Co-Movement Risk Premiums and Return Predictability

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Abstract

This paper develops a new measure of asset co-movement and studies the associated price of risk. Based on characteristic functions, our new measure captures all forms of co-movement, can isolate higher-order co-movement, is model free and can be computed under the risk neutral and physical measures allowing direct measurement of risk premia. We find a positive and statistically significant co-movement risk premium at all examined horizons whereas higher-order co-movement exhibits a risk premium that changes sign as the return horizon increases. We examine the determinants of co-movement risk premia and further show that these risk premiums can help to predict future returns on the S&P 100 index, complementing other classic predictors such as the P/E ratio, default spread and consumption-wealth ratio (CAY), with strongest predictability occurring at the one-year horizon.

1 Introduction

Understanding the co-movement between assets is a primary concern for financial economists. The way in which asset returns are related to one another determines an investor's ability to diversify and this in turn has significant consequences for portfolio choice, asset valuation and risk management. At present, the co-movement between assets is dominated by a single measure, covariance. One drawback of covariance as a measure of co-movement is that it only provides information about the second moment of the return distribution. However, it is well understood that asset returns are related to one another in non-linear ways with higher-order moments playing a significant role in determining the nature of return co-movement.[1]

In this paper, we develop an aggregated measure of asset co-movement, similar to the implied correlation index published by the Chicago Board Options Exchange (CBOE), that addresses shortcomings associated with standard co-movement measures. Using the entire distribution, our measure accommodates all forms of co-movement, including non-linearities and dependencies associated with higher-order moments, and is free from distributional assumptions. Our measure is closely related to robust dependence/co-movement measures, called distance correlation/covariance,

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[1]For example, Harvey and Siddique (2000) show that systematic coskewness carries a significant premium and is related to the momentum factor, Bakshi et al. (2003) show that skew is important for the pricing of individual options, Pan (2002) and Bollerslev and Todorov (2011) examines jump risk premia while Caporin et al. (2017) study co-jumps in stock returns and find that they provide short term predictive power and are strongly correlated with changes in the variance risk premium.

developed in Székely et al. (2007) and may be computed under both the risk neutral measure, via traded option prices, and the physical measure, via observed asset returns, allowing for direct computation of a price of co-movement risk.

We compute the price of co-movement risk from 1996 to 2021 using the S&P 100 index and its constituents, finding that co-movement exhibits significant time series variation, spiking in times of economic crisis. Measuring risk premia as the difference between risk neutral and physical measures, we find that there is, in general, a positive risk premium associated with both co-movement (all forms) and higher-order forms of co-movement (co-movement that excludes the contribution from correlation and variance). Similar results are found in the literature for variance (see Carr and Wu, 2008), skewness (see Kozhan et al., 2013) and correlation (see Driessen et al., 2009). Option implied risk premia, such as those in the aforementioned studies, are found to be positive (risk neutral values are higher than physical). However, we find that higher-order forms of co-movement exhibit a negative risk premium (physical values larger than risk neutral) at 30-day investment horizons, zero risk premium at 60-day horizons and positive risk premiums at all longer horizons. As far as the author is aware, this represents the first option implied risk premium that is negative and suggests that investors do not price in higher-order forms of co-movement risk at short investment horizons but require compensation for higher-order co-movement risk as the investment horizon increases. We identify several determinants of co-movement risk premia and find that while being strongly connected to variance, these premia are also connected to some commonly accepted asset pricing factors, specifically the excess return on the market, the high-minus-low (HML) and conservative-minus-agressive (CMA) factors of Fama and French (2015). This result lends evidence to the notion that underlying factors that have been shown to explain the cross-section of stock returns also explain the premium associated with co-movement. This finding suggests that co-movement premia can be used to assess asset pricing factors and is hence related to the protocol developed by Pukthuanthong et al. (2018) which suggests that relevant asset pricing factors must explain both the first and second moments of asset returns.

Finally, motivated by Bollerslev et al. (2009), we examine whether the risk premium associated with co-movement (higher-order co-movement) can predict returns. While we find evidence that risk premia associated with higher-order forms of co-movement can predict returns at the 365-day horizon in a univariate setting, we do not find predictability among any of our other option implied risk premia, including variance and correlation. This finding runs counter to those of Bollerslev et al. (2009) and Buss et al. (2019), though it must be noted that we examine the S&P 100 index whereas those authors study the S&P 500 index. However, in multivariate settings where we combine our option implied risk premia with other classic prediction variables such as those found in Lamont (1998), Lettau and Ludvigson (2001) and Ang and Bekaert (2007), we find that our option implied risk premia provide significant explanatory power, producing a predictive R^2 of over 40% at the 365-day horizon. Critically, we find that despite their relatively high correlation, multiple option implied risk premia are required to help predict returns suggesting that the combination of certain risk premia, such as variance and higher-order co-movement, capture signals regarding future returns.

Our co-movement measures are obtained from the characteristic function (CF) of returns. CFs are an alternative way of representing the distribution of a random variable with a one-to-one mapping existing between probability densities and CFs. However, CFs have particularly useful results for sums of independent random variables. While the probability density for sums of independent random variables is related to marginal densities via convolution, the equivalent calculation using CFs requires only multiplications. This makes CFs an ideal tool to study problems involving portfolio returns, which are simply weighted sums of asset returns. Our approach to measuring co-movement examines the difference between the CF of a portfolios return and the CF that would be obtained if all assets were independent, with the latter distribution being computed directly from marginal CFs. Additionally, CFs can be computed under the risk neutral measure, $\mathbb{Q}$, via option prices (see Todorov, 2019; Todorov and Zhang, 2023, and Appendix B for derivations) and under the physical measure, $\mathbb{P}$, via historical returns (see Carrasco and Florens, 2000; Singleton, 2001; Jiang and Knight, 2002; Chacko and Viceira, 2003; Malloch et al., 2021) via the empirical characteristic function (ECF).

There is presently little theoretical work that specifically examines the price of asset co-movement risk and the connection to equity risk premia. Some notable exceptions include Driessen et al. (2009) who examine correlation risk as a distinct component of the market variance risk premium and Buraschi et al. (2014) who develop a model that links the correlation risk premium to belief disagreement among investors. However, these authors use a framework based on Ito processes and hence cannot account for forms of higher-order co-movement that may be induced by (co-)jumps in asset returns. Martin (2013) addresses this issue by developing a general equilibrium model in an endowment economy where investors draw consumption from the dividends of multiple assets (or trees in the language of Lucas (1978)). The dividend stream from each asset is correlated and assumed to follow an iid Levy process. This model hence allows for nonlinear co-movement between assets based on features like jumps. Martin (2013) finds that there are a variety of non-trivial risk premia that arise through these nonlinear dependencies that cannot be accounted for in asset pricing models that focus on first and second moments only. The results in our paper provide empirical support to asset pricing theories that incorporate multiple assets in a general setting like that developed by Martin (2013). To illustrate, Martin (2013) shows that in an economy populated with a large and small firm, disasters, characterized by large jumps, spread from the large asset to the small asset even if the small assets cash flows remain relatively stable. This feature presents a channel through which significant higher-order co-movements contribute to risk premia. Our empirical results find that the price of risk associated with higher-order forms of co-movement, which captures features such as (co-)jumps, can predict returns at the 365-day horizon and hence is related to expected returns.

Our paper makes contributions along several dimensions. First, we contribute a new measure of asset co-movement to the literature. In financial economics, co-movement between assets has been dominated by correlation/covariance since the seminal work of Markowitz (1952) who developed the notion that return variance is a suitable proxy for risk. In the context of aggregated co-movement measures that can be computed under both risk neutral and physical measures, which is the focus of this paper, implied correlation developed by Skintzi and Refenes (2005) is the primary method of computing dependence (see Buss et al., 2019). An alternative form of dependence studied in the literature is asymmetric dependence; the finding that correlations change in different parts of the distribution. Key examples of this literature include Longin and Solnik (2001), Ang and Chen (2002), Patton (2004), Alcock and Hatherley (2016) and Jiang et al. (2018) among others. Our proposed co-movement goes beyond both implied correlation and asymmetric dependence in the sense that it captures all forms of dependence, including those associated with correlation/covariance and asymmetric dependence. Additionally, we demonstrate how to eliminate specific forms of co-movement from our complete measure allowing for the computation of co-movement measures that exclusively contain higher-order co-moments.

We also contribute new results regarding risk premiums derived from option prices. Carr and Wu (2008) study the variance risk premium; the finding that option implied variances are higher, on average, than realized variances, via the return on variance swaps. These authors find that the variance risk premium is positive (risk neutral variance larger than physical) for a variety of assets and indices.[2] Kozhan et al. (2013) similarly study the skew risk premium, finding that variance and skew risk premiums are strongly connected in the sense that earning the premium for one risk while hedging the other produces zero return on average. Estimating the risk premia associated with co-moments is more complicated than those associated with the moments of marginal distributions as we do not observe options whose value depends on pairs of assets. Nevertheless, the literature has developed methods to partially circumvent this issue. Driessen et al. (2009) study the correlation risk premium, finding that aggregated correlation under the risk neutral measure is systematically larger than under the physical and study this via a dispersion trading strategy. Importantly, these authors find the variance risk premium associated with indices derives almost entirely from the correlation risk premium.

The results in this paper also make contributions to option based measures of co-movement. For instance, Buss and Vilkov (2012) compute risk neutral betas by combining option implied variances with historical correlations that have been converted to risk neutral values via a parametric transform. These authors find that they can identify a monotonic risk-return relation not observed using historical beta estimates and that option implied betas can forecast realized betas. Chang et al. (2011) develop an alternative method of computing risk neutral betas that only require forward looking information, but must assume the skewness of the idiosyncratic error term is zero. Christoffersen et al. (2021) show that the price of coskewness (cokurtotis) risk can be linked to the markets variance (skewness) risk premium and hence may be computed from option prices on the market index. A further contribution of this work is related to the price of risk associated with alternative

[2] Carr and Wu (2008) refer to this finding as a negative risk premium as they measure the risk premium as the expected return on a long variance swap. In later literature (see Bollerslev et al., 2009, 2014, among others), these risk premia a typically defined as risk neutral values minus physical. I follow this newer convention and hence refer to the premium found in Carr and Wu (2008) as a positive risk premium.

forms of co-movement. Alcock and Hatherley (2016) find that asymmetric dependence carries a significant price of risk. In this paper, we examine the price of risk associated with all forms of co-movement, including those associated with option implied betas and coskewness/cokurtosis and asymmetric dependence, in a single measure. Additionally, our method for computing risk neutral co-movement requires no historical data and makes no parametric or distributional assumptions.

Our work contributes to the literature related to return predictability by showing that the price of co-movement risk can predict future index returns. Bollerslev et al. (2009), among others, show that the variance risk premium can predict returns of the S&P 500 index over horizons of up to a quarter while Bollerslev et al. (2014) find similar evidence in an international setting. Buss et al. (2019) show that the correlation risk premium predicts S&P 500 returns at longer horizons of around one year. Examining the S&P 100 index, we find weaker evidence for the variance risk premium being a predictor of future returns. However, we do find that higher-order dependence can predict S&P 100 returns at the 365-day horizon in a univariate setting and that when option implied risk premia, including our new co-movement measures, are combined with classic predictor variables found in Lamont (1998), Lettau and Ludvigson (2001) and Ang and Bekaert (2007), they provide significant additional predictive power, hence complementing these classic predictor variables.

Finally, we make contributions to literature employing CFs to address problems in financial economics. CFs have been used to address complex problems in asset/option pricing models where state variables follow affine jump diffusions (see Heston, 1993; Duffie et al., 2000, among many others) and address complex econometric estimations via the empirical characteristic function (ECF) (see Carrasco and Florens, 2000; Singleton, 2001; Jiang and Knight, 2002; Chacko and Viceira, 2003; Malloch et al., 2021, among others). More recently, Todorov (2019) and Todorov and Zhang (2023) use the risk neutral CF extracted from option prices to address issues associated with spot volatility estimation. We similarly use option-implied CFs and return-based ECFs to uncover co-movement between assets and compute an associated price of risk. Our estimates of these CFs require no parametric or distributional assumptions. Our work draws on insights from Székely et al. (2007) who develop robust measures of co-movement/dependence called distance covariance/correlation which accommodates non-linearities and higher-order features that cannot be captured with traditional measures such as covariance/correlation. It is well known that asset returns exhibit significant deviations with higher-order moments being significant in determining risk premiums. For instance, Bollerslev et al. (2015) find that much of the predictive power of the variance risk premium derives from the compensation investors require for jump tail risk while Caporin et al. (2017) show that asset returns do co-jump and that these movements are highly correlated with changes in the variance risk premium and contain predictive power int he short term. These types of risk naturally fall outside the second-order moment and hence measuring these risks and associated premia requires methods capable of capturing higher-order information, like those developed in this paper.

The remainder of this paper proceeds as follows. Section 2 introduces characteristic functions and outlines some of their main properties employed in our study. Section 3 develops our new measure of co-movement and provides background results on comparable measures such as implied correlation. We also illustrate in this section how one may obtain the characteristic function of returns under both the risk neutral and physical measures. Section 4 outlines our empirical analysis providing results on the presence of co-movement risk premia, the determinants of co-movement risk premia and return predictability tests while Section 5 concludes.

2 Characteristic Functions

In this section, we provide some background on CFs and how they may be used to define a measure of the aggregated co-movement between a portfolios constituent assets. Definition 2.1 defines the CF and lists some properties that will be used throughout the remainder of the paper.

Definition 2.1. *The t-conditional CF of a random variable, X_T $(T > t)$ under the measure $\mathbb{M}$, $\varphi_X^{\mathbb{M}}(t)$, is defined as*

$$\varphi_X^{\mathbb{M}}(s) = \mathbb{E}_t^{\mathbb{M}}\left[e^{isX_T}\right] \tag{1}$$

where $i = \sqrt{-1}$. Some useful properties of characteristic functions include:

1. *The CF of a real valued random variable always exists.*
2. $\varphi_X^{\mathbb{M}}(0) = 1$
3. *The CF is bounded: $|\varphi_X^{\mathbb{M}}(s)| \leq 1$ where $|z| = \sqrt{z\overline{z}}$ is the complex modulus and $\overline{z}$ is the complex conjugate of z.*
4. *If a random variable has moments up to order-k, then $\mathbb{E}_t^{\mathbb{M}}[X_T^k] = i^{-k}(\varphi_X^{\mathbb{M}})^{(k)}(0)$ where $(\varphi_X^{\mathbb{M}})^{(k)}$ is the kth derivative of the CF.*
5. *If $X_1, X_2, \ldots, X_n$ are independent and $a_1, a_2, \ldots, a_n$ are constants, then the CF of the linear combination is given by*

$$\varphi_{a_1X_1+a_2X_2+\ldots+a_nX_n}^{\mathbb{M}}(s) = \varphi_{X_1}^{\mathbb{M}}(a_1s) \times \varphi_{X_2}^{\mathbb{M}}(a_2s) \times \ldots \times \varphi_{X_n}^{\mathbb{M}}(a_ns)$$

6. *Let the random variable $Y = a + bX$ where X has CF $\varphi_X^{\mathbb{M}}(s)$. Then Y has the CF*

$$\varphi_Y^{\mathbb{M}}(s) = e^{isa}\varphi_X^{\mathbb{M}}(bs) \tag{2}$$

Equation (2) implies that one may compute a demeaned CF. Say the random variable X has $\mathbb{E}_t^{\mathbb{M}}[X] = \mu$. Then the CF of the random variable $Z = X - \mu$, which has $\mathbb{E}_t^{\mathbb{M}}[Z] = 0$, is given by $\varphi_Z^{\mathbb{M}}(s) = e^{-is\mu}\varphi_X^{\mathbb{M}}(s)$.

7. *The Taylor series representation of the characteristic function, expanded around 0, is given*

by

$$\varphi_X^{\mathbb{M}}(s) = 1 + \sum_{k=1}^{\infty} \frac{i^k s^k}{k!} \mathbb{E}_t^{\mathbb{M}}[X_T^k]. \tag{3}$$

Equation (3) shows that even moments (k) are captured by the real part of the characteristic function while odd moments are captured by the complex part.

Details regarding the computation of the characteristic function under the risk neutral measure ($\mathbb{M} = \mathbb{Q}$) and the physical measure ($\mathbb{M} = \mathbb{P}$) are provided in Sections 3.2.1 and 3.2.2 respectively. Going forward, we will omit the superscript identifying the measure when discussing results that hold under either measure. Next, we turn our attention to employing CFs in a new measure of co-movement.

3 A New Measure of Co-Movement

We use the results on characteristic functions, outlined in Definition 2.1, to develop a new measure of asset co-movement. To make our notation precise, let the price of asset j at time t be $S_{j,t}$. In our case j can identify a stock ($j = 1, 2, \ldots, n$) or a portfolio ($j = p$). The log return on asset j from t to T is hence given by $r_{j,t\to T} = \log \frac{S_{j,T}}{S_{j,t}}$. We refer to the volatility of returns at t as $\sigma_j = \sqrt{\text{Var}[r_{j,t\to T}]}$. Before developing our new co-movement measure, we first provide some background on the closely related dependence measure, implied correlation.

3.1 Implied Correlation

Implied correlation is a measure of aggregate asset dependence computed by equating the variance of a portfolio and the variance of its constituents through a single value for correlation. Portfolio variance, σ_p^2, is related to the variance of its n constituents via

$$\sigma_p^2 = \sum_{j=1}^{n} w_j^2 \sigma_j^2 + 2 \sum_{j=1}^{n-1} \sum_{k>j} w_j w_k \rho_{j,k} \sigma_j \sigma_k \tag{4}$$

where w_j is the weight of asset j in the portfolio and $\rho_{j,k,t}$ is the correlation between assets j and k. Assuming that all pairwise correlations are equal to a single implied correlation value, $\rho_{j,k} = IC_t$ for all j, k, we can write IC_t as a function of portfolio weights and volatilities,

$$IC_t = \frac{\sigma_p^2 - \sum_{j=1}^{n} w_j^2 \sigma_j^2}{2 \sum_{j=1}^{n-1} \sum_{k>j} w_j w_k \sigma_j \sigma_k}. \tag{5}$$

Implied correlation can be computed under either the risk neutral ($\mathbb{Q}$) or physical ($\mathbb{P}$) measure by inserting the corresponding volatilities into (5). We refer to risk neutral implied correlation as $IC_t^{\mathbb{Q}}$ and physical implied correlation as $IC_t^{\mathbb{P}}$. We note that only information regarding the second moment (namely volatilities) is used in the computation of implied correlation implying that dependence associated with high-order moments will not be included. The correlation risk premium studied by Buss et al. (2019) among others is defined as

$$CRP_t = IC_t^{\mathbb{Q}} - IC_t^{\mathbb{P}} \tag{6}$$

which holds the interpretation as the payoff of a short correlation swap.

3.2 Implied Co-Movement

We now develop our general measure of co-movement. First, denote the CF of log returns, for the portfolio p and its n constituents as $\varphi_{r_{j,t\to T}}(s)$ where $j = \{p, 1, 2, \ldots, n\}$. Next, let $\tilde{r}_{j,t\to T}$ be the demeaned log return. By property 6 of Definition 2.1 we have that

$$\varphi_{\tilde{r}_{j,t\to T}}(s) = e^{-is\mu}\varphi_{r_{j,t\to T}}(s) \tag{7}$$

where $\mu = \mathbb{E}_t[r_{j,t\to T}]$. Our measure of total, aggregated co-movement is stated in definition 3.1.

Definition 3.1. *The measure of total aggregate co-movement under the measure* $\mathbb{M}$ *at time* t *is given by*

$$D_t = \int_{-\infty}^{\infty} \left| \frac{\varphi_{\tilde{r}_{p,t}}(s) - \prod_{i=1}^{n} \varphi_{\tilde{r}_{i,t}}(w_i s)}{\prod_{i=1}^{n} \varphi_{\tilde{r}_{i,t}}(w_i s)} \right| \omega(s) ds \tag{8}$$

where $\omega(s)$ *is a positive valued weighting function with the property* $\int_{-\infty}^{\infty} \omega(s)ds < \infty$.

Our measure, D_t, exploits property 5 in Definition 2.1 to extract co-movement. To illustrate, if all assets were independent then the terms $\varphi_{\tilde{r}_{p,t}}(s)$ and $\prod_{i=1}^{n} \varphi_{\tilde{r}_{i,t}}(w_i s)$ would be equal for all s. In fact one may interpret $\prod_{i=1}^{n} \varphi_{\tilde{r}_{i,t}}(w_i s)$ as the characteristic function of demeaned returns under the assumption of independence. To make this point concrete we write this CF under the assumption of independence as

$$\varphi^{\perp}_{\tilde{r}_{p,t}}(s) = \prod_{i=1}^{n} \varphi_{\tilde{r}_{i,t}}(w_i s) \tag{9}$$

Our co-movement measure has the properties $D_t \geq 0$ with $D_t = 0$ meaning assets returns are independent. We emphasise that assets with an implied correlation of 0 can still exhibit co-movement as assets may be related non-linearly, a feature captured by higher-order co-moments. The measure D_t on the other hand is a true measure of co-movement, taking values of 0 only when assets are truly independent. One drawback of the measure D_t is that it is not signed; positive and negative forms of co-movement both produce a positive value. However, it is important to note that implied correlation suffers a similar problem. Because the covariance matrix must remain positive definite, as the number of assets increase the lower bound on allowed values of IC_t increases such that $\lim_{n\to\infty} IC_t \geq 0$, and hence, in this limit, implied correlation is also unsigned.

The weighting function $\omega(s)$ can be defined by the econometrician to control the degree to which certain moments impact D_t. It is well known, and can observed by examination of the Taylor expansion (see property 7 in Definition 2.1), that for small values of $|s|$, the CF is dominated by low-order moments with higher-order moments becoming more dominant as $|s|$ increases. Hence, by appropriately selecting $\omega(s)$, we can alter the influence that various moments have on the estimation of D_t. This is an important practical consideration as higher-order moment estimates are particularly noisy when there is insufficient data to probe the tails of the distribution.[3] We consider two choices for the weighting function $\omega(s)$,

$$\omega(s) = \omega_G(s) = e^{-\frac{s^2}{b}} \tag{10}$$

$$\omega(s) = \omega_T(s) = \mathbf{I}(|s| \leq B) \tag{11}$$

where $b > 0$ is a constant that governs the width of the Gaussian function ω_G, $\mathbf{I}$ is the indicator function and the constant $B > 0$ represents a truncation bound to the integral in (8). Using ω_G applies a smooth weighting across the entire domain of s while using ω_T truncates the limits of the integral in (8) to the interval $[-B, B]$. We examine the performance of each of these weighting functions in a simulation study presented in Appendix A. In our main results, we use $\omega(s) = \omega_G(s)$ for two main reasons. First, our simulation results suggest that we can incorporate a much larger part of the distribution without inducing additional error into our estimates when using $\omega(s) = \omega_G(s)$ relative to $\omega(s) = \omega_T(s)$ and second, we can efficiently compute integrals across the entire domain of s via Gauss-Hermite quadrature. Hence using $\omega(s) = \omega_G(s)$ allows us to gain extra information into our estimates with lower error and more computational efficiency.

Similar to the CRP defined in equation (6), we can also define a co-movement risk premium via

$$DRP_t = D_t^{\mathbb{Q}} - D_t^{\mathbb{P}}$$

Computing this risk premium requires that we compute characteristic functions under both risk neutral and physical measures which we address in Sections 3.2.1 and 3.2.2 respectively.

3.2.1 Risk Neutral Characteristic Functions

The characteristic function of log returns can be computed under $\mathbb{Q}$ from European option prices. This characteristic function is presented in Todorov (2019) and obtained using the spanning relations in Carr and Madan (2001).

[3]When working under the risk neutral measure, probing the tails of the distribution requires deep out-of-the-money options while under the physical measure it requires larger sets of observed returns to capture significant, but infrequent, observations.

Theorem 3.1. *The risk neutral characteristic function of log returns may be computed via*

$$\varphi^{\mathbb{Q}}_{r_{j,t\to T}}(s) = e^{isr_f(T-t)} - e^{r_f(T-t)}(s^2+is)\times \left[\int_0^{F_{t\to T}} \frac{1}{K^2} e^{is\log\left(\frac{K}{S_t}\right)} Put_{t,T}(K)dK + \int_{F_{t\to T}}^{\infty} \frac{1}{K^2} e^{is\log\left(\frac{K}{S_t}\right)} Call_{t,T}(K)dK\right] \tag{12}$$

where $Put_{t,T}(K)$ and $Call_{t,T}(K)$ are put and call option prices observed at t with maturity T and strike K, r_f is the continuously compounded risk-free rate and $F_{t\to T}$ is the forward price of the asset.

Proof. See Appendix B. □

3.2.2 Physical Characteristic Functions

To compute t-conditional, forward looking expectations at $T > t$ under $\mathbb{P}$ in a model free way, we follow the literature and use observed returns to estimate the empirical characteristic function to serve as our proxy for $\varphi^{\mathbb{P}}_t$. Specifically, let $\{r_{j,t\to t+\Delta t}, r_{j,t+\Delta t\to t+2\Delta t}, \ldots, r_{j,t+(N-1)\Delta t\to t+N\Delta t}\}$ be a series of N return observations that are assumed iid. We then have that

$$\varphi^{\mathbb{P}}_{r_{j,t\to t+\Delta t}}(s) = \frac{1}{N}\sum_{\tau=1}^{N} e^{isr_{j,t+(\tau-1)\Delta t\to t+\tau\Delta t}} \tag{13}$$

where $r_{j,t+(\tau-1)\Delta t\to t+\tau\Delta t}$ is the log return on asset j over the interval $[t+(\tau-1)\Delta t, t+\tau\Delta t]$ and $i = \sqrt{-1}$. In the limit as $N \to \infty$, the empirical characteristic function converges to the true characteristic function for iid observations (see Feuerverger and Mureika, 1977). Following Buss et al. (2019), we examine two possible choices for the series of returns used to compute $\varphi^{\mathbb{P}}_t(s)$.[4] The ex-ante estimate of $\varphi^{\mathbb{P}}_t(s)$ uses returns over the interval $[t-T, t]$ to produce an estimate of $\varphi^{\mathbb{P}}_t(s)$ over the horizon $[t, T]$. The ex-post estimator uses returns observed over the interval $[t+\Delta t, T]$. For the ex-post estimator, we assume the realized outcomes are equivalent to expectations while the ex-ante estimator assumes that future expectations match past realizations. While co-movement risk premia computed using ex-post estimators of $\varphi^{\mathbb{P}}_t(s)$ have the useful interpretation as the payoff from a swap arrangement (the co-movement swap), they cannot be used in forecasting exercises. While we study both estimators when computing co-movement risk premia, we follow Bollerslev et al. (2009) (among others) and use ex-ante estimators in return prediction studies and follow Carr and Wu (2008) (among others) by using ex-post estimators for studying the determinants of co-movement risk premia.

3.2.3 Isolating Higher-Order Co-Movement

Working with the characteristic function allows the econometrician to exclude specific moments from the co-movement measure. We hence develop an additional co-movement measure which

[4]This is the same approach used in other studies such as Bollerslev et al. (2009).

excludes the contribution from the second-order moments which we call a higher-order co-movement measure, H_t. Our methodology exploits the Taylor expansion provided in property 7 of Definition 2.1. To illustrate our approach, the Taylor series representation of the demeaned CF of log returns for the portfolio p is given by,

$$\varphi_{\tilde{r}_{p,t\to T}}(s) = 1 - \frac{s^2}{2} \underbrace{\mathbb{E}_t[(\tilde{r}_{p,t\to T})^2]}_{\mathrm{Var}_t[\tilde{r}_{p,t\to T}]} - \frac{is^3}{6} \mathbb{E}_t[(\tilde{r}_{p,t\to T})^3] + \dots \tag{14}$$

The term to order s^2 in (14) contains all information regarding the assets variance and hence

$$\mathrm{Var}_t[\tilde{r}_{p,t\to T}] = \sigma^2_{p,t\to T} = \sum_{j=1}^{n} w_j^2 \sigma_j^2 + 2 \sum_{j=1}^{n-1} \sum_{k>j} \rho_{j,k} \sigma_j \sigma_k. \tag{15}$$

Similarly, the demeaned CF of log returns for the product of the marginal CFs is given by,

$$\varphi^{\perp}_{\tilde{r}_{p,t\to T}}(s) = 1 - \frac{s^2}{2} \underbrace{\mathbb{E}^{\perp}_t[(\tilde{r}_{p,t\to T})^2]}_{\mathrm{Var}^{\perp}_t[\tilde{r}_{p,t\to T}]} - \frac{is^3}{6} \mathbb{E}^{\perp}_t[(\tilde{r}_{p,t\to T})^3] + \dots \tag{16}$$

where $\mathbb{E}^{\perp}_t$ denotes the t-conditional expectation assuming independence between the assets. Hence $\mathrm{Var}^{\perp}_t[\tilde{r}_{p,t\to T}]$ is variance computed under the assumption of independence meaning that

$$\mathrm{Var}^{\perp}_t[\tilde{r}_{p,t\to T}] = \sum_{j=1}^{n} w_j^2 \sigma_j^2. \tag{17}$$

We define the demeaned CF, excluding the second moment, via

$$\varphi^{\{\neg 2\}}_{\tilde{r}_{p,t\to T}}(s) = \varphi_{\tilde{r}_{p,t\to T}}(s) + \frac{s^2}{2} \mathrm{Var}_t[\tilde{r}_{p,t\to T}] \tag{18}$$

and similarly for the demeaned CF under the assumption of independence,

$$\varphi^{\perp\{\neg 2\}}_{\tilde{r}_{p,t\to T}}(s) = \varphi^{\perp}_{\tilde{r}_{p,t\to T}}(s) + \frac{s^2}{2} \mathrm{Var}^{\perp}_t[\tilde{r}_{p,t\to T}]. \tag{19}$$

We may then define the co-movement between higher-order moments only via

$$H_t = \int_{-\infty}^{\infty} \left| \frac{\varphi^{\{\neg 2\}}_{\tilde{r}_{p,t\to T}}(s) - \varphi^{\perp\{\neg 2\}}_{\tilde{r}_{p,t\to T}}(w_i s)}{\varphi^{\perp\{\neg 2\}}_{\tilde{r}_{p,t\to T}}(w_i s)} \right| \omega(s) ds. \tag{20}$$

Again, we may compute the price of higher-order co-movement via

$$HRP_t = H_t^{\mathbb{Q}} - H_t^{\mathbb{P}}. \tag{21}$$

We next turn our attention to addressing some practical issues associated with the computation of our co-movement measures.

3.3 Implementation Details

Implementing our co-movement measures requires that we address two issues. First, computing risk premia requires that our physical and risk neutral CFs are stated on the same time-scale. Risk neutral CFs will capture the distribution of returns over the same horizon as the maturity of the options used in its calculation (3-month options will yield a CF over a 3-month horizon) while physical CFs will capture the distribution of returns for a horizon equal to the frequency of returns used in its calculation (daily returns will yield a CF over a one day horizon). Hence, we need a method to time-scale CFs. Second, our co-movement measures require that we eliminate the mean of log returns in a model free setting. We address these issue in Sections 3.3.1 and 3.3.2 respectively.

3.3.1 Time-Scaling Characteristic Functions

One issue to address when directly comparing characteristic functions is the time-scale they represent. We require a rule, similar to that used to scale the variance of iid returns, for re-scaling characteristic functions of iid log returns. Such a rule is derived in Theorem 3.2.[5]

Theorem 3.2. *The characteristic function of iid log returns over the interval Δt, $r_{t \to t+\Delta t}$, may be converted to a characteristic function that represents returns over the interval $N\Delta t$, $r_{t \to t+N\Delta t}$, via*

$$\varphi_{r_{t \to t+N\Delta t}}(s) = \left(\varphi_{r_{t \to t+\Delta t}}(s)\right)^N. \tag{22}$$

Proof. Consider a sequence of iid log returns, $\{r_{t \to t+\Delta t}, r_{t+\Delta t \to t+2\Delta t}, \ldots, r_{t+(N-1)\Delta t \to t+N\Delta t}\}$. Each return is assumed to have the same distribution, hence the same characteristic function, and all returns are independent. Since log returns are additive, we have that

$$r_{t \to t+N\Delta t} = \sum_{j=1}^{n} r_{t+(j-1)\Delta t \to t+j\Delta t}.$$

Applying property 7 of Definition 2.1 and the iid assumption yields equation (22). □

3.3.2 Demeaning Characteristic Functions

Our proposed co-movement measures require that we eliminate the mean of log returns from the CF. We explore in this section how we may compute this mean in a model free setting. Under the physical measure, where CFs are computed via the ECF, computing the mean of log returns is

[5] Theorem 3.2 could be used to annualize any moment of the return distribution as moments can be recovered from characteristic functions via property 4 of Definition 2.1. It could also be used to annualize the entire distribution via the inverse Fourier transform, though this is not something we explore.

straightforward as we may use the sample mean of observed returns. Namely, for a set of N return observations, $\mu^{\mathbb{P}} = \mathbb{E}_t^{\mathbb{P}}[r_{i,t\to t+\Delta t}] = \frac{1}{N}\sum_{j=1}^{N} r_{t+(j-1)\Delta t\to t+j\Delta t}$. However, computing the expected value of log returns under the risk neutral measure in a model free setting is more complicated.

Recall that log returns are given by $r_{j,t\to T} = \log \frac{S_T}{S_t} = \log S_T - \log S_t$ and hence the desired risk neutral expectation is

$$\mu^{\mathbb{Q}} = \mathbb{E}_t^{\mathbb{Q}}[r_{j,t\to T}] = \mathbb{E}_t^{\mathbb{Q}}[\log S_T] - \log S_t. \tag{23}$$

While $\mathbb{E}_t^{\mathbb{Q}}[S_T] = e^{r_{f,t\to T}(T-t)}S_t$, we cannot directly use this result because of the convexity of the log function. Exchanging the log function and expectation operator requires a convexity correction. For example, in the setting of Black and Scholes (1973), this correction is equal to half the assets variance. To overcome this issue in a model free setting, we exploit property 4 in Definition 2.1. Setting $k = 1$, the first moment can be recovered from the CF via its first-order derivative. We compute this derivative numerically by computing the risk neutral CF on a fine grid of points around the origin. We then compute the centered first derivative and extract its value at $s = 0$. Multiplying by the imaginary unit, i, yields the expected log return.

4 Empirical Analysis

In this section, we compute our co-movement measures using observed returns and option prices. We conduct our analysis using the S&P 100 index, which we select for two main reasons. First, one of the criteria for inclusion in the S&P 100 is that all constituents have listed options. Second, as the S&P 100 consists of the 100 largest firms by market capitalization, we are examining firms with the most liquidly traded options. We extract option prices and associated information from the IvyDB OptionMetrics database. This includes all call/put prices for all constituent assets and the index itself. We also extract volatility surfaces, dividend projections and zero-curves from this database. Our option series begins in January 1996 and ends in December 2021. Asset returns and market capitalizations are extracted from the CRSP database. Because we also require returns on the S&P 100 index beyond the dates provided OptionMetrics, we also extract an extended S&P 100 series from Datastream covering the period 1984-2023.

To compute our co-movement measures, we use the weighting function given by (10) and hence evaluate the integrals in (8) via Gauss-Hermite quadrature. This quadrature procedure provides the CF arguments (s_j) and the associated weights (x_j). We elect to use 20 quadrature points and a width parameter $b = 1$ based on results from our simulation analysis provided in Appendix A. Ex-ante estimates of physical CFs, $\varphi^{\mathbb{P}}_{\tilde{r}_{i,t\to T}}$, are computed using daily returns available during the interval $[t-T, t]$ while ex-post estimates use returns available over the interval $[t+\Delta t, T]$ where Δt is one day. All CFs are annualized before computing our co-movement measures, which are hence also expressed in annualized terms.

Equation (12), used to compute CFs from options prices, requires that observed options have European exercise. Options on individual stocks, and the most liquid options on the S&P 100 index, all have American exercise. Hence, we cannot use these option prices directly in our calculations. To address this issue, we follow Carr and Wu (2008) and Martin and Wagner (2019) by using the Black-Scholes implied volatility surface to remove the early exercise premium. Specifically, we use the implied volatility surfaces provided by OptionMetrics which contain interpolated volatilities at a set of standardised maturities for each asset in our study. These implied volatility surfaces are computed using a binomial procedure that accounts for the early exercise premium. Following Carr and Wu (2008), we linearly interpolate these implied volatilities to create a a volatility surface across a fine grid of 2000 strike points with a range of ± 8 standard deviations. For strikes below (above) the lowest (highest) available strike, we use the implied volatility available at the lowest (highest) strike. The standard deviation used to compute the range of strikes is the average implied volatility. Using these implied volatilities and the dividend assumptions provided by OptionMetrics, we compute European option values using the Black-Scholes formula. These option values are then used to compute a discretized version of equation (12) at the CF arguments (s_j) provided by the Gauss-Hermite quadrature procedure.

4.1 Descriptive Statistics and Risk Premia

In this section we examine the statistical properties of our co-movement measures, computed at the daily frequency, to determine if their exists a statistically significant risk premium. To provide a point of comparison with existing literature, we compute the same statistics for implied correlation and index volatility. First, we compute descriptive statistics under both the physical and risk neutral measures at the 30, 60, 91, 182 and 365-day horizon for all measures used in our study. These statistics are presented in Table 1. Consistent with the literature, we find that risk neutral implied correlation and volatility is, on average, larger than their physical counterparts, indicative of the correlation risk premium found in Driessen et al. (2009) and the variance risk premium in Carr and Wu (2008). Risk neutral implied correlation and volatility have similar second-order moments to physical moments but risk neutral implied correlations have higher levels of skewness and kurtosis than physical implied correlation. Turning to our new co-movement measures, we observe that risk neutral co-movement, $D_t^{\mathbb{Q}}$, is on average larger than physical co-movement, has similar levels of volatility and skewness but tends to exhibit larger levels of kurtosis. Risk neutral higher-order co-movement, $H_t^{\mathbb{Q}}$, is higher on average, though not at the 30-day horizon, has similar volatility and higher skewness and kurtosis than the corresponding physical measure.

[INSERT TABLE 1 HERE]

In Figure 1, we present a timeseries plot of our co-movement measures and the volatility of the S&P 100 index. Figure 1a presents co-movement, $D_{t\to T}$, Figure 1b presents higher-order co-movement, $H_{t\to T}$ and Figure 1c presents the volatility, $\sigma_{t\to T}$. These measures are all computed under the risk neutral and ex-ante physical measures where $T = t + 91$ and $T = t + 365$. One thing that is immediately clear from Figure 1 is that our co-movement measure $D_{t\to T}$ is highly correlated with the index volatility. This finding is expected as our co-movement measures are not normalised like correlation is. Intuitively, our co-movement measures are more closely related to covariance than correlation, hence they contain the volatilities associated with all constituent assets.[6] Similar to volatility, we observe particularly strong measures of co-movement during crisis periods such as the global financial crisis (GFC) of 2007-2008 and the COVID-19 pandemic beginning in 2020. Interestingly, our higher-order co-movement measure exhibits very strong values during the COVID-19 pandemic, especially so under the risk neutral measure. This result suggests that investors were heavily concerned about higher-order forms of co-movement during this period, far more so than during the GFC. To illustrate this point, higher-order co-movement is approximately 20 (3) times larger during the pandemic than during the GFC when examined at the 365-day (91-day) horizon.

[INSERT FIGURE 1 HERE]

We next examine the unconditional risk premia associated with our co-movement measures. This is achieved by studying the statistical properties of the difference between the risk neutral and physical measures. We test for statistical significance of these risk premia via a t-test where standard errors are corrected for autocorrelation and heteroscedasticity using the method of Newey and West (1987). Results of these tests are presented in Table 2. Consistent with the literature (Carr and Wu, 2008; Driessen et al., 2009), we document a significantly positive volatility and correlation risk premium. We also find that generalized co-movement also exhibits a significant positive risk premium at all examined horizons. Higher-order co-movement provides some interesting counterpoints. We find at the 30-day horizon that higher-order co-movement exhibits a statistically significant negative risk premium while at the 60-day horizon we find no risk premium. This result suggests that, at the 30-day horizon, physical higher-order co-movement is, on average, larger than risk neutral higher-order co-movement. This result suggests that despite returns exhibiting higher-order co-movement, options markets are not adequately pricing this form of co-movement at short investment horizons. This mispricing doesn't occur at longer horizons suggesting that while investors do care about the risk associated with higher-order forms of co-movement, they are only willing to pay for protection against this kind of risk at longer horizons.

Given the strong connection between our co-movement measures and index volatility observed in Figure 1, we are interested in whether this connection is maintained across risk premia. Table 3 presents the correlation between our two co-movement risk premiums, DRP and HRP, the variance risk premium, VRP and the correlation risk premium, CRP. We observe that DRP and VRP are very highly correlated across all examined horizons. This result is expected as variance forms an important part of co-movement, hence making this correlation partially mechanical. Interestingly, we also observe relatively strong correlations between higher-order co-movement risk premiums and the variance risk premium. Because there is no mechanical interaction between the VRP and HRP, this result suggests that risk premia tend to move the same way. A similar observation is made by Kozhan et al. (2013) who find that variance and skew risk are tightly related. Perhaps

[6]See Appendix A for an analytic example of our co-movement measures in the Black-Scholes framework for additional details.

most interestingly, we find that CRP and HRP are weakly positively correlated at short horizons (30-91 days) and negatively correlated at longer horizons (182-365 days). This result suggests that while the risk premia associated with moments such as skewness and kurtosis is highly correlated, the risk premia associated with second-order and higher-order co-movement are far less tightly connected.

[INSERT TABLE 3 HERE]

4.2 Determinants of Co-Movement Risk Premiums

We examine here the determinants of the co-movement risk premium. Empirical asset pricing is dominated by factor models. Consequently, we are interested in which asset pricing factors, if any, determine the premium earned for bearing co-movement risk. To study the determinants of the risk premia associated with co-movement and high-order co-movement, we first compute the return earned on an asset that is exposed to these risk premia. To this end, we define the co-movement swap (DS) and higher-order co-movement swap (HS) as an asset which, when held long, pays the investor the realised value of co-movement/higher-order co-movement given by equations (8)/(20) respectively. Characteristic functions used to determine these payoffs are hence determined via equation (13) using returns observed form initiation at t to maturity at T. The cost of entering such a contract is given by the risk neutral expected value of future co-movement which is obtained using equations (8)/(20) but with characteristic functions computed via (12). The log return on these swap contracts is hence given by

$$r_{DS,t\to T} = \log\left(D^{\mathbb{P}}_{t\to T}\right) - \log\left(D^{\mathbb{Q}}_{t\to T}\right), \tag{24}$$

$$r_{HS,t\to T} = \log\left(DHO^{\mathbb{P}}_{t\to T}\right) - \log\left(DHO^{\mathbb{Q}}_{t\to T}\right). \tag{25}$$

To determine the return drivers associated with these swap contracts, and hence the drivers of the associated risk premia, we estimate a variety of regressions of the form,

$$r_{C,t\to T} = \alpha + \sum_{k=1}^{N} \beta_k F_{k,t\to T} + \epsilon_{t\to T} \tag{26}$$

where $C \in \{DS, HS\}$ and F_k are factors selected from common asset pricing models. We follow the literature (see for example Carr and Wu, 2008; Bollerslev et al., 2009, 2014) and estimate these regressions using monthly observations. We examine the standard Capital Asset Pricing Model (CAPM), the Fama and French (1992) three-factor model (FF3) and an extended model that includes the five factors of Fama and French (2015), the momentum factor of Carhart (1997) and the return on the VIX index.[7] We use monthly returns from January 1996 to December 2021

[7] All Fama and French (1992, 2015) factors and the momentum factor of Carhart (1997) are drawn from Ken French's website while the VIX index is downloaded from the CBOE website. All factors are adjusted to match the investment horizon.

to estimate these regressions with investment horizons ($t \to T$) of 30, 60, 91, 182 and 365-days. We compute robust standard errors of Newey and West (1987) with lag length equal to twice the return overlap to account for autocorrelation and heteroscedasticity. Results for regressions involving returns on co-movement swaps are presented in Table 4 while those for higher-order co-movement are presented in Table 5.

[INSERT TABLE 4 HERE]

[INSERT TABLE 5 HERE]

Starting with Table 4, we find that long co-movement swaps are negatively related to the excess market return and positively related to changes in the VIX index at all horizons. Given the strong connection between our co-movement measure and risk neutral variance, such a result is unsurprising and captures the "leverage effect" first identified in Black (1976). We also find that several other common asset pricing factors play a significant role in explaining the returns earned on a long co-movement swap. For instance, at the 182 and 365-day horizon, the factors HML, CMA and MOM are all statistically significant drivers of the return on a co-movement swap, though these results are not found at shorter horizons. While we typically study the ability of factor models to explain average rates of return, assuming a factor model also imposes an assumption on the co-movement between returns via the covariance matrix. Pukthuanthong et al. (2018) exploit this idea when developing a protocol to identify priced risk factors. Given our co-movement swap represents pure exposure to aggregated co-movement, factors that are related to an assets returns should also be related to the co-movement, including covariance and higher-order forms of co-movement. Given our co-movement measure contains second-order co-movement, it is unsurprising that classic asset pricing factors are related to co-movement risk premia.

We next turn our attention to higher-order co-movement swaps. Given we have removed the second-order component of the return distribution, the return on these assets represents the premium earned for pure exposure to higher-order forms of co-movement. Similar to co-movement swaps, we find that higher-order co-movement swaps are also significantly negatively related to excess market returns and positively related to changes in the VIX. This result suggests that there is commonality between these two measures of co-movement. Again, we emphasise that this commonality is not mechanical as we have explicitly removed second-order effects from the higher-order co-movement measures. We also find that HML and CMA factors provide explanatory power that the 30, 182 and 365-day horizons, further strengthening the argument that there is a source of commonality between co-movement, higher-order co-movement and asset pricing factors. This finding is consistent with Kozhan et al. (2013) who show that there is commonality between the variance and skew risk premia.

4.3 Return Predictability Tests

Motivated by the finding in Bollerslev et al. (2009) that the variance risk premium (VRP) can predict returns at the quarterly horizon, we examine the ability of our co-movement risk premia to predict the returns on the S&P 100 index. We also study the variance and correlation risk premiums. Following Bollerslev et al. (2009), we also include a set of traditional predictor variables that have been found to successfully predict returns in Lamont (1998), Lettau and Ludvigson (2001) and Ang and Bekaert (2007). Specifically, we include the price-earnings (PE) ratio for the S&P 100 index, defined as $\log(P/E)$, the price-dividend (PD) ratio for the S&P 100 index, defined as $\log(P/D)$, the the default spread (DS) defined as the difference between Moody's BAA and AAA corporate bond spreads, the term spread (TS) defined as the difference between the 10-year T-bond and the 3-month T-bill yields, the stochastically detrended risk-free rate ($SDRF$), defined as the 3-month T-bill yield minus its backward 12-month moving average and the consumption-wealth ratio (CAY) defined in Lettau and Ludvigson (2001).

We begin our analysis with the set of univariate regressions

$$r_{p,t\to t+T} - r_{f,t} = \alpha + \beta X_t + \epsilon_{t\to t+T} \tag{27}$$

where $X_t \in \{DRP_t, HRP_t, CRP_t, VRP_t, PE_t, PD_t, DS_t, TS_t, SDRF_t, CAY_t\}$ and T = 30, 60, 91, 182, 365-days. Following Bollerslev et al. (2015), we correct for autocorrelation caused by overlapping return observations using Newey and West (1987) standard errors with lag equal to twice the overlap for $T > 30$.[8] Results are presented in Table 6. At horizons of 30, 60, 91 and 182 days, we find no evidence of predictive power for any of the proposed predictors. However, at the 365-day horizon, the higher-order co-movement risk premium and price-dividend ratio both exhibit predictive power at the 5% level of significance with t-statistics of 2.15 and -2.15 respectively. We obtain a positive coefficient for HRP suggesting that higher prices of higher-order co-movement correspond to higher expected returns at the one-year horizon. This result implies that the risks associated with high-order co-movement are connected to expected returns over longer horizons and hence support the theory of Martin (2013) whos shows that co-jumps present a channel through which assets command risk premiums. Our results regarding the variance and correlation risk premiums stand in contrast to the existing literature. Studying the S&P 500, Bollerslev et al. (2009) find that variance risk premiums, when studied in a univariate setting, provide statistically significant predictive power at the 91-day horizon while Buss et al. (2019) find that the CRP has predictive power at 30 and 91-day horizons. Examining a different index (the S&P 100) over a longer time frame that includes the COVID-19 pandemic, we do not find that the VRP or CRP are statistically significant predictors of future returns on their own.

[INSERT TABLE 6 HERE]

Since univariate specifications are likely contaminated with omitted variable bias, we next examine four multivariate predictive regressions which include a host of predictor variables. In model

[8] We elect to use Newey and West (1987) standard errors rather than those of Hodrick (1992) because, as pointed out in Hodrick (1992) and Bollerslev et al. (2015), Hodrick-based t-statistics are formally valid under the null of no predictability by any of the variables. This makes interpretation in settings with more than one explanatory variable difficult. In contrast, Newey-West t-statistics are always (asymptotically) justified and interpretable.

1, we estimate a predictive regression utilising only option based information. Since the measure of co-movement contains the same information as variance, implied correlation and higher-order co-movement measures, we do not include these four variables together. Instead, model 1 contains VRP, CRP and HRP only. This specification captures second-order information through VRP and CRP and higher-order information through HRP. Specifically, model 1 is defined by,

$$r_{p,t\to t+T} - r_{f,t} = \alpha + \beta_{VRP}VRP_t + \beta_{CRP}CRP_t + \beta_{HRP}HRP_t + \epsilon_t \tag{28}$$

Next, in model 2, we examine the same "traditional" predictors studied in Bollerslev et al. (2009) by estimating

$$r_{p,t\to t+T-r_{f,t}} = \alpha + \beta_{PE}PE_t + \beta_{PD}PD_t + \beta_{DS}DS_t + \beta_{TS}TS_t + \beta_{SDRF}SDRF_t + \beta_{CAY}CAY_t + \epsilon_t. \tag{29}$$

In model 3, we combine the variables in models 1 and 2 and estimate

$$\begin{aligned} r_{p,t\to t+T} - r_{f,t} = \alpha + \beta_{VRP}VRP_t + \beta_{CRP}CRP_t + \beta_{DRP^{HO}}DRP_t^{HO} + \beta_{PE}PE_t + \beta_{PD}PD_t \\ + \beta_{DS}DS_t + \beta_{TS}TS_t + \beta_{SDRF}SDRF_t + \beta_{CAY}CAY_t + \epsilon_t, \end{aligned} \tag{30}$$

while in model 4 we estimate the same specification as model 3 but replace VRP, CRP and HRP with DRP to estimate,

$$\begin{aligned} r_{p,t\to t+T} - r_{f,t} = \alpha + \beta_{DRP}DRP_t + \beta_{PE}PE_t + \beta_{PD}PD_t + \beta_{DS}DS_t \\ + \beta_{TS}TS_t + \beta_{SDRF}SDRF_t + \beta_{CAY}CAY_t + \epsilon_t \end{aligned} \tag{31}$$

Results for these estimates are presented in Table 7.

[INSERT TABLE 7 HERE]

We find that higher-order co-movement, captured through HRP exhibits predictive power at the 30, 60 and 365-day horizon. Interestingly, the sign of the HRP coefficient changes from negative at the 30 and 60-day horizons to positive at the 365-day horizon. This change in sign is the opposite of what is found for the statistically significant variance risk premium which is positively related to future returns at the 30 and 60-day horizon but negatively related to returns at the 365-day horizon. We also identify a statistically significant correlation risk premium at the 60-day horizon which is negatively related to future realised returns. When all forms of co-movement are aggregated together, as measured by DRP, no predictability is found at any horizon. These results suggest that the premium associated with variance and that associated with higher-order co-movement (which also includes elements of skewness, kurtosis etc.) capture distinct elements of the drivers of future returns.

To better understand these results, Figures 2a and 2b present plots of the 60-day and 365-day estimates of HRP and VRP respectively. At the 60-day horizon, the high degree of correlation between HRP and VRP is immediately apparent. While smaller than the 60-day estimates, the 365-day results similarly exhibit high correlation. We emphasise however that this correlation is not mechanical as we have explicitly removed second-order information from the higher-order co-movement measure. This finding is consistent with Kozhan et al. (2013) who find a high degree of correlation between variance and skew risk premia for the S&P 500 index. We also observe from Figures 2a and 2b the familiar pattern to the variance risk premia presented in Bollerslev et al. (2014), though with a longer series that emphasises the impact both the financial crisis of 2007-2008 and the COVID-19 pandemic of 2020 had on financial markets.

[INSERT FIGURE 2 HERE]

Comparing the results in Table 7, where HRP and VRP collectively provide predictive power, with and Table 6, where these variables individually do not provide predictive power, suggests that return predictability found in VRP and HRP derives from the difference between these series. Hence, despite the strong correlations between these risk premia series, important information regarding future returns is contained in their difference. This result could be explained by different tolerances towards variance and high-order co-movement risks at different horizons. To elaborate, at shorter investment horizons, we find that returns are positively related to VRP but negatively related to HRP while we find the opposite for longer investment horizons. This suggests that at shorter horizons, investors require compensation for exposure to second-order risk (volatility) but are willing to accept higher levels of higher-order risk (skewness, kurtosis and associated co-moments). At longer investment horizons, the opposite effect takes place. Given higher-order shocks are more likely to be realized over longer investment horizons, investors require extra compensation when the premia associated with such shocks is high. However, over longer investment horizons, the impact of day-to-day volatility is reduced and hence the required compensation is similarly reduced.

To further examine how the difference between VRP and HRP provides predictive power, we rerun model 3 but omit either VRP or HRP. Results are presented in Table 8. When we omit HRP from the predictive regression, we find that the variance risk premium exhibits less predictive power than when it is accompanied by HRP, testing as weakly significant at the 365-day horizon only. Similarly, omitting VRP reduces the predictive power associated with HRP which is significant at the 365-day horizon only. However, removing either one of these variables significantly increases the significance of the correlation risk premium. With HRP omitted, CRP is significant at 60 to 365-day horizon and similarly so with HRP omitted. This result suggests that risk premia associated with both the moment and its associated dependencies provide channels that impact expected rates of return. It also suggests that CRP is subsumed by both the variance and higher-order co-movement risk premia.

Examining our option-based co-movement measures jointly with traditional predictor variables in model 3, we find that the explanatory power they provide is not subsumed by traditional predictors. Instead, our option-based measures complement the traditional set of predictors. This is evidenced by the significantly larger adjusted R^2 values obtained when combining option-based and traditional predictor variables. For example, at the 365-day horizon, adding the option-based risk premia to the traditional predictors increases the adjusted R^2 from 25% to over 40%. This is despite the option based risk premia generating only a 4% adjusted R^2 on their own. This dramatic increase in model fit implies that option based risk premia contain information regarding future rates of return not captured by traditional predictors.

5 Conclusion

This paper develops a new measure of asset co-movement based on characteristic functions of asset returns. We further show how to compute these characteristic functions under both the risk neutral measure via option prices and under the physical measure via observed returns. We find that risk neutral co-movement is typically higher than its physical value, leading to a negative return on a swap contract. However, we find that physical higher order co-movement is larger than risk neutral higher-order co-movement at horizons of 30-days. We also find no difference between risk neutral and physical higher-order co-movement at the 60-day horizon. This suggests that, at shorter horizons, investors do not command a premium for higher-order (co-)moment risk and instead only require compensation for risk associated with the second moment. As the investment horizon increases, option prices begin to reflect the price of risk associated with higher-order (co-)moments. In much the same way that investors may take short positions in variance swaps to increase the yield on their portfolio, having access to a traded product that captures higher-order co-movement would allow investors to earn yield via a long position in a short-term swap contract with longer term contracts providing yield via a short position.[9]

We examine the determinants of co-movement and higher-order co-movement risk premiums via a swap contract and find that while market volatility explains much of the return, other common asset pricing factors such as the excess return on the market, the high-minus-low and conservative-minus-aggressive factors of Fama and French (2015) also provide significant explanatory power. This result suggests that these factors, which have been previously identified as important for explaining the cross section of average stock returns, are also important for describing the co-movement between asset returns. Hence, we provide a new channel through which asset pricing factors can be examined. Finally, we examine the ability of risk premia associated with co-movement/higher-order co-movement to predict future returns. While we find that option implied higher-order co-movement premia can help predict returns at the 365-day horizon in univariate settings, we do not find predictive power among any of our other option implied risk premia, including the variance and correlation risk premiums. However, when combined together or with other classic predictor variables, we find that option implied risk premia, including those associated with our new co-movement measures, provide significant explanatory power with predictive R^2 of over 40% at the 365-day horizon.

[9]See McFarren (2013) for a discussion on using short VIX futures to increase portfolio yield.

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A A Study in the Black-Scholes Economy

In this section we provide an analysis of our proposed co-movement measure within the framework of Black and Scholes (1973). This is a particularly convenient model to work with for two main reasons: i) log returns are normally distributed and hence have a very simple characteristic function and ii) option prices can be computed quickly and easily in closed form for individual assets and via Monte-Carlo simulation for options on portfolios. We examine our prescribed approach across two dimensions: i) the ability to recover the characteristic function from returns observed at the daily frequency and ii) the ability to recover characteristic functions from a finite set of option prices. Before presenting these results in sections A.2 and A.3 respectively, we first derive the co-movement expression outlined in equation (8).

A.1 Co-Movement in the Black-Scholes Economy

In the Black-Scholes economy, asset prices are assumed to follow the geometric Brownian motion,

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{32}$$

An application of Ito's lemma show that log returns follow the arithmetic Brownian motion

$$d\log(S_t) = (\mu - \tfrac{1}{2}\sigma^2)dt + \sigma dW_t \tag{33}$$

and hence are normally distributed. This implies that at $T > t$, the CF of log returns is given by

$$\varphi_{r_{t\to T}}(s) = e^{is(\mu - \frac{1}{2}\sigma^2)(T-t) - \frac{1}{2}\sigma^2(T-t)s^2} \tag{34}$$

Under the risk neutral measure we have that $\mu = r_f$ and under the physical measure μ corresponds to the expected rate of return.

For simplicity, we normalise time such that $T - t = 1$ and examine a two-asset version of our co-movement measure. First, we have that the demeaned CF of asset js ($j = 1, 2, p$) log returns is given by

$$\varphi_{\tilde{r}_{j,t\to T}}(s) = e^{-\frac{1}{2}\sigma_j^2 s^2}. \tag{35}$$

Note also that $\sigma_p^2 = w_1^2\sigma_1^2 + w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2$ where $\rho = \text{Corr}[r_{1,t\to T}, r_{2,t\to T}]$ is the correlation between the two assets and w_j, $j = \{1, 2\}$ are the weights of each asset in the portfolio p. Hence, in this setting, we have that

$$\varphi_{\tilde{r}_{p,t\to T}}(s) - \prod_{j=1}^{2} \varphi_{\tilde{r}_{j,t\to T}}(w_j s) = e^{-\frac{1}{2}s^2(w_1^2\sigma_1^2 + w_1^2\sigma_1^2)}(e^{-s^2 w_1 w_2 \rho\sigma_1\sigma_2} - 1) \tag{36}$$

and hence

$$D_t = \int_{-\infty}^{\infty} |e^{-s^2 w_1 w_2 \rho \sigma_1 \sigma_2} - 1| \omega(s) ds. \tag{37}$$

It is clear that D_t is a deterministic function of the co-movement between assets 1 and 2 only, which in this special case is equal to the covariance between the assets.

A.2 Recovery of the CF from Returns

We simulate log returns at the daily frequency using the discretization

$$r_{t \to t+\Delta t} = \log(S_{t+\Delta t}) - \log(S_t) = \mu \Delta t + \sigma \sqrt{\Delta t} Z_{t+1} \tag{38}$$

where Δt is one day and $Z_t \sim N(0,1)$. The characteristic function of daily log returns at the is hence

$$\varphi_{r_{t \to t+\Delta t}}(s) = e^{is\mu\Delta t - \frac{1}{2}\sigma^2 \Delta t s^2}. \tag{39}$$

We generate normally distributed returns at the daily frequency via equation (38). We select the number of observations to match the average number of trading days that occurs within a 30, 60, 90, 180 and 360-day horizon, these being 21, 43, 64, 126 and 252-days respectively. The error associated with the CF is measured by the mean integrated relative modulus (MIRM) given by

$$MIRM = \frac{1}{N} \sum_{j=1}^{N} \frac{\int_{-\infty}^{\infty} \left| \hat{\varphi}_{r_{t \to t+dt}}(s) - \varphi_{r_{t \to t+dt}}(s) \right| \omega(s) ds}{\int_{-\infty}^{\infty} \left| \varphi_{r_{t \to t+dt}}(s) \right| \omega(s) ds} \tag{40}$$

where $\hat{\varphi}_{r_{t \to t+dt}}(s_j)$ is the empirical characteristic function estimated from the observed set of finite returns, $\varphi_{r_{t \to t+dt}}(s)$ is the true characteristic function given by (39) and we simulate N sets of returns. We examine the error using $\omega(s) = \mathbf{I}(|s| \leq B)$ across a variety of values for B and $\omega(s) = e^{-\frac{s^2}{b}}$, again using a variety of values of b. Increasing values of B (b) increase the impact that higher order moments have on the co-movement measure D. Hence we expect error to increase with B (b) as higher-order moments will have the highest error associated with them when measured from a finite set of observations. Our aim then is to identify a value of B (b) that is as large as possible while maintaining a reasonable level of error. We simulate 1000 sets of returns corresponding to observations over 30, 60, 91, 182 and 365-day intervals with $\mu = 0.06$ and $\sigma = 0.3$. Results are presented in in Table 9.

[INSERT TABLE 9 HERE]

As expected, error grows as b (B) increase as we are incorporating higher-order moments more significantly. However, we note that errors grow more slowly for Gaussian weighted CF estimates. This is despite the fact that the Gaussian weighted estimates incorporate more of the distribution.

To illustrate this point, we compute the coefficients in the Taylor expansion of the CF, $c_k(s) = \frac{s^k}{k!}$, where we have ignored the sign determined by i^k. It is these coefficients that govern how moments are incorporated into estimates of the CF at different values of s. In Figure 3, we plot the Gaussian weighted ($b = 1$) and raw versions of these coefficients for $k = 2, 3, \dots, 6$.[10]

[INSERT FIGURE 3 HERE]

We observe in Figure 3 that a far greater range of s is incorporated into the estimate of the CF relative to using the truncation $B = 1$, indicated with a black dashed line. This shows that using a Gaussian weighting function can incorporate a more significant amount of the distribution into CF/dependence estimates with less error than truncation.

A.3 Recovery of the CF from Options

We now examine the accuracy to which we can recover the characteristic function of log returns from observed option prices. Option prices are computed using the Black-Scholes model across a variety of strikes. We use the parameters $S_t = 1000$, $r_f = 0.03$, $\sigma = 0.3$ and $T - t = 1$. An important point to note is that, as mentioned previously, higher-order moments are more strongly represented at large values of $|s|$ and similarly for lower-order moments and small values of $|s|$. Hence producing accurate CF values at large values of $|s|$ requires probing the higher-order moments which in turn requires relatively deep out-of-the-money options. We investigate how accurately we can recover the CF when we have options available from 5% in/out-of-the-money to 20% in/out-of-the-money. To measure error associated with our option implied CF, we use the integrated relative modulus (IRM) given by

$$IRM = \frac{\int_{-\infty}^{\infty} |\hat{\varphi}_{r_{t\to T}}(s) - \varphi_{r_{t\to T}}(s)|\, \omega(s) ds}{\int_{-\infty}^{\infty} |\varphi_{r_{t\to T}}(s)|\, \omega(s) ds} \tag{41}$$

where $\hat{\varphi}_{r_{t\to T}}(s)$ is the characteristic function derived from option prices using equation (12) with one year maturity and $\varphi_{r_{t\to T}}(s)$ is the risk neutral characteristic function given by

$$\varphi_{r_{t\to T}}(s) = e^{is\left(r_f - \frac{1}{2}\sigma^2\right)(T-t) - \frac{1}{2}\sigma^2 (T-t) s^2}. \tag{42}$$

Results of our analysis are in Table 10.

[INSERT TABLE 10 HERE]

The results in Table 10 demonstrate that the CF can be more accurately recovered from option prices that observed returns with the relative errors being much smaller. Additionally, we observe that the Gaussian weighted value of integrated characteristic functions are generally more accurate that truncated integrals, further justifying our choice to use a Gaussian weighting scheme to compute our dependence measures.

[10] We ignore the first moment, $k = 1$, as this term is eliminated in our estimates of dependence.

B Derivation of the Risk Neutral CF of log Returns

Carr and Madan (2001) show that the time t present value of any twice differentiable payoff function, f, of the asset price S at T, $f(S_T)$, can be spanned by options via

$$f(S_{i,T}) = [f(S^0) - f'(S^0)S^0] + f'(S^0)S_{i,T} + \int_0^{S^0} f''(K)(K - S_{i,T})^+ dK + \int_{S^0}^{\infty} f''(K)(S_{i,T} - K)^+ dK$$

where $f'(x)$ denotes the first derivative with respect to x and $f''(x)$ the second. Setting $S^0 = S_{i,t}e^{r_{f,t\to T}} = F_{i,t\to T}$, the forward price, taking expectations under $\mathbb{Q}$ and discounting at the risk-free rate from t to T reduces the above to

$$e^{-r_{f,t\to T}(T-t)}\mathbb{E}_t^{\mathbb{Q}}[f(S_T)] = e^{-r_{f,t\to T}(T-t)} f(F_{i,t\to T}) + \int_0^{F_{i,t\to T}} f''(K)Put_{t\to T}(K)dK + \int_{F_{i,t\to T}}^{\infty} f''(K)Call_{t\to T}(K)dK$$

Making the substitution $f(S_{i,T}) = e^{isr_{i,t\to T}}$ where $r_{i,t\to T} = \log[S_{i,T}] - \log[S_{i,t}]$ and rearranging to make $\varphi_{r_{i,t\to T}(s)} = \mathbb{E}_t^{\mathbb{Q}}[f(S_{i,T})]$ the subject yields (12).

C Gauss-Hermite Quadrature

Gauss-Hermite quadrature provides an efficient and accurate means of approximating integrals of a specific form via a weighted sum. Specifically,

$$\int_{-\infty}^{\infty} f(x)e^{-x^2}dx \approx \sum_{j=1}^{N} w_j f(x_j) \tag{43}$$

where we have selected N weights and nodes, w_j and x_j respectively. Values for these nodes and weights can be found in Abramowitz and Stegun (1964). This quadrature method provides an efficient means of computing the integrals prescribed by our dependence measures. However, the form presented in equation (43) would impose the restriction $b = 1$ in equation (8). To allow any value of $b > 0$, we employ a change of variable so we can continue to use Gauss-Hermite quadrature. We wish to evaluate the integral

$$\int_{-\infty}^{\infty} f(x)e^{-\frac{x^2}{b}}dx. \tag{44}$$

Define $y = \frac{x}{\sqrt{b}}$, then $dx = \sqrt{b}dy$. Hence, we may write

$$\int_{-\infty}^{\infty} f(x)e^{-bx^2}dx = \sqrt{b}\int_{-\infty}^{\infty} f\left(\sqrt{b}y\right)e^{-y^2}dy \tag{45}$$

and the right-hand-side of (45) is in a form amenable to Gauss-Hermite quadrature.

	Physical (ex-ante)					Risk Neutral				
Panel A: D_t	30-Day	60-Day	91-Day	182-Day	365-Day	30-Day	60-Day	91-Day	182-Day	365-Day
Mean	0.0138	0.0141	0.0143	0.0144	0.0146	0.0190	0.0198	0.0203	0.0213	0.0228
Median	0.0073	0.0077	0.0080	0.0090	0.0108	0.0139	0.0151	0.0160	0.0180	0.0201
Std. Dev.	0.0243	0.0222	0.0204	0.0174	0.0141	0.0194	0.0178	0.0165	0.0147	0.0132
Skewness	6.1442	5.2295	4.5040	3.4151	2.3894	4.4870	4.0855	3.8151	4.0468	5.8524
Kurtosis	50.5412	36.1308	27.4332	16.4223	9.2364	35.1245	32.0497	30.2782	47.3704	141.8471
N	6538	6538	6538	6538	6538	6529	6529	6529	6529	6529
Panel B: H_t										
Mean	0.0015	0.0011	0.0008	0.0006	0.0005	0.0010	0.0012	0.0014	0.0019	0.0028
Median	0.0004	0.0002	0.0002	0.0001	0.0002	0.0004	0.0006	0.0008	0.0013	0.0021
Std. Dev.	0.0064	0.0042	0.0030	0.0017	0.0010	0.0033	0.0030	0.0028	0.0030	0.0038
Skewness	10.3875	8.1477	7.3119	5.3810	3.8756	13.0244	14.7985	16.2896	28.2867	48.4496
Kurtosis	126.7978	73.7873	62.7223	34.3301	18.6007	238.8689	399.3680	527.2891	1471.6744	3264.3517
N	6538	6538	6538	6538	6538	6529	6529	6529	6529	6529
Panel C: IC_t										
Mean	0.3338	0.3351	0.3368	0.3424	0.3477	0.4765	0.5165	0.5445	0.6004	0.6738
Median	0.3097	0.3155	0.3223	0.3242	0.3293	0.4625	0.5047	0.5340	0.5928	0.6782
Std. Dev.	0.1527	0.1371	0.1301	0.1230	0.1168	0.1494	0.1424	0.1421	0.1352	0.1111
Skewness	0.6799	0.7680	0.8130	0.8588	0.5966	2.1005	2.6924	3.2689	2.6076	2.8584
Kurtosis	3.1507	3.5438	3.7428	3.6825	2.8337	21.9708	31.8148	41.7102	31.7879	36.3161
N	6538	6538	6538	6538	6538	6529	6529	6529	6529	6529
Panel D: $\sigma_{p,t}$										
Mean	0.1625	0.1653	0.1671	0.1705	0.1741	0.1986	0.2046	0.2088	0.2158	0.2246
Median	0.1373	0.1385	0.1409	0.1483	0.1617	0.1822	0.1900	0.1954	0.2059	0.2170
Std. Dev.	0.1026	0.0963	0.0921	0.0846	0.0762	0.0844	0.0788	0.0744	0.0669	0.0601
Skewness	3.0052	2.7144	2.4241	1.9227	1.3165	2.0937	1.9069	1.7815	1.6584	1.4865
Kurtosis	17.2464	13.9394	11.3150	7.7226	4.7834	11.0005	9.8948	9.1900	9.7049	12.8648
N	6547	6547	6547	6547	6547	6529	6529	6529	6529	6529

Table 1 – **Descriptive Statistics:** This table presents the descriptive statistics for estimates of our dependence measures computed at the 30, 60, 91, 182 and 365-day horizon. Panel A provides statistics for dependence (D_t), Panel B for higher-order dependence (H_t), Panel C for implied correlation (IC_t) and Panel D for S&P 100 index volatility ($\sigma_{p,t}$). We provide these statistics for metrics computed under both the physical measure ($\mathbb{P}$), computed using ex-ante data, and under the risk neutral measure ($\mathbb{Q}$), computed from option prices. Statistics using ex-post data are almost identical to the ex-ante results and hence are omitted for clarity.

Panel A: ex-ante Risk Premia				
Maturity	DRP	HRP	CRP	VRP
30-Day	0.0052***	−0.0005*	0.1428***	0.0362***
60-Day	0.0056***	0.0002	0.1816***	0.0393***
91-Day	0.0060***	0.0006***	0.2078***	0.0418***
182-Day	0.0068***	0.0013***	0.2580***	0.0453***
365-Day	0.0082***	0.0023***	0.3261***	0.0505***

Panel B: ex-post Risk Premia				
Maturity	DRP	HRP	CRP	VRP
30-Day	0.0053***	−0.0005*	0.1448***	0.0368***
60-Day	0.0056***	0.0002	0.1845***	0.0392***
91-Day	0.0060***	0.0006**	0.2113***	0.0413***
182-Day	0.0066***	0.0013***	0.2615***	0.0437***
365-Day	0.0079***	0.0023***	0.3278***	0.0472***

Table 2 – **Risk Premiums:** This table presents the unconditional risk premia associated with dependence (DRP), higher-order dependence (HRP), correlation (IC) and volatility (VRP). The risk premium for metric M is defined as $M^{\mathbb{Q}} - M^{\mathbb{P}}$.

	30-Day	60-Day	91-Day	182-Day	365-Day
Corr(DRP, HRP)	0.8571	0.8878	0.8342	0.6653	0.6135
Corr(DRP, VRP)	0.9889	0.9946	0.9959	0.9947	0.9922
Corr(DRP, CRP)	0.2511	0.2683	0.2668	0.2326	0.2153
Corr(HRP, VRP)	0.9209	0.9237	0.8671	0.7195	0.6895
Corr(HRP, CRP)	0.1345	0.1194	0.0707	-0.0547	-0.1212
Corr(VRP, CRP)	0.2073	0.2267	0.2260	0.1848	0.1582

Table 3 – **Risk Premium Correlations:** This table presents the correlations observed between the risk premia associated with dependence (DRP), higher-order dependence (HRP), index variance (VRP) and implied correlation (CRP). We present results for all six pairwise correlations at the 30, 60, 91, 182 and 365-day horizons.

30-Day	α	MKT	SMB	HML	RMW	CMA	MOM	ΔVIX	R^2_{Adj} (%)
Mdl. 1	-0.574	-5.566							27.81
	(-16.632)	(-10.782)							
Mdl. 2	-0.573	-5.551	-0.140	-0.307					27.40
	(-16.497)	(-10.600)	(-0.191)	(-0.460)					
Mdl. 3	-0.579	-4.618	-0.034	0.129	-0.234	-1.254	-0.568	1.332	41.29
	(-17.686)	(-7.530)	(-0.046)	(0.138)	(-0.244)	(-0.969)	(-1.132)	(8.375)	
60-Day									
Mdl. 1	-0.494	-5.368							40.44
	(-12.556)	(-9.547)							
Mdl. 2	-0.491	-5.369	-0.202	-0.959					40.95
	(-12.412)	(-9.252)	(-0.396)	(-1.515)					
Mdl. 3	-0.512	-4.139	-0.033	-1.287	-0.327	0.439	-0.828	1.167	56.88
	(-14.610)	(-8.388)	(-0.059)	(-1.474)	(-0.489)	(0.431)	(-1.929)	(10.240)	
91-Day									
Mdl. 1	-0.477	-4.845							42.80
	(-9.927)	(-9.077)							
Mdl. 2	-0.471	-4.822	-0.405	-1.236					44.69
	(-9.948)	(-8.676)	(-0.883)	(-1.738)					
Mdl. 3	-0.502	-3.612	-0.200	-1.804	-0.331	1.218	-0.831	1.019	59.22
	(-12.293)	(-7.070)	(-0.416)	(-1.767)	(-0.459)	(1.136)	(-2.489)	(9.613)	
182-Day									
Mdl. 1	-0.425	-3.672							41.24
	(-6.205)	(-9.585)							
Mdl. 2	-0.407	-3.693	-0.391	-1.686					48.59
	(-7.055)	(-9.486)	(-0.764)	(-2.406)					
Mdl. 3	-0.499	-2.365	-0.045	-2.646	0.305	2.189	-0.779	0.979	64.56
	(-9.627)	(-4.533)	(-0.106)	(-2.728)	(0.381)	(2.096)	(-3.820)	(9.334)	
365-Day									
Mdl. 1	-0.390	-2.282							33.37
	(-3.587)	(-4.615)							
Mdl. 2	-0.342	-2.450	-0.549	-1.427					45.86
	(-4.373)	(-5.477)	(-0.794)	(-1.922)					
Mdl. 3	-0.573	-1.057	-0.134	-2.768	1.272	2.405	-0.688	0.848	66.11
	(-5.962)	(-1.685)	(-0.244)	(-3.489)	(1.653)	(2.738)	(-2.090)	(7.922)	

Table 4 – **Determinants of Co-Movement Risk Premiums:** This table presents estimates of contemporaneous regressions where the dependent variable is the return on a long dependence swap. Independent variables include the excess return on the market portfolio (MKT), the small-minus-big (SMB), high-minus-low (HML), conservative-minus-aggressive (CMA) and robust-minus-weak (RMW) factors of Fama and French (1992) and Fama and French (2015), the momentum (MOM) factor of Carhart (1997) and the log return of the VIX (ΔVIX). Values in parentheses are robust Newey and West (1987) standard errors with lag length equal to twice the return overlap.

30-Day	α	MKT	SMB	HML	RMW	CMA	MOM	ΔVIX	R^2_{Adj} (%)
Mdl. 1	0.132	-6.572							24.27
	(2.807)	(-8.879)							
Mdl. 2	0.129	-6.531	0.012	1.044					24.14
	(2.750)	(-8.794)	(0.012)	(1.087)					
Mdl. 3	0.111	-4.830	0.076	2.657	-0.523	-2.905	0.326	1.970	44.45
	(2.552)	(-5.813)	(0.077)	(2.157)	(-0.427)	(-1.760)	(0.552)	(9.837)	
60-Day									
Mdl. 1	-0.636	-7.980							42.90
	(-10.920)	(-9.048)							
Mdl. 2	-0.634	-8.000	0.024	-0.325					42.57
	-(10.650)	(-8.697)	(0.030)	(-0.321)					
Mdl. 3	-0.676	-6.008	0.282	-0.510	-0.444	0.491	-0.669	1.716	59.16
	(-12.793)	(-7.889)	(0.314)	(-0.384)	(-0.463)	(0.338)	(-1.093)	(9.813)	
91-Day									
Mdl. 1	-1.080	-7.803							45.70
	-(14.585)	(-8.976)							
Mdl. 2	-1.074	-7.787	-0.327	-1.047					46.00
	(-14.164)	(-8.551)	(-0.422)	(-0.892)					
Mdl. 3	-1.136	-5.838	-0.047	-2.060	-0.274	2.359	-0.922	1.449	57.94
	(-16.574)	(-6.899)	(-0.055)	(-1.254)	(-0.234)	(1.357)	(-1.634)	(7.979)	
182-Day									
Mdl. 1	-1.655	-6.511							43.60
	(-14.152)	(-9.216)							
Mdl. 2	-1.635	-6.573	-0.175	-2.161					47.15
	(-14.221)	(-8.265)	(-0.179)	(-1.693)					
Mdl. 3	-1.793	-4.474	0.054	-4.235	0.376	5.021	-1.067	1.360	58.51
	(-15.213)	(-4.167)	(0.054)	(-2.332)	(0.237)	(2.575)	(-1.860)	(6.484)	
365-Day									
Mdl. 1	-2.134	-4.340							36.47
	(-11.125)	(-5.182)							
Mdl. 2	-2.082	-4.615	0.146	-2.169					43.02
	(-11.606)	(-5.380)	(0.092)	(-1.617)					
Mdl. 3	-2.498	-2.220	0.189	-5.092	2.036	5.810	-1.015	1.181	58.81
	(-10.192)	(-1.713)	(0.128)	(-3.301)	(1.357)	(3.243)	(-1.126)	(4.963)	

Table 5 – Determinants of Higher-Order Co-Movement Risk Premiums: This table presents estimates of contemporaneous regressions where the dependent variable is the return on a long higher-order co-movement swap. Independent variables include the excess return on the market portfolio (MKT), the small-minus-big (SMB), high-minus-low (HML), conservative-minus-aggressive (CMA) and robust-minus-weak (RMW) factors of Fama and French (1992) and Fama and French (2015), the momentum (MOM) factor of Carhart (1997) and the log return of the VIX (ΔVIX). Values in parentheses are robust Newey and West (1987) standard errors with lag length equal to twice the return overlap.

30-Day	DRP	HRP	CRP	VRP	PE	PD	CS	TS	$SDRF$	CAY
α	0.004	0.005	0.007	0.004	0.041	0.073	0.018	0.009	0.006	0.004
	(0.912)	(1.805)	(2.004)	(1.252)	(1.592)	(1.373)	(1.706)	(1.853)	(2.221)	(1.408)
β	0.213	-0.039	-0.015	0.056	-0.011	-0.017	-0.013	-0.003	0.007	-0.2207
	(0.424)	(-0.032)	(-1.005)	(0.333)	(-1.370)	(-1.287)	(-1.188)	(-0.974)	(1.523)	(-1.507)
R^2_{adj} (%)	0.10	-0.32	0.06	-0.04	0.27	0.54	0.97	0.02	0.74	0.49
60-Day										
α	0.005	0.009	0.017	0.006	0.057	0.124	0.024	0.014	0.010	0.008
	(0.587)	(1.640)	(2.413)	(0.857)	(1.260)	(1.128)	(1.189)	(1.610)	(2.238)	(1.412)
β	0.779	2.555	-0.044	0.270	-0.015	-0.029	-0.016	-0.003	0.013	-0.251
	(0.957)	(0.793)	(-1.709)	(0.933)	(-1.023)	(-1.062)	(-0.681)	(-0.594)	(1.389)	(-1.016)
R^2_{adj} (%)	2.03	0.96	1.04	1.93	0.19	0.89	0.56	-0.05	1.43	0.20
91-Day										
α	0.010	0.011	0.029	0.011	0.071	0.200	0.026	0.021	0.016	0.013
	(0.902)	(1.239)	(3.089)	(1.043)	(1.026)	(1.257)	(1.006)	(1.595)	(2.409)	(1.557)
β	0.695	5.620	-0.073	0.278	-0.018	-0.047	-0.012	-0.004	0.018	-0.380
	(0.674)	(0.888)	(-1.867)	(0.703)	(-0.791)	(-1.181)	(-0.417)	(-0.559)	(1.360)	(-1.083)
R^2_{adj} (%)	0.86	1.67	2.34	1.04	0.22	2.03	0.08	0.07	2.12	0.57
182-Day										
α	0.027	0.019	0.057	0.026	0.106	0.416	0.022	0.034	0.030	0.023
	(1.397)	(0.899)	(2.760)	(1.374)	(0.673)	(1.478)	(0.597)	(1.207)	(2.103)	(1.311)
β	-0.212	5.224	-0.124	-0.051	-0.026	-0.098	0.004	-0.005	0.037	-0.609
	(-0.156)	(0.430)	(-1.369)	(-0.090)	(-0.490)	(-1.390)	(0.092)	(-0.388)	(1.383)	(-0.763)
R^2_{adj} (%)	-0.29	0.04	3.08	-0.31	0.17	4.40	-0.31	-0.06	4.59	0.71
365-Day										
α	0.052	-0.009	0.139	0.047	0.198	1.152	0.000	0.034	0.052	0.041
	(1.401)	(-0.200)	(2.620)	(1.211)	(0.610)	(2.641)	(-0.006)	(0.562)	(1.726)	(1.090)
β	-0.949	23.717	-0.290	-0.137	-0.049	-0.279	0.046	0.007	0.065	-1.090
	(-0.431)	(2.773)	(-1.342)	(-0.147)	(-0.457)	(-2.507)	(1.035)	(0.302)	(1.207)	(-0.651)
R^2_{adj} (%)	-0.06	3.80	6.45	-0.29	0.42	15.59	0.74	-0.16	6.02	1.06

Table 6 – Univariate Predictive Regressions: This table presents estimates of univariate predictive regressions at horizons of 30, 60, 91, 182 and 365-days. We examine the dependence risk premium (DRP), higher-order dependence risk premium HRP, correlation risk premium (CRP), variance risk premium (VRP), price-earnings ratio (P/E) defined as the log(P/E), price-dividend ratio (P/D) defined as the log(P/D), credit spread (CS) defined as the difference between Moody's BAA and AAA bond yield indices, term spread (TS) defined as the difference between the 10-year and 3-month Treasury yields, stochastically detrended risk-free rate ($SDRF$) defined as the 3-month T-bill rate minus its trailing twelve month moving averages and the consumption-wealth ratio (CAY) of Lettau and Ludvigson (2001), defined by the most recent quarterly observation. Numbers in parentheses are robust Newey and West (1987) t-statistics computed with lag equal to twice the return overlap.

30-Day	α	DRP	VRP	HRP	CRP	PE	PD	DS	TS	$SDRF$	CAY	R^2_{adj} (%)
Mdl. 1	0.001		0.534	-3.972	-0.025							2.59
	(0.311)		(2.034)	(-1.812)	(-1.708)							
Mdl. 2	0.156					0.005	-0.036	-0.021	-0.002	0.001	-0.110	2.57
	(2.677)					(0.491)	(-2.449)	(-1.724)	(-0.962)	(0.225)	(-0.737)	
Mdl. 3	0.156		0.655	-5.108	-0.023	0.004	-0.036	-0.022	-0.004	0.003	-0.045	6.36
	(2.785)		(2.409)	(-2.479)	(-1.491)	(0.381)	(-2.472)	(-1.840)	(-1.508)	(0.654)	(-0.307)	
Mdl. 4	0.166	0.259				0.004	-0.038	-0.020	-0.003	0.000	-0.113	2.85
	(2.947)	(0.517)				(0.376)	(-2.653)	(-1.676)	(-1.094)	(0.092)	(-0.756)	
60-Day												
Mdl. 1	0.011		1.109	-9.860	-0.076							5.68
	(1.505)		(2.547)	(-2.161)	(-2.826)							
Mdl. 2	0.222					0.012	-0.056	-0.024	-0.003	0.006	-0.086	2.74
	(2.076)					(0.660)	(-2.100)	(-1.176)	(-0.671)	(0.738)	(-0.326)	
Mdl. 3	0.168		1.349	-13.092	-0.114	0.021	-0.046	-0.019	-0.009	0.008	0.364	8.93
	(1.657)		(2.988)	(-2.631)	(-2.798)	(1.265)	(-1.739)	(-0.894)	(-2.037)	(1.009)	(1.358)	
Mdl. 4	0.241	0.843				0.008	-0.060	-0.016	-0.005	0.005	-0.091	4.78
	(2.281)	(0.998)				(0.471)	(-2.286)	(-0.877)	(-1.092)	(0.559)	(-0.350)	
91-Day												
Mdl. 1	0.028		0.247	2.811	-0.089							4.54
	(2.664)		(0.341)	(0.276)	(-2.062)							
Mdl. 2	0.290					0.024	-0.081	-0.022	-0.005	0.012	-0.173	4.70
	(1.770)					(0.935)	(-2.028)	(-0.892)	(-0.835)	(0.918)	(-0.468)	
Mdl. 3	0.192		0.320	1.790	-0.146	0.047	-0.066	-0.019	-0.012	0.012	0.397	9.19
	(1.114)		(0.456)	(0.195)	(-1.924)	(1.943)	(-1.599)	(-0.695)	(-1.712)	(1.009)	(0.942)	
Mdl. 4	0.298	0.829				0.022	-0.084	-0.012	-0.007	0.010	-0.182	5.75
	(1.844)	(0.723)				(0.880)	(-2.132)	(-0.452)	(-1.151)	(0.830)	(-0.501)	
182-Day												
Mdl. 1	0.052		-0.002	3.706	-0.121							2.63
	(2.044)		(-0.002)	(0.228)	(-1.171)							
Mdl. 2	0.411					0.060	-0.140	-0.002	-0.008	0.038	-0.288	9.66
	(1.095)					(1.222)	(-1.601)	(-0.041)	(-0.714)	(1.208)	(-0.356)	
Mdl. 3	0.383		-0.796	21.885	-0.180	0.092	-0.138	-0.044	-0.012	0.047	0.779	14.29
	(0.999)		(-0.696)	(0.964)	(-1.366)	(1.793)	(-1.541)	(-0.840)	(-1.065)	(1.668)	(1.097)	
Mdl. 4	0.406	0.312				0.061	-0.141	0.003	-0.009	0.038	-0.303	9.42
	(1.111)	(0.204)				(1.273)	(-1.587)	(0.073)	(-0.726)	(1.202)	(-0.384)	
365-Day												
Mdl. 1	0.073		-0.614	22.236	-0.208							7.95
	(1.073)		(-0.504)	(1.719)	(-0.841)							
Mdl. 2	1.049					0.161	-0.376	-0.001	-0.006	0.076	-0.421	25.19
	(1.689)					(1.605)	(-3.037)	(-0.015)	(-0.234)	(1.053)	(-0.274)	
Mdl. 3	1.396		-2.330	70.002	-0.195	0.197	-0.455	-0.182	-0.014	0.094	1.513	41.06
	(2.675)		(-1.812)	(3.560)	(-1.630)	(2.510)	(-4.046)	(-3.069)	(-0.769)	(1.512)	(1.627)	
Mdl. 4	1.007	1.314				0.178	-0.384	0.007	-0.005	0.078	-0.575	25.36
	(1.619)	(0.649)				(1.813)	(-3.237)	(0.140)	(-0.218)	(1.064)	(-0.393)	

Table 7 – **Multivariate Predictive Regressions:** This table presents estimates of the multivariate regressions presented in equations (28), (29), (30) and (31) for 30, 60, 91, 182 and 365-day returns. Numbers in parentheses are robust Newey and West (1987) t-statistics.

Panel A: Excl. HRP

	α	VRP	CRP	PE	PD	DS	TS	$SDRF$	CAY	R^2_{adj} (%)
30-Day	0.150	0.068	-0.015	0.007	-0.035	-0.021	-0.003	0.000	-0.076	2.44
	(2.497)	(0.351)	(-0.914)	(0.663)	(-2.988)	(-2.767)	(-1.380)	(0.105)	(-0.354)	
60-Day	0.170	0.330	-0.086	0.022	-0.047	-0.019	-0.007	0.006	0.253	6.18
	(1.317)	(1.079)	(-2.614)	(1.200)	(-1.895)	(-1.239)	(-1.758)	(0.545)	(0.812)	
91-Day	0.190	0.430	-0.150	0.048	-0.066	-0.019	-0.012	0.012	0.411	9.47
	(0.896)	(1.017)	(-2.351)	(1.634)	(-1.658)	(-0.685)	(-1.883)	(0.771)	(0.950)	
182-Day	0.293	0.343	-0.245	0.103	-0.123	-0.014	-0.014	0.043	0.863	13.06
	(0.768)	(0.537)	(-2.407)	(1.937)	(-1.527)	(-0.3710	(-1.194)	(1.217)	(1.146)	
365-Day	0.944	1.119	-0.497	0.252	-0.371	-0.039	-0.014	0.088	1.770	31.83
	(1.655)	(1.869)	(-3.389)	(2.503)	(-3.496)	(-0.695)	(-0.630)	(1.251)	(1.667)	

Panel B: Excl. VRP

	α	HRP	CRP	PE	PD	DS	TS	$SDRF$	CAY	R^2_{adj} (%)
30-Day	0.146	-0.142	-0.011	0.007	-0.034	-0.022	-0.003	0.001	-0.083	2.09
	(2.371)	(-0.120)	(-0.663)	(0.651)	(-2.832)	(-2.716)	(-1.153)	(0.293)	(-0.374)	
60-Day	0.171	2.443	-0.073	0.022	-0.047	-0.023	-0.006	0.006	0.199	4.43
	(1.301)	(0.837)	(-2.157)	(1.193)	(-1.847)	(-1.518)	(-1.404)	(0.565)	(0.614)	
91-Day	0.198	6.455	-0.134	0.047	-0.067	-0.022	-0.011	0.012	0.353	9.34
	(0.923)	(1.116)	(-2.110)	(1.596)	(-1.640)	(-0.892)	(-1.801)	(0.777)	(0.847)	
182-Day	0.319	10.360	-0.224	0.102	-0.130	-0.022	-0.014	0.045	0.855	14.01
	(0.786)	(0.853)	(-2.162)	(1.937)	(-1.519)	(-0.489)	(-1.299)	(1.376)	(1.264)	
365-Day	1.119	39.600	-0.370	0.253	-0.426	-0.112	-0.014	0.096	1.631	38.49
	(2.180)	(4.929)	(-2.985)	(2.701)	(-4.147)	(-2.300)	(-0.736)	(1.444)	(1.700)	

Table 8 – **Multivariate Predictive Regressions with Dropped Variables:** This table presents estimates of model 3, presented in equation (30) at the 30, 60, 91, 182 and 365-day horizon but with either DRP^{HO} (panel A) or VRP (panel B) omitted. Numbers in parentheses are robust Newey and West (1987) t-statistics.

Panel A: ω_G	$b = 1$	$b = 2$	$b = 3$	$b = 4$	$b = 5$
30-Day	0.4326	0.5616	0.6348	0.7002	0.7507
60-Day	0.3172	0.4053	0.5111	0.5494	0.5733
91-Day	0.2574	0.3502	0.4185	0.4556	0.5039
182-Day	0.1889	0.2521	0.3055	0.3435	0.3703
365-Day	0.1339	0.1918	0.2173	0.2535	0.2643

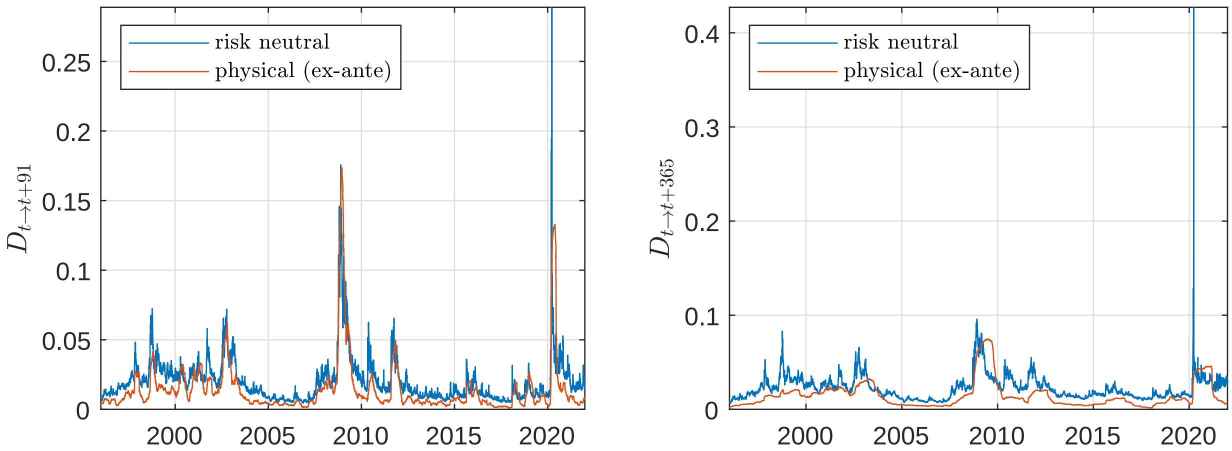
Panel B: ω_T	$B = 1$	$B = 2$	$B = 3$	$B = 4$	$B = 5$
30-Day	0.3948	0.6821	0.8350	0.9343	0.9869
60-Day	0.2856	0.5004	0.7092	0.7928	0.8359
91-Day	0.2304	0.4330	0.5979	0.6932	0.7682
182-Day	0.1681	0.3128	0.4467	0.5426	0.6080
365-Day	0.1187	0.2383	0.3214	0.4121	0.4510

Table 9 – **Error in ECF Estimates:** This table presents the mean-integrated-relative-modulus error associated with the estimation of the characteristic function from observed returns. We explore a variety of observation windows corresponding to those used in the empirical analysis. We also examine a variety of Gaussian weighing parameters (b) and truncation parameters (B). We simulate returns for 100 assets each with an average return of 3%, volatility of 25% and a constant correlation of 30%.

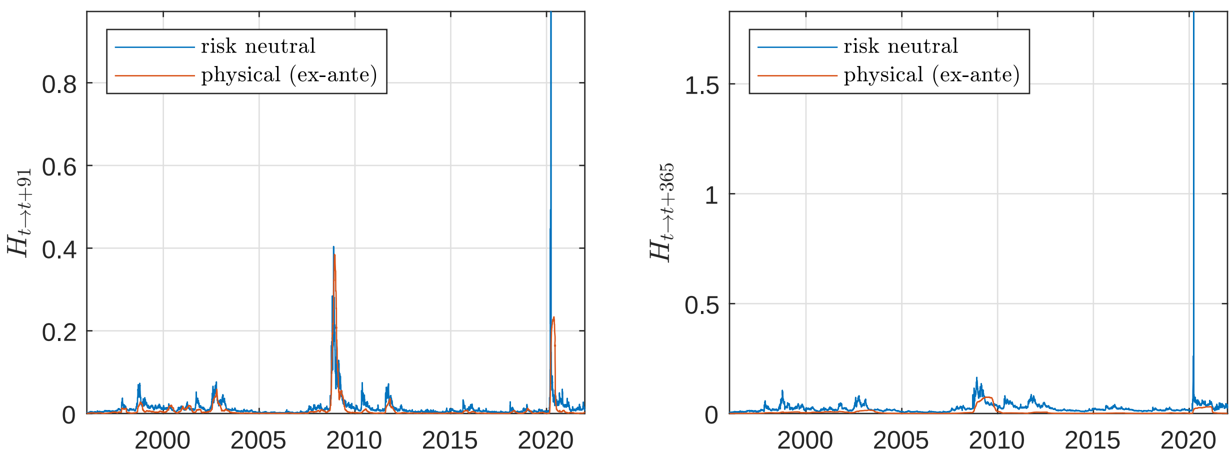
Panel A: ω_G	$b = 1$	$b = 2$	$b = 3$	$b = 4$	$b = 5$
OTM_5	0.0265	0.0441	0.0604	0.0757	0.0902
OTM_{10}	0.0196	0.0322	0.0435	0.0538	0.0633
OTM_{15}	0.0142	0.0230	0.0306	0.0372	0.0430
OTM_{20}	0.0101	0.0161	0.0209	0.0250	0.0286

Panel B: ω_T	$B = 1$	$B = 2$	$B = 3$	$B = 4$	$B = 5$
OTM_5	0.0209	0.0576	0.1136	0.1868	0.2724
OTM_{10}	0.0155	0.0422	0.0807	0.1269	0.1729
OTM_{15}	0.0113	0.0302	0.0557	0.0823	0.1018
OTM_{20}	0.0081	0.0211	0.0373	0.0511	0.0593

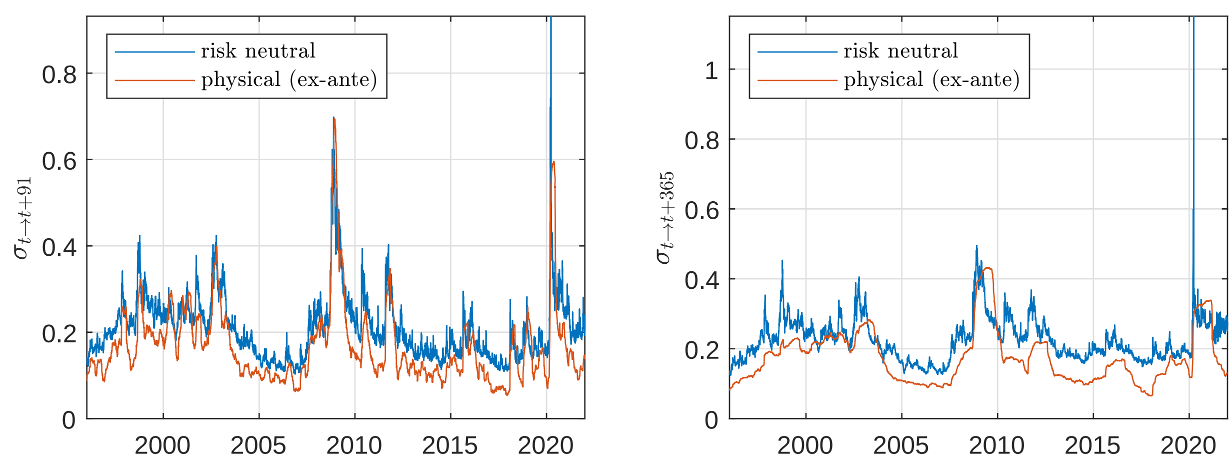
Table 10 – **Error in Option Implied CFs:** This table presents the integrated-relative-modulus error associated with the estimation of the characteristic function from observed option prices. We explore ranges of options from 5% in/out-of-the-money up to 20% in/out-of-the-money (OTM_5-OTM_{20}). We also examine a variety of Gaussian weighing parameters (b) and truncation parameters (B). We use the parameters: $S_t = 1000$, $r_f = 0.03$, $\sigma = 0.3$ and $T - t = 1$.



(a) D_t at the 91- and 365-day horizon.



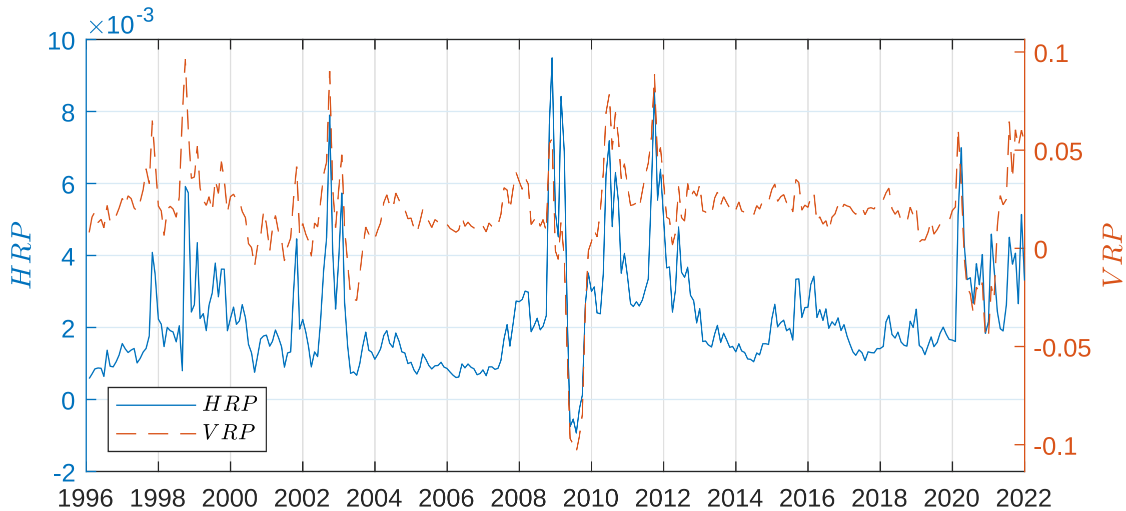
(b) H_t at the 91- and 365-day horizon.



(c) σ_t at the 91- and 365-day horizon.

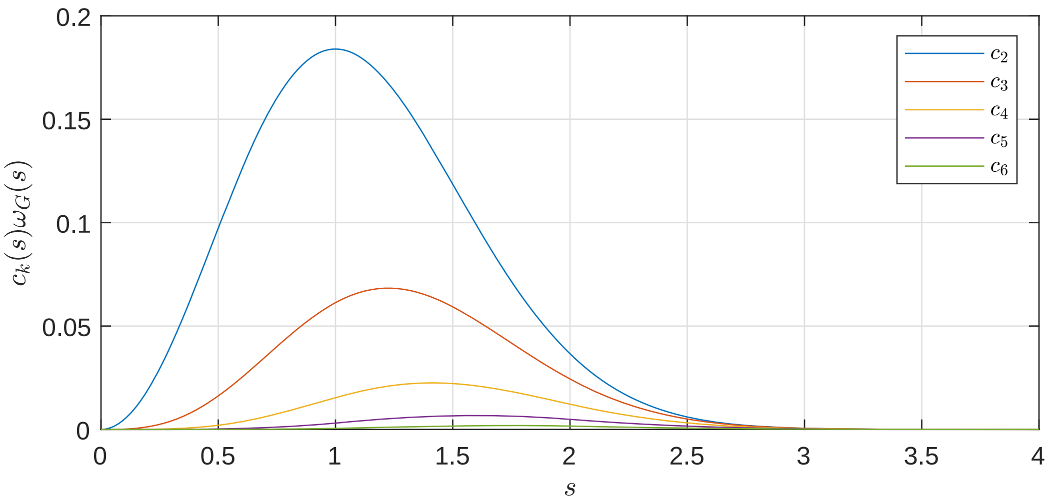
Figure 1 – **Time-Series Plots:** This figure presents time-series plots of the co-movement measures $D_{t\to T}$ (1a), $D^{HO}_{t\to t+T}$ (1b) and $\sigma_{t\to t+T}$ (1c) (the S&P 100 volatility) at the 91- and 365-day horizon under both the risk neutral and physical measure.

(a) Higher-order dependence risk premiums (HRP_t) and variance risk premiums (VRP_t) at the 60-day horizon.

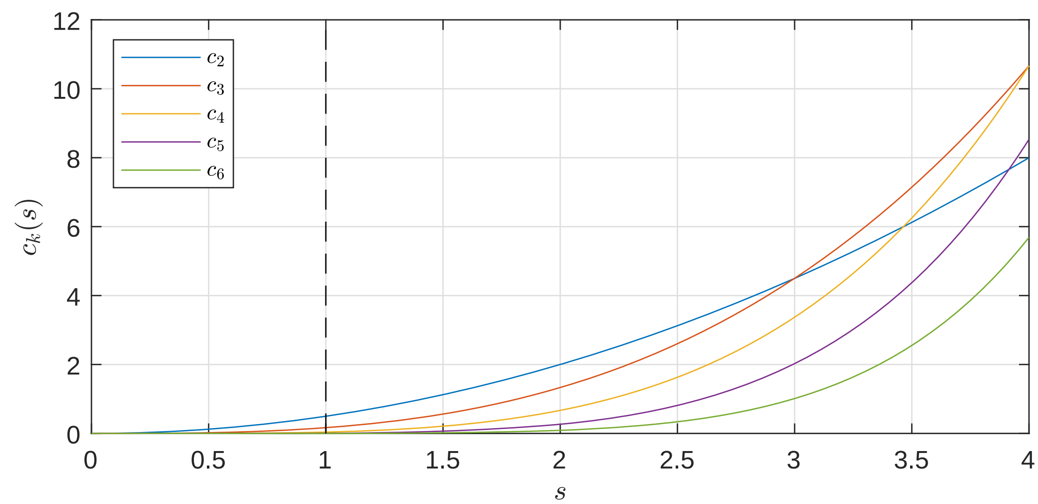


(b) Higher-order dependence risk premiums (HDR_t) and variance risk premiums (VRP_t) at the 365-day horizon.

Figure 2 – Higher-Order Co-Movement and Variance Risk Premiums: This figure presents plots of the higher-order co-movement (HRP) and variance (VRP) risk premiums at the 60-day (2a) and 365-day (2b) horizons.



(a) Gaussian weighted coefficients of the CF Taylor expansion.



(b) Raw coefficients of the CF Taylor expansion.

Figure 3 – Taylor Expansion Coefficients: This figure presents that Gaussian weighted 3a and raw 3b coefficient values, $c_k(s)$ for $k = 2, 3, \ldots, 6$ from the Taylor expansion of the CF. For Gaussian weighted coefficients, we use $b = 1$. In Figure 3a we indicate the truncation that would occur if $B = 1$ were selected.