VIX and SKEW Indices for SPX and VIX Options

Zhiguang Wang

Assistant Professor of Economics

South Dakota State University

Robert T. Daigler *

Knight Ridder Research Professor of Finance

Florida International University

JEL category: G13 Contingent Pricing

Keywords: Skew Index, VIX of VIX, Vol of Vol

Zhiguang Wang is Assistant Professor, South Dakota State University, Department of Economics, SSB118, Box 504, Brookings, SD 57007. Email: zhiguang.wang@sdstate.edu. Phone: 605-688-4861

Robert Daigler is Knight Ridder Research Professor of Finance, Florida International University, Department of Finance RB 206, College of Business, Miami, FL 33199. Email: daiglerr@fiu.edu. Phone: 305-348-3325 (or home 954-434-2412).

* Corresponding author

Current Draft: 5/6/2014

VIX and SKEW Indices for SPX and VIX Options

ABSTRACT

The CBOE "SKEW" and "VVIX" indices respectively measure the implied volatility skew of SPX options and the implied volatility of VIX options (the volatility of volatility). We compute intraday values of the SKEW index for SPX (SKEWSPX) and VIX options (SKEWVIX) as well as the VVIX, and then determine the empirical characteristics of these factors during the Fall 2008 financial crisis. After testing for unit roots and cointegration, we determine that changes in the VIX possess a bidirectional Granger causality with changes in the SKEWSPX; moreover, changes in the SKEWSPX leads changes in the SKEWVIX, and the SKEWVIX leads changes in the VVIX. We also confirm the positive relation between changes in the VIX and VVIX, document strong asymmetric GARCH effects for the VVIX and SKEWSPX, and symmetric GARCH effects for the SKEWVIX. Overall, we find evidence supporting the bidirectional information flow between SPX and VIX options markets.

During the past decade the CBOE has added a substantial number of new volatility based indices and tradable products to improve our knowledge of the behavior and characteristics of volatility. The most recent developments in this implied volatility information escalation are the addition of the SKEW index in 2011 and the VVIX index in 2012. The SKEW provides an indirect measure of the "slope" of the implied volatility curve, showing to what extent farther-out-of-the-money (put) options affect the value of the VIX as well as provide evidence of the fear of major downward potential movements in the S&P500 index. Thus, the SKEW (modeled after Bakshi, Kapadia, and Madan's (2003) concept of measuring the implied volatility curve) can be viewed as the market's estimate of a Black Swan (tail risk) event. The VVIX index, the VIX index for VIX options, provides a measure of the "fear of fear" in the market, another type of tail risk. The existence of the SKEW and VVIX indices provide information not previously available to examine how the market views future risk and tail events.

The VVIX and SKEW indices can help us understand market behavior during the financial crisis of 2008. In particular, the effect of the sub-prime crisis in late 2008 highlighted the importance of hedging against tail risk, as measured by the skewness and kurtosis of the market. Thus, understanding the behavior of the stochastic volatility of volatility and the implied volatility surface is essential to the pricing and hedging of VIX options and futures, in the same way as the implied volatility surface is essential to understanding SPX options. In fact, most dealers and traders use implied volatility surfaces to price and trade options. Therefore, an in-depth empirical analysis of the risk-neutral measures derived from SPX and VIX options can help uncover the information flow between these two markets. Furthermore, the understanding of the volatility surfaces of the SPX and VIX can lead to better models to price SPX and VIX derivatives jointly, which has been the focus of the recent literature (Cont and Kokholm 2013; Lin and Chang, 2010).

The literature has yet to show a dedicated interest in examining the issues of the volatility of volatility and the implied skewness of the S&P500 option series, especially in terms of intraday data.[1] Corsi et al. (2008) is an exception to the paucity of research in this area, as they employ the typical five-minute intraday time interval to determine and model the daily realized volatility of volatility for underlying cash assets. They use a GARCH-based realized vol of vol to show that realized vol of vol is time-varying and clusters on the daily interval.

This paper examines the characteristics and econometric relations among the VIX, VVIX, the SKEW index for the SPX (henceforth "SKEWSPX"), and the SKEW for VIX options (henceforth "SKEWVIX"). Our results provide a new perspective on the information provided

[1] Whereas little research exists on the vol of vol, especially using intraday data, a great deal of research does exist for high-frequency volatility data for equity data. For example, the "realized" volatility research typically involves using five-minute data to examine the behavior of intraday volatility or daily volatility measured from intraday data: See Andersen, Bollerslev, Diebold and Labys (2003); Barndorff-Nielsen and Shephard, (2002); Meddahi, (2002); Aït-Sahalia, Mykland and Zhang (2005), among others.

by index option prices. In particular, we concentrate on the Fall 2008 financial crisis period and compute the VVIX, SKEWSPX, and SKEWVIX indices at the 15-minute interval for our empirical analysis, using the CBOE methodology (or equivalently the theoretical approach developed by Bakshi et al. 2003).

We first investigate the Granger causality among the VIX, VVIX, SKEWSPX and SKEWVIX, using both pair-wise vector autoregression (VAR) and system-wide vector error correction (VECM) models. The following chain is found: ΔVIX ←→ ΔSKEWSPX → ΔSKEWVIX → ΔVVIX. We then study the factors that affect the VVIX, SKEWSPX and SKEWVIX using ordinary regression with EGARCH effect in the residuals, and quantile regressions to account for the potential bias in the Ordinary Least Squares (OLS) estimate. We document strong volatility clustering in the change in the VVIX, SKEWSPX and SKEWVIX, and an asymmetric response of their conditional volatilities to the shocks: a positive shock increases volatility more than a negative shock. We also find the change in the VIX has a dominating and positive effect on the change in the VVIX, linking "fear" in a positive relation to the "fear of fear." Alternatively, most of the changes in the SKEWSPX and the SKEWVIX are explained by their own lags. The overarching evidence regarding the information flow between the SPX and VIX options markets supports the value of the existence of the VIX options market.

Our research contributes to the literature in three areas. First, the intraday analysis of implied volatility of volatility adds to the existing literature of the volatility of realized volatility, alternatively referred to as quadratic variation (Corsi et al. 2008; Bollerslev et al. 2009). Second, Granger causality is established using two different VAR models, which relates to potential arbitrage between SPX and VIX options. Lastly, we provide new evidence for price discovery between the SPX and VIX option markets.

The paper is organized as follows. Section I provides background information on our measures of volatility of volatility and implied skewness. Section II discusses data and the main hypotheses regarding VIX, VVIX, SKEWSPX and SKEWVIX. Section III provides empirical results and tests the hypothesis. Section IV concludes with a direction for future research.

I. Volatility of Volatility and Implied Skewness

We employ a non-parametrical measure of realized and implied volatility of volatility and implied skewness of SPX and VIX options at each 15-minute interval to examine the time series behavior of these measures. The realized volatility of volatility is computed as a 30-day standard deviation of the VIX futures. In order to make this calculation we first compute the volatilities of the nearby and first deferred VIX futures separately, as follows:

$$RVVIXF_T = \sqrt{\sum_t \left(\log \frac{VIXF_{t,T}}{VIXF_{t-1,T}} \right)^2} \tag{1}$$

where $VIXF_{t,T}$ is the closing value for the VIX futures expiring at the future time T and calculated for each 15-minute interval *t*. The nearby ($T=T_1$) and deferred volatilities ($T=T_2$) are then linearly interpolated to obtain a fixed 30-day realized volatility of the VIX futures, calculated as follows:

$$RVVIXF = \sqrt{RVVIXF_{T_1}^2 \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} + RVVIXF_{T_2}^2 \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}} \tag{2}$$

where N_{30}, N_{T_1}, N_{T_2} are time intervals of 30 days, the nearby expiration, and the first deferred expiration.

The 30-day implied volatility of volatility is defined as the VIX of the VIX options at the 15-minute interval, or the VVIX per the CBOE. The 30-day VVIX that matches the time frame of the realized volatility of volatility has the following form:

$$VVIX = \sqrt{IVAR_{T_1}\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}} + IVAR_{T_2}\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}}} \tag{3}$$

where $IVAR_{T_1}$ and $IVAR_{T_2}$ are implied variances of the nearby and the first deferred VIX options. The detailed calculation procedure for $IVAR_T$ can be found in the CBOE white paper for the VIX index.[2]

The implied skewness is obtained by applying the methodology developed by Bakshi et al. (2003) and adapted by the CBOE for the SPX options (abbreviated as "SKEWSPX"), and determined separately here to the VIX options (abbreviated as "SKEWVIX"). The 30-day implied skewness that shares the same time frame with the realized and implied volatility of volatility has the following form:

$$SKEW = 100 - 10 * \left(S_1\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}} + S_2\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}} \right) \tag{4}$$

where S_1 and S_2 are the implied skewness of the nearby and the first deferred SPX or VIX options. The calculation procedure for S_1 and S_2 can be found in the CBOE white paper for the SKEW index.[3]

II. Data and Hypotheses

In order to investigate the behavior of the volatility of volatility during the 2008 crisis period we employ the intraday S&P500 index options, the cash VIX index, and the VIX futures and options instruments from September 2008 to December 2008. We also include the VIX

[2] http://www.cboe.com/micro/vix/vixwhite.pdf

[3] http://www.cboe.com/micro/skew/documents/SKEWwhitepaperjan2011.pdf

options in 2007 and the first eight months of 2008 as the "benchmark" normal period for comparison purposes relative to the financial crisis period of 2008. The minute-by-minute cash VIX and VIX futures values are obtained from TradeStation and the CQG DataFactory, respectively. The quote data for the SPX options and VIX options are from a direct feed from OPRA. The daily interest rate is obtained from the Federal Reserve of St. Louis website.

A. Summary Statistics

Figure I introduces our data by showing the level of both the VIX and SKEWSPX indices during the 1990-2013 time period. The mean and standard deviation for the VIX (SKEWSPX) are 20.2 (117.1) and 9.1(5.5), respectively. Both series show significant randomness as well as the mean reversion characteristic of volatility.

Table I reports the summary statistics for three groups of variables during the 2008 financial crisis. Group I is the cash VIX and its changes, which provides a benchmark for comparison; Group II covers measures of the volatility of volatility, including the realized volatility of the VIX futures (RVVIX) and the implied volatility of volatility using the VIX options (VVIX); Group III covers the SKEWSPX and SKEWVIX. For Groups I and II in Table I two pairs of values (VIX vs. VVIX) and (RVVIX vs. VVIX) provide interesting results. First, on average, the VVIX is almost twice as large as the VIX itself (see the top panel of Figure 2 for details). Second, on average VVIX is lower than RVVIX during the last quarter of 2008, showing a positive volatility for the volatility risk premium during the financial crisis period.[4] As shown in the middle panel of Figure 2, VVIX is significantly smaller than the realized volatility of VIX futures from September through mid-October, although the pattern is reversed in November. This same phenomenon during the financial crisis is documented for the S&P 500

[4] The volatility of volatility risk premium is defined as the (ex-post) realized volatility of VIX futures minus the (ex-ante) implied volatility of the VIX options during the same period.

index by Zhou (2009). This result contrasts with the more common negative volatility risk premium between the implied volatility (VIX) over the realized volatility of the S&P 500 index for both normal and crisis periods (See Bakshi and Kapadia (2003a, 2003b), Carr and Wu (2008), Todorov (2009), and Zhou (2009)).

For Group III we compare SKEWSPX with SKEWVIX based on Table 1 and the bottom panel of Figure 2. SKEWSPX has a minimum value of 104.61 and an average of 134.28 for the Fall of 2008. The SKEW index for the SPX options is consistently above 100, which is consistent with a negative skewness for the S&P 500 return distribution. For our results the SKEWVIX has an average of 96.50 for the Fall of 2008. This value is almost exclusively below 100, which shows a positive skewness for the VIX futures return distribution.

Finally, we compare the implied volatility and SKEW indices for the SPX and VIX option series. Table I shows that VVIX and SKEWSPX are the most and the least volatile, respectively. This finding is related to Bollen and Whaley (2004), who document that the level of implied volatility varies more than the slope of the implied volatility skew.[5]

B. Unit Root and Cointegration Tests

The presence of a unit root has important implications for econometric modeling of volatility and SKEW. Table II reports the unit root test results for VIX, VVIX, SKEWSPX, SKEWVIX and the first difference of these four variables. Five tests are included, namely the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Ng-Perron (NP), Elliott-Rothenberg-

[5] The difference between our results and those in Bollen and Whaley arises from the definition of implied volatility skew (IV slope). Their skew (slope) is based on the volatility difference between two arbitrary moneyness categories (OTM and ATM), whereas the SKEW index emphasized here is a weighted average of OTM (including deep OTM) and ATM options based on the CBOE/Bakshi, Kapadia and Madan (2003) method. Therefore, the Bollen and Whaley measure of the skew is somewhat ad-hoc due to their use of only two options to determine the skew value. Bollen and Whaley define the level of the implied volatility as the average implied volatility of near-the-money (NTM) options, with the absolute value of the delta of the options falling into the range of [0.375, 0.625]. The slope of the implied volatility curve is defined as the percentage difference between the average implied volatility of OTM options (with the absolute value of delta falling in the range of [0.125, 0.375]) and the average implied volatility of NTM options.

Stock (ERS), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. All tests show strong evidence of a unit root for the VIX and VVIX. However, there is mixed evidence for the SKEWSPX and SKEWVIX. More specifically, all tests except the KPSS test show stationarity for the SKEWSPX; the ADF, PP and ERS tests show stationarity for SKEWVIX whereas the NP and KPSS tests support the existence of unit root. Nonetheless, differencing these four variables is sufficient to make them stationarity. Therefore, our intraday unit root test results for VIX and SKEWSPX are largely consistently with Neumann and Skiadopoulos' (2012) and Shu and Zhang's (2012) daily results, although the evidence for SKEWSPX is weaker in our sample.

The Johansen cointegration tests, based on trace and maximum eigenvalue methodologies, show that cointegration of rank 2 exists among all four of VIX, VVIX, SKEWSPX and SKEWVIX when examined simultaneously. However, six *pair-wise* tests of each two variable combination show cointegration only between VVIX and SKEWSPX and between VVIX and SKEWVIX. Both types of tests are needed because of our subsequent examination of univarite, bivariate and multivariate models in the results section. Therefore, the above evidence of potential cointegration relations among the four variables calls for a cointegrated Vector Autoregression (VAR) model such as the Vector Error Correction Model (VECM).

C. Hypothesis Development

Prices of VIX futures and options are based on the implied volatilities from SPX options. Such a theoretical pricing connection naturally drives the integration of the SPX and VIX options markets, especially after the growth in VIX futures and options volume heading into the 2008 financial crisis. Alternatively, we hypothesize that the SPX and VIX options markets are still separately dominated by SPX and VIX traders. We further contend that the VIX and SKEWSPX indices are related to each other more closely than are the VVIX and SKEWVIX indices, since the former (latter) two are determined in the SPX (VIX) options market. Based on the above associations, we formulate seven hypotheses regarding the relations among VIX, VVIX, SKEWSPX and SKEWVIX and then briefly explain the basis for each hypothesis.

Since the VIX is based on SPX options, and since the price of the SPX options changes as volatility changes, we hypothesize that the information flow between SPX options and VIX options is bidirectional.

Hypothesis I: Bidirectional causality exists among the pairs of VIX, VVIX, SKEWSPX and SKEWVIX.

Corsi et al. (2008), and Bollerslev et al. (2009) document empirical evidence for the ARCH effect in realized volatility. We posit that a similar effect exists in the volatility of volatility and skewness variables examined here since volatility and skewness are computed with the same underlying data (SPX options and VIX options), as stated in Hypothesis II.

Hypothesis II: An ARCH effect exists in the residuals of the **Δ**VVIX, **Δ**SKEWSPX and **Δ**SKEWVIX regressions.

Price discovery in the VIX (SPX) options market determines the VVIX (VIX) and SKEWVIX (SKEWSPX). The VVIX and SKEWVIX are strongly linked through the prices of the VIX options, whereas the VIX and SKEWSPX are strongly linked through the prices of the SPX options. Consequently, the former essentially measure the characteristics of the VIX options market, whereas the latter measure the characteristics of the SPX options market. Although an influence exists from the SPX options market to the VIX options, we contend that the inter-market link is of secondary order in determining the VVIX and SKEWVIX relative to the intra-market link, and vice-versa.

Hypothesis III.1: A change in the SKEWVIX relates more closely to changes in the VVIX than to changes in the VIX and SKEWSPX.

Hypothesis III.2: A change in the VIX (VVIX) relates more closely to changes in the

SKEWSPX (SKEWVIX), than to changes in the VVIX (VIX) and SKEWVIX (SKEWSPX).

Barndorff-Nielsen and Shephard (2005) and Corsi, Mittnik, Pigorsch and Pigorsch (2008) support the concept that the volatility of (realized) volatility is positively associated with realized volatility for high-frequency returns. We therefore hypothesize that a positive relation holds for the changes under the risk-neutral measures investigated in this paper, as given in Hypothesis IV.

Hypothesis IV: After controlling for skewness via SKEWSPX and SKEWVIX, a positive relation exists between the changes in the VIX and the changes in VVIX.

The VIX and SKEWSPX are measured in terms of the SPX options, whereas the VVIX and SKEWVIX are measured in terms of VIX options. As with Hypothesis I, we conjecture that the two options markets are interconnected, with the associated information flow forming a bidirectional flow of information. We further hypothesize that lagged volatility and the lagged SKEW index measured in one option market help to explain current volatility and the SKEW index measured in another option market. More specifically:

Hypothesis V: Past changes in the VIX and SKEWSPX (representing the SPX options market) contribute to the changes in the VVIX and SKEWVIX (representing the VIX options market), and changes in the VVIX and SKEWVIX contribute to changes in the VIX and SKDWSPX.

Any increase in the (upward call dominated) implied volatility skew of VIX options that is driven by the excess demand for out-of-the-money (OTM) call options can also drive a higher level of uncertainty in the VVIX. The opposite is true for SPX options since they possess a put dominated downward implied volatility skew driven by excess demand for OTM puts. Note that the SKEWSPX increases when the skewness of the S&P 500 returns become more negative, whereas the SKEWVIX increases when the skewness of the VIX becomes more positive. Thus, we posit a positive relation between changes in the SKEWVIX and changes in the VVIX, and a negative relation between the changes in the SKEWSPX and changes in the VVIX.

Hypothesis VI: The changes in the SKEWVIX (SKEWSPX) are positively (negatively) associated with changes in the VVIX.

The 2008 financial crisis caused dramatic spikes in volatility and led the macro economy into the so-called Great Recession. The "outliers" in economic variables during the crisis can cause the linear relations among the VIX, VVIX, SKEWSPX and SKEWVIX to behave differently across the quantiles of their distribution. For example, the effect of the SKEWSPX on the VIX could be different on the right tail of the VIX distribution (a high level of fear) compared to the left tail (a low level of fear). A more complete view can be obtained through quantile regressions (i.e. absolute value regressions, Koenker and Hallock 2001; Badshah 2013), which we shall describe in Section III.B.

Hypothesis VII: Financial crises pose uneven responses from explanatory variables to dependent variables (such as volatility and skew indices) across the quantiles of the distribution of the dependent variable.

III. What Determines the VIX, VVIX, SKEWSPX and SKEWVIX?

The VIX and SKEWSPX represent proxies for two of the important statistical moments of the S&P500 return risk-neutral distribution. Correspondingly, the VVIX and SKEWVIX are proxies for the VIX futures return distribution. These proxy measures are crucial for characterizing SPX and VIX options. In particular, Poon and Granger (2003) show the importance of examining the volatility of volatility as a component in pricing derivatives on volatility. As the SPX and VIX options markets become more integrated (Cont and Kokholm, 2013; Lin and Chang, 2010), it is essential to understand how the risk-neutral moments relate to the other moments. Thus, in this section, we first examine the Granger causality among the VIX, SKEWSPX, VVIX and SKEWVIX indices. We then employ the ordinary mean-based regression analysis to investigate the factors associated with the VIX, VVIX, SKEWSPX and SKEWVIX, both individually and jointly. We further augment our analysis with a quantile regression in order to determine the impact of outliers during the financial crisis.

A. Least Squares Regression

We test the seven hypotheses proposed in Section II.C. The first hypothesis is to test for Granger-causality among VIX, VVIX, SKEWSPX and SKEWVIX indices via Model 1. Hypotheses II through VI examine the factors that affect the last three indices via Models 2 and 3. Similarly, Hypothesis VII examines the asymmetric impact on all four indices due to the financial crisis via Model 4.[6]

A.1 Granger-causality

Among the six pairs of indices, we are mostly interested in the relation between the changes in the level and slope of the volatility smile (ΔVIX_t, $\Delta SKEWSPX_t$), between the changes in the level of volatility and the vol of vol (ΔVIX_t, $\Delta VVIX_t$), between the *change* in the level of volatility and the change in the slope of the volatility smile ($\Delta VVIX_t$, $\Delta SKEWVIX_t$), and between the changes in the skews ($\Delta SKEWSPX_t$, $\Delta SKEWVIX_t$). The changes are used due to the existence of unit roots. The Granger causality relations are tested using the Vector Autoregressions (VAR) of Models 1.1 through 1.8, respectively.

Model 1

(M1.1) $$\Delta VIX_t = C_{11} + \sum_{j=1}^{p} \phi_{1j}^{1} \Delta VIX_{t-j} + \sum_{j=1}^{p} \theta_{1j}^{1} \Delta SKEWSPX_{t-j} + \epsilon_{1t}^{1} \tag{5a}$$

(M1.2) $$\Delta SKEWSPX_t = C_{12} + \sum_{j=1}^{p} \phi_{2j}^{1} \Delta SKEWSPX_{t-j} + \sum_{j=1}^{p} \theta_{2j}^{1} \Delta VIX_{t-j} + \epsilon_{2t}^{1} \tag{5b}$$

---

[6] All statistically significant values in this section are significant at the .05 level, unless specified otherwise.

$$\text{(M1.3)}\quad \Delta VIX_t = C_{21} + \sum_{j=1}^{p} \phi_{1j}^{2}\, \Delta VIX_{t-j} + \sum_{j=1}^{p} \theta_{1j}^{2}\, \Delta VVIX_{t-j} + \varepsilon_{1t}^{2} \qquad (5c)$$

$$\text{(M1.4)}\quad \Delta VVIX_t = C_{22} + \sum_{j=1}^{p} \phi_{2j}^{2}\, \Delta VVIX_{t-j} + \sum_{j=1}^{p} \theta_{2j}^{2}\, \Delta VIX_{t-j} + \varepsilon_{2t}^{2} \qquad (5d)$$

$$\text{(M1.5)}\quad \Delta VVIX_t = C_{31} + \sum_{j=1}^{p} \phi_{1j}^{3}\, \Delta VVIX_{t-j} + \sum_{j=1}^{p} \theta_{1j}^{3}\, \Delta SKEWVIX_{t-1}$$

$$+ \Pi_{11}\, VVIX_{t-1} + \Pi_{12}\, SKEWVIX_{t-1} + \varepsilon_{1t}^{3} \qquad (5e)$$

$$\text{(M1.6)}\quad \Delta SKEWVIX_t = C_{31} + \sum_{j=1}^{p} \phi_{2j}^{3}\, \Delta SKEWVIX_{t-j} + \sum_{j=1}^{p} \theta_{2j}^{3}\, \Delta VVIX_{t-j}$$

$$+ \Pi_{21}\, VVIX_{t-1} + \Pi_{22}\, SKEWVIX_{t-1} + \varepsilon_{2t}^{3} \qquad (5f)$$

$$\text{(M1.7)}\quad \Delta SKEWSPX_t = C_{12} + \sum_{j=1}^{p} \phi_{1j}^{4}\, \Delta SKEWSPX_{t-j} + \sum_{j=1}^{p} \theta_{1j}^{4}\, \Delta SKEWVIX_{t-j} + \varepsilon_{2t}^{1} \qquad (5g)$$

$$\text{(M1.8)}\quad \Delta SKEWVIX_t = C_{12} + \sum_{j=1}^{p} \phi_{2j}^{4}\, \Delta SKEWVIX_{t-j} + \sum_{j=1}^{p} \theta_{2j}^{4}\, \Delta SKEWSPX_{t-j} + \varepsilon_{2t}^{1} \qquad (5h)$$

It is worth to note that only the last two equations have additional terms of VVIX and SKEWVIX to correct for the cointegration relationship. Two lags of the independent variables are chosen, based on the lowest AIC (Akaike Information Criterion) and SBC (Schwarz Bayesian Criterion) values for the data examined.

The results for the six individual regressions and the Granger-causality tests are reported in Table III. Here we restrict our discussion of causality to the three pairs of variables noted above, as examined in (5a) to (5h). First, we choose Models 1.1 and 1.2 to investigate the causality between ΔVIX and ΔSKEWSPX. The Wald test statistic supports Granger-causality from the ΔSKEWSPX to the ΔVIX at the 0.05 level of significance. Consequently, changes in the SKEW index affect future changes in the VIX. Conversely, when we test for Granger causality from ΔVIX to ΔSKEWSPX we reject the null hypothesis of no causality at the 0.10 level. Therefore, the Granger-causality between ΔVIX and ΔSKEWSPX are bidirectional supporting Hypothesis I.

Second, we use Models 1.3 and 1.4 to investigate the causality between ΔVIX and ΔVVIX. We cannot reject the null hypothesis of no causality from ΔVVIX to ΔVIX, neither can we reject the null hypothesis of no causality from ΔVIX to ΔVVIX. Therefore, no Granger-causality exists between ΔVIX and ΔVVIX, therefore this result rejects Hypothesis I.

Third, we employ Models 1.5 and 1.6 to examine the ΔVVIX and the ΔSKEWVIX relation, while simultaneously considering the cointegration between them. Using an error correction model, we reject the null hypothesis of no causality from ΔSKEWVIX to ΔVVIX, but not the null hypothesis of no causality from ΔVVIX to ΔSKEWVIX. Thus, the results show that a uni-directional causality exists from ΔSKEWVIX to ΔVVIX. One possible strategy to profit from this lead-lag relation is to long (short) a variance swap on VIX futures when the VIX implied volatility skew is more (less) upwardly sloped.

Lastly, we use Models 1.7 and 1.8 to investigate the causality between ΔSKEWSPX and ΔSKEWVIX. We cannot reject the null hypothesis of no causality from ΔSKEWVIX to ΔSKEWSPX, but can reject the null hypothesis of no causality from ΔSKEWSPX to ΔSKEWVIX. Therefore, there is a uni-directional causality from ΔSKEWSPX to ΔSKEWVIX. In order to profit from this causality one can long (short) the bull spread in VIX call options when the SPX implied volatility skew is more (less) downwardly sloped.

To summarize, we find the following Granger-causality chain:

ΔVIX **←**→ ΔSKEWSPX **→** ΔSKEWVIX **→** ΔVVIX, where bold arrows show stronger links.

The economic implication of the chain effect is that price discovery in the SPX options market lead price discovery in the VIX options market. The feedback from VIX options market to the SPX options market appears to be weak, at least statistically. It is conceivable that option traders tend to respond first in the SPX options and then in the VIX options market, given that the latter is still a relatively new market for hedging volatility risk.

A.2 Determination of the Change in VVIX

Here we test Hypotheses II through VI to examine the factors associated with the change in the VVIX by using the benchmark Model 2, as follows:

Model 2

$$\Delta VVIX_t = \alpha + \beta_1 \Delta SKEWVIX_t + \beta_2 \Delta SKEWVIX_{t-1} + \beta_3 \Delta SKEWSPX_t + \beta_4 \Delta SKEWSPX_{t-1} + \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta VVIX_{t-1} + + \beta_7 \Delta VVIX_{t-2} + \epsilon_t \quad (6)$$

We estimate the complete specification and five reduced versions of Model 2, labeled as Models 2.1 to 2.6. In order to test for the explanatory power of ΔSKEWVIX, ΔSKEWSPX, ΔVIX, and past ΔVVIX, we sequentially remove from the regression each of the following four pairs of variables: ($\Delta SKEWVIX_t$, $\Delta SKEWVIX_{t-1}$), ($\Delta SKEWSPX_t$, $\Delta SKEWSPX_{t-1}$), (ΔVIX_t, ΔVIX_{t-1}), and lastly $\Delta VVIX_{t-1}$, to obtain Models 2.2 to 2.5 respectively. Estimation results are reported in Table IV. Associated hypotheses are reproduced and discussed below.

Regarding Hypothesis II, we test for the ARCH effect in the regression Model 2 for **Δ**VVIX. The Lagrange Multiplier (LM) test strongly supports the existence of an ARCH effect in the regression residual for **Δ**VVIX. We employ an EGARCH(1,1) model to account for potential asymmetric response of conditional variance to shocks in the residual. We augment equation (6) with the following conditional variance equation:

$$ln(h_t) = a_0 + a_1(\theta e_{t-1} + |e_{t-1}| - E(|e_{t-1}|) + b_1 ln(h_{t-1}) \quad (7)$$

where $\epsilon_t = h_t z_t$ and $z_t \sim i.i.d.N(0,1)$.

The GARCH parameters a_0, a_1, b_1 are statistically significant at the 0.05 level, showing volatility clustering in the change of VVIX and accepting the first part of Hypothesis II. This finding is an addition to the recent literature on realized volatility (Corsi et al. 2008, Bollerslev et al. 2009) in that the GARCH effect here is on (implied) volatility of (implied) volatility as opposed to realized volatility, i.e. volatility of volatility of volatility (vol^3) vs. volatility of realized volatility (vol^2). Furthermore, we find asymmetric response of vol^2 to vol^3, as evidenced by a positive and significant θ=0.21. The asymmetry lies in that positive shocks to vol^2 increase vol^3 more than do negative shocks.

Regarding Hypothesis III.1, we compare the R^2s of Models 2.1 to 2.5, as shown in Table IV. We find that the fit of the equation drops the most (and is significant) from an R^2 of 23.2% to 8.9% if we omit ΔVIX. Removing other variables from the equation impacts the R^2 of the equation, but with far less significance. Although the effect of removing ΔSKEWVIX and ΔSKEWSPX are of secondary order, removing the former causes more decrease in R^2 than does removing the latter. This means that the ΔVIX (measuring fear) contributes the most to the determination of the ΔVVIX (the fear of fear), rejecting Hypothesis III. The result further motivates the test of Hypothesis IV.

Regarding Hypothesis IV, we examine the coefficient of the ΔVIX in the regression for the ΔVVIX (Model 2). The coefficient of the ΔVIX is positive and significant, supporting the notion of the inverse leverage effect between contemporaneous changes of VIX and VVIX. More specifically, one percentage point increase in ΔVIX is associated with 0.23 percentage point increase in ΔVVIX. Our finding with risk-neutral measure of volatilities is consistent with empirical evidence found in the literature of volatility of realized volatility (Corsi et al. 2008; Bollerslev et al. 2009).

Regarding Hypothesis V, we examine the statistical significance of past changes in the VIX and SKEWSPX in the regression for ΔVVIX (Model 2). We note that Hypothesis V is along the same line with the Granger-causality test in Hypothesis I, but is examined using a standalone regression with ARCH effect, instead of a pair-wise VAR model. Table IV shows that only the coefficient for the lagged change in SKEWVIX is significantly negative, consistent with the Granger-causality test. The lagged changes in VIX and SKEWSPX (measured in SPX options market) do not cause significant impact on the change in VVIX (measured in VIX options market). Therefore, we reject the hypothesis that SPX options market leads VIX options market.

Regarding Hypothesis VI, we investigate the sign and significance of the coefficients for ΔSKEWSPX and ΔSKEWVIX in the regression examining ΔVVIX in Models 2.1, and 2.3-2.5. As reported in Table IV, the coefficients for ΔSKEWVIX are all significantly negative, whereas the coefficients for SKEWSPX are all significantly positive. However, the size of the former (0.37) is significantly larger than that of the latter (-0.02). We deem the primary effect on VVIX is still originated within VIX options market. Therefore, we accept Hypothesis V.

A.3 Determination of the Change in SKEWSPX and SKEWVIX

We employ Models 3a and 3b to test Hypotheses II, III.2 and V concerning SKEWSPX and SKEWVIX. The structure of both models follows that of Model 2 for VVIX. The estimation results are reported in Table V.

Model 3a: $$\Delta SKEWSPX_t = \alpha + \beta_1 \Delta SKEWVIX_t + \beta_2 \Delta SKEWVIX_{t-1} + \beta_3 \Delta VVIX_t + \beta_4 \Delta VVIX_{t-1} + \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta SKEWSPX_{t-1} + \beta_8 \Delta SKEWSPX_{t-2} + \varepsilon_t \quad (8a)$$

Model 3b: $$\Delta SKEWVIX_t = \alpha + \beta_1 \Delta SKEWSPX_t + \beta_2 \Delta SKEWSPX_{t-1} + \beta_3 \Delta VVIX_t + \beta_4 \Delta VVIX_{t-1} + \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta SKEWVIX_{t-1} + \beta_8 \Delta SKEWVIX_{t-2} + \varepsilon_t \quad (8b)$$

Regarding Hypothesis II, we test for the ARCH effect in the regression Models 3a and 3b for **Δ**SKEWSPX and **Δ**SKEWVIX, respectively. The LM tests strongly support the existence of the ARCH effect in the residuals of **Δ**SKEWSPX and **Δ**SKEWVIX regression. Using the same

EGARCH model in Equation (7), we quantify volatility clustering and asymmetry in the two SKEW indices, reported in Table V. The GARCH parameters a_0, a_1, b_1 are statistically significant at the 0.05 level, indicating volatility clustering in **Δ**SKEWSPX and **Δ**SKEWVIX and accepting the second part of Hypothesis II.

The volatility asymmetric effect is found in **Δ**SKEWSPX, but not in **Δ**SKEWVIX. Positive shocks to **Δ**SKEWSPX increase its volatility more than negative shocks. The difference in the SKEWVIX may be due to the lack of depth in the VIX options market for traders to respond differently for positive vs. negative shocks.

Regarding Hypothesis III.2, we compare the R^2s of Models 3a.1 through 3a.5 and 3b.1 through 3b.5, as shown in Table V. First of all, **Δ**SKEWSPX and **Δ**SKEWVIX do not correlate to each other in Model 3a and 3b, after conditioning on the lags of the respective variables. Additionally, we find negative and significant coefficients for contemporaneous **Δ**VIX and **Δ**VVIX in explaining **Δ**SKEWSPX. We further perform an F test for the equality between the two coefficients. We cannot reject the null of the equal effect between the **Δ**VIX and **Δ**VVIX (p value =0.25). Dropping **Δ**VIX from the regression (Model 3a.3) can hardly be distinguished from dropping **Δ**VVIX (Model 3a.4) in terms of R^2. Therefore, we reject Hypothesis III.2 regarding SKEWSPX on the basis that the **Δ**VIX and **Δ**VVIX possess an equal impact on the **Δ**SKEWSPX.

Similarly regarding **Δ**SKEWVIX, we find positive and significant coefficients for contemporaneous **Δ**VIX and **Δ**VVIX. The F test shows a stronger effect of **Δ**VVIX than **Δ**VIX (p value < 0.01). There is a noticeably more decrease in R^2 when dropping **Δ**VVIX (Model 3b.3) than dropping **Δ**VIX (Model 3b.4). Therefore, we accept Hypothesis III.2 regarding SKEWVIX.

Regarding Hypothesis V, we examine the statistical significance of past changes in the VVIX and SKEWVIX in the regression explaining ΔSKEWSPX, as well as the changes on VIX and SKEWSPX for the regression explaining ΔSKEWVIX (Models 3a and 3b). Table V shows that neither the coefficient of lagged ΔVVIX nor the coefficient of lagged ΔSKEWVIX is statistically significant in explaining ΔSKEWSPX. The same conclusion holds for ΔSKEWSPX — neither the coefficient of lagged ΔVIX or lagged ΔSKEWSPX is statistically significant in explaining ΔSKEWVIX. Thus, we reject Hypothesis V for the two SKEW indices. Further investigation of R^2 shows that the SKEW indices are mostly explained by their own lags.

A.4 Joint Determination of VIX, VVIX, SKEWSPX, and SKEWVIX

As a robustness check for the Granger-causality and the determination of each variable, we cast VIX, VVIX, SKEWSPX and SKEWVIX in a vector autoregression with error correction model (VECM), accounting for potential cointegration. Denote Y_t as the four-variable vector (VIX_t, $VVIX_t$, $SKEWSPX_t$, $SKEWVIX_t$). The following VECM model (Model 4) is employed to examine the correlation across the four variables:

$$\Delta Y_t = C + \sum_{i=1}^{p-1} \Phi\, Y_{t-i} + \Pi\, Y_{t-1} + \epsilon_t \quad (9)$$

where C, Φ and Π are the intercept, the vector autoregressive and error-correction coefficients, respectively. Estimation results are reported in Table VI.

We find that lagged ΔSKEWVIX explains ΔVVIX, lagged ΔSKEWSPX explains ΔSKEWVIX and ΔVIX, which is consistent with the earlier Granger causality results based on the pair-wise VAR. The only difference in the VECM model is the lack of explanatory power of ΔVIX for ΔSKEWSPX (as opposed to the 9.3% significance in the pair-wise VAR model). We contend that the remaining linkage between ΔVIX and ΔSKEWSPX is weakened once we account for the long-run cointegration effect.

We also find the statistical significance and sign of the explanatory variables are largely shared by the standard-alone regression model with EGARCH effect and the VECM model, although the model specifications are not exactly the same. The similarity in the results points to the consistency between the two models.

B. Quantile Regressions

The large kurtosis in Table I for ΔVIX, ΔVVIX, ΔSKEWSPX and ΔSKEWVIX (50, 26.2, 16.7 and 23.5, respectively) cause concern that extreme values can potentially bias the OLS regression results due to the squaring of the error terms, which motivates Hypothesis VII. In order to obtain a complete view of its impact on parameter estimate, we run a simple quantile regression of VIX, VVIX, SKEWSPX and SKEWVIX from 5% to 95% quantile incremented by 5%, where quantile regression employs an absolute value procedure. The three ordinary regression models (2.1, 3a.1 and 3b.1) are estimated under the quantile regression framework. In the interest of space, we report in Figures 3a and 3b the quantile plots for variables that are statistically significant in the ordinary regressions.[7]

A general observation is made about the difference between the ordinary and quantile regressions before we delve into the variation of each parameter estimate across quantiles. The signs of the coefficients of our interest remain the same under both regressions. In Figure 3a (ΔVVIX), we include $\Delta SKEWVIX_t$, $\Delta SKEWVIX_{t-1}$, $\Delta SKEWSPX_t$, ΔVIX_t, $\Delta VVIX_{t-1}$, and $\Delta VVIX_{t-2}$. The most significant variation across quantiles exists in $\Delta SKEWSPX_t$ and ΔVIX_t. $\Delta SKEWSPX_t$ responds more negatively to ΔVVIX in extreme quantiles. However, ΔVIX_t responds more positively to ΔVVIX in extreme quantiles. In the left panel of Figure 3b (ΔSKEWSPX), we include $\Delta VVIX_t$, ΔVIX_t, $\Delta SKEWSPX_{t-1}$, and $\Delta SKEWSPX_{t-2}$. The most significant variation across quantiles is also found in ΔVIX_t and $\Delta SKEWSPX_{t-1}$. ΔVIX_t and $\Delta SKEWSPX_{t-1}$ respond more negatively to ΔSKEWSPX in extreme and in median quantiles,

[7] The detailed estimates for the quantile regression are available upon request.

respectively. In the right panel of Figure 3b (ΔSKEWVIX), we include $\Delta VVIX_t$, ΔVIX_t, $\Delta SKEWVIX_{t-1}$, and $\Delta SKEWVIX_{t-2}$. The most significant variation across quantiles is found in ΔVIX_t, which responds more positively to ΔSKEWVIX in extreme quantiles.

In sum, ΔVIX_t and $\Delta SKEWSPX_t$ tend to show more impact at extreme and median values. The results may be associated with the fact that the former has the highest kurtosis while the latter has the lowest kurtosis.

IV. Conclusions

We examine the characteristics of the VIX index for VIX options (VVIX), as well as analyze the new SKEW index that determines a value of the slope of the implied volatility curve of the S&P500 (SKEWSPX) and VIX options (SKEWVIX). These measures are applied to the Fall of 2008 financial crisis data. We also investigate the Granger-causality among VIX, VVIX, SKEWSPX and SKEWVIX. Lastly we analyze the factors that determine the changes in VVIX, SKEWSPX and SKEWVIX. To the best of our knowledge, studying the intraday behavior of these four variables is new to the literature. The main conclusions are summarized below.

Granger causality: the following chain effect is found ΔVIX ←→ ΔSKEWSPX → ΔSKEWVIX → ΔVVIX. The relation is robust under the pair-wise VAR model and the system-wide VECM model. The economic implication is the slope of change in the volatility smile forecasts (at least predates) the change in implied volatility at the intraday level. The results apply in both SPX and VIX options markets. Option traders can design strategy to capture the lead-lag relation.

Volatility Clustering: we document the GARCH effect in the change of volatility of volatility (VVIX) and the SKEW indices (SKEWSPX and SKEWVIX). Conditional volatility of

VVIX (volatility-cubed) and SKEWSPX respond asymmetrically to the shocks to the change in VVIX and SKEWSPX: positive shock increases the conditional volatility more than negative shocks.

Explanatory Power: the change in VIX is positively related to and has the most explanatory power to the change in VVIX, whereas its own lags explain most of the change in SKEWSPX and SKEWVIX. The former result is termed “inverse leverage effect” that can be intuitively interpreted as “fear” moves in the same direction as “fear of fear”. The latter result indicates more independence of the risk-neutral skewness dynamics at the intraday level.

Information flow: after 4 (2) years of development of VIX futures (options) prior to the 2008 financial crisis, most evidence points to the influence of the mature SPX options market on the burgeoning VIX options market. There is some evidence that the latter provides additional price discovery information, for instance from VVIX to SKEWSPX.

The examination of the SKEW index and the volatility of volatility provided here is a starting point for analyzing the characteristics and relations associated with these variables. The results and comments in this paper suggest a number of directions for future research. One avenue is to examine the behavior and stochastic process of the SKEW index and the vol of vol (using both implied and realized values of this variable). A second avenue is to determine how the SKEW and the VIX of VIX can be employed to price VIX options, especially given the poor performance of the current VIX option pricing models (Wang and Daigler, 2011). A third, more practical, use of the SKEW and VIX of VIX is to determine if and how they can be used to help forecast market movements or option prices. Finally, one can develop strategies and pricing models for a volatility derivative on the SKEW index. Overall, the forecast for volatility derivative research seems to have a bright future.

REFERENCES

Aït-Sahalia, Y., Mykland, P.A. & Zhang, L. (2005). How often to sample a continuous-time process in the presence of market microstructure noise. Review of Financial Studies, 18, 351-416.

Andersen, T. G., Bollerslev, T., Diebold, F. X. & Labys, P. (2003). Modeling and forecasting realized volatility, Econometrica, 71, 579-625.

Badshah, I. U. (2013), Quantile regression analysis of the asymmetric return-volatility relation, Journal of Futures Markets, 33, 235-265.

Bakshi, G. and Kapadia, N. (2003a), Delta-hedged gains and the negative volatility risk premium. Review of Financial Studies, 16, 527-566.

Bakshi, G. & Kapadia, N. (2003b). Volatility risk premium embedded in individual equity options: some new insights. Journal of Derivatives, 11, 45-54.

Bakshi, G., Kapadia, N. & Madan, D. (2003) Stock return characteristics, skew laws, and differential pricing of individual equity option. Review of Financial Studies, 16, 101-143.

Barndorff-Nielsen, O. E. & Shephard, N. (2002). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society, Series B, 253-280.

Black, F. & Scholes, M. (1973). The pricing of options and corporate liabilities, Journal of Political Economy, 81, 637-654.

Bollen, N. & Whaley, R. (2004). Does the net buying pressure affect the shape of implied volatility functions? Journal of Finance 59, 711-753.

Bollerslev, T., Kretschmer, U., Pigorsch C. & Tauchen G. (2009). A discrete-time model for daily S&P 500 returns and realized variations: Jumps and leverage effects. Journal of Econometrics 150, 151-166.

Bossu, S., Strasser, E. & Guichard, R. (2005). Just what you need to know about variance swaps. JPMorgan Equity Derivatives report.

Breeden, D. T. & Litzenberger, R. H. (1978). Prices of state contingent claims implicit in options prices, Journal of Business, 51, 621-651.

Carr, P. & Wu, L. (2006). A tale of two indices, Journal of Derivatives, 13, 13-29.

Cont, R. & Kokholm, T. (2013). A consistent pricing model for index options and volatility derivatives. Mathematical Finance, 23(2), 248-274.

Corsi, F., Mittnik, S., Pigorsch, C. & Pigorsch, U. (2008). The volatility of realized volatility. Econometric Reviews, 27, 46-78.

Das, S. R. & Sundaram, R. K. (1997). Taming the skew: Higher-order moments in modeling asset price processes in finance. NBER working paper 5796.

Dacorogna, M. M., Gencay, R., Muller, U., Olsen, R. B. & Pictet, O. V. (2001). An introduction to high-frequency finance. Academic Press.

Demeterfi, K., Derman, E., Kamal, M. & Zou, J. (1999). A guide to volatility and variance swaps. Journal of Derivatives, 6, 9-32.

Gârleanu, N., Pedersen, L. H. & Poteshman, A. M. (2009). Demand-based option pricing. Review of Financial Studies, 22, 4259-4299.

Gatheral, J. (2006). The volatility surface: a practitioner's guide. John Wiley & Sons.

Hansen, P. R. & Lunde, A. (2006). Realized variance and market microstructure noise, Journal of Business and Economic Statistics, 24, 127-218.

Jiang, G. J. & Tian, Y. (2007). Extracting model-free volatility from option prices: An examination of the VIX index, The Journal of Derivatives, 14 (3), 35-60.

Koenker, R. & Hallock, K.F. (2001). Quantile Regression, Journal of Economic Perspectives, 15(4), 143-156.

Latane, H. & Rendleman, R. (1976). Standard deviations of stock price ratios implied in option prices, Journal of Finance, 31, 369-381.

Lin, Y.-N. & Chang, C.-H. (2009). VIX option pricing, Journal of Futures Markets, 29, 523-543.

Meddahi, N. (2002). A theoretical comparison between integrated and realized volatility, Journal of Applied Econometrics, 17, 479–508.

Mixon, S. (2011). What does implied volatility skew measure? The Journal of Derivatives, 18, 9-25.

Neuberger, A. (1994). The log contract. Journal of Portfolio Management, 20, 74-80.

Neumann, M. & Skiadopoulos, G. (2012). Predictable dynamics in higher order risk-neutral moments: evidence from the S&P 500 options, Journal of Financial and Quantitative Analysis, forthcoming.

Ni, S. X., (2009). Stock option returns: a puzzle, working paper, Hong Kong University of Science and Technology.

Oomen, R. A. (2005). Properties of bias-corrected realized variance under alternative sampling schemes. Journal of Financial Econometrics, 3, 555-577.

Padungsaksawasdi, C. & Daigler, R. T. (2014). The return-implied volatility relation for commodity ETFs, The Journal of Futures Markets, forthcoming 2014.

Poon, S.-H. & Granger, C. W. J. (2003). Forecasting volatility in financial markets: A review, Journal of Economic Literature, 41, 478-539.

Rubinstein, M. (1994). Implied binomial trees, Journal of Finance, 69, 771-818.

Shu, J. & Zhang, J. (2012). Causality in the VIX futures market, Journal of Futures Markets, 32, 24-46.

Todorov, V. (2010). Variance risk premium dynamics: the role of jumps. Review of Financial Studies, 23, 345-383.

Wang, Z. & Daigler, R. T. (2011). The performance of VIX option pricing models: Empirical evidence beyond simulation. Journal of Futures Markets, 31, 251-281.

Whaley, R. (1993). Derivatives on market volatility hedging tools long overdue, Journal of Derivatives, 1, 71-84.

Xing, Y., Zhang, X. & Zhao, R. (2010). What does individual option volatility smirk tell us about future equity returns? Journal of Financial and Quantitative Analysis, 45, 641-662.

Zhang, J. E., & Xiang, Y. (2008). The implied volatility smirk, Quantitative Finance, 8, 263-284.

FIGURES AND TABLES

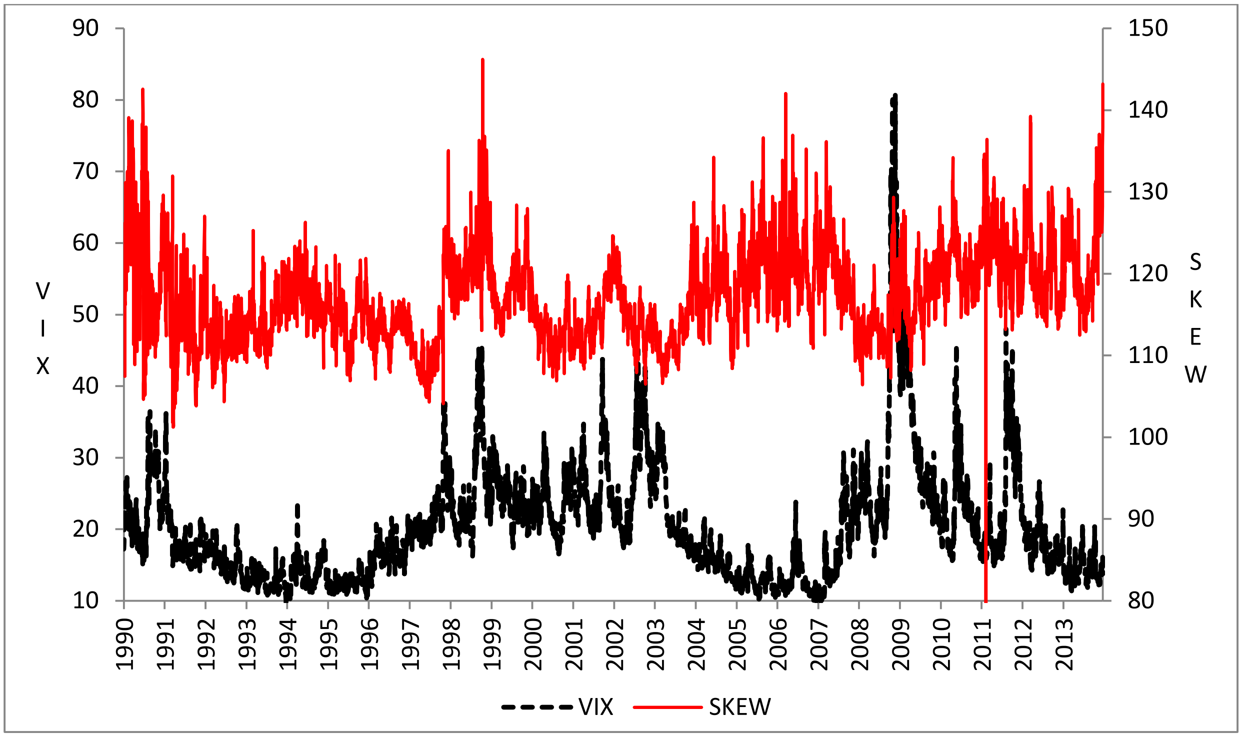


Figure 1 Daily VIX vs. SKEW for the S&P500 Options for 1990-2013

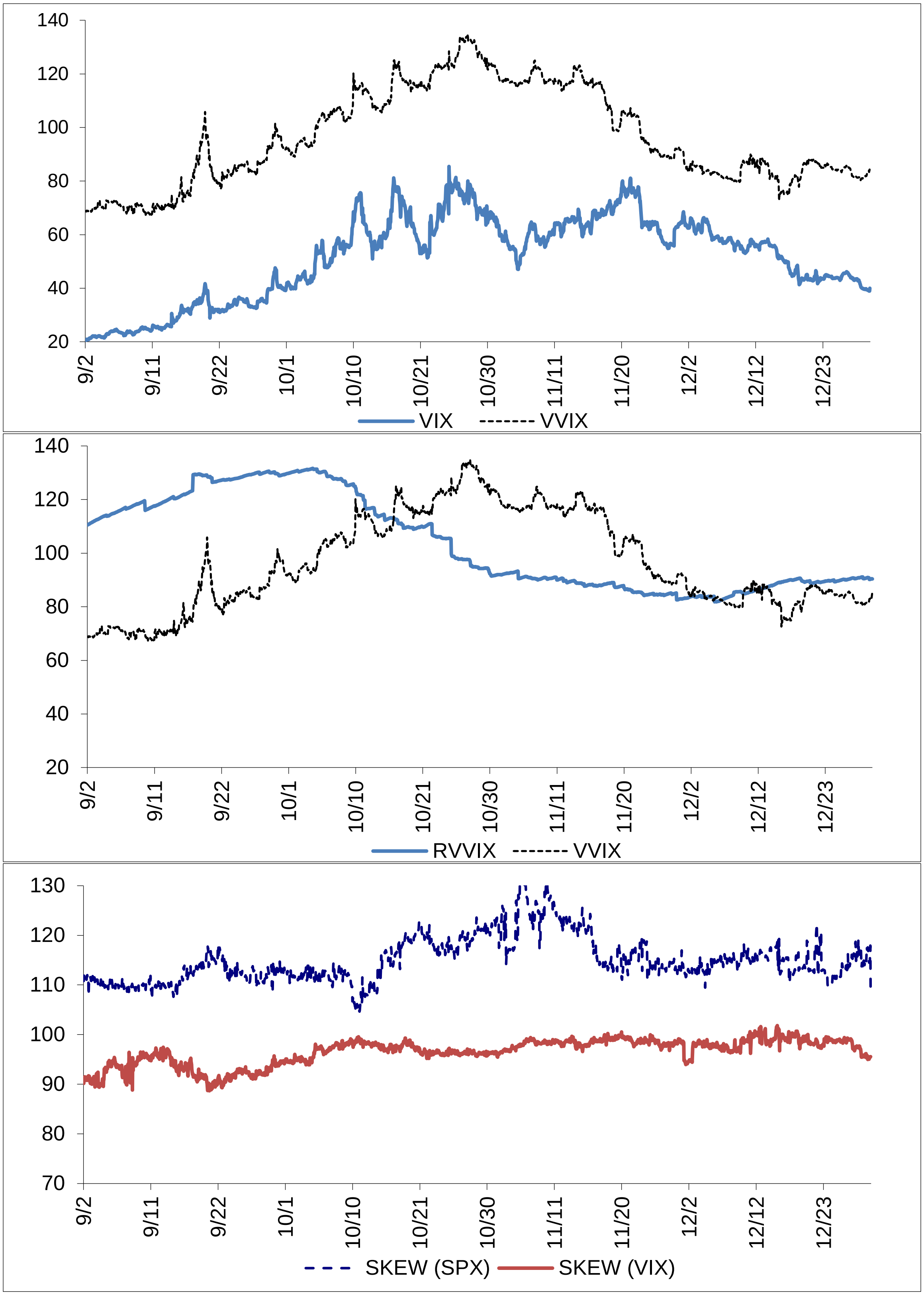
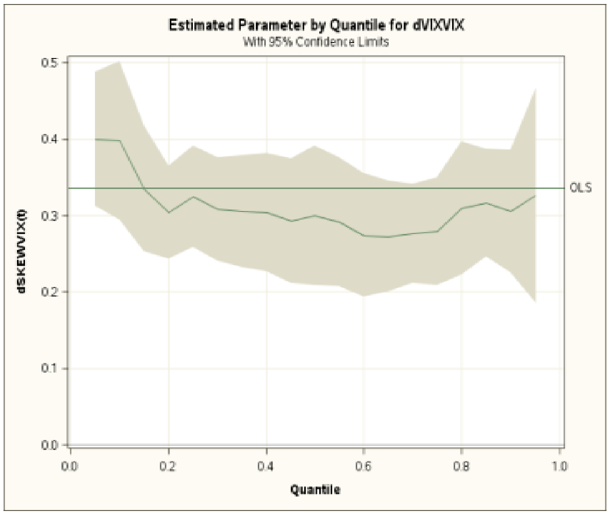
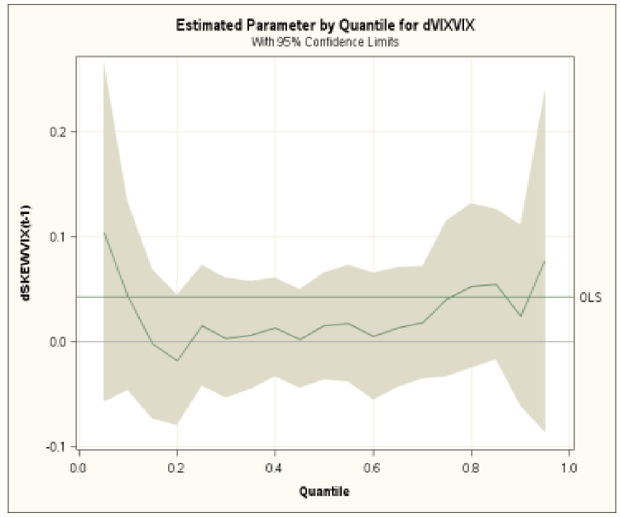


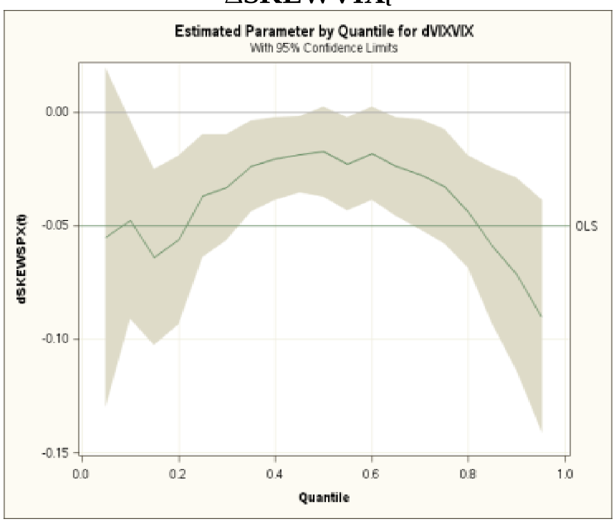
Figure 2 Intraday 15-minute VIX, VVIX, SKEWSPX and SKEWVIX for Fall 2008



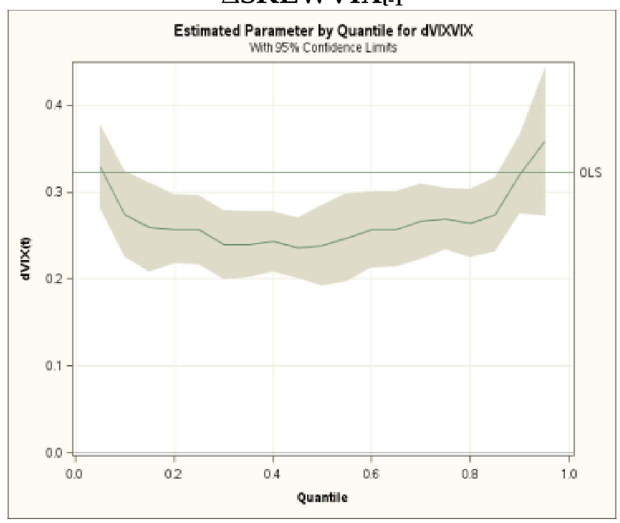
Δ**SKEWVIX**$_t$



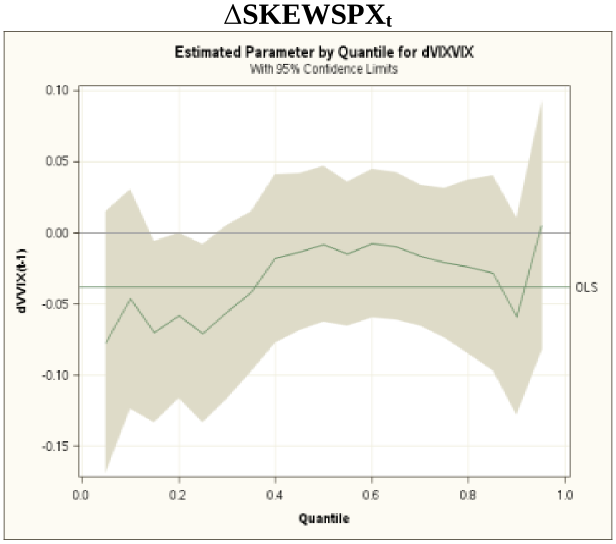
Δ**SKEWVIX**$_{t-1}$



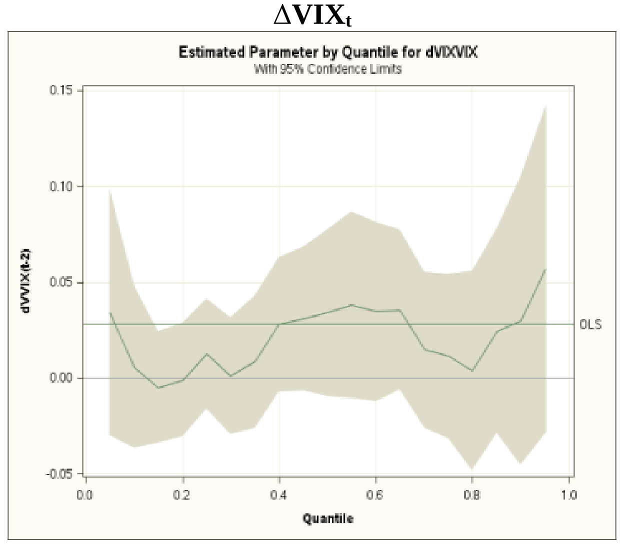
Δ**SKEWSPX**$_t$



Δ**VIX**$_t$

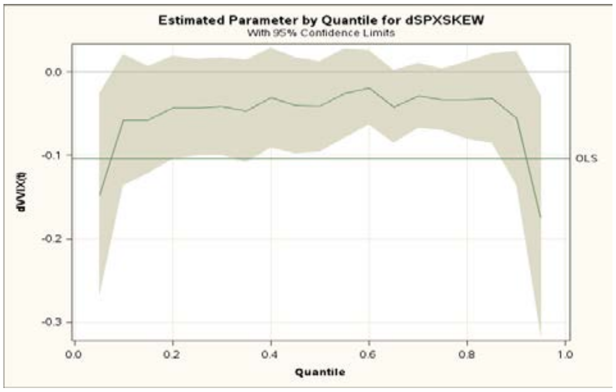


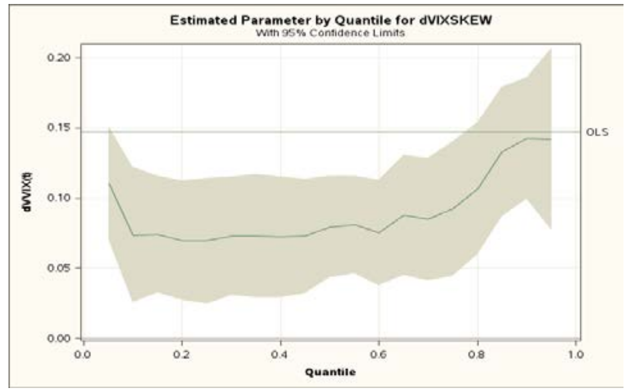
Δ**VVIX**$_{t-1}$



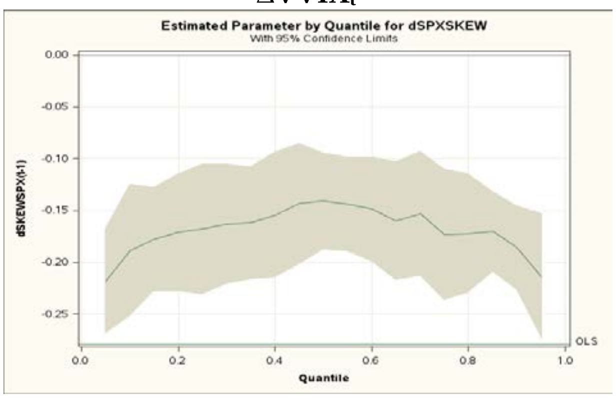
Δ**VVIX**$_{t-2}$

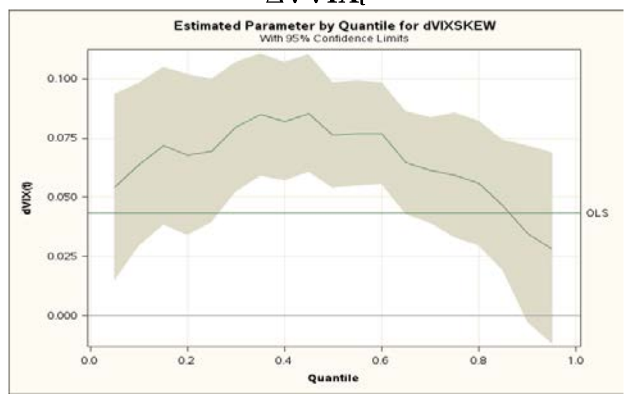
Figure 3a Quantile Plots for the Regressions of ΔVVIX



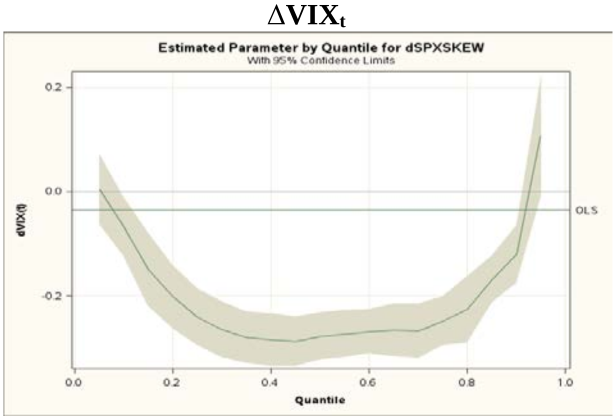


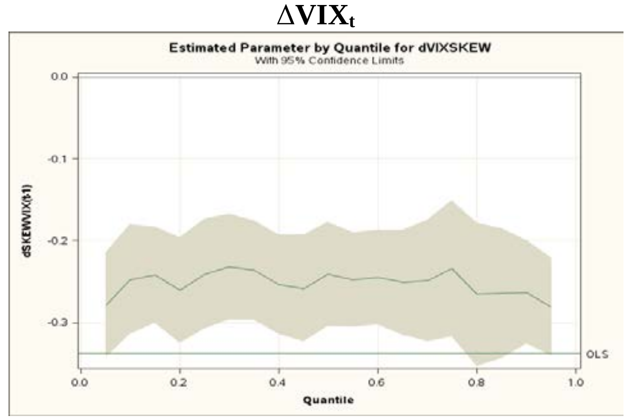
ΔVVIX$_t$ **ΔVVIX$_t$**



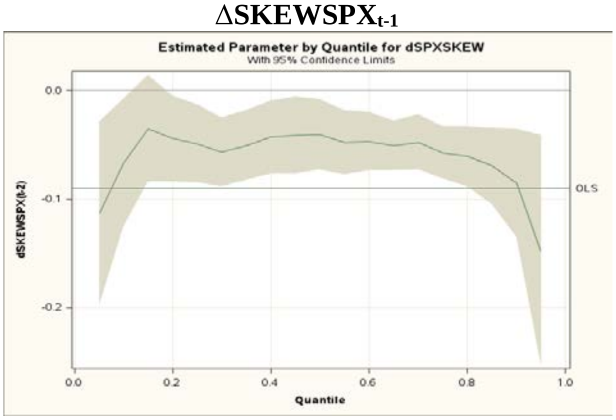


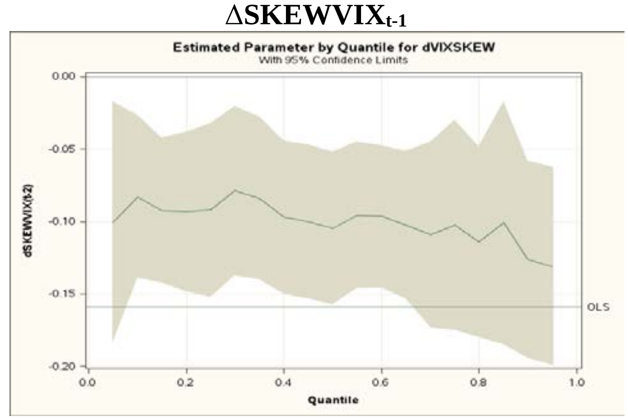
ΔVIX$_t$ **ΔVIX$_t$**





ΔSKEWSPX$_{t-1}$ **ΔSKEWVIX$_{t-1}$**





ΔSKEWSPX$_{t-2}$ **ΔSKEWVIX$_{t-2}$**

Figure 3b Quantile Plots for the Regressions of ΔSKEWSPX (left) and ΔSKEWVIX (right)

Table I Summary Statistics for Intraday Volatility and SKEW Indices (Fall of 2008)

Table I presents summary statistics for three groups of implied volatility variables using 15-minute data for the last quarter of 2008. Group I includes the VIX and the change in the VIX (ΔVIX); Group II includes the realized volatility of the VIX futures (RVVIX), the implied volatility of volatility (VVIX) and the change in the VVIX (ΔVVIX); Group III includes the SKEWSPX, the change in SKEWSPX (ΔSKEWSPX), the SKEWVIX, and the change in SKEWVIX (ΔSKEWVIX).

Group	Variable	N	Mean	Std Dev	Median	Skewness	Kurtosis	Min.	Max.
I	VIX	2212	51.54	15.71	55.57	-0.31	-0.92	20.58	85.46
I	ΔVIX	2211	0.01	1.04	0.00	2.62	50.00	-6.81	17.66
II	RVVIX	2212	103.67	17.26	94.46	0.35	-1.52	81.85	131.63
II	VVIX	2212	96.37	18.04	91.66	0.24	-1.20	66.95	134.86
II	ΔVVIX	2211	0.01	0.87	-0.01	1.41	26.22	-7.38	11.99
III	SKEWSPX	2212	115.01	4.68	113.99	1.02	1.29	104.61	134.28
III	ΔSKEWSPX	2211	0.00	1.18	0.01	-0.26	16.68	-10.39	9.35
III	SKEWVIX	2212	96.50	2.64	97.28	-0.92	0.04	88.65	101.83
III	ΔSKEWVIX	2211	0.00	0.56	0.01	-0.09	23.52	-6.12	6.50

Table II Unit Root Tests

Table II reports the unit root test results for VIX, VVIX, SKEWSPX and SKEWVIX. The tests included are Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Ng-Perron (NP), Elliott-Rothenberg- Stock (ERS), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. All tests are based on optimal lags that provides the lowest modified AIC for the ADF regression. *, ** and *** show the existence of unit root can be rejected at the 0.10, 0.05 and 0.01 significance levels, respectively. Otherwise "x" is marked as existence of unit root.

Variable	ADF	PP	NP	ERS	KPSS
VIX	x	x	x	x	x
ΔVIX	***	***	***	***	***
VVIX	x	x	x	x	x
ΔVVIX	***	***	***	***	***
SKEWSPX	*	***	**	***	x
ΔSKEWSPX	***	***	***	***	***
SKEWVIX	*	***	x	*	x
ΔSKEWVIX	***	***	***	***	***

Table III Pair-wsie Granger Causality Tests (*, ** and *** show significance at the 0.10, 0.05 and 0.01 levels, respectively.)

Model	Variable	Estimate	Std. Err.	Model	Variable	Estimate	Std. Err.
ΔVIX	Constant	0.009	0.022	ΔSKEWSPX	Constant	0.000	0.024
(1a)	ΔVIX(t-1)	0	0.021	(1b)	ΔVIX(t-1)	-0.042*	0.023
	ΔSKEWSPX(t-1)	-0.053***	0.020		ΔSKEWSPX(t-1)	-0.28***	0.021
	ΔVIX(t-2)	-0.033	0.021		ΔVIX(t-2)	0.029	0.023
	ΔSKEWSPX(t-2)	-0.016	0.020		ΔSKEWSPX(t-2)	-0.091***	0.022
H0: ΔSKEWSPX does not Granger cause ΔVIX.				H0: ΔVIX does not Granger cause ΔSKEWSPX.			
Wald Test	7.40**	P value	0.02	Wald Test	4.76*	P value	0.09
ΔVIX	Constant	0.009	0.022	ΔVVIX	Constant	0.008	0.018
(1c)	ΔVIX(t-1)	-0.006	0.024	(1d)	ΔVIX(t-1)	0.013	0.020
	ΔVVIX(t-1)	0.028	0.028		ΔVVIX(t-1)	-0.034	0.024
	ΔVIX(t-2)	-0.032	0.024		ΔVIX(t-2)	0.011	0.020
	ΔVVIX(t-2)	0.003	0.028		ΔVVIX(t-2)	0.010	0.024
H0: ΔVVIX does not Granger cause ΔVIX.				H0: ΔVIX does not Granger cause ΔVVIX.			
Wald Test	0.97	P value	0.61	Wald Test	0.75	P value	0.69
ΔVVIX	Constant	1.25*	0.7	ΔSKEWVIX	Constant	1.77***	0.43
(1e)	VVIX(t-1)	0.000	0.000	(1f)	VVIX(t-1)	0.001	0.000
	SKEWVIX(t-1)	-0.013	0.007		SKEWVIX(t-1)	-0.019	0.005
	ΔVVIX(t-1)	-0.016	0.022		ΔVVIX(t-1)	0.013	0.014
	ΔSKEWVIX(t-1)	-0.062*	0.035		ΔSKEWVIX(t-1)	-0.29***	0.021
H0: ΔSKEWVIX does not Granger cause ΔVVIX.				H0: ΔVVIX does not Granger cause ΔSKEWVIX.			
Wald Test	6.32	P value	0.043**	Wald Test	2.13	P value	0.35
ΔSKEWSPX	Constant	0.000	0.024	ΔSKEWVIX	Constant	0.003	0.011
(1g)	ΔSKEWSPX(t-1)	-0.27***	0.021	(1h)	ΔSKEWSPX(t-1)	-0.012	0.009
	ΔSKEWVIX(t-1)	-0.005	0.046		ΔSKEWVIX(t-1)	-0.34***	0.021
	ΔSKEWSPX(t-2)	-0.09***	0.022		ΔSKEWSPX(t-2)	0.019*	0.010
	ΔSKEWVIX(t-2)	-0.004	0.046		ΔSKEWVIX(t-2)	-0.16***	0.021
H0: ΔSKEWVIX does not Granger cause ΔSKEWSPX.				H0: ΔSKEWSPX does not Granger cause ΔSKEWVIX.			
Wald Test	0.02	P value	0.99	Wald Test	6.5**	P value	0.0387

Table IV Regression Results for the Determination of ΔVVIX

This table reports the estimates and standard errors (in parenthesis) of the regression coefficients from Model 2. Estimates that are statistically significant at the 5% level are shown in bold.

Model 2: $\Delta VVIX_t = \alpha + \beta_1 \Delta SKEWVIX_t + \beta_2 \Delta SKEWVIX_{t-1} + \beta_3 \Delta SKEWSPX_t + \beta_4 \Delta SKEWSPX_{t-1}$

$$+ \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta VVIX_{t-1} + \beta_8 \Delta VVIX_{t-2} + \epsilon_t$$

The conditional variance is specified by the EGARCH(1,1) equation $ln(h_t) = a_0 + a_1(\theta e_{t-1} + |e_{t-1}| - E(|e_{t-1}|) + b_1 ln(h_{t-1})$.

Model	α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	a_0	a_1	b_1	θ	R^2 (%)
2.1	0.006	**0.365**	**0.129**	**-0.017**	-0.013	**0.234**	0.015	**-0.148**	**0.042**	**-0.023**	**0.448**	**0.865**	**0.211**	23.2
	(0.012)	(0.017)	(0.025)	(0.008)	(0.009)	(0.007)	(0.012)	(0.024)	(0.017)	(0.007)	(0.025)	(0.008)	(0.034)	
2.2	0.004			-0.013	-0.012	**0.273**	0.002	**-0.114**	0.016	**-0.024**	**0.427**	**0.843**	**0.226**	19.0
	(0.013)	0.000	0.000	(0.007)	(0.010)	(0.008)	(0.014)	(0.025)	(0.018)	(0.007)	(0.024)	(0.009)	(0.036)	
2.3	0.000	**0.338**	**0.121**			**0.233**	0.011	**-0.144**	0.027	**-0.022**	**0.446**	**0.866**	**0.208**	22.8
	(0.013)	(0.015)	(0.025)	0.000	0.000	(0.007)	(0.012)	(0.023)	(0.017)	(0.007)	(0.025)	(0.007)	(0.034)	
2.4	0.013	**0.454**	**0.107**	**-0.023**	**-0.020**			**-0.076**	**0.042**	0.004	**0.472**	**0.864**	**0.115**	8.9
	(0.012)	(0.013)	(0.028)	(0.008)	(0.010)	0.000	0.000	(0.020)	(0.019)	(0.006)	(0.023)	(0.009)	(0.023)	
2.5	0.012	**0.366**	0.049	**-0.019**	-0.011	**0.233**	**-0.029**			**-0.028**	**0.419**	**0.859**	**0.213**	23.1
	(0.012)	(0.013)	(0.031)	(0.008)	(0.010)	(0.008)	(0.009)	0.000	0.000	(0.007)	(0.024)	(0.008)	(0.037)	

Table V Regression Results for the Determination of ΔSKEWSPX and ΔSKEWVIX

Table V reports the estimates and standard errors (in parenthesis) for the coefficients in Models 3a (Panel A) and 3b (Panel B), along with their R^2s. Estimates that are statistically significant at the 5% level are marked in bold.

Model 3a: $\Delta SKEWSPX_t = \alpha + \beta_1 \Delta SKEWVIX_t + \beta_2 \Delta SKEWVIX_{t-1} + \beta_3 \Delta VVIX_t + \beta_4 \Delta VVIX_{t-1} + \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta SKEWSPX_{t-1} + \beta_8 \Delta SKEWSPX_{t-2} + \varepsilon_t$

Model 3b: $\Delta SKEWVIX_t = \alpha + \beta_1 \Delta SKEWSPX_t + \beta_2 \Delta SKEWSPX_{t-1} + \beta_3 \Delta VVIX_t + \beta_4 \Delta VVIX_{t-1} + \beta_5 \Delta VIX_t + \beta_6 \Delta VIX_{t-1} + \beta_7 \Delta SKEWVIX_{t-1} + \beta_8 \Delta SKEWVIX_{t-2} + \varepsilon_t$

The conditional variance is specified by the EGARCH(1,1) equation $ln(h_t) = a_0 + a_1(\theta e_{t-1} + |e_{t-1}| - E(|e_{t-1}|)) + b_1 ln(h_{t-1})$.

Model	α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	a_0	a_1	b_1	θ	R^2 (%)
3a.1	0.001	0.021	-0.001	**-0.079**	-0.035	**-0.105**	-0.018	**-0.384**	**-0.248**	**0.060**	**0.301**	**0.842**	**0.086**	7.9
	(0.013)	(0.035)	(0.046)	(0.015)	(0.027)	(0.011)	(0.018)	(0.025)	(0.025)	(0.007)	(0.018)	(0.010)	(0.038)	
3a.2	0.001			**-0.074**	-0.034	**-0.104**	-0.020	**-0.386**	**-0.249**	**0.059**	**0.297**	**0.844**	**0.089**	7.9
	(0.013)	0.000	0.000	(0.015)	(0.026)	(0.011)	(0.018)	(0.025)	(0.024)	(0.007)	(0.018)	(0.010)	(0.038)	
3a.3	0.002	-0.005	-0.001			**-0.133**	**-0.035**	**-0.383**	**-0.252**	**0.058**	**0.285**	**0.849**	**0.096**	7.4
	(0.013)	(0.037)	(0.043)	0.000	0.000	(0.009)	(0.014)	(0.025)	(0.025)	(0.007)	(0.017)	(0.009)	(0.039)	
3a.4	-0.006	0.025	0.004	**-0.114**	**-0.047**			**-0.371**	**-0.245**	**0.054**	**0.291**	**0.856**	**0.108**	7.7
	(0.014)	(0.036)	(0.045)	(0.013)	(0.023)	0.000	0.000	(0.024)	(0.025)	(0.006)	(0.017)	(0.009)	(0.039)	
3a.5	0.001	0.015	-0.008	**-0.082**	0.033	**-0.107**	0.036			**0.060**	**0.294**	**0.845**	**0.095**	0.8
	(0.009)	(0.036)	(0.042)	(0.015)	(0.026)	(0.012)	(0.019)	0.000	0.000	(0.007)	(0.017)	(0.009)	(0.040)	
3b.1	0.005	-0.003	0.001	**0.102**	-0.001	**0.056**	0.007	**-0.321**	**-0.147**	**-0.012**	**0.198**	**0.983**	0.042	18.7
	(0.007)	(0.006)	(0.005)	(0.005)	(0.010)	(0.006)	(0.007)	(0.020)	(0.052)	(0.006)	(0.010)	(0.003)	(0.038)	
3b.2	0.005			**0.103**	0.000	**0.055**	0.007	**-0.321**	**-0.146**	**-0.012**	**0.197**	**0.983**	0.042	18.7
	(0.006)	0.000	0.000	(0.005)	(0.010)	(0.006)	(0.007)	(0.020)	(0.051)	(0.005)	(0.010)	(0.003)	(0.038)	
3b.3	0.003	-0.003	-0.003			**0.090**	0.007	**-0.329**	-0.099	-0.001	**0.161**	**0.990**	-0.046	14.4
	(0.006)	(0.005)	(0.005)	0.000	0.000	(0.006)	(0.007)	(0.020)	(0.057)	(0.004)	(0.009)	(0.002)	(0.035)	
3b.4	0.006	-0.002	-0.002	**0.145**	0.003			**-0.310**	**-0.196**	**-0.021**	**0.214**	**0.977**	**0.109**	18.2
	(0.007)	(0.006)	(0.005)	(0.005)	(0.010)	0.000	0.000	(0.019)	(0.022)	(0.007)	(0.012)	(0.004)	(0.042)	
3b.5	0.003	-0.002	0.004	**0.104**	**-0.047**	**0.056**	-0.008			**-0.013**	**0.202**	**0.982**	0.041	8.8
	(0.004)	(0.006)	(0.005)	(0.005)	(0.009)	(0.006)	(0.007)	0.000	0.000	(0.006)	(0.010)	(0.003)	(0.038)	

Table VI Vector Autoregression with Error Correction

Table VI reports the estimates and standard errors for coefficients in Model 4. Estimates that are statistically significant at the 1%, 5% and 10% level are marked in ***, ** and *, respectively.

Model 4: $\Delta Y_t = C + \sum_{i=1}^{p-1} \Phi \, \Delta Y_{t-i} + \Pi \, Y_{t-1} + \epsilon_t$

where $Y_t = (VIX_t, VVIX_t, SKEWSPX_t, SKEWVIX_t)$.

Equation	C	$VVIX_{t-1}$	SKEW VIX_{t-1}	VIX_{t-1}	SKEW SPX_{t-1}	$\Delta VVIX_{t-1}$	ΔSKEW VIX_{t-1}	ΔVIX_{t-1}	ΔSKEW SPX_{t-1}
VVIX	0.971	-0.001	-0.011	0.002	0.001	-0.025	-0.064*	0.015	-0.024
	(1.004)	(0.001)	(0.010)	(0.002)	(0.005)	(0.024)	(0.035)	(0.020)	(0.016)
SKEWVIX	2.614	-0.003	-0.031	0.006	0.004	0.013	-0.285***	0.001	-0.017*
	(0.611)	(0.001)	(0.006)	(0.001)	(0.003)	(0.015)	(0.021)	(0.012)	(0.010)
VIX	-1.180	0.001	0.014	-0.003	-0.001	0.026	-0.031	-0.005	-0.047**
	(1.203)	(0.002)	(0.012)	(0.002)	(0.006)	(0.029)	(0.042)	(0.024)	(0.019)
SKEWSPX	3.075**	0.009	0.013	-0.003	-0.044	-0.027	0.016	-0.032	-0.233***
	(1.311)	(0.002)	(0.013)	(0.002)	(0.007)	(0.032)	(0.046)	(0.026)	(0.021)