SHORT SELLING WITH COLLATERAL CONSTRAINTS AND RECALL RISK

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ABSTRACT. Short sales are represented as negative purchases in textbook asset pricing theory. In reality, however, the symmetry between purchasing and short selling an asset is broken by a variety of costs and risks that are peculiar to short sales. We develop a theoretical model in which the decision to cover a short position is the solution to an optimal stopping problem, and where short selling is subject to collateral constraints and recall risk. We analyse the effects of these short sale-specific frictions on the optimal strategy, and quantify the loss of value suffered by the short seller due to each.

1. INTRODUCTION

Short sales allow investors to obtain negative exposures to assets they do not own, by borrowing and selling them. The salient features of a short sale in the equity market may be summarised as follows.[1] First, a prospective short seller must identify a willing lender of the desired stock. He then borrows the stock, sells it, and posts collateral equal to its market value plus a haircut (usually 2% of its value) with the lender. The collateral is marked to market daily, so that an increase in the stock price prompts a margin call for more collateral, while a decrease entitles the short seller to withdraw some collateral. The lender invests the collateral at the prevailing interest rate, and pays part of the resulting income to the short seller, in the form of a negotiated rebate (the difference between the interest rate and the rebate rate represents the lending fee). In return, the short seller compensates the lender for all dividends forgone during the life of the loan. The transaction is completed when the short seller repurchases the stock, and returns it to the lender. Although this is likely to occur at the short seller's discretion, stock loan contracts generally include a recall clause that allows lenders to force short sellers to liquidate their positions involuntarily.

The previous overview highlights several costs and risks associated with short sales, which have no counterparts when securities are purchased. First, short sellers incur *search costs*, since the process of identifying willing security lenders can be costly. Second, the *lending fees* associated with security loans impose a cost on short sellers that is often quite significant. Third, short sellers are exposed to *margin risk*, due to the possibility that successive increases in the prices of borrowed securities may generate margin calls that eventually exhaust their collateral budgets. Finally, short sellers face *recall risk*, since lenders may recall borrowed securities at any time. Collectively, these frictions are referred to as *short selling constraints*.

Short selling constraints can have dramatic implications for textbook no-arbitrage relationships, since arbitrage portfolios invariably contain long and short positions. Lamont and Thaler (2003) examined the role of short selling constraints in preventing

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[1]See Cohen et al. (2004), D'Avolio (2002), and Reed (2013) for comprehensive overviews of the short sale process and the equity lending market.

arbitrageurs from exploiting mispriced equity carve-outs, while Mitchell et al. (2002) highlighted their effect in situations when companies trade at discounts relative to their subsidiaries. Short selling constraints have also been implicated in mispriced equity index futures by Fung and Draper (1999), and in put-call parity violations by Ofek et al. (2004).

Our study focusses on two specific short selling constraints, namely, margin risk and recall risk. We aim to quantify the loss of value suffered by a short seller due to these frictions, in the setting of a optimal stopping problem where a trader with an existing short position must choose the optimal time close-out his position. To capture the effect of margin risk, we assume that the trader only has a finite collateral budget to fund margin calls, while recall risk is modelled by assuming that forced close-out of his position occurs at some independent random time. We provide a detailed analysis of how these constraints affect the value of the trader's position, and his optimal strategy.

Shleifer and Vishny (1997) were the first to recognise the importance of margin risk. They demonstrated that hedge funds may be forced to close-out theoretically profitable arbitrage trades under loss-making circumstances, if interim losses trigger margin calls and investor withdrawals, since withdrawals exascerbate collateral constraints. Liu and Longstaff (2004) reinforced the intuition that margin risk detracts from the attractiveness of arbitrage, by showing that a collateral-constrained risk-averse trader will underinvest in arbitrage opportunities, due to the possibility that interim losses may exhaust his collateral before he realises a profit.

The significance of recall risk was recently highlighted by Chuprinin and Ruf (2015), who estimated that a perfectly informed short seller loses around 20% of his first-best profits due to recalls. Chung and Tanaka (2015) studied an optimal stopping problem in which a trader must choose the optimal time to close-out a short position, subject to recalls and lending fees. They obtained closed-form expressions for the short-seller's value function and optimal strategy, and showed that the possibility of recall leads to a substantial reduction in value.

Our optimal stopping model for short selling shares many features with optimal stopping formulations of the related problem of when to close-out a margin loan (see e.g. Cai and Sun 2014, Dai and Xu 2011, Grasselli and Gómez 2013, Liang et al. 2010, Liu and Xu 2010, Wong and Wong 2013, Xia and Zhou 2007). However, this modelling approach appears to have been applied to the problem of short selling under constraints only by Chung and Tanaka (2015).

The remainder of our paper is structured as follows. First, Section 2 presents the technical formalities, and formulates the problem of closing-out a short position under collateral constraints and recall risk as an optimal stopping problem. In Section 3 we formulate and solve a free-boundary problem for an associated optimal stopping problem,

2. Short Selling as an Optimal Stopping Problem

Fix $I := (0, \infty)$, and suppose the continuous functions $a : I \to (0, \infty)$ and $b : I \to \mathbb{R}$ satisfy the local integrability condition

$$\forall x \in I, \quad \exists \varepsilon > 0 \quad \text{s.t.} \quad \int_{x-\varepsilon}^{x+\varepsilon} \frac{1 + |b(y)|}{a^2(y)} \, \mathrm{d}y < \infty. \tag{2.1}$$

In that case, the stochastic differential equation

$$\mathrm{d}X_t = b(X_t)\,\mathrm{d}t + a(X_t)\,\mathrm{d}B_t, \tag{2.2}$$

for all $t \geq 0$, has a weak solution $(X, B) = (X_t, B_t)_{t\geq 0}$ that takes values in I, and which is unique (in the sense of probability law) until the explosion time of X (i.e. the first time it leaves I). Here the process B is a standard Brownian motion on a complete filtered probability space $(\Omega, \mathscr{F}, \mathfrak{F}, \mathsf{P})$, whose filtration $\mathfrak{F} = (\mathscr{F}_t)_{t\geq 0}$ satisfies the *usual conditions* of right continuity and completion by the null-sets of $\mathscr{F}$.[2]

Let $\mathfrak{S}$ denote the family of all stopping times, with respect to the filtration $\mathfrak{F}$. These include the first-exit times

$$\hat{\tau}_z := \inf\{t \geq 0 \,|\, X_t \geq z\} \qquad \text{and} \qquad \check{\tau}_z := \inf\{t \geq 0 \,|\, X_t \leq z\}, \tag{2.3}$$

for all $z \in I$. The first-passage times of X to the upper and lower boundaries of I are then given by

$$\hat{\tau}_\infty := \lim_{n\uparrow\infty} \hat{\tau}_n \qquad \text{and} \qquad \check{\tau}_0 := \lim_{n\uparrow\infty} \check{\tau}_{1/n},$$

respectively. Using this notation, the explosion time of X may be written as $\check{\tau}_0 \wedge \hat{\tau}_\infty$. Finally, we assume that X is absorbed at the origin, if it is accessible. That is to say, we require $\mathbf{1}_{\{\check{\tau}_0 \leq t\}} X_t = 0$, for all $t \in \mathbb{R}_+$.

Given $\alpha > 0$, let $\phi_\alpha, \psi_\alpha \in C^2(I)$ denote the unique (up to multiplication by a positive scalar) decreasing and increasing solutions, respectively, to the second-order ordinary differential equation

$$\frac{1}{2}a^2(x)f''(x) + b(x)f'(x) = \alpha f(x), \tag{2.4}$$

for all $x \in I$. The Laplace transforms of the first-exit times (2.3) have the following representations in terms of these functions:

$$\mathsf{E}_x\big(\mathrm{e}^{-\alpha\hat{\tau}_z}\big) = \frac{\psi_\alpha(x)}{\psi_\alpha(z)} \qquad \text{if } x \leq z,$$

and

$$\mathsf{E}_x\big(\mathrm{e}^{-\alpha\check{\tau}_z}\big) = \frac{\phi_\alpha(x)}{\phi_\alpha(z)} \qquad \text{if } x \geq z,$$

for all $\alpha > 0$ and $x, z \in I$, where $\mathsf{E}_x(\,\cdot\,)$ denotes the expected value operator with respect to the probability measure P_x, under which $X_0 = x$.

We consider a trader with collateral budget $c \geq 0$, who is short a non-dividend-paying stock, with price X. Forced close-out of his position, due to collateral exhaustion, occurs when the current price of the stock first exceeds the price at which he sold it by more than his budgeted collateral. We denote this stopping time by

$$\zeta := \inf\{t \geq 0 \,|\, X_t = X_0 + c\}. \tag{2.5}$$

Involuntary close-out of the trader's position, due to the stock being recalled, occurs at an exponentially distributed random time $\sigma \sim \mathrm{Exp}(\lambda)$, where $\lambda > 0$ specifies the recall intensity. We assume that σ is $\mathscr{F}$-measurable and independent of $\mathscr{F}_\infty := \bigvee_{t\geq 0} \mathscr{F}_t$, which implies that the recall event is independent of the stock price.

[2] See Karatzas and Shreve (1991, Definition 5.5.20) for a definition of the concept of a weak solution for a stochastic differential equation. Karatzas and Shreve (1991, Theorem 5.5.15) shows that the local integrability condition (2.1) implies that (2.2) admits a unique weak solution until explosion.

Having initiated the short sale, the trader's objective is to identify the close-out time that maximises his expected discounted profit, using a subjective discount rate $\rho \geq 0$, subject to the frictions outlined above. This gives rise to the optimal stopping problem

$$V(x) := \sup_{\tau \in \mathfrak{S}} \mathsf{E}_x\big(\mathrm{e}^{-\rho(\tau\wedge\zeta\wedge\sigma)}(x - X_{\tau\wedge\zeta\wedge\sigma})\big), \tag{2.6}$$

for all $x > 0$.

3. An Associated Free-Boundary Problem

Continuous-time optimal stopping problems are traditionally solved by first reformulating them as free-boundary problems. Unfortunately, this approach cannot be applied directly to (2.6). Instead, we proceed by embedding it in a more general optimal stopping problem, which can be associated with a free-boundary problem. To do so, we fix a new parameter $\kappa \in I$, and introduce the optimal stopping problem

$$\widetilde{V}(x) := \sup_{\tau \in \mathfrak{S}} \mathsf{E}_x\Big(\mathrm{e}^{-\rho(\tau\wedge\hat{\tau}_{\kappa+c}\wedge\sigma)}\big(\kappa - X_{\tau\wedge\hat{\tau}_{y+c}\wedge\sigma}\big)\Big), \tag{3.1}$$

for all $x \in I$. Note that $\zeta = \hat{\tau}_{x+c}$ P_x-a.s., for all $x \in I$, from which it follows that $V(x) = \widetilde{V}(x)$, if $\kappa = x$. Consequently, a solution for (3.1) yields a solution for (2.6), as a special case.

By virtue of the time-homogeneity of (3.1), we expect the optimal stopping time to be a threshold time. This leads us to speculate that

$$\begin{aligned}
\widetilde{V}(x) &= \sup_{z\geq 0} \mathsf{E}_x\Big(\mathrm{e}^{-\rho(\check{\tau}_z\wedge\hat{\tau}_{\kappa+c}\wedge\sigma)}\big(\kappa - X_{\check{\tau}_z\wedge\hat{\tau}_{y+c}\wedge\sigma}\big)\Big)\\
&= \mathsf{E}_x\Big(\mathrm{e}^{-\rho(\check{\tau}_{z_*}\wedge\hat{\tau}_{\kappa+c}\wedge\sigma)}\big(\kappa - X_{\check{\tau}_{z_*}\wedge\hat{\tau}_{y+c}\wedge\sigma}\big)\Big)\\
&= \int_0^\infty \lambda \mathrm{e}^{-\lambda t}\overline{V}(t,x)\,\mathrm{d}t,
\end{aligned} \tag{3.2}$$

for all $x \in I$, where $z_* \geq 0$ is the optimal threshold, and

$$\overline{V}(t,x) := \mathsf{E}_x\Big(\mathrm{e}^{-\rho(\check{\tau}_{z_*}\wedge\hat{\tau}_{\kappa+c}\wedge t)}\big(\kappa - X_{\check{\tau}_{z_*}\wedge\hat{\tau}_{\kappa+c}\wedge t}\big)\Big),$$

for all $(t,x) \in \mathbb{R}_+ \times I$. The latter function satisfies the partial differential equation

$$\frac{1}{2}a^2(x)\frac{\partial^2\overline{V}}{\partial x^2}(t,x) + b(x)\frac{\partial\overline{V}}{\partial x}(t,x) - \rho\overline{V}(t,x) = \frac{\partial\overline{V}}{\partial t}(t,x), \tag{3.3a}$$

for all $(t,x) \in \mathbb{R}_+ \times I$, subject to the boundary conditions

$$\overline{V}(t,z_*) = \kappa - z_* \qquad \text{and} \qquad \overline{V}(t,\kappa+c) = -c, \tag{3.3b}$$

for all $t \in \mathbb{R}_+$, and the initial condition

$$\overline{V}(0,x) = \kappa - x, \tag{3.3c}$$

for all $x \in I$.

The Laplace-Carson transform of a suitably integrable function $f : \mathbb{R}_+ \to \mathbb{R}$ is given by

$$\mathscr{L}_\alpha\{f\} := \int_0^\infty \alpha\mathrm{e}^{-\alpha t}f(t)\,\mathrm{d}t,$$

where $\alpha > 0$. Using this notation, we may write (3.2) as

$$\widetilde{V}(x) = \mathscr{L}_\lambda\big\{\overline{V}(\,\cdot\,,x)\big\},$$

for all $x \in I$. This suggests that the free-boundary value problem corresponding to (3.2) can be derived by applying Laplace-Carson transforms to (3.3a)–(3.3c). To begin with, applying the Laplace-Carson transform to (3.3a) yields

$$\begin{aligned}\mathscr{L}_\lambda\left\{\frac{1}{2}a^2(x)\frac{\partial^2\overline{V}}{\partial x^2}(\,\cdot\,,x)+b(x)\frac{\partial\overline{V}}{\partial x}(\,\cdot\,,x)-\rho\overline{V}(\,\cdot\,,x)\right\}&=\mathscr{L}_\lambda\left\{\frac{\partial\overline{V}}{\partial t}(\,\cdot\,,x)\right\}\\&=\lambda\big(\mathscr{L}_\lambda\{\overline{V}(\,\cdot\,,x)\}-\overline{V}(0,x)\big),\end{aligned}$$

for all $x \in I$, where the second equality is an application of integration by parts. Exchanging derivatives and integrals in the above equation, and applying the initial condition (3.3c), then gives

$$\frac{1}{2}a^2(x)\widetilde{V}''(x)+b(x)\widetilde{V}'(x)-(\rho+\lambda)\widetilde{V}(x)+\lambda(\kappa-x)=0, \tag{3.4a}$$

for all $x \in I$. Next, an application of the Laplace-Carson transform to the boundary conditions in (3.3b) yields

$$\widetilde{V}(\kappa+c)=\mathscr{L}_\lambda\big\{\overline{V}(\,\cdot\,,\kappa+c)\big\}=\mathscr{L}_\lambda\{-c\}=-c, \tag{3.4b}$$

and

$$\widetilde{V}(z_*)=\mathscr{L}_\lambda\big\{\overline{V}(\,\cdot\,,z_*)\big\}=\mathscr{L}_\lambda\{\kappa-z_*\}=\kappa-z_*. \tag{3.4c}$$

Finally, we impose the customary smooth pasting condition at the optimal stopping boundary, to get

$$\widetilde{V}'(z_*)=-1. \tag{3.4d}$$

Together, (3.4a)–(3.4d) specify the free-boundary problem associated with the optimal stopping problem (3.1), under the assumption that the optimal stopping policy is a threshold rule.

By comparing the inhomogeneous equation (3.4a) with the homogeneous equation (2.4), we observe that the general solution to the inhomogeneous equation takes the form

$$\widetilde{V}(x)=A\phi_{\rho+\lambda}(x)+B\psi_{\rho+\lambda}(x)+v(x),$$

for all $x \in I$, where $A, B \in \mathbb{R}$ and $v \in C^2(I,\mathbb{R})$ is a particular solution. Substituting this expression into (3.4b) and (3.4c), and solving for A and B, gives

$$\begin{aligned}\widetilde{V}(x)=\big(\kappa-z_*-v(z_*)\big)&\frac{\phi_{\rho+\lambda}(\kappa+c)\psi_{\rho+\lambda}(x)-\phi_{\rho+\lambda}(x)\psi_{\rho+\lambda}(\kappa+c)}{\phi_{\rho+\lambda}(\kappa+c)\psi_{\rho+\lambda}(z_*)-\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(\kappa+c)}\\&+\big(c+v(\kappa+c)\big)\frac{\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x)-\phi_{\rho+\lambda}(x)\psi_{\rho+\lambda}(z_*)}{\phi_{\rho+\lambda}(\kappa+c)\psi_{\rho+\lambda}(z_*)-\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(\kappa+c)}+v(x),\end{aligned} \tag{3.5}$$

for all $x \in I$. Substituting this expression above into (3.4d) yields the equation

$$\begin{aligned}\big(\kappa-z_*-v(z_*)\big)&\frac{\phi_{\rho+\lambda}(\kappa+c)\psi'_{\rho+\lambda}(z_*)-\phi'_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(\kappa+c)}{\phi_{\rho+\lambda}(\kappa+c)\psi_{\rho+\lambda}(z_*)-\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(\kappa+c)}\\&+\big(c+v(\kappa+c)\big)\frac{\phi_{\rho+\lambda}(z_*)\psi'_{\rho+\lambda}(z_*)-\phi'_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(z_*)}{\phi_{\rho+\lambda}(\kappa+c)\psi_{\rho+\lambda}(z_*)-\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(\kappa+c)}+v'(z_*)=-1,\end{aligned} \tag{3.6}$$

which implicitly characterizes the optimal stopping boundary z_*.

Our final task is to find an expression for the particular solution v. We begin by recalling that the resolvent operator $\mathscr{R}_\alpha$, for $\alpha > 0$, associated with X acts on suitably integrable functions $g : I \to \mathbb{R}$ as follows:

$$(\mathscr{R}_\alpha g)(x) := \mathsf{E}_x\biggl(\int_0^\infty \mathrm{e}^{-\alpha t} g(X_t)\,\mathrm{d}t\biggr),$$

for all $x \in I$. Moreover, $\mathscr{R}_\alpha g$ satisfies the ordinary differential equation

$$\frac{1}{2}a^2(x)(\mathscr{R}_\alpha g)''(x) + b(x)(\mathscr{R}_\alpha g)'(x) - \alpha(\mathscr{R}_\alpha g)(x) + g(x) = 0,$$

for all $x \in I$. Comparing this equation with (3.4a) gives the following expression for the particular solution to that equation:

$$\begin{aligned} v(x) := \bigl(\mathscr{R}_{\rho+\lambda}\lambda(\kappa - \,\cdot\,)\bigr) &= \mathsf{E}_x\biggl(\int_0^\infty \mathrm{e}^{-(\rho+\lambda)t}\lambda(\kappa - X_t)\,\mathrm{d}t\biggr) \\ &= \frac{\lambda\kappa}{\rho+\lambda} - \int_0^\infty \lambda\mathrm{e}^{-(\rho+\lambda)t}\mathsf{E}_x(X_t)\,\mathrm{d}t, \end{aligned} \tag{3.7}$$

for all $x \in I$.

Putting (3.5)–(3.7) together gives the value function $\widetilde{V}$ for the optimal stopping problem (3.2), with the optimal stopping time $\check{\tau}_{z_*}$ determined by (3.6). Replacing κ with x in these expressions gives the value function V for the original optimal stopping problem (2.6) and the optimal stopping time $\check{\tau}_{z_*}$, subject to the assumption that the optimal stopping policy is of threshold type. In particular, if the solution to (3.2) is determined by a threshold strategy, then the value function V for that problem is given by

$$\begin{aligned} V(x) = \bigl(x - z_* - v(z_*)\bigr)&\frac{\phi_{\rho+\lambda}(x+c)\psi_{\rho+\lambda}(x) - \phi_{\rho+\lambda}(x)\psi_{\rho+\lambda}(x+c)}{\phi_{\rho+\lambda}(x+c)\psi_{\rho+\lambda}(z_*) - \phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x+c)} \\ &+ \bigl(c + v(x+c)\bigr)\frac{\phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x) - \phi_{\rho+\lambda}(x)\psi_{\rho+\lambda}(z_*)}{\phi_{\rho+\lambda}(x+c)\psi_{\rho+\lambda}(z_*) - \phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x+c)} + v(x), \end{aligned} \tag{3.8}$$

for all $x \in I$, while the equation

$$\begin{aligned} \bigl(x - z_* - v(z_*)\bigr)&\frac{\phi_{\rho+\lambda}(x+c)\psi'_{\rho+\lambda}(z_*) - \phi'_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x+c)}{\phi_{\rho+\lambda}(x+c)\psi_{\rho+\lambda}(z_*) - \phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x+c)} \\ &+ \bigl(c + v(x+c)\bigr)\frac{\phi_{\rho+\lambda}(z_*)\psi'_{\rho+\lambda}(z_*) - \phi'_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(z_*)}{\phi_{\rho+\lambda}(x+c)\psi_{\rho+\lambda}(z_*) - \phi_{\rho+\lambda}(z_*)\psi_{\rho+\lambda}(x+c)} + v'(z_*) = -1, \end{aligned} \tag{3.9}$$

which determines the optimal threshold z_*.

4. An Example

This section presents a detailed analysis of the case when the stock price follows a geometric Brownian motion, with $a(x) := \sigma x$ and $b(x) := \mu x$, for all $x \in I$, where $\sigma \in (0, \infty)$ and $\mu \in \mathbb{R}$. In that case, given $\alpha > 0$, the fundamental solutions to (2.4) are given by

$$\phi_\alpha(x) := x^{-\sqrt{\nu^2 + 2\frac{r+\lambda}{\sigma^2}} - \nu} \qquad \text{and} \qquad \psi_\alpha(x) := x^{\sqrt{\nu^2 + 2\frac{r+\lambda}{\sigma^2}} - \nu},$$

for all $x \in I$, where

$$\nu := \frac{\mu}{\sigma^2} - \frac{1}{2}.$$

We perform a comparative static analysis, by comparing the trader's value function (3.8) and optimal strategy (3.9) with and without the constraints. The unconstrained

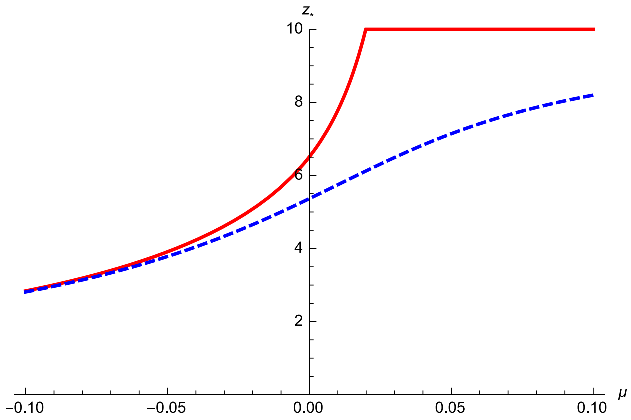

FIGURE 4.1. The optimal threshold z_* as a function of the drift rate μ, with constraints (solid red) and without constraints (blue dashed)

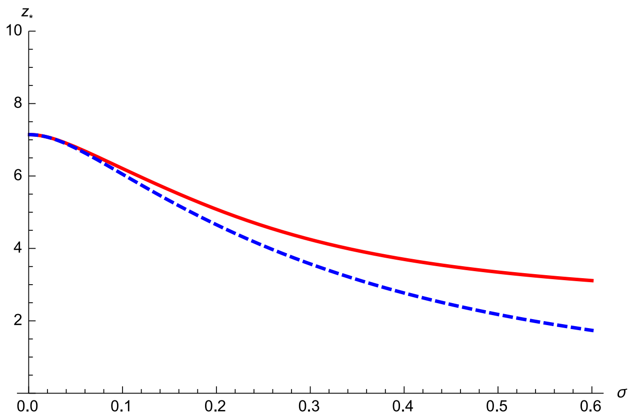

FIGURE 4.2. The optimal threshold z_* as a function of the volatility σ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

value function and optimal strategy are obtained by setting $c \uparrow \infty$ and $\lambda = 0$ in those expressions. Unless specified otherwise, we use the following parameter values: $x = 10$, $\mu = -0.02$ or $\mu = 0.02$, $\sigma = 0.2$, $\rho = 0.05$, $c = 75$, and $\lambda = 0.05$.

Figures 4.1–4.9 compare the optimal threshold for closing out the short position when the trader has limited collateral and recall is possible (solid red lines) with the optimal threshold when the trader's collateral is infinite and recall is impossible.

Figures 4.10–4.18 compare the optimal threshold for closing out the short position when the trader has limited collateral and recall is possible (solid red lines) with the optimal threshold when the trader's collateral is infinite and recall is impossible.

REFERENCES

Cai, N. and L. Sun (2014). Valuation of stock loans with jump risk. *J. Econ. Dynam. Control 40*, 213–241.

Chung, T.-K. and K. Tanaka (2015). Optimal timing for short covering of an illiquid security. *J. Oper. Res. Soc. Japan 58*(2), 165–183.

Chuprinin, O. and T. Ruf (2015). When pessimism doesn't pay off: Determinants and implications of stock recalls in the short selling market. Working paper.

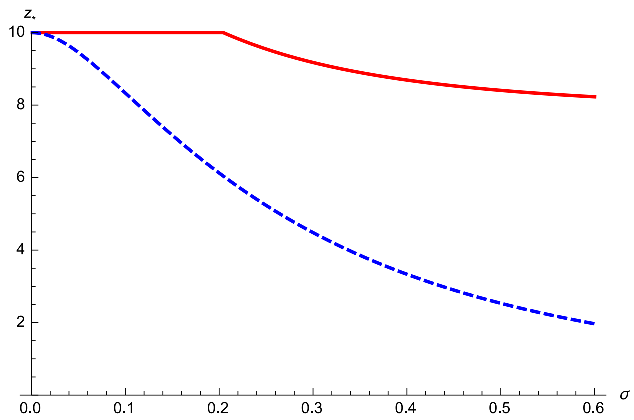

FIGURE 4.3. The optimal threshold z_* as a function of the volatility σ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

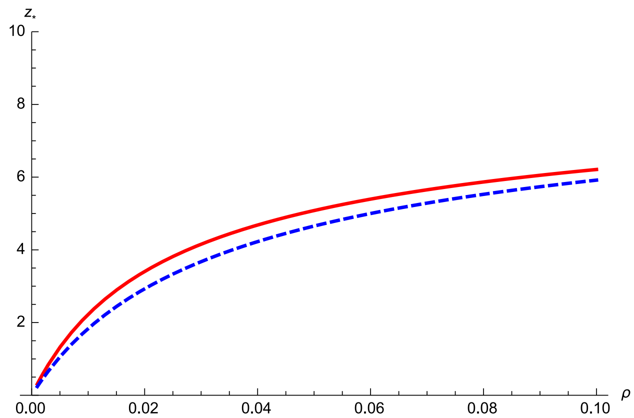

FIGURE 4.4. The optimal threshold z_* as a function of the cost of capital ρ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

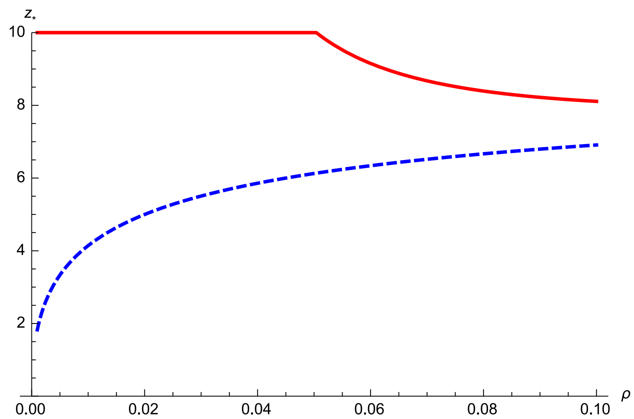

FIGURE 4.5. The optimal threshold z_* as a function of the cost of capital ρ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

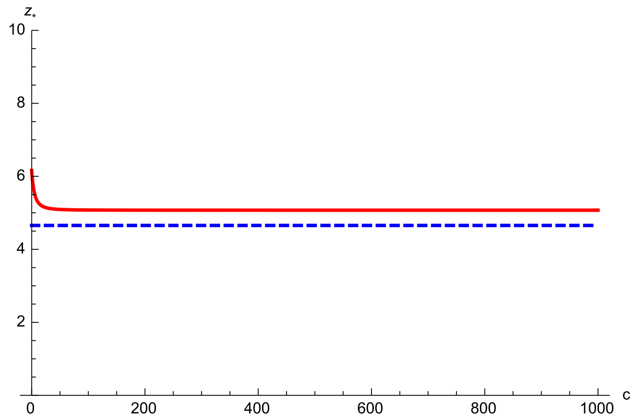

FIGURE 4.6. The optimal threshold z_* as a function of collateral c when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

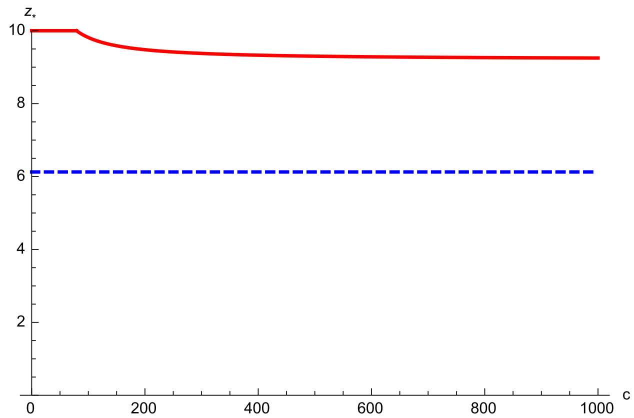

FIGURE 4.7. The optimal threshold z_* as a function of collateral c when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

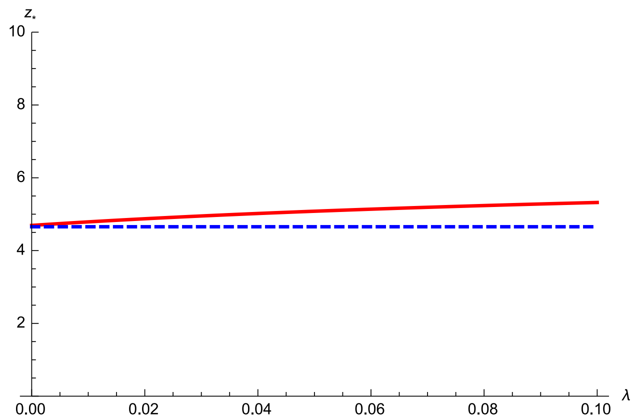

FIGURE 4.8. The optimal threshold z_* as a function of the recall intensity λ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

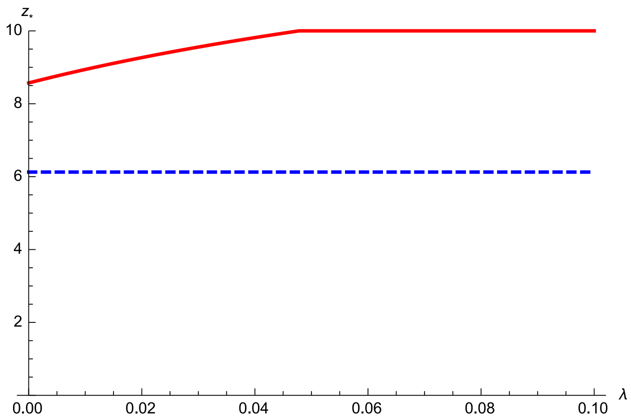

FIGURE 4.9. The optimal threshold z_* as a function of the recall intensity λ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

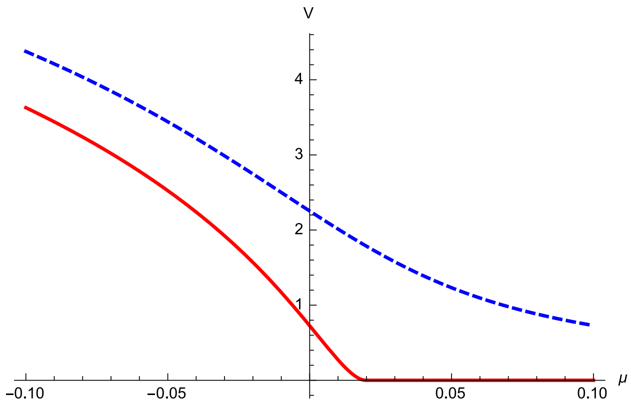

FIGURE 4.10. The value V as a function of the drift rate μ, with constraints (solid red) and without constraints (blue dashed)

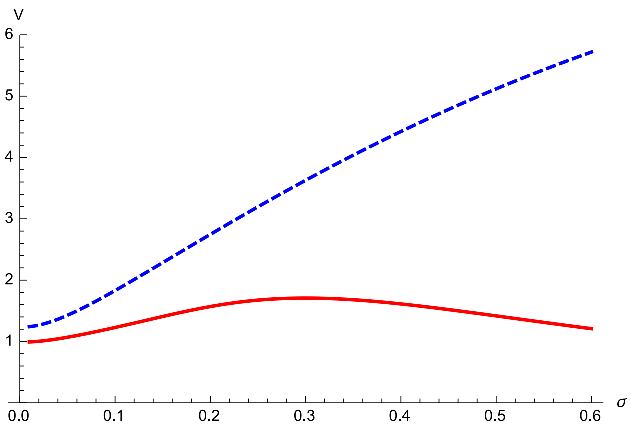

FIGURE 4.11. The value V as a function of the volatility σ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

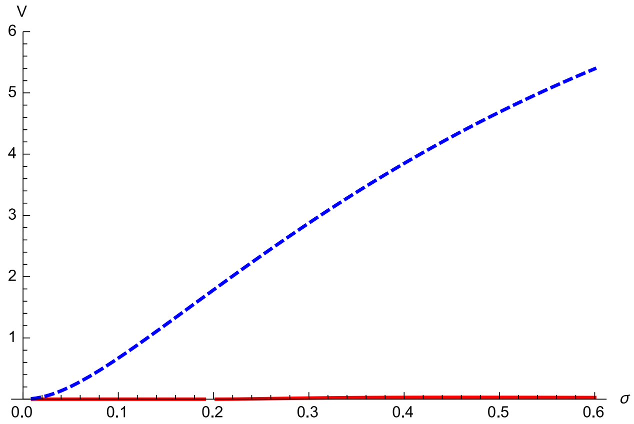

FIGURE 4.12. The value V as a function of the volatility σ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

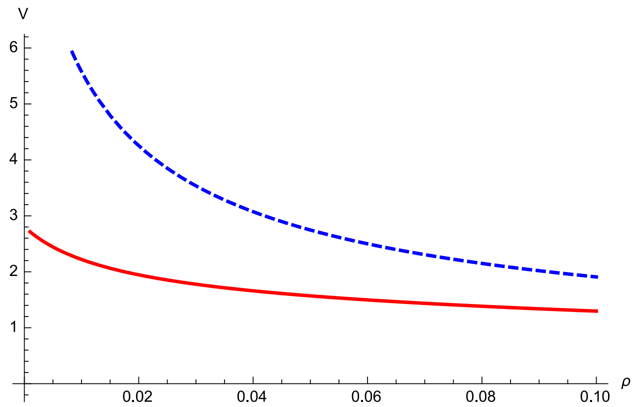

FIGURE 4.13. The value V as a function of the cost of capital ρ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

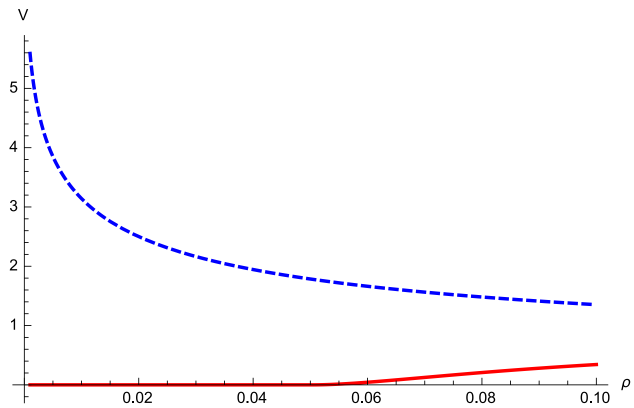

FIGURE 4.14. The value V as a function of the cost of capital ρ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

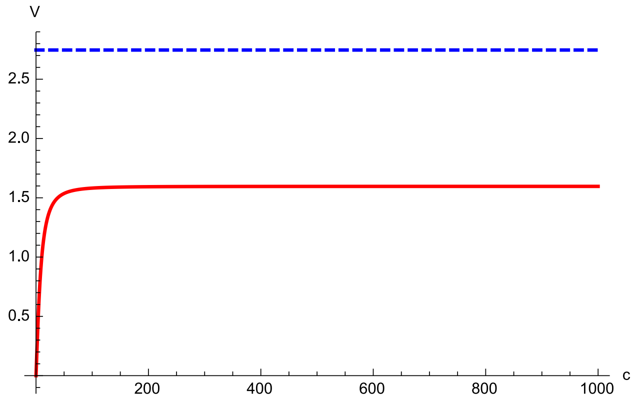

FIGURE 4.15. The value V as a function of the collateral c when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

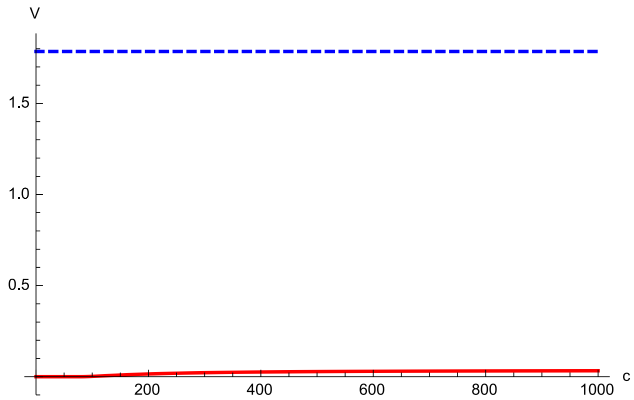

FIGURE 4.16. The value V as a function of the collateral c when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

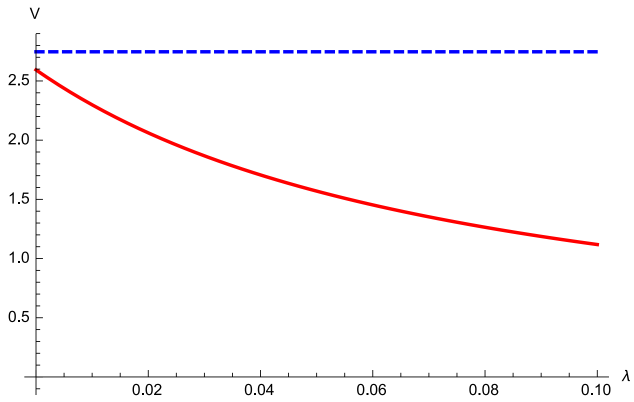

FIGURE 4.17. The value V as a function of the recall intensity λ when $\mu = -0.02$, with constraints (solid red) and without constraints (blue dashed)

Cohen, J., D. Haushalter, and A. V. Reed (2004). Mechanics of the equity lending market. In F. J. Fabozzi (Ed.), *Short Selling: Strategies, Risks, and Rewards*, Chapter 2, pp. 9–16. Hoboken, New Jersey: John Wiley and Sons.

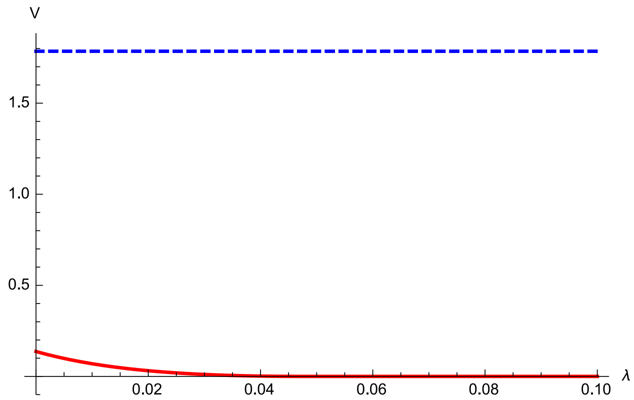

FIGURE 4.18. The value V as a function of the recall intensity λ when $\mu = 0.02$, with constraints (solid red) and without constraints (blue dashed)

Dai, M. and Z. Q. Xu (2011). Optimal redeeming strategy of stock loans with finite maturity. *Math. Finance 21*(4), 775–793.

D'Avolio, G. (2002). The market for borrowing stock. *J. Finan. Econ. 66*(2–3), 271–306.

Fung, J. K. W. and P. Draper (1999). Mispricing of index futures contracts and short sales constraints. *J. Futures Markets 19*(6), 695–715.

Grasselli, M. R. and C. Gómez (2013). Stock loans in incomplete markets. *Appl. Math. Finance 20*(2), 118–136.

Karatzas, I. and S. E. Shreve (1991). *Brownian Motion and Stochastic Calculus* (Second ed.). New York: Springer.

Lamont, O. A. and R. H. Thaler (2003). Can the market add and subtract? Mispricing in tech stock carve-outs. *J. Pol. Econ. 111*(2), 227–268.

Liang, Z., W. Wu, and S. Jiang (2010). Stock loan with automatic termination clause, cap and margin. *Comput. Math. Appl. 60*(12), 3160–3176.

Liu, G. and Y. Xu (2010). Capped stock loans. *Comput. Math. Appl. 59*(11), 3548–3558.

Liu, J. and F. A. Longstaff (2004). Losing money on arbitrage: Optimal dynamic portfolio choice in markets with arbitrage opportunities. *Rev. Financ. Stud. 17*(3), 611–641.

Mitchell, M., T. Pulvino, and E. Strafford (2002). Limited arbitrage in equity markets. *J. Finance 57*(2), 551–584.

Ofek, E., M. Richardson, and R. F. Whitelaw (2004). Limited arbitrage and short sales restrictions: Evidence from the options markets. *J. Finan. Econ. 74*(2), 305–342.

Reed, A. V. (2013). Short selling. *Annu. Rev. Financ. Econ. 5*, 245–258.

Shleifer, A. and R. W. Vishny (1997). The limits of arbitrage. *J. Finance 52*(1), 35–55.

Wong, T. W. and H. Y. Wong (2013). Valuation of stock loans using exponential phase-type lévy models. *Appl. Math. Comput. 222*, 275–289.

Xia, J. and X. Y. Zhou (2007). Stock loans. *Math. Finance 17*(2), 307–317.

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