Present value, commodity prices and exchange rates.

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Abstract

Both real exchange rates and real commodity prices are forward looking and reflect information about future economic conditions. This paper applies the present value approach to decompose contributions about future inflation, interest rate, or their differentials, and excess returns to both variables and to analyze the degree of their co-movement. It is found that some currencies are correlated with commodity prices mostly through expectations about future risk premiums and convenience yields. This is a novel explanation of the link between commodity prices and exchange rates, which is traditionally explained by terms-of-trade or inetreset rate differentials in the existing literature. The paper estimates a dynamic factor model applied jointly to 10 commodity prices and 7 exchange rates excess returns and recovers a common factor that explains the substantial co-movement among them.

Keywords: present value, commodity prices, exchange rates, excess returns

JEL: F31, F41, G15

1. Introduction

The present value approach is widely used in asset pricing. It says that an asset price is the present value of future "fundamentals."

The approach has being applied to commodity prices. Pindyck (1993) presents commodity prices as present value of future convenience yields which arise from benefits of holding inventories. Inventories can be viewed as insurance against future fluctuations in demand for final products and allow to smooth production by holding a reserve supply of primary commodities which are used as inputs. Gospodinov and Ng (2013) find that two principal components recovered from convenience yields of 23 commodities have predictive power for inflation. Convenience yields are viewed as variables that reflect information about future economic conditions.

The application of present value approach to exchange rates is substantially broader. Campbell and Clarida (1987) iterate forward the relationship between expected real exchange rate changes, expected real

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interest rate differentials, and risk premium. This allows to present real exchange rates as the expectation of the long-run equilibrium value, expected real interest rate differential, and expected risk premiums. Engel and West (2005) apply the present value approach to exchange rates and consider various fundamentals based on models of exchange rate determination. They use the model to explain why exchange rates are disconnected from fundamentals, and find that exchange rates help predicting these fundamentals in line with the present value approach. Froot and Ramadorai (2005) decompose currency returns into intrinsic-value shocks and expected returns shocks and find that currency flows are related to the latter. Engel (2016) postulates a puzzle in the relationship between currency risk premiums and interest rate differentials. On one hand, countries with higher interest rates have exchange rates that appreciate over short horizons. This is consistent with those currencies being relatively risker and demanding a positive risk premium. On the other hand, countries with high interest rates have currencies stronger in levels that what is implied by expected interest rate differentials. This implies that those currencies have lower risk premium. Dalhquist and Penasse (2022) propose to resolve the puzzle by incorporating the real exchange rate as the missing risk premium. Stronger currencies predict lower excess return in the long-run as exchange rates revert to the long-run equilibrium implied by the Purchasing Power Parity (PPP).

The recent literature investigates the role of convenience yield in explaining movements in exchange rates. Jiang et al. (2021) link exchange rates to the convenience yields foreign investors gain from holding safe U.S. assets. The paper derives the real exchange rate as the expected value of future interest rate differentials and convenience yields and finds that the convenience yield accounts for a substantial portion of exchange rates variation. U.S. safe assets play a special role in the International Monetary System. Engel and Wu (2023) emphasize that the liquidity yield is not just a U.S. story and find evidence for all G10 currencies.

In this paper I apply the present value approach to both commodity prices and exchange rate to investigate the source of high correlation between both. The high correlation is especially apparent for the so-called commodity currencies, the exchange rates of commodity exporting countries like Canada, Australia and New Zealand (Chen and Rogoff, 2003). Given that both exchange rates and commodity prices are forward-looking and reflect information about future fundamentals, a question arises if prices of currencies and commodities are correlated because they reflect the same information about the future.

The two most relevant papers to the current study are Chen et al. (2010) and Devereux and Smith (2021). Both consider the same question of why commodity currencies and commodity prices are highly correlated. Both apply the present value concept to exchange rates while treating commodity prices as exogenous. The latter assumption is relaxed in the current paper as I treat both commodity prices and exchange rates as endogenous. Chen et al. (2010) present nominal exchange rates as present value of future commodity prices for commodity exporting countries because commodity prices deflated by U.S. CPI represent terms of trade for these countries. Given that terms of trade react differently to supply and demand shocks and given that prices of differentiated products are sticky, commodity prices are better for empirical analysis than using terms of trade directly. Commodity prices are given exogenously for Australia, New Zealand, and Canada since these countries do not have impact on prices of most commodities even though these countries are big exporters of primary commodities. Commodity prices are also flexible so price stickiness does not play a role in obscuring terms of trade movements. Chen et al. (2010) find that exchange rates predict commodity prices in sample and out of sample, consistent with the implication of the present value approach. Bork et al. (2022) show that the predictability is driven by the use of commodity prices that are averages of daily values. Once more appropriate end-of-period measures are used, the result of predictability is weak at best. Devereux and Smith (2021) argue that commodity currencies are not that different from other currencies in the sense that they are present value of future interest rate differentials. As commodity prices are rising, monetary authorities in commodity exporting countries are expected to tighten monetary policy. The exchange rate appreciates today based on this expectation. This explain the correlation between commodity currencies and commodity prices through differences in expected interest rates.

I present real commodity prices as expected future U.S. interest rates, U.S. inflation, risk premiums, and convenience yields. The explicit assumption is that it is the U.S. monetary policy and U.S. inflation that matter for commodity prices denominated in U.S. dollars.[1] Real exchange rates are presented as expected future interest rate differentials, inflation differentials, risk premiums and convenience yields. I decompose the correlation between exchange rates and commodity prices into the contribution of future real interest rate differentials and future risk premiums and convenience yields. Seven currencies (Australian, Canadian and New Zealand's dollars, the euro, the Swiss franc, the British pound, and the Japanese yen all versus the U.S. dollar) and ten commodities are analyzed over the period from January of 1995 to March of 2024. Several currencies (Australian and New Zealand dollars, the euro and the Swiss franc) are correlated with commodity prices mostly through expectations about future risk premiums and convenience yields. This channel is a novel contribution to the literature. This is different from existing studies emphasising the link between commodity prices and exchange rates through the terms of trade (Chen and Rogoff, 2003; Chen et al., 2010; Ayres et al., 2020) or due to non-U.S. monetary policy (Devereux and Smith, 2021).

Given that commodity prices and exchange rates potentially reflect the same information about future economic conditions, this paper estimates a dynamic factor model applied jointly to all 10 commodity prices and 7 exchange rates. The recovered factor is statistically significant for almost all commodity prices and exchange rates and explains a substantial portion of co-movement among exchange rates and commodity prices. The estimation of the factor model is relevant to the literature identifying common factors in commodity prices (West and Wong, 2014; Chiaie et al., 2022) and exchange rates (Verdelhan, 2018; Greenway-McGrevy et al., 2018). Comparing to this literature, I identify a factor in commodity excess returns jointly with

[1] This is consistent with the analysis of Frankel (2006) who first consider U.S. interest rates as impacting commodity prices before adding interest rate differentials when considering commodity prices in local currencies. This is also consistent with Miranda-Agrippino and Rey (2020) who put the U.S. monetary policy at the center of the Global Financial Cycle.

currency excess returns while controlling for other variables like interest rates and real commodity prices since the analysis of this paper is driven by the present value representation. Liu et al. (2020) show that the average of monthly commodity returns is useful in predicting levels and excess returns of commodity currencies. This paper compliments their analysis by showing that the same factor existing in commodity excess returns is also present in currency excess returns and that the link goes beyond commodity currencies and applies to other exchange rates as well.

The paper is organized as follows. Section 2 outlines the present value approach applied to commodity prices and exchange rates. Given insights from the present value approach, Section 3 develops a methodology how to assess testable implications of the theory. Section 4 presents data and conducts an empirical analysis of the testable implications. Section 5 concludes.

2. Present value approach

2.1. A model of commodity prices

Consider a strategy that borrows and buys commodity i at the log price p_{it} to be held for one period and sold at the price p_{it+1}. The log excess return for this strategy is:

$$prx_{it+1} = p_{it+1} - p_{it} - r_t, \tag{1}$$

r_t is the nominal interest rate. The conditional expectation of the log excess return, $E_t(prx_{it+1})$, can be decomposed into the risk premium, ρ_{it}, and the convenience yield (net of storage costs), $\psi_{p,it}$, as follows:

$$E_t(prx_{it+1}) = \rho_{p,it} - \psi_{p,it},$$

where $\rho_{p,it} = E_t(p_{it+1}) - f_{p,it}$ is a risk premium, which risk-averse investors earn for the fluctuations in the future log spot price relative to the log forward price f_{it} of commodity i to be delivered in the next period. The convenience yield, $\psi_{p,it}$, reflects convenience drawn from holding a commodity to smooth production. This convenience yield can be calculated based on observed forward and spot rates as $-\psi_{p,it} = f_{p,it} - p_{it} - r_t$.

Let p_t denote the overall log price index and the tilde symbol above a variable to denote the real equivalent:

$$\tilde{p}_{it+1} = p_{it+1} - p_{t+1}.$$

Equation (1) can now be represented as:

$$prx_{it+1} = \tilde{p}_{it+1} - \tilde{p}_{it} + \pi_{t+1} - r_t, \tag{2}$$

where π_{t+1} is inflation in the general price level, $p_{t+1} - p_t$.

Taking conditional expectation of this equation and iterating forward, as in Campbell and Clarida (1987) and others, results in the following expression:

$$\tilde{p}_{it} - w_{it} = -\sum_{l=1}^{\infty} E_t(prx_{it+l}) + \sum_{l=1}^{\infty} E_t(\pi_{t+l}) - \sum_{l=0}^{\infty} E_t(r_{t+l}), \tag{3}$$

where $w_{it} = E_t(\lim_{l\to\infty} \tilde{p}_{it+l})$ is the long-run equilibrium of the real commodity price in expectation. This limit is assumed to exist as in Frankel (2006). Equation (3) indicates that the real price for commodity i is the undiscounted sum of expected future excess returns and real interest rates. The real commodity price increases relative to its long-run equilibrium if we expect subsequently lower excess returns in the future, if inflation expectations are revised upward, or we expect lower interest rates in the future. The first term in the equation represent the overshooting mechanism in Frankel (2006) which is similar to overshooting in exchange rates as in Dornbusch (1976). Commodity prices first increase if they are expected to go down over time.

2.2. A model of exchange rates

A strategy involving exchange rates is to borrow in U.S. dollars (assuming the USA is the home country) at r_t and invest in foreign currency j which earns the nominal interest rate r^*_{jt}. An asterisk symbol above variables indicates a foreign country. The log excess return of this strategy is:

$$rx_{jt+1} = s_{jt+1} - s_{jt} + r^*_{jt} - r_t, \tag{4}$$

where s_{jt} is the log U.S. price of foreign currency j (an increase in s_{jt} indicates depreciation of the U.S. dollar). The expected excess return conditional on information at time t can be similarly decomposed into risk premium, $\rho_{s,jt} = E_t(s_{jt+1} - f_{s,jt})$, where $f_{s,jt}$ is the log forward rate for currency j to be delivered in the next period, and the convenience yield differential, $\psi_{s,jt} = f_{s,jt} - s_{jt} + r^*_{jt} - r_t$. Engel and Wu (2023) interpret the latter as the payoff of a synthetic home government bond composed by buying the foreign government bond (offering r^*_{jt} return) and signing a forward contract to eliminate exchange rate risk minus the domestic government bond (offering r_t). Thus, the term represents the relative difference in liquidity offered by both bonds.[2]

Reworking equation (4) in real terms results in the following expression:

$$rx_{jt+1} = \tilde{s}_{jt+1} - \tilde{s}_{jt} + r^*_{jt} - r_t - \pi^*_{jt+1} + \pi_{t+1}, \tag{5}$$

where $\tilde{s}_{jt} = s_{jt} + p^*_{jt} - p_t$ is the real exchange rate.

Iterating equation (5) forward results in the following relationship:

[2]Convenience yield is often called "liquidity yield" in international finance.

$$\tilde{s}_{jt} - u_{jt} = \sum_{l=0}^{\infty} E_t(r^*_{jt+l} - r_{t+l}) + \sum_{l=1}^{\infty} E_t(\pi_{t+l} - \pi^*_{jt+l}) - \sum_{l=1}^{\infty} E_t(rx_{jt+l}), \tag{6}$$

where $u_{jt} = E_t(\lim_{l\to\infty} \tilde{s}_{jt+l})$ and it is assumed that this limit exists and is constant as in Engel (2016). The real exchange rate depreciates if there is an expectation of relatively higher interest rates abroad, if domestic inflation is expected to be relatively higher, and if expected returns are revised downwards.

3. Methodology

I analyze the implications of the present value approach for both commodity prices and exchange rates along several dimensions. The first questions is whether there is evidence that excess returns are predictable ahead of time. If predictability is found it will imply that prices of commodities and exchange rates move by more than implied by the cost of carry and that time-varying risk premiums and convenience yields are present. The predictability of excess returns is also needed to decompose commodity prices and exchange rate into expected real interest (differentials) and excess returns according to (3) and (6). A VAR model is used for the purpose of constructing expected variables. After analysing commodity prices and exchange rates separately, the paper proceeds by looking at both excess returns (prx_{it} and rx_{jt}) jointly.

3.1. No predictability of excess returns

Both excess returns are regressed on a set of lagged variables to investigate if there is any predictability. The objective here is not to include all possible variables that may be useful but rather to investigate if the null of no predictability can be rejected with a limited set of observables.

$$prx_{it} = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 \tilde{p}_{it-1} + \alpha_3 \pi_t + \varepsilon_{it}.$$

The inclusion of $\tilde{p}_{it-1}$ is motivated by equation (3). Real commodity prices should predict expected excess returns and demonstrate mean-reverting property. The equation for currency excess returns:

$$rx_{jt} = \beta_0 + \beta_1(r^*_{jt-1} - r_{t-1}) + \beta_2(\pi_{t-1} - \pi^*_{jt-1}) + \beta_3 \tilde{s}_{jt-1} + \epsilon_{jt}.$$

Regressions of currency excess returns on the interest differential are known as Fama regressions (Fama, 1984). The extension of these regressions to include real exchange rates is examined in Balduzzi and Chiang (2020); Dalhquist and Penasse (2022).

Testing predictability one month ahead could be asking too much of data as both exchange rates and commodity prices are known to be volatile. Consistent with the literature, I also consider predictability at longer horizons. The dependent variables are cumulative excess returns $\sum_{l=0}^{11} prx_{it+l}$ and $\sum_{l=0}^{11} rx_{jt+l}$. The same regressors as for one month ahead regressions are considered.

3.2. VAR-based predictions

Equations (3) and (6) can be used to decompose commodity prices and exchange rates, respectively, into expectations about future fundamentals and excess returns. Following Balduzzi and Chiang (2020), I adopt VAR(1) for this purpose:

$$z_t = Az_{t-1} + v_t. \tag{7}$$

The VAR is assumed to meet the stability condition and all variables are stationary and demeaned. The following sets of variables are considered:

$$\begin{aligned} z_{it} &= [prx_{it}, r_t, \pi_t, \tilde{p}_{it}]^\top \quad \text{for commodity } i \\ z_{jt} &= [rx_{jt}, r^*_{jt} - r_t, \pi_t - \pi^*_{jt}, \tilde{s}_{jt}]^\top \quad \text{for currency } j \end{aligned}$$

Using the VAR model, equation(3) can be represented as:

$$\tilde{p}_{it} - w_{it} = \mathbf{PRX}_t + \mathbf{\Pi}_t + \mathbf{R}_t,$$

where

$$\begin{aligned} \mathbf{PRX}_t &= -\sum_{l=1}^{\infty} E_t(prx_{it+l}) = -e'_{prx}A(I-A)^{-1}z_{it}, \\ \mathbf{\Pi}_t &= \sum_{l=1}^{\infty} E_t(\pi_{t+l}) = e'_{\pi}A(I-A)^{-1}z_{it}, \\ \mathbf{R}_t &= -\sum_{l=0}^{\infty} E_t(r_{t+l}) = -e'_{r}(I-A)^{-1}z_{it}, \end{aligned}$$

where e_k, $k = [prx, \pi, r]$ is the selection vector. The long-run equilibrium w_{it} is assumed to be constant and, since the variables are demeaned, is equal to zero.

The presentation allows to present the variance of $\tilde{p}_{it}$ as the contribution of expected excess returns, interest rate, and inflation:

$$\frac{cov(\tilde{p}_{it}, \mathbf{PRX}_t)}{var(\tilde{p}_{it})} + \frac{cov(\tilde{p}_{it}, \mathbf{\Pi}_t)}{var(\tilde{p}_{it})} + \frac{cov(\tilde{p}_{it}, \mathbf{R}_t)}{var(\tilde{p}_{it})} = 1. \tag{8}$$

Similarly, the variance of the real exchange rate can be decomposed into contributions of expected excess returns, and inflation and interest rate differentials:

$$\frac{cov(\tilde{s}_{jt}, \mathbf{RX}_t)}{var(\tilde{s}_{jt})} + \frac{cov(\tilde{s}_{jt}, \mathbf{\Pi_{diff}}_t)}{var(\tilde{s}_{jt})} + \frac{cov(\tilde{s}_{jt}, \mathbf{R_{diff}}_t)}{var(\tilde{s}_{jt})} = 1, \tag{9}$$

where $\mathbf{RX}_t = -\sum_{l=1}^{\infty} E_t(rx_{jt+l})$, $\mathbf{\Pi_{diff}}_t = \sum_{l=1}^{\infty} E_t(\pi_{t+l} - \pi^*_{jt+l})$, and $\mathbf{R_{diff}}_t = \sum_{l=0}^{\infty} E_t(r^*_{jt+l} - r_{t+l})$.

3.3. Comovement between currency and commodity excess returns

The unified approach to present both commodity prices and exchange rates as present values of future fundamentals and risk premiums allows to disentangle the channels by which commodity prices and exchange rates are interconnected. One possibility is that commodity prices and exchange rates are driven by monetary policy and especially by U.S. monetary policy. The other possibility is that both variables are affected by common risk premiums and liquidity (convenience) yields.

The presentation of commodity prices and exchange rates in equations (3) and (6) allows to decompose the covariance between the real commodity prices and real exchange rates as:

$$cov(\tilde{p}_{it}, \tilde{s}_{jt}) = cov(\tilde{p}_{it}, \Re_{\mathbf{diff}\,jt}) + cov(\tilde{p}_{it}, \mathbf{RX}_{jt}), \tag{10}$$

where $\Re_{\mathbf{diff}\,jt} = \mathbf{R}_{\mathbf{diff}\,jt} + \mathbf{\Pi}_{\mathbf{diff}\,jt}$ is the expectation of future real interest rates. The first channel represents the covariance between real commodity prices and future real interest differentials. This is the channel emphasized in Devereux and Smith (2021). The second channel represents the covariance through risk premiums and/or convenience yields.

3.4. Dynamic factor model

Given that there is a substantial comovement among currency and commodity excess returns, this subsection investigates the possibility that there exists a common factor in both excess returns. The following dynamic factor model is estimated jointly for all 10 commodities and 7 exchange rates:

$$\begin{aligned}
prx_{it} &= \alpha_{fi} f_t + \alpha_{1i} r_{t-1} + \alpha_{2i} \tilde{p}_{it-1} + \alpha_{0i} + \varepsilon_{it}, \\
rx_{jt} &= \beta_{fj} f_t + \beta_{1j}(r^*_{jt-1} - r_{t-1}) + \beta_{2j} \tilde{s}_{jt-1} + \beta_{0j} + \epsilon_{jt}, \\
f_t &= \gamma f_{t-1} + v_t, \qquad v_t \sim N(0, \sigma_v^2), \\
\varepsilon_{it} &= \varrho_i \varepsilon_{it-1} + u_{it}, \qquad u_{it} \sim N(0, \sigma_{ui}^2), \\
\epsilon_{jt} &= \varphi_j \epsilon_{jt-1} + \zeta_{jt}, \qquad \zeta_{jt} \sim N(0, \sigma_{\zeta j}^2), \\
i &\in [1, 10] \text{ of commodities, } j \in [1, 7] \text{ of currencies,}
\end{aligned}$$

where f_t is the unobserved common factor which is assumed to follow an AR(1) process. The disturbances v_t, u_{it}, ζ_{jt} are assumed to be uncorrelated across units and time. The model is set up in the state-space form and estimated by Maximum Likelihood by using the Kalman filter with the De Jong (1988) method for estimating initial values.

4. Empirical analysis

4.1. Data

The frequency of data are monthly. All variables except for Consumer Price Indices (CPI) are end-of-month values of daily data.

Interest rates are shadow short rates (Krippner, 2013, 2015).[3] The framework allows to measure stance of monetary policy when interest rates are near zero lower bound. The data cover the period from January, 1995 to May, 2024 for the United States of America, the Euro-zone, Japan, the United Kingdom, Switzerland, Canada, Australia and New Zealand.

Exchange rates are U.S. dollar per domestic currency, end of month, from the IMF IFS. The exchange rate for the Euro starts on January, 1999.

Prices are CPI, all items, from the IMF IFS with the exception of the Euro zone. Consumer price indices for Australia and New Zealand are available at the quarterly frequency only. They are converted to the monthly frequency using cubic interpolation. The consumer price index for the Euro zone is the harmonized CPI from the IMF IFS.

Commodity prices for crude oil, cotton, aluminum, copper, gold, hogs, sugar, lead, silver and zinc in nominal US dollars are from the Datastream Commodities Data assessed through Wharton Research Data Services. The choice of commodities is motivated by data availability. Appendix A provides the mnemonics of variables.

4.2. Descriptive statistics

Table 1(a) presents descriptive statistics for commodity excess returns. The excess returns for oil and hogs are the most volatile followed by cotton, silver and sugar. Many commodity excess returns exhibit mild autocorrelations with the correlation coefficients ranging from -0.09 for gold to 0.14 for aluminum. The first order autocorrelation is significant for 5 out of 10 commodities at the 10% significance level. Real commodity prices for oil, copper, and gold are twice more volatile than the rest of commodities. All real commodity prices are very persistent with the coefficient in the 0.88 to 0.99 range.

Table 1(b) presents descriptive statistics for currency excess returns. Currency excess returns have smaller volatility and exhibit no significant autocorrelation of the first order. In this regard, the behaviour of currency returns is somewhat different from commodity returns. Real exchange rates show substantial persistence with the autocorrelation coefficients in the range from 0.97 to 0.98.

[3]The data are downloaded on Jun. 13, 2024 from https://www.ljkmfa.com

Table 1: Descriptive statistics of excess returns and real prices

(a) Commodities

	OIL	ALM	COP	GLD	CTN	ZNC	SLV	LED	SGR	HGS
					Excess Returns					
mean	0.0027	-0.0013	0.0018	0.0035	-0.0023	0.0009	0.0037	0.0018	-0.0007	-0.0001
Std. Dev.	0.1077	0.0613	0.0722	0.0444	0.0853	0.0766	0.0835	0.0785	0.0843	0.1016
AC(1)	0.1278	0.1470	0.1031	-0.0939	-0.0300	0.0005	-0.0660	-0.0178	0.1182	-0.0351
p-value	0.0160	0.0055	0.0517	0.0764	0.5711	0.9923	0.2129	0.7377	0.0258	0.5072
					Real Commodity Prices (demeaned)					
mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.4703	0.2239	0.4876	0.5314	0.2964	0.3401	0.5175	0.4879	0.3233	0.2135
AC(1)	0.9716	0.9531	0.9887	0.9935	0.9525	0.9750	0.9847	0.9863	0.9633	0.8871
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(b) Currencies

	AUS	CAN	EUR	CHE	NZL	JPN	GBR
			Excess Returns				
mean	0.0011	0.0005	-0.0006	-0.0005	0.0018	-0.0041	-0.0001
Std. Dev.	0.0328	0.0234	0.0272	0.0285	0.0351	0.0293	0.0239
AC(1)	0.0643	-0.0690	0.0372	-0.0318	-0.0271	0.0837	0.0259
p-value	0.2254	0.1937	0.5140	0.5488	0.6100	0.1146	0.6251
			Real Exchange Rates (demeaned)				
mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.1877	0.1248	0.1365	0.1214	0.1795	0.2236	0.1281
AC(1)	0.9843	0.9805	0.9778	0.9703	0.9806	0.9747	0.9792
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Notes: (a) The log excess return prx_{it} and the real log price index for commodity, $\tilde{p}_i$. The list of commodity abbreviations: OIL=crude oil; ALM=aluminum; COP=copper; GLD=gold; CTN=cotton; ZNC=zinc; SLV=silver; LED=lead; SGR=sugar; HGS=hogs. (b) The log excess currency return rx_{jt} and the real log exchange rate, $\tilde{s}_j$. The list of country abbreviations: CAN=Canada; AUS=Australia; NZL=New Zealand; EUR=Euro zone; JPN=Japan; GBR=United Kingdom; CHE=Switzerland.

Table 2: Predictability of excess returns for commodities

	OIL	ALM	COP	GLD	CTN	ZNC	SLV	LED	SGR	HGS
(a) One month ahead										
r_{USA}	-6.069**	-2.487**	-3.765*	-0.293	-3.454*	-2.432	-2.392	-2.532	-3.608*	-2.324
	(-2.22)	(-2.04)	(-1.95)	(-0.26)	(-1.80)	(-1.46)	(-1.13)	(-1.08)	(-1.90)	(-1.07)
$\tilde{p}_i$	-0.042**	-0.037**	-0.017*	0.001	-0.043**	-0.024*	-0.016*	-0.018	-0.042***	-0.118***
	(-2.23)	(-2.02)	(-1.69)	(0.25)	(-2.05)	(-1.84)	(-1.84)	(-1.44)	(-2.79)	(-4.57)
π_{USA}	1.405	0.423	1.660	-1.805***	-1.038	-0.316	-2.270**	0.296	-0.181	2.827**
	(0.58)	(0.23)	(0.94)	(-3.15)	(-0.97)	(-0.23)	(-2.24)	(0.17)	(-0.15)	(2.40)
const	-0.020	0.105**	0.068*	0.006	-0.211*	0.075**	-0.023	0.051	-0.080**	0.003
	(-1.35)	(2.12)	(1.82)	(0.55)	(-1.93)	(2.05)	(-1.17)	(1.57)	(-2.49)	(0.44)
R^2	0.026	0.029	0.019	0.022	0.040	0.018	0.019	0.010	0.028	0.067
(b) Twelve months ahead										
r_{USA}	-56.59**	-18.33**	-48.89***	-8.35	-29.22*	-40.08***	-20.99	-39.88	-39.36***	-22.46*
	(-2.17)	(-1.98)	(-2.63)	(-0.62)	(-1.85)	(-2.64)	(-1.31)	(-1.62)	(-2.87)	(-1.97)
$\tilde{p}_i$	-0.41***	-0.54***	-0.23***	-0.01	-0.58***	-0.42***	-0.14**	-0.25**	-0.48***	-0.57***
	(-3.29)	(-3.29)	(-2.81)	(-0.25)	(-5.29)	(-3.73)	(-2.37)	(-2.14)	(-4.55)	(-4.46)
π_{USA}	-4.72	-4.99	-7.12	-2.93	-7.99*	-5.46	-9.72*	-9.86*	-1.96	5.14
	(-0.74)	(-1.36)	(-1.17)	(-0.94)	(-1.76)	(-0.93)	(-1.91)	(-1.68)	(-0.33)	(1.43)
const	-0.16*	1.55***	1.00***	0.08	-2.85***	1.29***	-0.21	0.75**	-0.91***	0.04
	(-1.68)	(3.33)	(3.00)	(0.82)	(-5.09)	(3.92)	(-1.50)	(2.34)	(-4.26)	(1.35)
R^2	0.247	0.313	0.195	0.019	0.401	0.291	0.079	0.168	0.280	0.327

Notes: The dependent variable is the log excess return prx_{it} in (a) and the cumulative log excess return $\sum_{l=0}^{11} prx_{it+l}$ in (b) regressed on lagged U.S. nominal interest rate r_{USA}, the real log price index for commodity, $\tilde{p}_i$, and U.S. inflation, π_{USA}. The list of commodity abbreviations: OIL=crude oil; ALM=aluminum; COP=copper; GLD=gold; CTN=cotton; ZNC=zinc; SLV=silver; LED=lead; SGR=sugar; HGS=hogs. Newey-West standard errors with 3 lags in (a) and 12 lags in (b) are used to calculate t statistics. N=350 observations in (a) and 341 in (b). t statistics in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.3. Predictability of excess returns

Turning to predictability of commodity excess returns one and twelve months ahead reveals substantial degree of predictability especially at longer horizons. Table 2(a) demonstrates the results for one month ahead predictability. The only commodity for which there is no variable that significantly affects excess returns is lead. Gold comes next with weak predictability as only inflation is found to significantly predicts excess return but the negative sign of the effect is unexpected. U.S. interest rates statistically significantly affect excess returns for five out of ten commodities, with the strongest impact for oil. There is evidence of mean reversion behaviour for almost all commodities with the exception of gold and lead. Inflation seems not to predict excess returns one month ahead, with the exception of gold, silver, and hogs.

Predictability improves when cumulative returns over 12 months are considered as shown in Table 2(b). Gold is the only commodity for which there are no statistically significant predictors among those considered. All other excess returns are strongly predicted by real commodity prices indicating mean reversion. Seven out of ten excess returns are also affected by interest rates. The only exceptions are gold, silver and lead where there is no statistically significant impact. Inflation still remains insignificant for most commodities.

Table 3: Predictability of excess returns for currencies

	CAN	AUS	NZL	EUR	JPN	GBR	CHE
(a) One month ahead							
$r_i - r_{USA}$	1.697	3.632***	2.647**	2.687***	1.433	2.266	3.219***
	(1.46)	(3.45)	(2.31)	(2.67)	(1.60)	(1.58)	(3.50)
$\pi_{USA} - \pi_i$	0.111	0.328	0.115	-0.383	-0.742**	-0.127	-0.918**
	(0.24)	(0.65)	(0.20)	(-1.15)	(-1.98)	(-0.42)	(-2.20)
$\tilde{s}_i$	-0.018	-0.030***	-0.023**	-0.026**	-0.001	-0.017	-0.037***
	(-1.58)	(-2.63)	(-2.07)	(-1.99)	(-0.12)	(-1.41)	(-2.84)
const	-0.004	-0.014***	-0.014**	0.005*	-0.003	0.006	0.001
	(-1.40)	(-2.76)	(-2.15)	(1.73)	(-0.09)	(1.22)	(0.32)
R^2	0.009	0.035	0.020	0.031	0.018	0.009	0.056
(b) Twelve months ahead							
$r_i - r_{USA}$	14.33	37.20***	32.42***	25.34***	9.43	20.74	29.60***
	(1.32)	(5.01)	(3.45)	(2.86)	(0.95)	(1.30)	(4.17)
$\pi_{USA} - \pi_i$	-2.56	-3.24	-4.60**	-2.33***	-1.18	-1.74**	-2.65***
	(-1.57)	(-1.35)	(-2.41)	(-2.66)	(-0.63)	(-2.51)	(-2.61)
$\tilde{s}_i$	-0.17	-0.35***	-0.31***	-0.33***	-0.06	-0.23	-0.40***
	(-1.43)	(-4.04)	(-3.24)	(-2.72)	(-0.67)	(-1.57)	(-4.89)
const	-0.04	-0.15***	-0.18***	0.06**	-0.31	0.09	-0.01
	(-1.06)	(-4.42)	(-3.97)	(2.51)	(-0.72)	(1.53)	(-0.85)
R^2	0.080	0.281	0.263	0.218	0.038	0.081	0.374

Notes: The dependent variable is the log excess return rx_{jt} in (a) and the cumulative log excess return $\sum_{l=0}^{11} rx_{jt+l}$ on (b) regressed on lagged nominal interest rate differential $r_j - r_{USA}$, the inflation differential, $\pi_{USA} - \pi_j$, and the real log exchange rate, $\tilde{s}_j$. The list of country abbreviations: CAN=Canada; AUS=Australia; NZL=New Zealand; EUR=Euro zone; JPN=Japan; GBR=United Kingdom; CHE=Switzerland. N=303 for the Euro zone, and N=350 for the other countries in (a) 293 and 340 in (b). Newey-West standard errors with 12 lags are used to calculate t statistics reported in parentheses in (b). $^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Predictability evidence for currency excess returns at one month horizon is mixed as well as shown in Table 3(a). There is strong evidence of predictability for Australia, New Zealand, the Euro, and Switzerland. Interest rate differentials have approximately three times stronger impact on excess returns than the direct impact of including the interest rate differential in the returns as in (4). This is in line with the literature that models risk premiums as proportional to the interest rate differential (e.g., Campbell and Clarida, 1987; Backus et al., 2001; Verdelhan, 2010; Farhi and Gabaix, 2016). In addition to the interest rate differential, the real exchange rate also plays an important role (as in Balduzzi and Chiang, 2020; Dalhquist and Penasse, 2022) showing evidence of mean reversion for these exchange rates. There is no predictability from individual regressors for Canada and the United Kingdom. The predictability for Japan is also weak with only inflation differential as significant predictor at 5%.

At 12 months ahead, there is still no predictability for Canada and Japan. This is surprising as predictability is expected to get better at longer horizons. The predictability is also weak for the United

Kingdom, with only inflation differential as statistically significant at 5%. Inflation differential also shows up as a predictor for three more countries (NZL, EUR, and CHE) but the sign of impact is unexpected as higher relative inflation in the USA is supposed to increase expected returns of investing abroad. This result is, however, consistent with the results reported in Balduzzi and Chiang (2020) for a panel of 34 currencies over the 1983 - 2012 period.[4]

4.4. VAR-based predictions

Given that the excess returns are predictable at least for some commodities and currencies, it is possible to impose structure on representations (3) and (6) and decompose the contributions of expected real interest rates and excess returns according to (8) and (9). VAR models are estimated separately for all commodity and currency excess returns. All models pass stability condition.[5]

Table 4(a) shows the decomposition for commodity prices. For all commodities the main channel through which prices are affected is the expected returns. The contribution of expected returns is higher for oil, copper, silver, lead, and sugar. The contribution is even higher than 1 for aluminum, cotton, zinc and hogs. The decomposition for gold indicates instability of the VAR model. Pindyck (1993) analyzes the implications of present value approach for heating oil, copper, lumber, and gold.[6] He concludes that the implication of present value are strongly rejected for gold. The only way to reconcile the behaviour of gold prices if they are strongly driven by expectations of future convenience yields and by perception that the yields would raise in the future. The analysis of this paper confirms that the VAR model based on the present value approach shows sign of instability when applied to gold. Inflation does not contribute much to any of commodities. This can be due to the sample of 1995 to 2024 used in estimation when inflation was low and stable with the exception of the last three years.

The decomposition of currencies in Table 4(b) also demonstrates some degree of heterogeneity. Several currencies like Australian and New Zealand dollars, the Euro, and the Swiss franc are driven mostly by expected excess returns. Australia and New Zealand are popular destinations for carry-trade strategies. The Swiss franc is often considered as safe haven currency, and Switzerland experienced substantial capital inflows following the 2007-2008 Financial Crisis. For other currencies, interest rate differentials and, sometimes, inflation differentials, play non-negligible role in explaining real exchange rate movements.

Figure 1 presents the visual representation of the decomposition for real commodity prices over time. The first row shows the price of oil and copper. Those two are well explained by the expected excess returns as

[4]Table 6. Their inflation differential is minus $(\pi_{USA} - \pi_i)$ in this paper and so the estimated coefficient is reported as positive. The coefficient is statistically significant.

[5]With the exception of gold for which the highest eigenvalue is 0.9999. While it passes the stability condition, it is extremely close to 1. This will show up in some results for gold due to possible instability of the model.

[6]Commodity prices are modeled as present value of future convenience yields. I do not include convenience yields explicitly but they are part of expected excess returns as discussed in sub-section 2.1.

Table 4: Decomposition of commodity prices and currencies

(a) Commodity prices

	OIL	ALM	COP	GLD	CTN	ZNC	SLV	LED	SGR	HGS
r_{USA}	0.397	-0.154	0.135	-179.84	-0.173	-0.163	0.389	0.371	0.354	-0.187
π_{USA}	-0.051	0.016	0.017	39.813	0.022	0.043	0.005	0.004	-0.022	0.018
prx_i	0.659	1.140	0.847	134.20	1.154	1.120	0.610	0.627	0.666	1.169

(b) Currencies

	CAN	AUS	NZL	EUR	JPN	GBR	CHE
$r_j - r_{USA}$	0.575	-0.080	-0.105	-0.082	0.771	0.265	-0.169
$\pi_{USA} - \pi_j$	0.014	0.061	0.050	0.066	0.329	0.112	0.021
rx_j	0.410	1.019	1.054	1.023	-0.115	0.622	1.150

Notes: (a) The decomposition of real commodity prices into contributions of expected future U.S. interest rate (r_{USA}), expected future inflation (π_{USA}), and expected future excess returns (prx_i). The list of commodity abbreviations: OIL=crude oil; ALM=aluminum; COP=copper; GLD=gold; CTN=cotton; ZNC=zinc; SLV=silver; LED=lead; SGR=sugar; HGS=hogs. (b) The decomposition of real exchange rate into contributions of expected future interest rate differentials ($r_j - r_{USA}$), expected future inflation differentials ($\pi_{USA} - \pi_j$), and expected future excess returns (rx_j). The list of country abbreviations: CAN=Canada; AUS=Australia; NZL=New Zealand; EUR=Euro zone; JPN=Japan; GBR=United Kingdom; CHE=Switzerland.

can be seen in the figure as the red (dashed) line tracks the black (solid) line very well. The second row is the example of aluminum and gold. For aluminum, the contribution of excess returns exceeds 100%. The figure demonstrates that this is because the expected real interest rate is negatively correlated with the real price. The figure for gold shows apparent sign of model instability. The next row is cotton and zinc. Similar to aluminum, the contribution of expected returns exceed 100% as expected real interest rate correlate negative with the price especially during the 2001-2008 period. The remaining two rows show decompositions for silver, lead, sugar and hogs. The decomposition of silver and lead is similar to oil and copper since the expected excess returns contribute the majority to the variance of those prices. The last row represents sugar and hogs. These commodities are representatives of foodstuffs commodities (sugar) and livestock and meat group (hogs). In this regard they are different from the other commodities included in the analysis (energy, metals, and industrials). Nevertheless, both are driven by expected excess returns.

The similar representation of decomposition is shown for currencies in Figure 2. The first two rows show countries for which the contribution of excess returns account for more than 100% of real exchange rate variance. The Australian and New Zealand's dollars, the Swiss franc and the Euro are well tracked by the expected excess returns (red dashed line). The behavior of the remaining three currencies in the last two rows of the Figure is explained by both expected real interest rate differential and expected excess returns. Sometimes expected returns and sometimes expected real interest rate differential (blue dotted line) get close to the real exchange rate.

The decompositions of this sub-section demonstrate that there is substantial heterogeneity among commodity prices and exchange rates. Some exchange rates like Australian and New Zealand dollars are driven

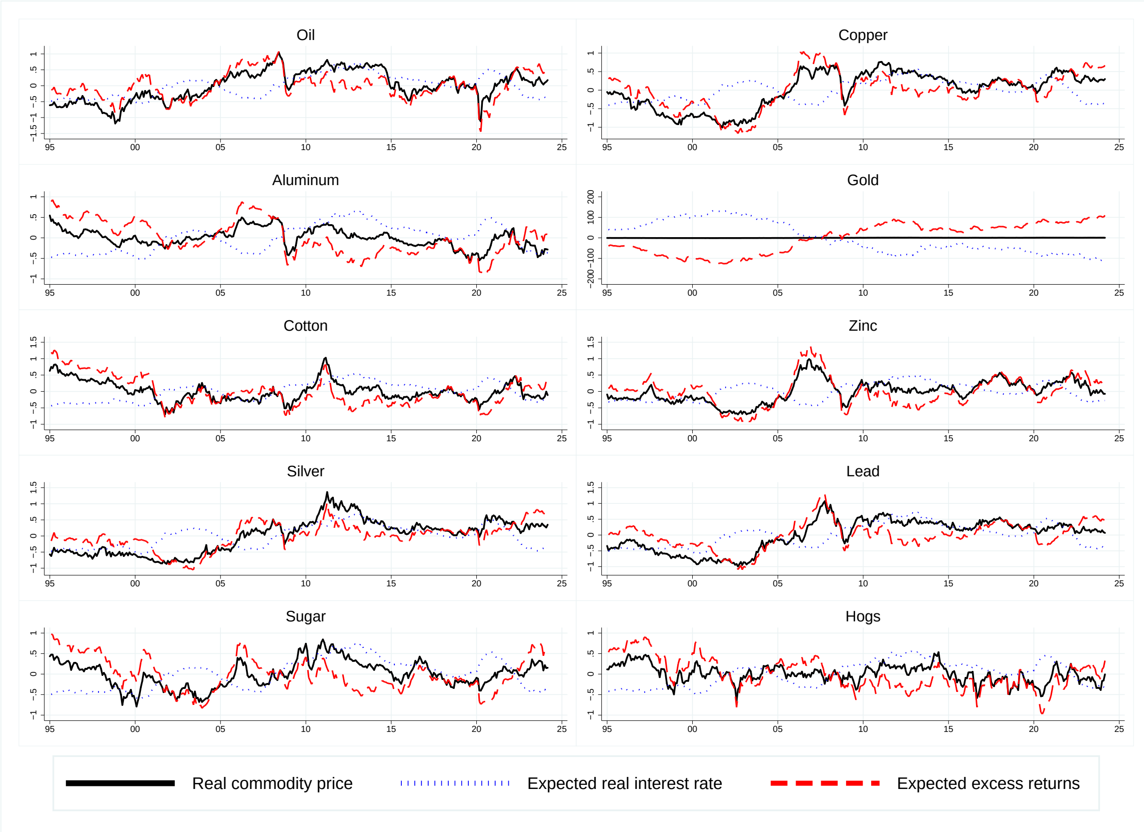

Figure 1: Decomposing the log real prices of commodities (demeaned). The contribution of expected real interest rates and excess returns

mostly by expected excess returns, or by risk premiums and convenience yields. The next sub-section analyzes the co-movement between exchange rates and commodity prices in greater detail.

4.5. Co-movement between excess returns

First, I analyze the correlation between currency and commodity excess returns in Table 5. What is striking about Table 5 is that there is substantial comovement across currency and commodity excess returns. This suggests that the risk premium (and/or convenience yields) is the possible channel through which exchange rates and commodities are related. Second, the correlation between commodities and exchange rates remains strong across commodity currencies (like Australian, New Zealand's and Canadian dollars) and other non-commodity currencies like the Euro, British pound, and the Swiss franc. Japanese yen is a clear outlier in the observed patterns of correlations. This can be potentially explained by Japan's interventions in the foreign exchange market to keep yen against appreciations during the period.[7] For all other currencies, correlation signs and magnitude remain relatively the same. This potentially causes doubts on a possible

[7]The historical data for foreign exchange rate intervention operations during the period can be found on the Ministry of Finance, Japan website. The Swiss National Bank also started to intervene in the foreign exchange market following the Great Financial Crisis. See Cwik and Winter (2024).

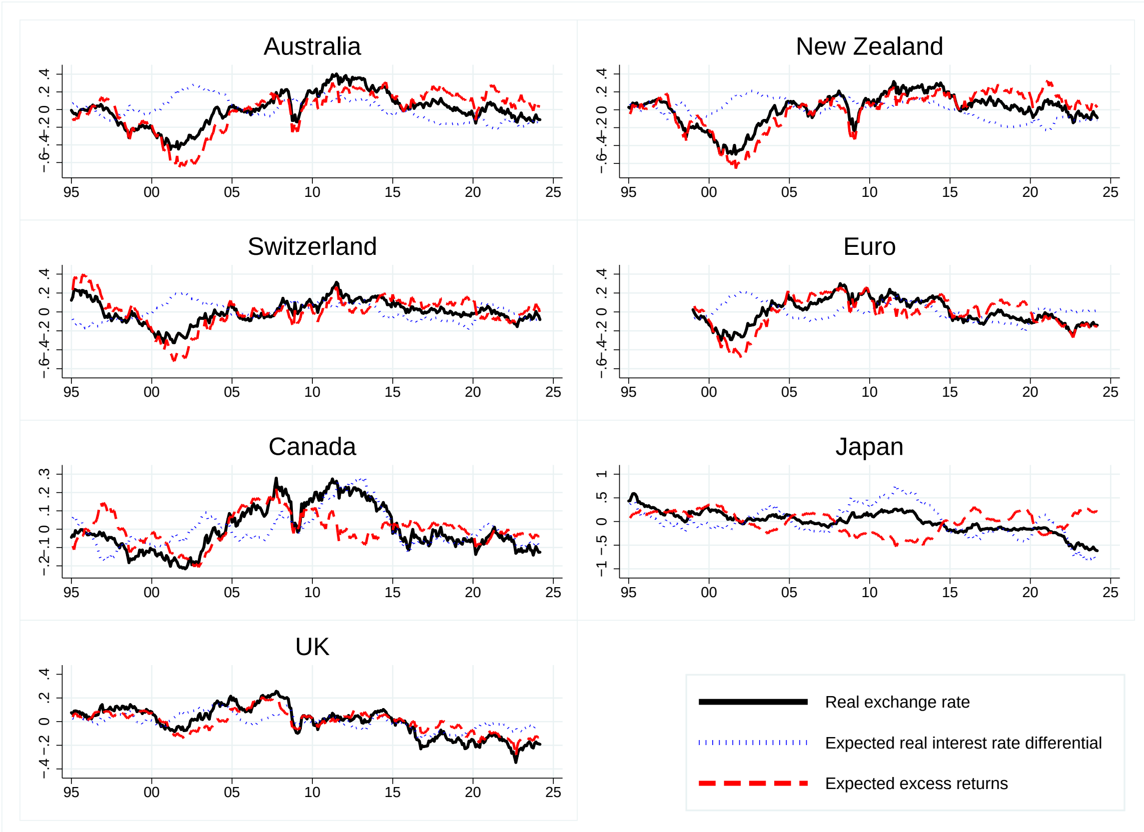

Figure 2: Decomposing the log real exchange rates (demeaned). The contribution of expected real interest differential and excess returns

explanation of the link between commodity currencies and prices of primary commodities through the terms of trade. The traditional explanation (i.e. Chen and Rogoff, 2003) is that higher commodity prices represent terms of trade improvement for commodity exporting countries and lead to appreciation of their currencies. We expect some variation in production shares of various commodities across the countries. We expect some countries to be a net importer of primary commodities. For example, New Zealand is not a major exporter of copper, gold, and cotton (see Chen and Rogoff, 2003, Table A.2.1). Yet, the New Zealand's dollar is highly correlated with real prices of these commodities. The United Kingdom is a net importer of aluminum and copper (see Ayres et al., 2020, Table A.2). Yet, the excess returns for the British pound are positively correlated with these commodity prices and the magnitude of the correlation is similar to other countries.

The next question is what is the source of high correlation between exchange rates (especially for commodity-exporting countries) and commodity prices. Since I can decompose the real exchange rates in the terms representing the expected real interest differential and expected excess returns, I can address this question by looking at the covariance of real commodity prices and those components. One possibility is that the U.S. monetary policy and expectations of the future U.S. interest rates drive the value of the U.S. dollar against other currencies and affect the values of commodity prices. Since exchange rates are rel-

Table 5: Correlation of currency and commodity excess returns

	OIL	ALM	COP	GLD	CTN	ZNC	SLV	LED	SGR	HGS
CAN	0.414^{***}	0.260^{***}	0.483^{***}	0.335^{***}	0.282^{***}	0.415^{***}	0.395^{***}	0.352^{**}	0.072	0.062
AUS	0.366^{***}	0.365^{***}	0.532^{***}	0.395^{***}	0.336^{***}	0.495^{***}	0.453^{***}	0.368^{***}	0.157^{***}	0.059
NZL	0.298^{***}	0.272^{***}	0.439^{***}	0.290^{***}	0.255^{***}	0.420^{***}	0.363^{***}	0.368^{***}	0.124^{**}	0.070
EUR	0.260^{***}	0.238^{***}	0.358^{***}	0.390^{***}	0.248^{***}	0.351^{***}	0.395^{***}	0.299^{***}	0.112^{*}	0.039
JPN	-0.013	0.004	0.048	0.319^{***}	0.023	0.061	0.192^{***}	0.037	0.103^{*}	-0.036
GBR	0.308^{***}	0.260^{***}	0.369^{***}	0.258^{***}	0.177^{***}	0.306^{***}	0.336^{***}	0.315^{***}	0.079	0.075
CHE	0.168^{***}	0.132^{***}	0.213^{***}	0.294^{***}	0.131^{**}	0.200^{***}	0.356^{***}	0.210^{***}	0.110^{**}	-0.004

Notes: The correlation of the log currency excess return rx_{jt} and the log commodity excess return prx_{it}. The list of country abbreviations: CAN=Canada; AUS=Australia; NZL=New Zealand; EUR=Euro zone; JPN=Japan; GBR=United Kingdom; CHE=Switzerland. The list of commodity abbreviations: OIL=crude oil; ALM=aluminum; COP=copper; GLD=gold; CTN=cotton; ZNC=zinc; SLV=silver; LED=lead; SGR=sugar; HGS=hogs. N=300 for the Euro zone, and N=347 for the other countries. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ for the null hypothesis that the correlation coefficient is zero.

ative prices and react to relative stances of monetary policy in the U.S. and abroad, exchange rates predict future interest rate differentials. Since the U.S. interest rate and the interest rate differential can be highly correlated, it is possible that commodity prices correlate with exchange rate because both predict future monetary policy.[8] Another possibility, explored by Devereux and Smith (2021), is that monetary authorities in commodity-exporting countries may, or expected to, react to commodity prices. So rising commodity prices lead to higher future interest rate and since the exchange rate is the present value of future interest rate differentials, the exchange rate appreciates today. In this case, commodity prices are correlated with expected interest rate differential. Since I can construct the expected interest rate differentials based on the VAR model, I can investigate to what degree commodity prices are correlated with the expected future interest rates.

Another possibility for the link between exchange rates and commodity prices is through expected excess returns. Such returns reflect risk premiums and convenience yields. It is possible that global attitudes towards risk drive both exchange rates and commodity prices. The results of the decomposition using equation (10) are presented in Table 6. The major channel of correlation between currencies of Australia, New Zealand, the Euro zone, Switzerland and commodity prices appears to be through expected returns. These results are somewhat not surprising given that these are the currencies which are driven mostly by excess returns in Table 4(b). The link between expected interest rate differentials and commodity prices is supported for Canada since for 6 out of 10 commodity prices the channel trough expected real interest differentials accounts for the majority of correlation. The Japanese yen and the British pound show mixed performance with equal dominance of both channels.

[8] Indeed, in the data used in this paper the correlation of the U.S. interest rate and interest rate differentials is the following: CAN=-0.72; AUS=-0.60; NZL=-0.39; EUR=-0.55; JPN=-0.86; GBR=0.02; CHE=-0.74.

Table 6: The channels of correlation between commodity prices and exchange rates

	CAN		AUS		NZL		EUR		JPN		GBR		CHE	
	τ_r	τ_{rx}	τ_r	τ_{rx}	τ_r	τ_{rx}	τ_r	τ_{rx}	τ_r	τ_{rx}	τ_r	τ_{rx}	τ_r	τ_{rx}
OIL	**0.60**	0.40	0.05	**0.95**	0.09	**0.91**	0.00	**1.00**	-1.42	**2.44**	-0.86	**1.87**	0.47	**0.53**
ALM	0.44	**0.56**	0.39	**0.61**	**0.56**	0.44	0.14	**0.86**	**1.43**	-0.45	0.33	**0.67**	0.16	**0.84**
COP	0.49	**0.51**	-0.27	**1.27**	-0.21	**1.21**	-1.45	**2.49**	-0.35	**1.36**	-0.35	**1.36**	-0.08	**1.08**
GLD	**0.70**	0.30	-0.35	**1.35**	-0.35	**1.35**	-0.47	**1.49**	-0.01	**1.02**	**0.57**	0.43	-0.07	**1.07**
CTN	**0.73**	0.27	-0.04	**1.04**	0.03	**0.97**	0.28	**0.72**	**1.17**	-0.18	0.17	**0.83**	-0.39	**1.39**
ZNC	0.07	**0.93**	-0.74	**1.74**	-0.58	**1.58**	-1.23	**2.26**	**0.90**	0.08	**2.22**	-1.23	-0.80	**1.81**
SLV	**0.69**	0.31	-0.19	**1.19**	-0.20	**1.20**	-0.14	**1.15**	-0.80	**1.82**	**0.82**	0.18	0.08	**0.92**
LED	0.46	**0.54**	-0.26	**1.26**	-0.21	**1.21**	-0.40	**1.41**	-0.27	**1.28**	**1.34**	-0.35	-0.13	**1.13**
SGR	**0.57**	0.43	-0.09	**1.09**	-0.01	**1.01**	0.12	**0.88**	**4.24**	-3.30	**1.64**	-0.65	-0.11	**1.12**
HGS	**0.55**	0.45	0.41	**0.59**	**0.66**	0.34	**0.51**	0.48	**1.22**	-0.24	0.35	**0.65**	-0.01	**1.01**

Notes: The contribution of the expected real interest rate differential to the covariance between real exchange rates and real commodity prices: $\tau_r = cov(\tilde{p}_{it}, \Re_{\mathbf{diff}\,jt})/cov(\tilde{p}_{it}, \tilde{s}_{jt})$. The contribution of expected currency excess returns: $\tau_{rx} = cov(\tilde{p}_{it}, \mathbf{RX}_{jt})/cov(\tilde{p}_{it}, \tilde{s}_{jt})$. The variables are defined in Section 3.

In light of the link between commodity prices and exchange rates through risk premiums the question that arises is if it is possible that exchange rates and commodity prices are driven by a common missing risk premium. Cutler et al. (1991) analyze correlations between excess returns across various asset classes such as bonds, stocks, exchange rates, metal commodity prices, houses and collectables. They address the question whether it is possible that all excess returns are driven by changing risk factors over time. The paper concludes that given that excess returns are only weakly correlated and that markets are distinct in their characteristics it is unlikely that they all driven by the same risk premium. The authors argue in favour of the speculative process that have similar features across different markets.

In what follows, I investigate if it is possible that commodity prices and exchange rate are driven by the same common factor. I estimate the dynamic factor model including all 10 commodities and all 7 exchange rates used in this study. The common factor has an AR(1) structure. Since commodity excess returns exhibit possible serial correlation as shown in Table 1, the model allows for AR(1) structure of disturbances. Table 7 presents the results of estimation by Maximum Likelihood method. First, there is mild autoregressive structure in the factor with the coefficient of autocorrelation of 0.089 (not statistically significant at 10%). The left part of the Table shows the estimated coefficients for commodity excess returns. Commodities are ordered by the magnitude of the coefficient showing loading on the unobserved factor. Copper loads the most followed by zinc, silver, and oil. These are the commodities that were found previously to be driven by expected excess returns (see Table 4). The estimated loading coefficient is significant for 9 out of 10 commodities. The only commodity that is not statistically affected by the common factor is hogs. This is also the commodity which excess returns were found to be uncorrelated with currency excess returns.

Interestingly, U.S. interest rates become insignificant for 8 our of 10 commodities once the common dynamic factor is introduced. It is possible that interest rates were capturing some of the effects of the common factor in previous analysis (Table 3). All commodity prices exhibit mean reversion with the coefficient for the error-correction term ranging from -0.004 for gold to -0.162 for hogs. Two commodity excess returns exhibit positive autocorrelation (oil and sugar) while other two show negative autocorrelation (silver and gold).

The right side of Table 7 shows estimated coefficients for currency excess returns. Similarly to commodities, currencies are ordered by the magnitude they load on the common factor. Top four currencies are those of New Zealand, Australia, the Euro, and Switzerland. These are the same currencies that were found to be driven mostly by expected excess returns (Table 4(b)). Interest rate differential are statistically significant in four out of seven currencies. Currencies exhibit mean reversion with the exception of Japan. Autocorrelation of disturbance is mostly insignificant with the exception of Canada, where residuals exhibit negative autocorrelation.

Small and statistically significant factor in excess returns can imply substantial deviation of asset prices from their equilibrium levels. The assumption of this paper that the long-run equilibrium for real exchange rate and real commodity prices are constant. To highlight the importance of the recovered factor I accumulate the factor over time and present it in Figure 3 along with top four commodity prices (on the left panel of the Figure) and exchange rates (on the right panel). The comparison of both panels reveal that deviations in asset prices from the equilibrium values are substantial. Commodity prices are more volatile than exchange rates and are more dispersed in the cross-sectional dimension. This highlights the difficulty of extracting a common factor from commodity prices.

The factor starts in 1999 since the Euro was included in the estimation. First, it drops substantially before and during the dot-com crisis in the U.S. in 2001. The risk premiums are expected to rise and convenience yields to drop for commodities. Producers of commodities are interested to take a short position which generates an increased risk premium for speculators. Convenience yields are expected to drop since it is less likely that there will be substantial rush in demand during a recession. In order for risk premiums to rise and convenience yields to drop, commodity prices need to decrease first. We see in the Figure that commodity prices are declining prior and during the dot-com crisis. After the recession, risk premiums start a gradual decline until the next recession of 2007-2009. Commodity prices are rising over this period as well resulting in a commodity boom. The risk premiums are rising fast in the financial crisis of 2007-2009. As with the previous recession, commodity prices drop to accommodate rising risk premiums. After the recession, commodity prices continue to rise until the end of 2014 when the cumulative common factor reaches the highest level. As the commodity prices are deviating from the long-run equilibrium, their high prices predict lower expected returns. To accommodate a positive risk premium requires a price correction and commodity

Table 7: Dynamic factor model of excess returns

Dynamic factor

$$f_t = 0.089 f_{t-1} + \hat{v}_t$$
(1.43)

Excess returns

Commodity: $prx_{it} = \alpha_{fi} f_t + \alpha_{1i} r_{t-1} + \alpha_{2i} \tilde{p}_{it-1} + \alpha_{0i} + \varepsilon_{it}$, $\varepsilon_{it} = \varrho_i \varepsilon_{it-1} + u_{it}$

Currency: $rx_{jt} = \beta_{fj} f_t + \beta_{1j}(r^*_{jt-1} - r_{t-1}) + \beta_{2j} \tilde{s}_{jt-1} + \beta_{0j} + \epsilon_{jt}$, $\epsilon_{jt} = \varphi_j \epsilon_{jt-1} + \zeta_{jt}$

Commodity	$\hat{\alpha}_{fi}$	$\hat{\alpha}_{1i}$	$\hat{\alpha}_{2i}$	$\hat{\alpha}_{0i}$	$\hat{\varrho}_i$	Currency	$\hat{\beta}_{fj}$	$\hat{\beta}_{1j}$	$\hat{\beta}_{2j}$	$\hat{\beta}_{0j}$	$\hat{\varphi}_j$
COP	**0.048**	0.085	**-0.024**	**0.097**	0.059	NZL	**0.030**	2.335	**-0.038**	**-0.019**	-0.082
	(12.48)	(0.05)	(-3.05)	(3.13)	(0.95)		(17.57)	(3.10)	(-4.78)	(-4.08)	(-1.18)
ZNC	**0.048**	0.504	**-0.033**	**0.099**	0.028	AUS	**0.030**	**2.517**	**-0.039**	**-0.014**	-0.030
	(11.26)	(0.29)	(-2.96)	(3.00)	(0.45)		(20.17)	(3.88)	(-5.26)	(-3.97)	(-0.37)
SLV	**0.048**	-0.213	**-0.021**	**-0.038**	**-0.111**	EUR	**0.020**	**2.883**	**-0.044**	**0.008**	0.021
	(10.61)	(-0.11)	(-2.28)	(-2.10)	(-1.86)		(15.07)	(3.88)	(-4.52)	(3.21)	(0.33)
OIL	**0.046**	-2.239	**-0.060**	**-0.029**	**0.132**	CHE	**0.018**	**2.616**	**-0.048**	-0.003	-0.102
	(7.38)	(-0.73)	(-3.45)	(-2.36)	(2.17)		(12.21)	(3.47)	(-4.15)	(-1.46)	(-1.65)
LED	**0.042**	0.162	**-0.024**	**0.066**	-0.077	CAN	**0.018**	1.150	**-0.028**	**-0.006**	**-0.126**
	(9.33)	(0.09)	(-2.68)	(2.69)	(-1.29)		(14.25)	(1.26)	(-3.26)	(-2.30)	(-2.02)
CTN	**0.032**	**-4.153**	**-0.063**	**-0.314**	-0.036	GBR	**0.016**	**3.200**	**-0.023**	**0.008**	0.002
	(6.52)	(-1.97)	(-3.20)	(-3.17)	(-0.59)		(12.58)	(2.70)	(-2.22)	(1.80)	(0.04)
ALM	**0.026**	-0.845	**-0.039**	**0.112**	0.041	JPN	**0.006**	0.368	0.005	0.023	0.082
	(6.88)	(-0.55)	(-2.53)	(2.55)	(0.64)		(3.98)	(0.38)	(0.65)	(0.59)	(1.42)
GLD	**0.021**	1.148	-0.004	0.013	**-0.106**						
	(8.27)	(0.99)	(-0.74)	(1.06)	(-1.82)						
SGR	**0.018**	-3.528	**-0.059**	**-0.113**	**0.153**						
	(3.57)	(-1.27)	(-2.93)	(-2.84)	(2.49)						
HGS	0.004	**-4.514**	**-0.162**	0.009	0.041						
	(0.67)	(-1.67)	(-4.26)	(1.31)	(0.59)						

Notes: The dynamic factor model is estimated by Maximum Likelihood. The observed information matrix estimates of standard errors are reported. Coefficients in bold are statistically significant at 10%. t statistics are reported in parentheses. The list of country abbreviations: CAN=Canada; AUS=Australia; NZL=New Zealand; EUR=Euro zone; JPN=Japan; GBR=United Kingdom; CHE=Switzerland. The list of commodity abbreviations: OIL=crude oil; ALM=aluminum; COP=copper; GLD=gold; CTN=cotton; ZNC=zinc; SLV=silver; LED=lead; SGR=sugar; HGS=hogs. Variables: the log excess return on currency j is rx_{jt}; the nominal interest rate differential is $r_j - r_{USA}$; the real log exchange rate is $\tilde{s}_j$; the log excess return on commodity i is prx_{it}; U.S. nominal interest rate is r_{USA}; the real log price index for commodity $\tilde{p}_i$. U.S. inflation and its differential are not included in estimation since they found mostly as insignificant in previous estimation results.

prices drop in 2015 and in 2018.

Turning to exchange rates in the right panel of the Figure reveals a similar pattern in exchange rates. As the risk premium is rising prior to 2001 there is a flight to quality and currencies depreciate versus the U.S. dollar. After the 2001 recession, there is a period of appreciation until the financial crisis. Following the depreciation of the currencies due to flight to quality, there is a period of currencies' appreciations. In 2015, currencies reach the same point as commodities: they are too risky going forward and they depreciate in 2015 and 2018.

Overall, there is a similar pattern of behavior of commodity prices and exchange rate. They are connected through risk premiums. As risk premiums are rising both prices drop (depreciate) to be followed by a period of predictable excess returns that result in dollar carry-trade strategies (Lustig et al., 2014). This link between commodity prices and exchange rates is not due to terms of trade. The Euro and Swiss franc show a pattern of behavior similar to Australian and New Zealand's dollars even though the former currencies are not known as commodity currencies since the Euro zone and Switzerland are not major commodity exporting countries (zones).[9] The link is not due to interest rate differentials, as these currencies tend to be dominated by expected excess returns. The dynamic factor model controls for interest rate differentials and their impact on risk premium. There is still a risk factor not accounted for with the interest differentials. I recover this missing factor from analyzing 10 commodities and 7 currencies together.

5. Conclusions

The paper proposes a novel explanation of why commodity prices and exchange rates are correlated. They are correlated through expectations of future risk premiums and convenience yields. This is a different channel comparing to existing explanations that exchange rates are correlated with commodity prices through terms of trade, or through interest rate differentials.

This conclusion is drawn from the application of the present value approach to both real commodity prices and real exchange rates. Real commodity prices are decomposed into the expected long-run equilibrium, expected real interest rates, and expected excess returns. Real exchange rates are represented as the expected long-run equilibrium, expected real interest rate differential, and expected excess returns. It is assumed in the analysis that the long-run equilibriums are constant in line with the literature. However, it is worth investigating if the results will hold under the assumption that the long-run equilibrium values are random walk processes. This analysis is left for future work.

Expected excess returns represent risk premiums and convenience yields. There is no distinction between the latter two in the current analysis. It may be worth introducing convenience yields, constructed based

[9]See the list of 58 commodity exporting countries in Cashin et al. (2004).

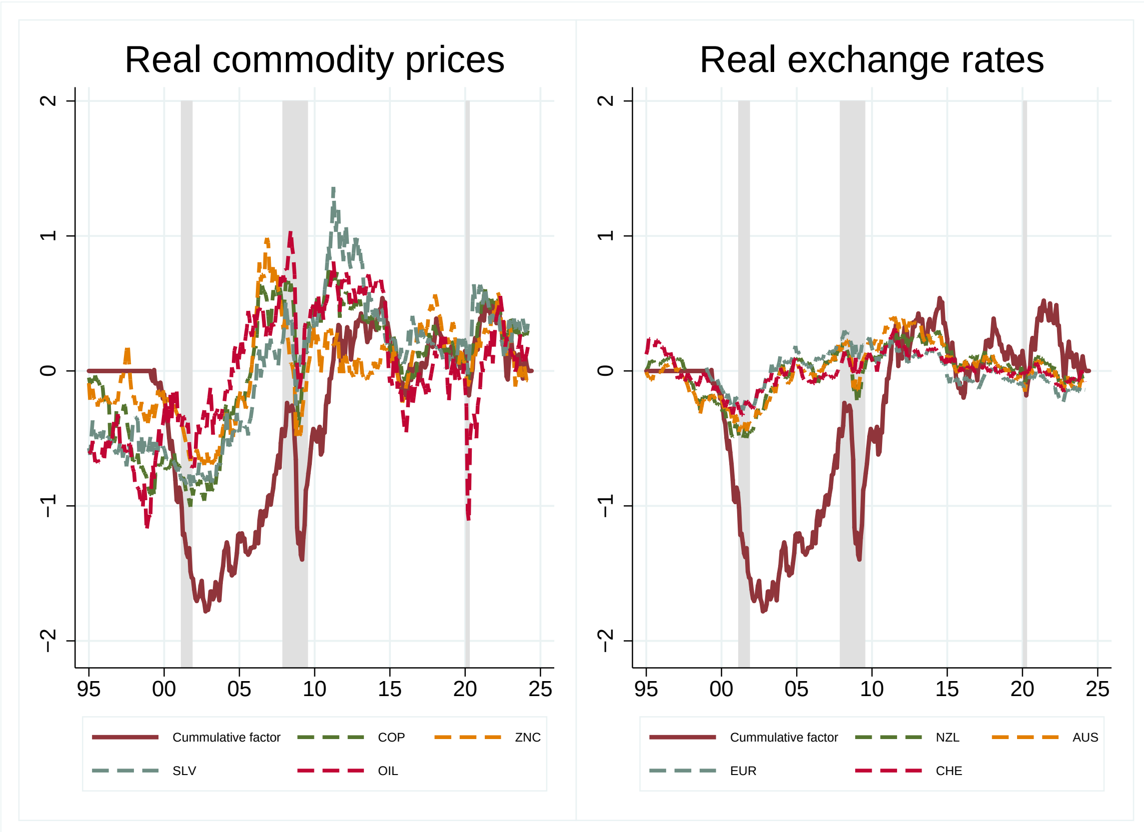

Figure 3: Accumulated common factor in real commodity prices and exchange rates (both are demeaned). Accumulated common factor is estimated in the dynamic factor model applied to 10 commodity prices and 7 exchange rates. The figure shows only top four commodities and exchange rates that load most on the factor. Grey shaded areas denote U.S. recessions

on forward rates, explicitly in the analysis in line with the recent literature on liquidity yields and exchange rates. This extension is left for future work as well.

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Appendix A.

Table A1: Mnemonics and description of commodity prices data

Short Name	Abbreviation	Mnemonic	Description
Crude Oil	OIL	CRUDOIL	Crude Oil-WTI Spot Cushing U$/BBL
Cotton	CTN	COTTONM	Cotton,1 1/16Str Low -Midl,Memph $/Lb
Aluminum	ALM	LADCASH	LME-Aluminium Alloy Cash U$/MT
Copper	COP	LCPCASH	LME-Copper Grade A Cash U$/MT
Gold	GLD	GOLDBLN	Gold Bullion LBM $/t oz DELAY
Hogs	HGS	GSLHSPT	S&P GSCI Lean Hogs Index Spot
Sugar	SGR	WSUGDLY	Raw Sugar-ISA Daily Price c/lb
Lead	LED	LEDCASH	LME-Lead Cash U$/MT
Silver	SLV	SILVUSL	LBMA Silver Price USD/t oz DELAY
Zinc	ZNC	LZZCASH	LME-SHG Zinc 99.995% Cash U$/MT

Source: Wharton Research Data Services.

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