \quad (5)$$

where  $g_{j,k}$  is the number of shortest paths between banks  $j$  and  $k$ , and  $g_{j,k}(b_i)$  is the number of shortest paths between banks  $j$  and  $k$ , along which bank  $i$  acts as a bridge.

Table 1 shows that, during the global financial crisis, labeled as of March 2009, the banks at nodes with the highest betweenness centrality (41.20%) are eight banks - five commercial banks belonging to one of three mega-bank groups, Resona bank, Norinchukin bank, and Citibank Japan. These banks are called “Major Banks, etc.” as the most important banks in the Japanese financial system. Contrastingly, as of March 2013, the highest betweenness centrality increased significantly to 69.36%. The banks with this centrality are four commercial banks belonging to one of three mega-bank groups, and the banks with the second highest figure (40.36%) are the other Major Banks, etc. including Resona bank, Saitama Resona bank, Norinchukin bank, Mitsubishi UFJ Trust and Banking, and Sumitomo Mitsui Trust and Banking.

Since the inauguration of the Abe administration in December 2012, the Japanese financial system has been stable, mainly owing to “the quantitative and qualitative monetary easing” by the Bank of Japan. As the result, the betweenness centralities demonstrate that the Japanese interbank transactions further concentrate on the three mega-bank groups, and the Japanese



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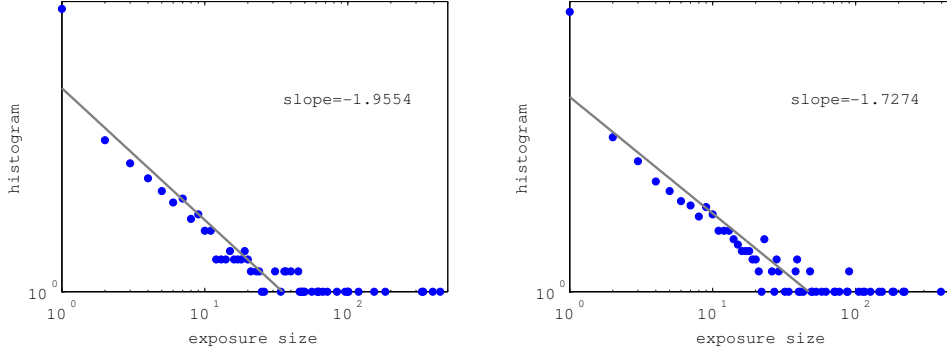


Figure 3: Histogram of contract sizes in the Japanese interbank network

Note: Data are from matrices for  $X$  as of March 2009 and March 2013. This histogram denotes the exposure size distribution within the Japanese network.

financial stability depends on the soundnesses of the three mega-bank groups. This fact proves that they have been selected as Globally Systemically Important Banks (G-SIBs) in Basel III regulation for three years in a row since November 2011.

*Closeness centrality.* The second centrality measure in a network is closeness centrality. It is defined as the function of farness, which represents the sum of distances to all other nodes. The closeness centrality for bank  $i$  is as follows:

$$C(b_i) = \sum_{j=1}^n d(b_i, b_j) / (n - 1) \quad (6)$$

where  $d(b_i, b_j)$  is the number of edges in the shortest path between banks  $i$  and  $j$ , and hence  $d \geq 1$ .

Closeness centrality tracks the proximity of a given node to another node. Because the distance between nodes in disconnected components of a network is infinite, this measure cannot be applied to networks with disconnected components. This post highlights a possible alternative, which allows the measure to be applied to disconnected component networks and, at the same time, maintain the original theory behind the measure. Table 1 denotes that banks order by this measure is similar to that of betweenness centrality.

However, there is no notable difference by this centrality between nodes.

*Eccentricity.* The third centrality measure is eccentricity, which is a measure of the maximum distance between a single node and any other node in the network. The distance  $E(b_i, b_j)$  between banks  $b_i$  and  $b_j$  is the sum of the edge weights on the shortest path from  $b_i$  to  $b_j$  in the network  $G$ . Thus, the eccentricity of a bank  $b_i$  is given as follows:

$$E(b_i) = \arg \max_{b_j \in G} d(b_i, b_j) \quad (7)$$

where  $E(b_i) \geq 1$ . Table 1 shows that this measure ranked three mega-bank groups as first, as well as two other closeness centralities in both fiscal years 2008 and 2012. This measure, however, ranked all other banks second. This measure's discriminant power, therefore, may be weaker than the two other centrality measures.

#### 4.2. Degree distribution

We analyze whether the Japanese interbank network is a small-world network (see Watts and Strogatz (1998)), similar to various real world networks, and follows a power law. In addition, we analyze whether the network is a scale-free network (see Barabási and Albert (1999)).

Four panels in Figure 4 denote the single logarithmic histograms of in-degree (assets) and out-degree (liabilities) distributions of the nodes in the interbank market as of March 2009 and March 2013. The two left panels of Figure 4 show that, for the in-degree distributions, the networks are small-world networks in the regions where  $2 \leq \text{degree} \leq 8$  as of March 2009 and  $2 \leq \text{degree} \leq 7$  as of March 2013, and are scale-free networks in regions where  $9 \leq \text{degree} \leq 26$  as of March 2009 and  $8 \leq \text{degree} \leq 26$  as of March 2013. The networks with greater degrees are major banks and top-tier regional banks (see Table 2). Consequently, after the global financial crisis, three mega-bank groups, Citibank Japan, Norinchukin Bank, and Bank of Yokohama played important roles in the Japanese interbank market.

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<sup>5</sup>Symbols are as follows: Bet: Betweenness centrality, Close: Closeness centrality, Ecc: Eccentricity. The data are sorted in descending order related to betweenness centrality. The aqua and gray parts, respectively, mark first and second banks in terms of either of the three centralities.

Table 1: Top 50 Japanese banks related to centrality measures

March 2009					March 2013				
Id	Label	Bet <sup>5</sup> (%)	Close	Ecc	Id	Label	Bet (%)	Close	Ecc
1	MIZUHO	41.20	1.00	1	1	MIZUHO	69.36	1.00	1
5	MUFJ	41.20	1.00	1	5	MUFJ	69.36	1.00	1
9	SMBC	41.20	1.00	1	9	SMBC	69.36	1.00	1
10	RESONA	41.20	1.00	1	16	MIZUHOCB	69.36	1.00	1
16	MIZUHOCB	41.20	1.00	1	10	RESONA	40.36	1.02	2
30	NOCHU	41.20	1.00	1	17	S-RESONA	40.36	1.02	2
288	MUFJT	41.20	1.00	1	30	NOCHU	40.36	1.02	2
401	CITI	41.20	1.00	1	288	MUFJT	40.36	1.02	2
398	AOZORA	26.45	1.01	2	294	SMTRUST	40.36	1.02	2
17	S-RESONA	13.45	1.03	2	134	CHIBA	28.14	1.03	2
125	77	13.45	1.03	2	138	YOKOHAMA	28.14	1.03	2
138	YOKOHAMA	13.45	1.03	2	149	SHIZUOKA	28.14	1.03	2
143	82	13.45	1.03	2	170	YAMAGUCHI	28.14	1.03	2
158	KYOTO	13.45	1.03	2	177	FUKUOKA	28.14	1.03	2
177	FUKUOKA	13.45	1.03	2	289	MIZUHOT	28.14	1.03	2
294	STRUST	13.45	1.03	2	397	SHINSEI	28.14	1.03	2
397	SHINSEI	13.45	1.03	2	398	AOZORA	28.14	1.03	2
130	JOYO	7.10	1.04	2	130	JOYO	22.69	1.05	2
134	CHIBA	7.10	1.04	2	143	82	22.69	1.05	2
144	HOKURIKU	7.10	1.04	2	144	HOKURIKU	22.69	1.05	2
149	SHIZUOKA	7.10	1.04	2	172	AWA	22.69	1.05	2
152	OGAKI	7.10	1.04	2	174	IYO	22.69	1.05	2
170	YAMAGUCHI	7.10	1.04	2	175	SHIKOKU	22.69	1.05	2
173	114	7.10	1.04	2	158	KYOTO	20.96	1.05	2
182	HIGO	7.10	1.04	2	157	SHIGA	20.18	1.05	2
289	MIZUHOT	7.10	1.04	2	167	SANINGODO	19.57	1.05	2
291	CMTRUST	7.10	1.04	2	125	77	16.39	1.03	2
554	KANSAI-U	7.10	1.04	2	128	GUNMA	14.51	1.10	2
167	SANINGODO	6.83	1.04	2	119	AKITA	12.12	1.05	2
129	ASHIKAGA	3.85	1.06	2	155	105	12.09	1.11	2
142	YAMANASHI-C	3.85	1.06	2	168	CHUGOKU	12.09	1.11	2
168	CHUGOKU	3.85	1.06	2	173	114	12.09	1.11	2
174	IYO	3.85	1.06	2	190	NISHINIHON-C	12.09	1.11	2
183	OITA	3.85	1.06	2	152	OGAKI	10.45	1.13	2
190	W-NIHON-CITY	3.85	1.06	2	169	HIROSHIMA	10.45	1.13	2
543	NAGOYA	3.85	1.06	2	116	HOKKAIDO	9.19	1.05	2
155	105	3.62	1.06	2	543	NAGOYA	8.78	1.11	2
123	IWATE	1.77	1.07	2	151	SHIMIZU	6.25	1.11	2
128	GUNMA	1.77	1.07	2	34	SEVEN	5.12	1.19	2
153	16	1.77	1.07	2	183	OITA	4.73	1.17	2
169	HIROSHIMA	1.77	1.07	2	117	AOMORI	4.22	1.11	2
172	AWA	1.77	1.07	2	304	NOMURAT	4.15	1.21	2
180	18	1.77	1.07	2	179	SAGA	4.12	1.08	2
304	NOMURAT	1.77	1.07	2	185	KAGOSHIMA	4.07	1.19	2
307	ORIX	1.77	1.07	2	569	MOMIJI	3.86	1.05	2
544	CHUKYO	1.77	1.07	2	534	TOYAMA	2.83	1.24	2
569	MOMIJI	1.57	1.07	2	180	18	1.80	1.27	2
573	KAGAWA	1.57	1.07	2	307	ORIX	1.23	1.21	2
175	SHIKOKU	0.65	1.10	2	123	IWATE	1.05	1.03	2
594	MINAMINIHON	0.28	1.07	2	133	MUSASHINO	0.81	1.10	2

Contrastingly, for the out-degree distributions, the networks as of March 2009 are scale-free networks that follow a power law with an exponent 0.0584 (determination coefficient  $R^2 = 88.2\%$  in single regression analysis) in the regions of  $2 \leq \text{degree} \leq 33$ , and the ones as of March 2013 follow a power law with an exponent 0.1224 (determination coefficient  $R^2 = 65.7\%$  in single regression analysis) in the region of  $2 \leq \text{degree} \leq 31$ . However, the small-world networks are not observed for both fiscal years.

Additionally, compared to the in-degree distributions, the frequency of each major bank is strictly one. A bank with a high out-degree owes debts to many banks (see Figure 2) and there may be no substitute for the bank. It is important that the lack of readily available substitutes is an evaluation criterion of G-SIBs. Therefore, the banks listed in Table 2 are systemically important lenders in the interbank network, and represent Domestically Systemically Important Banks (D-SIBs)<sup>6</sup> as well as G-SIBs.

#### 4.3. Average path length

We calculated the average length of the networks as of March 2009 and March 2013. Each length is, respectively, 1.07 and 1.14. We need to note the possibility that all nodes cannot be reached in a direct graph. The average path length in the undirected interbank network is reported as  $2.26 \pm 0.03$  in Boss et al. (2004). In comparison, the Japanese interbank network resembles a small-world.

#### 4.4. SIR model

The Susceptible-Infected-Removable (SIR) model is the most widely used model in the epidemic spreading literature (for reference, see Jackson (2010)). In this type of model, populations are classified into different states according to different spreading activities and different contracts. These states are mainly susceptible, infected, and recovered. Once a node arrives at the infected state, it has either recovered and is no longer susceptible, or it has died. On a network graph, the nodes represent banks that are in one of the states, whereas the edges represent the interbank contracts between banks.

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<sup>6</sup>D-SIBs are banks whose failure or impairment would have external effects that would damage the domestic real economy. The purpose of the D-SIB regulation is to limit those effects, as well as the likelihood of failure or impairment.

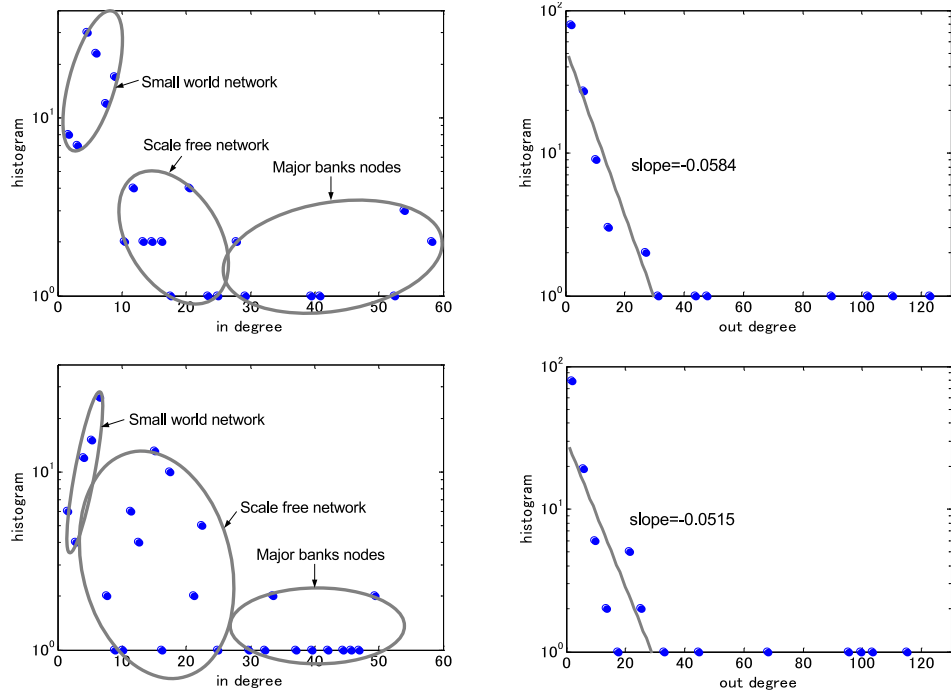


Figure 4: Degree distributions

Note: upper left panel: in-degree in March 2009, upper right panel: out-degree in March 2009, lower left panel: in-degree in March 2013, lower right panel: out-degree in March 2013

Table 2: Banks at major banks nodes of degree distributions

	March 2009	March 2013
in-degree	Mizuho Financial Group (Mizuho Bank, Mizuho Corporate Bank), Mitsubishi UFJ Financial Group (Bank of Tokyo-Mitsubishi UFJ, Mitsubishi UFJ Trust Bank), Sumitomo Mitsui Financial Group (Sumitomo Mitsui Banking Corporation), Resona Bank, Aozora Bank, Citibank Japan, Norinchukin Bank, Bank of Yokohama, Bank of Kyoto (total 11 banks)	Mizuho Financial Group (Mizuho Bank, Mizuho Corporate Bank), Mitsubishi UFJ Financial Group (Bank of Tokyo-Mitsubishi UFJ, Mitsubishi UFJ Trust Bank), Sumitomo Mitsui Financial Group (Sumitomo Mitsui Banking Corporation), Sumitomo Mitsui Trust Bank, Resona Bank, Saitama Resona Bank, Citibank Japan, Norinchukin Bank, 77 Bank, Bank of Yokohama (total 12 banks)
out-degree	Mizuho Financial Group (Mizuho Bank, Mizuho Corporate Bank, Mizoho Trust Bank), Mitsubishi UFJ Financial Group (Bank of Tokyo-Mitsubishi UFJ, Mitsubishi UFJ Trust Bank), Sumitomo Mitsui Financial Group (Mitsui Sumitomo Bank), Norinchukin Bank (total 7 banks)	Mizuho Financial Group (Mizuho Bank, Mizuho Corporate Bank, Mizoho Trust Bank), Mitsubishi UFJ Financial Group (Bank of Tokyo-Mitsubishi UFJ), Sumitomo Mitsui Financial Group (Sumitomo Mitsui Bank), Norinchukin Bank, Shizuoka Bank (total 7 banks)

The SIR model is described as follows:

$$\begin{aligned}
\frac{dS_k(t)}{dt} &= -\lambda I_k(t) S_k(t) \\
\frac{dI_k(t)}{dt} &= \lambda I_k(t) S_k(t) - \mu I_k(t) \\
\frac{dR_k(t)}{dt} &= \mu I_k(t)
\end{aligned} \tag{8}$$

where  $k$  denotes the degree of a bank in the interbank network,  $S_k(t) + I_k(t) + R_k(t) = 1$ ,  $S_k(t)$ : a solvent bank,  $I_k(t)$ : a defaulted bank, and  $R_k(t)$ : a recovered bank. Solvent banks default at a rate proportional to both the number of solvent banks  $S_k(t)$  and the number of defaulted banks  $I_k(t)$ . Hence, the  $\lambda I_k(t) S_k(t)$  represents the speed at which a solvent bank defaults. The  $\lambda$  represents the contagious rate at which a solvent bank and an insolvent bank have a connection. Contrastingly, defaulted banks recover at a rate of  $\mu$ . The  $\mu I_k(t)$  is the speed at which a defaulted bank recovers and acquires immunity. It is to be noted that, as a recovered bank neither cause a contagious default to an other bank nor be affected by a defaulted bank, the recovered bank is considered as a bank which liquidated and left the interbank market.

Default in the interbank market happens when a bank does not fulfill the debt obligation. According to the recovery and resolution plan (RRP) framework in Basel III regulation (see FSB (2011)), the defaulted bank can be saved only by means such as merger and acquisitions from other financially solvent banks. An official capital injection by the government is now out of consideration. The SIR model framework hence is in consistent with the RRP framework.

The critical contagious rate  $\lambda_c$  in a steady state for the interbank network can then be solved analytically as follows:

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle} \tag{9}$$

where  $\langle k \rangle$  and  $\langle k^2 \rangle$ , respectively, denote the average degree and the average of the square of degrees in the interbank network. Systemic default happens in the absence of a non-zero contagious threshold  $\lambda_c$ . In the case of  $\lambda > \lambda_c$ , the systemic defaults spread and a financial crisis occurs, whereas in the case of  $\lambda < \lambda_c$ , banks rarely default and no systemic default happens.

As a calculation result, the values of  $\lambda_c$  for out-degree and in-degree in



the Japanese interbank network are 0.01680 and 0.04079 in March 2009, and 0.01642 and 0.04486 in March 2013, respectively. After the global financial crisis, the infection threshold for out-degree is almost unchanged, but the contagious threshold for in-degree has ascended by 10%. The ascent of the contagious threshold for in-degree implies that contributors or contributions to systemic risk of the Japanese interbank market increased. This assertion is affirmed by Table 2, in section 4.2.

## 5. Conclusions

We studied the assessment of systemic risk of the Japanese interbank system network structure. The analysis of the Japanese interbank market enabled the systemic risk analysis in a closed network. The results validated the selection of three mega-bank groups as G-SIBs in terms of size, inter-connectedness, and substitutability. Additionally, they provided the list of D-SIB candidates.

In contrast to Western banks, Japanese banks are considered to have been impacted to a lesser extent by the global financial crisis. The recovery and resolution plan is imposed on G-SIBs in Basel III regulation; however, this regulation does not resemble previous official capital injections previously instituted by the Japanese government.

Finally, we are convinced of the need to utilize the systemic risk assessment methodologies based on the network theory, in order to maintain the stability of the financial system.

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## Appendix A. RAS algorithm

A problem in equation (4) can be solved by the RAS algorithm (see Censor and Zenios (1998)). Given an initial value  $L^0$ , the algorithm works as follows:

**Step 1** : row scaling

$$x_{i,j}^U \leftarrow x_{i,j}^U \rho_{i,j}^U, \quad \text{where } \rho_{i,j}^U = \frac{a_i}{\sum_{\forall j | x_{i,j}^0 > 0} x_{i,j}^U} \quad (\text{A.1})$$

**Step 2** : column scaling

$$x_{i,j}^{U+1} \leftarrow x_{i,j}^U \rho_{i,j}^U, \quad \text{where } \sigma_{i,j}^U = \frac{l_i}{\sum_{\forall i | x_{i,j}^0 > 0} x_{i,j}^U} \quad (\text{A.2})$$

**Step 3** :  $U \leftarrow U + 1$ , and return to Step 1

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