Using Alpha To Generate Alpha

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ABSTRACT

We test whether the alpha of an investment relative to one's existing portfolio can be used to improve out-of-sample performance (Sharpe ratio; Four-factor alpha). For the period 2000-2014, we confirm this for the Vanguard S&P 500 Index Fund and the Growth and Small Index Fund, which we extend by adding various Exchange Traded Funds. If one considers that our baseline funds may be proxies of the market portfolio, our results indirectly demonstrate that prices do not adjust (fast enough) to make those proxies mean-variance optimal, and hence, for the Capital Asset Pricing Model (CAPM) to emerge. Our findings also provide a foundation for recent studies that claim to be able to extract, from asset flows, the portfolio that investors use as benchmark.

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I. Introduction

It has long been known (Blume (1984); Dybvig and Ross (1985)) that, in principle, alpha can be used to improve the Sharpe ratio of one's portfolio. All one has to do is to *marginally change portfolio weights of individual holdings in proportion to their alphas*. Importantly, the alphas should be computed with one's own portfolio as benchmark, and not some other, arbitrary benchmark. While the approach is mathematically correct, it is not obvious that it will work in practice (though we discuss related work below). To our knowledge, nobody has provided systematic evidence whether the technique produces economically significant results. This is what we set out to test.

There are number of reasons why adjusting weights in proportion to alphas may not work in practice. Estimation error immediately comes to mind: alphas are merely estimated, and the resulting sampling error may destroy the improvement in Sharpe ratio that one could obtain if one had known the true alphas. But perhaps the most important reason is that expected returns change over time (e.g., Conrad and Kaul (1988)). By the time alphas are estimated accurately, expected returns have moved, to the extent that the obtained alphas are no longer relevant. One is effectively chasing a moving target (Gârleanu and Pedersen (2013)).

A particularly interesting case to consider, we think, is where one starts from a broadly diversified index, i.e., some proxy of the market portfolio. That is what we report on here. We shall refer to the portfolio that one obtains after adjusting weights in proportion to alphas as the *alpha-adjusted index*. We think that our case is interesting because of two reasons. (i) We know that broad indices generally are not mean-variance optimal, so that alphas of individual securities are indeed nonzero. This means that our exercise makes sense. (ii) When one takes the index as a proxy of the market portfolio, the emergence of nonzero alphas implies that the CAPM fails. If however markets constantly move *in the direction of* CAPM, then our investment strategy may fail after all. Indeed, prices adjust to ensure that alphas converge to zero (this is what it means for markets to move in the direction of CAPM).

Hence, the index one started from becomes mean-variance optimal, while the alpha-adjusted index becomes mean-variance sub-optimal. As such, one should have remained invested in the index, rather than adjusting its weights.

The latter remark suggests that our exercise could be viewed as a test of *whether markets* move towards CAPM. It is well known that CAPM fails empirically (Fama and French (1992)), but traditional tests assume that one always observes prices when the market is in equilibrium. Common sense instead suggests that markets take a long time to equilibrate, and chances that observations always coincide with equilibrium are slim. Experimental evidence confirms this: even if traditional CAPM tests may fail, markets do have a strong tendency to move towards CAPM (Bossaerts and Plott (2004); Asparouhova, Bossaerts, and Plott (2003)). Of course, real-world financial markets are more complex than laboratory markets, encounter far more friction, and participants know much less than in a controlled setting (e.g., they do not know the true distribution of future payoffs). So, additional forces may be at work which the stylized setting of the laboratory ignores. If we find that our alphaadjusted index does not improve the mean-variance efficiency of our index, one possible cause is that prices adjust in the direction of CAPM. Indeed, in that case, it is beneficial to stick to the original index, even if, based on prior return data alone, the index is inefficient (i.e., there exist nonzero alphas). One effectively lets the markets do the adjustments towards mean-variance optimality.

One way to gauge the economic significance of our exercise is to appeal to a result in Dybvig and Ross (1985). There, it is shown that, to generate a positive alpha with respect to *any* (necessarily mean-variance sub-optimal) benchmark or collection of benchmarks, it suffices to acquire a mean-variance optimal portfolio. Admittedly, our investment strategy does not guarantee full mean-variance optimality. At best, the strategy improves efficiency. Still, one can pose the following question: will improvement in mean-variance efficiency be sufficient to generate a (significantly) positive alpha with respect to benchmarks traditionally used in the academic literature? The benchmarks we have in mind are the Fama-French/Carhart four factor portfolios (Carhart (1997)). That is, we test whether our alpha-adjusted portfolio is capable of generating positive alpha with respect to the traditional Fama-French/Carhart factor portfolios.

Putting everything together, evidence that our alpha-adjusted portfolio generates positive alpha with respect to the Fama-French/Carhart model would not only demonstrate that our technique is economically relevant. It would also vindicate the claim in Dybvig and Ross (1985). At the same time, it would demonstrate that markets do not move to CAPM, or that markets move towards CAPM sufficiently slow for there to be exploitable mean-variance inefficiencies. This is exactly what we find.

To ensure that our strategy would work in practice, we do not use an academic index as benchmark (e.g., the CRSP index), but instead focus on investable indices, namely, two of Vanguard's ETF (Exchange Traded Fund) indices. In addition, we use a number of ETFs as candidate extensions of those indices. As such, our results are not only aimed at an academic audience, but should be of interest to practitioners as well.

Concurrent with our analysis, Levy and Roll (2015) have investigated alpha-based strategies for portfolio improvement. There are a number of key differences between their and our investigations. First, Levy and Roll (2015) determine whether weights on component stocks of an index can be changed in order to improve performance, while we focus on *additions* of various types of ETFs to a given index. There are two differences, as a result: (i) we look at extending the index, while Levy and Roll (2015) investigate changing weights; (ii) we consider (diversified) ETFs rather than individual stock. The latter is important because alphas of individual stock cannot be estimated precisely, while those of ETFs, because of their lower volatility, are far more precise. Second, Levy and Roll (2015) aim at improving in-sample performance: alphas are estimated over a ten-year period, new weights are computed based on those alphas, and Sharpe ratio improvements, *over the same ten-year period*, are recorded. Instead, our analysis is entirely out-of-sample: we use alphas that are estimated over the prior sixty-month period in order to determine weights to be applied over the subsequent month; we then move our sixty-month estimation window and determine weights for the next month. Etc. Third, Levy and Roll (2015) study to what extent alpha-based adjustment can provide a globally optimal portfolio, while we are merely interested in marginal improvement. Mathematically, alpha-based adjustment is meant only for marginal improvements, and then only when alphas are relatively stable over time. Levy and Roll (2015) find that, for the purpose of finding globally optimal portfolios, alpha-based adjustment does not work.¹.

If our procedure works (which it does), then the following academic exercise makes sense. Assume that investors are interested in improving the mean-variance efficiency of their portfolio. In that case, observed asset flows should correlate with alpha. If an asset has a positive alpha, then investors increase exposure, while if an asset has a negative alpha, then investors decrease exposure. We don't know which portfolio investors use as benchmark, though. Is it some market proxy? Or the Fama-French factor portfolios? One can infer the benchmark from the asset flows: the benchmark should be such that it generates positive alphas for assets toward which investors move, while it ought to generate negative alphas for assets from which investors retreat. Implications of such an exercise are discussed in Berk and Van Binsbergen (2014); Barber, Huang, and Odean (2014). The approach makes sense only if investors believe that alpha improves mean-variance efficiency. Our results suggest that such beliefs are warranted.

The remainder of the paper is organized as follows. Section II describes out empirical methodology. We present main results in Section III, and discuss the results in Section IV. Section V concludes.

¹In the spirit of Newtonian hill climbing, one should re-estimate alphas and re-determine weights after each marginal adjustment, to eventually end up with the optimal portfolio. Instead, Levy and Roll (2015) merely scale the adjustments, conjecturing that alphas do not need to be re-estimated.

II. Methods

We assume the investor starts from a benchmark index fund. Each period, she is considering several additions. So, each period, our investor is deciding how much to allocate to her benchmark index fund, and to alternative assets. Whether to invest in these alternatives will depend on their alphas, as estimated over a finite past history, with the index fund as benchmark. If alpha is estimated to be positive, the corresponding asset is *added* to the index; if the estimated alpha is negative, the corresponding asset is *shorted* (if the asset is part of the index, this effectively means that its weight is reduced). As mentioned before, the resulting portfolio will be referred to as *alpha-adjusted index*.

As benchmarks, we use various equity index funds, such as the Vanguard S&P 500. We consider Exchange Traded Funds (ETFs) as potential additional investments. Our choices ensure tradability. Indeed, funds such as the Vanguard S&P 500 are probably among the most widely used index vehicles in the marketplace, as they are available for a low management fee and, because of their liquidity, involve less trading costs. We here follow a recent trend in the academic literature (Berk and Van Binsbergen, 2014) to substitute tradeable funds for the previously more popular, but academic, factor portfolios such as the Fama-French factors. Nevertheless, we will evaluate performance of our alpha-adjusted index with respect to these academic portfolios.

There is another, no less crucial reason why we use ETFs. In contrast to individual stock, their volatility is usually much lower, and hence, alphas are estimated with more precision.

As benchmark indices, we used the following.

- Vanguard S&P500 Fund (ticker symbol: VFINX; CRSP Fund Number: 31432);
- Vanguard Growth and Small Fund (ticker symbol: VISGX; CRSP Fund Number: 31471).

ETFs data are from the CRSP Monthly Stock File. They carry Share Code 73. We applied filters to ensure ETFs to be tradable and to be liquid. Here are specifics.

- Average Daily Dollar Volume exceeds 1 million.
- ETF must have at least 72 monthly observations to be included.
- We start our sampling in 2000 (January).²
- We run our alpha-adjustment investment strategy till December 2014.

To determine the alpha-adjusted index for a particular month t, we ran a time series regression over the prior sixty months, with the excess return on a candidate investment (ETF) as dependent variable, and the excess return of the benchmark index as independent variable. We require the ETF to have at least 24 months return observations over the estimation period. Excess returns are computed relative to the one-month Treasury Bill Rate. The intercept of this regressions for ETF i, denoted alpha $\alpha_{i,t}$, is used subsequently to determine the ETF's weight $x_{i,t}$ in the alpha-adjusted portfolio, as follows:

$$x_{i,t} = \begin{cases} \frac{\alpha_{i,t}}{\sum_{\{j:\alpha_{j,t}>0\}} \alpha_{j,t}} & \text{if } \alpha_{i,t} > 0, \\ \frac{\alpha_{i,t}}{\sum_{\{j:\alpha_{j,t}<0\}} \alpha_{j,t}} & \text{otherwise.} \end{cases}$$

As a result, the month-t return on the alpha-adjusted index equals:

$$I_t + \sum_{\{i:\alpha_{i,t}>0\}} x_{i,t} E_{i,t} + \sum_{\{i:\alpha_{i,t}<0\}} x_{i,t} E_{i,t},$$

where I_t denotes the month-t return on the index, and $E_{i,t}$ is the month-t return on ETF i.

In principle, our alpha-adjustment would need plenty of rebalancing each month. In fact, as we will demonstrate, estimated alphas were quite persistent, so that monthly weight adjustments were minimal, and hence, trading costs should be reasonable.

We used the following *performance measures*. First, we looked at the cumulative wealth over the investment period January 2005-December 2014 and compared it to that of buying and holding the benchmark index. This does not correct for risk, and hence, the results

 $^{^{2}}$ ETFs only started to get popular around 2000, at which point the CRSP dataset reported on 31 funds. Given that we need 60 months to estimate alpha, this implies that the first return observation for our alpha-adjusted index is for January of 2005.

are relevant only for a risk-neutral investor. Second, we computed Sharpe ratios, which are relevant for someone with quadratic (or mean-variance) preferences. Third, we estimated alphas of our alpha-adjusted indices with respect to the Fama-French four-factor benchmark. This is relevant for the academic community, which traditionally uses the four-factor model to determine abnormal performance of an investment strategy.

Significance of improvements in Sharpe ratio relative to buying and holding the benchmark indices is determined using bootstrap estimation of the empirical distribution of Sharpe ratios. There, we randomly (with replacement) drew weight vectors $[x_{i,t}, i = 1, ..., N]$ from our histories of estimated weights, randomly permuting vector elements in order to avoid hindsight bias.³ Significance of alphas relative to the Fama-French four-factor benchmark is determined by standard time series z-statistics.

III. Results

Figure 1 plots the evolution of wealth from the beginning of our exercise (January 2005) till the end (December 2014). For both benchmark indices, wealth (blue line) increases at a much faster pace than when merely buying and holding the benchmark index (red line). This increase does come at the cost of additional volatility, but the average return more than compensates: the Sharpe ratios (at 0.23 and 0.22, respectively) are substantially higher than those for the benchmark indices (0.12 and 0.14).

Figure 2 displays the empirical distribution of the bootstrapped (10,000 times) Sharpe ratios based on random drawing and scrambling of weight vectors. In both cases, the Sharpe ratios of the alpha-adjusted benchmarks are comfortably above the 99th percentile of the empirical distribution, suggesting that they are significant at the 1% level (Alpha-Adjusted VFINX monthly Sharpe ratio = 0.226; Bootstrapped Sharpe ratio at p = 0.01 [right tail] = 0.194. Alpha-Adjusted VISGX monthly Sharpe ratio = 0.245; Bootstrapped Sharpe ratio at

³When the *t*th weight vector is drawn and applied to adjust the benchmark index for period $t - \tau$, where $\tau = 1, ..., 60$, spurious increases in the Sharpe ratio emerge because the *t*th weight vector is based on estimates of alphas over the sixty months prior to *t*.

p = 0.01 [right tail] = 0.176).

Table I presents time series regressions of excess returns on the VFINX index and the alpha-adjusted VFINX index onto the three "Fama-French factor portfolio" excess returns (Market mktrf, Size smb, Value hml) and the Carhart Momentum portfolio (umd). The intercept, i.e., the "four-factor alpha" 4FF alpha, is significantly negative for the index, and at 84bp per month, significantly positive (p < 0.01) for our alpha-adjusted index. Alpha-adjustment therefore increases alpha by a sigificant 87bp (model (3)).

Table II replicates the previous table for the VISGX index. The intercept, i.e., the "fourfactor alpha," is insignificant for the index, and at 2bp per month, significantly positive for alpha-adjusted strategy (p < 0.05). Alpha-adjustment increases alpha by 79bp (right most column).

IV. Discussion

Alpha adjustment using a selection of ETFs appears to have significant effects on the performance of the VFINX and VISGX indices. Final wealth increases dramatically, Sharpe ratios rise significantly and Fama-French Four-Factor alphas are significantly positive. The economic magnitudes of the improvements are substantial: Sharpe ratios double or increase by 1/3, respectively; alphas increase on a monthly basis by 87bp and 66bp, respectively.

Behind our "alpha adjustment" is the idea that an investment's alpha relative to one's base holdings provides an indication of whether the investment is worth adding to one's portfolio (positive alpha) or worth shorting (negative alpha). Alphas for candidate investments are generally not computed for an individual's own base holdings, but for standard benchmarks. When the standard benchmarks are closely related to the individual's own base holdings, the latter alphas may still provide a good indication on how to invest. To determine whether this is the case for our benchmarks, the VFINX and VISGX, we replicated our alpha adjustment procedure, but used Fama-French Four-Factor alphas instead of alphas relative to VFINX and VISGX. As expected, the improvements were not as good, but nevertheless quite satisfactory. See Figure 1.

In our exercise, we decided each month whether to invest in, or short, an ETF based on that ETF's alpha with respect to the benchmark index (VFINX or VISGX), as estimated over the past 60 months. It is important to note that we did not decide based on an ETF's alpha with respect to the alpha-adjusted index from the prior month. This could have been a plausible alternative, whereby one adjusts each month the alpha-adjusted index, but the alternative approach estimates alphas for a portfolio whose weights in the ETFs are noisy because these weights are based on estimated alphas. As a result, errors in estimating alphas are compounded. Not surprisingly, when we implemented the alternative approach, outperformance of the alpha-adjusted index disappeared entirely.⁴

It is worth investigating to what extent the weights on the ETF investments in our alphaadjusted index change over time. If these weights change too much, then the outperformance may be lost in transaction costs. Closer inspection of the evolution of weights suggests, however, that they change only little from one month to another, and hence, that adjustments to the alpha-adjusted portfolio are minimal. Figure 3 plots the evolution of weights for our two indices. This figure demonstrates that the weights are persistent over time.

Altogether, our findings confirm the practical validity of the alpha-based performance improvement technique advocated in Blume (1984); Dybvig and Ross (1985). Specifically, alphas are sufficiently stable over time for extensions of one's portfolio based on these alphas to lead to better out-of-sample performance (Sharpe ratios; Fama-French Four-Factor alphas).

At the same time, if the benchmarks we used (VFINX and VISGX) can be considered proxies of the market portfolio, our findings discredit the CAPM – or, at a minimum, prices do not adjust fast enough to eliminate alpha within our investment horizon, which was one month. If CAPM had consistently obtained within a month, we should not have been able to

⁴Results can be obtained from the authors upon request.

use alphas estimated from prior months and generate outperformance (higher Sharpe ratios).

Of course, it could be that CAPM does hold but our benchmarks are bad proxies for the market. It would be interesting to investigate more closely the weights on individual investments of our alpha-adjusted portfolio. Since performance (Sharpe ratios) did improve, since only the market portfolio (together with a risk free security) is optimal under CAPM, and since the market portfolio weighs all individual investments positively Green (1986), the signs of the weights in our alpha-adjusted portfolio should be positive. We leave such an exercise for future investigation.

V. Conclusion

Because of our findings, we advocate the use of alpha to obtain marginal out-of-sample improvements to one's investments. Importantly, alpha is to be measured with respect to one's own benchmark, and not to someone else's investments, or to a set of portfolios of interest to academics (e.g., the Fama-French factor portfolios).

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Figure 1. Red solid lines: evolution of wealth invested in the alpha-adjusted index (top: VFINX; bottom: VISGX), starting from one dollar; alpha adjustment is based on alphas estimated against VFINX (top) or VISGX (bottom). Blue dashed lines: evolution of wealth invested in the alpha-adjusted index (top: VFINX; bottom: VISGX), starting from one dollar; alpha adjustment is based on Fama-French Four-Factor alphas, and not alpha relative to the indices. Purple dotted line: evolution of wealth invested in the index (top: VFINX; bottom: VISGX), starting from one dollar.



Figure 2. Histograms of bootstrapped Sharpe ratios under the null hypothesis that alpha-based adjustment has no impact. Bootstraps obtained by drawing randomly from the time series of weight adjustments, randomly permuting weights across ETFs. Text annotation in the graph indicates the Sharpe Ratio of the Index (VFINX on top; VISGX on bottom), 99 Percentile of the bootstrapped empirical distribution of Sharpe Ratio, and Sharpe Ratio of alpha adjusted portfolio; Alpha adjustment is based on the alpha estimated against the indices.



Figure 3. Evolution of monthly weights on the ETFs (identified by color) in the alphaadjusted index (top: VFINX; bottom: VISGX).

Table I. Alpha-Adjusted VFINX Index.

Fama-French-Carhart Four-Factor time series regressions for different portfolios. Model (1) is for VFINX Index (Vanguard S&P500 Index Fund); Model (2) is for the Alpha-Adjusted VFINX Index; Model (3) is for the difference between Alpha-Adjusted VFINX Index and VFINX Index; Model (4) is for the Alpha-Adjusted VFINX where alphas are computed relative to the Fama-French-Carhart four factor portfolios; Finally, Model (5) is for the difference between Four-Factor Alpha-Adjusted VFINX Index and VFINX Index. Standard errors are reported in parentheses. ***: significant at p = 0.01; **: significant at p = 0.05.

	VFINX (1)	Alpha-Adjusted (2)	(2)-(1) (3)	4F-Alpha-Adjusted (4)	(4)-(1) (5)
Intercept	-0.0003^{*} (0.0002)	0.0084^{**} (0.0040)	0.0087^{**} (0.0040)	0.0057 (0.0036)	0.0060^{*} (0.0036)
mktrf	$\begin{array}{c} 0.9987^{***} \\ (0.0047) \end{array}$	$1.8950^{***} \\ (0.1084)$	$\begin{array}{c} 0.8962^{***} \\ (0.1085) \end{array}$	$1.7280^{***} \\ (0.0979)$	$\begin{array}{c} 0.7292^{***} \\ (0.0976) \end{array}$
smb	-0.1469^{***} (0.0083)	-0.3571^{*} (0.1930)	-0.2101 (0.1931)	-0.3269^{*} (0.1743)	-0.1799 (0.1737)
hml	0.0226^{***} (0.0079)	-0.0705 (0.1829)	-0.0931 (0.1830)	-0.2363 (0.1651)	-0.2589 (0.1646)
umd	-0.0012 (0.0039)	$0.1245 \\ (0.0915)$	$0.1256 \\ (0.0915)$	$0.0952 \\ (0.0826)$	0.0963 (0.0823)
Observations Adjusted R ²	$\begin{array}{c} 120\\ 0.9981 \end{array}$	$\begin{array}{c} 120\\ 0.7609\end{array}$	$\begin{array}{c} 120\\ 0.3870\end{array}$	$\begin{array}{c} 120\\ 0.7601\end{array}$	$\begin{array}{c} 120\\ 0.3270\end{array}$

Table II. Alpha-Adjusted VISGX Index.

Fama-French-Carhart Four-Factor time series regressions for different portfolios. Model (1) is for VISGX Index (Vanguard Small and Growth Index Fund); Model (2) is for the Alpha-Adjusted VISGX Index; Model (3) is for the difference between Alpha-Adjusted VISGX Index and VISGX Index; Model (4) is for the Alpha-Adjusted VISGX where alphas are computed relative to the Fama-French-Carhart four factor portfolios; Finally Model (5) is for the difference between Four-Factor Alpha-Adjusted VISGX Index and VISGX Index. Standard errors are reported in parentheses. ***: significant at p = 0.01; **: significant at p = 0.05.

	VISGX (1)	Alpha-Adjusted (2)	(2)-(1) (3)	4F-Alpha-Adjusted (4)	(4)-(1) (5)
Intercept	$0.0002 \\ (0.0009)$	0.0082^{**} (0.0032)	0.0079^{***} (0.0029)	$0.0062 \\ (0.0038)$	0.0060^{*} (0.0036)
mktrf	$\frac{1.1084^{***}}{(0.0233)}$	1.3915^{***} (0.0876)	$\begin{array}{c} 0.2831^{***} \\ (0.0789) \end{array}$	$\begin{array}{c} 1.8377^{***} \\ (0.1041) \end{array}$	$\begin{array}{c} 0.7292^{***} \\ (0.0976) \end{array}$
smb	$0.7894^{***} \\ (0.0414)$	$\begin{array}{c} 0.5036^{***} \\ (0.1559) \end{array}$	-0.2858^{**} (0.1404)	0.6095^{***} (0.1853)	-0.1799 (0.1737)
hml	-0.1440^{***} (0.0392)	-0.4303^{***} (0.1477)	-0.2863^{**} (0.1330)	-0.4030^{**} (0.1756)	-0.2589 (0.1646)
umd	-0.0069 (0.0196)	0.1428^{*} (0.0739)	0.1497^{**} (0.0665)	$0.0895 \\ (0.0879)$	$0.0963 \\ (0.0823)$
Observations Adjusted R ²	$\begin{array}{c} 120\\ 0.9749\end{array}$	120 0.7607	$\begin{array}{c} 120\\ 0.1200\end{array}$	120 0.8013	$\begin{array}{c} 120\\ 0.3270\end{array}$