# Using Equity and Index Options to Obtain Forward-Looking Measures of Beta and Idiosyncratic Variance

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# Abstract

This paper presents a parsimonious and theoretically-sound basis for extracting forward-looking measures of equity betas and idiosyncratic variance.

Defining forward-looking betas and idiosyncratic variance as perturbations of historical estimates, we use the market prices of equity and index options under a single-factor market model to compute forward-looking term structures of equity betas and idiosyncratic variance. Accordingly, we are able to discern the market's perceptions regarding these oil companies' prospective beta, and hence signaling their future sensitivity to market changes. In turn, the prospective fraction of idiosyncratic variance relative to total variance provides a forward-looking market measure for onset of crises, when idiosyncratic risk fades relative to systematic, and complementing the information conveyed by VIX and the CBOE's equity implied correlation.



Key Words: Implied volatilities, implied correlations and implied market betas

JEL Classification: G12 – Asset Pricing; G13 – Contingent Pricing

# 1 Introduction

The search for forward-looking indicators is a natural one in finance, as one of the primary roles of the discipline is to utilize market information to discern participants' views and expectations. This paper attempts to apply that principle to the analysis of implied volatility, implied correlation and implied betas, and their impact on investment analysis and practice.

Equity implied volatility dates back to Latané and Rendleman (1976).[1] Implied correlations have been more challenging, originally requiring the simultaneous pricing of individual *and* (relatively illiquid) spread options so as to permit isolation of the two vols and the implied correlation using the Margrabe (1978) formula. More recently, CBOE (2009) derived an implied correlation index requiring only the implied vols of individual stocks.

Implied equity betas are even more recent, and with several requiring alternative modifications on the "hybrid"-model use of option and historical data. Thus, French, Groth and Kolari (1983) use option-implied volatility with historical correlations. Siegel (1995) uses a hypothetical Margrabe-style exchange option to price "implicit betas." Assuming the skewnewss of the idiosyncratic shock is zero, Chang, Christoffersen, Jacobs and Vainberg (2012) use option-implied volatility and skewness measures from out-of-the-money equity and index options to derive forward-looking betas. In turn, Buss and Vilkov (2012) use forward-looking information from option prices to estimate option-implied correlations, construct option-implied predictors of factor betas and find a monotonically increasing risk-return relation. Fouque and Kolman (2011) used a continuous-time CAPM with stochastic volatilities and forward-looking betas based on second and third risk neutral moments obtained from call option prices. Finally, Broadie, Chernov and Johannes (2007) use an affine jump-diffusion model to estimate risk premia using S&P futures options.

In contrast, our approach is a distinctly different hybrid model: We use histor-

[1] Some of the earlier literature referred to this volatility with the adjective "implicit."

ical betas as well as historical idiosyncratic variances as inputs in order to obtain the *perturbations* or *adjustment factors* (to these historical measures) implied by observed equity and index option prices. It is important to note that, in so doing, the information forwarded by option prices is the information *incremental to* what is observed in the historical estimates. By using at-the-money (ATM) options, our approach obviates the need to compute skewness, an issue that can be especially acute in shorter-dated options where the vol skew is known to be more pronounced. Moreover, by utilizing the entire term structure of volatilities (for each stock and the S&P), we are able to compute a term structure of betas and idiosyncratic variances out to the most-distant option expiration date. The inclusion of time-to-maturity terms is designed to lessen any bias that would otherwise permeate the intercept terms.

Finally, it is instructive to compare and contrast the oil-company equity betas reported here with those obtained by Chang, Christoffersen, Jacobs and Vainberg (CCJV) (2012) and Buss and Vilkov (2012). CCJV generally report far-lower betas (0.33 to 0.80), but that may well be due to the earlier 1996 – 2004 time period covered in their sample: As is well-known, crude-oil prices exhibited a far-lower correlation with the S&P 500 prior to the Great Recession. Whether the heightened correlation exhibited more recently is due to the disparate perspectives of "financialization of the energy industry" or to "integrated capital markets" remains an issue to be resolved. Buss and Vilkov (2012), in turn, report a marginally higher implied beta relative to the historical beta. One of the findings of the current paper is that the relationship between historical and implied is quite sensitive to the time period being analyzed — specifically, whether we are in "crisis" or calm mode.

The paper now proceeds as follows. Considering the format of standard equity options, Section 2 demonstrates the empirical methodology by presenting the model for equity options and its corresponding econometric specification. Section 3 reports the corresponding empirical results. Section 4 concludes.

# 2 Using Equity Options to Obtain Forward-Looking Equity Betas

## 2.1 Specification of Econometric Tests — Equity Options

Consider the one-factor market-model equation,

$$R_i = a_i + \beta_{i,\,\mathrm{SPX}} R_{\mathrm{SPX}} + e_i \tag{1}$$

where we assume $\mathrm{Corr}(R_{\mathrm{SPX}},\, e_i) = 0$. Applying the variance operator to (1) yields

$$\Sigma_i^2 = \beta_{i,\,\mathrm{SPX}}^2 \sigma_m^2 + \sigma_i^2, \tag{2}$$

where

$\Sigma_i^2 \equiv \mathrm{Var}(R_i)$, the variance of the return on stock $i$

$\sigma_m^2 \equiv \mathrm{Var}(R_{\mathrm{SPX}})$, the variance of the return on the S&P 500 market index

$\sigma_i^2 \equiv \mathrm{Var}(e_i)$, the idiosyncratic variance

Eq. (5) applies to *historical* $\left\{\widehat{\Sigma}_i,\, \widehat{\beta}_{i,\,\mathrm{SPX}},\, \widehat{\sigma}_i\right\}$ data with $\widehat{\beta}_{i,\,\mathrm{SPX}} \equiv \mathrm{Cov}\,(R_i,\, R_{\mathrm{SPX}})/\mathrm{Var}\,(R_{\mathrm{SPX}})$ over some specified time interval (such as a 60-day moving window).[2]

Relationship (5) also holds *prospectively,* that is, to implied vols $\{\Sigma_i,\, \sigma_i\}$ extracted from option prices on the individual equities $i$ and the market portfolio $m \equiv$ SPX. Now consider a specification that explicitly models the relationship between historical estimates $\left\{\widehat{\beta}_{i,\,\mathrm{SPX}},\, \widehat{\sigma}_i\right\}$ and forward-looking ones $\{\beta_{i,\,\mathrm{SPX}},\, \sigma_i\}$. In theory, the difference between historical and *ex-ante* statistics arises from two sources:

1. The information set. The historical returns, variances and covariances are due to a specific realization of uncertainty. That information set is (part of) investors' perceptions of the future, but investors may and presumably do consider other sources of information in forming expectations of the future.

2. A risk premium, aka the "market price of volatility risk." Technically, implied vols are risk-neutral expectations of future realized vols, but as is well-known,

[2]Throughout this paper, variables with a "carat" ^ denote historical estimates.

there can be a non-zero market price of volatility risk that separates the statistical from the risk-neutral expectations. While the literature is not unanimous, most researchers have found risk-neutral implied vols exceed their statistical-expectations counterparts: See Jackwerth and Rubinstein (1996), Pan (2002), Bakshi and Kapadia (2003a, 2003b), Low and Zhang (2005), Doran and Ronn (2008) and Bollerslev, Gibson and Zhou (2011). Since both the LHS and RHS of eq. (5) pertain to risk-neutral expectations, the forward-looking betas we obtain here are risk-neutral.

The econometric model we posit is one which utilizes *cross-sectional daily tests* while allowing for a term structure of betas: In this test we employ a cross-sectional analysis at a given point in time, taking into explicit consideration the entire term structure of betas. Since equity implied vols are provided on a daily basis out to 24 mos. maturities, we fill in the missing observations by assuming forward vols are constant between observable expiration dates.[3]

The hybrid portion of the model links the historical estimates $\left\{\widehat{\beta}_i, \widehat{\sigma}_i\right\}$ to their forward-looking analogues $\{\beta_{iT}, \sigma_{iT}\}$ via a linear additive (3) correction: At any date $t$,

$$\begin{cases} \beta_{itT} &= \widehat{\beta}_{it} + \alpha_{1t} + \alpha_{2t}T \\ \sigma_{itT} &= \widehat{\sigma}_{it} + \alpha_{3t} + \alpha_{4t}T \end{cases} \tag{3}$$

[3]Using the principle variance is additive across maturities — since S&P returns are uncorrelated across time — for both individual stocks and market index, the algorithm is the following. Assume we observe implied vols for monthly maturities 1, 2, 3 and 6 mos.

For the maturity $m = 4$, we solve for $\sigma_4$ using the observable $\sigma_3$ and $\sigma_6$ :

$$(4/12)\sigma_4^2 = (3/12)\,\sigma_3^2 + \left[\sigma_6^2\,(6/12) - \sigma_3^2\,(3/12)\right]/3.$$

We repeat this for successive maturities out to 24 mos. for the S&P, and 18 mos. for those stocks whose actively-traded options do not extend to 24 mos.

where the coefficients $\{\alpha_{1t},\, \alpha_{2t}\, \alpha_{3t},\, \alpha_{4t}\}$ are stock-$i$ *in*dependent. Accordingly, the non-linear optimization performed to estimate the four coefficients is:

$$\min_{\{\alpha_{1t},\, \alpha_{2t}\, \alpha_{3t},\, \alpha_{4t}\}} \sum_i \sum_T \left[\Sigma_{iT}^2 - \left(\widehat{\beta}_{it} + \alpha_{1t} + \alpha_{2t} T\right)^2 \sigma_{mT}^2 - \left(\widehat{\sigma}_{it} + \alpha_{3t} + \alpha_{4t} T\right)^2\right]^2 \quad (4)$$

For each stock $i$ on date $t$ we have (at most) $T - 4 = 20$ d.f. Although we naturally retain the $i$-dependence of the historical estimates $\left\{\widehat{\beta}_{i,\mathrm{SPX}},\, \widehat{\sigma}_i\right\}$, to increase the number of degrees of freedom, we make the simplifying assumptions the estimated coefficients $\{\alpha_{1t},\, \alpha_{2t},\, \alpha_{3t},\, \alpha_{4t}\}$ are all $i$-*in*dependent: This increases the number of degrees of freedom by the number of stocks $N = 8$ to $24N - 4 = 188$.

The coefficients $\{\alpha_{1t},\, \alpha_{2t},\, \alpha_{3t},\, \alpha_{4t}\}$ should be interpreted as the information option prices provide *above and beyond* the historical information incorporated into the historical estimates of beta $\widehat{\beta}_{it}$ and idiosyncratic risk $\widehat{\sigma}_{it}$. There are at least two reasons we might be interested whether an analysis of the type eq. (4) hybrid model we are considering here gives rise to meaning/interesting results:

1. Obviously, across the entire market, the weighted average beta for any maturity $T$ is by definition 1.0. However, that does not imply a unit beta for any specific sub-section of stocks. Thus, the question is whether the beta time-to-maturity coefficient $\alpha_{4t}$ is non-zero for the oil equities under consideration here.

2. By virtue of the forward-looking nature of implied vols, the prices of options provide a contemporaneous "Message from Markets." In this case, we can examine a measure of forward-looking *idiosyncratic* risk, with respect to whatever information these contain *above and beyond* the comparable historical estimate.

## 2.2 Data

The empirical results reported here cover the period Oct. 1, 2007 – June 30, 2017. Covering the period from the pre-Great Recession S&P stock-market high (Oct. 9, 2007), the data include the Great Recession and its recovery, the period of price increase associated with the "Arab Spring" 12/1/10 – 4/1/11,[4] and the precipitous

[4]The choice of the "terminal date" April 1, 2011 will subsequently be justified.

decline in oil prices in 2014. $N = 8$ stocks are utilized. They are the stocks included in Bloomberg's BUSOILP Index: Anadarko Petroleum Corp., Apache Corp., ConocoPhillips Co., Chevron Corp., Hess Corp., Occidental Petroleum Corp., Marathon Oil Corp. and Exxon Mobil Corp.[5]

We need specify the empirical proxies for the three sets of historical variables $\left\{\widehat{\Sigma}_{it},\, \widehat{\beta}_{i,\text{SPX},t},\, \widehat{\sigma}_{it}\right\}$ as well as the nine ATM implied vols on the S&P 500 and the individual stocks $i$, $\{\sigma_{mt},\, \Sigma_{it}\}$ :

1. The historical estimates $\left\{\widehat{\Sigma}_{it},\, \widehat{\beta}_{i,\text{SPX},t},\, \widehat{\sigma}_{it}\right\}$ on each stock $i$ are obtained from 60-day rolling regressions of eq. (1) culminating on date $t$.

2. The nine ATM implied vols for maturities $T$ $\{\sigma_{mtT},\, \Sigma_{itT}\}$ are obtained from ATM options on the S&P 500 Index and the eight individual BUSOILP stocks as observed on date $t$.

## 3 Empirical Results

The empirical results pertain to the time-series of the estimated parameters for betas, $\alpha_{1t}$ and $\alpha_{2t}$, and the corresponding parameters for idiosyncratic volatility $\alpha_{3t}$ and $\alpha_{4t}$.

### 3.1 Results — $\alpha_{1t}$ and $\alpha_{2t}$

The time-series of $\alpha_{1t}$ addresses the issue of whether the option-beta is materially different from the historical beta. In turn, the estimated coefficients of $\alpha_{2t}$ speaks to the issue of whether there is, at time $t$, a statistically-significant upward- or

[5] The Anadarko Petroleum Corp., a member of the BUSOILP eight, owned a 25% interest in Deepwater Horizon, the oil rig operated by BP PLC in the Gulf of Mexico. As a consequence of the catastrophic sinking of that oil rig on April 20, 2010 Anadarko stock underperformed the BUSOILP index by as much as 42% by June 30, 2010. To eliminate the dependence on that unique event, APC was removed from the analysis effective April 1, 2010.

downward-sloping term-structure of Betas in oil stocks.[6]

Table 1 — Time-Series Analysis of $\alpha_{1t}$ and $\alpha_{2t}$
Data Period: Oct. 1, 2007 – June 30, 2017
Number of Observations: $N = 2414$

| | $\alpha_{1t}$ | $\alpha_{2t}$ |
|---|---|---|
| Oct. 1, 2007 – June 30, 2017; $N = 2414$ | | |
| Percentage Statistically Non-Zero | 0.753 | 0.541 |
| $\frac{\text{No. of Positive Coefficients}}{\text{No. of Positive Coefficients} + \text{No. of Negative Coefficient}}$ | 0.458 | 0.292 |
| Inverse-Variance $\frac{1/\hat{\sigma}^2_{\alpha t}}{\sum_t (1/\hat{\sigma}^2_{\alpha t})}$ Weighted Average | $0.118^{**}$ | $-.0048^{**}$ |
| Oct. '07 – Dec. '10; $N = 779$ | | |
| Percentage Statistically Non-Zero | 0.810 | 0.633 |
| $\frac{\text{No. of Positive Coefficients}}{\text{No. of Positive Coefficients} + \text{No. of Negative Coefficient}}$ | 0.575 | 0.371 |
| Inverse-Variance $\frac{1/\hat{\sigma}^2_{\alpha t}}{\sum_t (1/\hat{\sigma}^2_{\alpha t})}$ Weighted Average | $0.143^{**}$ | $-.0036^{**}$ |

$**$ : Significant at 1%

In weighing the import of these results, it should be borne in mind there are two possible reasons for a non-zero value to the estimated parameter values $\alpha_{1t}$ and $\alpha_{2t}$ (and, indeed, $\alpha_{3t}$ and $\alpha_{4t}$). The first is that option prices incorporate newly-arrived information that is not within the historical $\widehat{\beta}_{i,\,\mathrm{SPX},\,t}$ and $\widehat{\sigma}_{it}$ on each stock $i$ obtained from 60-day rolling regressions. And the second explanation is that there are differences between the subjective $P$ and risk-neutral $Q$ probability

[6] The statistical significance of the estimated weighted-average of $\alpha_{1t}$ and $\alpha_{2t}$ was tested using the variance of the inverse-variance weighted average of each coefficient given by their respective $1 \Big/ \sum_i 1\,/\sigma_i$ .

measures. Finally, it is critical to note that, across the entire stock-market (encompassing, but obviously not limited to, the eight oil-sector stocks analyzed here), the weighted average value of historical and forward-looking betas should both be unity — implying that, across all stocks, the weighted average of $\alpha_{1t}$ and $\alpha_{2t}$ is zero.

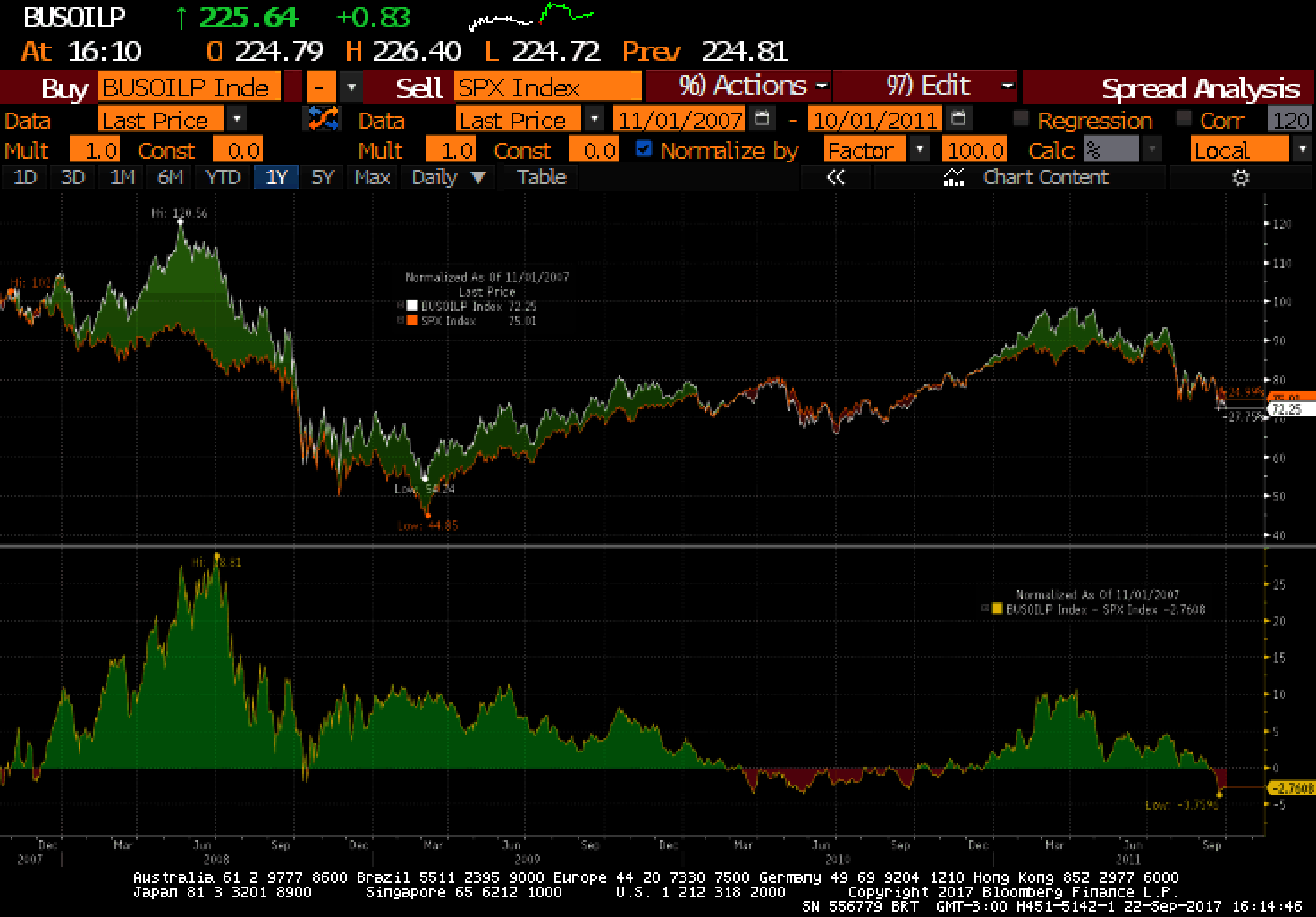
Of noteworthy interest here is the time-series of statistically-significant estimates of $\alpha_{2t}$ during two periods:

1. The period immediately at the onset of the financial crisis, and the subsequent recovery

2. The period surrounding the "Arab Spring" Dec. 2010 – April 2011

Fig. 1, extracted from the Bloomberg system, plots the relative performance of the eight-stock BUSOILP index and the S&P 500 over two critical periods: The 11/1/07 – 10/1/09 financial crisis and the 12/1/10 – 6/1/11 period during which the "Arab Spring" affected oil prices. To lend greater relevance to the data, Fig. 2 depicts the historical 60-day moving correlations between the BUSOILP index and the S&P 500.

These two graphs demonstrate the differential behavior of the BUSOILP index in these two periods: Oil equities peaked on May 20, 2007 (oil prices kept increasing until July 2008), even as the S&P began its precipitous decline; the 2011 "Arab Spring" caused oil prices to rise, as well as a short-dated negative correlation of oil equities with the S&P, until markets determined there would not be substantial contagion from North Africa to the Persian Gulf.


BUSOILP 225.64 +0.83
At 16:10 O 224.79 H 226.40 L 224.72 Prev 224.81
Buy BUSOILP Inde
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96) Actions
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Spread Analysis
Data Last Price
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Regression
Corr 120
Mult 1.0 Const 0.0
Mult 1.0 Const 0.0 Normalize by Factor 100.0 Calc % Local
1D 3D 1M 6M YTD 1Y 5Y Max Daily Table
Chart Content
Hi: 120.56
Hi: 102
Normalized As Of 11/01/2007
Last Price
BUSOILP Index 72.25
SPX Index 75.01
Low: 54.24
Low: 44.85
72.25
Normalized As Of 11/01/2007
BUSOILP Index - SPX Index -2.7608
-2.7608
Low: -3.75%
Dec Mar Jun Sep Dec Mar Jun Sep Dec Mar Jun Sep Dec Mar Jun Sep
2007 2008 2009 2010 2011
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2017 Bloomberg Finance L.P.
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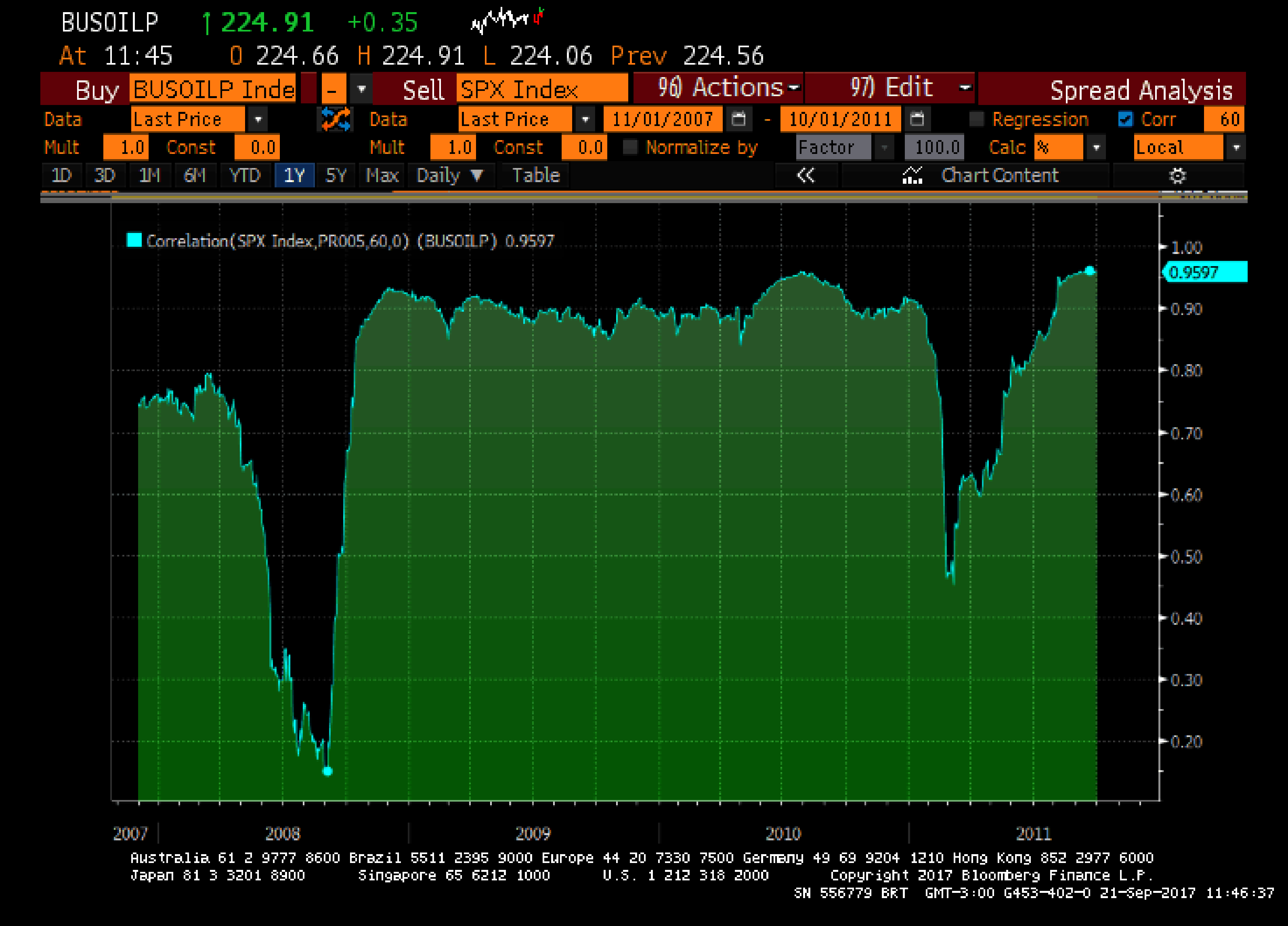
BUSOILP 224.91 +0.35
At 11:45 O 224.66 H 224.91 L 224.06 Prev 224.56
Buy BUSOILP Inde
Sell SPX Index
96) Actions
97) Edit
Spread Analysis
Data Last Price
Data Last Price 11/01/2007 - 10/01/2011
Regression
Corr 60
Mult 1.0 Const 0.0
Mult 1.0 Const 0.0
Normalize by Factor 100.0
Calc %
Local
1D 3D 1M 6M YTD 1Y 5Y Max Daily Table
Chart Content
Correlation(SPX Index,PR005,60,0) (BUSOILP) 0.9597
1.00
0.9597
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
2007
2008
2009
2010
2011
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2017 Bloomberg Finance L.P.
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Fig. 3 depicts a plot plot *forward* BUSOILP betas, computed simply as

$$\widehat{\beta}_{\text{BUSOILP}} + \alpha_{1t}$$

over the period 11/29/07 – 10/1/09. From Fig. 3, we infer two phenomena:

1. As Murphy and Ronn (2014) documented in reporting volatility of estimated parameters during the Great Recession, the market for options on crude-oil futures struggled to understand the depth and severity of the Great Recession.[7] In the current paper, with respect to the eight oil companies, this is shown in the volatile nature of the estimated $\alpha_{1t}$'s over this period.

2. A turning point appears to occur on Sep. 18, 2008, at which point the forward-looking beta is significantly greater than the moving-window historical.

Turning to the examination of the period covered by the onset of the "Arab Spring" and its attendant lower betas for oil companies, a review of Fig. 4 reveals the turning-point is Jan. 20, 2011, at which the forward beta declined more than a full point from 1.41 to 0.14.

[7] To quote from their paper on oil-futures options,

> "The market appeared to be significantly 'challenged' in understanding the ramifications to the oil price of the Great Recession. The [model's parameter] values display significant variation from one day to the next . . . ."

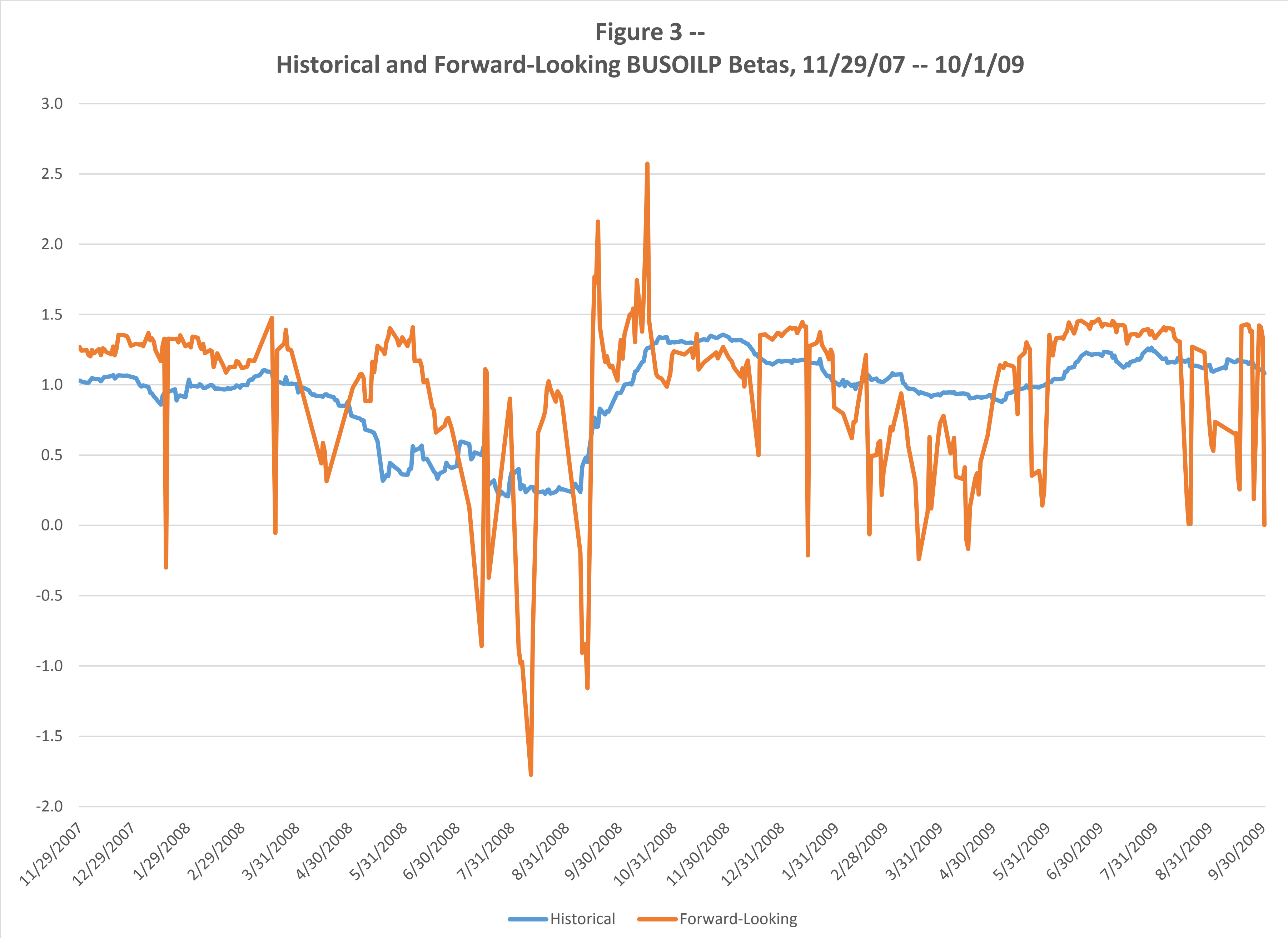


Figure 3 --
Historical and Forward-Looking BUSOILP Betas, 11/29/07 -- 10/1/09
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
11/29/2007
12/29/2007
1/29/2008
2/29/2008
3/31/2008
4/30/2008
5/31/2008
6/30/2008
7/31/2008
8/31/2008
9/30/2008
10/31/2008
11/30/2008
12/31/2008
1/31/2009
2/28/2009
3/31/2009
4/30/2009
5/31/2009
6/30/2009
7/31/2009
8/31/2009
9/30/2009
Historical
Forward-Looking

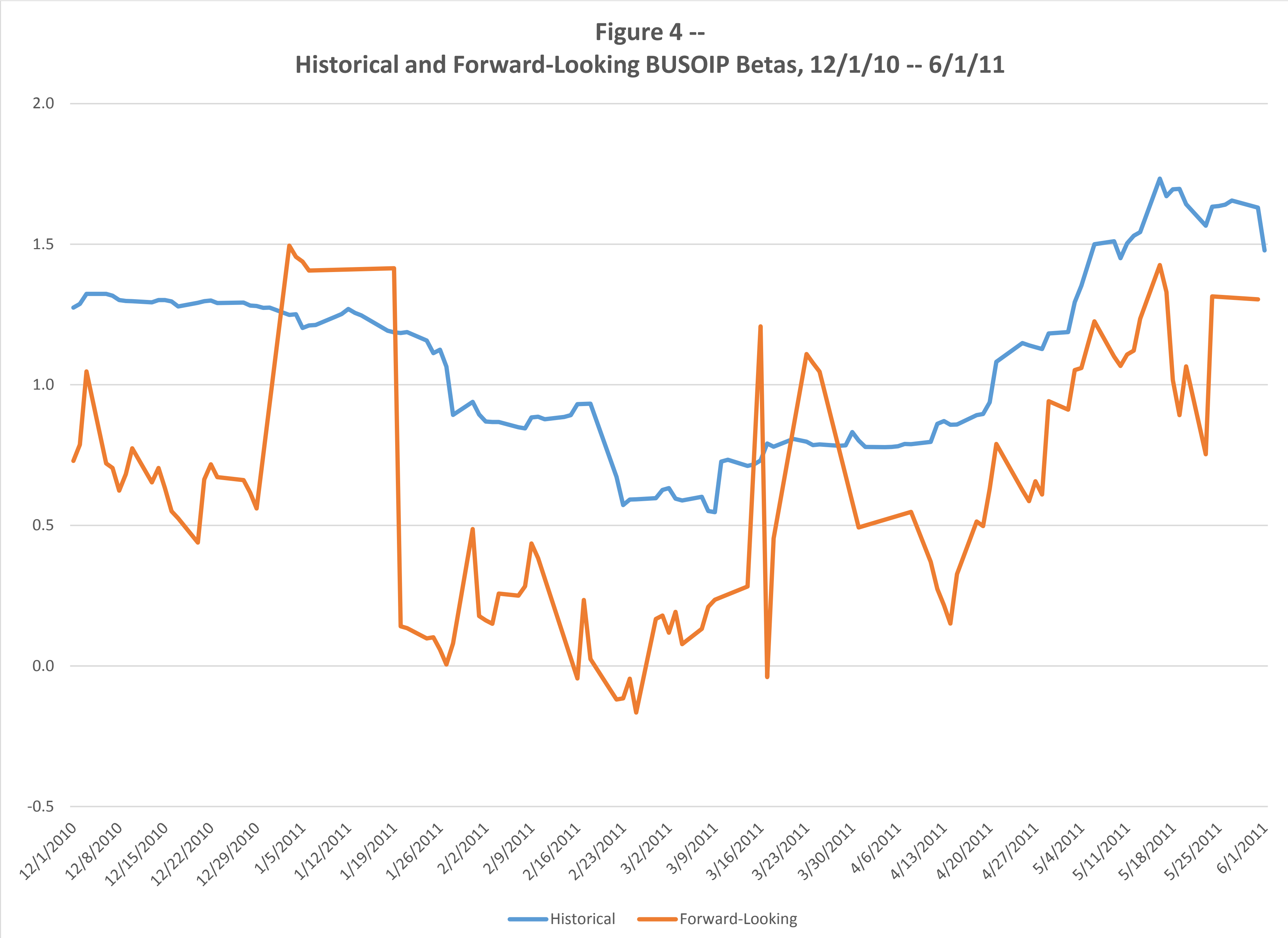


Figure 4 --
Historical and Forward-Looking BUSOIP Betas, 12/1/10 -- 6/1/11
2.0
1.5
1.0
0.5
0.0
-0.5
12/1/2010
12/8/2010
12/15/2010
12/22/2010
12/29/2010
1/5/2011
1/12/2011
1/19/2011
1/26/2011
2/2/2011
2/9/2011
2/16/2011
2/23/2011
3/2/2011
3/9/2011
3/16/2011
3/23/2011
3/30/2011
4/6/2011
4/13/2011
4/20/2011
4/27/2011
5/4/2011
5/11/2011
5/18/2011
5/25/2011
6/1/2011
Historical
Forward-Looking

### 3.2 Results — $\alpha_{3t}$ and $\alpha_{4t}$

By way of background, a timeline of the major events in Sep. 2008 includes:

1. Sep. 7: The Federal Government takes over Fannie Mae and Freddie Mac
2. Sep. 15: Lehman Brothers files for bankruptcy
3. Sep. 16: The Fed bails out insurance giant AIG

In addition to these news reports and the concurrent level/return of equity prices, there are two forward-looking market indicators for the advent of the financial crisis:

1. The level of VIX, the implied vol on the S&P 500 Index. Naturally, such an index rises during times of financial, economic or political crises. While the S&P index bottomed out during the financial crisis on March 3, 2009, VIX peaked on Nov. 20, 2008 at a level of 80.9%.[8]
2. The level of ICJ, the implied correlation in the S&P 500 Index. Assuming an identical correlation across all stocks, the implied correlation reported by the CBOE is:[9]

$$\rho_{\text{Average}} = \frac{\sigma^2_{\text{Index}} - \sum_i w_i^2 \sigma_i^2}{2\sum_i \sum_{j>i} w_i w_j \sigma_i \sigma_j}. \tag{6}$$

[8]The all-time high for equity implied vol occurred on Black Monday Oct. 19, 1987, when VIX's S&P 100 predecessor VXO spiked to 150.2%.

[9]Under appropriate assumptions, the implied correlation can easily be derived. For any portfolio, the variance property holds by definition:

$$\sigma^2_{\text{Index}} = \sum_i w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}, \tag{5}$$

where

$\sigma_i =$ volatility of asset $i$

$w_i =$ weight of asset $i$ in the index

$\rho_{ij} =$ correlation coefficient between assets $i$ and $j$

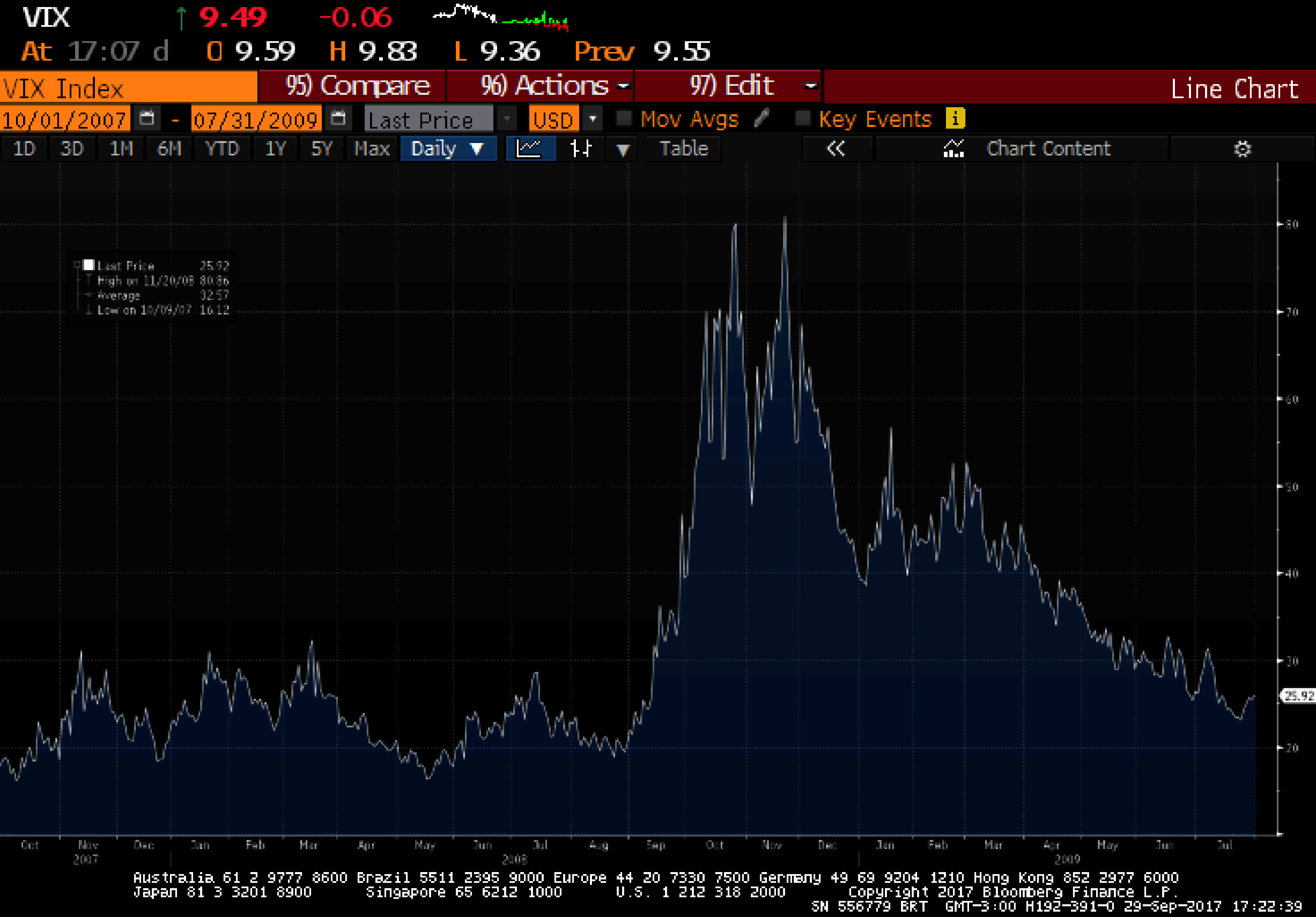
If we now set $\rho$ equal across all assets $i$ and $j$, we can solve for the $\rho_{\text{Average}}$ in eq. (6) in the text.

$\rho_{\text{Average}}$ is computed using implied vols $\sigma_i$ extracted from "SPX options prices, together with the prices of options on the 50 largest stocks in the S&P 500 Index" (`http://www.cboe.com/micro/impliedcorrelation/`). Whereas ICJ was as low as .428 on Oct. 10, 2008, it reached its financial-crisis high of 0.721 on Oct. 29, 2008.

The reason within-market correlations spike during a market crash is well-known: During a market decline, especially when it is sharp, equities' systematic risk dominates their unsystematic portions.

3. Although the following is not an equity-based measure, it is nevertheless of interest to report what was happening concurrently in the *bond market:* The Merrill Lynch C0A0 Corp. Bond Index peaked on Oct. 30, 2008 at a yield of 9.00%.

The values of the two indicia, VIX and ICJ, are reported in the next two figures, 5 and 6. These form the backdrop and benchmarks for the infomation set available in the time-series of the coefficient $\alpha_{3t}$, the values of which are designed to answer the question: By signaling forward-looking lower idiosyncratic risk lower than its historical value — i.e., $\alpha_{3t} < 0$ — when do equity options signal the arrival of a financial crisis?


VIX
9.49
-0.06
At 17:07 d
O 9.59
H 9.83
L 9.36
Prev 9.55
VIX Index
95) Compare
96) Actions
97) Edit
Line Chart
10/01/2007 - 07/31/2009
Last Price
USD
Mov Avgs
Key Events
1D 3D 1M 6M YTD 1Y 5Y Max
Daily
Table
Chart Content
Last Price 25.92
High on 11/20/08 80.86
Average 32.57
Low on 10/09/07 16.12
80
70
60
50
40
30
25.92
20
Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul
2007
2008
2009
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2017 Bloomberg Finance L.P.
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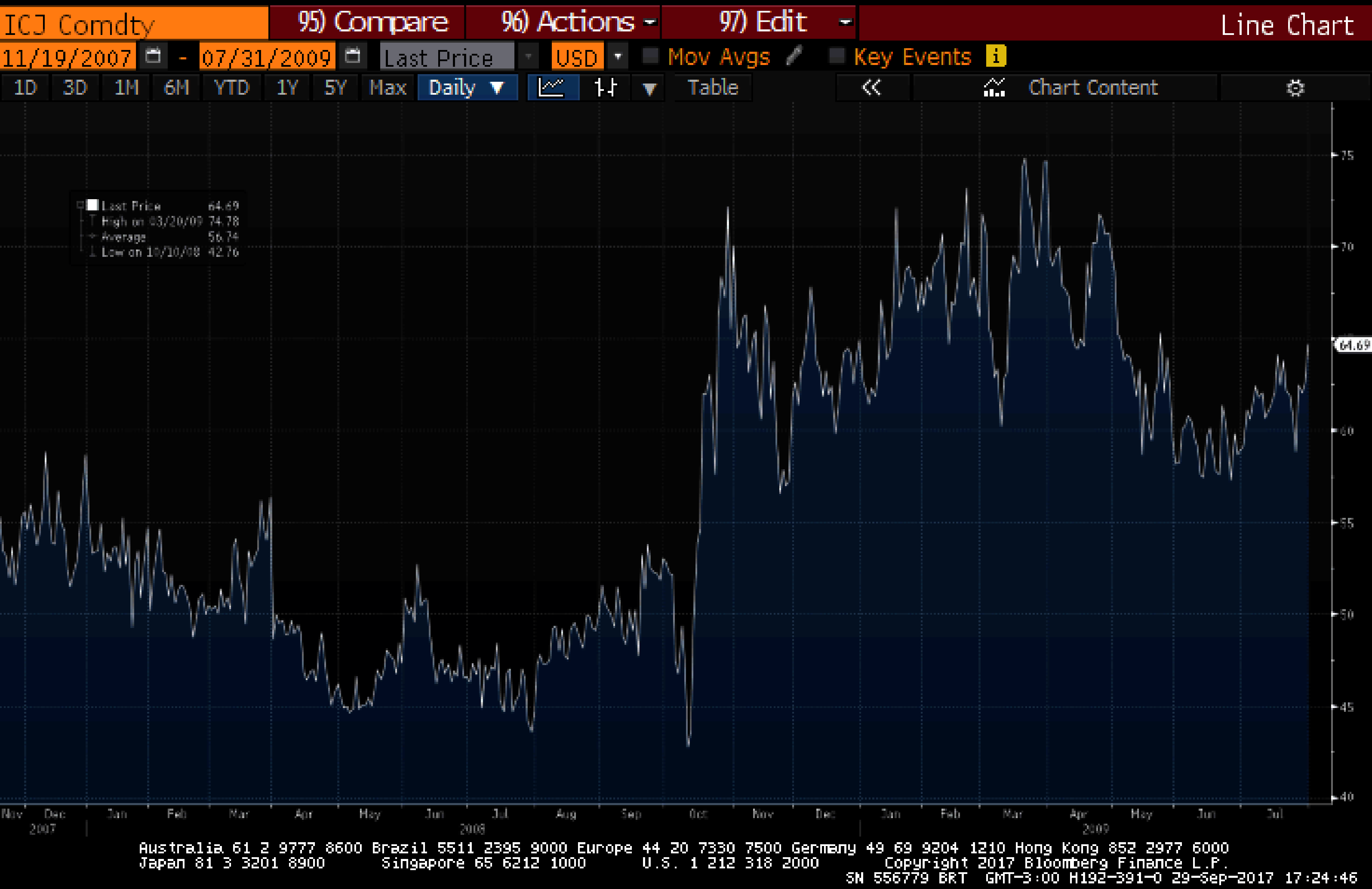
ICJ Comdty
95) Compare
96) Actions
97) Edit
Line Chart
11/19/2007 - 07/31/2009
Last Price
USD
Mov Avgs
Key Events
1D 3D 1M 6M YTD 1Y 5Y Max Daily
Table
Chart Content
Last Price 64.69
High on 03/20/09 74.78
Average 56.74
Low on 10/10/08 42.76
75
70
64.69
60
55
50
45
40
Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul
2007 2008 2009
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2017 Bloomberg Finance L.P.
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The data plotted in Fig. 7 displays $\alpha_{3t}$ over the period 11/29/07 – 7/31/09. In this time-series, we can observe several episodes of negative $\alpha_{3t}$'s: Dec. 4, 2007, Feb. 8, 2008, Aug. 21, 2008, Sep. 19, 2008, Nov. 10, 2008, Dec. 19, 2008 and Jan. 8, 2009.

In comparison, during the period of the financial crisis, VIX exceeded 30% for the first time on Sep. 15, 2008, whereas the implied-correlation value exceeded 0.5 on Oct. 16, 2008.

Of substantial interest is the behavior of $\alpha_{3t}$ on the *recovery phase* of the financial recession. As is well-known, VIX peaked on 11/20/08, and the S&P bottomed out on 3/9/09. In terms of $\alpha_{3t}$, the first large positive numbers are evidenced on 2/9/09, six weeks after the VIX peak but a month before the S&P trough.

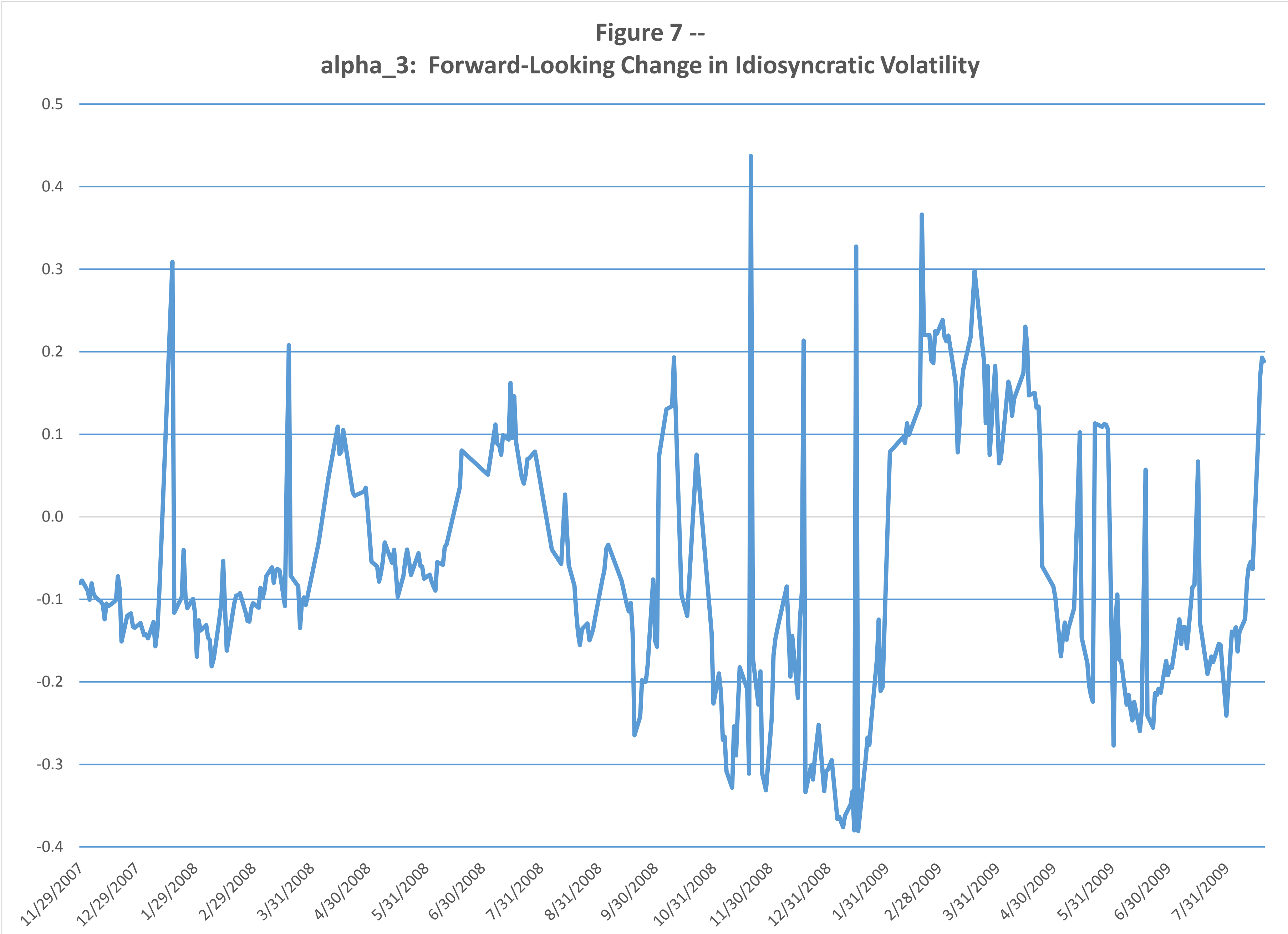
Figure 7 --
alpha_3: Forward-Looking Change in Idiosyncratic Volatility
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
11/29/2007
12/29/2007
1/29/2008
2/29/2008
3/31/2008
4/30/2008
5/31/2008
6/30/2008
7/31/2008
8/31/2008
9/30/2008
10/31/2008
11/30/2008
12/31/2008
1/31/2009
2/28/2009
3/31/2009
4/30/2009
5/31/2009
6/30/2009
7/31/2009

Revisiting the above results from a slightly different perspective, as a final test we examine the relative informativeness of historical and forward-looking idiosyncratic variances. To do so, consider the two measures denoted $\text{Stat}_1$ and $\text{Stat}_2$ :

$$\text{Stat}_{1t} = \frac{1}{8}\sum_{i=1}^{8}\left(\frac{\widehat{\sigma}_{it}}{\widehat{\Sigma}_{it}}\right)^2 \tag{7}$$

$$\text{Stat}_{2t} = \frac{1}{8}\sum_{i=1}^{8}\left(\frac{\widehat{\sigma}_{it}+\alpha_{3t}}{\Sigma_{it,\,T=2/12}}\right)^2 \tag{8}$$

$\text{Stat}_{1t}$ should be interpreted as the average fraction of historical idiosyncratic variance relative to historical total variance, whereas $\text{Stat}_{2t}$ computes the numerator as a forward-looking measure relative to the two-mo. implied vol $\Sigma_{it,\,T=2/12}$.

Fig. 8 depicts the time-series of these two variables. While we observe substantial variation in $\text{Stat}_{2t}$ relative to $\text{Stat}_{1t}$, note that beginning in the critical month of Sep. 2008, $\text{Stat}_{1t}$ declined slowly from 97.3% to an eventual low of 14.4% on Nov. 26, 2008, whereas the forward-looking $\text{Stat}_{2t}$ broke sharply on Sep. 12. By Sep. 22 $\text{Stat}_{2t}$ had already reached 25.8%, whereas $\text{Stat}_{1t}$ remained at a still-elevated 68.0%.

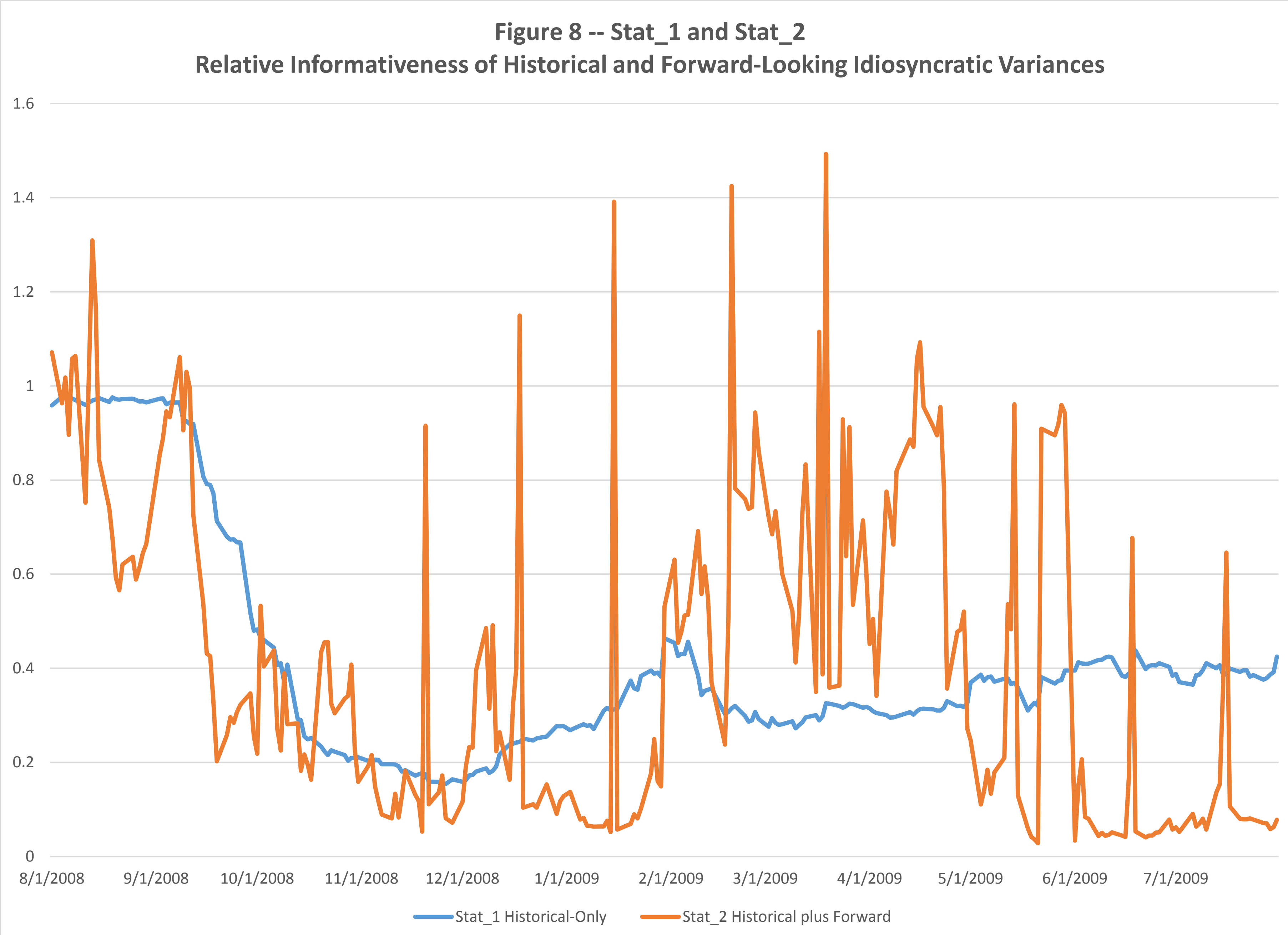
Figure 8 -- Stat_1 and Stat_2
Relative Informativeness of Historical and Forward-Looking Idiosyncratic Variances
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
8/1/2008
9/1/2008
10/1/2008
11/1/2008
12/1/2008
1/1/2009
2/1/2009
3/1/2009
4/1/2009
5/1/2009
6/1/2009
7/1/2009
Stat_1 Historical-Only
Stat_2 Historical plus Forward

# 4 Conclusions

The paper has documented the potential for using options on equities and a market index to infer forward-looking statistics of relevance to investors and portfolio managers: Applying a single-factor model to equity and index options, we were able to use historical data parsimoniously to obtain meaningful forward-looking equity betas and idiosyncratic variances. In comparing and contrasting these signals with other forward-looking measures, such as VIX and ICJ, we are able to extract what might be termed the "Message from Markets."

The work here admits of extensions in several interesting complementary directions:

1. Add additional stocks, so as to permit credible and accurate measurement of the slope and intercept coefficients
2. Extend the model from one-factor (the stock market) to a two-factor (the second factor is the return on oil-futures contracts) model: A method for confirming the robustness of the forward-looking equity betas and idiosyncratic variance
3. Extend the work to apply the same model to correlations and betas of a traded index — such as an oil-futures price — to extract meaningful forward-looking information.

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