Risk-insensitive Regulation*

Alexander Bleck
UBC

This version: November 2016

Abstract

Banking is risky and prone to failure. Yet banking regulation is surprisingly not all that risk-sensitive. When the bank has an informational advantage over the regulator, designing banking regulation both affects and reflects bank behavior. This gives rise to a trade-off in risk-sensitive regulation: relying on the banking market for information to refine regulation improves bank risk-taking but also aggravates the market’s allocative failure (increases risk and instability) and could undermine its informativeness. This tension could explain why Basel capital regulations are often coarse or information-light.

Keywords: Bank regulation, risk-taking, mechanism design.

JEL classifications: D61, D62, D82, G21, G28.

* I thank Jeremy Bertomeu (discussant), Jim Brander, Sandra Chamberlain, Vitor Farinha Luz, Michael Peters, Ralph Winter and workshop participants at UBC.
1 Introduction

Financial stability is a public good. The 2007-8 financial crisis however underscored the instability of the banking sector and reignited a decades-long debate about its regulation. The latest response by regulators gave rise to the 2010 Basel III Accord by the Bank of International Settlements (BIS) that promulgates rules for the centerpiece of bank regulation: the regulation of bank capital. Banks do not internalize all the consequences of their risk-taking resulting in a social cost inflicted on the banking sector as a whole. Such market failure justifies regulation that requires banks to put more skin in the game (equity capital) when taking risks. Greater risk (taking) then requires stricter regulation: risk-sensitive regulation.

Yet observed regulation is not all that risk-sensitive. The first form of risk-based bank capital regulation came into effect in 1989. The standard known as Basel I was based on classifications of bank risk to which the regulation assigned risk weights (tax). The higher the risk weight, the stricter the regulation. Assets received one of four possible risk weights (0, 20%, 50%, 100%), with deemed safest assets (like U.S. treasuries) receiving the lowest weight, and deemed riskiest assets (like ordinary loans) receiving the highest weight. This “flat tax” design treats any two assets in a category the same irrespective of their actual risk. Such crudeness in ignoring potentially more relevant information about risk seems inefficient, if not puzzling. In 2004, Basel II was enacted to overcome this apparent inefficiency by improving “risk-sensitivity” (BIS (2001, p.1)). Since Basel II, the bank has had a choice between subjecting itself to finer but still coarse classifications or rely on its own information to set risk weights.\(^1\) In response to the apparent ineffectiveness of such regulation in preventing the recent financial crisis, the BIS has been working on amending its

---

\(^1\) A 2015 survey by the BIS finds that all 27 Basel Committee on Banking Supervision member countries had implemented enhanced risk-based capital regulations by the end of 2013 (www.bis.org/bcbs/publ/d345.htm).
standards again toward lower risk-sensitivity.\textsuperscript{2} This development of bank capital regulation poses at least two questions. Should the regulator use all available information to design the regulation? Is perfectly risk-sensitive regulation desirable? This paper shows that these questions are intimately related and that the answers to them reveal a conflict.

This paper studies the (mechanism) design of macroprudential capital regulation. Designing risk-sensitive regulation requires information. When the bank has an informational advantage over the regulator, designing banking regulation both affects and reflects bank behavior, and its efficiency should be evaluated in this context. This gives rise to a tradeoff in risk-sensitive regulation. Eliciting information from banks to refine regulation could improve bank risk-taking but also aggravate the allocative failure in the banking market and undermine its informativeness.

Banks engage in risk-taking when making loans. Yet the individual bank does not bear all the consequences of its risk-taking. When bank quality is public information, the regulator can perfectly and costlessly correct the market failure from socially excessive risk-taking. However, when the bank is better informed about its risk than the regulator, this information asymmetry complicates the design of regulation. In designing incentives to elicit the information from banks, the regulator faces a tradeoff: requiring the bank to retain critical exposure to the risk of its loans (“skin in the game”) reveals information about its risk but also generates the negative externality.\textsuperscript{3} In the attempt to reduce such systemic risk, risk-sensitive regulation increases risk. Optimal regulation balances the social benefits from risk-taking with its social costs. Just when the benefits of regulation could be greatest (when the social cost from risk-taking is high), regulation becomes insensitive to the very risk it seeks to control. This paradoxical result is reminiscent of the theory


\textsuperscript{3} The classic applications of risk retention as a solution to adverse selection and moral hazard problems include among others trading in markets for real goods (Akerlof (1970)), insurance (Rothschild and Stiglitz (1976)), financial securities (Leland and Pyle (1977); Glosten (1989); Biais, Rochet, and Martimort (2000); Chemla and Hennesey (2014)) and labor (Hölmstrom (1979)).
of the second best from welfare economics. When there is more than one imperfection in
the market (information asymmetry and negative externality), removing one imperfection
(information asymmetry) could exacerbate another (negative externality) to the detriment
of overall welfare.

This insight could explain why real-world bank capital regulations, like the (early and
latest versions of the) Basel Accords, are often coarse or not all that risk-sensitive. The
result is also consistent with the “information-light” policies promoted by Tirole (2015). In
his 2014 Nobel Prize speech, Tirole urged economists to offer “policies that do not require
information that is unlikely to be available to regulators”. This paper argues that the reason
regulators may choose to implement information-light policies is to avoid the unintended,
adverse distortions to the behavior of the regulated firms that information-sensitive policies
would cause.

As such, my theory could not just shed light on the observed design of capital regulation
but also on its puzzling empirical performance, as evidenced by the relationship between
measured and actual risk. While risk-sensitive regulation is designed to reflect and control
risk, the observed link between measured risk and actual risk is low (Van Hoose (2007);
Haldane (2013); Acharya, Engle, and Pierret (2014)) or even negative (Behn, Haselmann,
and Vig (2014); Becker and Opp (2014); Begley, Purnanandam, and Zheng (2016)). That
is, at best measures of bank risk appear to be uninformative about actual risk. At worst,
there are instances in which capital regulation seems to have increased risk in the banking
and insurance sectors, consistent with my theory. While the effect of capital-adequacy
requirements is to decrease risk-taking in the theoretical literature, the reverse has also
been shown due to distorted portfolio choice (Koehn and Santomero (1980); Lam and Chen
(1985); Koehn and Santomero (1988); Flannery (1989); Genotte and Pyle (1991); Rochet
(1992); Acharya (2009)) and lower portfolio quality through reduced monitoring incentives
(Besanko and Kanatas (1996); Boot and Greenbaum (1993)). The contribution of my
theory is to demonstrate that increased risk-taking could be part of an optimal regulatory
mechanism.4

Finally, my theory offers a caveat to the coordinated design of bank regulation. Bank capital regulation comes in two forms (see Hansen, Kashyap, and Stein (2011)): micro-prudential (aimed at the soundness of the individual institution) and macro-prudential (aimed at the stability of the system as a whole). My theory caveats the familiar truism that any effort to stabilize the individual institution must necessarily stabilize the system (see also Morris and Shin (2008)). I argue that these regulations could be in conflict: attempting to reduce the fragility of the individual institution could exacerbate the instability of the system.

Another closely related strand of literature studies the use of information in the design of bank regulation. The underlying rationale is that using additional information gleaned from the market prices of securities of the bank should improve the regulation. Indeed, McDonald (2010) and Hart and Zingales (2012) argue that using equity and credit default swap prices of the bank improves the efficient intervention by the regulator. In contrast, Faure-Grimaud (2002); Bond, Goldstein, and Prescott (2010) and Lehar, Seppi, and Strobl (2011) caution against the use of market-based regulation by arguing that intervening in the market based on the information content of market prices alters their information content and thus undermines their usefulness. In a similar spirit to my paper, Lehar, Seppi, and Strobl (2011) show that not only does the attempt to learn information from the bank affect the information content but it also distorts the risk-taking of the bank. My paper complements both sides of the arguments by demonstrating that the use of information by the regulator distorts the very activity that produces the information. The resulting tradeoff could then lead the regulator to optimally ignore information. While in the existing theories the source of information is the market price of bank securities, I show that the counterproductive consequences of regulation can be part of an optimal contracting

4 For surveys on the literature on bank regulation and risk-taking, see Bhattacharya, Boot, and Thakor (1999); Allen (2004); Van Hoose (2007).
mechanism. Since the “skin in the game” mechanism results in the bank retaining more risky assets on its publicly available balance sheet, any outsider to the bank could use this information to refine their interaction with the bank. As a consequence, it is possible that this mechanism is the original source of information that is reflected in any transaction outcome, such as market prices, not just that between the regulator and the bank.

The rest of the paper is organized as follows. Section 2 describes the model, section 3 presents the equilibria, section 4 states the main results, and section 5 discusses policy implications. Appendix A includes the proofs that are not in the text.

2 Model

There are three dates, \( t = 0, 1, 2 \), and a continuum of banks, each of which manages a risky loan portfolio. The bank’s loan portfolio generates a random cash flow \( \tilde{\theta} + \tilde{x} \) at \( t = 2 \). Henceforth I will simply refer to the portfolio as a single loan. The single loan is a stand-in for a portfolio of loans, each with cash flow \( \tilde{\theta} + \tilde{x} + \tilde{\varepsilon}_n \), where \( \tilde{\theta} + \tilde{x} \) and \( \tilde{\varepsilon}_n \) are the systematic and the idiosyncratic components of loan risk respectively, mutually independent and with zero mean except for \( \tilde{\theta} \). Thus, the aggregate cash flow of the loan portfolio is \( \tilde{\theta} + \tilde{x} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} (\tilde{\theta} + \tilde{x} + \tilde{\varepsilon}_n) \). \( \theta \), the realization of \( \tilde{\theta} \), is privately observed by the bank at \( t = 1 \). \( \theta \) is drawn from distribution \( F(\theta) \) with density \( f(\theta) \) in \([0, \theta]\). In contrast, \( \tilde{x} \) cannot be influenced by anyone and its realization is not revealed to anyone, including the bank, until \( t = 2 \). \( \tilde{x} \) has density \( g(x) \) and distribution \( G(x) \) in \([x, \bar{x}]\), with \(-\infty \leq x < \bar{x} \leq \infty \). The interpretation is that banking is risky. The expected cash flow per unit of loan conditional on the bank’s private information at \( t = 1 \) is thus \( E_{1}[\tilde{\theta} + \tilde{x}] = \theta \).

At \( t = 1 \), after observing its loan quality \( \theta \), the bank makes its risk-taking decision \( k(\theta) \). Risk-taking by the bank is captured by its decision to retain loan risk. In addition to

\[\text{5} \text{ Alternatively, } \theta \text{ could reflect the privately known benefits of a risky loan to the bank protected by limited liability. Risk-taking by the bank is then influenced by the classic risk-shifting incentive.}\]
the expected benefit of the loan cash flow, the bank also faces a convex risk-management cost \( \frac{1}{2c}k(\theta)^2 \) on loan retention, where \( c \) controls the cost sensitivity to retention.\(^6\) In the rest of the paper, I will use the terms risk-taking and risk retention interchangeably.

To motivate bank regulation, I assume that the risk-taking by a (type of) bank generates systemic risk by adversely affecting other banks.\(^7\) The negative aggregate externality in the banking sector is captured by

\[
\int_0^{\theta_0} K(\theta) f(\theta) d\theta, \tag{1}
\]

where \( K(\theta) = \int \xi(\omega, \theta) k(\omega) f(\omega) d\omega \) measures the contribution to the aggregate externality by the risk-taking of bank type \( \theta \), and \( \xi(\omega, \theta) > 0 \) is the per-unit externality of the risk-taking of bank type \( \theta \) on bank type \( \omega \). Let \( \kappa(\theta) \equiv \int \xi(\omega, \theta) f(\omega) d\omega \). The greater the aggregate risk-taking in the banking sector \( \{k(\theta)\} \), the greater the aggregate externality. Since \( \theta \) is the only source of uncertainty about bank quality (loan quality and severity of externality), I refer to \( \theta \) as the quality (type) of the bank. The value of the bank of type \( \theta \) at \( t=1 \) is then

\[
U(k, \tau; \theta) = V(k; \theta) - \tau = \theta k - \frac{1}{2c}k^2 - \tau, \tag{2}
\]

where \( \tau \geq 0 \) is a tax set by the regulator. The bank values satisfy single-crossing

\[
-\frac{\partial}{\partial \theta} \frac{U_k}{U_\tau} = V_{k\theta} > 0, \quad \text{for all } \theta, k, \tau.
\]

In words, higher-quality banks benefit more (after tax) from risk-taking.

In designing the regulation \( (k(\theta), \tau(\theta)) \), the regulator’s objective is to maximize social welfare at \( t=0 \), the value of the banking sector plus total tax revenue with welfare weight

\(^6\) An alternative interpretation is a private risk-return tradeoff underlying the bank’s risk-taking choice. With bank value being increasing in the average loan cashflow and decreasing in the total cashflow risk, bank value can be written \( U = \mu(\theta) k - \frac{\sigma^2(\omega)k^2}{2c} \). The normalization \( h(\theta) = \frac{\mu(\theta)}{\sigma^2(\theta)} \) reconciles the bank value in the text for \( h(\theta) = \theta \). Our results rely on any increasing function \( h \).

\(^7\) See Greenwood, Landier, and Thesmar (2015) for sources of negative externalities in the banking sector.
\[ \lambda \in (0, 1), \]
\[ \int_{0}^{\bar{\theta}} [U(k, \tau; \theta) - \gamma K + \lambda \tau] f(\theta) d\theta. \]

where \( \gamma > 0 \) controls the sensitivity of the banking sector to the aggregate externality.

3 Equilibrium

To illustrate the failure in the competitive market outcome and its regulatory solution, I first present the outcomes when bank quality is public information. This will serve as the benchmark for the analysis under asymmetric information about bank quality. The equilibrium concept is Perfect Bayesian Equilibrium. Hats denote quantities in the analysis under asymmetric information.

3.1 Risk-taking and regulation under full information

In the unregulated competitive equilibrium, defined as \( \{k^M(\theta)\}, K^M \), banks choose their risk-taking ignoring the impact of their risk-taking \( k \) on the aggregate externality

i) \( k^M(\theta) = \arg \max_{k} \{ \theta k - \frac{1}{2 \theta} k^2 \} \) for all \( \theta \in [0, \bar{\theta}] \),

ii) \( K^M = \int_{0}^{\bar{\theta}} \xi(\theta, \omega) k^M(\theta) f(\theta) d\theta. \)

**Proposition 1.** In the unregulated competitive equilibrium under complete information, bank risk-taking is \( k^M(\theta) = c \theta \).

Trading off the benefits of risk-taking against its risk-management cost, the individual bank chooses the size of its loan portfolio, which determines the privately optimal size of the bank. The collective risk-taking behavior of all banks taken together however produces the social cost from the aggregate externality. The resulting welfare loss from this market
failure motivates the introduction of regulation.

In the regulator equilibrium, defined as \(\{k^R(\theta)\}, K^R\), the regulator maximizes social welfare through the choice of banks' collective risk-taking

\[
\{k^R(\theta)\}, K^R = \arg \max \{k(\theta)\} \int_0^\theta \left[ \theta k(\theta) - \frac{1}{2} k^2(\theta)^2 - \gamma K \right] f(\theta) d\theta \\
s.t. \quad K = \int_0^\theta \xi(\theta, \omega) k(\theta) f(\theta) d\theta.
\]

After substituting the aggregate externality into the objective function, the first-order condition to the program becomes

\[
\theta = \frac{1}{c} k^R(\theta) + \gamma k(\theta).
\]

**Proposition 2.** In the regulated equilibrium under full information, bank risk-taking is \(k^R(\theta) = c [\theta - \gamma k(\theta)]\).

The social optimum of risk-taking in the economy is reached where the marginal social benefit of risk-taking (the left-hand side of (3)) equals its marginal social cost (the right-hand side of (3)). When bank quality is public information, the regulator can exploit this information to perfectly and costlessly correct the market failure. The regulator chooses the banks’ risk-taking on their behalf taking into account the impact of their collective behavior on the aggregate externality. As such, the regulator internalizes the externality in his choices.

### 3.2 Regulation under asymmetric information

Bank quality is now private information of the bank, its type. This information asymmetry between the bank and the regulator complicates the design of regulation. The regulator must
elicit information from the bank to be able to use it in setting the regulation. To achieve this, the regulation needs to provide the bank with incentives to reveal its information and to subject itself to the regulation: regulation must be incentive compatible, reflected in constraints (4), and leave banking profitable, reflected in constraints (5).

Among all such feasible mechanisms, the optimal regulatory mechanism solves

$$\max_{k(\theta) \geq 0, \tau(\theta) \geq 0} \int_{0}^{\theta} \left[ U(k(\theta), \tau(\theta); \theta) - \gamma K(\{k(\theta)\}) + \lambda \tau(\theta) \right] f(\theta) d\theta$$

s.t. \quad $$U(k(\theta), \tau(\theta); \theta) \geq U(k(\theta'), \tau(\theta'); \theta) \quad \text{for} \quad \theta, \theta' \in [0, \theta] \quad \text{(4)}$$

and

$$U(k(\theta), \tau(\theta); \theta) \geq 0 \quad \text{for} \quad \theta \in [0, \theta] \quad \text{(5)}$$

This program can be simplified in a series of steps. The preference of higher-quality banks for higher risk-taking implies that, for banks to reveal themselves through their choice of risk-taking, risk-taking must be non-decreasing in \(\theta\). This allows me to replace the global incentive compatibility (IC) constraints in (4) with local IC constraints and a monotonicity constraint on risk-taking (Laffont and Maskin (1980))

$$\max_{\theta} U(k(\theta), \tau(\theta); s) \bigg|_{s=\theta}, \quad \text{(6)}$$

and

$$k'(\theta) \geq 0. \quad \text{(7)}$$

Define the indirect bank value \(U(\theta) \equiv U(k(\theta), \tau(\theta); \theta)\). Differentiating the indirect bank value, and using the envelope theorem with (6), gives \(U'(\theta) = U_{\theta}(k(\theta), \tau(\theta); \theta) = V(\theta)(k(\theta); \theta)\). Once optimal risk-taking is determined, the bank’s indirect value is obtained by integration as \(U(\theta) = U(0) + \int_{0}^{\theta} V(\theta)(k(s); s) ds\), where \(U(0) = 0.\)

To see this, note that, since taxes must be non-negative, constraints (5) impose that regulated banking be profitable. Then, there exists a break-even threshold of quality \(\theta\) below which regulated banking is unprofitable, characterized by \(V(k^*(\theta); \theta) = 0\). As a result, any type \(\theta\) in \([0, \theta]\) will not participate in banking (that is, not engage in risk-taking and not be taxed), that is \(k^*(\theta) = \tau^*(\theta) = 0\), and thus obtain zero value, that is \(V(k^*(\theta); \theta) = 0\). Thus, \(U(\theta) = 0\) for \(\theta \in [0, \theta]\).
indirect bank value gives the optimal tax

\[
\tau (\theta) = V (k (\theta) ; \theta) - \int_0^\theta V_\theta (k (s) ; s) \, ds.
\] (8)

Intuitively, the optimal tax is set so as to charge the bank for revealing itself through its risk-taking.

By substituting out the tax and simplifying, we can rewrite social welfare as

\[
\max_{k(\theta) \geq 0} \int_0^\theta \left[ V (k (\theta) ; \theta) - (1 - \lambda) \left( V (k (\theta) ; \theta) - \int_0^\theta V_\theta (k (s) ; s) \, ds \right) - \gamma K (\theta) \right] f (\theta) \, d\theta
= \max_{k(\theta) \geq 0} \int_0^\theta \left[ \lambda V (k (\theta) ; \theta) + (1 - \lambda) V_\theta (k (\theta) ; \theta) \frac{1 - F (\theta)}{f (\theta)} - \gamma K (\theta) \right] f (\theta) \, d\theta,
\] (9)

where the equality follows from integration by parts. The regulator’s simplified objective function is the sum of the bank’s so-called virtual surplus less the aggregate externality. The second term reflects the incentives for information revelation. Optimal risk-taking is now solution to maximizing social welfare subject to risk-taking being non-decreasing. I will solve the relaxed problem in (9) without the monotonicity constraint (7), and then check whether the constraint is satisfied at the solution. The first-order condition of the relaxed program is

\[
Y_k (k^* (\theta) ; \theta) \triangleq \lambda V_k (k^* (\theta) ; \theta) + (1 - \lambda) V_{\theta k} (k^* (\theta) ; \theta) \frac{1 - F (\theta)}{f (\theta)} - \gamma K_k (\theta) = 0.
\] (10)

Implicitly differentiating this condition gives

\[
\frac{d k^* (\theta)}{d \theta} = - \frac{Y_{\theta k} (k^* (\theta) ; \theta)}{Y_{kk} (k^* (\theta) ; \theta)}.
\] (11)

The second-order condition for a maximum, \( Y_{kk} < 0 \), implies that the sign of \( Y_{\theta k} (k^* (\theta) ; \theta) \)
determines the sign of \( \frac{d}{d\theta} k^*(\theta) \). Expanding the numerator gives

\[
Y_{k\theta} = \lambda V_{k\theta} + (1 - \lambda) V_{\theta k} \left[ 1 - F(\theta) \right]' + (1 - \lambda) V_{\theta k} \frac{1 - F(\theta)}{f(\theta)} - \gamma K_{k\theta}
\]

\[
= \lambda \left[ v'(\theta) V_{k\theta} - e'(\theta) \right]. \tag{12}
\]

where \( v'(\theta) \triangleq 1 + \left( \frac{1 - \lambda}{\lambda} \right) \left[ \frac{1 - F(\theta)}{f(\theta)} \right]' \) is the marginal virtual value of the bank, and \( e'(\theta) \triangleq \frac{\gamma K_{k\theta}}{\lambda} \) is the change in the marginal contribution to the externality by bank type \( \theta \). Thus, expression (12) reveals that, with single-crossing \( V_{k\theta} > 0 \), an increasing marginal virtual bank value is a necessary (but not sufficient) condition for \( k^*(\theta) \) to be non-decreasing - in contrast to the classic mechanism design problem. For risk-taking to be non-decreasing in the design of an optimal mechanism with externalities (from screening), the marginal social surplus of bank risk-taking must be higher for higher bank types, that is \( Y_{k\theta} > 0 \) for all \( \theta \). When \( Y_{k\theta} \) is not increasing for all bank types, risk-taking is not always increasing. As a result, the monotonicity constraint on risk-taking will bind for some bank types. Optimal risk-taking is then “ironed” out across some bank types resulting in pooling or bunching regions. In general, there can exist multiple such pooling intervals. Guesnerie and Laffont (1984) show how bunching can occur in classic mechanism design problems. The following lemma highlights a necessary condition for the existence of pooling.

**Lemma 1.** *(Bunching)* Given increasing marginal virtual bank values, heterogeneity of the externality in bank quality is a necessary condition for bank regulation to be risk-insensitive under asymmetric information.

Expression (11) shows that risk-taking is decreasing for some levels of bank quality when the marginal social surplus is decreasing in those intervals. From expression (12), it is straightforward to see that a necessary condition for this is that the contribution to the aggregate externality depends on bank quality.

Proposition 3 below illustrates a generalization of this analysis to mechanisms with
externalities (from screening). The solution to the regulator problem with a binding monotonicity constraint at the top of the bank-quality distribution is determined as follows:\(^9\)

\[
\max_{k(\theta), \bar{k}, \bar{\theta}} \int_{\theta}^{\bar{\theta}} Y\left(k(\theta) ; \theta \right) d\theta + \int_{\theta}^{\bar{\theta}} Y\left(\bar{k}; \theta \right) d\theta.
\]

Optimal risk-taking for any bank type \(\theta \in [\hat{\theta}, \theta]\) is determined by the pooling interval \([\hat{\theta}, \bar{\theta}]\) and the pooled level of risk-taking \(\bar{k}\), characterized by the following two conditions

\[
k^*(\bar{\theta}) = \bar{k}, \tag{13}
\]

\[
\int_{\hat{\theta}}^{\bar{\theta}} Y_k\left(\bar{k}; \theta \right) d\theta = 0. \tag{14}
\]

The first condition imposes that at the starting point of the bunching interval the separating level of risk-taking \(k^*\) coincide with the pooled level \(\bar{k}\). The second condition shows that the pooling interval is determined so that the marginal social surplus from risk-taking \(\bar{k}\) averages to zero over this interval.

### 4 Analysis

In this section, I will illustrate the general analysis of the preceding section with the following simple structure of bank quality, which I maintain in the rest of the paper.

**Assumption 1. (Bank quality)** Bank quality is uniformly distributed, \(\theta \sim U\left[0, \bar{\theta}\right]\), the externality is quadratic, \(\xi = 2\omega \left(\frac{\theta}{\bar{\theta}}\right)^2\), banking is moderately important, \(\lambda \in \left(\lambda_r, \lambda\right)\), and not too sensitive to the externality, \(\gamma < \frac{1}{2}\).

This structure of bank quality ensures a closed-form solution to the optimal regulation problem under asymmetric information, stated in Proposition 3 and depicted in Figure 1.

---

\(^9\) An alternative proof based on optimal control theory is included in the Appendix.
Proposition 3. (Optimal regulation) When bank quality is private information of the bank, regulated risk-taking and taxes are

\[
\hat{k}^R(\theta) = \begin{cases} 
0, & \text{for } \theta \in [0, \bar{\theta}) \\
c \left[ v(\theta) - e(\theta) \right], & \text{for } \theta \in \left[ \bar{\theta}, \hat{\theta} \right), \\
k, & \text{for } \theta \in \left[ \hat{\theta}, \bar{\theta} \right]
\end{cases}
\]

\[
\hat{\tau}^R(\theta) = \begin{cases} 
0, & \text{for } \theta \in [0, \bar{\theta}) \\
c \left[ v(\theta) - e(\theta) \right] \left[ \theta - \frac{1}{2} (v(\theta) - e(\theta)) \right], & \text{for } \theta \in \left[ \bar{\theta}, \hat{\theta} \right), \\
\hat{\tau} - \frac{1}{2c} k^2 - c \int_{\theta}^{\hat{\theta}} \left[ v(s) - e(s) \right] ds, & \text{for } \theta \in \left[ \hat{\theta}, \bar{\theta} \right]
\end{cases}
\]

where \( \bar{\theta} = \left[ \frac{1}{2} \sqrt{4\gamma(1-\lambda)+1-1} \right] \bar{\theta}, \hat{\theta} = \left[ \frac{3(2\lambda-1)-2\gamma}{4\gamma} \right] \bar{\theta}, k = c \left[ \frac{3(2\lambda-1)^2-4\gamma(2\lambda-3-\gamma)}{16\lambda\gamma} \right] \bar{\theta}, v(\theta) = \theta + \frac{1-\lambda}{\lambda} (\bar{\theta} - \theta) \) and \( e(\theta) = \frac{\gamma \theta^2}{2\lambda}. \)

Figure 1: Regulated risk-taking. The figure plots regulated risk-taking under full information (red) and asymmetric information (second-best (blue) and first-best (dashed)). Parameter values are \( \bar{\theta} = 4.2, \gamma = 0.43, \lambda = 0.75, c = 1. \)
The salient feature of Proposition 3 is that private incentives for information production exist (single-crossing holds). Yet, Proposition 3 shows that information production is limited even if the incentives for information production are perfect.

Producing information requires incentives to both supply and demand this information. Banks can supply information to the regulator through their risk-taking. Supplying information to the regulator then requires that a higher-quality bank retain more risk (the monotonicity constraint (7)). Intuitively, a higher-quality bank derives greater benefits from risk-taking and thus reveals itself through higher risk-taking. However, even though the potential supply of information by banks thus exists (equivalently, the ability by the regulator to elicit information from banks), the demand for this information does not always. A case in point is the interval $[\hat{\theta}, \theta^\ast]$ in Figure 1. This interval contains bank types that are willing to supply information (whose risk-taking is increasing). Yet, the regulator chooses not to demand this information (risk-taking is constant). The reason that the regulator does not always choose to elicit the information is because doing so would distort the allocation of risk (taking). From Proposition 3, the distortion of risk-taking (relative to the complete information market outcome) has two components. To see this, rewrite the first-order condition (10) as

$$\theta + \frac{1 - \lambda}{\lambda} \frac{1 - F(\theta)}{f(\theta)} = \frac{k^\ast(\theta)}{c} + \frac{\gamma}{\lambda} \kappa(\theta).$$  (15)

Optimal regulation balances the (marginal) social benefits of risk-taking, the left-hand side of (15), with its (marginal) social costs, the right-hand side. The two distortions to bank risk-taking are captured in the last term on either side. The last term on the LHS reflects an upward distortion in the risk-taking by bank type $\theta$ to achieve information revelation. The interpretation is that the regulation requires the bank to retain excessive risk (“skin in the game”) to separate itself from other bank types. The last term on
the RHS captures a downward distortion in bank $\theta$’s risk-taking due to its contribution to the aggregate externality. Relative to the regulatory outcome under full information characterized by (3), the bank takes additional risk to convey its riskiness to the regulator but in so doing exacerbates the market failure that motivates the regulation. This illustrates a new tension between the informational and the allocative efficiency that complicates the design of regulation under asymmetric information: the micro-economic role of risk retention (information production) conflicts with its macro-economic consequences (negative externalities).

4.1 Risk-insensitivity and risk weights

When bank quality is private information to the bank, conventional wisdom would hold that the efficient design of bank regulation would attempt to overcome the information asymmetry. As we saw in the previous subsection, the regulation indeed forces the bank to retain skin in the game to convey its private loan information to the regulator. The following lemma states that, as information asymmetry increases, so does its solution (skin in the game). Figure 2 illustrates.

**Lemma 2. (Regulated risk-taking)** The risk-taking of the regulated individual bank increases with the degree of information asymmetry, that is $\frac{\partial}{\partial \theta} k^R(\theta) > 0$ for $\theta \in [\overline{\theta}, \overline{\theta}]$, at an increasing rate, that is $\frac{\partial^2}{\partial \theta^2} k^R(\theta) > 0$ for $\theta \in [\overline{\theta}, \overline{\theta}]$.

However, as the information asymmetry increases, the regulation also uses less information. This paradoxical result is stated in Lemma 3. When the marginal social costs increase faster than the marginal social benefits as bank quality increases, the regulation ceases to use information and becomes information- and risk-insensitive. Risk-insensitivity of the regulation manifests in the interval $[\hat{\theta}, \overline{\theta}]$, in which both risk-taking and the tax are independent of bank quality $\theta$. This interval exists as long as the regulator places a moderate weight on banking, that is $\hat{\theta} \in [0, \overline{\theta}]$ for $\lambda \in (\overline{\lambda}, \overline{\lambda})$. The economic implication
Figure 2: Regulated risk-taking under increasing information asymmetry. The figure plots levels of regulated risk-taking \( \hat{k}^R(\theta) \) under two values of the information asymmetry parameter \( \overline{\theta} (\overline{\theta} = 4.2 \text{ (blue)}, \overline{\theta}' = 5.7 \text{ (red)}) \). Other parameter values are \( \gamma = 0.43, \lambda = 0.75, c = 1. \)

is that, in the pooling interval, the optimal tax is constant, as can be seen from (6): \( \tau'(\theta) = V_k \hat{k}'(\theta) = 0 \) for \( \theta \in [\hat{\theta}, \overline{\theta}] \). In words, revealing no additional information is free.

As such, the constant tax resembles the use of the same risk weight for a bucket of risky assets, much like in the Basel bank capital regulations. A generalization of Proposition 3 shows that, when the severity of externality varies strongly with bank quality \( \theta \), optimal regulation features multiple risk buckets, each endowed with a progressively higher tax.

This feature seems to fit real-world bank capital regulation quite well.

**Lemma 3. (Risk-insensitive regulation)** When bank quality is private information of the bank, regulation is risk-insensitive for high bank types \( \theta \in [\hat{\theta}, \overline{\theta}] \), and more likely so when the aggregate externality is costlier, that is \( \frac{\partial}{\partial \gamma}(\overline{\theta} - \hat{\theta}) > 0 \), and the informational friction is more severe, that is \( \frac{\partial}{\partial \theta}(\overline{\theta} - \hat{\theta}) > 0 \).
Lemma 3 highlights the extreme downside of risk-sensitive regulation. When banks with higher-quality loans (high expected cash flow) are also the ones whose risk-taking causes greater social harm (large externality), the regulator chooses not to make the regulation sensitive to the risk of these banks. That is, when the social benefits and the social costs from risk-taking are positively correlated but the social surplus (the difference) decreases with bank quality (as is the case with a quadratic externality), regulation becomes risk-insensitive at the top of the bank-quality distribution. The prediction that larger banks (from (7), risk-taking is (weakly) increasing in $\theta$) are the bigger troublemakers for the system is consistent with the concept of “systemically important financial institutions”, a recent regulatory development in the U.S. (see section 5).

$\gamma$ controls how harmful the externality is to the banking system and thus the speed at which risk-taking reduces social surplus as bank quality $\theta$ increases. A higher $\gamma$ reduces social surplus faster making risk-taking less socially desirable. Regulation discourages such risk-taking by pushing down the threshold of the bunching interval thus enlarging the interval. When $\gamma$ is not too large, such that risk-taking is still socially valuable, increasing the set of bank types (increasing $\bar{\theta}$) expands the bunching interval: as $\bar{\theta}$ increases, the threshold $\hat{\theta}(\bar{\theta})$ increases at a lower rate than the upper bound $\bar{\theta}$. In sum, as the market frictions (of information asymmetry, measured by an increase in $\bar{\theta}$, and the allocative externality, measured by $\gamma$) worsen, regulation becomes less sensitive to the very market failure (risk) it seeks to control.

4.2 Financial stability

The goal of bank regulation is to maximize the social value of banking and improve stability in the banking system, defined as minimizing the negative externalities that banks cause one another through their risk-taking activities. Lemma 3 highlights the main result of the paper that the informational friction could make regulation insensitive to very risk it is
meant to reduce. Despite the fact that bank regulation is designed to reduce the social cost from risk-taking, bank regulation could counterproductively increase this market failure when bank quality is private information to the bank.

**Proposition 4. (Financial stability)** Bank regulation increases financial instability under asymmetric information (relative to the full information case) if the banking sector is not too sensitive to the aggregate externality.

Proposition 4 underscores the delicate nature of regulation. In an attempt to reduce risk and improve financial stability, regulation could increase risk and exacerbate instability. The intuition is simple. To deal with the informational advantage of the bank and overcome the information asymmetry, the regulator forces the bank to retain excessive exposure to the risk of its assets, or “skin in the game”. Regulation thus tolerates this elevated social cost in its design. This paradoxical result is reminiscent of the theory of the second best from welfare economics. When there is more than one imperfection in the market (information asymmetry and negative externality), removing one imperfection (information asymmetry) could exacerbate another (negative externality) to the detriment of overall welfare.

5 Discussion

In this section, I discuss the empirical relevance of the mechanism and two policy implications of my theory.

5.1 “Skin in the game” risk retention

The incentives of financial intermediaries were at the center of the debate about the causes and fixes of the crisis. One solution to align incentives and overcome market imperfections is market participants holding “skin in the game”. Such retention of residual interests in
transactions could convey the quality of the transaction to the counterparty, and thus overcome the market failure from adverse selection.

While it is still debated whether “skin in the game” actually worked as intended, there is both theoretical and empirical support for this assumption. Retaining partial interests by the bank could be a solution to both its information advantage over investors or its unobservable incentive to improve the value of loans (e.g. Leland and Pyle (1977); Gorton and Pennacchi (1995); DeMarzo and Duffie (1999); Fender and Mitchell (2009); Chemla and Hennessy (2014); Vanasco (2015)). There is also empirical evidence indicating that banks do have private information and use retention as a signal (e.g. Simons (1993); Higgins and Mason (2004); Sufi (2007); Keys, Mukherjee, Seru, and Vig (2009); Acharya and Schnabl (2010); Loutskina and Strahan (2011); Erel, Nadauld, and Stulz (2014); Ashcraft, Gooriah, and Kermani (2014)). Acharya and Schnabl (2010) presents evidence that banks were exposed to the risk of their transferred assets ex post through investor recourse, consistent with the “skin in the game” mechanism. Higgins and Mason (2004) finds similar results and additionally shows that the recourse to sponsors of structuring transactions improved the long-run operating performance of sponsors as well as their stock prices in the short and long run following recourse events.

Retention is typically a large component on a bank’s balance sheet and exerts important influences on a bank’s income statement. Using the data from regulatory filings (e.g. schedules HC-S in Y-9C and RC-S in Call Reports) that U.S. bank holding companies file quarterly with the Federal Reserve, Chen, Liu, and Ryan (2008) report that on average the value of interest-only strips and subordinated asset-backed securities, two components of retention, accounts for about 11% of the outstanding principal balance of private label securitized loans. The information about a bank’s position in retention interest is also available from SEC filings (e.g. 10-Q and 10-K) if the position is material.
5.2 Macro-prudential regulation

My theory applies to macro-prudential regulations of financial institutions, that is any regulation designed to reduce negative externalities, most directly in the financial sector. The most notable example is bank capital regulation known as the Basel Accords promulgated by the Bank for International Settlements (BIS). In particular, in its latest accord known as Basel III, the BIS introduced additional capital regulations to target institutions whose activities are most likely to cause (greater) negative externalities, so-called systemically important financial institutions (SIFIs), including non-bank financial institutions.\textsuperscript{10} In addition, the Financial Stability Board (FSB), an international body of finance ministers and central bankers, monitors SIFIs as part of its goal of promoting financial stability.\textsuperscript{11}

A key feature of capital regulation is its risk-sensitivity. Conventional wisdom holds that efficient regulation should exploit all relevant information in its design. Put differently, regulation should reflect and be sensitive to the risk it seeks to control. Yet, a glance at the evolution of capital regulation offers a puzzle. The first such accord known as Basel I, introduced in 1989, allocated risks into buckets with varying capital charges. Accordingly, any two risky assets in the same bucket received the same charge. This crudeness in reflecting risk motivated the 1996 Basel II Accord to refine “risk-sensitivity” and improve “incentive compatibility” of the regulation (BIS (2001, p.1)). The Accord was designed to achieve this by offering an alternative approach to the bank by which it exploits its own information in setting risk charges. The puzzling empirical relationship between measured (regulated) risk and realized risk in the aftermath of Basel II led policymakers to question the effectiveness of this approach, and its inherent conflict of interest. Van Hoose (2007); Haldane (2013) and Acharya, Engle, and Pierret (2014) note that risk-sensitive measures were poor predictors of actual risk, and no better than risk-insensitive alternatives. Begley,\textsuperscript{12}

\textsuperscript{10}See BIS release http://www.bis.org/publ/bcbs207.htm.
Purnanandam, and Zheng (2016) finds that banks underreport their actual risk while Behn, Haselmann, and Vig (2014) and Becker and Opp (2014) show that risk-sensitive regulation actually increased risk in banks and insurance companies, respectively. In response to this counterproductive effect of risk-sensitive regulation on risk-taking experienced around the recent Financial Crisis, the BIS is working on amending its standards toward lower risk-sensitivity.¹²

My theory provides an explanation for the risk-insensitivity of observed capital regulation, and its counterproductive effects in Haldane (2013); Behn, Haselmann, and Vig (2014); Becker and Opp (2014). Eliciting information from banks to set regulation forces banks to retain excessive risk, thereby exacerbating the very market failure the regulation seeks to correct. Optimal regulation becomes risk-insensitive when the social cost from such skin-in-the-game becomes excessive.

5.3 Micro- in conflict with macro-prudential regulation

My theory also offers a caveat for the coherent design of bank regulation. Financial regulation comes in two forms: micro-prudential (aimed at the soundness of the individual institution) and macro-prudential (aimed at the soundness of the system as a whole). The distinction was first highlighted by Crockett (2000) and Hansen, Kashyap, and Stein (2011). In similar spirit to Morris and Shin (2008) albeit with a different mechanism, I argue that these regulations could be in conflict: attempting to reduce the fragility of the individual institution could exacerbate the instability of the system.

In response to the recent Financial Crisis, regulators in the U.S. and Europe have adopted micro-prudential regulations to require minimum risk retention by various financial institutions. The most prominent example is the 2010 Dodd-Frank Act in the U.S., which

has been adopted by virtually all U.S. financial agencies,\textsuperscript{13} and the Capital Requirements Regulation in Europe.\textsuperscript{14} The regulation received support from then U.S. Secretary of Treasury Tim Geithner, in his role as the chairman of the Financial Stability Oversight Council, who highlighted the “macro-economic effects of risk retention requirements”.\textsuperscript{15}

The caveat my theory offers is simple. Risk retention could improve the individual institution but destabilize the collective. The micro-economic rationale for risk retention is to overcome conflicts of interest arising from informational frictions that harm the individual institution. The macro-economic consequence of risk retention is the negative externality such risk poses to the financial system. The tradeoff in my theory revolves precisely around these two factors and demonstrates a limit to the use of risk retention as a solution to market frictions.

6 Conclusion

Banking is risky and prone to failure. Yet banking regulation is surprisingly not all that risk-sensitive. When the bank has an informational advantage over the regulator, designing banking regulation both affects and reflects bank behavior. This gives rise to a trade-off in risk-sensitive regulation: relying on the banking market for information to refine regulation aggravates its failure and could undermine its informativeness. This tension could explain why Basel capital regulations are often coarse or information-light.

\textsuperscript{13}http://www.sfindustry.org/images/uploads/pdfs/Risk_Retention_Final_Rule.pdf
References


A Appendix

**Implications of Assumption 1**

The restriction \( \lambda \in \left( \hat{\lambda}, \bar{\lambda} \right) = \left( \frac{1}{2} + \frac{1}{3} \gamma, \frac{1}{2} + \gamma \right) \) ensures the existence of the pooling interval, that is \( \hat{\theta} \in [0, \bar{\theta}] \). The restriction \( \gamma < \frac{1}{2} \) is a sufficient condition for \( \left( \hat{\lambda}, \bar{\lambda} \right) \subset (0, 1) \). I restrict \( \lambda \) to lie in a smaller interval \( \left( \lambda, \bar{\lambda} \right) \), where \( \lambda, \bar{\lambda} > \hat{\lambda} \), is given in the proof of Proposition 3.

The restriction \( \lambda > \bar{\lambda} \) is also sufficient for the virtual bank value \( v(\theta) = \theta + \frac{1 - \lambda}{\lambda} (\bar{\theta} - \theta) \) to be increasing. That is, \( v'(\theta) = 1 - \frac{1 - \lambda}{\lambda} > 0 \) requires \( \lambda > \frac{1}{2} \) but \( \bar{\lambda} > \frac{1}{2} \).
Proof of Proposition 3

The proof is broken into three parts: the proofs of the exclusion, the pooling and the separating regions.

The exclusion region $[0, \theta]$ is characterized by the threshold $\theta$, which is implicitly given by the bank’s break-even condition $V (k^* (\theta) ; \theta) = 0$. Substituting for $k^* (\theta)$ from (15) into this condition and simplifying shows that the threshold $\theta$ is the positive solution to $\theta^2 + \frac{\theta}{\gamma} - (1 - \lambda) \frac{\theta^2}{\gamma} = 0$, or $\theta = \left[ \sqrt{4\gamma (1 - \lambda) + 1} - 1 \right] \gamma$. Thus, set $\hat{k} R (\theta) = 0$ for $\theta \in [0, \theta]$.

The pooling region $[\hat{\theta}, \theta]$ is characterized by the threshold $\hat{\theta}$ that, together with the pooling level $\bar{k}$, is solution to conditions (13) and (14). Substituting for $k^* (\theta)$ evaluated at $\theta = \hat{\theta}$ in condition (13) gives $c \left[ \hat{\theta} + \frac{1}{1 - \lambda} \left( \theta - \hat{\theta} \right) - \frac{\bar{k}}{K} \right] = \bar{k}$. Condition (14) gives $\int_{\hat{\theta}}^{\theta} \left[ \lambda \left( \theta - \frac{\bar{k}}{c} \right) + (1 - \lambda) (\theta - \theta) - \gamma K_k \right] \frac{1}{\theta} d\theta = 0$. Solving these two equations for $(\bar{k}, \hat{\theta})$ gives the result in the Proposition. Thus, set $\hat{k} R (\theta) = \bar{k}$ for $\theta \in [\hat{\theta}, \theta]$.

The separating region follows from the previous result. In $[\theta, \hat{\theta}]$, $k^* (\theta)$ is non-decreasing and so the optimal risk-taking is solution to condition (10). The second-order condition is negative, that is $Y_{kk} = -\frac{\lambda}{c} < 0$, so the first-order condition gives a maximum. Thus, set $\hat{k} R (\theta) = k^* (\theta)$ for $\theta \in [\theta, \hat{\theta}]$.

An alternative, unifying proof of the separating and bunching solutions is based on optimal control theory. The control problem is defined by the Hamiltonian

$$H = Y (k (\theta), \tau (\theta); \theta) f (\theta) + \alpha (\theta) V_k (k (\theta) ; \theta) q (\theta) + \beta (\theta) q (\theta),$$

where $Y = V (k (\theta) ; \theta) - \tau (\theta) - \gamma K (\{k (\theta)\}) + \lambda \tau (\theta)$ is social welfare, $\tau' (\theta) = V_k (k (\theta) ; \theta) q (\theta)$ follows from incentive compatibility (6), with $q (\theta) = k' (\theta)$, and $q (\theta) \geq 0$ from the monotonicity constraint (7). This is a classical non-autonomous control problem with free
boundaries and an inequality constraint on the control \(q(\theta)\) is the control variable and \(\tau(\theta)\) and \(k(\theta)\) are the costate variables). The Pontryagin maximum principle provides the necessary and sufficient conditions for a solution

\[
\frac{\partial H}{\partial q} = 0 \iff \alpha(\theta) V_k + \beta(\theta) = 0, \quad (16)
\]

\[
\frac{\partial H}{\partial \tau} = -\alpha'(\theta) \iff Y_\tau f(\theta) = -\alpha'(\theta), \quad (17)
\]

\[
\frac{\partial H}{\partial k} = -\beta'(\theta) \iff Y_k f(\theta) + \alpha(\theta) V_{kk} q(\theta) = -\beta'(\theta), \quad (18)
\]

and the transversality conditions \(\alpha(\theta) = \alpha(0) = \beta(\theta) = \beta(0) = 0\). Condition (17) along with the transversality condition implies

\[
\alpha(\theta) = Y_\tau (1 - F(\theta)). \quad (19)
\]

Substituting (19) into condition (16) gives

\[
\beta(\theta) = -Y_\tau V_k (1 - F(\theta)). \quad (20)
\]

When the monotonicity constraint is binding, \(q(\theta) = 0\) and we have from condition (18)

\[
Y_k f(\theta) + \beta'(\theta) = 0 \iff (V_k - \gamma K_k) f(\theta) - Y_\tau \frac{d}{d\theta} [V_k (1 - F(\theta))] = 0
\]

\[
\iff (V_k - \gamma K_k) f(\theta) + (1 - \lambda) \left[ (1 - F(\theta)) \frac{d}{d\theta} V_k - f(\theta) V_k \right] = 0
\]

\[
\iff \left[ \lambda \left( \theta - \frac{k(\theta)}{c} \right) + (1 - \lambda) \frac{1 - F(\theta)}{f(\theta)} \left( 1 - \frac{q(\theta)}{c} \right) - \gamma K_k \right] f(\theta) = 0, \quad (21)
\]

where in the last equality I used the definitions of \(V\) and \(K\) from (2) and (1).

Note that in the set in which the monotonicity constraint is binding, call it \([\hat{\theta}, \bar{\theta}]\), we have for all \(\theta\) in \([\hat{\theta}, \bar{\theta}]\) \(q(\theta) = k'(\theta) = 0\) and thus \(k(\theta) = \bar{k}\), a constant. Integrating condition (21) over this set, we obtain the solution of the pooling region defined by the
following two conditions

\[ \int_{\hat{\theta}}^{\theta} \left[ \lambda \left( \theta - \bar{\kappa} \right) + (1 - \lambda) \frac{1-F(\theta)}{f(\theta)} - \gamma K_k \right] f(\theta) d\theta = 0, \]

\[ k(\hat{\theta}) = \bar{k}. \]

When the monotonicity constraint is not binding on some interval, \( q(\theta) > 0 \), condition (18) with expressions (19) and (20) gives

\[ 0 = Y_k f(\theta) + Y_\tau (1 - F(\theta)) V_{kkq}(\theta) - Y_\tau \frac{d}{d\theta} \left[ V_k (1 - F(\theta)) \right] \]

\[ \Leftrightarrow 0 = (V_k - \gamma K_k) f(\theta) + Y_\tau (1 - F(\theta)) V_{kkq}(\theta) - Y_\tau \left[ (1 - F(\theta)) \frac{d}{d\theta} V_k - f(\theta) V_k \right] \]

\[ \Leftrightarrow 0 = ((1 + Y_\tau) V_k - \gamma K_k) f(\theta) + Y_\tau (1 - F(\theta)) \left[ V_{kkq}(\theta) - \frac{d}{d\theta} V_k \right] \]

\[ \Leftrightarrow 0 = ((1 + Y_\tau) V_k - \gamma K_k) f(\theta) + Y_\tau (1 - F(\theta)) \left[ -\frac{q(\theta)}{c} - \left( 1 - \frac{q(\theta)}{c} \right) \right] \]

\[ \Leftrightarrow 0 = \lambda V_k - \gamma K_k + (1 - \lambda) \frac{1-F(\theta)}{f(\theta)} \]

\[ \Leftrightarrow k(\theta) = c \left[ \theta + \frac{1-\lambda}{\lambda} \frac{1-F(\theta)}{f(\theta)} - \frac{\gamma}{\lambda} K_k \right]. \]

This concludes the proof.

Finally, the thresholds are ordered as \( \theta < \hat{\theta} < \theta^* < \bar{\theta} \), where \( \theta^* \) is the maximizer of \( k^*(\theta) \) or \( \theta^* = (\lambda - \frac{1}{2}) \frac{2}{\gamma} \). Note that \( \lambda < \bar{\lambda} \) ensures that \( \theta^* < \bar{\theta} \) and \( \hat{\theta} < \theta^* \). Moreover, \( \theta < \hat{\theta} \) holds for \( \lambda > \lambda = (\sqrt{22} + 2) \frac{1}{9} \). For \( \gamma < \frac{1}{2} \), \( \hat{\lambda} < \lambda < \bar{\lambda} \) so requiring \( \lambda \in (\Delta, \bar{\lambda}) \) yields the ordering.

**Proof of Proposition 4**

Let \( \triangle_k \equiv \bar{K}^R - K^R \). \( \bar{K}^R \) and \( K^R \) are given by \( \bar{K}^R(\theta) = \int_{\hat{\theta}}^{\theta'} \xi(\omega, \theta) \bar{K}^R(\omega) f(\omega) d\omega \) and \( K^R(\theta) = \int_{\hat{\theta}}^{\theta'} \xi(\omega, \theta) K^R(\omega) f(\omega) d\omega \), where the break-even thresholds are \( \hat{\theta} = \left[ \sqrt{4\gamma (1 - \lambda) + 1} - 1 \right] \frac{2}{\sqrt{\gamma}} \)

and \( \theta' = \frac{\bar{\theta}}{\gamma} \). Since \( \gamma < \frac{1}{2} \), \( \theta' > \bar{\theta} \) and no bank is excluded under full information. Substituting and simplifying gives \( \triangle_k \propto 192 (16\lambda - 7) \gamma^5 - 80 (60\lambda - 43) \gamma^4 + 1080 \gamma^2 \psi^3 - 540 \gamma^4 \psi^4 + 81 \psi^5 \), where \( \psi \equiv 2\lambda - 1 > 0 \). Analytical results using Mathematica show that \( \int_{\hat{\theta}}^{\theta} \triangle_k f(\theta) d\theta > 0 \) under Assumption 1 and for \( \gamma < \frac{1}{3} \).