Default and liquidation timing under asymmetric information *

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Abstract

We consider a dynamic model in which shareholders delegate a manager, who observes private information about running and liquidation costs of the firm, to operate the firm. We analytically derive the shareholders' optimal contract contingent on the cost structure of the firm. The information asymmetries change the high-cost firm's default and liquidation timing. Even if the liquidation value is higher than the face value of debt, the shareholders of the high-cost firm, unlike in the symmetric information case, can choose default rather than liquidation in order to reduce the information rent to the manager. The information asymmetries accelerate negative liquidation and delay positive liquidation, while they accelerate default. Although the information asymmetries decrease the equity and firm values, they may increase the debt value. The optimal leverage ratio of the asymmetric information case becomes higher than that of the symmetric information case because more debt mitigates the loss due to the information asymmetries. Our results can potentially account for many empirical findings.

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1 Introduction

Since the seminal works by Leland (1994), Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000) many papers have investigated the dynamic models of default, liquidation, and debt renegotiation. Combining this literature with the real options literature (e.g., McDonald and Siegel (1986) and Dixit and Pindyck (1994)), many papers, including Mauer and Sarkar (2005), Sundaresan and Wang (2007), Shibata and Nishihara (2012), and Sundaresan, Wang, and Yang (2015), have developed the dynamic theory of investment and financing. Most of the previous research, however, has focused mainly on the conflicts between the equity holders and debt holders.

On the other hand, in recent years, several papers also have investigated the manager-shareholders conflicts in the dynamic investment models. For instance, Grenadier and Wang (2005) showed how the investment timing is distorted when the shareholders delegate the investment decision to the manager who has private information. In their framework, Shibata and Nishihara (2010) also derived the investment timing of the levered firm with the optimal capital structure.[1]

In this paper, we investigate how the information asymmetries between the manager and shareholders affect the firm's default and liquidation. In other words, we examine the conflicts among the manager, shareholders, and debt holders on default and liquidation. Our model builds largely on Mella-Barral and Perraudin (1997), Goldstein, Ju, and Leland (2001), and Grenadier and Wang (2005). Following Mella-Barral and Perraudin (1997), we consider the shareholders who choose between default (stopping the coupon payments) and liquidation (scrapping and/or selling the assets along with retiring the face value of debt).[2] Following Goldstein, Ju, and Leland (2001), we also assume the trade-off between the tax shield and default costs of debt. Last and most importantly, as in Grenadier and Wang (2005), we consider the information asymmetries between the manager and shareholders. The shareholders delegate the manager, who observes private information about running and liquidation costs of the firm (high-cost or low-cost firm), to operate the firm. The shareholders offer a contract, which consists of the default or liquidation timing and compensation contingent on the firm's cost structure, to the manager so that they can maximize the ex-ante equity value. The asymmetric information model approximates firms that have diffuse ownership as well as a lower level of transparency and disclosure.

In the model, we analytically derive the shareholders' optimal contract. As in the

[1]In other perspectives, Sannikov (2008), Grenadier and Malenko (2011), Morellec and Schürhoff (2011), and Delaney and Thijssen (2015) among others, examined the investment models under asymmetric information.

[2]Although Mella-Barral and Perraudin (1997) also examined the possibility of debt renegotiation, we do not consider debt renegotiation but focus on the effects of the information asymmetries on the default and liquidation timing.

previous papers, including Grenadier and Wang (2005) and Shibata and Nishihara (2010), we show that only the high-cost firm's behavior changes from that of the symmetric information case. Although, as in the symmetric information case, the firm with more existing debt (scrap value) tends to proceed to default (liquidation), the shareholders' choice of default in the high-cost firm is encouraged by the information asymmetries. This is because the choice of default in the high-cost firm reduces the information rent to the manager of the low-cost firm. If this effect is stronger, the shareholders causes the high-cost firm to default even when the liquidation value is higher than the face value.

In another perspective, we argue that more existing debt, which tends to cause default rather than liquidation, can play a positive role in reducing the information rent to the manager and alleviating the loss due to the information asymmetries. This result is similar to that of Lambrecht and Myers (2008). They showed that risky debt can mitigate manager-shareholder conflicts, although their model does not include the manager's private information but considers the shareholders' possibility of firing the manager and closing or managing the firm by themselves.

The default timing of the high-cost firm is accelerated due to the information asymmetries because the shareholders can decrease the information rent to the manager by accelerating default. This is contrary to the result in the previous papers, such as Grenadier and Wang (2005) and Shibata and Nishihara (2010), showing that the high-cost firm's investment timing is delayed. However, the opposite result is straightforward because our model is not an investment timing model but a default timing model. Our result is also in line with the empirical evidence by Anderson, Mansi, and Reeb (2003). Actually, they found that family firms are more likely than firms with diversified ownership to value firm survival and hence better protect the debt holders' interests.

More notably, the effects of the information asymmetries on liquidation are equivocal. The liquidation timing is accelerated (delayed) when the information asymmetries in the running costs are larger (smaller) than those of the liquidation costs. Our results are contrasted with the monotonic results in the previous literature, such as Grenadier and Wang (2005) and Shibata and Nishihara (2010). When the effect of the running costs is larger, the high-cost firm liquidates earlier than the low-cost firm to save the high running cost. We hereafter call this case negative liquidation, which can be related to a case of scrapping the firm. When the effect of the liquidation costs is larger, the low-cost firm liquidates earlier to gain the high liquidation value. We hereafter call this case positive liquidation, which can be related to a case of selling the firm. Our result suggests that the information asymmetries accelerate negative liquidation and delay positive liquidation.

The information asymmetries reduce the equity and firm values and increase the manager's value. The debt and total values may increase due to the information asymmetries.

Below, we explain how the debt holders can potentially take advantage of the manager-shareholder conflicts under asymmetric information. Due to the default timing accelerated by the contract, the debt holders take over the firm earlier than in the symmetric information case. The early-defaulted firm may have a higher value than the debt value in the symmetric information case. For example, the shareholders can choose default of the high-cost firm to save the information rent even when the liquidation value is higher than the face value of debt. In this case, the debt holders can take over the firm which has a higher value than the face value. This also means that the debt value of the high-cost firm can be higher than that of the low-cost firm. The results also suggest the possibility of counter-intuitive market reactions as follows: The debt value rises as the firm approaches default, and it jumps downward (upward) when the firm's type appears to be low-cost (high-cost). Our results can potentially account for excess returns of the vulture investors' strategy that they buy a large block of debt of a distressed firm and thereafter control the firm (e.g., Hotchkiss and Mooradian (1997)).

Furthermore, we show several results in the comparative statics. Higher costs of the low-cost firm can increase the equity value. In the asymmetric information case, higher costs of the low-cost firm play a positive role in decreasing the manager's private information. When this positive effect dominates the direct and negative effect of the higher costs, the higher costs increase the equity value. A higher volatility tends to cause the wealth transfer from the debt holders and manager to the equity holders. The asset substitution between the equity holders and debt holders is consistent with the standard results (e.g., Jensen and Meckling (1976)). The asset substitution between the manager and equity holders is also found in Shibata (2009) and Shibata and Nishihara (2010).

In the optimal capital structure, we find that the initial coupon of debt, leverage, and credit spread in the asymmetric information case are higher than those of the symmetric information case. This is because issuing more risky debt reduces the information rent to the manager. This result is consistent with Lambrecht and Myers (2008), who argued that risky debt can mitigate manager-shareholder conflicts. Several empirical studies, including Agrawal and Nagarajan (1990) and McConaughy, Matthews, and Fialko (2001), showed the positive relation between the ownership dispersion and leverage. Our result is also related to the empirical evidence by John and Litov (2010) that firms with weaker governance tend to use more debt.

The remainder of this paper is organized as follows. As a benchmark, Section 2 presents the results in the symmetric information case. Section 3 shows the key results in the asymmetric information case. In Section 4, we discuss several implications regarding market reactions and the optimal capital structure as well as the comparative statics. Section 5 concludes the paper.

2 Symmetric information

2.1 Setup

The symmetric information model builds on the seminal works of Mella-Barral and Perraudin (1997) and Goldstein, Ju, and Leland (2001). We consider a firm that is receiving EBIT (earnings before interests and taxes) $X(t) - (w + w_i)$ at time t, where $X(t)$ is a stochastic component and $w + w_i$ stands for the firm's running costs. We also suppose that the firm issued console debt with coupon $C(\geq 0)$ and is paying the coupon C continuously. Following the standard real options literature, we assume that $X(t)$ follows a geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. We assume that the initial value, $X(0) = x$, is sufficiently large to exclude the firm's default or liquidation at time 0. A positive constant r denotes the interest rate, and for convergence we assume that $r > \mu$.

The running costs consist of two components: a public component $w(\geq 0)$ and a private component $w_i(\geq 0)$. The private component potentially takes on two types: w_L ($i = L$: Low-cost firm) and w_H ($i = H$: High-cost firm), which satisfies $\Delta w = w_H - w_L > 0$. In liquidation (scrapping and/or selling the assets), the firm receives the value $\theta - \theta_i(\geq 0)$ depending on the type.[3] The component $\theta(\geq 0)$ is publicly observed, while the cost $\theta_i(\geq 0)$ is privately observed. Assume that $\Delta\theta = \theta_H - \theta_L \geq 0$.[4] We assume that the shareholders delegate a manager to run the firm until the default or liquidation time.[5] When default takes place, the former debt holders take over, own, operate, and liquidate the firm.[6] For simplicity, we consider neither delegation nor debt financing after the changes in the ownership. After default, the firm has no debt and receives EBIT $(1 - \alpha)(X(t) - w - w_i)$ until liquidation, where $\alpha \in (0, 1)$ denotes the contraction parameter introduced in Mella-Barral and Perraudin (1997) and also corresponds to the default costs in Leland (1994) and

[3]For simplicity, this paper does not consider the possibility of partial liquidation, which is typically accompanied by debt renegotiation. For instance, Nishihara and Shibata (2016) examined the firm's optimal choice between full liquidation and partial liquidation with debt restructuring.

[4]Following Shibata and Nishihara (2010) and Grenadier and Malenko (2011), we assume the manager's private information on the firm's cost structure. The results are unchanged if we assume the manager's private information on the firm's profit structure.

[5]In the real world, a manager who was chosen by the former shareholders, is usually replaced by the new owners.

[6]To focus on the information asymmetries between the manager and shareholders, this paper does not consider debt renegotiation, which has been studied in Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b), among others.

Goldstein, Ju, and Leland (2001). For the details of the default costs, see also Hotchkiss, John, Mooradian, and Thorburn (2008). Following Mella-Barral and Perraudin (1997), we assume that the liquidation value $\theta - \theta_i$ is not changed by the transfers in the ownership.

Throughout the paper, we assume the manager, shareholders, and debt holders are all risk-neutral and observe all information except for the firm's type i, w_i, and θ_i. Before contracting, all agents know that the probability of drawing a low-cost type $i = L$ equals $P \in (0, 1)$. In this section, we consider a symmetric information case in which the manager and shareholders observe the realized type i, private costs w_i, and θ_i immediately after contracting, while in Section 3 we consider an asymmetric information case in which only the manager observes the realized type i as well as private costs w_i and θ_i immediately after contracting.

The symmetric (asymmetric) information case corresponds to a case without (with) separation of ownership and management (e.g., Jensen and Meckling (1976)). For instance, the symmetric information case can capture the behavior of family-owned-and-managed firms and firms with concentrated equity holdings, while the asymmetric information case can capture the behavior of firms with diffuse equity holdings. In another perspective, the symmetric (asymmetric) information case can approximate firms that have a higher (lower) level of transparency and disclosure. Although Anderson, Duru, and Reeb (2009) showed the positive relation between ownership concentration and corporate opacity, they focused mainly on the information asymmetries between controlling shareholders and minority shareholders rather than the information asymmetries between managers and shareholders. Thus, throughout this paper, we relate the asymmetric information case to firms with more diversified ownership.

Following the standard literature (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)), we assume a positive corporate tax rate τ and no taxation to the debt holders to highlight the tax benefits of debt. Following Shibata and Nishihara (2010), we also assume no taxation to the manager, but the results can be easily extended into the case with a positive tax rate to the manager.

2.2 Solutions

Because all agents observe the realized type i as well as private costs w_i and θ_i, our problem is essentially the same as that of Mella-Barral and Perraudin (1997). In the first place, we solve the problem after the shareholders' default. At the default time, the former debt holders take over and run the firm. They can optimize the liquidation time of the firm. The firm value right after default becomes

$$
\begin{aligned}
&A_i(X(t))\\
=&(1-\tau)\left\{\frac{(1-\alpha)X(t)}{r-\mu}-\frac{(1-\alpha)(w+w_i)}{r}+\max_{x_i}\left(\frac{X(t)}{x_i}\right)^{\gamma}\left(-\frac{(1-\alpha)x_i}{r-\mu}+\frac{(1-\alpha)(w+w_i)}{r}+\theta-\theta_i\right)\right\}\\
=&(1-\tau)\left\{\frac{(1-\alpha)X(t)}{r-\mu}-\frac{(1-\alpha)(w+w_i)}{r}+\left(\frac{X(t)}{\hat{x_i}}\right)^{\gamma}\left(-\frac{(1-\alpha)\hat{x_i}}{r-\mu}+\frac{(1-\alpha)(w+w_i)}{r}+\theta-\theta_i\right)\right\}
\end{aligned}
$$

for $X(t) > \hat{x_i}$, which is defined by

$$
\hat{x_i}=\frac{\gamma(r-\mu)}{\gamma-1}\left(\frac{w+w_i}{r}+\frac{\theta-\theta_i}{1-\alpha}\right), \tag{1}
$$

where $\gamma = 1/2-\mu/\sigma^2-\sqrt{(\mu/\sigma^2-1/2)^2+2r/\sigma^2}(<0)$. Note that $\hat{x_i}$ stands for the liquidation trigger which is determined by the former debt holders. The former debt holders run the firm until $X(t)$ hits the liquidation trigger $\hat{x_i}$. On the other hand, for $X(t) \leq \hat{x_i}$, we have $A_i(X(t)) = (1-\tau)(\theta-\theta_i)$, which means that the former debt holders liquidate immediately after they take over the firm.

At time 0, the shareholders offer the menu (x_L, x_H, l_L, l_H) to the manager, where for the firm's type i, $x_i(\geq 0)$ is the liquidation or default threshold and l_i is an indicator function of liquidation ($l_i = 1$ for liquidation and $l_i = 0$ for default). For simplicity, we assume that the reservation value for the manager equals 0. Then, the shareholders do not need to offer any payment to the manager when they observe all information. Following Mella-Barral and Perraudin (1997), among others, we assume the absolutely priority rule that the shareholders repay the face value C/r to the debt holders on liquidation. The shareholders receive $\theta-\theta_i-C/r$ on liquidation and receive 0 on default.

At time 0, the shareholders' problem of optimizing the contract is expressed as follows:[7]

$$
\begin{aligned}
E^*(x) =& \max_{x_L,x_H,l_L,l_H}(1-\tau)\left\{\frac{x}{r-\mu}+P\left(-\frac{w+w_L+C}{r}+\left(\frac{x}{x_L}\right)^{\gamma}\left(-\frac{x_L}{r-\mu}+\frac{w+w_L+C}{r}\right.\right.\right.\\
&\left.\left.\left.+l_L\left(-\frac{C}{r}+\theta-\theta_L\right)\right)\right)+(1-P)\left(-\frac{w+w_H+C}{r}+\left(\frac{x}{x_H}\right)^{\gamma}\left(-\frac{x_H}{r-\mu}+\frac{w+w_H+C}{r}\right.\right.\\
&\left.\left.\left.+l_H\left(-\frac{C}{r}+\theta-\theta_H\right)\right)\right)\right\}.
\end{aligned} \tag{2}
$$

Throughout the paper, we use the superscript $*$ for the optimum in the symmetric information case. Equation (2) is equal to $(1-\tau)\times$

$$
\begin{aligned}
&\frac{x}{r-\mu}-\frac{w+Pw_L+(1-P)w_H+C}{r}+\max_{x_L,l_L}\left(\frac{x}{x_L}\right)^{\gamma}\left(-\frac{x_L}{r-\mu}+\frac{w+w_L+C}{r}+l_L\left(-\frac{C}{r}+\theta-\theta_L\right)\right)\\
&+\max_{x_H,l_H}\left(\frac{x}{x_H}\right)^{\gamma}\left(-\frac{x_H}{r-\mu}+\frac{w+w_H+C}{r}+l_H\left(-\frac{C}{r}+\theta-\theta_H\right)\right).
\end{aligned} \tag{3}
$$

[7] It is not meaningful to consider a contract that includes C because $C = 0$ maximizes the equity value. It is not meaningful even if we consider a contract that includes C and maximizes the firm value. In this case, the firm can receive tax advantages without default costs and the firm value becomes infinity when we choose $C = \infty$ and $x_i = 0$. In other words, our model, like in the standard literature such as Leland (1994) and Goldstein, Ju, and Leland (2001), presumes the ex-post conflicts between the equity holders and debt holders.

The maximization problems in (3) are that of Mella-Barral and Perraudin (1997). When we fix at $l_i = 0$, our problems also correspond to that of Goldstein, Ju, and Leland (2001). Clearly, the optimal solution l_i^* is 1 if and only if $C/r \leq \theta - \theta_i$. As in Mella-Barral and Perraudin (1997) and Goldstein, Ju, and Leland (2001), we have the following solutions to problem (3) by the first order condition.

Proposition 1

Case (I-S): $C/r < \theta - \theta_H$ (Both types liquidate.)

$$x_L^* = \frac{\gamma(r-\mu)}{\gamma - 1}\left(\frac{w + w_L}{r} + \theta - \theta_L\right) \tag{4}$$
$$x_H^* = \frac{\gamma(r-\mu)}{\gamma - 1}\left(\frac{w + w_H}{r} + \theta - \theta_H\right) \tag{5}$$
$$l_L^* = l_H^* = 1$$

The ex-post equity values are

$$E_L^*(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w + w_L + C}{r} + \left(\frac{x}{x_L^*}\right)^\gamma \left(-\frac{x_L^*}{r-\mu} + \frac{w + w_L}{r} + \theta - \theta_L\right)\right\} \tag{6}$$
$$E_H^*(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w + w_H + C}{r} + \left(\frac{x}{x_H^*}\right)^\gamma \left(-\frac{x_H^*}{r-\mu} + \frac{w + w_H}{r} + \theta - \theta_H\right)\right\} \tag{7}$$

The ex-post debt values are

$$D_L^*(x) = D_H^*(x) = C/r.$$

Case (II-S): $\theta - \theta_H \leq C/r < \theta - \theta_L$ (Only the low-cost type liquidates.)

$$x_L^* = (4)$$
$$x_H^* = \frac{\gamma(r-\mu)}{\gamma - 1}\frac{w + w_H + C}{r} \tag{8}$$
$$l_L^* = 1, l_H^* = 0$$

The ex-post equity values are

$$E_L^*(x) = (6)$$
$$E_H^*(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w + w_H + C}{r} + \left(\frac{x}{x_H^*}\right)^\gamma \left(-\frac{x_H^*}{r-\mu} + \frac{w + w_H + C}{r}\right)\right\}. \tag{9}$$

The ex-post debt values are

$$
\begin{aligned}
&D_L^*(x) = C/r \\
&D_H^*(x) \ (< C/r) \\
&= \begin{cases}
\dfrac{C}{r} + \left(\dfrac{x}{x_H^*}\right)^\gamma \left(-\dfrac{C}{r} + (1-\tau)(\theta - \theta_H)\right) & (C/r \le (\theta-\theta_H)/(1-\alpha)) \\
\dfrac{C}{r} + \left(\dfrac{x}{x_H^*}\right)^\gamma \left(-\dfrac{C}{r} + (1-\tau)\left(\dfrac{(1-\alpha)x_H^*}{r-\mu} - \dfrac{(1-\alpha)(w+w_H)}{r}\right)\right) & \\
+(1-\tau)\left(\dfrac{x}{\hat{x_H}}\right)^\gamma \left(-\dfrac{(1-\alpha)\hat{x_H}}{r-\mu} + \dfrac{(1-\alpha)(w+w_H)}{r} + \theta - \theta_H\right) & ((\theta-\theta_H)/(1-\alpha) < C/r)
\end{cases}
\end{aligned} \tag{10}
$$

Case (III): $\theta - \theta_L \le C/r$ (Both types default.)

$$
\begin{aligned}
&x_L^* = \frac{\gamma(r-\mu)}{\gamma-1}\frac{w+w_L+C}{r} \\
&x_H^* = (8) \\
&l_L^* = 0, l_H^* = 0
\end{aligned} \tag{11}
$$

The ex-post equity values are

$$
\begin{aligned}
&E_L^*(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_L+C}{r} + \left(\frac{x}{x_L^*}\right)^\gamma \left(-\frac{x_L^*}{r-\mu} + \frac{w+w_L+C}{r}\right)\right\} \\
&E_H^*(x) = (9).
\end{aligned} \tag{12}
$$

The ex-post debt values are

$$
\begin{aligned}
&D_L^*(x) \\
&= \begin{cases}
\dfrac{C}{r} + \left(\dfrac{x}{x_L^*}\right)^\gamma \left(-\dfrac{C}{r} + (1-\tau)(\theta - \theta_L)\right) & (\theta-\theta_L < C/r \le (\theta-\theta_L)/(1-\alpha)) \\
\dfrac{C}{r} + \left(\dfrac{x}{x_L^*}\right)^\gamma \left(-\dfrac{C}{r} + (1-\tau)\left(\dfrac{(1-\alpha)x_L^*}{r-\mu} - \dfrac{(1-\alpha)(w+w_L)}{r}\right)\right) & \\
+(1-\tau)\left(\dfrac{x}{\hat{x_L}}\right)^\gamma \left(-\dfrac{(1-\alpha)\hat{x_L}}{r-\mu} + \dfrac{(1-\alpha)(w+w_L)}{r} + \theta - \theta_L\right) & ((\theta-\theta_L)/(1-\alpha) < C/r)
\end{cases}
\end{aligned} \tag{13}
$$

$$D_H^*(x) = (10)$$

Note that ex-ante equity and debt values are $E^*(x) = PE_L^*(x) + (1-P)E_H^*(x)$ and $D^*(x) = PD_L^*(x) + (1-P)D_H^*(x)$, respectively. Also, note that the ex-post and ex-ante manager's values, denoted by $M_i^*(x)$ $(i = L, H)$ and $M^*(x)$, respectively, are equal to 0. As in the standard literature (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)), Proposition 1 entails the conflicts between the equity holders and debt holders, while it does not entail the conflicts between the manager and equity holders. Depending on whether the face value of debt, C/r, is higher than the liquidation value $\theta - \theta_i$, we can classify the results into Cases (I-S), (II-S), and (III). These results are essentially the same as those of Mella-Barral and Perraudin (1997). Proposition 1 leads to the straightforward result that the firm with more existing debt (liquidation value) tends to proceed to default (liquidation). In the liquidation case, i.e., $C/r < \theta - \theta_i$, the debt value $D_i^*(x)$ is equal to the face value C/r because the debt holders receive C/r at the liquidation time. In the default case, i.e., $\theta - \theta_i \leq C/r$, the ownership of the firm is transferred to the former debt holders at the default time, and they decide whether they continue to operate or immediately liquidate the firm. In this case, the debt value is less than C/r (see Mella-Barral and Perraudin (1997)). As will be shown by Proposition 4 in Section 3.2, in the asymmetric information case, the debt values can be higher than the face value C/r.

3 Asymmetric information

3.1 Setup

In this section, we consider the asymmetric information case in which only the manager observes the realized type i as well as private costs w_i and θ_i. The model adds the asymmetric information model by Grenadier and Wang (2005) to the symmetric information model in Section 2. The asymmetric information case is more likely to apply to firms that have more diversified ownership and/or a lower level of transparency and disclosure.

At time 0, the equity holders offer the menu $(x_L, x_H, l_L, l_H, s_L, s_H)$ to the manager, where for the manager's report j, x_j is the liquidation or default threshold, l_j is the indicator function of liquidation, and s_j is the severance pay on the liquidation or default timing.[8] The manager accepts the offer and starts running the firm if it satisfies the participation condition, which will be explained later. Immediately after taking over the firm, the manager observes the firm's type i and cost w_i and reports the firm's type j and cost w_j to the equity and debt holders. If $l_j = 1$, the manager liquidates at the threshold x_j. Only the manager observes the true liquidation cost θ_i. If $l_j = 0$, the manager stops paying the coupon C at the threshold x_j. On the default timing, the manager is fired and the firm is taken over by the former debt holders.

The equity and debt holders cannot verify whether the manager's report j is equal to the real type i.[9] Thus, the manager may have incentive to falsely report $j \neq i$, receive

[8] The assumption of the severance pay is not essential. Actually, the results are unchanged when we assume the payments to the manager at time 0 instead of the severance pay.

[9] Unlike in our paper, Delaney and Thijssen (2015) examined the impact of the manager's voluntary disclosure

$w_j - w_i$ until the default or liquidation time, and receive $l_j(\theta_j - \theta_i)$ and s_j at the default or liquidation time.[10] However, we can easily check that the revelation principle holds. Therefore, the shareholders can find the optimal menu $(x_L^{**}, x_H^{**}, l_L^{**}, l_H^{**}, s_L^{**}, s_H^{**})$ within the menus that lead the manager to truthfully report $j = i$. Throughout the paper, we use the superscript $**$ for the optimum in the asymmetric information case.

Although the ex-ante commitment may lead to ex-post inefficiency in default and liquidation decisions, it increases the ex-ante equity value. When costs of renegotiation are lower than costs of the ex-post inefficiency, the manager and shareholders may renegotiate the contract and execute the first-best default and liquidation policy.[11] Alternatively, when costs of auditing the manager's false report is low and/or the penalty for the false report is strong (e.g., the manager is fired), the first-best default and liquidation policy in Section 2 can be executed under asymmetric information (for such auditing mechanisms, see Nishihara and Shibata (2008), Shibata (2009), and Shibata and Nishihara (2011)). However, in order to focus how information asymmetries distort the default and liquidation decisions, we assume that no opportunity for renegotiation and audit exists. Our model approximate situations with high costs of renegotiation and audits. It should also be noted that management compensation tends to depend more on the firm's performance in a distressed firm than in an ordinary firm (e.g., Hotchkiss, John, Mooradian, and Thorburn (2008)).

3.2 Solutions

At time 0, the shareholders' problem is expressed as

$$
\begin{aligned}
E^{**}(x) = \max_{\substack{x_L, x_H, l_L, l_H \\ s_L, s_H}} (1-\tau)\Bigg\{ \frac{x}{r-\mu} + P\left(-\frac{w+w_L+C}{r} + \left(\frac{x}{x_L}\right)^{\gamma} \left(-\frac{x_L}{r-\mu} + \frac{w+w_L+C}{r} \right.\right. \\
\left.\left. + l_L\left(-\frac{C}{r} + \theta - \theta_L \right) - s_L \right) \right) + (1-P)\left(-\frac{w+w_H+C}{r} + \left(\frac{x}{x_H}\right)^{\gamma} \left(-\frac{x_H}{r-\mu} + \frac{w+w_H+C}{r} \right.\right. \\
\left.\left. + l_H\left(-\frac{C}{r} + \theta - \theta_H \right) - s_H \right) \right) \Bigg\} \qquad (14)
\end{aligned}
$$

on the firm's investment decision assuming that the manager's voluntary disclosure is creditable.

[10] As in the standard contract theory, the terms Δw and $\Delta\theta$ can be interpreted not only as the manager's profits by outright frauds such as embezzlement and hiding property and assets on liquidation but also as some increases in the manager's utility. For instance, the manager may run and liquidate the firm with costs w_H and θ_H by putting less efforts into the low-cost firm than into the high-cost firm.

[11] In general, the possibility of renegotiation greatly changes equilibrium results. For example, in a real option model of venture financing and investment, Lukas, Mölls, and Welling (2016) showed renegotiation between an entrepreneur and a venture capitalist greatly changes the outcome.

subject to the incentive compatibility conditions

$$\left(\frac{x}{x_L}\right)^{\gamma} s_L \geq \frac{\Delta w}{r} + \left(\frac{x}{x_H}\right)^{\gamma}\left(-\frac{\Delta w}{r} + l_H \Delta\theta + s_H\right), \tag{15}$$

$$\left(\frac{x}{x_H}\right)^{\gamma} s_H \geq -\frac{\Delta w}{r} + \left(\frac{x}{x_L}\right)^{\gamma}\left(\frac{\Delta w}{r} - l_L \Delta\theta + s_L\right) \tag{16}$$

and the ex-post participation conditions

$$s_L \geq 0 \tag{17}$$

$$s_H \geq 0. \tag{18}$$

Under condition (15), the manager who observes the low cost reports truthfully because the expected payoff with the truthful report, i.e., the left-hand side value of (15), is larger than the expected payoff with the false report, i.e., the right-hand side value of (15). Note that the manager who falsely reports the high-cost type gains $\Delta\theta$ if and only if the shareholder chooses default of the high-cost firm, i.e., $l_H = 1$. Similarly, condition (16) ensures that the manager who observes the high-cost reports truthfully. Note that the objective function (14) is equal to (2) minus the severance pay. The ex-ante participation condition follows from (17) and (18).

We can remove (17) because it is always satisfied under (15) and (18). We can readily show that in optimum, $s_H^{**} = 0$, which means the manager's value $M_H^{**}(x)$ is equal to 0 in the high-cost firm. In addition, we can easily see that the problem with $(l_L, l_H) = (0, 1)$ is dominated by the problem with $(l_L, l_H) = (1, 1)$ for $C/r \leq \theta_H$ and dominated by the problem with $(l_L, l_H) = (0, 0)$ for $C/r > \theta_H$. Thus, we need to solve the three remaining cases and compare the maximal values. In the following proposition, Cases (I), (II), and (III) correspond to $(l_L^{**}, l_H^{**}) = (1, 1), (1, 0)$, and $(0, 0)$, respectively. Note that in optimum, the equality holds in (15). For the proof, refer to Appendix A.

Proposition 2 The optimal contract $(x_L^{**}, x_H^{**}, l_L^{**}, l_H^{**}, s_L^{**}, s_H^{**})$ is as follows. In all cases, $x_L^{**} = x_L^{*}$ and $s_H^{**} = 0$.
Case (I): $C/r < \theta - \theta_H - \Delta\theta P/(1-P)$ (Both types liquidate.)

$$x_H^{**} = \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w + w_H}{r} + \theta - \theta_H + \frac{P}{1-P}\left(\frac{\Delta w}{r} - \Delta\theta\right)\right) \tag{19}$$

$$l_L^{**} = l_H^{**} = 1$$

$$s_L^{**} = \left(\frac{x_L^{*}}{x}\right)^{\gamma}\frac{\Delta w}{r} + \left(\frac{x_L^{*}}{x_H^{**}}\right)^{\gamma}\left(-\frac{\Delta w}{r} + \Delta\theta\right) \ (> 0) \tag{20}$$

The ex-post equity values are

$$E_L^{**}(x) = E_L^*(x) - (1-\tau)\left\{\frac{\Delta w}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{\Delta w}{r} + \Delta\theta\right)\right\} \tag{21}$$

$$E_H^{**}(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H\right)\right\} \tag{22}$$

The ex-post debt values are

$$D_L^{**}(x) = D_H^{**}(x) = C/r.$$

The ex-post manager's value is

$$M_L^{**}(x) = \frac{\Delta w}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{\Delta w}{r} + \Delta\theta\right) \tag{23}$$

Case (II): $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_L$ (Only the low-cost type liquidates.)

$$x_H^{**} = \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right) \tag{24}$$

$$l_L^{**} = 1, l_H^{**} = 0$$

$$s_L^{**} = \left(\frac{x_L^*}{x}\right)^\gamma \frac{\Delta w}{r} - \left(\frac{x_L^*}{x_H^{**}}\right)^\gamma \frac{\Delta w}{r} \ (>0) \tag{25}$$

The ex-post equity values are

$$E_L^{**}(x) = E_L^*(x) - (1-\tau)\left\{\frac{\Delta w}{r} - \left(\frac{x}{x_H^{**}}\right)^\gamma \frac{\Delta w}{r}\right\} \tag{26}$$

$$E_H^{**}(x) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H+C}{r}\right)\right\}. \tag{27}$$

The ex-post debt values are

$$D_L^{**}(x) = D_L^*(x) = C/r$$

$$D_H^{**}(x)$$

$$= \begin{cases} \frac{C}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{C}{r} + (1-\tau)(\theta-\theta_H)\right) \quad (C/r \leq (\theta-\theta_H)/(1-\alpha) - \Delta w P/r(1-P)) \\ \frac{C}{r} + \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{C}{r} + (1-\tau)\left(\frac{(1-\alpha)x_H^{**}}{r-\mu} - \frac{(1-\alpha)(w+w_H)}{r}\right)\right) \\ +(1-\tau)\left(\frac{x}{\hat{x_H}}\right)^\gamma \left(-\frac{(1-\alpha)\hat{x_H}}{r-\mu} + \frac{(1-\alpha)(w+w_H)}{r} + \theta - \theta_H\right) \\ ((\theta-\theta_H)/(1-\alpha) - \Delta w P/r(1-P) < C/r). \end{cases} \tag{28}$$

The ex-post manager's value is

$$M_L^{**}(x) = \frac{\Delta w}{r} - \left(\frac{x}{x_H^{**}}\right)^\gamma \frac{\Delta w}{r} \tag{29}$$

Case (III): $\theta - \theta_L \leq C/r$ (Both types default.)

$$x_H^{**} = (24),\ l_L^{**} = l_H^{**} = 0,\ s_L^{**} = (25).$$

The ex-post equity values are

$$E_L^{**}(x) = (26),\ E_H^{**}(x) = (27).$$

The ex-post debt values are

$$D_L^{**}(x) = D_L^{*}(x),\ D_H^{**}(x) = (28).$$

The ex-post manager's value is

$$M_L^{**}(x) = (29)$$

Note that ex-ante equity, debt, and manager's values are $E^*(x) = PE_L^{**}(x) + (1 - P)E_H^{**}(x)$, $D^{**}(x) = PD_L^{**}(x) + (1-P)D_H^{**}(x)$, and $M^{**}(x) = PM_L^{**}(x)$, respectively. As in Proposition 1, we classified the results into three cases, and we have the straightforward result that the firm with more existing debt (liquidation value) tends to proceed to default (liquidation).

A most notable result is that the threshold between Cases (I) and (II) is lower than the threshold between Cases (I-S) and (II-S) in Proposition 1 by $\Delta\theta P/(1-P)$. To be more precise, for $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_H$, the high-cost firm's strategy is default under asymmetric information, while it is liquidation under symmetric information (see Table 1). This means that in the asymmetric information case, the high-cost firm can default even if the liquidation value $\theta - \theta_H$ is higher than the face value of debt, C/r. The intuition is as follows. The manager's private information consists of two components: Δw for the running costs and $\Delta\theta$ for the liquidation costs. In Case (I), the shareholders pay the information rent for both Δw and $\Delta\theta$ to the low-cost firm's manager, while in Case (II) or (III), the shareholders pay the information rent only for Δw. In Case (II) or (III), the shareholders can save the information rent for $\Delta\theta$ because the manager of the low-cost firm cannot gain $\Delta\theta$ by feigning the high-cost type. For $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_H$, the effect of saving the information rent by choosing default of the high-cost firm dominates the direct gain $\theta - \theta_H - C/r$ by choosing liquidation of the high-cost firm. This is why under asymmetric information the shareholders can choose default even when $\theta - \theta_H$ is higher than C/r.

Now, we look at Proposition 2 from the viewpoint of existing debt C/r. In Case (I), the manager's value $M_L^{**}(x)$ (see (23)) is independent of C/r because x_H^{**} (see (19)) is independent of C/r. When C/r increases to $\theta - \theta_H - \Delta\theta P/(1-P)$, $M_L^{**}(x)$ jumps downward to (29) from (23) because the information rent $\Delta\theta$ disappears at this point. From this point, $M_L^{**}(x)$ (see (29)) monotonically decreases with C/r because x_H^{**} (see (24)) monotonically increases with C/r. Thus, we argue that more existing debt can play a role in alleviating the asymmetric information problem and decreasing the manager's value. In other words, a debt financing constraint can worsen the manager-shareholder conflicts. Our result is similar to the finding of Lambrecht and Myers (2008). They showed that risky debt can mitigate manager-shareholder conflicts, although their model does not include the manager's private information but considers the shareholders' possibility of firing the manager and closing or managing the firm by themselves. A similar result is also found in Lambrecht and Myers (2012). We will turn back to this argument in the analysis of the optimal capital structure in Section 4.3.

In this paper, we consider the severance pay s_j on the liquidation or default timing, but we have other types of compensation schemes, which achieve the same maximum for the equity holders.[12] In the following corollary, we present the compensation that does not depend on the initial state variable x.

Corollary 1 The following compensation scheme achieves the same results as in Proposition 2.

Case (I): $C/r < \theta - \theta_H - \Delta\theta P/(1-P)$

If $x_H^{**} \geq x_L^*$, the shareholders continuously pay Δw until $X(t) \geq x_H^{**}$ and pay the severance pay $(x_L^*/x_H^{**})^\gamma \Delta\theta$ at x_L^* to the the low-cost firm's manager. Otherwise, the shareholders continuously pay Δw until $X(t) \geq x_L^*$ and pay the severance pay $\Delta w/r + (x_L^*/x_H^{**})^\gamma(-\Delta w/r + \Delta\theta)$ at x_L^* to the low-cost firm's manager.

Case (II) or (III): $\theta - \theta_H \Delta\theta P/(1-P) \leq C/r$

If $x_H^{**} \geq x_L^*$, the shareholders continuously pay Δw until $X(t) \geq x_H^{**}$ to the low-cost firm's manager. Otherwise, the shareholders continuously pay Δw until $X(t) \geq x_L^*$ and pay the severance pay $\Delta w/r - (x_L^*/x_H^{**})^\gamma \Delta w/r$ at x_L^* to the low-cost firm's manager.

In the compensation scheme of Corollary 1, the equity holders pay $w + w_L + \Delta w$, which is exactly equal to the payment $w + w_H$ to the high-cost firm's manager, to the low-cost firm's manager until $X(t)$ hits $\max(x_L^*, x_H^{**})$. Thus, we can implement this optimal contract by assuming that the manager reports the firm's type at the default or liquidation time rather than at the initial time. In this case, until $X(t) > \max(x_L^*, x_H^{**})$, the equity and debt holders cannot observe the firm's type, and hence the equity and debt values remain the ex-ante values $E^{**}(x)$ and $D^{**}(x)$, respectively. In Section 4.1, we will relate

[12] We cannot increase the equity value beyond $E^{**}(x)$ of Proposition 2 because there are only two types $i = L$ and H.

this situation to equity and debt market reactions.

As to the default and liquidation thresholds, we can show the following proposition. For the proof, see Appendix B.

Proposition 3 The default and liquidation thresholds satisfy the following relation.
Case (I): $C/r < \theta - \theta_H - \Delta\theta P/(1-P)$

$$x_H^{**} \begin{cases} > x_H^* > x_L^{**} = x_L^* & (\Delta\theta < \Delta w/r) \\ = x_H^* = x_L^{**} = x_L^* & (\Delta\theta = \Delta w/r) \\ < x_H^* < x_L^{**} = x_L^* & (\Delta\theta > \Delta w/r) \end{cases}$$

Cases (I-S) and (II) : $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_H$
If $\Delta\theta \leq \Delta w/r$,

$$x_H^{**} > x_H^* \geq x_L^{**} = x_L^*$$

Otherwise,

$$x_H^{**} \begin{cases} < x_H^* < x_L^{**} = x_L^* & (\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_H - \Delta w P/r(1-P)) \\ = x_H^* < x_L^{**} = x_L^* & (C/r = \theta - \theta_H - \Delta w P/r(1-P)) \\ > x_H^* < x_L^{**} = x_L^* & (\theta - \theta_H - \Delta w P/r(1-P) < C/r \leq \theta - \theta_H) \end{cases}$$

Case (II-S): $\theta - \theta_H \leq C/r < \theta - \theta_L$

$$x_H^{**} \begin{cases} > x_H^* < x_L^{**} = x_L^* & (\theta - \theta_H \leq C/r < \theta - \theta_L - \Delta w/r) \\ > x_H^* = x_L^{**} = x_L^* & (C/r = \theta - \theta_L - \Delta w/r) \\ > x_H^* > x_L^{**} = x_L^* & (\theta - \theta_L - \Delta w/r < C/r < \theta - \theta_L) \end{cases}$$

Case (III): $\theta - \theta_L < C/r$

$$x_H^{**} > x_H^* > x_L^{**} = x_L^*$$

The liquidation and default timing of the low-cost firm is not changed by the information asymmetries. The default timing of the high-cost firm is always earlier in the asymmetric information case than in the symmetric information case.[13] This is because the shareholders can decrease the incentive for the low-cost firm's manager to feign the high cost by increasing the default threshold x_H^{**} beyond x_H^*. The logic behind our result is the same as that of the previous papers, such as Grenadier and Wang (2005) and Shibata and Nishihara (2010). They showed that the high-cost firm's investment time is delayed due to the information asymmetries. However, we have the opposite result that

[13]Proposition 3 shows that $x_H^{**} > x_H^*$ in Case (II-S) or (III). On the other hand, the relation between x_H^{**} and x_H^* is ambiguous when both Cases (I-S) and (II) are satisfied. This is because x_H^* is the liquidation trigger, while x_H^{**} is the default trigger.

the high-cost firm's default time is accelerated by the information asymmetries because we consider not investment timing but default timing.

On the other hand, in Case (I), the high-cost firm's liquidation timing can be earlier or later in the asymmetric information case than in the symmetric information case. The sensitivity depends on whether the information asymmetries about running costs, $\Delta w/r$, are larger than those of liquidation costs, $\Delta\theta$. An increase in the liquidation threshold x_H^{**} decreases the information rent for $\Delta w/r$ but increases the information rent for $\Delta\theta$. For $\Delta\theta < \Delta w/r$, the shareholders increase x_H^{**} beyond x_H^* because the former effect is stronger than the latter. In this case, the high-cost firm liquidates earlier than the low-cost firm, and, hence, liquidation is regarded as negative due to the high running cost. For instance, negative liquidation may occur in the case of scrapping a firm piecemeal. Our result suggests that the information asymmetries accelerate negative liquidation. On the other hand, for $\Delta\theta > \Delta w/r$, the shareholders decrease x_H^{**} below x_H^* because the latter effect is stronger than the former. In this case, the low-cost firm liquidates earlier than the high-cost firm, and, hence, liquidation is regarded as the positive consequence of the high liquidation value. For instance, positive liquidation may occur in the case of selling a firm as a going concern. Our result suggests that the information asymmetries delay positive liquidation.

For $\Delta\theta < \Delta w/r$ or $\Delta\theta > \Delta w/r$, the shareholders succeed in reducing the information rent to the manager by adjusting the liquidation trigger x_H^{**}. However, for $\Delta\theta = \Delta w/r$, the shareholders cannot decrease the information rent by adjusting x_H^{**}. Thus, the liquidation timing remains unchanged from that of the symmetric information case. Our equivocal results about the liquidation timing are contrasted with the monotonic results in the previous literature, including Grenadier and Wang (2005) and Shibata and Nishihara (2010). The novel result stems from the fact that our model, unlike those in the previous literature, includes two different types of private information, Δw and $\Delta\theta$.

Except for Case (II), the distance between x_H^{**} and $x_L^{**} = x_L^*$ is larger than the distance between x_H^* and x_L^*. This property is the same as in the existing literature such as Grenadier and Wang (2005) and Shibata and Nishihara (2010). However, in Case (II), the distance between x_H^{**} and $x_L^{**} = x_L^*$ can be smaller than the distance between x_H^* and x_L^*. The reason is that in Case (II), x_H^{**} is the default threshold in the asymmetric information case, while $x_L^{**} = x_L^*$ is the liquidation threshold.[14] Thus, we have ambiguous results, which are different from the previous findings.

To summarize, we have the following empirical predictions: Firms with more diversified ownership and/or a lower level of transparency and disclosure tend to default earlier. Firms with more diversified ownership and/or a lower level of transparency and disclo-

[14] x_H^{**} can be either the default or liquidation threshold

sure tend to do negative (positive) liquidation earlier (later). Our result regarding the default timing aligns with the empirical evidence in Anderson, Mansi, and Reeb (2003). They showed that firms with family ownership are more likely than firms with diversified ownership to value firm survival and hence better protect the debt holders' interests.

We now turn to the values. We define the firm value, denoted by $F(x)$, and total value, denoted by $TV(x)$, as the sum of the equity and debt values and the sum of the equity, debt, and manager's values, respectively. Clearly, we have $F_i^*(x) = TV_i^*(x)$ $(i = L, H)$ and $F^*(x) = TV^*(x)$ in the symmetric information case. On the other hand, by Proposition 2 we have $F_H^{**}(x) = TV_H^{**}(x)$ and $TV_L^{**}(x) = E_L^{**}(x) + D_L^*(x) + M_L^{**}(x) = TV_L^*(x) + \tau M_L^{**}(x)$ in the asymmetric information case. As to the equity, debt, firm, manager's, and total values, we have the following proposition. See Appendix C for the proof.

Proposition 4 In all cases, the equity, manager's, and total values satisfy the following equations:[15]

$$\begin{aligned}
&E_H^{**}(x) \leq E_H^*(x) \leq E_L^{**}(x) < E_L^*(x), && E^{**}(x) < E^*(x),\\
&M_H^{**}(x) = M_H^*(x) = M_L^*(x) = 0 < M_L^{**}(x), && M^*(x) = 0 < M^{**}(x),\\
&F_H^{**}(x) \leq F_H^*(x) < F_L^*(x),\ F_L^{**}(x) < F_L^*(x), && F^{**}(x) < F^*(x),\\
&TV_H^{**}(x) \leq TV_H^*(x) < TV_L^*(x) < TV_L^{**}(x).
\end{aligned}$$

Case (I): $C/r < \theta - \theta_H - \Delta\theta P/(1-P)$

$$\begin{aligned}
&D_H^{**}(x) = D_L^{**}(x) = D_H^*(x) = D_L^*(x) = D^*(x) = D^{**}(x) = C/r, \\
&F_H^*(x) \leq F_L^{**}(x).
\end{aligned} \tag{30}$$

Case (II) or (III): $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r$

$$D_H^*(x) \leq D_L^{**}(x) = D_L^*(x) \leq C/r. \tag{31}$$

When $A_H(x_H^{**}) > C/r$, the debt values satisfy

$$D_H^*(x) \leq D_L^{**}(x) = D_L^*(x) \leq C/r < D_H^{**}(x), D^*(x) \leq C/r < D^{**}(x). \tag{32}$$

The relations of the equity values are straightforward. The equity value in the symmetric information case is not lower than that of the asymmetric information case, and the ex-post equity value in the low-cost firm is not lower than that of the high-cost firm. In Case (I), the debt and firm values also follow the straightforward relation. As to the total

[15] The relations (e.g., the relation between $TV^{**}(x)$ and $TV^*(x)$) other than described in Proposition 4 depend on the parameter values.

value, the ex-post total value of the low-cost firm in the asymmetric information case is higher than that of the symmetric information case because the payment to the manager, like the coupon payment to the debt holders, generates the tax advantage. Accordingly, the ex-ante total value in the asymmetric information case can be higher than that of the symmetric information case.

The results of debt and firm values are ambiguous in Case (II) or (III). For a wide range of parameter values, we have straightforward relations as follows: The debt and firm values in the symmetric information case are not lower than those of the asymmetric information case. The ex-post values of the low-cost firm are not lower than those of the high-cost firm. The straightforward results are consistent with empirical findings of the agency costs of debt in Anderson, Mansi, and Reeb (2003). They found that family ownership decreases the conflicts between the equity holders and debt holders and hence increases the debt values.

More notably, however, the debt and firm values in Case (II) or (III) may deviate from the straightforward relations for some parameter values (especially for a high P). Especially for $A_H(x_H^{**}) > C/r$, we have (32), which means that the information asymmetries increase the ex-post debt value of the high-cost firm beyond the face value of debt, C/r. Thus, the debt value in the asymmetric information case is higher than that of the symmetric information case, and the ex-post debt value of the high-cost firm is higher than that of the low-cost firm.[16]

This counter-intuitive results can happen in the following two situations. One situation is where Cases (I-S) and (II) are both satisfied, i.e., $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r < \theta - \theta_H$. As explained after Proposition 2, in the high-cost firm the shareholders choose default and decrease the information rent to the low-cost firm's manager though the liquidation value $\theta - \theta_H$ is higher than C/r. Then, the debt holders take over the firm that has a higher value than C/r. The counter-intuitive result can happen even if $C/r \geq \theta - \theta_H$. As explained after Proposition 3, the shareholders increase the default threshold x_H^{**} of the high-cost firm to decrease the information rent. When x_H^{**} is sufficiently large, the firm which the debt holders take over can generate a lot of profits until $X(t)$ hits the liquidation trigger $\hat{x}_H$. The relation $A_H(x_H^{**}) > C/r$ can be satisfied in this situation.

Thus, in the high-cost firm, the debt holders can take advantage of the manager-shareholder conflicts under asymmetric information. Our result suggests the possibility that debt holders can benefit from more diversified ownership and a lower level of transparency and disclosure. Relatedly, Shibata and Nishihara (2010) also showed that the information asymmetries can increase the debt value of the high-cost firm. In their model,

[16] In this case, with respect to the firm values, we may also have $F_H^{**}(x) > F_L^{**}(x)$, but we can show that $F^{**}(x) < F^*(x)$.

the information asymmetry about the investment costs increases the investment threshold and then increases the optimal coupon at the investment time. They argue that because of the increased coupon, the debt value can be higher than that of the symmetric information case. However, their model does not lead to a distortion in the default time and debt value from that of Goldstein, Ju, and Leland (2001) because they do not consider any information asymmetry after investment. Unlike in Shibata and Nishihara (2010), our model, which focuses on the information asymmetries about running and liquidation costs of the distressed firm with a fixed coupon, can explain how the information asymmetries distort the default time and debt value.

4 Further analysis

4.1 Jumps in equity and debt values

In this section, we assume that following the contract in Corollary 1 investors do not observe the firm's type until $X(t)$ hits $\max(x_L^*, x_H^{**})$. Investors evaluate the equity and debt prices by the ex-ante values $E^{**}(X(t))$ and $D^{**}(X(t))$, respectively, until $X(t) > \max(x_L^*, x_H^{**})$. At the trigger, they observe the realized type i and change the valuations to the ex-post values $E_i^{**}(\max(x_L^*, x_H^{**}))$ and $D_i^{**}(\max(x_L^*, x_H^{**}))$. Clearly, the uninformed equity price $E^{**}(X(t))$ decreases as $X(t)$ decreases to $\max(x_L^*, x_H^{**})$. At that point, as proved in Proposition 4, the equity value jumps upward to $E_L^{**}(\max(x_L^*, x_H^{**}))$ (downward to $E_H^{**}(\max(x_L^*, x_H^{**}))$) when the firm type is low-cost (high-cost).

On the other hand, according to Proposition 4, in Case (II) or (III) the debt price can jump downward (upward) when the firm type is low-cost (high-cost). In addition, the uninformed debt price $D^{**}(x)$ can increase as $X(t)$ decreases to $\max(x_L^*, x_H^{**})$. Figure 1 shows the three types of results about the debt values. The parameter values are set at Table 2[17] except for P and C. In the top panel, $D^{**}(x)$ increases as $X(t)$ decreases to the trigger x_H^{**}, and at the point, the debt value jumps downward (upward) when the firm type is low-cost (high-cost). According to our computations, this counter-intuitive result can occur in Case (II) or (III) with a high P and a low C. In the middle panel, $D^{**}(x)$ straightforwardly decreases as $X(t)$ decreases to the trigger x_H^{**}, and at that point, the debt value jumps downward (upward) when the firm type is low-cost (high-cost). We also find that this result can occur in Case (II) or (III) for a high P and a low or medium C. The bottom panel shows the straightforward result: $D^{**}(x)$ decreases as $X(t)$ decreases to the trigger x_H^{**}, and at that point, the debt value jumps upward (downward) when the firm type is low-cost (high-cost). We tend to have this straightforward result for most of

[17] We set the base parameter values following Nishihara and Shibata (2016).

the parameter values.

Our results can potentially account for a lot of types of excess returns of distressed firms. In particular, the results are closely associated with the following strategy of vulture investors. Typically, vulture investors purchase a large block of debt of a distressed firm and attempt to control the firm as large shareholders after bankruptcy (e.g., Hotchkiss and Mooradian (1997)). We can explain excess returns of the vulture investors' strategy not only when the firm type results in low-cost (cf. the bottom panel of Figure 1) but also when the firm type results in high-cost (cf. the top and middle panels of Figure 1).

4.2 Comparative statics

In this section, we explain several comparative statics results. We focus mainly on counter-intuitive and non-monotonic results. First, we can analytically show the non-monotonic impact of the costs of the low-cost firm, w_L and θ_L, on the equity and firm values $E^{**}(x)$ and $F^{**}(x)$ in Case (I). For the proof, see Appendix D.

Proposition 5 Case (I) : $C/r < \theta - \theta_H - \Delta\theta P/(1-P)$
In the symmetric information case, we have the monotonic relation

$$\frac{\partial E^*(x)}{\partial w_L} = \frac{\partial F^*(x)}{\partial w_L} < 0$$
$$\frac{\partial E^*(x)}{\partial \theta_L} = \frac{\partial F^*(x)}{\partial \theta_L} < 0,$$

while in the asymmetric information case, we have the non-monotonic relation

$$\frac{\partial E^{**}(x)}{\partial w_L} = \frac{\partial F^{**}(x)}{\partial w_L} \begin{cases} < 0 & (w_L < w_H - r\Delta\theta) \\ = 0 & (w_L = w_H - r\Delta\theta) \\ > 0 & (w_L > w_H - r\Delta\theta) \end{cases}$$

$$\frac{\partial E^{**}(x)}{\partial \theta_L} = \frac{\partial F^{**}(x)}{\partial \theta_L} \begin{cases} < 0 & (\theta_L < \theta_H - \Delta w/r) \\ = 0 & (\theta_L = \theta_H - \Delta w/r) \\ > 0 & (\theta_L > \theta_H - \Delta w/r). \end{cases}$$

In Proposition 5, in the presence of information asymmetries, higher w_L and θ_L have non-monotonic effects on $E^{**}(x)$, while in the absence of information asymmetry, higher w_L and θ_L monotonically decrease $E^*(x)$. In Case (I), the firm values follow the same sensitivities as those of the equity values because the firm value is the sum of the equity value and C/r. In the asymmetric information case, higher w_L and θ_L can play a positive role in decreasing the manager's private information Δw and $\Delta\theta$, respectively. When $w_L > w_H - r\Delta\theta$ ($\theta_L > \theta_H - \Delta w/r$), this indirect and positive effect dominates the direct and negative effect and improves the equity value. If $\Delta w = r\Delta\theta$, the ex-ante equity value $E^{**}(x)$ is exactly equal to the worst-case value $E^{**}_H(x)$ because of $x^*_L = x^{**}_H$. This implies that there is no efficient compensation to resolve the information asymmetries. It is worth noting that higher w_H and θ_H monotonically decrease $E^{**}(x)$ and $F^{**}(x)$ because they increase Δw and $\Delta\theta$, respectively. We numerically found similar non-monotonic impacts of w_L and θ_L on $E^{**}(x), D^{**}(x)$, and $F^{**}(x)$ in Case (II) or (III) with a low C, although we cannot analytically prove the non-monotonic sensitivities. As to the total value, we also numerically found that $TV^{**}(x)$ monotonically decreases with w_L and θ_L.

We also examine the comparative statics with respect to σ although we cannot analytically prove the results. Figures 2 and 3 show the comparative statics with respect to a volatility σ. We present the results in Case (III). We see from Figure 2 that a higher σ decreases the default triggers (and the manager's values) and increases the equity values in the symmetric (asymmetric) information cases. The results regarding the default trigger and equity values are straightforward. Because the manager's values stem only from the private information about the running cost until default, a higher σ, which shortens the average default time, reduces the manager's values. We see from Figure 3 that the debt values decrease with σ, while the firm and total values are U-shaped with with σ in both symmetric and asymmetric information cases. The result regarding the debt values is straightforward. The non-monotonic results regarding the firm and total values stem from the trade-off between increases in the equity values and decreases in the debt and manager's values. According to our computations, in Case (I) or (II), we found the same results regarding the default and liquidation triggers as well as the equity and manager's values. However, the firm and total values tend to monotonically increase with σ because debt can be risk-less and constant in Case (I) or (II).

In summary, we argue that a higher σ causes the wealth transfer from the manager (and the debt holders) to the equity holders (when debt is risky). The asset substitution between the shareholders and manager is also consistent with previous findings in Shibata (2009) and Shibata and Nishihara (2010). The asset substitution between the equity holders and debt holders is consistent with the standard results (e.g., Jensen and Meckling (1976)).

4.3 Optimal capital structure

So far, we have examined the problem with an initial coupon C fixed. In this subsection, we explore the optimal capital structure case, where C is chosen so as to maximize the ex-ante firm value. Table 3 shows the results, where the parameter values are set at Table 2 except for C. In addition to the symmetric and asymmetric information cases, denoted by "Sym." and "Asym." in Table 3, respectively, we also present the case "Asym-TV." in which C is chosen so as to maximize the ex-ante total value. In every case, we obtained an inverted U-shaped function of C and found that the optimal C is high enough to lead to Case (III).

In Table 3, the coupon, leverage (denoted by LV), and credit spread (denoted by CS), in the asymmetric information case are higher than those of the symmetric information case. This is because the shareholders decrease the information rent $M_L^{**}(x)$ to the manager by increasing C and LV. For a high $\theta - \theta_i$, the optimal C can lead to Case (I) or (II). The results remain unchanged when the optimal C leads to Case (II). As was discussed after Proposition 2, $M_L^{**}(x)$ (see (29) and (24)) decreases with C in Case (II) or (III). Thus, the shareholders can decrease the information rent to the manager by increasing C and LV. In some cases, in the symmetric information case, the optimal C leads to Case (I-S), whereas in the asymmetric information case, the optimal C leads to Case (II) or (III). In these cases, the differences in C, LV, and CS between the symmetric and asymmetric cases become larger.

On the other hand, the results regarding C, LV, and CS change from Table 3 when the optimal C leads to Case (I), i.e., the risk-less debt case. Actually, in this case, the optimal C agrees with the threshold between Cases (I) and (II) (Cases (I-S) and (II-S)) in the asymmetric (symmetric) information case. The difference arises mainly because in Case (I), debt is risk-less and the shareholders cannot decrease $M_L^{**}(x)$ (see (23) and (19)) by increasing C and LV.

In summary, we argue that the shareholders can reduce the loss due to the information asymmetries by increasing risky debt, leverage, and credit spread. This result aligns with Lambrecht and Myers (2008) and Lambrecht and Myers (2012), who argued that risky debt can potentially mitigate manager-shareholder conflicts, although their model does not include the information asymmetries. As a testable prediction, our result suggests that a firm, that has more diffuse equity holdings as well as a lower level of transparency and disclosure, tends to have a higher leverage ratio. Although there have been ambiguous results about the relation between ownership and leverage (e.g., Anderson and Reeb (2003), King and Santor (2008)), several papers, such as Agrawal and Nagarajan (1990) and McConaughy, Matthews, and Fialko (2001), empirically support our results. Actually, they found that ownership dispersion is positively related to leverage. Our result is also consistent with the empirical evidence by John and Litov (2010) that firms with weaker governance use more debt.

Lastly, we note that the key results tend to remain unchanged from Propositions 3 and 4 even when we compare the results in the symmetric and asymmetric information cases for the optimal C instead of a fixed C. Indeed, we can see the following results in

Table 3. The information asymmetries accelerate the default timing. The information asymmetries decrease the equity and firm values, while they can increase the debt value. We also have an equivocal result in the liquidation timing when the optimal C leads to Case (I) or (II), although we omit a numerical example.

5 Conclusion

We examined the default and liquidation timing of a firm in which the shareholders delegate the manager, who observes private information about running and liquidation costs of the firm, to operate the firm. We analytically derived the shareholders' optimal contract, which consists of the default or liquidation timing and compensation contingent on the firm's cost structure. The main results in the asymmetric information case are summarized as follows.

As in the symmetric information case, the firm with more existing debt (liquidation value) tends to proceed to default (liquidation). However, unlike in the symmetric information case, the shareholders can choose default of the high-cost firm and reduce the compensation to the manager even when the liquidation value is higher than the face value of debt. More existing debt decreases the compensation to the manager and alleviates the loss due to the information asymmetries.

Although the information asymmetries do not change the low-cost firm's default and liquidation timing, they change the high-cost firm's default and liquidation timing. Indeed, they accelerate (delay) negative (positive) liquidation, while they accelerate default. In particular, our result regarding the default timing aligns with the empirical evidence regarding family firms.

While the information asymmetries straightforwardly decrease the equity and firm values, they may increase the debt value. In other words, the debt holders can take advantage of the manager-shareholder conflicts. In terms of market reactions, our model can generate a variety of jumps in the equity and debt values. In particular, our results can potentially explain excess returns of the vulture investors, who target distressed firms.

We also show several comparative statics results. Higher costs of the low-cost firm can play a positive role in decreasing the manager's private information. A higher volatility causes wealth transfer from the debt holders and manager to the equity holders. In the optimal capital structure, the initial coupon of debt, leverage, and credit spread in the asymmetric information case are higher than those of the symmetric information case. This is because an increase in risky debt decreases the loss due to the information asymmetries. Our results are consistent with empirical findings of the positive relation between the ownership dispersion and firm leverage.

A Proof of Proposition 2

We solve the problem (14) subject only to (15) and show that the solution satisfies (16). In the optimal solution, we have $s_H^{**} = 0$ and (15) is binding because otherwise we can increase the objective value by decreasing s_H and/or s_L. Below, we will solve the following problem:

$$\max_{\substack{x_L, x_H, \\ l_L, l_H, s_L}} (1-\tau)\left\{\frac{x}{r-\mu} + P\left(-\frac{w+w_L+C}{r} + \left(\frac{x}{x_L}\right)^\gamma \left(-\frac{x_L}{r-\mu} + \frac{w+w_L+C}{r} + l_L\left(-\frac{C}{r} + \theta - \theta_L\right.\right.\right.\right.$$
$$\left.\left.\left.\left.-s_L\right)\right) + (1-P)\left(-\frac{w+w_H+C}{r} + \left(\frac{x}{x_H}\right)^\gamma \left(-\frac{x_H}{r-\mu} + \frac{w+w_H+C}{r} + l_H\left(-\frac{C}{r} + \theta - \theta_H\right)\right)\right)\right\} \tag{33}$$

$$\text{subject to } \left(\frac{x}{x_L}\right)^\gamma s_L = \frac{\Delta w}{r} + \left(\frac{x}{x_H}\right)^\gamma \left(-\frac{\Delta w}{r} + l_H \Delta\theta\right), \tag{34}$$

The problem (33) with $(l_L, l_H) = (0,1)$ is dominated by the problem with $(l_L, l_H) = (1,1)$ for $C/r \leq \theta_H$ and dominated by the problem with $(l_L, l_H) = (0,0)$ for $C/r > \theta_H$. Then, we will solve the three remaining cases: $(l_L, l_H) = (1,1), (1,0)$, and $(0,0)$, and choose the maximal value.

Solution with $(l_L, l_H) = (1,1)$:

By substituting (34) into the objective function (33), the problem (33) can be reduced to

$$\begin{aligned}
&\max_{x_L, x_H} (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L}\right)^\gamma\left(-\frac{x_L}{r-\mu} + \frac{w+w_L}{r} + \theta - \theta_L\right)\right. \\
&\left. + (1-P)\left(\frac{x}{x_H}\right)^\gamma\left(-\frac{x_H}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H - \frac{P}{1-P}\left(-\frac{\Delta w}{r} + \Delta\theta\right)\right)\right\} \qquad (35) \\
=&(1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L^*}\right)^\gamma\left(-\frac{x_L^*}{r-\mu} + \frac{w+w_L^*}{r} + \theta - \theta_L\right)\right. \\
&\left. + (1-P)\left(\frac{x}{x_H^{**}}\right)^\gamma\left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H - \frac{P}{1-P}\left(-\frac{\Delta w}{r} + \Delta\theta\right)\right)\right\},
\end{aligned}$$

where by the first order condition x_L^* and x_H^{**} are derived as (4) and (19), respectively. The optimal servant pay s_L^{**} is derived by (34). Then, we have $(x_L^*, x_H^{**}, s_L^{**})$ in Case (I) of Proposition 2.

Solution with $(l_L, l_H) = (1,0)$:

In the same fashion as above, we have

$$\begin{aligned}
&\max_{x_L, x_H} (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L}\right)^\gamma\left(-\frac{x_L}{r-\mu} + \frac{w+w_L}{r} + \theta - \theta_L\right)\right. \\
&\left. + (1-P)\left(\frac{x}{x_H}\right)^\gamma\left(-\frac{x_H}{r-\mu} + \frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right)\right\} \qquad (36) \\
=&(1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L^*}\right)^\gamma\left(-\frac{x_L^*}{r-\mu} + \frac{w_L^*}{r} + \theta - \theta_L\right)\right. \\
&\left. + (1-P)\left(\frac{x}{x_H^{**}}\right)^\gamma\left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right)\right\},
\end{aligned}$$

where by the first order condition, x_L^* and x_H^{**} are derived as (4) and (24), respectively. The optimal servant pay s_L^{**} is derived by (34). Therefore, we have $(x_L^*, x_H^{**}, s_L^{**})$ in Case (II) of Proposition 2.

Solution with $(l_L, l_H) = (0,0)$:

In the same fashion as above, we can derive x_L^* and x_H^{**} as (11) and (24), respectively. We can derive the optimal servant pay s_L^{**} by (34). Then, we have $(x_L^*, x_H^{**}, s_L^{**})$ in Case (III) of Proposition 2 and the objective value:

$$\begin{aligned}&(1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L^*}\right)^\gamma\left(-\frac{x_L^*}{r-\mu} + \frac{w+w_L+C}{r}\right)\right.\\ &\left. + (1-P)\left(\frac{x}{x_H^{**}}\right)^\gamma\left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right)\right\},\end{aligned}$$

Next, we compare the objective values in the solutions above. Clearly, the objective value with $(l_L, l_H) = (0,0)$ is larger than that of $(l_L, l_H) = (1,0)$ if and only if $\theta - \theta_L < C/r$. By comparing the objective functions in (35) and (36), we have the objective value with $(l_L, l_H) = (1,0)$ is larger than that of $(l_L, l_H) = (1,1)$ if and only if

$$\theta - \theta_H - \frac{P}{1-P}\Delta\theta < \frac{C}{r}. \tag{37}$$

Then, we have the optimal solutions $(l_L^{**}, l_H^{**}) = (1,1), (1,0)$, and $(0,0)$ in Cases (I), (II), and (III), respectively.

Finally, we need to check that the solutions satisfy (16). Using (34), we have

$$\begin{aligned}(16) \Leftrightarrow & -\frac{\Delta w}{r} + \left(\frac{x}{x_L^*}\right)^\gamma\left(\frac{\Delta w}{r} - l_L^{**}\Delta\theta + s_L^{**}\right) \le 0\\ \Leftrightarrow & x_L^{*-\gamma}\left(\frac{\Delta w}{r} - l_L^{**}\Delta\theta\right) + x_H^{**-\gamma}\left(-\frac{\Delta w}{r} + l_H^{**}\Delta\theta\right) \le 0.\end{aligned} \tag{38}$$

Case (I): $C/r \le \theta - \theta_H - \Delta\theta P/(1-P)$

Note that

$$(38) \Leftrightarrow (x_L^{*-\gamma} - x_H^{**-\gamma})\left(\frac{\Delta w}{r} - \Delta\theta\right) \le 0.$$

We can show this inequality because using (19), we have

$$\begin{aligned}& x_L^* \ge x_H^{**}\\ \Leftrightarrow & \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_L}{r} + \theta - \theta_L\right) \ge \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H}{r} + \theta - \theta_H + \frac{P}{1-P}\left(\frac{\Delta w}{r} - \Delta\theta\right)\right)\\ \Leftrightarrow & \frac{\gamma(r-\mu)}{(\gamma-1)(1-P)}\left(-\frac{\Delta w}{r} + \Delta\theta\right) \ge 0\\ \Leftrightarrow & \frac{\Delta w}{r} - \Delta\theta \le 0.\end{aligned}$$

Case (II): $\theta - \theta_H - \Delta\theta P/(1-P) < C/r \le \theta - \theta_L$

Note that

$$(38) \Leftrightarrow x_L^{*-\gamma}\left(\frac{\Delta w}{r} - \Delta\theta\right) - \frac{x_H^{**-\gamma}\Delta w}{r} \le 0.$$

If $\Delta w/r - \Delta\theta \leq 0$, we immediately obtain this inequality. Otherwise, we can show this inequality because using (24), we have

$$\begin{aligned} x_H^{**} =& \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right) \\ >& \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H+C}{r} + \frac{P}{1-P}\Delta\theta\right) && (39) \\ >& \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H}{r} + \theta - \theta_H\right) && (40) \\ >& \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_L}{r} + \theta - \theta_L\right) = x_L^*, && (41) \end{aligned}$$

where in (39) and (41) we used $\Delta w/r > \Delta\theta$ and in (40) we used $\theta - \theta_H - \Delta\theta P/(1-P) < C/r$.

Case (III): $\theta - \theta_L < C/r$

Note that

$$(38) \Leftrightarrow x_L^{*-\gamma} - x_H^{**-\gamma} \leq 0.$$

By (24) we can show this inequality as follows:

$$\begin{aligned} x_H^{**} =& \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right) \\ >& \frac{\gamma(r-\mu)(w+w_H+C)}{(\gamma-1)r} = x_H^* \\ >& \frac{\gamma(r-\mu)(w+w_L+C)}{(\gamma-1)r} = x_L^*. \end{aligned}$$

It is straightforward to obtain the equity, debt, and manager's values in each case. The proof is completed.

B Proof of Proposition 3

Case (I): $C/r \leq \theta - \theta_H - \Delta\theta P/(1-P)$

Note that Case (I) is included in Case (I-S). Then, we can immediately show the relations by comparing (4), (5), and (19).

Case (II): $\theta - \theta_H - \Delta\theta P/(1-P) < C/r \leq \theta - \theta_L$

We have

$$\begin{aligned} & x_H^{**} \geq x_H^* \\ \Leftrightarrow & \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H+C}{r} + \frac{P}{1-P}\frac{\Delta w}{r}\right) \geq \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H}{r} + \max\left\{\frac{C}{r}, \theta-\theta_H\right\}\right) \\ \Leftrightarrow & \frac{C}{r} \geq \theta - \theta_H - \frac{P}{1-P}\frac{\Delta w}{r} && (42) \end{aligned}$$

and

$$
\begin{aligned}
&x_H^* \geq x_L^* \\
\Leftrightarrow &\frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_H}{r}+\max\left\{\frac{C}{r},\theta-\theta_H\right\}\right) \geq \frac{\gamma(r-\mu)}{(\gamma-1)}\left(\frac{w+w_L}{r}+\theta-\theta_L\right) \\
\Leftrightarrow &\max\left\{\frac{C}{r},\theta-\theta_H\right\} \geq \theta-\theta_L-\frac{\Delta w}{r} \qquad (43)
\end{aligned}
$$

Using (42) and (43), we can show the relations in Case (II).

Case (III): $\theta - \theta_L < C/r$

We can immediately show the relations by comparing (11), (8), and (24).

C Proof of Proposition 4

Equity values.

By Proposition 2, we can immediately show that $E_L^{**}(x) < E_L^*(x)$. By Proposition 2 and the optimality of x_H^* and l_H^*, we also have $E_H^{**}(x) \leq E_H^*(x)$. By the revelation principle, we have $E^{**}(x) \geq E_H^*(x)$. Then, we have

$$
\begin{aligned}
E_H^*(x) &\leq PE_L^{**}(x) + (1-P)E_H^{**}(x) = E^{**}(x) \\
&\leq PE_L^{**}(x) + (1-P)E_H^*(x),
\end{aligned}
$$

which leads to $E_H^*(x) \leq E_L^{**}(x)$.

Manager's values.

The relation immediately follows from Proposition 2.

Debt, firm and total values.

Case (I): $C/r \leq \theta - \theta_H - \Delta\theta P/(1-P)$

By Proposition 2, we immediately get (30). Thus, the firm values satisfy the same relations as those of the equity values. We can easily show the relations of the total values using $TV_H^*(x) = F_H^*(x), TV_L^*(x) = F_L^*(x), TV_H^{**}(x) = F_H^{**}(x)$, and $TV_L^{**}(x) = TV_L^*(x) + \tau M_L^{**}(x)$.

Case (II) or (III): $\theta - \theta_H - \Delta\theta P/(1-P) \leq C/r$

It is easy to check (31) in general. Using (31) and the relation between the equity values, we have

$$
F_H^*(x) < F_L^*(x),\ F_L^{**}(x) < F_L^*(x),\ TV_H^*(x) < TV_L^*(x).
$$

Because of $TV_L^{**}(x) = TV_L^*(x) + \tau M_L^{**}(x)$, we also have $TV_L^*(x) < TV_L^{**}(x)$.

Lastly, we will show that $F_H^{**}(x) < F_H^*(x)$ below. Note that if we have $F_H^{**}(x) < F_H^*(x)$, $F^{**}(x) < F^*(x)$ and $TV_H^{**}(x) < TV_H^*(x)$ immediately follow.

— **Case (II-S) or (III):** $\theta - \theta_H \le C/r$

We define $f(y)$ by

$$
\begin{aligned}
f(y) &= \left(\frac{x}{y}\right)^{\gamma}\left(-\frac{(1-\tau)y}{r-\mu} + \frac{(1-\tau)(w+w_H)}{r} - \frac{\tau C}{r} + A_H(y)\right) \\
&= \begin{cases}
\left(\frac{x}{y}\right)^{\gamma}\left(-\frac{(1-\tau)y}{r-\mu} + \frac{(1-\tau)(w+w_H)}{r} - \frac{\tau C}{r} + (1-\tau)(\theta-\theta_H)\right) & (y \le \hat{x_H}) \\
\left(\frac{x}{y}\right)^{\gamma}\left(-\frac{(1-\tau)\alpha y}{r-\mu} + \frac{(1-\tau)\alpha(w+w_H)}{r} - \frac{\tau C}{r}\right) & \\
+(1-\tau)\left(\frac{x}{y}\right)^{\gamma}\left(-\frac{(1-\alpha)\hat{x_H}}{r-\mu} + \frac{(1-\alpha)(w+w_H)}{r} + \theta - \theta_H\right) & (y > \hat{x_H})
\end{cases}
\end{aligned}
\tag{44}
$$

Note that

$$
F_H^*(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + f(x_H^*) \tag{45}
$$

$$
F_H^{**}(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + f(x_H^{**}). \tag{46}
$$

We have only to show that $f(x_H^{**}) < f(x_H^*)$. By computing the derivatives of (44), we can easily show that

$$
f'(y) < 0 \quad (y > \tilde{y}), \tag{47}
$$

where $\tilde{y}$ is defined by

$$
\tilde{y} = \frac{\gamma(r-\mu)}{\gamma-1}\left(\frac{w+w_H}{r} + \theta - \theta_H - \frac{\tau C}{1-\tau}\right) \ (< \hat{x_H}). \tag{48}
$$

By (8), (24), (48), and $\theta - \theta_H \le C/r$, we have

$$
\tilde{y} \le x_H^* < x_H^{**}. \tag{49}
$$

By (47) and (49), we have $f(x_H^{**}) < f(x_H^*)$.

— **Cases (I-S) and (II):** $\theta - \theta_H - \Delta\theta P/(1-P) \le C/r < \theta - \theta_H$

In the symmetric information case, the firm chooses liquidation and the liquidation trigger x_H^* becomes (5). We have

$$
\begin{aligned}
F_H^*(x) &= \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + (1-\tau)\left(\frac{x}{x_H^*}\right)^{\gamma}\left(-\frac{x_H^*}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H\right) \\
&> \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + f(x_H^*).
\end{aligned}
\tag{50}
$$

If $C/r + P\Delta w/(1-P)r > \theta - \theta_H$, we have

$$
(50) > \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + f(x_H^{**}) = F_H^{**}(x)
$$

by $\tilde{y} \leq x_H^* < x_H^{**}$ and (47). If $C/r + P\Delta w/(1-P)r \leq \theta - \theta_H$, we have $x_H^{**} < \hat{x_H}$, and hence, we have

$$
\begin{aligned}
F_H^{**}(x) = &\frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} \\
&+ \left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{(1-\tau)x_H^{**}}{r-\mu} + \frac{(1-\tau)(w+w_H)}{r} - \frac{\tau C}{r} + (1-\tau)(\theta-\theta_H)\right) \\
&< \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + (1-\tau)\left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H\right) \\
&\leq \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+w_H)}{r} + \frac{\tau C}{r} + (1-\tau)\left(\frac{x}{x_H^{*}}\right)^\gamma \left(-\frac{x_H^{*}}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H\right) \\
&= F_H^*(x).
\end{aligned}
$$

The proof is completed.

D Proof of Proposition 5

In Case (I), we have $F^*(x) = E^*(x) + C/r$ and $F^{**}(x) = E^{**}(x) + C/r$. Then, we have only to show the results of $E^*(x)$ and $E^{**}(x)$. In the symmetric information case, we have

$$
\begin{aligned}
E^*(x) =& PE_L^*(x) + (1-P)E_H^*(x) \\
=& (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+C}{r} - \frac{Pw_L}{r} + P\left(\frac{x}{x_L^*}\right)^\gamma \left(-\frac{x_L^*}{r-\mu} + \frac{w+w_L}{r} + \theta - \theta_L\right)\right. \\
&\left. - \frac{(1-P)w_H}{r} + (1-P)\left(\frac{x}{x_H^*}\right)^\gamma \left(-\frac{x_H^*}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H\right)\right\}.
\end{aligned} \tag{51}
$$

By (51) and the envelope theorem, we have

$$
\begin{aligned}
\frac{\partial E^*(x)}{\partial w_L} &= P(1-\tau)\left\{-\frac{1}{r} + \left(\frac{x}{x_L^*}\right)^\gamma \frac{1}{r}\right\} < 0, \\
\frac{\partial E^*(x)}{\partial \theta_L} &= -P(1-\tau)\left(\frac{x}{x_L^*}\right)^\gamma < 0
\end{aligned}
$$

On the other hand, in the asymmetric information case, we have

$$
\begin{aligned}
E^{**}(x) =& PE_L^{**}(x) + (1-P)E_H^{**}(x) \\
=& (1-\tau)\left\{\frac{x}{r-\mu} - \frac{w+w_H+C}{r} + P\left(\frac{x}{x_L^*}\right)^\gamma \left(-\frac{x_L^*}{r-\mu} + \frac{w+w_L}{r} + \theta - \theta_L\right)\right. \\
&\left. + (1-P)\left(\frac{x}{x_H^{**}}\right)^\gamma \left(-\frac{x_H^{**}}{r-\mu} + \frac{w+w_H}{r} + \theta - \theta_H + \frac{P}{1-P}\left(\frac{\Delta w}{r} - \Delta\theta\right)\right)\right\}.
\end{aligned} \tag{52}
$$

By (52) and the envelope theorem, we have

$$
\begin{aligned}
\frac{\partial E^{**}(x)}{\partial w_L} &= (1-\tau)\left\{P\left(\frac{x}{x_L^*}\right)^\gamma \frac{1}{r} - (1-P)\left(\frac{x}{x_H^{**}}\right)^\gamma \frac{P}{(1-P)r}\right\} \\
&= \frac{(1-\tau)P}{r}\left\{\left(\frac{x}{x_L^*}\right)^\gamma - \left(\frac{x}{x_H^{**}}\right)^\gamma\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial E^{**}(x)}{\partial \theta_L} &= (1-\tau)\left\{-P\left(\frac{x}{x_L^*}\right)^{\gamma} + (1-P)\left(\frac{x}{x_H^{**}}\right)^{\gamma}\frac{P}{1-P}\right\} \\
&= (1-\tau)P\left\{\left(\frac{x}{x_H^{**}}\right)^{\gamma} - \left(\frac{x}{x_L^*}\right)^{\gamma}\right\}.
\end{aligned}
$$

Then, the results follows from Proposition 3.

References

Agrawal, A., and N. Nagarajan, 1990, Agency costs, and ownership control: The case of all-equity firms, *Journal of Finance* 45, 1325–1331.

Anderson, R., A. Duru, and D. Reeb, 2009, Founders, heirs, and corporate opacity in the united states, *Journal of Financial Economics* 92, 205–222.

Anderson, R., S. Mansi, and D. Reeb, 2003, Founding family ownership and the agency cost of debt, *Journal of Financial Economics* 68, 263–285.

Anderson, R., and D. Reeb, 2003, Founding-family ownership, corporate diversification, and firm leverage, *Journal of Law and Economics* 46, 653–684.

Delaney, L., and J. Thijssen, 2015, The impact of voluntary disclosure on a firm's investment policy, *European Journal of Operational Research* 242, 232–242.

Dixit, A., and R. Pindyck, 1994, *Investment Under Uncertainty* (Princeton University Press: Princeton).

Fan, H., and S. Sundaresan, 2000, Debt valuation, renegotiation, and optimal dividend policy, *Review of Financial Studies* 13, 1057–1099.

Goldstein, R., N. Ju, and H. Leland, 2001, An EBIT-based model of dynamic capital structure, *Journal of Business* 74, 483–512.

Grenadier, S., and A. Malenko, 2011, Real options signaling games with applications to corporate finance, *Review of Financial Studies* 24, 3993–4036.

Grenadier, S., and N. Wang, 2005, Investment timing, agency, and information, *Journal of Financial Economics* 75, 493–533.

Hotchkiss, E., K. John, R. Mooradian, and K. Thorburn, 2008, Bankruptcy and the resolution of financial distress, Available at SSRN: 1086942.

Hotchkiss, E., and R. Mooradian, 1997, Vulture investors and the market for control of distressed firms, *Journal of Financial Economics* 43, 401–432.

Jensen, M., and W. Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of Financial Economics* 3, 305–360.

John, K., and L. Litov, 2010, Corporate governance and financing policy: New evidence, SSRN Working Paper 637341.

King, M., and E. Santor, 2008, Family values: Ownership structure, performance and capital structure of canadian firms, *Journal of Banking and Finance* 32, 2423–2432.

Lambrecht, B., and S. Myers, 2008, Debt and managerial rents in a real-options model of the firm, *Journal of Financial Economics* 89, 209–231.

Lambrecht, B., and S. Myers, 2012, A Lintner model of payout and managerial rents, *Journal of Finance* 67, 1761–1810.

Leland, H., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Lukas, E., S. Mölls, and A. Welling, 2016, Venture capital, staged financing and optimal funding policies under uncertainty, *European Journal of Operational Research* 250, 305–313.

Mauer, D., and S. Sarkar, 2005, Real options, agency conflicts, and optimal capital structure, *Journal of Banking and Finance* 29, 1405–1428.

McConaughy, D., C. Matthews, and A. Fialko, 2001, Founding family controlled firms: Performance, risk, and value, *Journal of Small Business Management* 39, 31–49.

McDonald, R., and D. Siegel, 1986, The value of waiting to invest, *Quarterly Journal of Economics* 101, 707–727.

Mella-Barral, P., and W. Perraudin, 1997, Strategic debt service, *Journal of Finance* 52, 531–556.

Morellec, E., and N. Schürhoff, 2011, Corporate investment and financing under asymmetric information, *Journal of Financial Economics* 99, 262–288.

Nishihara, M., and T. Shibata, 2008, The agency problem between the owner and the manager in real investment: The bonus-audit relationship, *Operations Research Letters* 36, 291–296.

Nishihara, M., and T. Shibata, 2016, Asset sale, debt restructuring, and liquidation, *Journal of Economic Dynamics and Control* 67, 73–92.

Sannikov, Y., 2008, A continuous-time version of the principal-agent problem, *Review of Economic Studies* 75, 957–984.

Shibata, T., 2009, Investment timing, asymmetric information, and audit structure: a real options framework, *Journal of Economic Dynamics and Control* 33, 903–921.

Shibata, T., and M. Nishihara, 2010, Dynamic investment and capital structure under manager-shareholder conflict, *Journal of Economic Dynamics and Control* 34, 158–178.

Shibata, T., and M. Nishihara, 2011, Interactions between investment timing and management effort under asymmetric information: Costs and benefits of privatized firms, *European Journal of Operational Research* 215, 688–696.

Shibata, T., and M. Nishihara, 2012, Investment timing under debt issuance constraint, *Journal of Banking and Finance* 36, 981–991.

Shibata, T., and M. Nishihara, 2015a, Investment-based financing constraints and debt renegotiation, *Journal of Banking and Finance* 51, 79–92.

Shibata, T., and M. Nishihara, 2015b, Investment timing, debt structure, and financing constraints, *European Journal of Operational Research* 241, 513–526.

Sundaresan, S., and N. Wang, 2007, Investment under uncertainty and strategic debt service, *American Economic Review, Papers & Proceedings* 97, 256–261.

Sundaresan, S., N. Wang, and J. Yang, 2015, Dynamic investment, capital structure, and debt overhang, *Review of Corporate Finance Studies* 4, 1–42.

Table 1: Differences between the symmetric and asymmetric information cases.

C/r	$[0, \theta - \theta_H - \Delta\theta P/(1-P))$	$[\theta - \theta_H - \Delta\theta P/(1-P), \theta - \theta_H)$	$[\theta - \theta_H, \theta - \theta_L)$	$[\theta - \theta_L, +\infty)$
Sym.	Case (I-S)		Case (II-S)	Case (III)
Asym.	Case (I)	Case (II)		Case (III)

Table 2: Base parameter values.

r	μ	σ	τ	x	w	w_H	w_L	θ	θ_H	θ_L	α	P	C
0.06	0.01	0.2	0.15	2	0.5	0.1	0	5	1	0	0.3	0.5	1

Table 3: Optimal capital structure.

	C	LV	CS	x_L	x_H	$E(x)$	$D(x)$	$M(x)$	$F(x)$	$TV(x)$
Sym.	1.12	0.54	0.013	0.81	0.86	12.9	15.26	0	28.16	28.16
Asym.	1.14	0.56	0.014	0.82	0.92	12.2	15.37	0.57	27.57	28.14
Asym-TV	1.07	0.53	0.013	0.79	0.89	12.91	14.65	0.59	27.56	28.15

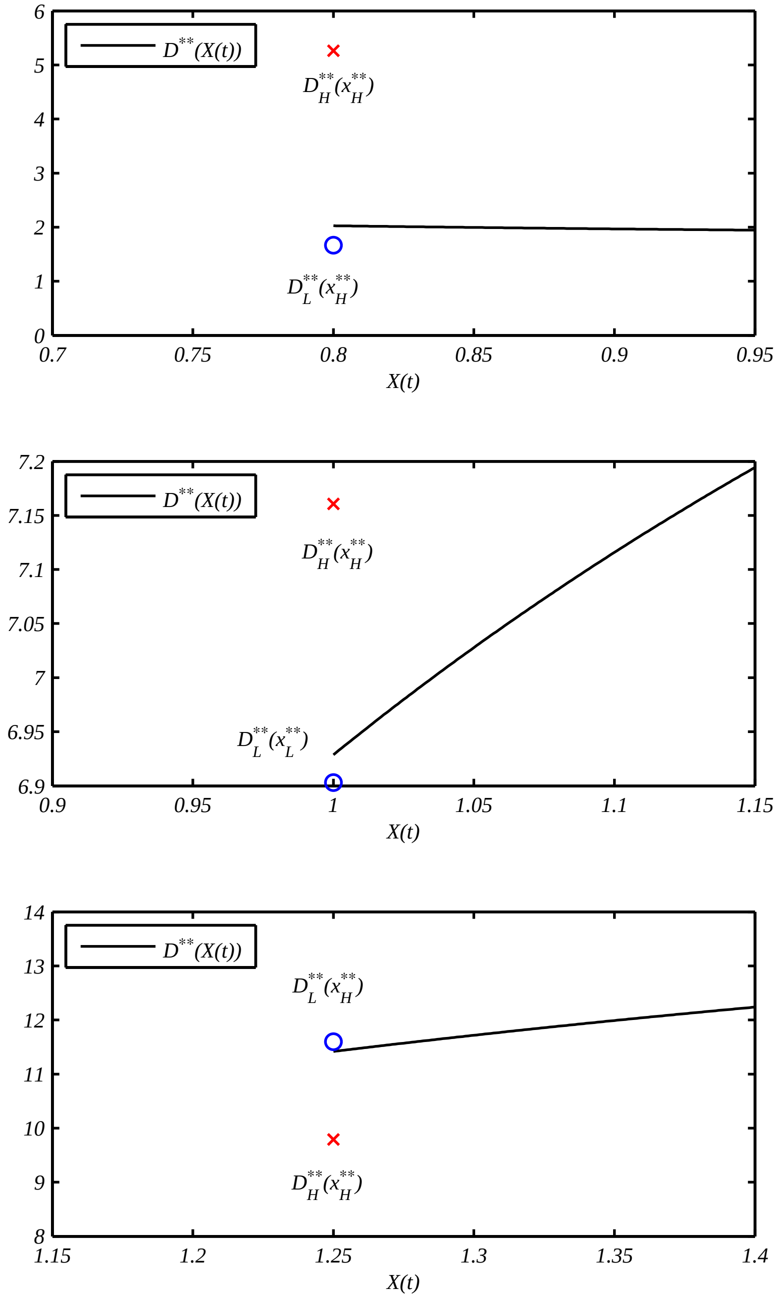

Figure 1: Jumps in the debt value. We set $P = 0.9$. In the top, middle, bottom panels, C are set at $0.1, 0.5$, and 1, respectively. The other parameter values are set at Table 2.

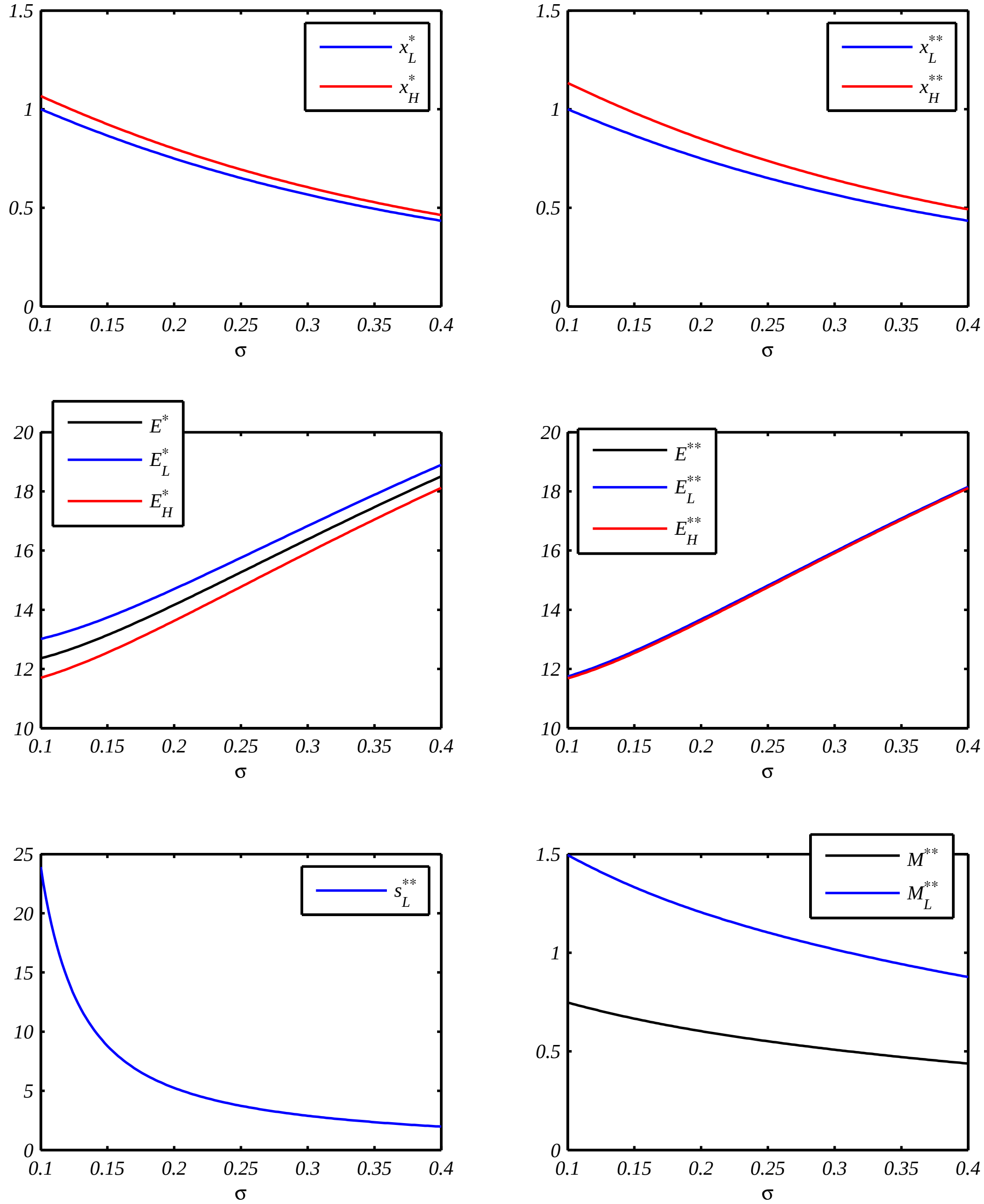

Figure 2: Comparative statics with respect to σ in Case (III). The other parameter values are set at Table 2. Except for the bottom panels, the left-hand and right-hand panels show the results in the symmetric and asymmetric information cases, respectively.

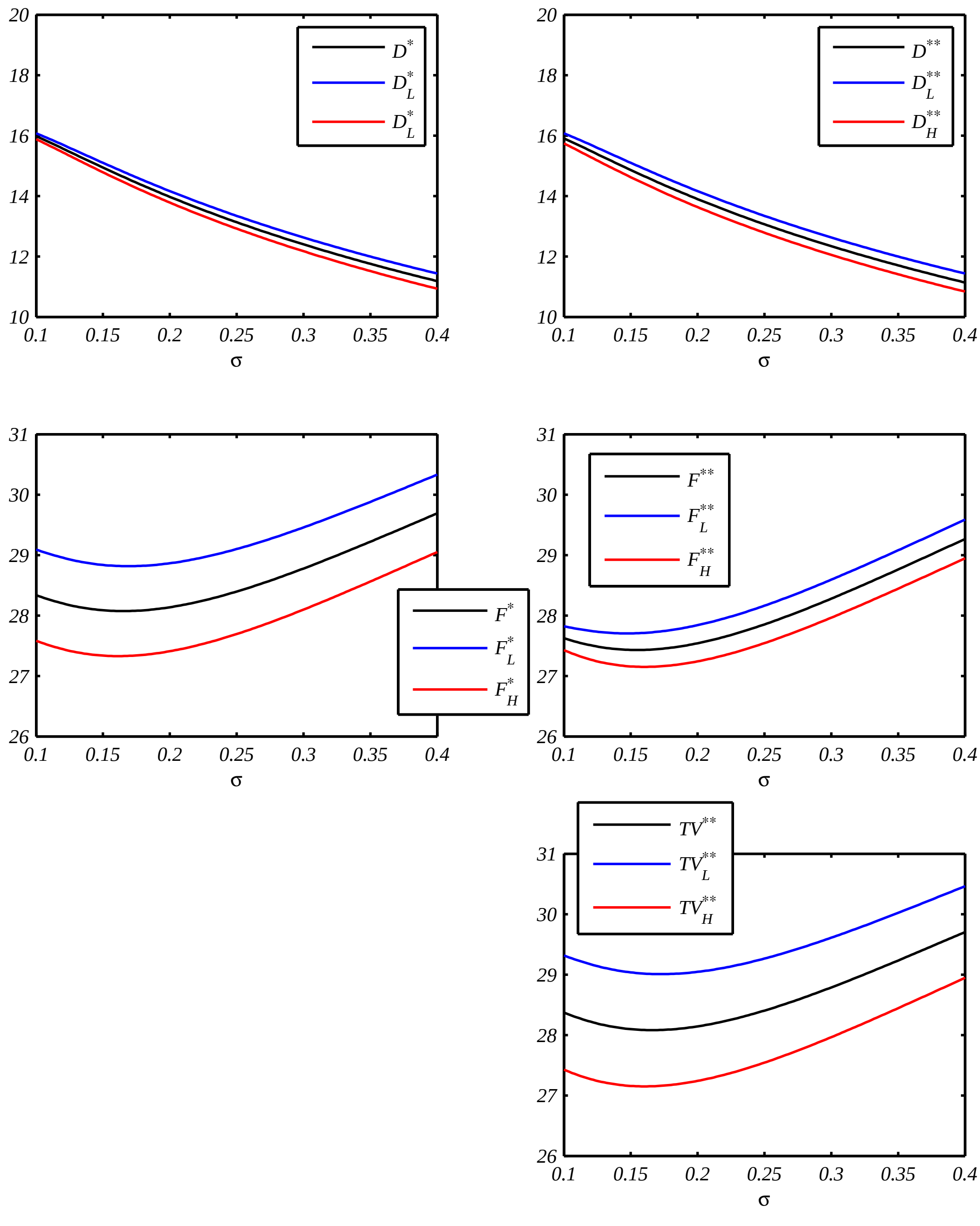

Figure 3: Comparative statics with respect to σ in Case (III). The other parameter values are set at Table 2. The left-hand and right-hand panels show the results in the symmetric and asymmetric information cases, respectively. Note that in the symmetric information case we omit the total values, which are exactly equal to the firm values.