Where to hide in bad times: Or should one still diversify internationally?

Redouane Elkamhi

University of Toronto

Denitsa Stefanova[1]

University of Luxembourg

Luxembourg School of Finance

[1]Corresponding author: Denitsa Stefanova, University of Luxembourg, Luxembourg School of Finance, 4, rue Albert Borschette, L-1246 Luxembourg; telephone: +352 46 66 44 5589. E-mail: denitsa.stefanova@uni.lu

Where to hide in bad times: Or should one still diversify internationally?

Abstract

Among the stylized features of international equity markets is the pronounced asymmetric nonlinear dependence and upward trend in correlations. Such features call into question investors' efforts to diversify internationally. We propose a model to capture those well understood characteristics of international equity index returns. Casting them in a dynamic portfolio problem, we evaluate the gains for a home-biased investor from including foreign assets in her portfolio. We find that accounting for the optimal dynamic demand for hedging on top of a standard mean-variance portfolio policy brings substantial benefits from international portfolio exposure. Such benefits become increasingly sizeable over long investment horizons.

(*JEL* C15, C51, G11)

Keywords: Asset allocation, dynamic correlations, asymmetric dependence, diversification.

The premise for investing in international assets is typically driven by the benefits that diversification offers to an investor. Constructing a portfolio of foreign indices has both traditionally increased total returns and decreased volatility. There has, however, been a trend of increased correlations between international equity markets.[2] This increase in correlations and of asset co-movement in general has called the concept of diversification into question.

There is a large body of work documenting that international equities are characterized by time-varying co-movements, which are especially pronounced in extreme market downturns, while also trending upwards. Typically, such trends have been found for developed markets but not for emerging markets (see Eiling and Gerard 2007; Baele, Bekaert, and Inghelbrecht 2010; King, Sentana, and Wadhwani 1990; Karolyi and Stulz 1996; Forbes and Rigobon 2002; Brooks and Del Negro 2006; Lewis 2006). In a recent article, Elkamhi and Stefanova (2015) demonstrate that if these features of the data are ignored by the investor in her portfolio policy, the resulting economic loss is substantial. Importantly, they provide evidence that the portfolio losses stem primarily from intertemporal hedging motives.

The aim of this paper is to show that there are still significant diversification benefits internationally from the view of a US investor even if international equities' correlations are characterized by an upward trend and asset co-movements are substantial, especially pronounced in market downturns. We show that in order to capture the diversification benefits the investor has to (1) allow for such time variation and asymmetry in the data generation process that underlies her optimal portfolio decisions; and (2) solve for the optimal portfolio in a way to capture and hedge this dynamics. The gain we document in this article is economically significant, especially over a long investment horizon.

Specifically, we follow the modeling approach in Elkamhi and Stefanova (2015) and adopt a state-invariant tail co-movement (SITC) process with time-varying conditional correlations for the evolution of the prices of international equity indices. It accommodates asymmetric tail dependence (potentially higher in extreme market downturns than in market upturns) on top of a flexible model for the dynamics of each univariate series based on

[2]There is consensus in recent literature that correlations between international equity markets has increased over time (see Goetzmann, Li, and Rouwenhorst 2005). Christoffersen et al. (2012), show that many measures of international dependence have also increased significantly over the course of time.

the normal inverse Gaussian (NIG) distribution that accounts for higher-order moments in the distribution of asset returns. Our model accounts for well-documented stylized features of international equity returns (see Longin and Solnik 2001; Ang and Bekaert 2002; Das and Uppal 2004; Christoffersen, Errunza, Jacobs, and Langlois 2012).

To investigate the benefits from investing internationally, we solve for the portfolio problem of an investor who maximizes expected utility over terminal wealth by selecting a dynamic portfolio policy over a finite horizon. The investor allocates her wealth between a broad U.S. equity market index and an alternative equity index from a developed or an emerging market (we consider the MSCI equity market indices for the world ex U.S., Europe, Emerging markets and Asia Pacific). In addition, she invests in two default-free bonds (a long-term bond and a bond that matches the investment horizon) and cash.

For all pairs of equity market indices, we document a pronounced upward trend in conditional correlations. In line with previous literature, we also find prevailing evidence that equity market co-movements are asymmetric. However, we do not obtain a matching downward trend in the benefits from diversifying internationally. To the contrary, we document substantial gains for a diversifying investor compared to a home-biased one. These gains vary considerably over time but we do not find evidence that they are consistently diminishing. The grounds for this finding stem both from the properties of the data as well as from the nature of the portfolio problem.

First, while international equity return correlations show a significant upward sloping trend, the correlations between higher moments of returns are in fact rather low (see also Ghysels, Plazzi, and Valkanov 2011). Correlations between rolling sample estimates of skewness vary between 0.3 for the U.S. and World ex U.S. index pair and 0.1 for the U.S. and emerging markets pair. The evidence of such low higher-order correlations suggests the potential to ensure significant portfolio gains from investing internationally even in the presence of trending return correlations.

Second, conditional correlation is a linear measure of dependence that affects the instantaneous covariance matrix of asset returns. Hence, patterns in conditional correlations are directly reflected in the mean-variance demand of the investor. Consistent with the increasing trend in conditional correlations, we find that the certainty equivalent (CEQ) costs of not investing internationally are rather modest for short investment horizons of about a year. They range between zero and five cents per dollar for a CRRA investor with relative risk aversion of five.

This finding is consistent with the evidence brought forward by Christoffersen et al. (2012) that diversification benefits are disappearing. However, the CEQ results change dramatically when we increase the investment horizon. For a three-year investment horizon, the CEQ costs for not investing in a foreign asset rise to about 20 cents per dollar. For a nine-year horizon, a home-biased investor needs twice as much initial wealth to be able to obtain the same utility of terminal wealth as an investor who diversifies in emerging markets.

Third, we find that the benefits from investing internationally stem both from diversification and hedging motives. Comparing the costs of following a sub-optimal home-biased portfolio policy for a myopic investor and an investor who dynamically hedges changes in the investment opportunity set, we find that for shorter horizons both incur similar costs. For example, it would cost a home-biased investor 21 cents per dollar to raise her initial wealth in order to reach the utility of terminal wealth over a five-year period of an investor who diversifies with the World ex U.S. index for relative risk aversion of five. The corresponding certainty equivalent cost for a dynamically hedging investor amounts to 25 cents per dollar. For a nine-year period however, a dynamically hedging investor would require substantially higher initial wealth than a myopic investor, especially for higher levels of risk aversion. While a myopic investor would need to raise her initial wealth by about a third, the dynamically hedging one would need 50% more initial capital to render her indifferent between investing in the home asset or internationally for relative risk aversion of five.

Fourth, our results show that intertemporal hedging with international equity adds substantial value to a home-biased market timer who rebalances her portfolio every period. The certainty equivalent costs of not investing in the foreign asset are 50% higher than in the case when both the home-biased and the diversifying investor rebalance their portfolios every period with a one-period ahead horizon over nine years for the lowest level of risk aversion. The benefits of investing internationally become dramatically more important for the dynamic investor when her risk aversion increases, up to five times higher compared to the myopic case when the foreign asset is the World ex U.S. equity index.

Fifth, in our portfolio exercise we consider shifts towards the risk-free asset, potentially driven by changes in (extreme) asset co-movement. For example, intertemporal market price of risk hedging reduces the demand for equities in an internationally diversified portfolio for the period 2007-2008, inducing a more pronounced shift to the risk-free asset compared to the home-biased portfolio. Hedging considerations raise the demand for equity in the beginning of the sample period around 2002, while mean-variance demand shifts equity holdings upwards towards the end of the sample period relative to the holdings of a home-biased investor. Such substantial shifts in the portfolio composition for an investor who diversifies internationally are reflected in the relative performance of either portfolio strategy.

The long investment horizon over which we evaluate the portfolio performance of the home-biased and the internationally diversifying investor is itself characterized by an upward trend in conditional correlations. Thus, the costs associated with home bias may reflect just that trend. To investigate the effect of trending correlations, we consider an alternative portfolio exercise where we reset every year the portfolio problem of both the home-biased and the diversifying investor with an annual horizon. We then compute the portfolio improvement from holding a foreign asset in terms of the certainty equivalent wealth each year (or the annualized continuously compounded return). We find that the portfolio improvement is sizeable, going up to 20% when comparing the least risk-averse U.S.-only investor with one who includes an emerging market index in her portfolio. Importantly, we do not find any monotonic pattern of reduction in the portfolio improvement as we move forward in time. To the contrary, the portfolio improvement from investing internationally varies substantially over different periods. It attains it lowest levels in the 2007-2009 period but picks up considerably in 2011 to reach even higher levels than the pre-2007 period, most notably for emerging markets but also when the foreign asset is the World ex U.S. equity index. Thus, even though the rising trend in conditional correlations potentially plays a role in finding higher benefits of holding a foreign asset for longer horizons, accounting for dynamics in co-movement and hedging against changes in the investment opportunity set plays a substantial part in uncovering those benefits.

Overall we examine the fundamental question of whether there is any gain of international exposures in the investment portfolio after capturing stylized dynamics of the data and accounting for intertemporal hedging. The answer is yes and the gain is substantial over a broad spectrum of international equity indices.

Our paper contributes to a long strand of literature that investigates the benefits of international diversification (see Solnik and Roulet (2000); Errunza (1977), and more recently, Erb, Harvey, and Viskanta (1994); Santis and Gerard (1997); Errunza, Hogan, and Hung (1999); Bekaert, Harvey, and Ng (2005); Christoffersen et al. (2012)).

While the recent study of Christoffersen et al. (2012) shows that diversification benefits are disappearing except for the case of investing within emerging markets, we demonstrate that the gains of investing internationally are still a wide-spread phenomenon that varies substantially through time. While a measure of diversification benefits based on a coherent measure of risk like the expected shortfall as proposed by Christoffersen et al. (2012) still shows a downward trend consistent with the increased dependence across international assets, we demonstrate that an investor who dynamically hedges changes in the investment opportunity set, accounting for high or low states of tail dependence or conditional correlation, has still the potential to achieve substantial benefits from holding a foreign asset, evaluated using a utility-based criterion.

Our study also draws on the literature of tail risk in fundamentals and asset prices, see Barro (2006) and the extensions to a dynamic setting in Gabaix (2012); Gourio (2012); Wachter (2013). In terms of measuring the dynamics of tail risk in asset returns, one strand of literature relies on an option-based approach (see for example Bakshi, Kapadia, and Madan (2003) who investigate risk-neutral skewness and kurtosis and Backus, Chernov, and Martin (2011) who study disaster risk premia implied by index options). Another line of research uses high-frequency data to estimate tail risk (see Bollerslev and Todorov 2011). More recently, Moore et al. (2013) investigate the cross-sectional differences in downside tail risk of stock returns, Kelly and Jiang (2014) estimate aggregate tail risk from the cross-section and Chabi-Yo, Ruenzi, and Weigert (2014) use a parametric model to estimate tail dependence of individual stocks from their return history and analyze the ability of lower-tail dependent stocks to serve as a hedge during crisis periods.

The paper proceeds as follows. Section 1 discusses the data and presents some preliminary findings on the current trends in equity market co-movements. Section 2 presents the model for the equity price dynamics and the evolution of the interest rate. Section 3 introduces the optimal portfolio policy of an investor who diversifies internationally. Section 4 discusses the results on the portfolio composition and the economic gains from diversification realized by an investor who includes in her portfolio a foreign asset. Section 5 concludes.

1 Data and Preliminary Observations

We use daily returns on five international equity market indices over the period 1994 - 2011. They are provided by Morgan Stanley Capital Indices (MSCI) and are expressed in U.S. dollars. Our sample covers the returns on equity indices of the United States (U.S.), Europe (EU), emerging markets (EM), Asia-Pacific (AP) and World ex U.S.

Descriptive statistics of the data are given in Table 1. Panel A reports the univariate descriptive statistics. We confirm the negative skewness and the high levels of excess kurtosis documented in prior research on international diversification (see Ghysels, Plazzi, and Valkanov 2011)). Also, we see an increase in the levels of volatility and excess kurtosis over the most recent period.

International equity return correlations have also increased over time. In Panel B of Table 1 we document higher correlations between U.S. equity returns and other developed and emerging market equity indices over the most recent period. As well, an upward trend in correlations is illustrated in Figure 1, where we plot dynamic conditional correlation estimates for pairs of international index returns over the sample period. This evidence corroborates with the findings in Christoffersen, Errunza, Jacobs, and Langlois (2012), who link it to evidence of decreasing diversification benefits over time.

It is important to notice, however, that conditional correlations are considerably volatile (e.g. correlation levels increase from close to 0% to about 40% in the mid-nineties for the U.S.- Asia Pacific pair, and drop back to about 5% two years later). A sound portfolio allocation policy should be designed to capture the high variation of correlation levels over time.

In order to gauge the degree of correlations in the tail regions of the distribution of international index returns, we compute correlations between returns exceeding a set threshold, or exceedance correlations, following Longin and Solnik (2001). We plot the correlations between return exceedances for thresholds that cover the whole range of the distribution (see Figure 2). For all pairs of indices, exceedance correlations are higher in the left quadrants of the distribution. This evidence points towards higher dependence in down markets, which represents a clear departure from the assumption that international stock returns have a symmetric elliptical distribution.

Panel B of Table 1 presents further descriptive statistics of the multivariate distribution of the international index returns we consider. It reports quantile dependence coefficients at the 5th and the 95th percentile levels. The quantile dependence measures the dependence between each pair of return series in the lower or upper tails of their joint distribution. For both tails we document significant levels of quantile dependence, highest for the

U.S.-EU pair and lowest for the U.S.-Asia Pacific pair of returns. As well, dependence has increased in the most recent period, in line with the evidence on return correlations.

This empirical evidence of asymmetries in the dependence structure of international return distributions confirms previous findings in Longin and Solnik (2001), Christoffersen, Errunza, Jacobs, and Langlois (2012) and Elkamhi and Stefanova (2015) among others who document significant left tail dependence in the data.[3] It also motivates our modeling choice for the dependence structure between assets, where we follow Elkamhi and Stefanova (2015) in modeling both long-run and short-run dependence. In order to account for asset co-movements in the long run, we model the stationary distribution of the underlying state variables using a dependence function that allows for asymmetric upper and lower tail dependence. To account for instantaneous return co-movements, we allow for dynamic conditional correlations (DCC). Although conditional correlation is a linear measure, Engle and Colacito (2006) argue that an asymmetric DCC model gives rise to higher tail dependence in the multiperiod density. Conditional correlations in our model vary depending on the level of risky asset return volatility and the level of the state variables underlying the price process of risky assets.

Correlations that are trending up and asset returns that tend to be more dependent in down markets may be indicative of lower diversification benefits (see Christoffersen, Errunza, Jacobs, and Langlois 2012). However, the correlation between higher moments of returns may still be low to ensure significant portfolio gains (see Ghysels, Plazzi, and Valkanov 2011). In order to gauge the degree of such correlations, we compute two popular unconditional measures of asymmetry of the univariate distribution of returns - the unconditional skewness and Hinkley (1975) robust coefficient of asymmetry (skewness). The latter is defined as:

$$RA_{\theta}\left(r_{t}\right)=\frac{\left(q_{\theta}\left(r_{t}\right)-q_{0.5}\left(r_{t}\right)\right)-\left(q_{0.5}\left(r_{t}\right)-q_{1-\theta}\left(r_{t}\right)\right)}{q_{\theta}\left(r_{t}\right)-q_{1-\theta}\left(r_{t}\right)}$$

where $q_{\theta}\left(r_{t}\right)=F^{-1}\left(r_{t}\right)$ is the θ unconditional quantile of return r_t for $\theta \in (0,1]$. It measures the asymmetry of quantiles $q_{\theta}\left(r_{t}\right)$ and $q_{1-\theta}\left(r_{t}\right)$ with respect to the median, $q_{0.5}\left(r_{t}\right)$. We consider the inter-quartile range, setting $\theta = 0.75$ in which case the robust coefficient of asymmetry is also known as the Bowley (1920) statistic. When $RA_{\theta}\left(r_{t}\right)=0$, the distribution is symmetric, while values closer to -1 or 1 indicate skewness to the left or to the

[3] Tail dependence is a measure that describes the amount of dependence that exists in the far-left or the far-right quadrant of the multivariate distribution.

right. This measure of asymmetry is known to be robust to outliers, unlike the moment-based skewness.

In Figure 3 we plot 250-day rolling estimates of the skewness and robust asymmetry measures of U.S., World ex U.S. and emerging markets daily index returns. Both measures of asymmetry are highly time-varying. Interestingly, they are not as highly correlated as the returns themselves. The unconditional correlation of rolling skewness estimates is 0.31 for the U.S. and the World ex U.S., and 0.09 for the U.S. and emerging markets returns. The unconditional correlations between the robust asymmetry measures for both pairs of index returns are close to zero and even negative. This evidence indicates the potential for portfolio gains, even when correlations between returns are high and increasing.

2 Model

In our model, the state vector $X_t = (X_{1,t}, X_{2,t})^\intercal$ drives the dynamics of the price process of each pair of equity indices. We assume that the price of each risky asset i can be expressed as $S_{i,t} = \exp(k_i t + X_{i,t}), i = \{1, 2\}$, where k_i is a trend parameter.

Following Elkamhi and Stefanova (2015), we parameterize the process of the state vector X_t using a probability density function q which represents the invariant density of the process, and a state-dependent local covariance matrix Σ_t, which is continuously differentiable and positive definite. The drift of X_t is obtained through the Fokker–Planck equation (see Arnold 1974), setting the partial derivative of q with respect to time to zero and equating all remaining terms:

$$\mu_j(X_t) = \frac{1}{2q(X)_{X=X_t}} \sum\nolimits_{i=1}^{d} \frac{\partial\left(v_{ij}(X_{i,t}, X_{j,t})\, q(X)_{X=X_t}\right)}{\partial X_{i,t}}$$

where $\mu_j(X_t)$ is the j^{th} element of the drift μ and $v_{ij}(X_{i,t}, X_{j,t})$ are the entries of the local covariance matrix Σ_t. Thus we obtain the following process for the state vector X_t:

$$dX_t = \mu(X_t)\, dt + \Lambda(X_t)\, dB_t^X \tag{1}$$

where $\Sigma = \Lambda\Lambda^\intercal$ and B_t^X is a two-dimensional standard Brownian motion.

In our setup, co-movements between each pair of equity indices are determined by the choice of the invariant distribution and the local covariance matrix. We present our modeling choice for each of them below.

2.1 Tail-dependent co-movement

The invariant density q is parameterized using the following decomposition formula given by Sklar's theorem:

$$q(X) \equiv c(X|\Theta^c) \prod_{i=1}^{2} f^i\left(X_i|\Theta^{i,M}\right),$$

where $c(X|\Theta^c)$ is a dependence function with parameters Θ^c and $f^i\left(X_i|\Theta^{i,M}\right)$ is proportional to the marginal density of each state variable X_i with parameters $\Theta^{i,M}$. The latter is assumed to be normal inverse Gaussian (NIG). This specification allows us to address the extreme co-movements between the state variables independently of the marginal properties of each X_i.

The choice of the NIG distribution is warranted by its ability to accommodate both negative and positive skewness, as well as higher excess kurtosis that are present in the data.

As well, it can accommodate any tail behavior ranging from power to exponential decline, and can account for univariate tail asymmetries. The NIG marginals in the process given by Eq. (1) allow also to account for persistence in autocorrelation for the squared increments of log prices, similar to stochastic volatility or GARCH models. See Appendix A for an overview of the form and properties of the NIG distribution.

The dependence function $c(X|\Theta^c) \equiv c\left(F^1(X_1), F^2(X_2)|\Theta^c\right)$ is the density of the copula C, where F^i are probability integral transforms corresponding to each univariate series. The copula parameters Θ^c model dependence between the state variables X_i. Given the evidence of asymmetric tail dependence brought forward in Section 1, we assume the following copula function:

$$C^{Ga-SJC}\left(\upsilon_1, \upsilon_2|\rho^{Ga}, \tau_L, \tau_U\right) = \omega^{SJC} C^{SJC}(\upsilon_1, \upsilon_2|\tau_L, \tau_U) + (1-\omega)\, C^{Ga}\left(\upsilon_1, \upsilon_2|\rho^{Ga}\right) \tag{2}$$

where C^{Ga} stands for a bivariate Gaussian copula function with correlation parameter $\rho^{Ga} \in (0,1)$ and C^{SJC} is a bivariate symmetrized Joe–Clayton (SJC) copula with parameters $\tau_L \in (0,1)$ and $\tau_U \in (0,1)$ that measure lower- and upper-tail dependence, respectively. A mixing parameter $\omega^{SJC} \in [0,1]$ determines the weight of the SJC copula and hence the degree of tail dependence. See Appendix B for details on the copula functions.

2.2 Conditional correlation dynamics

The entries of the local covariance matrix $\Sigma = \Lambda (\Lambda)^{\intercal}$ are given by:

$$\begin{aligned} \nu_{ij} (X_t) &= \Upsilon_{ij} (X_t) \, \sigma_i^X (X_{i,t}) \, \sigma_j^X (X_{j,t}) \\ \sigma_i^X (X_{i,t}) &= \sigma_i f^i (X_{i,t})^{-\frac{1}{2}\kappa_i} \end{aligned} \tag{3}$$

where the function $f^i (X_{i,t})$ is proportional to the i^{th} univariate marginal NIG distribution, $\Upsilon_{ij} (X_t)$ is a time-varying pairwise conditional correlation parameter, $\sigma_i > 0$, and $\kappa_i \in [0, 1], i = 1, 2$. This specification extends the constant conditional correlation model in Elkamhi and Stefanova (2015). It has also been used in Bibby and Sørensen (2003) in a univariate setting.

By expressing the volatility term as the inverse of a power function of the density, we obtain the familiar U-shape for the local volatility, typical for stationary processes. As well, the drift function implied by this specification exhibits strong mean reversion in the tails.

We model conditional correlation dynamics via Υ_{ij}, which we parameterize as a function of the stochastic state variables X_t. In order to keep the correlation coefficient in $[-1, 1]$, we apply the following logistic transform:

$$\Upsilon_{ij} (X_t) = \mathcal{A} (h_{ij} (X_t)) = \frac{1 - \exp(-h_{ij} (X_t))}{1 + \exp(-h_{ij} (X_t))}.$$

Further, in order to replicate the stylized fact that correlation between asset returns increases in volatile periods and in extreme market downturns, we parameterize the function h as follows:

$$h_{ij} (X_t) = \gamma_{ij,0} + \gamma_{ij,1} \max \left(\sigma_1^X (X_{1,t}), \sigma_2^X (X_{2,t}) \right) + \gamma_{ij,2} \prod_{i=1}^{2} F^i (X_{i,t}) \tag{4}$$

2.3 The model for the interest rate

We use the two-factor essentially affine model of Duffee (2002) with the $A_1(2)$ specification as in Driessen (2005), following the notation of Dai and Singleton (2000). The instantaneous interest rate r_t is a function of two factors $X_t^* = \left(X_{1,t}^* \; X_{1,t}^* \right)^{\intercal} : r_t = \delta_0 + \delta_1 X_{1,t}^* + \delta_2 X_{2,t}^*$. Under the physical measure they follow:

$$\begin{bmatrix} dX_{1,t}^* \\ dX_{2,t}^* \end{bmatrix} = \begin{bmatrix} \kappa_{11}^* & 0 \\ \kappa_{21}^* & \kappa_{22}^* \end{bmatrix} \begin{bmatrix} \theta_1^* - X_{1,t}^* \\ -X_{2,t}^* \end{bmatrix} dt + \begin{bmatrix} \sqrt{X_{1,t}^*} & 0 \\ 0 & \sqrt{1 + \beta_{21}^* X_{1,t}^*} \end{bmatrix} dB_t^r \tag{5}$$

where B_t^r is a two-dimensional vector of independent Brownian motions. The Brownian vector $\widetilde{B}_t^r$ under the risk-neutral probability measure satisfies $\widetilde{B}_t^r = \theta^r(X_t^*)\,dt + dB_t^r$ where the price of risk vector $\theta^r(X_t^*)$ is modeled according to the essentially affine framework in Duffee (2002):

$$\begin{aligned}
\theta^r(X_t^*) &= \begin{bmatrix} \sqrt{X_{1,t}^*} & 0 \\ 0 & \sqrt{1+\beta_{21}^* X_{1,t}^*} \end{bmatrix} \begin{bmatrix} \lambda_{11} \\ \lambda_{12} \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 & 0 \\ 0 & \left(1+\beta_{21}^* X_{1,t}^*\right)^{-1/2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \lambda_{2(21)} & \lambda_{2(22)} \end{bmatrix} \begin{bmatrix} X_{1,t}^* \\ X_{2,t}^* \end{bmatrix} \\
&= \begin{bmatrix} \lambda_{11}\sqrt{X_{1,t}^*} \\ \lambda_{12}\sqrt{1+\beta_{21}^* X_{1,t}^*} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_{2(21)}\left(1+\beta_{21}^* X_{1,t}^*\right)^{-1/2} & \lambda_{2(22)}\left(1+\beta_{21}^* X_{1,t}^*\right)^{-1/2} \end{bmatrix} \begin{bmatrix} X_{1,t}^* \\ X_{2,t}^* \end{bmatrix}
\end{aligned} \tag{6}$$

This leads to the exponentially affine formula for the price of a zero-coupon bond at time t with maturity at time $t+\tau$:

$$P(t,\tau) = \exp\left[A(\tau) - B(\tau)\,X_t^*\right] \tag{7}$$

where $A(\tau)$ is a scalar function and $B(\tau)$ is a vector-valued function. Both functions can be obtained numerically by solving a series of ordinary differential equations.

The instantaneous bond price dynamics can be expressed as:

$$\frac{dP(t,\tau)}{P(t,\tau)} = \left(r_t - B(\tau)^\intercal\,\Xi_t\theta^r(X_t^*)\right)dt - B(\tau)^\intercal\,\theta^r(X_t^*)\,dB_t^r$$

where Ξ_t is the instantaneous volatility matrix in Equation (5) with diagonal elements given by $\Xi_{11,t} = \sqrt{X_{1,t}^*}$ and $\Xi_{22,t} = \sqrt{1+\beta_{21}^* X_{1,t}^*}$.

3 The Optimal Portfolio of a Diversifying Investor

The setup of the optimal portfolio problem follows the lines of Elkamhi and Stefanova (2015). When solving for the optimal dynamic portfolio policy, we follow the martingale approach in Cox and Huang (1989) and Ocone and Karatzas (1991). The Monte Carlo with Malliavin derivatives methodology introduced in Detemple, Garcia, and Rindisbacher (2003) allows us to obtain an explicit portfolio solution for the mean-variance demand as well as the intertemporal hedging terms. The explicit expressions for the portfolio terms involve conditional expectations of random variables that depend on the drift and variance of the state variables in the model.

Specifically, we consider the portfolio choice problem of an investor who diversifies internationally over a finite horizon $[0, T]$. She maximizes expected utility over terminal wealth by selecting a dynamic portfolio policy, consisting of $d = 2$ international equity indices, two zero-coupon default-free bonds and cash. One of the bonds is a long-term bond with constant maturity of 30 years. The other bond serves as the risk-free asset, as its maturity matches the investment horizon.

The price S_i of stock index i follows an Itô process given by

$$dS_{it} = S_{it}\mu_i^S(X_t)\,dt + (\Lambda_i(X_t))^\intercal\,dB_t^X \tag{8}$$

where the drift $\mu_i^S(X_t) = \mu_i(X_t) + k_i + \frac{1}{2}\sum_{j=1}^{d}\Lambda_{ij}(X_t)^2$ is the i^{th} element of $\mu^S(X_t)$ and $\Lambda_i(X_t)$ is the vector of volatility coefficients, $i = 1, ..., d$. Given the process in Equation (8), the price of stock market risk is given by:

$$\theta^X(X_t) \equiv \Lambda(X_t)^{-1}\left[\mu^S(X_t) - r_t\mathbf{1}\right] \tag{9}$$

where $\mathbf{1}$ is a d-dimensional vector of ones. We discuss the properties of θ^X in Section 4.1.The price of risk vector $\theta^r(X_t^*)$ associated with innovations in B_t^r is given in Equation (6). The associated state price density is given by:

$$\xi_{0,t} \equiv \exp\left\{-\int_0^t r_s ds - \int_0^t \theta_s^\intercal dB_s - \frac{1}{2}\int_0^t \theta_s^\intercal\theta_s ds\right\} \tag{10}$$

where $\theta \equiv \left(\theta^X\ \theta^r\right)^\intercal$.

The investor maximizes the expected utility of terminal wealth by selecting a dynamic portfolio policy that consists of the $d + 2$ risky asset and the riskless asset:

$$\max_{\alpha}\mathcal{U}(W_T) \equiv E\left[u(W_T)\right] \quad s.t. \tag{11}$$

$$\begin{aligned} &dW_t = r_tW_tdt + W_t\alpha_t^\intercal\left[\left(\widetilde{\mu}_t - r_t\mathbf{1}\right)dt + \widetilde{\Lambda}_t dB_t\right], \\ &W_0 = \overline{W} \text{ and } W_t \geq 0 \text{ for all } t \in [0, T] \end{aligned} \tag{12}$$

where W_t represents the wealth of the investor at time t, $\overline{W}$ is the initial wealth, α_t is a $(d + 2) \times 1$ vector of proportions invested in the risky assets at date t, $\widetilde{\mu}_t$ and $\widetilde{\Lambda}_t$ are the drift and the instantaneous covariance matrix of the price process of the d +2 risky assets. Uncertainty is driven by a $(d+2)$-dimensional standard Brownian motion $B = \left(B^X \; B^r\right)^\intercal$, where B^X and B^r represent risk factors impacting stock returns and the interest rate, respectively. This is a complete market setting since there are $d+2$ risky assets available to invest in and all risks can be hedged.

We assume time-separable von Neumann–Morgenstern preferences. Specifically, we consider the class of hyperbolic absolute risk aversion (HARA) utility functions of the form $u\left(W\right) = \frac{1}{1-R}\left(W+b\right)^{1-R}$, where R and b are exogenous constants.[4] The utility function $u\left(\cdot\right)$ is strictly increasing, concave, and differentiable with limits $\lim_{W\to\infty} u'\left(W\right) = 0$, and $\lim_{W\to 0} u'\left(W\right) \leq \infty$. Relative risk aversion is dependent on the level of wealth and on the two exogenous constants. It is given by $R\left(W\right) = RW/\left(W+b\right)$. A negative sign for the b coefficient implies intolerance towards wealth shortfalls for an investor with HARA utility. In that case $R\left(W\right)$ is decreasing and convex in wealth and it becomes infinite when wealth approaches $-b$. This specification is consistent with a case when the investor seeks portfolio insurance or has a minimum wealth constraint.

Following Theorem 1 in Detemple, Garcia, and Rindisbacher (2003), the optimal dynamic portfolio policy is given by:

$$\alpha_t = \left(\widetilde{\Lambda}_t\left(Y_t\right)^\intercal\right)^{-1}\left(\frac{1}{R\left(W_t\right)}c^{MV}\left(W_t, Y_t\right) - a^{IRH}\left(W_t, Y_t\right) - b^{MPRH}\left(W_t, Y_t\right)\right) \tag{13}$$

where $R\left(W_t\right)$ is the coefficient of relative risk aversion and $Y_t = \left(X_t^\intercal \; X_t^*\right)^\intercal$ is a $(d+2)$-dimensional vector of the state variables in the model that drive the equity price dynamics and the instantaneous interest rate. The first term $c^{MV}\left(W_t, Y_t\right)$ in the brackets is the conditional expectation of a random variable that involves the vector of state variables Y_t and the state-price density associated with the market price of risk θ. It contributes towards the mean-variance (MV) demand of the investor, given by $\left(\widetilde{\Lambda}_t\left(Y_t\right)^\intercal\right)^{-1}\frac{1}{R(W_t)}c^{MV}\left(W_t, Y_t\right)$. The other two terms in the brackets $a^{IRH}\left(W_t, Y_t\right)$ and $b^{MPRH}\left(W_t, Y_t\right)$ determine the intertemporal demand of the investor that provides hedging against fluctuations in the short rate and the market price of risk, respectively. They are expressed as conditional expectations that involve Malliavin derivatives of the relevant state variables. The expression for the optimal portfolio holdings as well as the implementation of the portfolio solution using the Monte Carlo with

[4]The Constant Relative Risk Aversion (CRRA) utility function is obtained by setting $b = 0$.

Malliavin derivatives approach of Detemple, Garcia, and Rindisbacher (2003) is discussed in Appendix D.

4 Results

This section presents our empirical results on the benefits of diversifying internationally when returns are characterized by extreme asset co-movements. Details of the estimation procedure are provided in Appendix C.

It is important to note that the optimal portfolio holdings given in Equation (13) depend on the specification of the market price of risk and the interest rate. In our model, we do not set exogenously the market price of risk. Instead, it is implied by the process for the state variables. Therefore, we first investigate the implied properties of the market price of risk that would be reflected in the portfolio policies of a home-biased and a diversifying investor.

4.1 Properties of the market price of risk (implied by the data-generating process considered by a diversifying and a home-biased investor)

The properties of the market price of risk that follows from a process of extreme asset co-movement derived along similar lines as the one defined in Equations (1), (2) and (3) have been extensively studied in Elkamhi and Stefanova (2015). The market price of risk has been found to be well-behaved and bounded. For completeness and given the particular choice of dependence function and correlation dynamics in Equations (2) and (3), as well as the set of test assets, we investigate the properties of the market price of risk implied by our model.[5]

The portfolio policy of the diversifying investor depends on the level of the multivariate MPR process associated with the data-generating process defined in (1), (2) and (3) (for the mean-variance component of the portfolio demand) and on the Malliavin derivative of the MPR, i.e. on the persistence of a shock to the underlying sources of risk (for the intertemporal hedging component). Thus, in order to analyze the portfolio policy of the investor, it is important to understand the underlying properties of the MPR.

We investigate the sensitivity of the MPRs to the main parameters that drive the dependence between all the pairs of assets we consider in our empirical exercise (see Figure 5). Namely, we investigate for each pair and for each level of quantiles of the state variables X_i and X_j, the sensitivities with respect to the copula weight

[5] The optimal portfolio solution depends also on the dynamics of the interest rate. Its properties implied by the two-factor essentially affine process are well-understood (see Duffee 2002 and Driessen 2005).

parameter ω^{SJC}, the upper/lower tail dependence parameters τ_U and τ_L, and the level of conditional correlation $\gamma_{ij,0}$. The components of the MPR behave well, they are bounded and are not explosive for all levels of the parameters considered.

Higher levels of the copula weight parameter ω^{SJC} and hence higher extreme co-movement imply strongly expressed nonlinearities in the MPR. It changes from a concave to a convex function of the state variables as their quantiles go from 0 to 1. We observe a similar behavior of the MPR when varying the level of tail dependence via the τ_U and τ_L parameters. Higher levels of conditional correlation also imply higher sensitivity of the MPR towards changes in the X quantiles.

A home-biased investor solves for her optimal portfolio given the univariate NIG diffusion for the evolution of the U.S. equity index. The univariate NIG process in this case is obtained by assuming an independent (product) copula for $c\left(X|\Theta^c\right) = \Pi_{i=1}^2 \left(F^i\left(X_i\right)\right)$ and zero conditional correlation in the instantaneous covariance matrix Σ. Thus, the process for each X_i reduces to:

$$dX_{it} = \widetilde{\mu_i}\left(X_{it}\right)dt + \sigma_i^X\left(X_{it}\right)dB_t^{X_i}, \tag{14}$$

where the drift $\widetilde{\mu_i}\left(X_{it}\right) = 0.5\left(1-\kappa_i\right)\left(\sigma_i^X\left(X_{it}\right)\right)^2 f\left(X_{it}\right)^{-1-\kappa_i} f'\left(X_{it}\right)$ and $\sigma_i^X\left(X_{it}\right)$ is the i^{th} diagonal element of the matrix Λ given in Equation (3).

Figure 4 plots the univariate MPR implied by the NIG process for the relevant state variable X_i for the U.S. equity index over the whole range of its quantiles and for different levels of the corresponding volatility coefficient σ_i. The MPR generally increase with the level of volatility. As well, it is more sensitive to changes in the state variable for high levels of volatility.

Comparing the MPR implied by that univariate model to the one obtained under the multivariate specification considered by a diversifying investor, a number of observations appear. The nonlinearities of the multivariate MPR are much more strongly expressed than in the univariate case of a home-biased investor. In addition, episodes of high correlation or extreme co-movement render the MPR much more sensitive to changes in the state variables. While the response of the univariate MPR to changes in the state variables is much more uniform across quantile ranges, the multivariate MPR is much more sensitive to changes in X close to the tail regions.[6]

[6]For some parameter combinations in the sensitivity exercises, the MPRs can take negative values. In order to investigate the range of

Over the following sections we investigate the portfolio composition of both investors, as well as the economic costs related to not diversifying internationally.

4.2 Portfolio terms

We solve for the portfolio problem of a diversifying investor outlined in Equations (11) and (12) along realized trajectories of the state variables and a diminishing investment horizon. The investor allocates her wealth between a U.S. equity index, a foreign equity index, a long-term bond, a horizon-matching bond and cash. We also obtain the optimal portfolio holdings for a home-biased investor who chooses between a U.S. equity index, a long-term bond, a horizon-matching bond and cash. A similar setup in which the portfolio solution is obtained along realized state variable trajectories has been used in Detemple, Garcia, and Rindisbacher (2003) and Elkamhi and Stefanova (2015).

Specifically, we divide our sample period in two sub-periods, 1994-2001 used only for estimating the parameters of the model, and 2002-2011 used for both updating parameter estimates and solving the portfolio allocation exercise. Each week in the allocation period, we solve for the optimal portfolio holdings using Equation (13), given the data generating process for interest rate state variables X^* in Equation (5) and the state variables X that govern the equity price process in Equation (1) for the diversifying investor and Equation (14) for the home-biased investor respectively. In this portfolio exercise we do not have any look-ahead bias since we re-estimate the model at each rebalancing period.

To obtain portfolio holdings, we follow the implementation of the portfolio solution described in Appendix D. We also decompose the portfolio weights in mean-variance weights and intertemporal hedging terms. We set the investor horizon at the end of 2011. Each week between 2002 and 2011 we obtain optimal portfolio holdings for both investors, using reestimated parameters given data up to that week.

This approach allows us to gauge the importance of having an internationally diversified portfolio at any moment in time during the 10-year allocation period. It also allows us to investigate the relevance of the investment

values that the MPRs typically take for the estimated parameters of the model, we plot the MPRs over the whole range of X quantiles. For the case of the univariate NIG process, the MPR for the U.S. is concentrated around 0.3 (see Figure AI in the Internet Appendix). For the case of the multivariate MPRs implied by our model of extreme co-movement, the mass of the MPRs is concentrated around 0.2 for the World ex U.S. index, around 0.1 for the EU index, around 0.15 for the emerging markets and 0.4 for Asia-Pacific (see Figure AII in the Internet Appendix). Overall, they all stay within acceptable bounds over the whole range of X quantiles, given the estimated parameters.

horizon and the hedging motives when gauging the benefits of investing internationally.

Table 2 reports the medians of the risky asset portfolio weights for a home-biased U.S.-only investor as well as an investor who diversifies internationally. We consider the standard case of a CRRA utility function, as well as the more general HARA case, where the parameter b is set to either -0.3 or 0.3. Thus, we consider both the case when the investor has a subsistence constraint (when b is negative) and when her relative risk aversion in instead decreasing and convex in wealth ($b > 0$).

Generally, a home-biased investor would allocate a similar proportion of her wealth to the U.S. equity index as the internationally diversifying investor, taken as median values across the allocation period (and for all investment horizons of up to 10 years). However, the latter would also allocate a significant portion of her wealth to the foreign equity index. She would invest a higher proportion of her wealth in the EU index, she would invest to a lesser extent in emerging markets, and would take a short position on average in the Asia Pacific equity index. As the level of risk aversion increases, the part invested in equities diminishes. Interestingly, for relative risk aversion of 10, the proportion of wealth invested in the equity portfolio is similar for both the home-biased and the diversifying investor, when we consider the World ex U.S. as the foreign equity investment. However, the equity mix is quite different for emerging or developed markets. As well, the allocation to the bond portfolio differs substantially across investors. A diversifying investor would generally invest more in bonds for higher levels of risk aversion. Specifically, the holdings of the horizon-matching bond (which is essentially the risk-free asset in our setup) are higher for the international investor.

Tables 4 and 3 give the break-up of the median portfolio holdings in mean-variance and hedging demand. Overall, both hedging and diversification considerations lead to different equity demand for a home-biased investor vs. one with an international portfolio. The relative importance of intertemporal hedging in explaining the differences in equity holdings between both types of investors is higher for low levels of risk aversion.

Given the dynamic nature of our portfolio exercise, it is difficult to draw a comparison between the portfolio holdings of both investors when looking at median weights only. We therefore plot on Figures 6 and 7 the evolution of the market price of risk hedging demand and the mean-variance equity holdings for equities along the allocation period. Both components of the optimal portfolio demand display much higher variability for the international investor in comparison to the home-biased one. Variations in the investment opportunity set or resetting the portfolio exercise at different points in time (and thus different states of the market) contribute to that greater variability in the portfolio terms. For example, a U.S.-only investor would considerably under-invest in equities in the years 2002-2003 while she would over-invest during the period of the recent financial crisis, compared to an investor who chooses to diversify her portfolio with an emerging markets equity index. While both diversification and hedging motives drive the difference in the earlier part of the sample, it is only because of hedging considerations that the investor lowers her equity demand over the 2007-2008 period. The home-biased investor is not hedging equity correlation risk or the risk of increased extreme asset co-movement, both very relevant during the latter period.

For a myopic investor, there are periods when the mean-variance demand for equity is almost identical in both cases of home bias and international diversification. If in the case of a diversifying investor this demand is dominated by the U.S. equity index, diversification benefits over these periods should be rather slim. Such periods of similar mean-variance demand for equity occur between mid 2007 and mid 2008 for the U.S. - World ex U.S. pair, and are substantially extended when the European or the emerging market indices are considered as the foreign asset, spanning between 2005 and mid 2008. However, outside these periods the mean-variance demand for equity diverges substantially for both types of investors.

Whether these differences in the optimal portfolio composition have a significant economic impact is a question that we address in the following section.

4.3 The economic cost of home bias

In order to assess the economic impact of investing internationally, we first compute the certainty equivalent (CEQ) cost of home bias. It represents the additional initial wealth a home-biased investor needs in order to generate the same utility of terminal wealth as the investor who diversifies internationally. Formally, it is defined as follows:

$$E_0\left[u\left(W_T^* \mid W_0 = 1\right)\right] = E_0\left[u\left(W_T \mid W_0 = (1 + CEQ)\right)\right] \tag{15}$$

where W_T^* is the optimal terminal wealth of the diversifying investor and W_T is the terminal wealth of the home-biased one.

Table 5 reports the cost of not diversifying internationally for all four alternative foreign indices we consider. Portfolio weights and the associated terminal wealth are obtained along realized paths of the state variables, following the portfolio exercise described in the previous section. We report CEQ costs for horizons from one to nine years and different levels of risk aversion of a CRRA investor. Across all pairs of indices, we find substantial home bias costs, going up to 3.73 dollars when diversifying with an emerging markets index for an investment horizon of nine years and relative risk aversion of 2. More risk-averse investors lose less when not investing internationally, in line with the portfolio composition results reported in the previous section. But even for them the costs range between 20 and 55 cents per dollar of initial wealth for the nine-year horizon. For short investment horizons the CEQ costs get significantly reduced and range between 0 and 13 cents per dollar for a one-year horizon. The latter result is consistent with the findings in Christoffersen, Errunza, Jacobs, and Langlois (2012) who find low benefits of international diversification for myopic investors.

In order to appreciate the impact of the form of the utility function on the benefits from investing internationally, on Figure 8 we plot the CEQ costs for the three utility specifications considered in the previous section, obtained following the same portfolio exercise with a diminishing investment horizon as in Table 2 for an investment period of up to nine years and $R = 5$. The gains from diversifying internationally are rather modest for investment horizons of up to a year (they range between 0 and 5 cents per dollar). Such low CEQ costs are consistent with the findings in earlier literature that, in recent years, a myopic investor would not gain much if she invests internationally (the end of the investment period in our portfolio exercise is set at 2011). However, CEQ costs increase significantly with the investment horizon for all three utility specifications. For example, a CRRA home-biased investor would need to double her initial wealth in order to achieve the same utility of terminal wealth as an investor who diversifies with an emerging markets equity index over a nine-year horizon. CEQ costs are highest for a HARA investor with a positive b coefficient who allocates more of her wealth to an investment in equities relative to a CRRA investor or an investor with a subsistence level (i.e. negative b coefficient).

The CEQ generally increase with the investment horizon for all three utility specifications. However, the slope of the CEQ cost as a function of the investment horizon changes substantially over time. To understand the reasons behind this time variation, we need to disentangle the sources behind the gains from adding a foreign asset to the portfolio. First, a home-biased investor loses by not being able to diversify her portfolio strategy in a mean-variance sense. Second, losses come from not hedging correlation risk or the tail risk of extreme asset co-movement, while investing in international equities benefits additionally from low correlations of higher moments.

In order to disentangle both effects, in addition to the portfolio exercise outlined above, we solve for the optimal portfolio holdings of two myopic investors who do not hedge intertemporally–a home-biased one and one that invests internationally. These investors take the changing investment opportunity set into account only to the extent that they incorporate weekly updated model parameters in their one-period ahead portfolio problem. Panel A of Table 6 reports the CEQ costs of home bias for such a myopic investor. These costs can be interpreted as the loss of benefits from diversification. In order to bring them to a comparable scale of those reported in Table 5, we track the evolution of wealth for periods of up to 9 years, while implementing a one-period ahead portfolio strategy of a market timer. The benefits from diversification only are lower than those found for dynamic investors that in addition hedge intertemporally. They are however still sizeable. CEQ costs reach 2.13 dollars per dollar of initial wealth for an investor who diversifies with an emerging market index for nine years but can reach below one cent per dollar for a one-year horizon. The magnitude of the diversification benefits we report is in line with the argument that a market-timer would benefit from low correlations between higher moments of returns, even when return correlations are high (see Ghysels, Plazzi, and Valkanov 2011).

In Panel B of Table 6 we present the benefits that stem both from diversification and from intertemporal hedging. Terminal wealth W_T^* in this case is obtained for a dynamic investor who invests internationally, as in Table 5, while W_T corresponds to the terminal wealth of a myopic home-biased investor who updates her one-period ahead portfolio rule weekly. The CEQ costs we obtain are the most substantial, rising to up almost five times initial wealth for the least risk-averse investor who invests in emerging markets and has a nine-year investment horizon. For a five-year horizon and the least risk averse investor, they are between 36% higher than mean-variance diversification costs when forgoing an investment in the European equity index and 66% higher when not benefitting from an emerging markets investment. For the most risk-averse investor, the percentage increase in costs is more dramatic. The loss of benefits from diversification and intertemporal hedging together is three times higher on average than the loss of benefits from mean-variance diversification alone.

We should note that the long investment horizon over which we evaluate the portfolio performance of the home-biased and the internationally diversifying investor is characterized by an upward trend in conditional correlations. Thus, the costs associated with home bias may reflect just that trend. To investigate the effect of trending correlations, we consider an alternative portfolio exercise where we reset every year the portfolio problem of both the home-biased and the diversifying investor with an annual horizon. We then compute the portfolio improvement from holding a foreign asset in terms of the annualized continuously compounded return in certainty-equivalent wealth, R^{CEW} (see Liu, Longstaff, and Pan (2003) for a similar treatment for derivative strategies). The certainty-equivalent wealth W^I for an investor who diversifies internationally is defined by $u\left(W^I\right) = E_0\left[u\left(W_T^* \mid W_0\right)\right]$. Similarly, we obtain the certainty-equivalent wealth of a home-biased investor, W^{HB}. Then the portfolio improvement is calculated as follows:

$$R^{CEW} = \frac{\ln W^I - \ln W^{HB}}{T}. \tag{16}$$

To abstract from the investment horizon effect, we perform a series of portfolio exercises throughout the allocation period. Each year between 2003 and 2001, we solve for the portfolio problem of a dynamic investor with a one-year horizon who invests internationally in any of the four equity indices or who is home-biased. We then compute the portfolio improvement from investing internationally following Equation (16) for each year in the allocation period.

We document the portfolio improvement in terms of certainty equivalent wealth in Table 7. In line with the CEQ costs reported previously, the portfolio improvement is sizeable, going up to 20% when comparing the least risk-averse U.S.-only investor with one who includes an emerging market index in her portfolio. The portfolio improvement diminishes with the level of risk aversion. It also varies substantially over different periods. It is generally higher during the 2004-2006 period, diminishes over the years of the recent financial crisis and picks up again after 2010. This dynamics reflects the changing levels of tail dependence in the data, which attain highest levels during the 2007-2009 period. These results are in line with the changing slope of the CEQ costs plotted on Figure 8 over different investment horizons.

The last column of Table 7 reports the annualized continuously compounded return in certainty-equivalent wealth for a long-horizon investor over the whole period between 2002 and 2011. The portfolio improvement reaches 4% for the least risk-averse investor and drops substantially when we increase the level of risk aversion.

We also consider the most traditional measure of performance evaluation - the Sharpe ratio. In Table 8 we compute Sharpe ratios for home-biased and diversifying investors, tracking the performance of their portfolios over the 2002-2011 period. In line with our previous results, Sharpe ratios of internationally diversifying investors are substantially higher than those of home-biased ones across utility functions. The largest difference is found for the least risk-averse investors, but even for levels of risk-aversion of 10, Sharpe ratios of portfolios with international equity may exceed twice the Sharpe ratio of a portfolio with U.S. equity only.

An alternative approach to evaluate the gains from holding a foreign asset is to consider as a benchmark the 1/n allocation across equities. This benchmark has been advocated by DeMiguel, Garlappi, and Uppal (2009) who show that none of the mean-variance optimal portfolio allocation strategies widely used in literature can consistently outperform naïve diversification. To analyze whether intertemporal hedging adds value within the context of an international portfolio, we form the following benchmark. To abstract from flight-to-quality effects, we keep the bond/equity/cash mix from the optimal portfolio policy of an investor with foreign exposure. Within the equity portfolio, we assign equal weights to the domestic and the foreign asset. We then compute the CEQ costs of naïve equity diversification along the nine-year investment horizon. Figure 9 plots the CEQ costs for a CRRA investor with relative risk aversion of 5 and two HARA investors with positive and negative b coefficients. We find consistent evidence across all pairs of equity indices of substantial gains from optimally diversifying with a foreign asset. They increase with the investment horizon and reach 0.7 cents per dollar of initial wealth for the U.S. & World ex U.S. portfolio. Compared to the results in Table 6 which evaluates the loss of benefits from both diversification and intertemporal hedging for a home-biased investor, the costs we document for the 1/n strategy are about 30% lower. These results suggest that even though there are benefits to be achieved even with naïve diversification relative to a home-biased portfolio, they are dominated by the benefits that come from dynamic hedging in an optimal international portfolio.

5 Conclusion

In this paper, we address the fundamental question of whether there exist sizeable benefits of international diversification in the context of increased dependence among assets. We take the viewpoint of a home-biased investor and evaluate the gains from adding a foreign asset to her portfolio. To model asset co-movement, we employ a model in the lines of Elkamhi and Stefanova (2015) that incorporates short-run dynamics in dependence via a specification for dynamic conditional correlations, as well as asymmetric extreme co-movement via the dependence structure of the invariant distribution of the state variables underlying the stock price process. Despite correlations trending upward for both developed and emerging markets, we find consistent evidence of substantial gains from diversifying internationally. The benefits stem both from diversification and hedging motives.

Our results have important implications for portfolio management in the context of international diversification and risk sharing. They also add to the debate of the poor performance of optimal portfolio strategies relative to a naive diversification rule. In addition, the importance of the intertemporal hedging demand in the presence of extreme asset co-movements raises the question of understanding tail risk premia at different horizons. We leave such asset pricing implications for future research.

Appendix A Form and Properties of the Univariate Normal Inverse Gaussian (NIG) Distribution

The normal inverse Gaussian (NIG) distribution is a member of the family of generalized hyperbolic distributions, constructed as normal mean-variance mixtures with the generalized inverse Gaussian (GIG) as the mixing distribution. Its density is given by:

$$\begin{aligned} f_{NIG}\left(x;\alpha^{NIG},\beta^{NIG},\delta^{NIG},\mu^{NIG}\right) &= c\left(\alpha^{NIG},\delta^{NIG}\right)\left(\left(\delta^{NIG}\right)^2+\left(x-\mu^{NIG}\right)^2\right)^{-\frac{1}{2}}\times \\ & K_1\left(\alpha^{NIG}\sqrt{\left(\delta^{NIG}\right)^2+\left(x-\mu^{NIG}\right)^2}\right) \\ & e^{\delta^{NIG}\sqrt{\left(\alpha^{NIG}\right)^2-\left(\beta^{NIG}\right)^2}+\beta^{NIG}\left(x-\mu^{NIG}\right)} \\ \text{where } c\left(\alpha^{NIG},\delta^{NIG}\right) &= \frac{\alpha^{NIG}\delta^{NIG}}{\pi} \\ x &\in \mathbb{R} \end{aligned}$$

where $\delta^{NIG}>0$, $\alpha^{NIG}\geq\left|\beta^{NIG}\right|\geq 0$, $\mu^{NIG}\in\mathbb{R}$, and K_λ is the modified Bessel function of the third kind with index λ, defined as :

$$K_\lambda\left(x\right)=\frac{1}{2}\int_0^\infty y^{\lambda-1}e^{-\frac{x}{2}\left(y+y^{-1}\right)}dy,\quad x>0$$

Its tail behavior is given by

$$\lim_{x\to\pm\infty} f_{NIG}\left(x;\alpha^{NIG},\beta^{NIG},\delta^{NIG},\mu^{NIG}\right)\sim|x|^{-3/2}\,e^{\left(\mp\alpha^{NIG}+\beta^{NIG}\right)x},$$

and it has the interesting property of being closed under convolution, so that the sum of two independent random variables that have a NIG distribution is also NIG-distributed.

In the portfolio application, we use several properties of the modified Bessel function that we summarize below (following Bibby and Sørensen 2003):

$$\begin{aligned} K_{-\lambda}\left(x\right) &= K_\lambda\left(x\right) \\ K_\lambda'\left(x\right) &= -\frac{\lambda}{x}K_\lambda\left(x\right)-K_{\lambda-1}\left(x\right) \\ K_{n+\frac{1}{2}}\left(x\right) &= \sqrt{\frac{\pi}{2x}}\exp\left(-x\right)\left[1+\sum\nolimits_{i=1}^n\frac{(n+i)!}{(n-i)!i!}\left(2x\right)^{-i}\right],\quad n=0,1,2,... \end{aligned}$$

The tail behavior of the NIG distribution can be represented as:

$$\lim_{x\to\pm\infty} f_{NIG}\left(x;\alpha^{NIG},\beta^{NIG},\delta^{NIG},\mu^{NIG}\right)\sim|x|^{-3/2}\exp\left\{\left(\mp\alpha^{NIG}+\beta^{NIG}\right)x\right\}$$

for a set of parameters $\Theta^M=\left\{\alpha^{NIG},\beta^{NIG},\delta^{NIG},\mu^{NIG}\right\}$(Prause 1999; Barndorff-Nielsen and Blaesild 1981).

Appendix B Copula Functions

We consider the following copula functions for the mixture copula in Equation (2):

1. Gaussian copula

The Gaussian dependence function is given by:

$$
\begin{aligned}
& C^{Ga}\left(\upsilon_1, \upsilon_2, ..., \upsilon_d \mid \rho^{Ga}\right) && (17) \\
= & \int_{-\infty}^{\Phi^{-1}(\upsilon_1)} ... \int_{-\infty}^{\Phi^{-1}(\upsilon_d)} \frac{R_{Ga}1}{2\pi \left|R_{Ga}\right|^{1/2}} \exp\left\{-\frac{1}{2}x^{\intercal} R_{Ga}^{-1/2} x\right\} dx_1 ... dx_d
\end{aligned}
$$

where $\upsilon_i \equiv F^{(i)}\left(X_i\right), i = 1, ..., d$ are probability integral transforms corresponding to each univariate series X_i, $\Phi^{-1}\left(\upsilon_1\right)$ is the inverse of the standard normal CDF and R_{Ga} is the correlation matrix. The Gaussian copula has no tail dependence.

2. Symmetrized Joe–Clayton (SJC) copula

The SJC copula, introduced by Patton (2006), is based on the bivariate Joe–Clayton (JC) copula. The latter is a two-parameter dependence function with parameters τ_L and τ_U. The JC copula C^{JC} has the following form:

$$
\begin{aligned}
& C^{JC}\left(\upsilon_1, \upsilon_2 \mid \tau_L, \tau_U\right) && (18) \\
= & 1 - \left\{1 - \left[\left(1 - \left(1 - \upsilon_1\right)^{\kappa}\right)^{-\gamma} + \left(1 - \left(1 - \upsilon_2\right)^{\kappa}\right)^{-\gamma} - 1\right]^{-\frac{1}{\gamma}}\right\}^{\frac{1}{\kappa}} \\
\text{where } \kappa = & \frac{1}{\log_2\left(2 - \tau_U\right)} \\
\gamma = & -\frac{1}{\log_2\left(2 - \tau_L\right)}
\end{aligned}
$$

The symmetrized version of the copula C^{SJC} is designed to render it completely symmetric for equal values of the lower- and upper-tail dependence parameters. It has the following form:

$$
\begin{aligned}
& C^{SJC}\left(\upsilon_1, \upsilon_2 \mid \tau_L, \tau_U\right) && (19) \\
= & \frac{1}{2}\left[C^{JC}\left(\upsilon_1, \upsilon_2 \mid \tau_L, \tau_U\right) + C^{JC}\left(1 - \upsilon_1, 1 - \upsilon_2 \mid \tau_U, \tau_L\right) + \upsilon_1 + \upsilon_2 - 1\right]
\end{aligned}
$$

Appendix C Estimating the Model

C.1 The International Stock Indices

To estimate the model for the price dynamics of the international stock indices given by Equation (8), we perform a two-step procedure. First, we estimate each univariate series assuming an independent copula and zero conditional correlations. The univariate process for the stock indices is given by Equation (14). Second, we estimate the multivariate model for the stock indices implied by Equation (1), taking the estimates of the univariate NIG parameters as given. For each step of the estimation procedure, we apply the sequential Markov chain Monte Carlo algorithm of Golightly and Wilkinson (2006).

We estimate the model sequentially, so that all parameters are reestimated each week in the period 1997 - 2011, reflecting the available information up to that moment. In each subsequent estimation, the sample period is expanded by one week. The set of parameter estimates of the model that corresponds to the full sample is given in Tables A.I and A.II in the Internet Appendix.

C.2 The Instantaneous Interest Rate

The model for the instantaneous interest rate is estimated with the unscented Kalman filter of (see Julier and Uhlmann 2004). We use constant maturity treasuries with maturities ranging from three months up to ten years. We obtain parameter estimates for each week in the period 1997 - 2011. In each subsequent estimation, the sample period is expanded by one week. The set of parameter estimates of the model is given in Table A.III in the Internet Appendix.

Appendix D The Optimal Dynamic Portfolio Policy

Following Theorem 1 in Detemple, Garcia, and Rindisbacher (2003), the optimal dynamic portfolio policy is given by:

$$\alpha_t = \left(\widetilde{\Lambda}_t \left(Y_t\right)^{\intercal}\right)^{-1} \left(\frac{1}{R\left(W_t\right)} c^{MV}\left(W_t, Y_t\right) - a^{IRH}\left(W_t, Y_t\right) - b^{MPRH}\left(W_t, Y_t\right)\right) \tag{20}$$

where $R\left(W_t\right)$ is the coefficient of relative risk aversion and $Y_t = \left(X_t^{\intercal}\ X_t^{*}\right)^{\intercal}$ is a $(d+2)$-dimensional vector of the state variables in the model that drive the equity price dynamics and the instantaneous interest rate. The terms $c^{MV}\left(W_t, Y_t\right)$, $a^{IRH}\left(W_t, Y_t\right)$ and $b^{MPRH}\left(W_t, Y_t\right)$ are given by:

$$\begin{aligned} c^{MV}\left(W_t, Y_t\right) &= \theta\left(Y_t\right) E_t\left[\xi_{t,T} \frac{W_T}{W_t} \frac{R\left(W_t\right)}{R\left(W_T\right)} \mathbf{1}_{W_T>0}\right] \\ a^{IRH}\left(W_t, Y_t\right) &= E_t\left[\xi_{t,T} \frac{W_T}{W_t}\left(1 - R\left(W_T\right)^{-1}\right) \mathbf{1}_{W_T>0} \int_t^T \mathcal{D}_t r_s\right] \\ b^{MPRH}\left(W_t, Y_t\right) &= E_t\left[\xi_{t,T} \frac{W_T}{W_t}\left(1 - R\left(W_T\right)^{-1}\right) \mathbf{1}_{W_T>0} \int_t^T \left(dB_s + \theta_s ds\right)^{\intercal} \mathcal{D}_t \theta_s\right]. \end{aligned} \tag{21}$$

Optimal wealth is given by $W_t = E_t\left[\xi_{t,T} I\left(y\xi_T\right)^{+}\right]$ where I is the inverse marginal utility function with $I\left(\cdot\right)^{+} = \max\left\{I\left(\cdot\right), 0\right\}$. As well, y solves $\overline{W} = E\left[\xi_T I\left(y\xi_T\right)^{+}\right]$. $\mathcal{D}_t r$ and $\mathcal{D}_t \theta$ are the Malliavin derivatives of the short rate r and the MPR θ. $\widetilde{\Lambda}_t\left(Y_t\right)$ is a block-diagonal matrix with elements $\widetilde{\Lambda}_{1:2,1:2,t}\left(Y_t\right) = \Lambda\left(X_t\right)$ and $\widetilde{\Lambda}_{3:4,3:4,t}\left(Y_t\right) = -B\left(\tau\right)^{\intercal} \theta^{r}\left(X_t^{*}\right)$.

The first term in Equation (21) contributes towards the mean-variance demand. The latter is given by $\left(\widetilde{\Lambda}_t\left(Y_t\right)^{\intercal}\right)^{-1} \frac{1}{R(W_t)} c^{MV}$
The intertemporal interest rate hedging term is given by $-\left(\widetilde{\Lambda}_t\left(Y_t\right)^{\intercal}\right)^{-1} a^{IRH}\left(W_t, Y_t\right)$, and the intertemporal market price of risk hedging term is given by $-\left(\widetilde{\Lambda}_t\left(Y_t\right)^{\intercal}\right)^{-1} b^{MPRH}\left(W_t, Y_t\right)$.

We follow the Monte Carlo with Malliavin derivatives methodology of Detemple, Garcia, and Rindisbacher (2003) to implement the portfolio solution. We use randomized quasi–Monte Carlo for variance reduction. Details of the implementation of the portfolio policy are given in Elkamhi and Stefanova (2015). We replicate them below for completeness.

D.0.1 Implementation of the Portfolio Formulas via Quasi–Monte Carlo (QMC)

The implementation of the portfolio formulas in Equation (21) requires computing conditional expectations of the state vector $Y_t = (X_t^\intercal \; X_t^* \; Z_t)^\intercal$ and its Malliavin derivatives $\mathcal{D}_t Y_s = (\mathcal{D}_{1,t} Y_s, ..., \mathcal{D}_{d+2,t} Y_s)$. Both the state variables and their Malliavin derivatives solve stochastic differential equations (SDEs) that can be simulated using standard discretization techniques. In particular, $\mathcal{D}_{kt} Y_s, k = 1, ..., d+2$ solves:

$$d\left(\mathcal{D}_{kt} Y_s\right) = \frac{\partial \mu^Y\left(Y_s\right)}{\partial Y} \mathcal{D}_{kt} Y_s ds + \left(\sum\nolimits_{j=1}^{d+1} \frac{\partial \sigma_{\cdot j}^Y\left(Y_s\right)}{\partial Y} dB_{js}\right) \mathcal{D}_{kt} Y_s \tag{22}$$

subject to $\lim_{s \to t} \mathcal{D}_{kt} Y_s = \sigma_{\cdot k}^Y\left(Y_s\right)$, where μ^Y is obtained by stacking the drift of X_t, X_t^*, and Z_t, and similarly for σ^Y, and $\sigma_{\cdot j}^Y\left(Y_s\right)$ is the j^{th} column of the volatility term for the state vector Y_t.

An expansion of the state space to $\left(Y_s, D_t Y_s, \xi_{t,s}, H_{t,s}^r, H_{t,s}^\theta\right)$ and a standard application of Itô's lemma gives us a system of SDEs that can be simulated using standard discretization techniques.

We simulate M trajectories of the solutions of these SDEs using the Euler discretization scheme with N discretization points. We rely on randomized quasi-Monte Carlo (QMC) using a Sobol sequence, following Matousek (1998).

We simulate Y using 65,536 Monte Carlo paths and daily discretization intervals ($N = 256$).

We obtain a set of M estimates of the random variables: $(Y_s^{N,m}, D_t^{N,m} Y_s, \xi_{t,s}^{N,m}, H_{t,s}^{N,m,r}, H_{t,s}^{N,m,\theta})$. Averaging over M, we obtain estimates of the conditional expectations in $\left(\alpha_t^{IRH}, \alpha_t^{MPRH}\right)$. Thus, for the mean-variance and the hedging terms, we have the following estimates:

$$\begin{aligned}
\widehat{\alpha}_t^{MV} &= \left(\Lambda_t\left(Y_t\right)^\intercal\right)^{-1} \theta\left(Y_t\right) \left(\sum_{m=1}^{M} \xi_{t,T}^{N,m} I\left(y\xi_T^{N,m}\right)^+\right)^{-1} \sum_{m=1}^{M} \xi_{t,T}^{N,m} \frac{I\left(y\xi_T^{N,m}\right)^+}{R\left(I\left(y\xi_T^{N,m}\right)^+\right)} 1_{I\left(y\xi_T^{N,m}\right)>0} \\
\hat{\alpha}_t^{IRH} &= -\left(\Lambda_t\left(Y_t\right)^\intercal\right)^{-1} \left(\sum_{m=1}^{M} \xi_{t,T}^{N,m} I\left(y\xi_T^{N,m}\right)^+\right)^{-1} \\
&\quad \sum_{m=1}^{M} \left[\xi_{t,T}^{N,m} I\left(y\xi_T^{N,m}\right)^+ \left(1 - R\left(I\left(y\xi_T^{N,m}\right)^+\right)^{-1}\right) 1_{I\left(y\xi_T^{N,m}\right)^+>0} H_{t,T}^{N,m,r}\right] \\
\hat{\alpha}_t^{MPRH} &= -\left(\Lambda_t\left(Y_t\right)^\intercal\right)^{-1} \left(\sum_{m=1}^{M} \xi_{t,T}^{N,m} I\left(y\xi_T^{N,m}\right)^+\right)^{-1} \\
&\quad \sum_{m=1}^{M} \left[\xi_{t,T}^{N,m} I\left(y\xi_T^{N,m}\right)^+ \left(1 - R\left(I\left(y\xi_T^{N,m}\right)^+\right)^{-1}\right) 1_{I\left(y\xi_T^{N,m}\right)^+>0} H_{t,T}^{N,m,\theta}\right]
\end{aligned}$$

References

Ang, A., and Bekaert, G. (2002). International asset allocation with regime shifts. *Review of Financial Studies* 15(4):1137–87.

Arnold, L. (1974). *Stochastic differential equations: Theory and applications*. New York: Wiley.

Backus, D., Chernov, M., and Martin, I. (2011). Disasters implied by equity index options. *The Journal of Finance* 66(6):1969–2012.

Baele, L., Bekaert, G., and Inghelbrecht, K. (2010). The determinants of stock and bond return comovements. *Review of Financial Studies* 23:2374–428.

Bakshi, G., Kapadia, N., and Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies* 16(1):101–43.

Barndorff-Nielsen, O., and Blaesild, P. (1981). Hyperbolic distributions and ramifications: Contributions to theory and application. In C. Taillie, G. P. Patil, and B. A. Baldessari (Eds.), *Statistical distributions in scientific work*. Dordrecht: Reidel.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics* 823–66.

Bekaert, G., Harvey, C. R., and Ng, A. (2005). Market integration and contagion. *Journal of Business* 78(1).

Bibby, B. M., and Sørensen, M. (2003). Hyperbolic processes in finance. In *Handbook of heavy tailed distributions in finance,* 211–48. New York: Elsevier.

Bollerslev, T., and Todorov, V. (2011). Tails, fears, and risk premia. *The Journal of Finance* 66(6):2165–211.

Bowley, A. L. (1920). *Elements of statistics*, vol. 2. PS King.

Brooks, R., and Del Negro, M. (2006). Firm-level evidence on international stock market comovement. *Review of Finance* 10(1):69–98.

Chabi-Yo, F., Ruenzi, S., and Weigert, F. (2014). Crash sensitivity and the cross-section of expected stock returns. *Available at SSRN 2011746* .

Christoffersen, P., Errunza, V., Jacobs, K., and Langlois, H. (2012). Is the potential for international diversification disappearing? A dynamic copula approach. *Review of Financial Studies* 25(12):3711–51.

Cox, J. C., and Huang, C. (1989). Optimal consumption and portfolio policies when asset prices follow a diffusion process. *Journal of Economic Theory* 49(1):33–83.

Dai, Q., and Singleton, K. J. (2000). Specification analysis of affine term structure models. *Journal of Finance* 55(5):1943–78.

Das, S. R., and Uppal, R. (2004). Systemic risk and international portfolio choice. *Journal of Finance* 59(6): 2809–34.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *Review of Financial Studies* 22(5):1915–53.

Detemple, J. B., Garcia, R., and Rindisbacher, M. (2003). A Monte Carlo method for optimal portfolios. *Journal of Finance* 58(1):401–46.

Driessen, J. (2005). Is default event risk priced in corporate bonds? *Review of Financial Studies* 18(1):165–95.

Duffee, G. R. (2002). Term premia and interest rate forecasts in affine models. *Journal of Finance* 57(1):405–43.

Eiling, E., and Gerard, B. (2007). Dispersion, equity returns correlations and market integration. Working Paper, University of Toronto.

Elkamhi, R., and Stefanova, D. (2015). Dynamic hedging and extreme asset co-movements. *Review of Financial Studies* 28(3):743–90.

Engle, R., and Colacito, R. (2006). Testing and valuing dynamic correlations for asset allocation. *Journal of Business & Economic Statistics* 24(2):238–53.

Erb, C. B., Harvey, C. R., and Viskanta, T. E. (1994). Forecasting international equity correlations. *Financial analysts journal* 50(6):32–45.

Errunza, V., Hogan, K., and Hung, M.-W. (1999). Can the gains from international diversification be achieved without trading abroad? *The Journal of Finance* 54(6):2075–107.

Errunza, V. R. (1977). Gains from portfolio diversification into less developed countries' securities. *Journal of International Business Studies* 83–99.

Forbes, K., and Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements. *Journal of Finance* 57(5):2223–61.

Gabaix, X. (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly Journal of Economics* 127:645–700.

Ghysels, E., Plazzi, A., and Valkanov, R. I. (2011). Conditional skewness of stock market returns in developed and emerging markets and its economic fundamentals. *Swiss Finance Institute Research Paper* (11-06).

Goetzmann, W. N., Li, L., and Rouwenhorst, K. G. (2005). Long-term global market correlations. *Journal of Business* 78:1–38.

Golightly, D., and Wilkinson, J. (2006). Bayesian sequential inference for nonlinear multivariate diffusions. *Statistics and Computing* 16:323–38.

Gourio, F. (2012). Disaster risk and business cycles. *American Economic Review* 102(6):2734–66.

Hinkley, D. V. (1975). On power transformations to symmetry. *Biometrika* 62(1):101–11.

Julier, S. J., and Uhlmann, J. K. (2004). Unscented filtering and nonlinear estimation. *Proceedings of the Institute of Electrical and Electronics Engineers* 92(3):401–22.

Karolyi, G., and Stulz, R. (1996). Why do markets move together? An investigation of U.S.-Japan stock return comovements. *Journal of Finance* 51(3):951–86.

Kelly, B., and Jiang, H. (2014). Tail risk and asset prices. *Review of Financial Studies* 27(10):2841–71.

King, M., Sentana, E., and Wadhwani, S. (1990). Volatiltiy and links between national stock markets. Tech. rep., National Bureau of Economic Research.

Lewis, K. K. (2006). Is the international diversification potential diminishing? Foreign equity inside and outside the U.S. Technical Report, National Bureau of Economic Research.

Liu, J., Longstaff, F., and Pan, J. (2003). Dynamic asset allocation with event risk. *Journal of Finance* 58:231–59.

Longin, F., and Solnik, B. (2001). Extreme correlation of international equity markets. *Journal of Finance* 56: 649–75.

Matousek, J. (1998). On the L2-discrepancy for anchored boxes. *Journal of Complexity* 14:527–56.

Moore, K., Sun, P., de Vries, C. G., and Zhou, C. (2013). The cross-section of tail risks in stock returns .

Ocone, D., and Karatzas, I. (1991). A generalized Clark representation formula with applications to optimal portfolios. *Stochastics and Stochastic Reports* 34:187–220.

Patton, A. (2006). Modeling asymmetric exchange rate dependence. *International Economic Review* 47(2): 527–56.

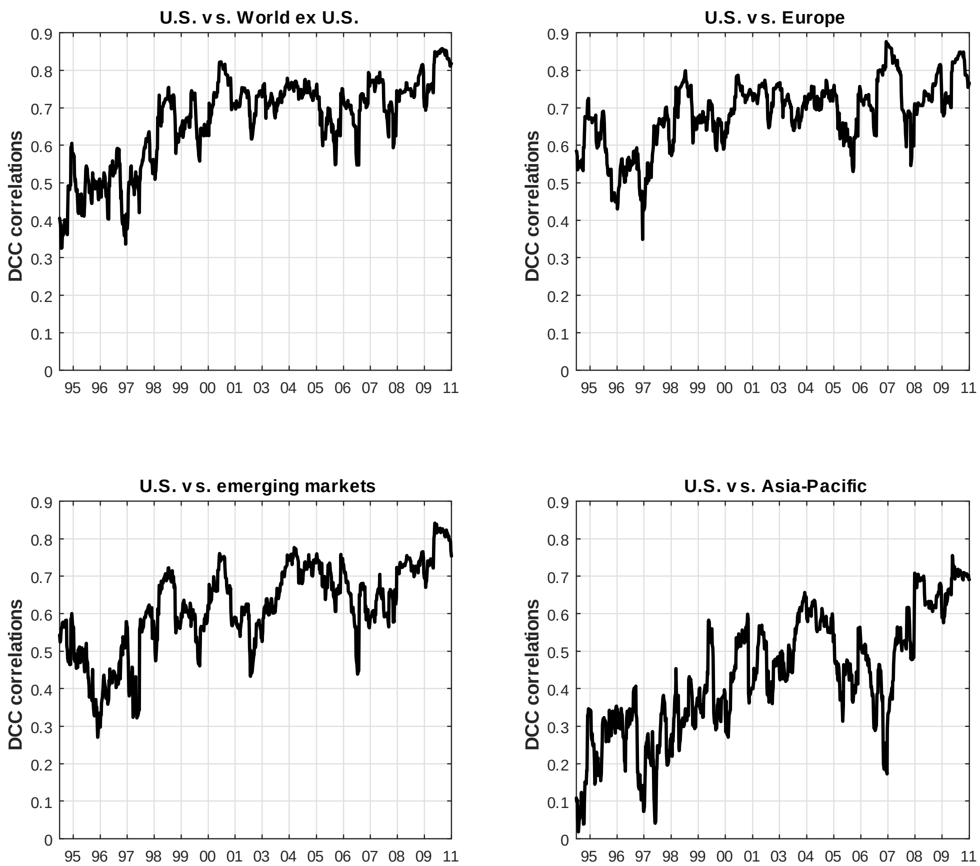
Prause, K. (1999). *The generalized hyperbolic model: Estimation, financial derivatives, and risk measures*. Ph.D. thesis, University of Freiburg.

Santis, G., and Gerard, B. (1997). International asset pricing and portfolio diversification with time-varying risk. *The Journal of Finance* 52(5):1881–912.

Solnik, B., and Roulet, J. (2000). Dispersion as cross-sectional correlation. *Financial Analysts Journal* 56(1): 54–61.

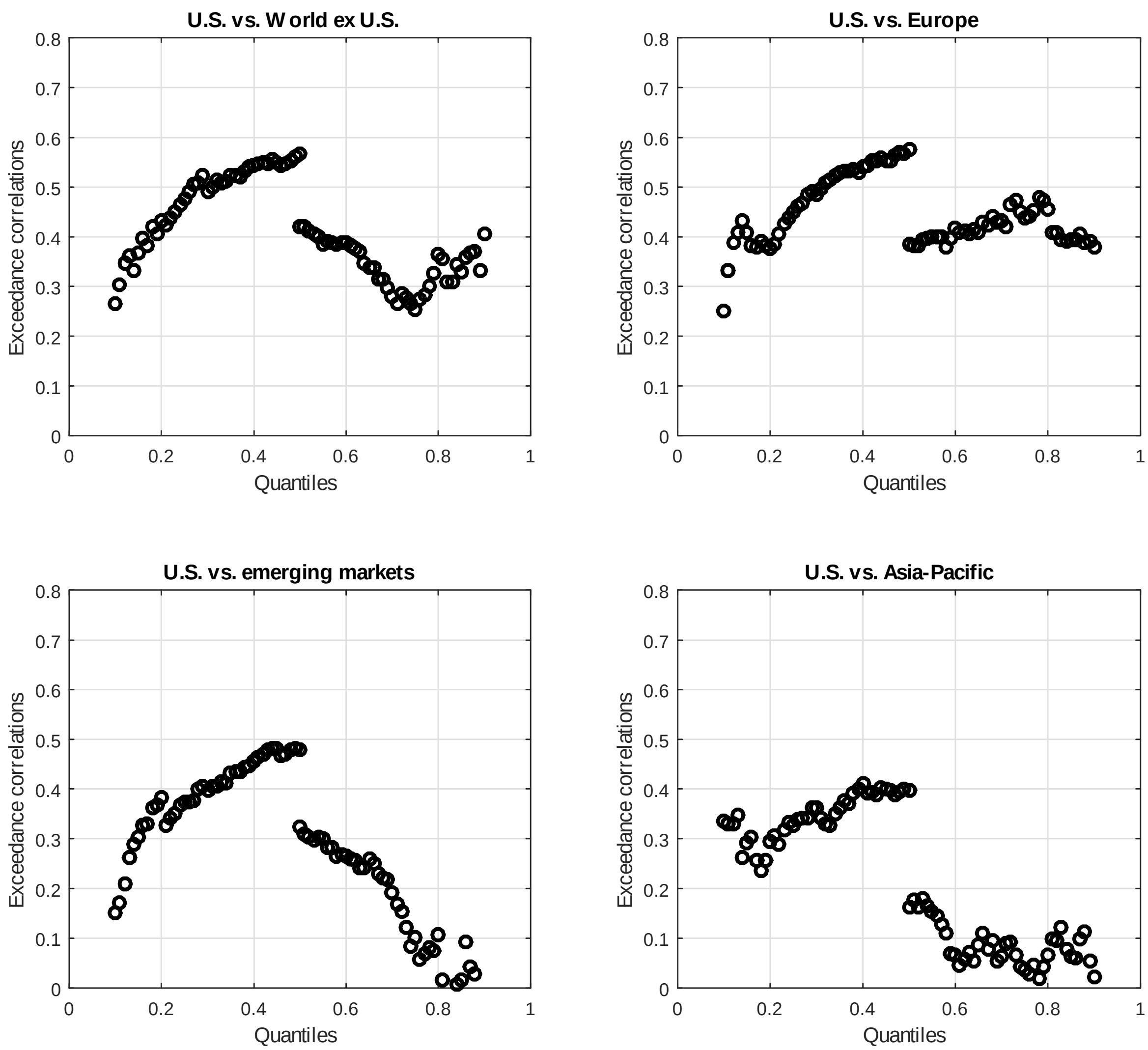
Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance* 68(3):987–1035.

Figure 1
Dynamic conditional correlations



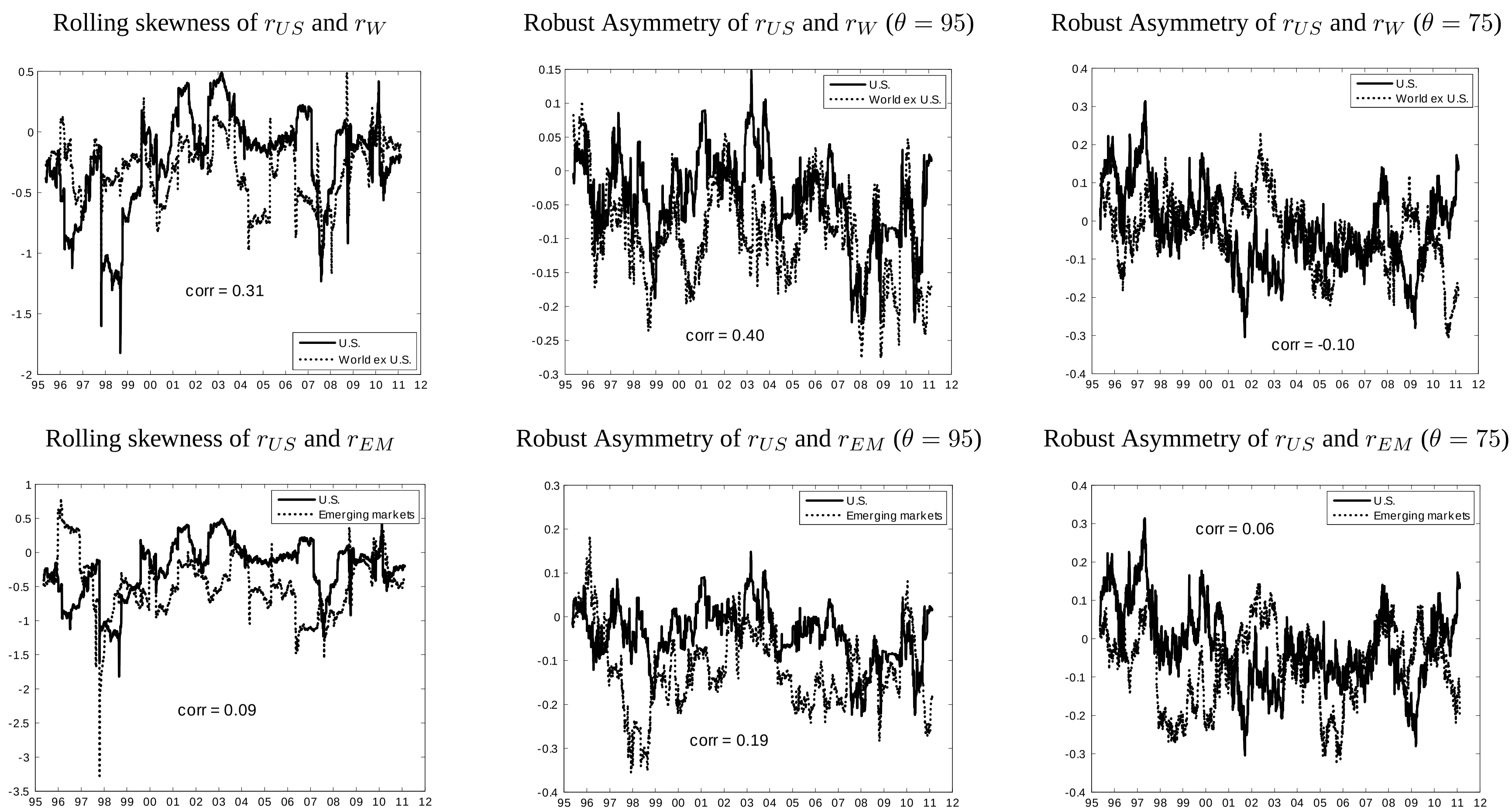
The figure plots estimates of dynamic conditional correlations (DCC) following Engle () between the daily returns of a U.S. equity index and four international indices (World ex U.S., Europe, emerging markets and Asia-Pacific) over the period between July 1994 and June 2011.

Figure 2
Exceedance correlaions



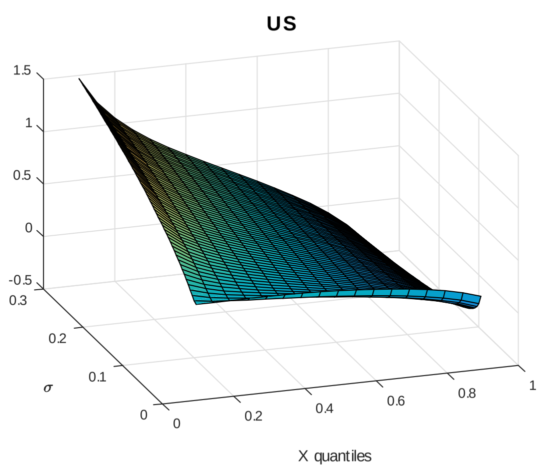
The figure displays exceedance correlation plots of the daily returns of the U.S. equity index and four international indices (World ex U.S., Europe, emerging markets and Asia-Pacific) over the period between July 1994 and June 2011.

Figure 3
Rolling sample estimates of skewness and robust asymmetry



The figure plots 250-day rolling estimates of the unconditional skewness and Hinkley's (1975) robust coefficient of asymmetry of U.S., World ex U.S. and emerging markets daily index returns. For computing the robust asymmetry measure, we use the unconditional 95th and 75th return quantiles The sample period is between July 1994 and June 2011. We also report the unconditional correlation between the rolling estimates of skewness and robust asymmetry. The upper three panels plot the results for the U.S.–World ex. U.S. pair, and the bottom three panels plot the rolling estimates for the U.S.–emerging markets pair of index returns.

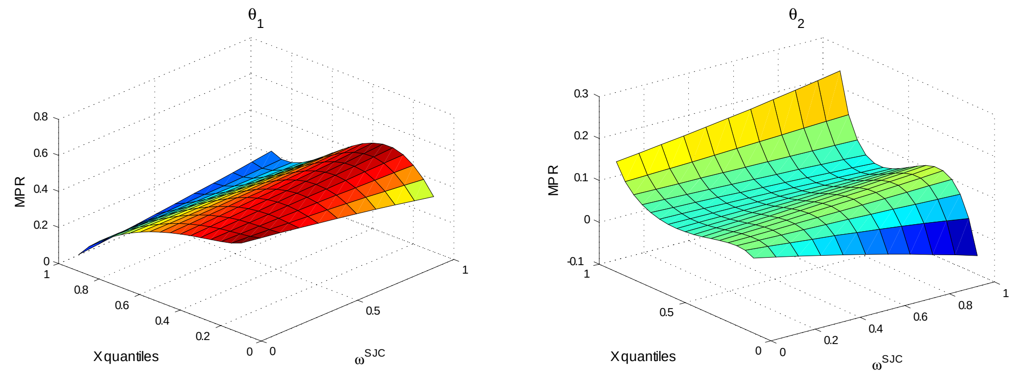
Figure 4
Sensitivity of the market price of risk towards changes in volatility for the U.S. equity index



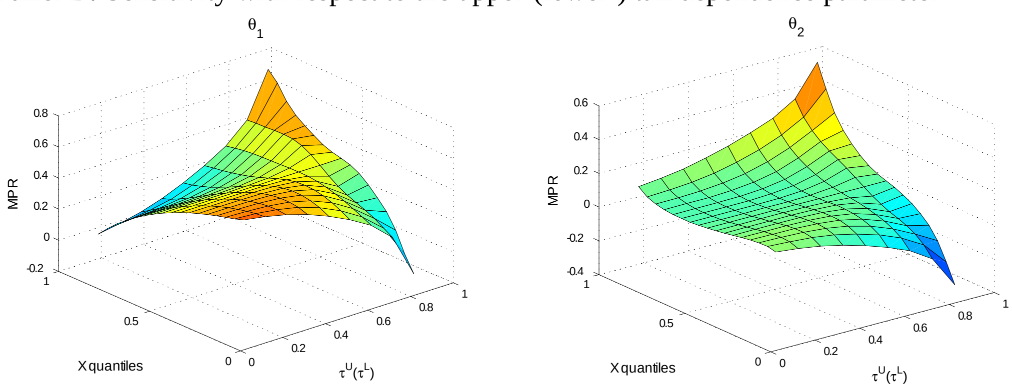
The figure plots the market prices of risk (MPR), implied by the univariate NIG process in Equation (14) for the U.S. equity index over a range of levels of the volatility coefficient σ_i, and for different quantile levels of the corresponding state variable X_i.

Figure 5
Sensitivity analysis of the market prices of risk implied by the multivariate model

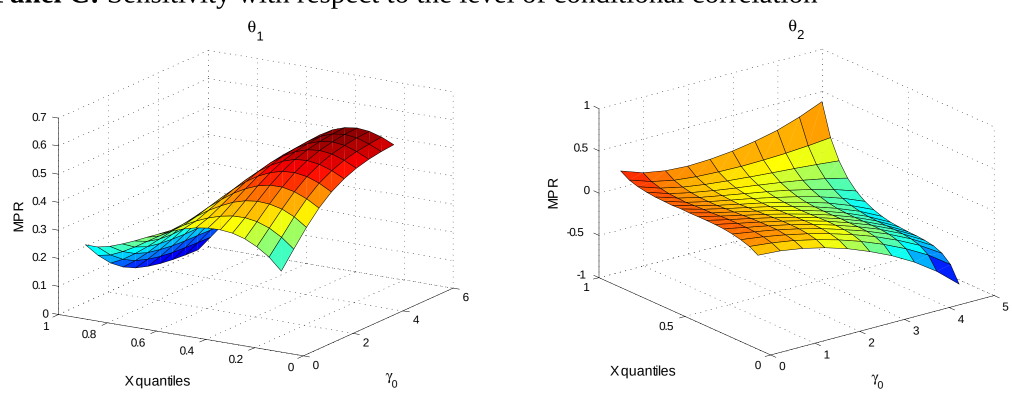
Panel A: Sensitivity with respect to the SJC weight parameter



Panel B: Sensitivity with respect to the upper-(lower-) tail dependence parameter



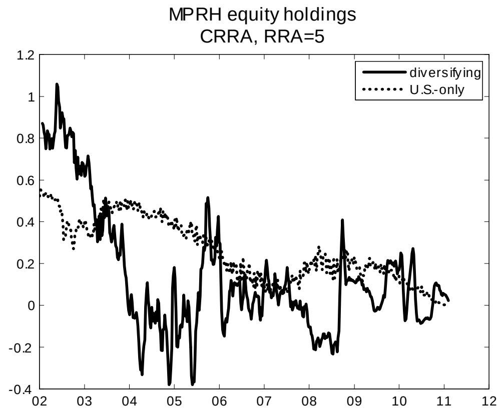
Panel C: Sensitivity with respect to the level of conditional correlation



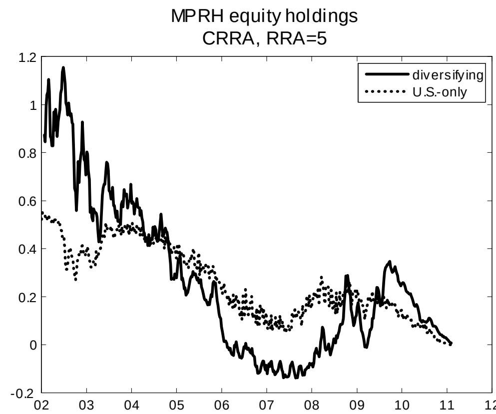
The figure plots comparative statics for the market price of risk (MPR) θ given by Equation (10) corresponding to the U.S.–World ex U.S. equity index pair. The two components θ_1 and θ_2 are plotted for different quantile levels of the state variables $X_i, i = 1, 2$, and for different levels of (a) the copula weight parameter ω^{SJC} (Panel A); (b) the upper-(lower-) tail dependence parameter $\tau^U \left(\tau^L\right)$, both set to be equal (Panel B) and (c) different levels of conditional correlation via the γ_0 parameter (Panel C). The rest of the parameters in the MPR specification are set at their estimated levels at the end of the sample period.

Figure 6
Market price of risk hedge equity holdings

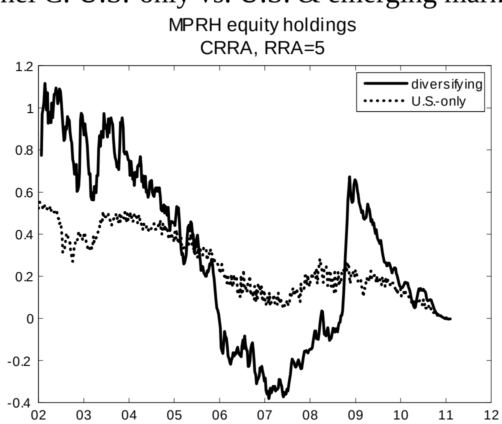
Panel A: U.S.-only vs. U.S. & World ex U.S.



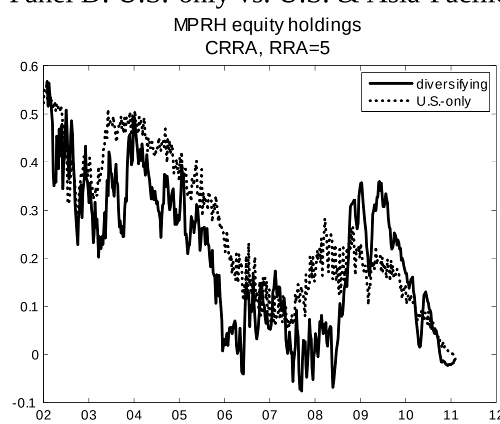
Panel B: U.S.-only vs. U.S. & Europe



Panel C: U.S.-only vs. U.S. & emerging markets



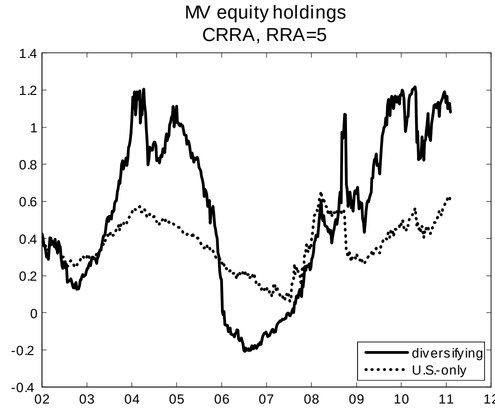
Panel D: U.S.-only vs. U.S. & Asia-Pacific



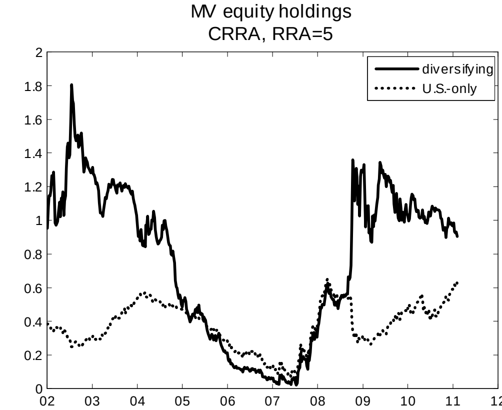
Plots of the total market price of risk hedging (MPRH) equity holdings for each pair of domestic (U.S.) and foreign (World ex U.S., Europe, emerging markets, Asia-Pacific) equity indices are set against a diminishing investment horizon, set at 2011, along realized trajectories of the market prices of risk and the interest rate. We consider an investor who optimizes expected utility of terminal wealth, choosing between each pair of equity indices, a long-term bond, a horizon-matching bond and the risk-free asset. Superimposed are the MPRH U.S. equity holdings of a home-biased investor who chooses between the U.S. equity index, a long-term bond, a horizon-matching bond and the risk-free asset. Each MPRH term is obtained by restarting the investor's problem at a different date with reestimated parameters using the MCMC sequential filter of Golightly and Wilkinson (2006). The investment horizon is fixed at the end of the allocation period in 2011. The investor's utility function is CRRA with relative risk aversion of 5.

Figure 7
Mean-variance equity holdings

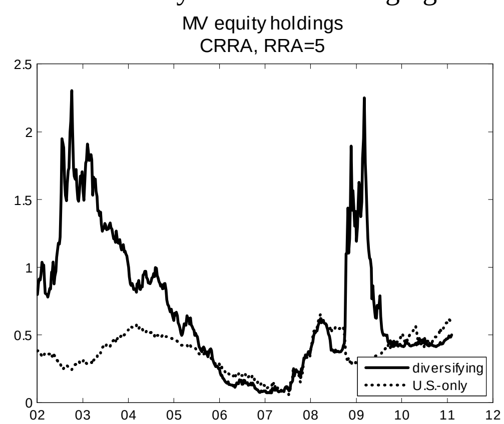
Panel A: U.S.-only vs. U.S. & World ex U.S.



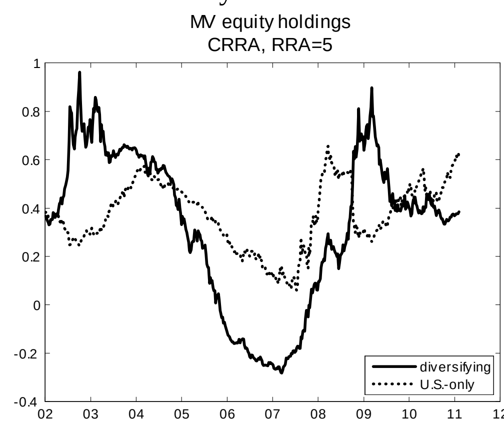
Panel B: U.S.-only vs. U.S. & Europe



Panel C: U.S.-only vs. U.S. & emerging markets

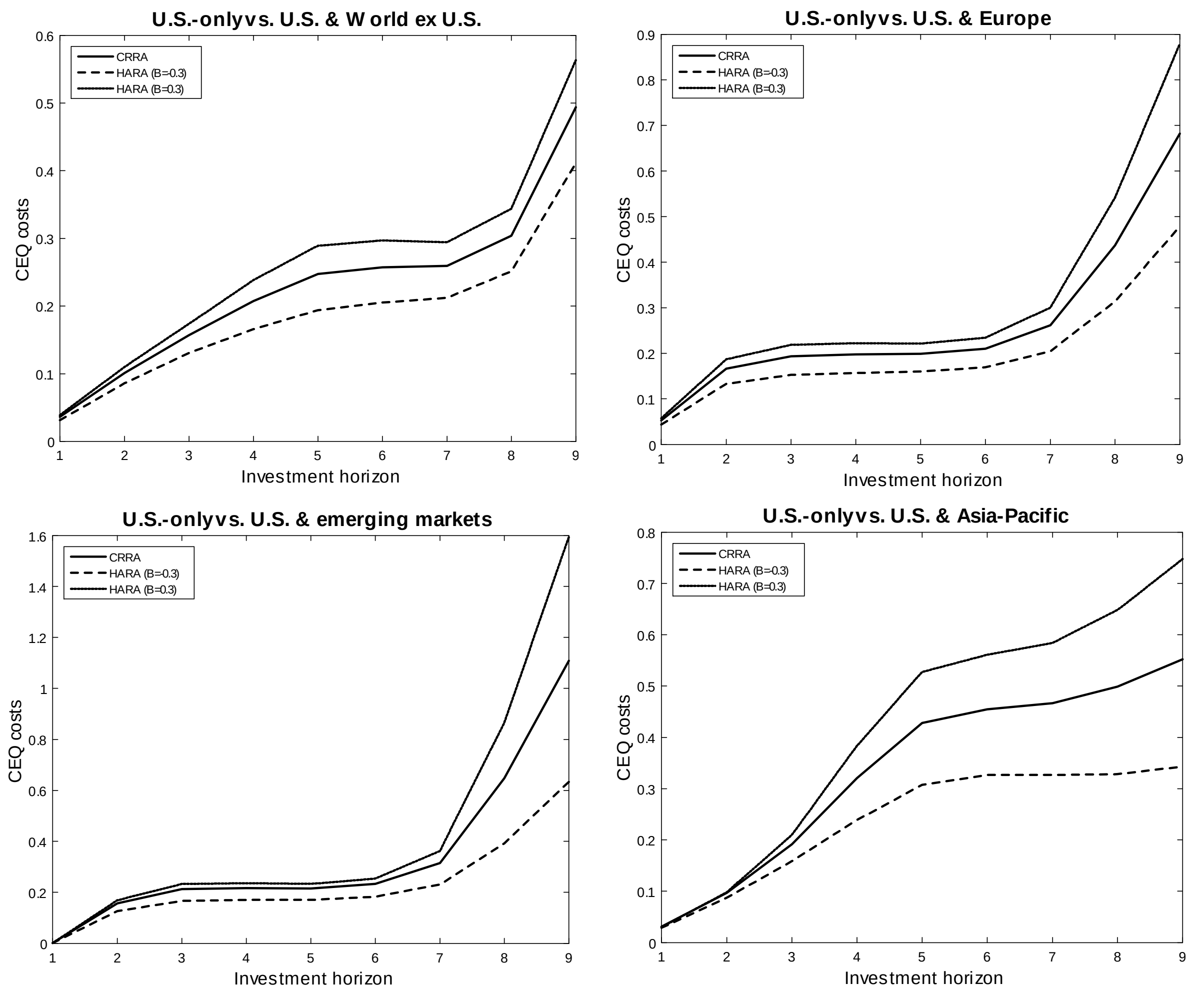


Panel D: U.S.-only vs. U.S. & Asia-Pacific



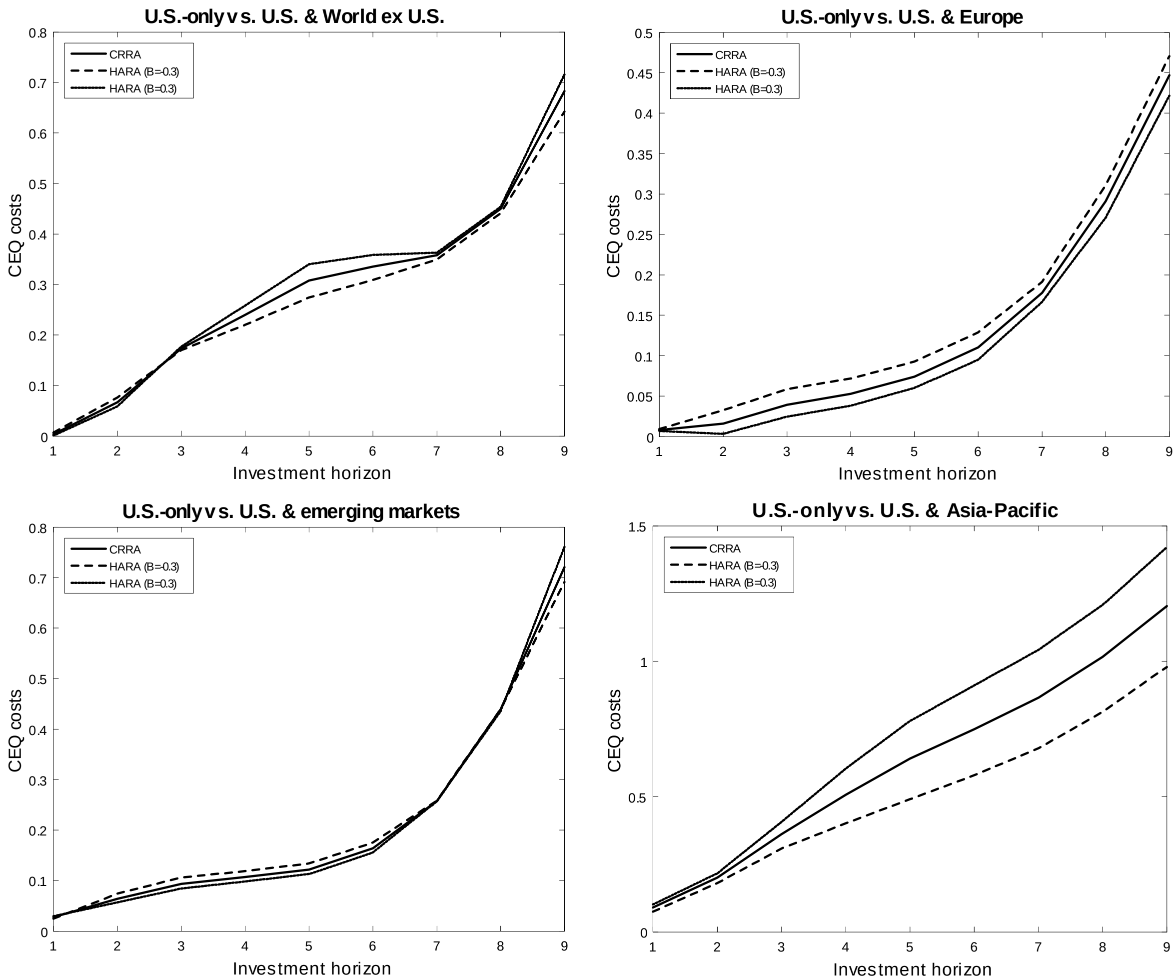
Plots of the total mean-variance (MV) equity holdings for each pair of domestic (U.S.) and foreign (World ex U.S., Europe, emerging markets, Asia-Pacific) equity indices are set against a diminishing investment horizon, set at 2011, along realized trajectories of the market prices of risk and the interest rate. We consider an investor who optimizes expected utility of terminal wealth, choosing between each pair of equity indices, a long-term bond, a horizon-matching bond and the risk-free asset. Superimposed are the MV U.S. equity holdings of a home-biased investor who chooses between the U.S. equity index, a long-term bond, a horizon-matching bond and the risk-free asset. Each MPRH term is obtained by restarting the investor's problem at a different date with reestimated parameters using the MCMC sequential filter of Golightly and Wilkinson (2006). The investment horizon is fixed at the end of the allocation period in 2011. The investor's utility function is CRRA with relative risk aversion of 5.

Figure 8
CEQ costs of home bias



The figure plots the CEQ costs of home bias for all four alternative foreign indices we consider. Portfolio weights and the associated terminal wealth are obtained along realized paths of the state variables. The investor optimizes expected utility of terminal wealth for an investment horizon between one and nine years, choosing between the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The CEQ costs are obtained following Equation (15), where the home-biased portfolio is composed of the U.S. equity index, a long-term bond, a horizon-matching bond, and the risk-free asset. We report CEQ costs in cents per dollar for three specifications of the HARA utility function with relative risk aversion of 5.

Figure 9
CEQ costs of following a 1/N strategy



The figure plots the CEQ costs of following a 1/N strategy for all four alternative foreign indices we consider. Portfolio weights and the associated terminal wealth are obtained along realized paths of the state variables. The investor optimizes expected utility of terminal wealth for an investment horizon between one and nine years, choosing between the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The CEQ costs are obtained following Equation (15). The suboptimal 1/N portfolio is obtained by keeping the bond/equity mix from the optimal allocation and modifying the equity holdings to be equally divided between the U.S. index and the corresponding foreign asset. We report CEQ costs in cents per dollar for three specifications of the HARA utility function with relative risk aversion of 5.

Table 1
Descriptive statistics

	First Estimation Window (1994–2001)					Allocation Window (2002–2011)					Whole period (1994–2011)				
Univariate	*US*	*W*	*EU*	*EM*	*AP*	*US*	*W*	*EU*	*EM*	*AP*	*US*	*W*	*EU*	*EM*	*AP*
Mean	0.04	-0.01	0.02	0.07	-0.03	0.03	0.04	0.04	0.04	0.05	0.03	0.03	0.03	0.03	0.02
Min	-6.66	-3.74	-5.12	-9.30	-6.35	-9.24	-7.83	-9.96	-7.83	-8.46	-9.24	-7.83	-9.96	-9.30	-8.46
Max	5.42	3.12	4.69	10.19	9.70	11.40	7.76	11.11	7.76	9.05	11.40	7.76	11.11	10.19	9.70
STD	1.28	0.94	1.08	1.32	1.36	1.36	1.19	1.47	1.19	1.26	1.33	1.11	1.35	1.28	1.30
Variance	1.64	0.89	1.16	1.74	1.84	1.84	1.41	2.16	1.41	1.60	1.77	1.24	1.83	1.64	1.68
Skewness	-0.18	-0.26	-0.25	-0.41	0.27	0.02	-0.28	0.11	-0.28	-0.31	-0.04	-0.27	0.06	-0.44	-0.10
Kurtosis	2.67	1.16	1.49	8.34	3.51	9.03	7.59	8.18	7.59	5.60	7.26	6.89	8.02	6.92	4.77
Multivariate		*US–W*	*US–EU*	*US–EM*	*US–AP*		*US–W*	*US–EU*	*US–EM*	*US–AP*		*US–W*	*US–EU*	*US–EM*	*US–AP*
Correlation		0.41	0.42	0.44	0.11		0.50	0.54	0.44	0.14		0.48	0.50	0.44	0.13
QD (5th)		0.26	0.34	0.27	0.07		0.41	0.49	0.27	0.13		0.37	0.42	0.26	0.11
QD (95th)		0.25	0.21	0.28	0.09		0.36	0.43	0.28	0.17		0.30	0.35	0.28	0.13
RS corr		0.34	0.45	0.13	-0.19		0.41	0.31	0.17	0.49		0.31	0.40	0.09	-0.18
RA corr															
(95th)		0.41	0.64	0.23	0.17		0.37	0.39	0.20	0.48		0.40	0.46	0.19	0.42
(75th)		-0.18	-0.45	0.25	-0.14		-0.41	0.12	0.02	-0.31		-0.1	-0.36	0.06	-0.11

This table gives the descriptive statistics of the daily returns in U.S. dollars for the MSCI equity indices of the United States (US), World ex U.S. (W), Europe (EU), emerging markets (EM) and Asia-Pacific (AP) for the whole estimation period between July 1994 and June 2011, which consists of 4,447 end-of-day values of the five equity indices, as well as for two subperiods: the first estimation window between July 1994 and December 2001, which consists of 1,979 end-of-day values of the five equity indices, and the allocation period between January 2002 and June 2011, which consists of 2,467 end-of-day values of the equity indices. Panel A reports descriptive statistics of each univariate series. Panel B gives correlation coefficients, quantile dependences (QD) at the 5th and the 95th percentiles of the daily returns, as well as estimates of the rolling skewness (RS) correlation and of the robust asymmetry (RA) correlationa the 95th and the 75th percentile levels.

Table 2
Median portfolio weights

	HARA ($b = -0.3$)				*CRRA*				*HARA* ($b = 0.3$)			
Index	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*
Relative risk aversion = 2												
U.S. only	-	1.00	0.35	-0.04	-	1.18	0.43	-0.43	-	1.31	0.51	-0.76
U.S.–World ex U.S.	0.50	0.94	0.28	0.20	0.66	1.07	0.48	-0.47	0.79	1.18	0.62	-1.06
U.S.–Europe	1.36	0.63	0.33	0.18	1.76	0.74	0.52	-0.63	1.95	0.81	0.64	-1.38
U.S.–emerging markets	0.33	0.67	0.39	-0.12	0.37	0.79	0.55	-0.92	0.40	0.85	0.62	-1.42
U.S.–Asia Pacific	-0.56	1.42	0.33	0.14	-0.63	1.66	0.48	-0.34	-0.67	1.83	0.60	-0.81
Relative risk aversion = 5												
U.S. only	-	0.52	0.17	0.61	-	0.62	0.22	0.40	-	0.71	0.27	0.19
U.S.–World ex U.S.	0.10	0.52	0.09	1.10	0.15	0.61	0.15	0.78	0.19	0.69	0.23	0.47
U.S.–Europe	0.45	0.39	0.08	1.14	0.67	0.43	0.16	0.83	0.84	0.47	0.26	0.50
U.S.–emerging markets	0.11	0.40	0.14	1.03	0.17	0.48	0.22	0.66	0.21	0.54	0.29	0.31
U.S.–Asia Pacific	-0.31	0.65	0.11	0.84	-0.32	0.79	0.19	0.60	-0.34	0.91	0.26	0.37
Relative risk aversion = 10												
U.S. only	-	0.35	0.08	0.97	-	0.41	0.11	0.84	-	0.48	0.14	0.71
U.S.–World ex U.S.	0.03	0.39	0.02	1.42	0.03	0.43	0.04	1.28	0.07	0.48	0.08	1.15
U.S.–Europe	0.14	0.29	0.03	1.57	0.28	0.33	0.04	1.37	0.39	0.37	0.08	1.22
U.S.–emerging markets	0.04	0.25	0.06	1.37	0.08	0.30	0.10	1.23	0.09	0.35	0.14	1.06
U.S.–Asia Pacific	-0.15	0.44	0.05	1.09	-0.22	0.51	0.07	0.98	-0.26	0.58	0.10	0.86

The table gives medians of the risky asset portfolio weights along realized trajectories of the market prices of risk and the interest rate over the whole nine-year investment horizon for an investor who optimizes expected utility of terminal wealth, choosing between the domestic U.S. equity index (US), a foreign index (W), a long-term (LT) bond, a horizon-matching (HM) bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. Results are given for three levels of risk aversion (2, 5, and 10), and three specifications of the HARA utility function (for $b = -0.3$, $b = 0$, corresponding to CRRA utility, and $b = 0.3$). The first line of results for each level of risk aversion gives the optimal allocation for a home-biased investor. Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.

Table 3
Median mean-variance weights

	HARA ($b = -0.3$)				CRRA				HARA ($b = 0.3$)			
Index	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*
Relative risk aversion = 2												
U.S. only	-	0.74	0.21	-0.30	-	0.92	0.28	-0.38	-	1.10	0.33	-0.45
U.S.–World ex U.S.	0.71	0.69	0.20	-0.29	1.04	0.81	0.28	-0.38	1.28	0.89	0.33	-0.44
U.S.–Europe	1.29	0.48	0.20	-0.28	1.69	0.57	0.28	-0.38	1.84	0.64	0.33	-0.43
U.S.–emerging markets	0.26	0.57	0.21	-0.30	0.31	0.65	0.28	-0.38	0.33	0.70	0.32	-0.43
U.S.–Asia Pacific	-0.47	1.20	0.21	-0.30	-0.55	1.47	0.28	-0.38	-0.59	1.66	0.32	-0.44
Relative risk aversion = 5												
U.S. only	-	0.29	0.08	-0.11	-	0.37	0.11	-0.15	-	0.45	0.14	-0.19
U.S.–World ex U.S.	0.27	0.25	0.07	-0.11	0.42	0.32	0.11	-0.15	0.56	0.39	0.14	-0.20
U.S.–Europe	0.46	0.17	0.08	-0.10	0.68	0.23	0.11	-0.15	0.81	0.28	0.14	-0.19
U.S.–emerging markets	0.09	0.20	0.07	-0.11	0.12	0.26	0.11	-0.15	0.14	0.30	0.14	-0.20
U.S.–Asia Pacific	-0.17	0.44	0.07	-0.11	-0.22	0.59	0.11	-0.15	-0.26	0.72	0.14	-0.19
Relative risk aversion = 10												
U.S. only	-	0.14	0.04	-0.06	-	0.18	0.06	-0.08	-	0.23	0.07	-0.09
U.S.–World ex U.S.	0.13	0.12	0.04	-0.05	0.21	0.16	0.06	-0.08	0.29	0.20	0.07	-0.10
U.S.–Europe	0.22	0.08	0.04	-0.05	0.34	0.11	0.06	-0.08	0.43	0.14	0.07	-0.10
U.S.–emerging markets	0.04	0.10	0.04	-0.05	0.06	0.13	0.06	-0.08	0.07	0.16	0.07	-0.10
U.S.–Asia Pacific	-0.08	0.21	0.04	-0.05	-0.11	0.29	0.06	-0.08	-0.14	0.37	0.07	-0.10

The table gives medians of the risky asset portfolio mean-variance weights along realized trajectories of the market prices of risk and the interest rate over the whole nine-year investment horizon for an investor who optimizes expected utility of terminal wealth, choosing between the domestic U.S. equity index (US), a foreign index (W), a long-term (LT) bond, a horizon-matching (HM) bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The mean-variance weights are obtained using the portfolio decomposition formula given by Equation (13). Results are given for three levels of risk aversion (2, 5, and 10), and three specifications of the HARA utility function (for $b = -0.3$, $b = 0$, corresponding to CRRA utility, and $b = 0.3$). The first line of results for each level of risk aversion gives the optimal mean-variance allocation for a home-biased investor. Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.

Table 4
Median market price of risk hedging terms

	HARA ($b = -0.3$)				CRRA				HARA ($b = 0.3$)			
Index	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*	*W*	*US*	*LT bond*	*HM bond*
Relative risk aversion = 2												
U.S. only	-	0.19	0.04	-0.45	-	0.19	0.06	-0.61	-	0.18	0.08	-0.77
U.S.–World ex U.S.	-0.20	0.21	0.06	-0.26	-0.24	0.19	0.08	-0.68	-0.27	0.18	0.10	-0.97
U.S.–Europe	0.01	0.21	0.06	-0.29	0.01	0.22	0.08	-0.86	0.00	0.23	0.09	-1.41
U.S.–emerging markets	0.06	0.17	0.07	-0.50	0.08	0.17	0.08	-0.81	0.08	0.17	0.08	-0.89
U.S.–Asia Pacific	-0.14	0.20	0.05	-0.34	-0.13	0.20	0.06	-0.57	-0.13	0.20	0.07	-0.64
Relative risk aversion = 5												
U.S. only	-	0.22	0.01	-0.22	-	0.22	0.02	-0.33	-	0.22	0.03	-0.45
U.S.–World ex U.S.	-0.10	0.26	0.00	0.01	-0.16	0.26	0.03	-0.09	-0.20	0.25	0.06	-0.20
U.S.–Europe	-0.02	0.22	0.01	0.18	-0.01	0.23	0.03	-0.01	0.00	0.25	0.06	-0.18
U.S.–emerging markets	0.03	0.16	0.03	0.01	0.05	0.19	0.05	-0.17	0.06	0.20	0.06	-0.38
U.S.–Asia Pacific	-0.05	0.24	0.02	-0.24	-0.09	0.24	0.04	-0.31	-0.12	0.24	0.05	-0.37
Relative risk aversion = 10												
U.S. only	-	0.21	0.00	-0.07	-	0.22	0.00	-0.14	-	0.22	0.01	-0.20
U.S.–World ex U.S.	-0.02	0.27	0.00	0.21	-0.06	0.26	0.00	0.11	-0.08	0.26	0.00	0.04
U.S.–Europe	-0.02	0.19	0.00	0.42	-0.02	0.21	0.00	0.31	-0.02	0.22	0.01	0.22
U.S.–emerging markets	0.01	0.14	0.01	0.25	0.02	0.15	0.02	0.14	0.03	0.17	0.03	0.04
U.S.–Asia Pacific	-0.02	0.25	0.00	-0.13	-0.03	0.25	0.00	-0.19	-0.04	0.25	0.01	-0.23

The table gives medians of the risky asset portfolio market price of risk hedging terms along realized trajectories of the market prices of risk and the interest rate over the whole nine-year investment horizon for an investor who optimizes expected utility of terminal wealth, choosing between the domestic U.S. equity index (US), a foreign index (W), a long-term (LT) bond, a horizon-matching (HM) bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The market price of risk hedging terms are obtained using the portfolio decomposition formula given by Equation (13). Results are given for three levels of risk aversion (2, 5, and 10), and three specifications of the HARA utility function (for $b = -0.3$, $b = 0$, corresponding to CRRA utility, and $b = 0.3$). The first line of results for each level of risk aversion gives the optimal allocation due to hedging the market price of risk for a home-biased investor. Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.

Table 5
The cost of home bias

		1 year	3 years	5 years	7 years	9 years
US-only vs. US–World ex U.S.	RRA=2	0.09	0.37	0.65	0.66	1.10
	RRA=5	0.04	0.16	0.25	0.26	0.49
	RRA=10	0.02	0.09	0.13	0.15	0.34
US-only vs. US–Europe	RRA=2	0.13	0.53	0.51	0.69	1.95
	RRA=5	0.05	0.19	0.20	0.26	0.68
	RRA=10	0.03	0.10	0.11	0.15	0.40
US-only vs. US–emerging markets	RRA=2	0.00	0.50	0.50	0.76	3.73
	RRA=5	0.00	0.21	0.21	0.32	1.11
	RRA=10	0.00	0.11	0.12	0.17	0.55
US-only vs. US–Asia Pacific	RRA=2	0.07	0.47	1.41	1.52	1.99
	RRA=5	0.03	0.19	0.43	0.47	0.55
	RRA=10	0.02	0.10	0.17	0.19	0.20

The table reports the CEQ costs of portfolio home bias for a CRRA investor who optimizes expected utility of terminal wealth for an investment horizon between one and nine years, choosing between the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The CEQ costs are obtained following Equation (15), where the home-biased portfolio is composed of the U.S. equity index, a long-term bond, a horizon-matching bond, and the risk-free asset. The weights of that portfolio are obtained following Equation (13), where the state variables driving the equity price dynamics follow the process in Equation (14). Results are given for three levels of risk aversion (2, 5, and 10). Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.

Table 6
Decomposing the cost of home bias: The loss of benefits from diversification and intertemporal hedging

		1 year	3 years	5 years	7 years	9 years
Panel A: Loss of benefits from diversification						
US-only vs. US–World ex U.S.	RRA=2	0.09	0.37	0.59	0.74	1.13
	RRA=5	0.04	0.13	0.21	0.25	0.35
	RRA=10	0.02	0.06	0.10	0.12	0.16
US-only vs. US–Europe	RRA=2	0.12	0.50	0.48	0.62	1.53
	RRA=5	0.05	0.17	0.17	0.20	0.42
	RRA=10	0.02	0.08	0.08	0.10	0.19
US-only vs. US–emerging markets	RRA=2	0.00	0.38	0.39	0.54	2.13
	RRA=5	0.00	0.14	0.14	0.19	0.59
	RRA=10	0.00	0.07	0.07	0.09	0.26
US-only vs. US–Asia Pacific	RRA=2	0.07	0.41	1.13	1.24	1.55
	RRA=5	0.03	0.15	0.36	0.38	0.46
	RRA=10	0.01	0.07	0.16	0.18	0.21
Panel B: Loss of benefits from diversification and intertemporal hedging						
US-only vs. US–World ex U.S.	RRA=2	0.10	0.50	0.83	1.00	1.65
	RRA=5	0.05	0.30	0.43	0.59	1.08
	RRA=10	0.03	0.23	0.31	0.47	0.91
US-only vs. US–Europe	RRA=2	0.14	0.62	0.68	0.90	2.65
	RRA=5	0.07	0.32	0.36	0.55	1.50
	RRA=10	0.04	0.22	0.26	0.43	1.18
US-only vs. US–emerging markets	RRA=2	0.01	0.61	0.65	1.00	4.94
	RRA=5	0.01	0.36	0.39	0.64	2.25
	RRA=10	0.01	0.25	0.29	0.49	1.52
US-only vs. US–Asia Pacific	RRA=2	0.08	0.59	1.59	1.92	3.03
	RRA=5	0.04	0.33	0.62	0.83	1.39
	RRA=10	0.03	0.24	0.35	0.51	0.92

The table reports the CEQ costs of portfolio home bias for a CRRA investor who optimizes expected utility of terminal wealth for an investment horizon between one and nine years, choosing between the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The CEQ costs are obtained following Equation (15), where the home-biased portfolio is composed of the U.S. equity index, a long-term bond, a horizon-matching bond, and the risk-free asset. Panel A reports the CEQ costs from not investing internationally for a myopic mean-variance investor who applies sequentially a one-period ahead portfolio rule. The CEQ costs reported in Panel B are obtained for a dynamic and internationally diversified investor for whom the alternative home-bias stragegy is obtained by sequentially applying a one-period ahead myopic portfolio rule. Results are given for three levels of risk aversion (2, 5, and 10). Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic and static portfolio problems are also initialized and solved for weekly.

Table 7
The portfolio improvement from diversifying internationally

		2003	2004	2005	2006	2007	2008	2009	2010	2011	2002-2011
US-only vs. US–World ex U.S.	RRA=2	5.63	7.26	5.85	8.13	4.88	2.01	4.61	8.45	12.60	4.27
	RRA=5	2.22	1.71	2.38	3.20	2.00	0.75	1.67	0.06	5.51	0.61
	RRA=10	1.05	2.21	1.17	1.50	0.98	0.37	0.79	0.10	3.21	0.81
US-only vs. US–Europe	RRA=2	8.16	14.74	4.22	0.67	0.53	0.21	4.27	12.79	13.85	3.53
	RRA=5	2.97	2.85	1.54	0.27	0.20	0.00	1.70	3.64	5.03	1.01
	RRA=10	1.34	1.05	0.70	0.13	0.09	0.00	0.84	0.74	1.93	0.66
US-only vs. US–emerging markets	RRA=2	0.09	14.52	6.23	0.89	0.02	1.35	6.58	20.23	20.31	2.85
	RRA=5	0.11	2.13	2.40	0.35	0.00	0.53	2.53	4.07	6.95	0.74
	RRA=10	0.05	1.61	1.15	0.16	0.00	0.23	1.21	1.16	2.33	0.43
US-only vs. US–Asia Pacific	RRA=2	3.76	6.35	11.49	17.33	12.60	2.75	1.17	5.53	5.60	3.82
	RRA=5	1.68	1.39	4.53	6.77	5.05	1.19	0.67	1.82	2.19	0.53
	RRA=10	0.84	1.75	2.18	3.26	2.46	0.60	0.40	0.87	0.93	0.41

The table reports the portfolio improvement from investing internationally in terms of the annualized continously compounded return in certainty-equivalent wealth for a CRRA investor who optimizes expected utility of terminal wealth, choosing between the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset. The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. The portfolio improvement is obtained following Equation (16), where the alternative home-biased portfolio is composed of the U.S. equity index, a long-term bond, a horizon-matching bond, and the risk-free asset. Results are given for three levels of risk aversion (2, 5, and 10) and one-year investment horizon for the years 2003–2011. The last column reports the annualized continously compounded return in certainty-equivalent wealth over the whole investment horizon. Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.

Table 8
Sharpe ratios

	HARA $B=-0.3$	HARA $B=-0.2$	HARA $B=-0.1$	CRRA	HARA $B=0.1$	HARA $B=0.2$	HARA $B=0.3$
Panel A: US-only investor							
RRA=2	0.25	0.26	0.27	0.28	0.29	0.30	0.31
RRA=5	0.18	0.19	0.19	0.20	0.20	0.21	0.21
RRA=10	0.16	0.16	0.17	0.17	0.17	0.17	0.18
Panel B: Diversifying investor (US–World ex U.S.)							
RRA=2	0.57	0.61	0.64	0.67	0.70	0.73	0.74
RRA=5	0.27	0.29	0.30	0.32	0.34	0.36	0.37
RRA=10	0.18	0.19	0.20	0.21	0.22	0.23	0.24
Panel C: Diversifying investor (US–Europe)							
RRA=2	0.86	0.95	1.03	1.10	1.17	1.23	1.26
RRA=5	0.39	0.43	0.47	0.52	0.56	0.59	0.63
RRA=10	0.25	0.27	0.29	0.32	0.34	0.36	0.38
Panel D: Diversifying investor (US–emerging markets)							
RRA=2	1.07	1.21	1.34	1.46	1.56	1.65	1.70
RRA=5	0.46	0.53	0.60	0.67	0.73	0.79	0.85
RRA=10	0.28	0.32	0.36	0.39	0.43	0.47	0.50
Panel E: Diversifying investor (US–Asia Pacific)							
RRA=2	0.88	0.94	1.00	1.06	1.11	1.15	1.18
RRA=5	0.37	0.41	0.45	0.48	0.52	0.55	0.58
RRA=10	0.21	0.23	0.25	0.27	0.29	0.31	0.33

The table reports Sharpe ratios for a dynamic investor who optimizes expected utility of terminal wealth over a nine-year period with a diminishing investment horizon, set in 2011, choosing between the domestic U.S. equity index, a long-term bond, a horizon-matching bond, and the risk-free asset (Panel A) or investing in the domestic U.S. equity index, a foreign index, a long-term bond, a horizon-matching bond, and the risk-free asset (Panels B through E). The foreign equity indices considered are World ex U.S., Europe, emerging markets and Asia-Pacific. Results are given for three levels of risk aversion (2, 5, and 10), and five specifications of the HARA utility function. Parameters for the data-generating processes for the U.S. index and for each equity index pair are reestimated every week. The dynamic portfolio problem is also initialized and solved for weekly.