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Asset Pricing with Affect Investing, Gambling, and Overconfidence: Theory and Evidence

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Abstract

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We present a multi-asset model with three investor types: Gamblers, who derive direct utility from large stock positions, overconfident investors, who underestimate the precision of public information (that they do not produce themselves), and affect investors, whose attitude towards a firm's products impacts their investment in the firm's stock. We consider the joint impact of these investors on trading activity, measured systematic risk, and expected returns. We find that gambling amplifies trading volume and mitigates overconfidence-induced excess return co-movement and underreaction. Further, risk-adjusted returns decrease in the strength of the affect heuristic; but this relation attenuates when gambling propensity is high. Empirical evidence supports these implications.

"The game of investing is intolerably boring and over-exacting to any one who is entirely exempt from the gambling instinct; whilst he who has it must pay to this propensity the appropriate toll."

–Keynes (1936)

"Perhaps the most robust finding in the psychology of judgment is that people are overconfident"

–De Bondt and Thaler (1995)

"Another common assumption in asset pricing models is that investors are concerned only with the payoffs from their portfolios; that is, investment assets are not also consumption goods. Apparent violations are plentiful. "

–Fama and French (2007)

1 Introduction

There is reliable evidence that financial market investors may not be characterized by neoclassical notions of risk and reward. For instance, investors appear to treat financial markets as gambling venues (Gao and Lin (2015)), and investment in a firm's stock (such as Tesla or Apple) appears to be influenced by individuals' personal preferences for the firm's activities and products (Billett, Jiang, and Rego (2014)).[1] In addition, investors tend to be overconfident; indeed, Lichtenstein, Fischhoff, and Phillips (1982) discuss the pervasiveness of the overconfidence bias. What are the joint implications of such behaviors for financial market equilibrium? We address this question by studying a multi-asset setting that *combines* these elements of investment within a unified framework. By integrating the elements, we allow for a first treatment of the question: what does the financial market equilibrium look like when behavioral elements are jointly considered in the same setting, as opposed to being modeled in isolation? We also derive and empirically test the implications of our model for the cross-section of equities.

[1] For more information on gambling incentives and how brand perceptions influence investment, see Coventry and Brown (1993) and Frieder and Subrahmanyam (2005).

In our setting, each stock's future payoff is influenced by both a common factor and a firm-specific component. Further, public signals are available about both systematic and firm-specific components of cash flows. Our model consists of three types of investors. First, due to a form of overconfidence (Odean (1998) and Luo, Subrahmanyam, and Titman (2021)), a class of investors underestimate the precision of the public signals, as they do not produce the signals themselves.[2] Second, affect investors perceive a stock to produce a non-monetary payoff beyond its actual level; the incremental payoff varies across stocks. This notion captures the idea that the stock can evoke positive or negative affect, which may arise from brand perceptions, loyalty (Schoenbachler, Gordon, and Aurand (2004); Frieder and Subrahmanyam (2005)), or societal norms (e.g., against "sin" companies (Geczy, Stambaugh, and Levin (2005); Hong and Kacperczyk (2009))). Finally, gambling investors (or just "gamblers") derive direct utility from the absolute scale of their stock positions, beyond wealth maximization. This type of gambling propensity represents a "betting-big" tendency, which reflects the idea that a large position (either positive or negative) in a stock increases the "thrill" of gambling in financial markets. Our perspective is supported by neuro-psychological research which shows that the ex ante uncertainty of the reward from gambling triggers the secretion of dopamine, a pleasure-inducing hormone (Preuschoff, Bossaerts, and Quartz (2006); Anselme and Robinson (2007)). In our analysis, we allow the gambling propensity parameter to vary in the cross-section.

In our model, trading is triggered because of "agreement to disagree" on the model's parameters (as also demonstrated by Odean (1998)). Further, gamblers act as de facto liquidity providers, selling/buying at a low demanded premium when other investors are on net buying/selling. This "market making" occurs because gamblers' consumption value arises from the scale of the position in the risky asset but not the position's sign. We show that gamblers' liquidity provision amplifies trading volume in equilibrium. Our explanation for high trading volume is consistent with Liu, Peng, Xiong, and Xiong (2022), who find evidence that overconfidence and gambling are primary drivers of trading.[3]

[2] Overconfidence also involves overestimating the precision of private information signals. As this aspect is hard to measure, we do not test its implications, but consider this case in the Internet Appendix.

[3] Dorn and Sengmueller (2009), Markiewicz and Weber (2013), and Gao and Lin (2015) also demonstrate the influence of gambling on trading activity.

While overconfidence results in underreaction to fundamentals-related variables, gambling attenuates this underreaction. The reason is as follows: Overconfident investors underestimate the precision of public signals, they willingly take the opposite side of the rational investors' reaction to the public signal; hence there is overtrading. Gamblers provide liquidity and help mitigate price movements caused by this extra trading. In the cross-section, if gamblers have a stronger preference for a particular stock, they provide more liquidity in that stock, resulting in mitigated underreaction to public information.

Overconfidence also leads to a divergence between the return beta (representing a stock's return co-movement with the market) and the cash flow beta; and gambling attenuates this divergence as well. The intuition here is similar: overconfident investors underestimate the precision of the signal about the common factor, and gamblers provide liquidity and mitigate the effects of overtrading due to overconfidence. In the cross-section, if gamblers have a stronger preference for a particular stock, they provide more liquidity in that stock, resulting in lower return co-movement with the market.

We also show that in the cross-section, risk-adjusted returns are lower when the affect heuristic is higher; furthermore, this negative relationship attenuates when the gambling propensity is high. To understand the intuition behind this, consider the stocks for which investors have a high positive affect. There is high demand for such a stock, leading to overpricing and thus lower future returns relative to other stocks. For stocks with a higher gambling propensity, gamblers trade more aggressively, mitigating the pressures on stock prices and risk-adjusted returns to a greater extent.

Based on our theoretical analyses, we formulate four main empirical implications:

(i) In the cross-section of stocks, turnover increases in gambling propensity.

(ii) The relative beta (i.e., the ratio of the return beta to the cash flow beta) decreases in gambling propensity.

(iii) Underreaction to public signals attenuates to a greater extent when gambling propensity is higher.

(iv) The risk-adjusted return decreases in the affect heuristic; further, this negative relationship attenuates when gambling propensity is high.

We test these implications for U.S. equities, using empirical proxies for gambling propensity and affect suggested by previous literature.[4]

Our gambling proxy is motivated by the work of Kumar, Page, and Spalt (2011), who observe that there is a positive correlation between an investor's gambling propensity and religion (specifically, Catholics are more pre-disposed towards gambling than Protestants). If investors tend to invest in local stocks (Coval and Moskowitz (1999)), then investors in firms with headquarters located in areas with a high Catholic-to-Protestant (CP) ratio are likely to have a high gambling propensity. Therefore, we measure gambling propensity in a stock by the CP ratio in the county where the firm of the stock is headquartered. We find that in the cross-section of stocks, turnover increases in the CP ratio, while the relative beta decreases in this ratio. The findings remain robust after we control for other variables including past stock performance, firm size, and book-to-market ratio.[5] This evidence supports Implications (i) and (ii). Novy-Marx (2013) finds that firms with high gross margins generate significantly higher returns than unprofitable firms. We find that this underreaction to profit margins decreases in the CP ratio, even after we control for other variables including past stock performance, firm size, and book-to-market ratio. This evidence supports Implication (iii).

Next, with regard to the affect proxy, Grullon, Kanatas, and Weston (2004) and Lou (2014) propose that a firm's product market advertising promotes brand visibility and attracts investor attention. Therefore, in our cross-sectional test of stock returns, we measure the affect heuristic for a stock via the corresponding firm's advertising spending. We find that risk-adjusted returns (relative to CAPM and standard factor models) decrease in advertising spending; further, this negative relationship attenuates when the CP ratio is high. These findings remain robust after we control for various variables including past stock performance, firm size, and book-to-market ratio. We also use portfolio analysis to find supportive evidence. These findings support Implication (iv).

[4]Note that overconfidence plays a key role in Implication (ii) by causing the relative beta to diverge from unity. The reader may wonder if proxies for overconfidence could be used to test for this divergence but we were unable to identify effective overconfidence proxies in the cross-section that are not also endogenous (e.g., trading volume, as analyzed in Odean (1998)).

[5]In the regression analysis for turnover, we also control for firm age, leverage, earnings volatility, the number of analysts following the firm, and analyst forecast dispersion. For relative beta, we control for operating leverage.

There are other models of investors' gambling incentives based on misperceptions of probabilities to generate a preference for lottery-type stocks, characterized by positively skewed returns. For example, Barberis and Huang (2008) show that if investors overweight small probabilities, the cumulative prospect theory of Tversky and Kahneman (1992) can generate a preference for lottery stocks. Bordalo, Gennaioli, and Shleifer (2012) demonstrate that investors may exaggerate the probability of salient (e.g., extreme) payoffs, leading to a preference for lottery stocks.

We explore a different dimension of gambling propensity, specifically the "betting-big" tendency, which arises from the "thrill" of gambling in financial markets. That individuals may derive direct utility from gambling is well-documented in psychology (see, e.g., Kuley and Jacobs (1988)). Black (1986) argues for the inclusion of "direct utility of trading" (p. 531) in financial market models to account for the higher trading volume observed in real-world markets compared to that predicted by rational portfolio rebalancing.[6] Our analysis shows that, consistent with Black's conjecture, the betting-big preference amplifies trading activity driven by heterogeneous beliefs, such as overconfidence, as demonstrated by Odean (1998). As such, we provide a rationale for the generally high volume in speculative markets (Hong and Stein (2007)).

Liu, Peng, Xiong, and Xiong (2022) provide evidence indicating that retail investors' turnover is significantly correlated with their responses to a survey question on "blockbusters," which reads as follows: "When I trade stocks, I aim to select those stocks whose price would rise sharply in a short period of time so that I can make a lot of money quickly." It is plausible that investors' responses to the "blockbusters" question are related to their "betting-big" preference, which extends to large total *positions* in a given stock, the value of which would also "rise sharply" in dollar terms as prices rise. Our model implies such a preference amplifies trading, and therefore accords with the arguments of Liu, Peng, Xiong, and Xiong (2022). In other related work, Grinblatt and Keloharju (2009) provide evidence that sensation seeking and turnover are correlated at-

[6]Other biases are discussed in Tversky and Kahneman (1974). Investors' naïve extrapolation from past outcomes, as shown in important survey-based work by Greenwood and Shleifer (2014), or failing to condition on market prices (Hong and Stein (1999); Eyster, Rabin, and Vayanos (2019)) are other well-studied biases in the literature. For parsimony, we focus on overconfidence, and understudied phenomena based on the affect heuristic and a gambling propensity.

tributes across investors. Dorn and Sengmueller (2009) show that investors who enjoy trading (as evidenced in a survey response) exhibit greater share turnover.

Components of our theory have been discussed in previous studies, although not in the same place. For instance, Daniel, Hirshleifer, and Subrahmanyam (1998) study the link between overconfidence and return patterns. Nagy and Obenberger (1994) find that an important reason for stock investment is an individual's feelings for a firm's products and services. Keloharju, Knüpfer, and Linnainmaa (2012) empirically connect stockholdings with product usage and argue that a "setup in which customer-investors regard stocks as consumption goods, not just as investments, seems to best explain [their] results." Friedman and Heinle (2016) explore investors' preferences for particular types of stocks, using the affect heuristic (see also Luo and Subrahmanyam (2019a)). Luo and Subrahmanyam (2019b) introduce the idea that investors derive direct utility from the magnitude of their unsigned positions.

There also are alternative theoretical approaches that depart from neoclassical models with rational investors. For instance, Grinblatt and Han (2005) and Da, Gurun, and Warachka (2014) present rationales for underreaction and momentum via the disposition effect, and the idea that investors pay less attention to news that arrives gradually, as opposed to discrete chunks. Bordalo, Gennaioli, Ma, and Shleifer (2020) show that momentum obtains under extrapolative expectations. Puri (2018) states that the complexity of an asset's payoff function dictates its desirability to an investor. While most previous papers, including the preceding ones, examine one behavioral aspect at a time, our contribution is to combine the elements of gambling, affect investing, and overconfidence to provide new insights on trading activity and asset pricing. Furthermore, we demonstrate that key implications of our theory are supported by empirical evidence.

The rest of this paper is organized as follows. In Section 2, we describe the setting of our model. In Section 3, we derive and analyze the equilibrium. In Section 4, we test the predictions of our model using empirical analysis. Section 5 concludes.

2 The Model

We next present the structure of our model, which consists of two dates, 0 and 1. A continuum of investors with mass unity trade at Date 0, and consume at Date 1. Each investor is an atomistic price-taker. There are three types of investors indexed by κ (overconfident investors), A (affect investors), and G (gamblers). The three types have masses of η_κ, η_A, and η_G, respectively, where the η's are positive constants that sum to one.

Assets and Information: There are J stocks. At Date 1, the j'th ($j = 1, ..., J$) stock pays a liquidating cash flow of

$$V_j = \bar{V}_j + \ell_j F + \theta_j, \tag{1}$$

where $\bar{V}_j$ is a positive constant denoting the unconditional mean, ℓ_j is a constant parameter denoting the loading on a common factor F, and θ_j denotes the firm-specific cash flow. The random variables F and θ_j follow independent normal distributions with a mean of zero. Throughout the paper, we assume that the variance of a generic random variable χ is given by a constant ν_χ, unless otherwise specified. For simplicity, we assume that the per capita supply of each stock is unity, as a normalization. There is also a risk-free asset with a constant gross rate of return $R_f > 1$.

At Date 0, a public signal about the common factor, $\tau = F + \zeta$, is released, where the noise term ζ is drawn from a normal distribution with mean zero and variance ν_ζ. Also, a public signal about the j'th stock's firm-specific cash flow, $s_j = \theta_j + \epsilon_j$, is released, where the noise term ϵ_j is drawn from a normal distribution with zero mean and variance ν_{ϵ_j}.

Overconfidence: As in Odean (1998) and Luo, Subrahmanyam, and Titman (2021), we assume that overconfident investors are skeptical about the quality of information they do not produce themselves. Specifically, overconfident investors are overly attached to their prior cash flow expectation relative to a rational Bayesian, so that they underestimate the precision of public information. Thus, in assessing the public signal about the common factor, $\tau = F + \zeta$, they believe that ζ has a larger variance $\rho\nu_\zeta$ than the actual variance ν_ζ, where $\rho > 1$ is a constant parameter representing the scale of the overconfident bias. Denote the unbiased (skeptical) belief about the variance of τ as $\nu_\tau = \nu_F + \nu_\zeta$ ($\kappa_\tau = \nu_F + \rho\nu_\zeta$). Overconfident investors also underestimate the precision of the public signal about the firm-specific cash flow, $s_j = \theta_j + \epsilon_j$, in that they believe that ϵ_j has a larger variance $\rho\nu_{\epsilon_j}$ than the actual variance ν_{ϵ_j}. Denote the unbiased (skeptical) belief about the variance of s_j as $\nu_{s_j} = \nu_{\theta_j} + \nu_{\epsilon_j}$ ($\kappa_{s_j} = \nu_{\theta_j} + \rho\nu_{\epsilon_j}$).[7]

The i'th overconfident investor has a standard exponential utility function given by:

$$U(W_{i1}) = -\exp(-\gamma W_{i1}),$$

where W_{i1} is the wealth at Date 1 and γ is a positive constant representing the absolute risk-aversion coefficient.

Affect Investors: Affect investors hold unbiased beliefs about the variances ν_ζ and ν_{ϵ_j}. The i'th affect investor also has a standard exponential utility function with a risk aversion parameter γ. As in Friedman and Heinle (2016), affect investors perceive that the j'th stock produces an extra non-monetary payoff A_j beyond the actual level V_j. This reflects the idea that the stock evokes positive or negative emotions. This affect heuristic can arise from brand perceptions and loyalty (Schoenbachler, Gordon, and Aurand (2004); Frieder and Subrahmanyam (2005)) and from societal norms against "sin" companies such as alcohol, tobacco, and gaming companies (Geczy, Stambaugh, and Levin (2005); Hong and Kacperczyk (2009)). In this setting, A_j denotes the strength and direction of the affect heuristic. If A_j is positive (negative), then affect investors have a positive (negative) emotional attachment to the stock.

Let X_{ij} denote the position of the i'th affect investor in the j'th stock, and let W_{i1} denote their wealth at Date 1. Then, the i'th affect investor's utility function is the following:

$$U(W_{i1}^A) = -\exp(-\gamma W_{i1}^A),$$

where

$$W_{i1}^A \equiv W_{i1} + \sum_{j=1}^{J}(X_{ij}A_j).$$

Gamblers: These investors have unbiased beliefs about the variances ν_ζ and ν_{ϵ_j}, but obtain direct consumption benefits from their stockholdings. Specifically, the i'th gambler

[7]In the Internet Appendix, we consider a setting with the traditional form of overconfidence (i.e., the informed investors over-assess the precision of an additional private signal they receive, as in Daniel, Hirshleifer, and Subrahmanyam (1998)). While this model does not permit an analytic solution, we show that our main results still hold. Further, our reasoning also applies when the uninformed underassess the precision of information signals, or fail to condition on market prices (Hong and Stein (1999), Eyster, Rabin, and Vayanos (2019)). However, such models are also complex and have no analytic solutions.

has the following utility function:

$$U_G(W_{i1}, X_i) = -\exp[-\gamma W_{i1} - C_G(X_i)],$$

where $C_G(X_i)$ is the extra benefit of holding stocks derived from non-wealth-related considerations. We define $C_G(X_i)$ as follows:

$$C_G(X_i) = 0.5 \sum_{j=1}^{J} \left(G_j X_{ij}^2\right),$$

where G_j is a positive constant representing investors' gambling propensity in the j'th stock, and X_{ij} denotes investor i's holdings in the j'th stock.

Thus, in the above specification, we model the idea that increasing the scale of the stock position increases the excitement derived from gambling in financial markets (Dorn and Sengmueller (2009)).[8] Based on this notion, the greater the holdings (in absolute terms), the higher the utility derived from investing. Therefore, we can think of this type of gambling propensity as a "betting-big" preference. Note that if G_j is high relative to γ (risk aversion), the investor, in effect, becomes a risk seeker, so there is no interior optimum for risky asset positions. We assume that the gambling propensity G_j is at a moderate level, so that there is indeed an interior optimum for the investor's demand for the j'th stock.[9] It is worth observing that gamblers derive utility from the size, but not the sign, of their position. Therefore, they are willing to absorb the opposite side of the net position from other investors at ever lower premia as G_j increases. This activity is what we term de facto liquidity provision.

We will use the following vector and matrix notations:

- The mean cash flows $\bar{V}$, the loadings ℓ, and the firm-specific cash flows θ and its

[8]In a survey of retail investors, Dhar and Goetzmann (2006) report that more than 25% of investors view stock market investing as a hobby. Grinblatt and Keloharju (2009) provide empirical evidence that sensation-seeking personalities (e.g., those who get speeding citations) may get a thrill from the act of trading.

[9]In our model, investors are afflicted with either a gambling motive, or overconfidence, or an affect heuristic, but the same investor does not possess more than one of these attributes. This assumption is for tractability, and we believe the thrust of our intuition would go through under alternative models where some affect investors are also overconfident gamblers, for example.

variance-covariance matrix ν_θ are denoted in vector and matrix notations as:

$$\bar{V} = \begin{pmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_J \end{pmatrix}, \; \ell = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \vdots \\ \ell_J \end{pmatrix}, \; \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_J \end{pmatrix}, \text{ and } \nu_\theta = \begin{pmatrix} \nu_{\theta_1} & 0 & \cdots & 0 \\ 0 & \nu_{\theta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nu_{\theta_J} \end{pmatrix}.$$

- The information signal vector s and its variance-covariance matrix based on unbiased (skeptical) beliefs, ν_s (κ_s), are denoted as:

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_J \end{pmatrix}, \; \nu_s = \begin{pmatrix} \nu_{s_1} & 0 & \cdots & 0 \\ 0 & \nu_{s_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nu_{s_J} \end{pmatrix}, \text{ and } \kappa_s = \begin{pmatrix} \kappa_{s_1} & 0 & \cdots & 0 \\ 0 & \kappa_{s_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_{s_J} \end{pmatrix}. \quad (2)$$

- The prices P, supplies $\mathbf{1}$, and the i'th investor's demands X_i are denoted as:

$$P = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_J \end{pmatrix}, \; \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \text{ and } X_i = \begin{pmatrix} X_{i1} \\ X_{i2} \\ \vdots \\ X_{iJ} \end{pmatrix}.$$

- The affect parameters A and gambling propensity G are denoted as

$$A = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_J \end{pmatrix} \text{ and } G = \begin{pmatrix} G_1 & 0 & \cdots & 0 \\ 0 & G_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & G_J \end{pmatrix}.$$

The cash flow can be expressed in vector form as $V = \bar{V} + \ell F + \theta$. Affect investors perceive the payoffs to be $A + V$. The direct utility of the i'th gambler from the risky asset position is denoted by $C_G(X_i) = 0.5 X_i' G X_i$.

3 The Equilibrium

In this section, we determine the equilibrium of the setting described in Section 2. All proofs, unless otherwise stated, are in Appendix A. Define the following parameters:

$$\begin{aligned}
\varDelta &\equiv \frac{\nu_F}{\kappa_\tau} - \frac{\nu_F}{\nu_\tau}, \qquad \kappa_{F|\tau} \equiv \nu_F - \frac{\nu_F^2}{\kappa_\tau}, \qquad \text{and } \nu_{F|\tau} \equiv \nu_F - \frac{\nu_F^2}{\nu_\tau}; \\
\varOmega &\equiv \nu_\theta \kappa_s^{-1} - \nu_\theta \nu_s^{-1}, \quad \kappa_{\theta|s} \equiv \nu_\theta - \nu_\theta^2 \kappa_s^{-1}, \qquad \text{and } \nu_{\theta|s} \equiv \nu_\theta - \nu_\theta^2 \nu_s^{-1}, \\
\phi_\kappa &\equiv \ell\ell' \kappa_{F|\tau} + \kappa_{\theta|s}, \qquad \phi \equiv \ell\ell' \nu_{F|\tau} + \nu_{\theta|s}, \qquad \phi_G \equiv \phi - G/\gamma^2, \text{ and} \\
\varPhi &\equiv \eta_\kappa \phi_\kappa^{-1} + \eta_A \phi^{-1} + \eta_G \phi_G^{-1}.
\end{aligned} \quad (3)$$

We have the following result:

Theorem 1 *In equilibrium:*

(i) Prices are given by:

$$P = \frac{1}{R_f}\left[\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s + \Phi^{-1}\left[\eta_\kappa\phi_\kappa^{-1}\left(\ell\Delta\tau + \Omega s\right) + \eta_A\phi^{-1}A - \gamma\mathbf{1}\right]\right].$$

(ii) Each overconfident and affect investor's, and each gambler's, demands for a stock are respectively given by:

$$X_\kappa = \frac{\phi_\kappa^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \nu_\theta\kappa_s^{-1}s - PR_f\right),$$
$$X_A = \frac{\phi^{-1}}{\gamma}\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right),$$
$$X_G = \frac{\phi_G^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right).$$

The term $\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s$ in the equilibrium price vector P represents the unbiased expectation of the final payoff V conditional on the public signals τ and s. The remaining terms reflect investors' behavior that goes beyond risk-reward considerations. Essentially a positive A (positive affect) implies a high mean price and vice-versa, whereas overconfidence causes underweighting of the information signals in the realized price. Finally, a high G lowers the premia demanded by gamblers to absorb the trades of the other investors, which mitigates the effect of biases on market prices. Thus, prices are the result of affect, gambling, and overconfidence interacting with each other.

However, the setting generally precludes analyzing the model in closed form. Therefore, in what follows, we first study a special case, where we are able to examine the equilibrium analytically; this allows us to obtain clear intuitions about the equilibrium. We then numerically examine the general case, and verify these intuitions.

3.1 A simple case with a closed-form solution

We consider a simple economy where firm-specific payoff variances are identical in the cross-section of stocks (i.e., $\forall j\ \nu_{\theta_j} = \nu$), and signals about the firm-specific component θ_j, s_j, are uninformative (i.e., $\forall j\ \nu_{\epsilon_j} \to \infty$). We then obtain the following Theorem:

Theorem 2 *In the simplified economy where firm-specific payoff variances are identical across firms, and there are no information signals about firm-specific payoffs, the following results hold:*

(i) The equilibrium prices are given by:

$$P = \frac{1}{R_f}\left[\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \Phi^{-1}\left(\eta_\kappa\phi_\kappa^{-1}\ell\Delta\tau + \eta_A\phi^{-1}A - \gamma\mathbf{1}\right)\right].$$

(ii) The demands of each overconfident and affect investor, and each gambler, are respectively given by:

$$X_\kappa = \frac{\phi_\kappa^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau - PR_f\right), \quad X_A = \frac{\phi^{-1}}{\gamma}\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau - PR_f\right),$$
$$\text{and} \quad X_G = \frac{\phi_G^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau - PR_f\right).$$

Since the signal about the firm-specific payoff θ_j, s_j, is not informative in the simple economy, the prices and demands for stocks do not depend on s_j as in the general case (see Theorem 1). The compensation for risk is represented by the term associated with $\gamma\mathbf{1}$ in the price vector P. The other terms in P, which are related to the masses of overconfident and affect investors, and gamblers (η_κ, η_A, and η_G, respectively) directly and/or through Φ, are the result of their trading behaviors grounded in overconfidence, or other nontraditional ways of defining utility and beliefs.

Overconfident investors underestimate the precision of the signal τ (i.e., $\rho > 1$ and therefore $\kappa_\tau > \nu_\tau$). This underreaction is reflected in the term $\Delta < 0$ in the price vector P. Affect investors obtain an additional component A in their payoff, and their demand for a given stock increases if the affect parameter A_j for that stock (i.e., the j'th element of the vector A) is higher. This tendency of affect investors to demand more of a stock with a higher A_j value contributes to a higher equilibrium price for that stock on average. Finally, gamblers help mitigate deviations of stock prices from their fundamental values. The influence of gamblers on stock prices is captured by the term Φ in the pricing equation.

We proceed below to analyze the influence of investors on turnover, the co-movement of stock returns with the market, and the risk-adjusted return. In the remainder of this section, we assume that we are in the economy described within Theorem 2. We further make a technical assumption that in this simple economy, the factor payoff exhibits sufficiently lower volatility compared to the firm-specific payoff (i.e., $\nu_F^{-1}\nu$ is sufficiently high). This assumption further allows us to conduct an analytical examination of the equilibrium.

3.1.1 Turnover

We first consider stock turnover. Theorem 2 shows that each overconfident and affect investor's and gambler's, demand for the j'th stock, denoted by $X_{\kappa j}$, X_{Aj}, and X_{Gj}, respectively, are simply the j'th elements of the vectors X_κ, X_A, and X_G. Let $\mathbf{1}_j$ be a $J \times 1$ vector where the j'th element equals 1 and the other elements equal 0. Using Theorem 2, we can write:

$$X_{\kappa j} = \mathbf{1}_j' X_\kappa, \quad X_{Aj} = \mathbf{1}_j' X_A, \quad \text{and} \quad X_{Gj} = \mathbf{1}_j' X_G.$$

Suppose that each of the three investor types respectively starts with an endowment of $Y_{\kappa j}$, Y_{Aj}, and Y_{Gj} shares of the stock. The expected turnover of the j'th stock is given by half of the sum of the absolute differences between each type of investor's demand for that stock and their initial endowment:

$$E(T_j) \quad = \quad 0.5\left[\eta_\kappa E\left(|X_{\kappa j} - Y_{\kappa j}|\right) + \eta_A E\left(|X_{Aj} - Y_{Aj}|\right) + \eta_G E\left(|X_{Gj} - Y_{Gj}|\right)\right]. \tag{4}$$

Note that our static setting can be interpreted as the reduced-form of a steady-state setting, where investors begin Date 0 with positions equal to the ex-ante means of their demands (i.e., $\forall j\ Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$) as a result of prior rebalancing. As new information arrives, trading occurs. Specifically, the expected turnover of the j'th stock takes the following form:

$$E(T_j^*) = 0.5\Big[\eta_\kappa E\left(|X_{\kappa j} - E(X_{\kappa j})|\right) + \eta_A E\left(|X_{Aj} - E(X_{Aj})|\right) + \eta_G E\left(|X_{Gj} - E(X_{Gj})|\right)\Big]. \tag{5}$$

We obtain the following result:

Proposition 1 *Suppose that investors begin with positions that are equal to the ex-ante means of their demands (i.e., $\forall j\ Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$).*

(i) If there is no overconfidence (i.e., $\rho = 1$), then the expected turnover $E(T_j^*) = 0$.

(ii) With overconfidence ($\rho > 1$), the expected turnover $E(T_j^*)$ increases in the gambling propensity G_j (assuming $\ell_j \neq 0$) with the cross-section. $E(T_j^*)$ also increases in the overconfidence parameter ρ, and decreases in the risk-aversion parameter γ.

In the proof of Proposition 1, we derive the following equations for the trades of overconfident investors, affect investors, and gamblers:

$$X_{\kappa j} - E(X_{\kappa j}) = \psi_{\kappa j}\Delta\tau, \quad X_{Aj} - E(X_{Aj}) = \psi_{Aj}\Delta\tau \;\text{ and }\; X_{Gj} - E(X_{Gj}) = \psi_{Gj}\Delta\tau,$$

where the ψ's are constant parameters. The affect parameter A_j does not affect trade. The reason for this is that affect investors overbuy or oversell depending on the level of the constant A_j. This affects the mean position $E(X_j)$. However, if investors start at the mean position, A_j does not induce any further trading.

Note that trade is triggered by the overconfidence bias (see also Odean (1998)). Specifically, if overconfident investors assess the precision of the signal τ properly (i.e., if $\rho = 1$), then $\Delta = 0$. Thus, without overconfidence, even if there is affect investing and gambling, there is no trade. However, if overconfident investors underestimate the precision of the signal τ (i.e., if $\rho > 1$), then $\Delta < 0$; trade occurs. In this case, overconfident investors perceive that a good (bad) signal τ is not that good (bad), and the resulting high (low) stock prices cannot be justified; therefore, they sell (buy). In the cross-section, if a stock has a higher gambling propensity G_j, gamblers buy (sell) more aggressively, amplifying trading activity.

If overconfident investors underestimate the precision of the signal τ to a greater extent (i.e., with a higher ρ), they sell (buy) based on τ to a greater extent. This further intensifies trading activity. If investors are more risk-averse (i.e., with a higher γ), then investors trade more conservatively; in this case, trading activity is subdued.

Of course, investors can start with positions that are not identical to the ex ante means of their demands. In this case, we define $\lambda_{XY,j}$ as endowment-driven trade, which represents the deviation of endowment Y_j from the ex ante mean $E(X_j)$; specifically:

$$\lambda_{XY,j} \;=\; 0.5\Big[\eta_\kappa\left(|E(X_{\kappa j}) - Y_{\kappa j}|\right) + \eta_A\left(|E(X_{Aj}) - Y_{Aj}|\right) + \eta_G\left(|E(X_{Gj}) - Y_{Gj}|\right)\Big].$$

It is straightforward to show that the expected turnover in Equation (4)

$$E(T_j) \in \left[E(T_j^*) - \lambda_{XY,j}, E(T_j^*) + \lambda_{XY,j}\right];$$

this means that $E(T_j^*)$ (the component of expected turnover due to information alone), represents the mid-point between the lower and upper bounds of $E(T_j)$. We are not able to further analyze the expected turnover $E(T_j)$ in closed from; we investigate it through numerical study in Section 3.2.

3.1.2 Return co-movement with the market

Using Theorem 2, we can express the j'th stock's excess return (i.e., the return above the risk-free interest rate) as follows:[10]

$$r_j = \mathbf{1}_j'(V - PR_f) = \ell_j \left(F - \frac{\nu_F}{\nu_\tau}\tau \right) + \theta_j - \mathbf{1}_j' \Phi^{-1} \left(\eta_\kappa \phi_\kappa^{-1} \ell \Delta\tau + \eta_A \phi^{-1} A - \gamma \mathbf{1} \right). \tag{6}$$

We represent the market excess return using the cross-sectional average of stock returns:

$$r_M = \frac{1}{J}\mathbf{1}'(V - PR_f) = \ell_M \left(F - \frac{\nu_F}{\nu_\tau}\tau \right) + \theta_M - \Psi_M \times \Delta\tau - A_M + \gamma \Sigma_M, \tag{7}$$

where

$$\begin{aligned} \ell_M &= \frac{1}{J}\mathbf{1}'\ell, \quad \theta_M = \frac{1}{J}\mathbf{1}'\theta, \quad \Psi_M = \frac{1}{J}\mathbf{1}'\Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\ell, \\ A_M &= \frac{1}{J}\mathbf{1}'\Phi^{-1}\eta_A\phi^{-1}A, \quad \text{and} \quad \Sigma_M = \frac{1}{J}\mathbf{1}'\Phi^{-1}\mathbf{1}. \end{aligned} \tag{8}$$

We assume that the loading of the market portfolio on the common factor equals 1, i.e.: $\ell_M = 1$. From Equation (7), the market excess return can then be expressed as:

$$r_M = F - \frac{\nu_F}{\nu_\tau}\tau + \theta_M - \Psi_M \times \Delta\tau - A_M + \gamma \Sigma_M. \tag{9}$$

As the market portfolio represents the factor, we interpret the loading of the j'th stock's cash flow on the common factor F, ℓ_j (see Equation (1)), as the cash flow beta. The co-movement of the stock with the market return is measured by the return beta:

$$\beta_{jM} = \frac{\text{Cov}(r_j, r_M)}{\text{Var}(r_M)}. \tag{10}$$

We scale the return beta by the cash flow beta and the resulting "relative beta" is β_{jM}/ℓ_j. We obtain the following result:

Proposition 2 *(i) For a sufficiently large number of securities (J), if there is no overconfidence bias (i.e., $\rho = 1$), then the relative beta $\beta_{jM}/\ell_j = 1$.*

[10] As is standard in exponential-normal settings, we measure the return via the price change (viz. Hong and Stein (1999)).

(ii) If there is overconfidence bias (i.e., $\rho > 1$), then in the cross-section of stocks, the relative beta β_{jM}/ℓ_j decreases in the gambling propensity G_j (assuming that the cross-sectional average factor loading, ℓ_j, is positive).

The intuition for Part (i) of Proposition 2 is as follows. If the number of stocks, J, is sufficiently high, then $\theta_M \to 0$ due to the diversification effect. In this case, if overconfident investors assess the precision of the signal factor τ properly (i.e., $\rho = 1$) and condition their trades on it, then stock prices respond to τ in a way that precisely reflects the information conveyed by the signal. Consistent with this intuition, $\Delta = 0$ in the expressions of r_j and r_M in Equations (6) and (9). This implies that without overconfidence, and when the number of securities J is large, a stock return's co-movement with the market depends almost completely on its cash flow loading ℓ_j.

If overconfident investors underestimate the precision of the signal τ (i.e., $\rho > 1$), then $\Delta < 0$, and therefore, $\beta_{jM} \neq \ell_j$; in this case, there is a divergence between the return beta and the cash flow beta. Part (ii) of Proposition 2 describes how gamblers affect this divergence.[11] The intuition is as follows: Suppose the information signal τ is positive. Since overconfident investors underestimate the precision of τ, they underreact to the information, leading to a market-wide price underreaction followed by a subsequent price continuation, as reflected in the term $-\Psi_M \times \Delta\tau > 0$ within the expression of r_M in Equation (9) (note that $\Delta < 0$, and we show $\Psi_M > 0$ in the proof of Proposition 2). When stock prices are below their fundamental levels due to overconfident investors underreacting to the information, gamblers buy stocks in equilibrium. If a stock consists of investors with a higher G_j, gamblers buy aggressively and cause an increase in the stock's price to a larger extent, resulting in a diminished level of the subsequent price continuation and return co-movement with the market; this implies an even smaller relative beta (that tends towards zero). A similar argument holds for negative signals.

Noting that the divergence between the return beta β_{jM} and the cash flow beta ℓ_j arises from stock price underreaction due to the overconfidence bias, we conjecture that

[11] It is also notable from Equations (6), (9), and (10) that the affect parameter A_j does not affect β_{jM}. The reason is that β_{jM} captures the return co-movement between r_j and r_M that is driven by the information about the common factor F, τ. The affect parameter A_j decreases the mean level of r_j, but does not affect its sensitivity to τ.

if overconfident investors underestimate the precision of the signal τ to a greater extent (i.e., a higher ρ), then there should be a greater deviation of β_{jM} from ℓ_j. If investors are more risk averse (i.e., have a higher γ), then overconfident investors are more conservative in their trading based on τ; this leads to a lesser deviation of β_{jM} from ℓ_j. On the other hand, affect investors and gamblers are more conservative in providing liquidity to overconfident investors, correcting mispricing due to the overconfident bias; this leaves a greater deviation β_{jM} from ℓ_j. We are not able to examine the net effect of risk aversion analytically; therefore, we investigate this effect through numerical study in Section 3.2.

3.1.3 Underreaction to public information

Consider a public signal regarding the j'th stock:

$$S_j = \mu s_j + (1 - \mu)\tau, \tag{11}$$

where $\mu \in (0, 1]$ is a constant parameter. This public signal has two components. The first component is related to the firm-specific signal s_j. The second component is related to the signal about the common factor, τ. This hybrid-type of public signal accounts for the fact that such signals would typically contain information about both the common and firm-specific components of value. Note that in the simple economy considered here, s_j is assumed to be uninformative; we will allow s_j to be informative in the numerical study in Section 3.2.

We use Equations (6) and (11) to compute $\mathrm{Cov}(r_j, S_j)$, which we use to measure post-information returns.[12]

Proposition 3 *There is underreaction to the public signal ($\mathrm{Cov}(r_j, S_j) > 0$) if and only if there is an overconfidence bias (i.e., $\rho > 1$). Further, in the cross-section of stocks, $\mathrm{Cov}(r_j, S_j)$ decreases in the gambling propensity G_j (assuming that ℓ_j is positive).*

If overconfident investors assess the precision of the signal factor τ properly (i.e., $\rho = 1$) and condition their trades on it, then stock prices respond to the signal, $S_j = \mu s_j + (1 - \mu)\tau$ (note that s_j is assumed to be uninformative here), in a way that precisely reflects the

[12] Luo, Subrahmanyam, and Titman (2021) show that this covariance can be interpreted as the return on a long-short portfolio formed based on S_j.

information conveyed by the announcement. This implies that without overconfidence, there is no misreaction to the signal.

If overconfident investors underestimate the precision of the signal τ (i.e., $\rho > 1$), then $\Delta < 0$, and therefore, $\mathrm{Cov}(r_j, S_j) > 0$; in this case, there is underreaction. Proposition 3 describes how gamblers affect this underreaction.[13] The intuition is as follows: Suppose the information signal τ is positive; this implies that the announcement $S_j = \mu s_j + (1-\mu)\tau$ is also likely to be positive. Since overconfident investors underestimate the precision of τ, they underreact to the information. This implies that they are willing to take the opposite side of the rational investors' positions, thus creating additional volume. If a stock consists of investors with a higher G_j, gamblers provide more liquidity, resulting in a diminished effect of underreaction on prices.

3.1.4 The risk-adjusted expected return

From Equations (6) and (9), we can express the expected excess returns of the j'th stock and the market as follows:

$$E(r_j) \;\; = \;\; \mathbf{1}_j' \Phi^{-1} \left(\gamma \mathbf{1} - \eta_A \phi^{-1} A \right) \;\; \text{and} \;\; E(r_M) = \gamma \Sigma_M - A_M. \tag{12}$$

The expected return of the stock adjusted for beta is:

$$\alpha_j \;\; = \;\; E(r_j) - \beta_{jM} E(r_M), \tag{13}$$

where β_{jM} is computed in Equation (10).[14] We obtain the following result:

Proposition 4 *(i) If overconfidence, affect investing, and gambling are absent (i.e., $\rho = 1$ and $\forall j \; A_j = G_j = 0$), then the risk-adjusted expected return $\alpha_j = 0$.*

(ii) If overconfidence, affect investing, and gambling are present (i.e., $\rho > 1$ and $\exists j$ such that $A_j \neq 0$ and $G_j > 0$), then in the cross-section of stocks, the derivative of α_j with respect to the affect parameter A_j is negative. The absolute magnitude of this derivative

(a) decreases in the gambling propensity G_j, and

[13] For a similar reason as described in Footnote 11, the affect parameter A_j does not affect the drift measure $\mathrm{Cov}(r_j, S_j) > 0$.

[14] For convenience, our model assumes only one risk factor. However, the model can be easily extended to include multiple risk factors using the technique in Luo, Subrahmanyam, and Titman (2021).

(b) increases in the risk-aversion parameter γ.

Proposition 4 shows that in the benchmark case where all investors are rational (specifically, when there is no overconfidence bias, affect investing, or gambling), the risk-adjusted expected return α_j equals zero. However, with overconfidence bias, affect investing, and gambling, $\alpha_j \neq 0$ and depends on the affect and gambling parameters, A_j and G_j. To provide further insight, consider stocks with a relatively high affect parameter A_j. Affect investors have an abnormally high demand for such stocks, causing them to become overpriced compared to others. This leads to a low risk-adjusted expected return α_j. In contrast, for higher G_j, gamblers mitigate the rise in the stock price P_j as well as the decrease in α_j. A reverse intuition holds for negative A_j.

If investors are more risk averse (i.e., with a higher γ), then both gamblers and overconfident investors are more conservative in their trading against affect investors, correcting the mispricing due to affect investing. In this case, affect investing has a more pronounced (negative) effect on the risk-adjusted return α_j.

3.2 The general case: Numerical illustration

In this section, we move away from the special case of the previous section, which means we lose analytical solutions. We therefore examine the general case using numerical study. We use this study to verify the implications of the equilibrium, which we have obtained using the special case, for turnover, stock return co-movement with the market, the post-public-announcement drift, and risk-adjusted expected returns.

3.2.1 Turnover

We can use Theorem 1 to obtain the demand for the j'th stock by each overconfident and affect investor, and gambler, denoted by $X_{\kappa j}$, X_{Aj}, and X_{Gj}, respectively, which are the j'th elements of the vectors X_κ, X_A, and X_G. We can then use Equation (4) to compute the expected turnover of the j'th stock.

We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$. The overconfidence parameter ρ

takes values from 1 to 9.[15] Figure 1 plots the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, for different values of overconfident investors' overconfidence ρ ranging from 1 to 9.[16] As in Proposition 1, we assume here that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j$ $Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$); in this case, the turnover is not affected by the affect parameter A_j.

Figure 1 supports findings from the previous analysis (see Section 3.1.1 and Proposition 1). Specifically, when overconfident investors correctly assess the precision of the signals (i.e., $\rho = 1$), no trading occurs. Conversely, if they underestimate the precision of the signals (i.e., $\rho > 1$), trading does occur, and an increase in the gambling propensity G_j leads to an increase in $E(T_j)$. Additionally, an increase in the overconfidence parameter ρ results in an increase in the expected turnover $E(T_j)$.

Our analysis also suggests that both the overconfidence bias and gambling play distinct roles in trading activity. Specifically, the overconfidence bias triggers trading, while gambling amplifies it. This implication is consistent with the findings of Liu, Peng, Xiong, and Xiong (2022), which suggest that overconfidence in having an information advantage and a preference for gambling are the primary drivers of trading.

Figure 2 displays the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, when investors' risk aversion γ varies from 2 to 2.5 (fixing $\rho = 9$). Consistent with the previous analysis (see Proposition 1), as γ increases, the expected turnover $E(T_j)$ decreases. Additionally, in line with the results from Figure 1, an increase in the gambling propensity G_j leads to an increase in $E(T_j)$ in the cross-section of stocks.

If, unlike in Proposition 1, each type of investors' endowments do not equal the ex ante means of their optimal demands, then in the cross-section of stocks, turnover can also depend on the affect parameter A_j. In Figure 3, we consider two other scenarios of endowments. In Panel A, we assume that each investor's endowment equals the per capita

[15] Our value for risk aversion, $\gamma = 2$, is the same as that used in Leland (1992) and Holden and Subrahmanyam (2002). We assume a significant mass of each type of investor, with $\eta_\kappa = 0.5$, $\eta_A = 0.2$, and $\eta_G = 0.3$. The values of $\ell_j = 1$ and $\nu_{\theta_j} = 1$ for all j are normalization constants. We choose $\nu_F = 0.01$ (with $\mathrm{SD}(F) = 0.1$), so that the common factor is less volatile than the firm-specific cash flow. We set $\nu_{\epsilon_j} = \nu_{\theta_j}/2$ and $\nu_\zeta = \nu_F/2$ to indicate a significant magnitude of noise in the signals s_j and τ. Our results are generally robust to different parameter values.

[16] This range is consistent with that of Odean (1998) (pp. 1909–10).

supply (i.e., $\forall j, Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 1$), and in Panel B, investor endowments are identically zero (i.e., $\forall j, Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 0$). We plot the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j and the affect parameter A_j. It is notable that the cross-sectional relationship between $E(T_j)$ and A_j is non-monotonic; $E(T_j)$ is higher for stocks with extreme positive/negative values of A_j. In other words, investors with strong positive/negative affect toward these stocks tend to trade them more actively. This heightened response drives them to buy or sell more intensely to reach their optimal holdings, intensifying trading activity. The relationship between gambling and turnover also remains consistent with the previous analysis. Specifically, in the cross-section of stocks, the expected turnover $E(T_j)$ increases as gambling propensity G_j increases.

3.2.2 Return co-movement with the market

Using Theorem 1, we can express for the general case the realized excess returns of the j'th stock and the market as

$$r_j = \mathbf{1}_j'(V - PR_f) \text{ and } r_M = \frac{1}{J}\mathbf{1}'(V - PR_f), \tag{14}$$

where

$$V - PR_f = \ell\left(F - \frac{\nu_F}{\nu_\tau}\tau\right) + \theta - \nu_\theta \nu_s^{-1} s - \Phi^{-1}\left[\eta_\kappa \phi_\kappa^{-1}\left(\ell \Delta\tau + \Omega s\right) + \eta_A \phi^{-1} A - \gamma \mathbf{1}\right].$$

We can compute for the stock $\beta_{jM} = \dfrac{\mathrm{Cov}(r_j, r_M)}{\mathrm{Var}(r_M)}$.

Figure 4 depicts the relative beta β_{jM}/ℓ_j for each stock as a function of gambling propensity G_j, for different levels of overconfidence (ρ). In line with our previous analysis (Proposition 2), when overconfident investors assess the precision of the signals properly (i.e., $\rho = 1$), we observe $\beta_{jM}/\ell_j = 1$, indicating a perfect alignment between the return beta β_{jM} and the cash flow beta ℓ_j. However, if overconfident investors underestimate the precision of the signals (e.g., $\rho = 9$), a discrepancy arises between β_{jM} and ℓ_j. Specifically, in the cross-section of stocks, we observe that β_{jM}/ℓ_j decreases as G_j increases, which accords with Proposition 2. It is also worth noting from Figure 4 that as overconfident investors underestimate the precision of the signals to a greater extent (i.e., a higher ρ), a greater discrepancy arises between β_{jM} and ℓ_j. This result aligns with the reasoning in Proposition 2 that the discrepancy is a consequence of stock price underreaction due to overconfidence.

Figure 5 depicts the relative beta β_{jM}/ℓ_j for each stock as a function of gambling propensity G_j, when investors' risk aversion (indicated by the parameter γ) varies. Consistent with Figure 4, as G_j increases, β_{jM}/ℓ_j decreases. Note that as investors become more risk averse (i.e., as γ increases), the discrepancy between β_{jM} and ℓ_j diminishes. According to the previous analysis (see also the discussion following Proposition 2), a higher level of risk aversion has two effects. On the one hand, overconfident investors are more cautious in their trading. This increased caution leads to reduced stock price underreaction and, consequently, a decreased disparity between β_{jM} and ℓ_j. On the other hand, affect investors and gamblers are more cautions in providing liquidity to overconfident investors, correcting mispricing due to the overconfident bias; this leaves an increased disparity between β_{jM} and ℓ_j. Our numerical study indicates that the first effect dominates.

3.2.3 Underreaction to public signals

Recall that the public signal for the j'th stock (see Equation (11)) takes the following form:

$$S_j = \mu s_j + (1 - \mu)\tau,$$

where $\mu \in (0, 1]$ is a constant parameter. We can use r_j given in Equation (14) to compute the measure $\text{Cov}(r_j, S_j)$.

Figure 6 plots the underreaction measure, $\text{Cov}(r_j, S_j)$, for each stock as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We set $\mu = 0.8$ (our results are robust to different values of μ); the other parameters are the same as those used in the earlier figures. Confirming our previous analysis (Proposition 3), when overconfident investors assess the precision of the signals properly (i.e., $\rho = 1$), we observe $\text{Cov}(r_j, S_j) = 0$ indicating no underreaction. However, if overconfident investors underestimate the precision of the signals s and τ (i.e., $\rho > 1$), then underreaction arises, i.e., $\text{Cov}(r_j, S_j) > 0$, and an increase in the gambling propensity G_j leads to a decrease in $\text{Cov}(r_j, S_j)$. It is also worth noting from Figure 6 that as overconfident investors underestimate the precision of the signals to a greater extent (i.e., a higher ρ), $\text{Cov}(r_j, S_j)$ is greater. This result aligns with the discussion following Proposition 3.

Figure 7 depicts the underreaction measure, $\text{Cov}(r_j, S_j)$, for each stock as a function of the gambling propensity G_j, when investors' risk aversion (indicated by the parameter γ) varies. Consistent with Figure 6, as G_j increases, $\mathrm{Cov}(r_j, S_j)$ decreases. Note that as investors become more risk averse (i.e., as γ increases), $\mathrm{Cov}(r_j, S_j)$ increases. According to the previous analysis (see also the discussion following Proposition 3), a higher level of risk aversion has two effects. On the one hand, overconfident investors are more cautious in their trading. This increased caution leads to reduced stock price undereaction. On the other hand, affect investors and gamblers are more cautious in providing liquidity to overconfident investors and thus correcting mispricing due to overconfidence. Our numerical study indicates that the second effect dominates (we have verified this is the case for most parameter values).

3.2.4 The (unconditional) risk-adjusted expected return

Using Theorem 1 and Equation (14), we can express for the general case the expected excess returns of the j'th stock and the market as

$$E(r_j) = \mathbf{1}_j' \Phi^{-1} \left(\gamma \mathbf{1} - \eta_A \phi^{-1} A\right), \quad \text{and} \quad E(r_M) = \frac{1}{J} \mathbf{1}' \Phi^{-1} \left(\gamma \mathbf{1} - \eta_A \phi^{-1} A\right);$$

note that these expected returns are unconditional on the public information τ and s. We can compute the (unconditional) risk-adjusted expected returns of the stocks as

$$\alpha_j = E(r_j) - \beta_{jM} E(r_M),$$

Figure 8 plots the risk-adjusted return α_j for each stock as a function of its gambling propensity G_j and affect parameter A_j. In line with our previous analysis (see Section 3.1.4 and, specifically, Proposition 4), we observe that in the cross-section of stocks, α_j decreases as the affect parameter A_j increases. Additionally, the negative relationship between α_j and A_j attenuates when gambling propensity G_j is high.

In Panel A of Figure 9, we set γ to a higher value (2.5), compared to Figure 8, where $\gamma = 2$. Consistent with our previous analysis (see Proposition 4), we observe that the negative relationship between α_j and the affect parameter A_j becomes more pronounced for higher γ. We corroborate this finding in Panel B of Figure 9, where we plot the difference between the α_j's when $\gamma = 2.5$ and when $\gamma = 2$. The difference decreases as the affect parameter A_j increases.

3.3 Empirical implications

Based on our theoretical analyses, we formulate four main empirical implications which we test using available data for U.S. equities. We provide these implications below, along with references to the corollaries and/or figures that support them:[17]

(i) (Proposition 1 and Figures 1-3) In the cross-section of stocks, turnover increases in gambling propensity.

(ii) (Proposition 2 and Figures 4-5) The relative beta (i.e., the ratio of the return beta to the cash flow beta) decreases in gambling propensity.

(iii) (Proposition 3 and Figures 6-7) Underreaction to public information is mitigated when gambling propensity is high.

(iv) (Proposition 4 and Figures 8-9) The risk-adjusted return decreases in the affect heuristic; this negative relationship attenuates when gambling propensity is high.

4 Empirical Analysis

We now test the empirical implications of our model for turnover, return co-movement with the market, post-public-announcement drift, and risk-adjusted expected returns, using data on U.S. equities.

4.1 Data

We obtain our sample of common stocks traded on NYSE, NASDAQ, and AMEX from the CRSP database. We retrieve accounting information from the Compustat database. To avoid potential market microstructure issues, we exclude stocks with prices below $1 as of the end of month $t-1$, where t is the current month.[18] Further, in accordance with Fama and French (1992), we exclude firms in the financial and utility industries.

[17]While Propositions 1, 2, and 4 used for the implications are derived under the assumption that, after accounting for conditional information, the factor payoff is sufficiently less volatile than the firm-specific payoff, we have verified that these results hold for the parameter values used in the calibration in Section 3.2.

[18]Our empirical results are not significantly affected when we instead use a $5 threshold for penny stocks. We have also tried excluding microcaps, defined as stocks with market capitalization below the 10% NYSE breakpoint (Fama and French (2008)), and found similar results.

For stock-month t, we use monthly share turnover as reported by CRSP, scaled by the number of outstanding shares at the end of the past month $t-1$. To address NASDAQ double counting due to the recording of inter-dealer trades in turnover, we follow the method proposed by Gao and Ritter (2010) and divide the turnover of NASDAQ stocks by 2.0 before January 2001, by 1.8 for the rest of 2001, by 1.6 for 2002-2003, and leave it unchanged thereafter.

We compute $Relative\ beta$ as the ratio of the estimated return beta to cash flow beta in month t. We estimate the return beta using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$). If month t is in quarter q, we estimate the cash flow beta as the coefficient in the regression of the firm's ROE on the value-weighted market-level ROE using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$). When computing the firm's ROE in a quarter, we use the sum of the earnings in the quarter and in the past three quarters to remove seasonality effects; we scale this sum using the book equity as of the past quarter end. To ensure the reliability of our regressions involving relative betas, we require both the estimated return and cash flow betas to be positive to retain a stock-month in our sample.

We calculate the risk-adjusted return as the stock's excess return over the risk-free interest rate or the difference between the stock return and the benchmark-implied return in month t. We estimate our benchmark models, including the CAPM, the Fama and French (1993) and Carhart (1997) four-factor model (FFC4), and the Fama and French (2015) five-factor model (FF5), using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$).[19]

Novy-Marx (2013) finds evidence that stock returns exhibit preditctability in the direction of profitability (specifically, firms with high announced gross margins generate significantly higher returns than firms with the opposite characteristic). We focus on this phenomenon to test the relation between returns and public information.[20] For stock-

[19] We obtain the market, size, and value factors through Wharton Research Data Services, and the five Fama and French (2015) factors from Kenneth French's website (https://tinyurl.com/336hyekj).

[20] Earnings drift (Bernard and Thomas (1989)) has virtually completely attenuated in recent years (Martineau (2021)); hence we chose not to use this anomaly; though, in unreported analysis we find similar results. when we use earnings surprises in place of profits. Subrahmanyam (2024) shows that profitability is one of the few main anomalies that survives in more recent years.

month t, we use the quarterly report that is released in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter.[21]

We next turn to our gambling and affect proxies. We identify gambling propensity in a stock as follows. Kumar, Page, and Spalt (2011) argue that investors' proclivity to gamble is related to their religion (i.e., Catholicism vs. Protestantism). If investors tend to invest in local stocks (Coval and Moskowitz (1999)), then stocks of companies headquartered in counties with high Catholic-to-Protestant (CP) ratios would tend to attract investors with a high gambling propensity. We calculate a county's CP for each year between 1980 and 2020 using survey data of the religious profile of all U.S. counties in 1980, 1990, 2000, and 2010 from the Association of Religion Data Archives and in 2020 from the U.S. Religion Census.[22] For years with missing data, we estimate CP using linear interpolation.[23] For stock-month t in the period from July of year y to June of the next year $y+1$, we use the CP ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$.

We identify the affect heuristic for a stock as follows. According to Campbell and Keller (2003), and Grullon, Kanatas, and Weston (2004), a firm's product market advertising increases brand visibility and attracts investor attention. We thus use a firm's advertising expenditure in the product market as a proxy for the affect heuristic for a stock. We retrieve the annual advertising expense (XAD) from the Compustat database for the firm.[24] For stock-month t in the period from July of year y to June of the next year $y+1$, we use the firm's three-year moving average of advertising expense (to accommodate for periodic fluctuations in advertising expenditure) as of the end of the past fiscal year $y-1$, denoted as XAD_3. We adjust XAD_3 by subtracting the industry average advertising expense (where the industry is based on the first two digits of the listed SIC code for the firm in month $t-1$), in order to mitigate potential industry effects on advertising expen-

[21]We use item RDQ, the reporting date for quarterly profits in Compustat, as the release date for gross profits each quarter (see Novy-Marx (2013)).

[22]See http://www.thearda.com/ and https://www.usreligioncensus.org.

[23]For years with missing data, we also consider using the CP based on the most recent survey data, and find similar empirical results.

[24]We ignore firm-years with missing XAD information because we are not able to tell whether the firm fails to report XAD, or its advertising expense is actually zero. We exclude firm-years with advertising expenses below $\$100,000$, as suggested by Lou (2014). Our results remain robust to this exclusion.

diture. Figure 10 plots the cross-sectional dispersion of XAD_3 over time, and shows that this quantity has been increasing in the past decades; so using XAD_3 directly in the Fama-MacBeth regression may affect inferences from the regression test. Therefore, we employ a standardized advertising measure ADV, which is computed as the cross-sectional percentile rank of the industry-adjusted XAD_3.

In our empirical analyses, we also control for several firm characteristics. For stock-month t, ME denotes the stock's market value of equity (i.e., the stock price $\times$ the number of outstanding shares) as of the end of the past month $t-1$; $Firm\ age$ is the number of months since the stock's first CRSP appearance as of the end of the past month $t-1$; ANA is the number of analysts who follow the firm and report forecasts to the I/B/E/S database in the past month $t-1$; and $FDISP$ is the stock analysts' forecast dispersion, which is computed as the standard deviation of earnings per share (EPS) forecasts reported by analysts in the I/B/E/S database in the past month $t-1$. For stock-month t in the period from July of year y to June of the next year $y+1$, BM is the book-to-market ratio computed using the book equity as of the end of the past fiscal year $y-1$, and the market value of equity as of the end of the past calendar year $y-1$.[25] D/A is the stock's book debt (i.e., the sum of short-term and long-term debt) divided by total assets as of the end of the past fiscal year $y-1$. OL is the operating leverage computed as the yearly sum of the stock's cost of goods sold and selling, general and administrative expenses divided by total assets as of the end of the past fiscal year $y-1$ (we follow the method of Novy-Marx (2011)). For stock-month t in quarter q, we compute $EVOLA$, the earnings volatility of the stock, as the standard deviation of EPS using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$).

Appendix B provides detailed definitions of the variables. Our sample period is from July 1981 to December 2021. To address extreme values, we winsorize all continuous variables (i.e., all variables except $Firm\ age$ and ANA) at the 1% and 99% levels,[26] using each of their distributions every month. The exceptions are risk-adjusted returns and the standardized advertising measure ADV, which are not winsorized.

[25]We follow Fama and French (1993) and require both the market equity and the book equity to be positive. Our empirical results still hold if we do not impose this requirement.

[26]We also try winsorizing at the 0.5% and 99.5% percentiles as a robustness check; our results continue to obtain.

Table 1 presents the time-series averages of the cross-sectional statistics of selected variables. The mean (median) $Turnover$ is 12.8% (8.7%) per month. The mean (median) $Relative\ beta$ is 2.344 (0.777). The mean stock return is 1.4% per month. The mean CP ratio across firms, 1.705, is higher than the mean county-level ratio across all U.S. counties, which is 0.694 during the sample period. The reason for the difference is that the headquarters of most firms in our sample are located in counties with a high CP ratio.[27] The mean advertising expense is $\$89.687$ million per year. The mean gross profitability is 0.099% per quarter. The average market equity ME is $\$3.16$ billion, and the means of firm age, analyst following, and EPS forecast dispersion are 178 months, 4.97, and $\$0.121$, respectively. The average value of the book-to-market ratio BM is 0.663, the mean leverage D/A is 0.219, and the mean operating leverage OL is 1.113. The mean EPS volatility is $\$0.508$ per quarter.

4.2 Gambling and turnover

Implication (i) from Section 3.3 is that in the cross-section of stocks, turnover should increase in gambling propensity. We test this implication using the following regression:

$$Turnover_{j,t} = b_0 + b_1 \times CP_{j,t} + b_2 \times Controls + \epsilon_{j,t}. \tag{15}$$

The dependent variable is the turnover of the j'th stock in month t. If month t is in the period from July of year y to June of the next year $y+1$, $CP_{j,t}$ is the Catholic-to-Protestant ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We control for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME and firm age as of the end of the past month $t-1$, book-to-market ratio BM and leverage D/A as of the end of the past year $y-1$, earnings volatility $EVOLA$, and the number of analysts following the firm and reporting forecasts, ANA, and analyst forecast dispersion $FDISP$ in the past month $t-1$. According to Chordia, Huh, and Subrahmanyam (2007), firm size (i.e., market equity ME), firm age, and book-to-market ratio BM proxy for a firm's visibility; earnings volatility $EVOLA$ proxies for uncertainty about fundamental values; the number of analysts following the firm, ANA, proxies for the mass of informed

[27] Kumar, Page, and Spalt (2011) provide details of counties with high CP ratios (see their Panel B of Figure 1 on p. 673).

agents; and leverage D/A and analysts' forecast dispersion $FDISP$ proxy for differences of opinion. We use past stock returns $r_{j,t-1}$, $r_{j,t-2}$, and $r_{j,t-12\ to\ t-3}$ to capture possible trend-chasing (Hong and Stein (1999)).

We estimate Regression (15) using the Fama and MacBeth (1973) procedure and correct standard errors using the Newey-West method with up to three lags. Table 2 presents the results. We find consistent evidence that the coefficient on the CP ratio is significantly positive. Specifically, in Column 1, the CP coefficient is 0.097 (t-stat$=$ 3.48) after controlling for past stock returns $r_{j,t-1}$, $r_{j,t-2}$, and $r_{j,t-12\ to\ t-3}$, market equity ME, firm age, book-to-market ratio BM, leverage D/A, and earnings volatility $EVOLA$. In Column 2, there is an increase to 0.26 (t-stat$=$ 8.98) after controlling for the number of analysts following the firm and reporting forecasts, ANA, and analyst forecast dispersion $FDISP$. The latter coefficient suggests that a one-standard-deviation shift in CP (1.428 from Table 1) corresponds to a $(0.26/100) \times 1.428 = 0.371\%$ increase in turnover. Relative to the median turnover (8.7% from Table 1), this represents a $0.371\%/8.7\% = 4.27\%$ increase.

Table 2 also presents additional evidence on turnover. Specifically, the coefficients on past stock returns $r_{j,t-1}$, $r_{j,t-2}$, and $r_{j,t-12\ to\ t-3}$ are significantly positive, indeed suggesting trend chasing or portfolio rebalancing. Young stocks and growth stocks have high turnover (the coefficients on $\ln(Firm\ age)$ and BM are significantly negative). Uncertainty about fundamental values increases turnover (the coefficient on earnings volatility $EVOLA$ is positive). There is evidence of analyst-induced trading (i.e., the coefficient on the number of analysts following the firm, ANA, is significantly positive). Greater differences of opinion increases turnover (analysts' forecast dispersion $FDISP$ is positively related to turnover). These findings are in line with previous research on trading activity (e.g., Chordia, Huh, and Subrahmanyam (2007)).

4.3 Gambling and the relative beta

Implication (ii) from Section 3.3 suggests that in the cross-section of stocks, the relative beta should be decreasing in gambling propensity. We test this implication using the following regression:

$$Relative\ beta_{j,t} = b_0 + b_1 \times CP_{j,t} + b_2 \times Controls + \epsilon_{j,t}. \quad (16)$$

The dependent variable is the relative beta of the j'th stock (i.e., the ratio of the return beta to the cash flow beta). We control for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM, leverage D/A, and operating leverage OL as of the end of the past year $y-1$.

Table 3 presents the results from estimating Regression (16) using the Fama-MacBeth procedure. We find consistent evidence that the coefficient on the CP ratio is significantly negative. Specifically, in Column 1, the CP coefficient is significantly negative at -0.031 (t-stat= -2.68) after controlling for past stock returns $r_{j,t-1}$, $r_{j,t-2}$, and $r_{j,t-12\ to\ t-3}$, market equity ME, book-to-market ratio BM, and leverage D/A. In Column 2, the coefficient on CP is also significantly negative at -0.021 (t-stat= -1.81), after further controlling for operating leverage OL. This suggests that a one standard deviation shift in CP (1.428 from Table 1) corresponds to a $0.021 \times 1.428 = 0.030$ decrease in relative beta. Relative to the median relative beta (0.777 from Table 1), this represents an economically significant $0.030/0.777 = 3.86\%$ decrease.

4.4 The Profitability Effect and Gambling

Implication (iii) from Section 3.3 is that underreaction to public signals attenuates when gambling propensity is high. As motivated in Section 4.1, we use gross margins (Novy-Marx (2013)) to test this implication. To accurately measure the level of underreaction to gross margins, we measure the relation of returns to profit margins announced one, two, and three months ago, relative to the current month. Specifically, we use the following regression:

$$\begin{aligned} \textit{Risk-adjusted return}_{j,t} \quad = \quad \sum_{m=1}^{3} (\; & b_{0,m} \times D_{-m} \\ & + b_{1,m} \times Profitability_{j,t} \times D_{-m} \\ & + b_{2,m} \times Profitability_{j,t} \times CP_{j,t} \times D_{-m} \\ & + b_{3,m} \times CP_{j,t} \times D_{-m}) + b_4 \times Controls + \epsilon_{j,t}. \quad (17) \end{aligned}$$

In the above regression, the dependent variable $\textit{Risk-adjusted return}_{j,t}$ is either the j'th stock's excess return over the risk-free interest rate, or the difference between the stock return and the benchmark-implied return in month t. For stock-month t, we use the quarterly report that is announced in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter; D_{-m} (where $m = 1, 2,$ or 3) is a dummy variable indicating that the announcement is made in month $t-m$. We control for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$),[28] market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM as of the end of the past year $y-1$.

Table 4 presents the results from the Fama-MacBeth estimation of Regression (17). The evidence is consistent with the notion that underreaction attenuates when gambling propensity (proxied by CP here) is high. For example, consider the FF5-$adjusted\ return$ in Columns 7 and 8. In Column 7, the coefficient on $Profitability \times D_{-1}$ (D_{-2}) (D_{-3}) is significantly positive; this evidence indicates that the underreaction lasts at least for three months. A simple comparison of the coefficients suggests that the underreaction is most pronounced in the first subsequent month. Further, the coefficient on $Profitability \times CP \times D_{-1}$ is significantly negative at -1.221 (t-stat$=-2.83$), while the coefficient on $Profitability \times CP \times D_{-2}$ (D_{-3}) is not; this evidence is consistent with the notion that gamblers attenuate underreaction, especially in the first subsequent month when the underreaction is most pronounced. We find consistent evidence in Column 8, where we control for past stock returns $r_{j,t-1}$, $r_{j,t-2}$, and $r_{j,t-12\ to\ t-3}$, market equity ME, and book-to-market ratio BM. Particularly, the coefficient on the term $Profitability \times CP \times D_{-1}$ remains significantly negative at -1.367 (t-stat$=-3.27$). The evidence is similar in other columns of Table 4 where we use the excess return and in turn, risk-adjusted returns based on the CAPM and FFC4 models as the dependent variable.

Table 4 also presents evidence of the short-term reversal effect (i.e., the coefficient on $r_{j,t-1}$ is significantly negative), the size effect (i.e., the coefficient on $\ln(ME)$ is significantly negative), and the value effect (i.e., the coefficient on BM is significantly positive). We also find evidence of momentum (e.g., Column 2 shows that the coefficient of the excess

[28] We use the stock return in month $t-1$ ($r_{j,t-1}$) to control for short-term reversals (Jegadeesh (1990)), and the stock return from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$) to control for the longer-term momentum effect (Jegadeesh and Titman (1993)). We control for the stock return in month $t-2$ ($r_{j,t-2}$) separately because Goyal and Wahal (2015) document a carryover of short-term reversals from month -2.

return on $r_{j,t-12\ to\ t-3}$ is positive).[29]

4.5 Gambling, affect investing, and risk-adjusted expected returns

Implication (iv) from Section 3.3 is that in the cross-section of stocks, risk-adjusted expected returns should decrease in the affect heuristic, and this negative relationship should attenuate when gambling propensity is high.

Recall from Section 4.1 that we measure the affect heuristic for a stock using the standardized advertising-spending information ADV. We test the effect of gambling propensity (i.e., the CP ratio) on the relation between risk-adjusted return and ADV using the following regressions:

$$\begin{aligned} Risk\text{-}adjusted\ return_{j,t} &= b_0 + b_1 \times ADV_{j,t} + b_2 \times ADV_{j,t} \times CP_{j,t} \\ &\quad + b_3 \times CP_{j,t} + b_4 \times Controls + \epsilon_{j,t}. \end{aligned} \tag{18}$$

The dependent variable $Risk\text{-}adjusted\ return_{j,t}$ is either the j'th stock's excess return over the risk-free interest rate, or the difference between the stock return and the benchmark-implied return in month t. If month t is in the period from July of year y to June of the next year $y+1$, $ADV_{j,t}$ is the cross-sectional percentile rank of the industry-adjusted XAD_3 (i.e., the difference between the firm's three-year moving average of advertising expense, XAD_3, and the industry average) as of the end of the past year $y-1$. We consider controlling for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM as of the end of the past year $y-1$. We also control for profitability as suggested by the empirical analysis in Section 4.4.

Table 5 presents the results from the Fama-MacBeth estimation of the regressions. The evidence is consistent with the notion that the negative relation between risk-adjusted returns and ADV attenuates when gambling propensity (proxied by CP here) is high.

[29] Daniel and Moskowitz (2016) show that the momentum effect reverses following recessions; specifically, during 2001-2002 and 2009 (see their Table 2 on p. 227). If we exclude these specific years and the immediate post-pandemic period of 2021 from our sample period, then the statistical significance of the momentum effect improves. See also Jegadeesh, Luo, Subrahmanyam, and Titman (2023); they show that the coefficient on $r_{j,t-1}$ is negative and significant, indicating the presence of familiar monthly reversals (as highlighted in Jegadeesh (1990)); the coefficient on $r_{j,t-12\ to\ t-3}$ is positive and significant, pointing to momentum, a phenomenon extensively studied by Jegadeesh and Titman (1993) and others; and the coefficient on $r_{j,t-2}$ is insignificant, suggesting a gradual transition from reversals to momentum.

For example, consider the FF5-$adjusted\ return$ in Columns 6 and 7. Column 6 shows that the coefficient on ADV alone is significantly negative at -0.620 (t-stat= -4.24), while the coefficient on the interaction term $ADV \times CP$ is significantly positive at 0.190 (t-stat= 2.68). Column 7 shows that after controlling for additional variables, the coefficient on the interaction term $ADV \times CP$ remains significantly positive at 0.247 (t-stat= 3.39). The evidence is similar in other columns where we use the excess return and in turn, risk-adjusted returns based on the CAPM and FFC4 models as the dependent variable.

Table 5 also presents evidence consistent with the previous analysis in Section 4.4: That is, the underreaction to profitability attenuates when gambling propensity (proxied by CP here) is high. Specifically, the coefficient on $Profitability \times D_{-1}$ (D_{-2}) (D_{-3}) is mostly significantly positive; the coefficient on $Profitability \times CP \times D_{-1}$ is significantly negative.

Portfolio analysis

We next use portfolio analysis to test for the relations between advertising expense, the CP ratio, and risk-adjusted expected returns. For month t from July of year y to June of the next year $y + 1$, we divide the sample of stocks into two equal-size groups by the median of the advertising expense variable ADV from the past fiscal year $y - 1$, and independently into two equal-size groups based on the median of the Catholic-to-Protestant (CP) ratio from the past fiscal year $y - 1$. This leads to 2×2 portfolios. We compute the return of each portfolio by weighting the stocks in the portfolio equally. We then compute the risk-adjusted return for the portfolio using the excess return over the risk-free interest rate and the benchmark-adjusted returns based on the CAPM, the Fama and French (1993) and Carhart (1997) 4-factor model (FFC4), and the Fama and French (2015) 5-factor model (FF5).

Table 6 presents the results. Consistent with the regression results in Table 5, the negative relation between the risk-adjusted return and advertising spending attenuates when gambling propensity is high. Consider, for example, the FF5-$adjusted\ return$ in Columns 7 and 8. Column 7 shows that conditional on low CP, the FF5-$adjusted\ return$ of the high-ADV minus low-ADV portfolio is significantly negative at -19.6 bps per month (t-stat= -2.27). By contrast, Column 8 shows that conditional on high CP, the FF5-

$adjusted\ return$ of the high-ADV minus low-ADV portfolio, 12.3 bps per month, is insignificant. The difference is significantly positive at $12.3 - (-19.6) = 31.9$ bps per month (t-stat$= 2.53$). The evidence is similar in other columns where we use excess returns, and, in turn, risk-adjusted returns based on the CAPM and FFC4 models.

5 Conclusions

We address the joint impact of non-cashflow considerations and overconfidence on investors' demands, and in turn, on expected returns, within a multi-asset setting. Specifically, we consider the effects of a direct preference or affect for a firm's stock, direct consumption utility from trading (a gambling motive), and a form of overconfidence wherein investors underestimate the precision of public information they do not produce themselves. Our setting takes a first step towards integrating various behavioral elements within a common setting, as opposed to considering the elements in isolation. We also test the model's implications using data on U.S. equities, along with proxies for gambling propensity and affect investing.

In our model, gamblers amplify volume that results from agreements to disagree on the model's parameters. Further, they are willing to absorb the net positions of other traders at low premia. This liquidity provision reduces excess stock return co-movement with the market and the stock price underreaction caused by overconfident investors. In turn, this implies that the relative beta (the ratio of return beta to cash flow beta), and the degree of underreaction to public information, both decrease in the propensity of investors to gamble. Our analysis also suggests that in the cross-section of stocks, the risk-adjusted return decreases in the affect heuristic, and this relation attenuates when gambling propensity is high.

To measure the scale of the gambling propensity in a stock, we adopt the approach of Kumar, Page, and Spalt (2011) and use the Catholic-to-Protestant (CP) ratio of the county where the firm's headquarters is situated. Our findings support our theory: in the cross-section of stocks, turnover increases in the CP ratio; the relative beta decreases in the CP ratio; and the underreaction to profit margins (Novy-Marx (2013)) attenuates when the CP ratio is high. We follow Grullon, Kanatas, and Weston (2004) and Lou (2014) and use the firm's advertising spending in the product market as a metric for the strength of the affect heuristic. Again, our results are in line with our theory: in the cross-section of stocks, risk-adjusted return decreases in advertising spending; further, this negative relation attenuates when the CP ratio is high.

Our results underscore the value of considering attributes of investing that go beyond traditional wealth maximization. In follow-up work, it may be useful to consider dynamic extensions of our setting where gambling propensities vary with asset moments such as volatility and skewness. The role of rational arbitrageurs in attenuating pricing effects caused by affect investing, gambling, and overconfidence also deserves a full investigation. These and other issues are left for future research.

References

Anselme, P. and M. Robinson (2007), "What motivates gambling behavior? Insight into dopamine's role," *Neuronal and Psychological Underpinnings of Pathological Gambling*, page 52.

Barberis, N. and M. Huang (2008), "Stocks as lotteries: The implications of probability weighting for security prices," *American Economic Review*, vol. 98, 2066–2100.

Bernard, V. and J. Thomas (1989), "Post-earnings-announcement drift: delayed price response or risk premium?" *Journal of Accounting research*, pages 1–36.

Billett, M., Z. Jiang, and L. Rego (2014), "Glamour brands and glamour stocks," *Journal of Economic Behavior and Organization*, vol. 107, 744–759.

Black, F. (1986), "Noise," *Journal of Finance*, vol. 41, 528–543.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2020), "Overreaction in macroeconomic expectations," *American Economic Review*, vol. 110, 2748–82.

Bordalo, P., N. Gennaioli, and A. Shleifer (2012), "Salience theory of choice under risk," *Quarterly Journal of Economics*, vol. 127, 1243–1285.

Campbell, M. and K Keller (2003), "Brand familiarity and advertising repetition effects," *Journal of Consumer Research*, vol. 30, 292–304.

Carhart, M. (1997), "On persistence in mutual fund performance," *Journal of Finance*, vol. 52, 57–82.

Chordia, T., S. Huh, and A. Subrahmanyam (2007), "The cross-section of expected trading activity," *Review of Financial Studies*, vol. 20, 709–740.

Coval, J. and T. Moskowitz (1999), "Home bias at home: Local equity preference in domestic portfolios," *Journal of Finance*, vol. 54, 2045–2073.

Coventry, K. and R. Brown (1993), "Sensation seeking, gambling and gambling addictions," *Addiction*, vol. 88, 541–554.

Da, Z., U. Gurun, and M. Warachka (2014), "Frog in the pan: Continuous information and momentum," *Review of Financial Studies*, vol. 27, 2171–2218.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998), "Investor psychology and security market under-and overreactions," *Journal of Finance*, vol. 53, 1839–1885.

Daniel, K. and T. Moskowitz (2016), "Momentum crashes," *Journal of Financial Economics*, vol. 122, 221–247.

De Bondt, W. and R. Thaler (1995), "Financial decision-making in markets and firms: A behavioral perspective," *In Jarrow, R., V. Maksimovic, and W. Ziemba (eds.), Finance, Handbooks in Operations Research and Management Science, North Holland, Amsterdam*, pages 385–410.

Dhar, R. and W. Goetzmann (2006), "Bubble investors: What were they thinking?" *Yale ICF working paper*.

Dorn, D. and P. Sengmueller (2009), "Trading as entertainment?" *Management Science*, vol. 55, 591–603.

Eyster, E., M. Rabin, and D. Vayanos (2019), "Financial markets where traders neglect the informational content of prices," *Journal of Finance*, vol. 74, 371–399.

Fama, E. and K. French (1992), "The cross-section of expected stock returns," *Journal of Finance*, vol. 47, 427–465.

Fama, E. and K. French (1993), "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics*, vol. 33, 3–56.

Fama, E. and K. French (2007), "Disagreement, tastes, and asset prices," *Journal of Financial Economics*, vol. 83, 667–689.

Fama, E. and K. French (2008), "Dissecting anomalies," *Journal of Finance*, vol. 63, 1653–1678.

Fama, E. and K. French (2015), "A five-factor asset pricing model," *Journal of Financial Economics*, vol. 116, 1–22.

Fama, E. and J. MacBeth (1973), "Risk, return, and equilibrium: Empirical tests," *Journal of Political Economy*, vol. 81, 607–636.

Frieder, L. and A. Subrahmanyam (2005), "Brand perceptions and the market for common stock," *Journal of Financial and Quantitative Analysis*, vol. 40, 57–85.

Friedman, H. and M. Heinle (2016), "Taste, information, and asset prices: Implications for the valuation of CSR," *Review of Accounting Studies*, vol. 21, 740–767.

Gao, X. and T. Lin (2015), "Do individual investors treat trading as a fun and exciting gambling activity? Evidence from repeated natural experiments," *Review of Financial Studies*, vol. 28, 2128–2166.

Gao, X. and J. Ritter (2010), "The marketing of seasoned equity offerings," *Journal of Financial Economics*, vol. 97, 33–52.

Geczy, C., R. Stambaugh, and D. Levin (2005), "Investing in socially responsible mutual funds," *working paper, University of Pennsylvania*.

Goyal, A. and S. Wahal (2015), "Is momentum an echo?" *Journal of Financial and Quantitative Analysis*, vol. 50, 1237–1267.

Greenwood, R. and A. Shleifer (2014), "Expectations of returns and expected returns," *Review of Financial Studies*, vol. 27, 714–746.

Grinblatt, M. and B. Han (2005), "Prospect theory, mental accounting, and momentum," *Journal of Financial Economics*, vol. 78, 311–339.

Grinblatt, M. and M. Keloharju (2009), "Sensation seeking, overconfidence, and trading activity," *Journal of Finance*, vol. 64, 549–578.

Grullon, G., G. Kanatas, and J. Weston (2004), "Advertising, breadth of ownership, and liquidity," *Review of Financial Studies*, vol. 17, 439–461.

Holden, C. and A. Subrahmanyam (2002), "News events, information acquisition, and serial correlation," *Journal of Business*, vol. 75, 1–32.

Hong, H. and M. Kacperczyk (2009), "The price of sin: The effects of social norms on markets," *Journal of Financial Economics*, vol. 93, 15–36.

Hong, H. and J. C. Stein (1999), "A unified theory of underreaction, momentum trading, and overreaction in asset markets," *Journal of Finance*, vol. 54, 2143–2184.

Hong, H. and J. C. Stein (2007), "Disagreement and the stock market," *Journal of Economic Perspectives*, vol. 21, 109–128.

Jegadeesh, N. (1990), "Evidence of predictable behavior of security returns," *Journal of Finance*, vol. 45, 881–898.

Jegadeesh, N., J. Luo, A. Subrahmanyam, and S. Titman (2023), "Short-term reversals and longer-term momentum around the world: Theory and evidence," *Available at SSRN: https://ssrn.com/abstract=4069575*.

Jegadeesh, N. and S. Titman (1993), "Returns to buying winners and selling losers: Implications for stock market efficiency," *Journal of Finance*, vol. 48, 65–92.

Keloharju, M., S. Knüpfer, and J. Linnainmaa (2012), "Do investors buy what they know? Product market choices and investment decisions," *Review of Financial Studies*, vol. 25, 2921–2958.

Keynes, J. M. (1936), *The general theory of interest, employment and money*, London: Macmillan.

Kuley, N. and D. Jacobs (1988), "The relationship between dissociative-like experiences and sensation seeking among social and problem gamblers," *Journal of Gambling Behavior*, vol. 4, 197–207.

Kumar, A., J. Page, and O. Spalt (2011), "Religious beliefs, gambling attitudes, and financial market outcomes," *Journal of Financial Economics*, vol. 102, 671–708.

Leland, H. (1992), "Insider trading: Should it be prohibited?" *Journal of Political Economy*, vol. 100, 859–887.

Lichtenstein, S., B. Fischhoff, and L. Phillips (1982), "Calibration of probabilities: The state of the art to 1980." *In D. Kahneman, P. Slovic, and A. Tversky (eds.), Judgement under uncertainty: Heuristics and biases, New York, Cambridge University Press*, pages 306–334.

Liu, H., C. Peng, W. Xiong, and W. Xiong (2022), "Taming the bias zoo," *Journal of Financial Economics*, vol. 143, 716–741.

Lou, D. (2014), "Attracting investor attention through advertising," *Review of Financial Studies*, vol. 27, 1797–1829.

Luo, J. and A. Subrahmanyam (2019a), "The affect heuristic and stock ownership: A theoretical perspective," *Review of Financial Economics*, vol. 37, 6–37.

Luo, J. and A. Subrahmanyam (2019b), "Asset pricing when trading is for entertainment," *Review of Behavioral Finance*, vol. 11, 220–264.

Luo, J., A. Subrahmanyam, and S. Titman (2021), "Momentum and reversals when overconfident investors underestimate their competition," *Review of Financial Studies*, vol. 34, 351–393.

Markiewicz, Ł. and E. Weber (2013), "DOSPERT's gambling risk-taking propensity scale predicts excessive stock trading," *Journal of Behavioral Finance*, vol. 14, 65–78.

Martineau, C. (2021), "Rest in peace post-earnings announcement drift," *Critical Finance Review, Forthcoming*.

Nagy, R. A. and R. W. Obenberger (1994), "Factors influencing individual investor behavior," *Financial Analysts Journal*, vol. 50, 63–68.

Novy-Marx, R. (2011), "Operating leverage," *Review of Finance*, vol. 15, 103–134.

Novy-Marx, R. (2013), "The other side of value: The gross profitability premium," *Journal of financial economics*, vol. 108, 1–28.

Odean, T. (1998), "Volume, volatility, price, and profit when all traders are above average," *Journal of Finance*, vol. 53, 1887–1934.

Preuschoff, K., P. Bossaerts, and S. Quartz (2006), "Neural differentiation of expected reward and risk in human subcortical structures," *Neuron*, vol. 51, 381–390.

Puri, I. (2018), "Preference for simplicity," *Available at SSRN 3253494*.

Schoenbachler, D., G. Gordon, and T. Aurand (2004), "Building brand loyalty through individual stock ownership," *Journal of Product and Brand Management*, vol. 13, 488–497.

Sherman, J. and W. J. Morrison (1950), "Adjustment of an inverse matrix corresponding to a change in one element of a given matrix," *The Annals of Mathematical Statistics*, vol. 21, 124–127.

Subrahmanyam, A. (2024), "Keeping it simple: The disappearance of premia for standard non-market factors," *Available at SSRN 4584638*.

Tversky, A. and D. Kahneman (1974), "Judgment under uncertainty: Heuristics and biases," *Science*, vol. 185, 1124–1131.

Tversky, A. and D. Kahneman (1992), "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, vol. 5, 297–323.

Woodbury, MA (1950), "Inverting modified matrices. memorandum rep. 42," *Statistical Research Group, Princeton University*.

Appendix A

Proof of Theorem 1: (a) The i'th overconfident investor believes that

$$\begin{aligned} F|\tau &\sim N\left(\frac{\nu_F}{\kappa_\tau}\tau, \kappa_{F|\tau}\right), \text{ where } \kappa_{F|\tau} = \nu_F - \frac{\nu_F^2}{\kappa_\tau}; \text{ and} \\ \theta|s &\sim N\left(\nu_\theta {\kappa_s}^{-1} s, \kappa_{\theta|s}\right), \text{ where } \kappa_{\theta|s} = \nu_\theta - \nu_\theta^2 {\kappa_s}^{-1}. \end{aligned}$$

Write the investor's wealth at Date 1 as

$$W_{i1} = W_{i0}R_f + X_i'(V - PR_f) = W_{i0}R_f + X_i'(\bar{V} + \ell F + \theta - PR_f).$$

The investor's demand X_i maximizes

$$\begin{aligned} & \hat{E}\left[U(W_{i1})|\tau, s\right] \\ = \; & \hat{E}\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(\bar{V} + \ell F + \theta - PR_f)\right]|\tau, s\right] \\ = \; & -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \nu_\theta\kappa_s^{-1}s - PR_f\right) + 0.5\gamma^2 X_i'\phi_\kappa X_i\right], \end{aligned}$$

where $\hat{E}()$ indicates taking expectations based on the investor's biased beliefs, $\phi_\kappa = \ell\ell'\kappa_{F|\tau} + \kappa_{\theta|s}$, and the second equality is based on the normality assumption. The first-order condition (f.o.c.) with respect to (w.r.t.) X_i implies that the optimal demand is:

$$X_\kappa = \frac{\phi_\kappa^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \nu_\theta\kappa_s^{-1}s - PR_f\right). \tag{A.1}$$

The second-order condition holds here and other cases that follow, so we omit referencing it in the rest of the proofs.

(b) The i'th affect investor perceives that in addition to the actual liquidation values $V = \bar{V} + \ell F + \theta$, there are extra non-monetary payoffs as indicated by A, and has unbiased beliefs that

$$\begin{aligned} F|\tau &\sim N\left(\frac{\nu_F}{\nu_\tau}\tau, \nu_{F|\tau}\right), \text{ where } \nu_{F|\tau} = \nu_F - \frac{\nu_F^2}{\nu_\tau}; \text{ and} \\ \theta|s &\sim N\left(\nu_\theta {\nu_s}^{-1} s, \nu_{\theta|s}\right), \text{ where } \nu_{\theta|s} = \nu_\theta - \nu_\theta^2 {\nu_s}^{-1}. \end{aligned}$$

The investor's demand X_i maximizes:

$$\begin{aligned} & E\left[U(W_{i1}^A)|\tau, s\right] \\ = \; & E\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(A + \bar{V} + \ell F + \theta - PR_f)\right]|\tau, s\right] \\ = \; & -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right) + 0.5\gamma^2 X_i'\phi X_i\right], \end{aligned}$$

where $\phi = \ell\ell'\nu_{F|\tau} + \nu_{\theta|s}$. The f.o.c. implies that the demand is expressed as:

$$X_A = \frac{\phi^{-1}}{\gamma}\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right). \tag{A.2}$$

(c) The i'th gambler also has unbiased beliefs. The investor's demand X_i maximizes:

$$\begin{aligned}
& E\left[U_G(W_{i1})|\tau, s\right] \\
= \; & E\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(\bar{V} + \ell F + \theta - PR_f) - 0.5X_i'GX_i\right]|\tau, s\right] \\
= \; & -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right) + 0.5\gamma^2 X_i'\left(\phi - \frac{G}{\gamma^2}\right)X_i\right].
\end{aligned}$$

The f.o.c. implies that the demand is expressed as:

$$X_G = \frac{\phi_G^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + \nu_\theta\nu_s^{-1}s - PR_f\right), \tag{A.3}$$

where $\phi_G = \phi - G/\gamma^2 > 0$.

(d) From Equations (A.1), (A.2), and (A.3), the market-clearing condition, $\eta_\kappa X_\kappa + \eta_A X_A + \eta_G X_G = \mathbf{1}$, implies that the equilibrium prices P take the form as given in this theorem. □

Proof of Proposition 1: Prior to proving this proposition, we first lay out the groundwork. This groundwork also serves as a foundation for proving Propositions 2 and 4.

Note that in the simple economy, the variance of the firm-specific payoff (denoted by ν) is identical across stocks and the information signal about θ_j, denoted by s_j, is not informative. See Equation (3). According to the formula in Sherman and Morrison (1950),[30] we have:

$$\begin{aligned}
\phi_\kappa^{-1} &= \frac{1}{\nu}\left(\mathbf{I} - \frac{\ell\ell'}{\kappa_{F|\tau}^{-1}\nu + \ell'\ell}\right), \quad \phi^{-1} = \frac{1}{\nu}\left(\mathbf{I} - \frac{\ell\ell'}{\nu_{F|\tau}^{-1}\nu + \ell'\ell}\right), \quad \text{and} \\
\phi_G^{-1} &= \frac{1}{\nu}\left[(\mathbf{I} - G^*)^{-1} - \frac{(\mathbf{I} - G^*)^{-1}\ell\ell'(\mathbf{I} - G^*)^{-1}}{\nu_{F|\tau}^{-1}\nu + \ell'(\mathbf{I} - G^*)^{-1}\ell}\right],
\end{aligned} \tag{A.4}$$

where $G^* = G/(\gamma^2\nu)$, and $\mathbf{I}$ is a $J \times J$ identity matrix. We can express $\Phi = \nu^{-1}(\Phi_1 + \Phi_0)$,

[30] Specifically, let M be a $J \times J$ matrix and Λ be a $J \times 1$ vector. Suppose that M is invertible and that $1 + \Lambda' M^{-1}\Lambda \neq 0$. Then, $(M + \Lambda\Lambda')^{-1} = M^{-1} - \dfrac{M^{-1}\Lambda\Lambda' M^{-1}}{1 + \Lambda' M^{-1}\Lambda}$.

where

$$\Phi_1 = (\eta_\kappa + \eta_A)\mathbf{I} + \eta_G (\mathbf{I} - G^*)^{-1}, \tag{A.5}$$

$$\Phi_0 = -\left[\eta_\kappa \frac{\ell\ell'}{\kappa_{F|\tau}^{-1}\nu + \ell'\ell} + \eta_A \frac{\ell\ell'}{\nu_{F|\tau}^{-1}\nu + \ell'\ell} + \eta_G \frac{(\mathbf{I} - G^*)^{-1}\ell\ell'(\mathbf{I} - G^*)^{-1}}{\nu_{F|\tau}^{-1}\nu + \ell'(\mathbf{I} - G^*)^{-1}\ell}\right]. \tag{A.6}$$

From the matrix identity in Woodbury (1950),[31] we have

$$\Phi^{-1} = \nu\left[\Phi_1^{-1} - \Phi_1^{-1}\Phi_0(\mathbf{I} + \Phi_1^{-1}\Phi_0)^{-1}\Phi_1^{-1}\right]. \tag{A.7}$$

Suppose that in the simple economy, the conditional factor payoff is significantly less volatile than the conditional firm-specific payoff. For notational convenience, let $\nu_F^{-1}\nu \to \infty$; this implies $\nu_{F|\tau}^{-1}\nu \to \infty$ and $\kappa_{F|\tau}^{-1}\nu \to \infty$. We can use Equations (A.6) and (A.7) to show that:

$$\Phi_0 \to \mathbf{0}_{J\times J} \text{ and } \Phi^{-1} \to \nu\Phi_1^{-1}. \tag{A.8}$$

Equation (A.5) implies that Φ_1 is a diagonal matrix: It is straightforward to show:

- $d\Phi_1/d\rho = \mathbf{0}_{J\times J}$;
- $d(\Phi_1^{-1})/d\gamma$ is a diagonal matrix where each diagonal element is positive;
- $d(\Phi_1^{-1})/dG_j \propto -\mathbf{I}_{jj}$ where $\mathbf{I}_{jj}$ is a $J \times J$ matrix where the j'th diagonal element equals 1 and the other elements equal 0.

Since $\nu_{F|\tau}^{-1}\nu \to \infty$ and also $\kappa_{F|\tau}^{-1}\nu \to \infty$, it follows from Equation (A.6) that:

- $d\Phi_0/d\rho \to \mathbf{0}_{J\times J}$, and $d\Phi_0/d\gamma \to \mathbf{0}_{J\times J}$.

See the expression of Φ^{-1} in Equation (A.7). Note from the previous derivation that $\Phi_0 \to \mathbf{0}_{J\times J}$, $d\Phi_0/d\rho \to \mathbf{0}_{J\times J}$, and $d\Phi_1/d\rho \to \mathbf{0}_{J\times J}$; it follows that

$$\frac{d(\Phi^{-1})}{d\rho} \to \mathbf{0}_{J\times J}. \tag{A.9}$$

[31]Specifically, let M and N be two $J \times J$ matrices. Suppose that M is invertible and that $\mathbf{I} + M^{-1}N$ is also invertible. Then, $(M+N)^{-1} = M^{-1} - M^{-1}N(\mathbf{I} + M^{-1}N)^{-1}M^{-1}$.

Note from the previous derivation that $\Phi_0 \to \mathbf{0}_{J\times J}$, $d\Phi_0/d\gamma \to \mathbf{0}_{J\times J}$, and $d(\Phi_1^{-1})/d\gamma$ is a diagonal matrix where each diagonal element is positive; it follows that

$$\frac{d(\Phi^{-1})}{d\gamma} \text{ is a diagonal matrix where each diagonal element is positive.} \tag{A.10}$$

Next, we consider the derivative of Φ^{-1} w.r.t. G_j. See the expression of Φ_0 in Equation (A.6). Note that in the simple economy, $\nu_{F|\tau}^{-1}\nu \to \infty$ and $\kappa_{F|\tau}^{-1}\nu \to \infty$; it follows that $d\Phi_0/dG_j \to \mathbf{0}_{J\times J}$. Also note from the previous derivation that $\Phi_0 \to \mathbf{0}_{J\times J}$ and $d(\Phi_1^{-1})/dG_j \propto -\mathbf{I}_{jj}$; then, it follows from Equation (A.7) that

$$\frac{d(\Phi^{-1})}{dG_j} \;\to\; \nu\frac{d(\Phi_1^{-1})}{dG_j} \propto -\mathbf{I}_{jj}. \tag{A.11}$$

This completes the preparation for the subsequent analysis.

Now we prove Proposition 1 using Theorem 2. We substitute the equilibrium price into the demands of each investor type, which yields:

$$\begin{aligned}
X_\kappa &= \frac{\phi_\kappa^{-1}}{\gamma}\left[\left(\mathbf{I} - \Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\right)\ell\Delta\tau - \Phi^{-1}\left(\eta_A\phi^{-1}A - \gamma\mathbf{1}\right)\right], \\
X_A &= \frac{\phi^{-1}}{\gamma}\left[A - \Phi^{-1}\left(\eta_\kappa\phi_\kappa^{-1}\ell\Delta\tau + \eta_A\phi^{-1}A - \gamma\mathbf{1}\right)\right], \\
X_G &= -\frac{\phi_G^{-1}}{\gamma}\Phi^{-1}\left(\eta_\kappa\phi_\kappa^{-1}\ell\Delta\tau + \eta_A\phi^{-1}A - \gamma\mathbf{1}\right).
\end{aligned}$$

Note that the demand for the j'th stock by each of the three investor types, denoted as $X_{\kappa j}$, X_{Aj}, and X_{Gj}, is simply the j'th element of the vectors X_κ, X_A, and X_G, respectively.

$$X_{\kappa j} - E(X_{\kappa j}) = \psi_{\kappa j}\Delta\tau, \;\; X_{Aj} - E(X_{Aj}) = \psi_{Aj}\Delta\tau \;\text{ and }\; X_{Gj} - E(X_{Gj}) = \psi_{Gj}\Delta\tau, \tag{A.12}$$

where

$$\begin{aligned}
\psi_{\kappa j} &= \frac{1}{\gamma}\mathbf{1}_j'\phi_\kappa^{-1}\left(\mathbf{I} - \Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\right)\ell, \\
\psi_{Aj} &= -\frac{1}{\gamma}\mathbf{1}_j'\phi^{-1}\Phi^{-1}\left(\eta_\kappa\phi_\kappa^{-1}\right)\ell, \;\text{ and }\; \psi_{Gj} = -\frac{1}{\gamma}\mathbf{1}_j'\phi_G^{-1}\Phi^{-1}\left(\eta_\kappa\phi_\kappa^{-1}\right)\ell.
\end{aligned} \tag{A.13}$$

It is straightforward to verify that $\eta_\kappa\psi_{\kappa j} + \eta_A\psi_{Aj} + \eta_G\psi_{Gj} = 0$.

If $\rho = 1$, then it follows from Equation (3) that $\Delta = 0$. We can use Equations (5) and (A.12) to show that $E(T_j^*) = 0$. This proves Part (i) of this proposition.

If $\rho > 1$, suppose that $\ell_j > 0$ (we can modify the derivation here to show that the results also apply to the case with $\ell_j < 0$). Then, from Equation (A.13), we have

$$\begin{aligned}
\psi_{\kappa j} &= \frac{1}{\gamma\nu}\mathbf{1}_j'\left(\mathbf{I} - \frac{\ell\ell'}{\kappa_{F|\tau}^{-1}\nu + \ell'\ell}\right)\left[\mathbf{I} - \Phi^{-1}\frac{\eta_\kappa}{\nu}\left(\mathbf{I} - \frac{\ell\ell'}{\kappa_{F|\tau}^{-1}\nu + \ell'\ell}\right)\right]\ell \qquad \text{(A.14)} \\
&\to \frac{1}{\gamma\nu}\mathbf{1}_j'\left(\mathbf{I} - \Phi_1^{-1}\eta_\kappa\right)\ell \\
&\propto \left[1 - \frac{\eta_\kappa}{\eta_\kappa + \eta_A + \eta_G(1 - G_j/(\gamma^2\nu))^{-1}}\right]\ell_j \\
&> 0
\end{aligned}$$

where the equality follows from the expression of ϕ_κ in Equation (A.4), the $\to$ follows from Equation (A.8) and the fact that $\kappa_{F|\tau}^{-1}\nu \to \infty$ in the simple economy, and the inequality follows from the assumption that $\ell_j > 0$. A similar derivation can be used to show that

$$\psi_{Aj} < 0 \text{ and } \psi_{Gj} < 0.$$

See the expression of $\psi_{\kappa j}$ in Equation (A.14). It follows from the fact $\kappa_{F|\tau}^{-1}\nu \to \infty$ and the previously derived $d(\Phi^{-1})/d\rho \to \mathbf{0}_{J\times J}$ (see Equation (A.9)) that

- $d\psi_{\kappa j}/d\rho \to 0$.

Note from the previous derivation that $d(\Phi^{-1})/d\gamma$ is a diagonal matrix where each diagonal element is positive (see Equation (A.10)); it follows that

- $d\psi_{\kappa j}/d\gamma < 0$.

Note from the previous derivation that $d(\Phi^{-1})/dG_j \propto -\mathbf{I}_{jj}$ (see Equation (A.11)); it follows that given $\ell_j > 0$,

- $d\psi_{\kappa j}/dG_j > 0$.

Consider Equations (5) and (A.12). Using the previously derived results that $\psi_{\kappa j} > 0$, $\psi_{Aj} < 0$, $\psi_{Gj} < 0$, and $\eta_\kappa\psi_{\kappa j} + \eta_A\psi_{Aj} + \eta_G\psi_{Gj} = 0$, we obtain

$$\begin{aligned}
E(T_j^*) &= 0.5\eta_\kappa\psi_{\kappa j}E(|\Delta\tau|) - 0.5\eta_A\psi_{Aj}E(|\Delta\tau|) - 0.5\eta_G\psi_{Gj}E(|\Delta\tau|) \\
&= \eta_\kappa\psi_{\kappa j}E(|\Delta\tau|). \qquad \text{(A.15)}
\end{aligned}$$

Note from the previous derivation that $d\psi_{\kappa j}/d\rho \to 0$; it follows that

$$\frac{dE(T_j^*)}{d\rho} \quad \to \quad \eta_\kappa \psi_{\kappa j} \frac{dE(|\mathit{\Delta} \tau|)}{d\rho} \propto \frac{d|\mathit{\Delta}|}{d\rho} > 0,$$

where the last inequality obtains from the expression of $\mathit{\Delta}$ in Equation (3). Note from the previous derivation that $d\psi_{\kappa j}/d\gamma < 0$; it follows that

$$\frac{dE(T_j^*)}{d\gamma} \quad \propto \quad \frac{d\psi_{\kappa j}}{d\gamma} < 0.$$

Note from the previous derivation that $d\psi_{\kappa j}/dG_j > 0$ given that $\ell_j > 0$; it follows that

$$\frac{dE(T_j^*)}{dG_j} \quad \propto \quad \frac{d\psi_{\kappa j}}{dG_j} > 0.$$

This proves Part (ii) of this proposition. □

Proof of Proposition 2: Suppose that $\rho = 1$. It follows from Equation (3) that $\mathit{\Delta} = 0$. For a sufficiently large J, θ_M in Equation (8) converges to zero due to the diversification effect. In this case, we can use Equations (6), (9), and (10) to show that

$$\beta_{jM} = \frac{\ell_j \nu_{F|\tau}}{\nu_{F|\tau}} = \ell_j.$$

Thus, $\beta_j/\ell_j = 1$. This proves Part (i) of this proposition.

Now suppose that $\rho > 1$. See the expression of r_j in Equation (6). We can use a similar derivation as in the proof of Proposition 1 to show that the expression

$$\begin{aligned} \mathbf{1}_j' \mathit{\Phi}^{-1} \eta_\kappa \phi_\kappa^{-1} \ell \quad &\to \quad \mathbf{1}_j' \mathit{\Phi}_1^{-1} \eta_\kappa \left(\mathbf{I} - \frac{\ell \ell'}{\kappa_{F|\tau}^{-1} \nu + \ell' \ell} \right) \ell \to \mathbf{1}_j' \mathit{\Phi}_1^{-1} \eta_\kappa \ell \\ &= \quad \frac{\eta_\kappa}{\eta_\kappa + \eta_A + \eta_G (1 - G_j/(\gamma^2 \nu))^{-1}} \ell_j, \end{aligned} \tag{A.16}$$

where the first $\to$ follows from Equation (A.8) and the expression for ϕ_κ^{-1} in Equation (A.4), and the second $\to$ follows from the fact that $\kappa_{F|\tau}^{-1} \nu \to \infty$ in the simple economy. Using Equation (8), it follows that:

$$\mathit{\Psi}_M = \frac{1}{J} \sum_{j=1}^{J} (\mathbf{1}_j' \mathit{\Phi}^{-1} \eta_\kappa \phi_\kappa^{-1} \ell) \to \frac{1}{J} \sum_{j=1}^{J} \left[\frac{\eta_\kappa}{\eta_\kappa + \eta_A + \eta_G (1 - G_j/(\gamma^2 \nu))^{-1}} \ell_j \right] > 0,$$

where the inequality holds under regular parameter values, with ℓ_j on average being positive. Thus, in the simple economy, we can assert that $\mathit{\Psi}_M > 0$.

From Equations (6), (9), and (10), we obtain

$$\frac{\beta_{jM}}{\ell_j} = \frac{\text{Cov}(r_j, r_M)}{\ell_j}\frac{1}{\text{Var}(r_M)},$$

where

$$\begin{aligned}\frac{\text{Cov}(r_j, r_M)}{\ell_j} &= \frac{\ell_j \nu_{F|\tau} + \nu/J + \Psi_M \mathbf{1}_j' \Phi^{-1}\eta_\kappa \phi_\kappa^{-1}\ell \Delta^2 \nu_\tau}{\ell_j} \\ &= \nu_{F|\tau} + \frac{\nu}{J\ell_j} + \frac{1}{\ell_j}\Psi_M \mathbf{1}_j' \Phi^{-1}\eta_\kappa \phi_\kappa^{-1}\ell \Delta^2 \nu_\tau.\end{aligned}$$

Note that $\text{Var}(r_M)$ and $\Psi_M > 0$ are identical across all stocks. For Part (ii) of this proposition, it suffices to show that

$$\begin{aligned}\frac{d}{dG_j}\left(\frac{\mathbf{1}_j'\Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\ell\Delta^2\nu_\tau}{\ell_j}\right) &\propto \frac{1}{\ell_j}\mathbf{1}_j'\frac{d(\Phi^{-1})}{dG_j}\phi_\kappa^{-1}\ell = \frac{1}{\ell_j}\mathbf{1}_j'\frac{d(\Phi^{-1})}{dG_j}\left(\mathbf{I} - \frac{\ell\ell'}{\kappa_{F|\tau}^{-1}\nu + \ell'\ell}\right)\ell \\ &\to \frac{1}{\ell_j}\mathbf{1}_j'\nu\frac{d(\Phi_1^{-1})}{dG_j}\ell \propto \frac{1}{\ell_j}\mathbf{1}_j'(-\mathbf{I}_{jj})\ell = -1 < 0, \qquad \text{(A.17)}\end{aligned}$$

where the first equality follows from the expression for ϕ_κ in Equation (A.4), the $\to$ follows from Equation (A.11) and the fact that $\kappa_{F|\tau}^{-1}\nu \to \infty$ in the simple economy, and the second $\propto$ follows from Equation (A.11). This completes the proof for Part (ii) of this proposition. □

Proof of Proposition 3: Note that in the simple economy, the firm-specific signal s_j is not informative. Using Equations (6) and (11), we can compute

$$\text{Cov}(r_j, S_j) = -(1-\mu)\mathbf{1}_j'\Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\ell\Delta\nu_\tau.$$

If $\rho = 1$, then it follows from Equation (3) that $\Delta = 0$; in this case, $\text{Cov}(r_j, S_j) = 0$.

Suppose $\rho > 1$. Then, $\kappa_\tau > \nu_\tau$; it follows from Equation (3) that $\Delta < 0$. In this case,

$$\text{Cov}(r_j, S_j) \propto \mathbf{1}_j'\Phi^{-1}\eta_\kappa\phi_\kappa^{-1}\ell \to \frac{\eta_\kappa}{\eta_\kappa + \eta_A + \eta_G(1 - G_j/(\gamma^2\nu))^{-1}}\ell_j,$$

where the $\to$ can be obtained by using the same derivation as in the proof of Proposition 2 (see Equation (A.16)). Therefore, given $\ell_j > 0$, $\text{Cov}(r_j, S_j) > 0$. We can use a similar derivation as in the proof of Proposition 2 (see Equation (A.17)) to show that $d\text{Cov}(r_j, S_j)/dG_j < 0$.□

Proof of Proposition 4: Suppose that $\rho = 1$ and $\forall j$ $A_j = G_j = 0$. It follows from Equation (3) that $\Delta = 0$ and $\Phi = \phi^{-1}$. Using Theorem 2, we can express the excess return vector as

$$r = V - PR_f = \ell\left(F - \frac{\nu_F}{\nu_\tau}\tau\right) + \theta + \gamma\phi\mathbf{1}.$$

Note that by definition, $r_M = \mathbf{1}'r/J$. It follows that

$$E(r) = \gamma\phi\mathbf{1}, \quad E(r_M) = \gamma\mathbf{1}'\phi\mathbf{1}/J,$$
$$\text{Var}(r) = \phi, \quad \text{Cov}(r, r_M) = \phi\mathbf{1}/J, \quad \text{and} \quad \text{Var}(r_M) = \mathbf{1}'\phi\mathbf{1}/J^2.$$

We can express the risk-adjusted expected return vector as:

$$\alpha = E(r) - \beta_m E(r_M) = \gamma\phi\mathbf{1} - \frac{\phi\mathbf{1}/J}{\mathbf{1}'\phi\mathbf{1}/J^2}\gamma\mathbf{1}'\phi\mathbf{1}/J = \mathbf{0}_{J\times 1};$$

it follows that $\alpha_j = \mathbf{1}_j'\alpha = 0$. This proves Part (i) of this proposition.

Now suppose that $\rho > 1$ and $\exists j$ such that $A_j \neq 0$ and/or $G_j > 0$. We can use a similar derivation as in the proof of Proposition 2 to compute β_{jM} and show that it does not depend on A_j.

Note from Equations (12) and (13) that

$$\alpha_j = E(r_j) - \beta_{jM}E(r_M), \text{ where } E(r_j) = \mathbf{1}_j'\Phi^{-1}\left(\gamma\mathbf{1} - \eta_A\phi^{-1}A\right).$$

Note that $E(r_M)$ is identical across all stocks and that β_{jM} does not depend on A_j, as proved above. For Part (ii) of this proposition, we just need to show that

$$\begin{aligned}\frac{\partial E(r_j)}{\partial A_j} &= -\mathbf{1}_j'\Phi^{-1}\eta_A\phi^{-1}\mathbf{1}_j \\ &\to -\mathbf{1}_j'\Phi_1^{-1}\eta_A\left(\mathbf{I} - \frac{\ell\ell'}{\nu_{F|\tau}^{-1}\nu + \ell'\ell}\right)\mathbf{1}_j \to -\mathbf{1}_j'\Phi_1^{-1}\eta_A\mathbf{1}_j \\ &= -\frac{\eta_A}{\eta_\kappa + \eta_A + \eta_G(1 - G_j/(\gamma^2\nu))^{-1}} < 0,\end{aligned}$$

where the first $\to$ follows from Equation (A.11) and the expression for ϕ^{-1} in Equation (A.4), and the second $\to$ follows from the fact that $\nu_{F|\tau}^{-1}\nu \to \infty$ in the simple economy.

$$\begin{aligned}\frac{\partial}{\partial G_j}\left(\frac{\partial E(r_j)}{\partial A_j}\right) &\propto -\mathbf{1}_j'\frac{d(\Phi^{-1})}{dG_j}\phi^{-1}\mathbf{1}_j \to -\mathbf{1}_j'\frac{d(\Phi^{-1})}{dG_j}\left(\mathbf{I} - \frac{\ell\ell'}{\nu_{F|\tau}^{-1}\nu + \ell'\ell}\right)\mathbf{1}_j \\ &\to -\mathbf{1}_j'\nu\frac{d(\Phi_1^{-1})}{dG_j}\mathbf{1}_j \propto \mathbf{1}_j'\mathbf{I}_{jj}\mathbf{1}_j = 1 > 0,\end{aligned}$$

where the first $\rightarrow$ follows from the expression for ϕ^{-1} in Equation (A.4), the second $\rightarrow$ follows from Equation (A.11) and the fact that $\nu_{F|\tau}^{-1}\nu \rightarrow \infty$ in the simple economy, and the second $\propto$ follows from Equation (A.11).

$$\begin{aligned}
\frac{\partial}{\partial \gamma}\left(\frac{\partial E(r_j)}{\partial A_j}\right) &\propto -\mathbf{1}_j' \frac{d(\varPhi^{-1})}{d\gamma}\phi^{-1}\mathbf{1}_j \rightarrow -\mathbf{1}_j' \frac{d(\varPhi^{-1})}{d\gamma}\left(\mathbf{I} - \frac{\ell\ell'}{\nu_{F|\tau}^{-1}\nu + \ell'\ell}\right)\mathbf{1}_j \\
&\rightarrow -\mathbf{1}_j' \frac{d(\varPhi^{-1})}{d\gamma}\mathbf{1}_j < 0,
\end{aligned}$$

where the first $\rightarrow$ follows from the expression for ϕ^{-1} in Equation (A.4), the second $\rightarrow$ follows from the fact that $\nu_{F|\tau}^{-1}\nu \rightarrow \infty$ in the simple economy, and the inequality follows from the fact that $d(\varPhi^{-1})/d\gamma$ is a diagonal matrix where each diagonal element is positive (see Equation (A.10) in the proof of Proposition 1). □

Appendix B

This table describes how we compute the variables used in our empirical analysis.

Variables	Definition
$Turnover$	For stock-month t, we use monthly shares traded in month t as reported by CRSP, scaled by the number of outstanding shares at the end of the past month $t-1$. To address double counting in NASDAQ turnover, due to the recording of inter-dealer trades, we follow the method proposed by Gao and Ritter (2010) and divide the turnover of NASDAQ stocks by 2.0 before January 2001, by 1.8 for the rest of 2001, by 1.6 for the years 2002-2003, and leave it unchanged thereafter.
$Relative\ beta$	For stock-month t, we compute $Relative\ beta$ as the ratio of the estimated return beta to cash flow beta in that month. We estimate the return beta using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$). For month t is in quarter q, we estimate the cash flow beta as the coefficient of the market-level ROE in the regression of the firm's ROE on the value-weighted market-level ROE, using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$). When computing the firm's ROE as of the end of a quarter, we use the sum of the earnings in the quarter and in the past three quarters to remove seasonality effects; we scale this sum using the book equity as of quarter-end. To enhance the reliability of our regressions involving relative betas, we further require both the estimated return and cash flow betas to be positive in order to retain a stock-month in our sample.
$Risk\text{-}adjusted\ return$	For stock-month t, we use the stock's excess return over the risk-free interest rate or the difference between the stock return and the benchmark-implied return in month t. We estimate benchmark models, including the CAPM, the Fama and French (1993) and Carhart (1997) 4-factor model (FFC4), and the Fama and French (2015) 5-factor model (FF5), using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$).
$Profitability$	For stock-month t, we use the quarterly report that is announced in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter.
CP	For stock-month t in the period from July of year y to June of the next year $y+1$, we use the Catholic-to-Protestant (CP) ratio in the county where the firm of the stock is headquartered, as of the end of the past fiscal year $y-1$. We compute the CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data. For any missing year between 1980 and 2020, we compute CP using linear interpolation.

XAD_3	For stock-month t in the period from July of year y to June of the next year $y+1$, we use the firm's three-year moving average of advertising expense as of the end of the past fiscal year $y-1$.
ADV	For stock-month t, we adjust XAD_3 by subtracting the industry average (where the industry is based on the first two digits of the SIC code in the past month $t-1$). ADV represents the cross-sectional percentile rank of the industry-adjusted XAD_3.
ME	For stock-month t, we use the stock's market value of equity (i.e., the stock price $\times$ the number of outstanding shares) as of the end of the past month $t-1$.
$Firm\ age$	For stock-month t, we use the number of months since the stock's first CRSP appearance as of the end of the past month $t-1$.
ANA	For stock-month t, we use the number of analysts who follow the firm and report forecasts to the I/B/E/S database in the past month $t-1$.
$FDISP$	For stock-month t, we use the stock's analyst forecast dispersion, which is computed as the standard deviation of EPS forecasts reported by analysts in the I/B/E/S database in the past month $t-1$.
BM	For stock-month t in the period from July of year y to June of the next year $y+1$, we compute the book-to-market ratio using the book equity as of the end of the past fiscal year $y-1$, and the market value of equity as of the end of the past calendar year $y-1$. We follow Fama and French (1993) and require both the market equity and the book equity to be positive.
D/A	For stock-month t in the period from July of year y to June of the next year $y+1$, we use the stock's book debt (i.e., the sum of short-term and long-term debt) divided by total assets as of the end of the past fiscal year $y-1$.
OL	For stock-month t in the period from July of year y to June of the next year $y+1$, we use the operating leverage computed as the yearly sum of the stock's cost of goods sold and selling, general and administrative expenses divided by total assets as of the end of the past fiscal year $y-1$.
$EVOLA$	For stock-month t in quarter q, we compute the earnings volatility of the stock as the standard deviation of EPS using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$).

Figure 1: Expected turnover as a function of gambling propensity, for different levels of overconfidence

This graph plots the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$. We assume that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j\ Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$).

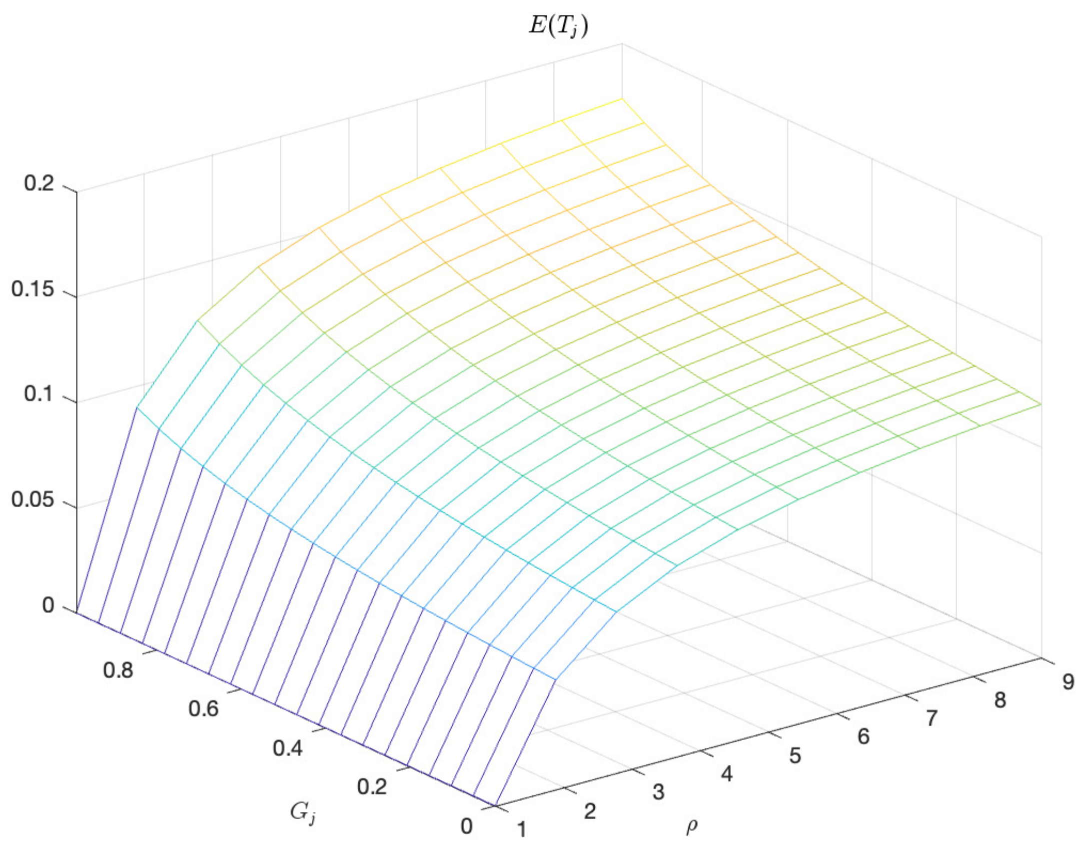

Figure 2: Expected turnover as a function of gambling propensity, for different levels of investors' risk aversion

This graph plots the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. We assume that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j$ $Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$).

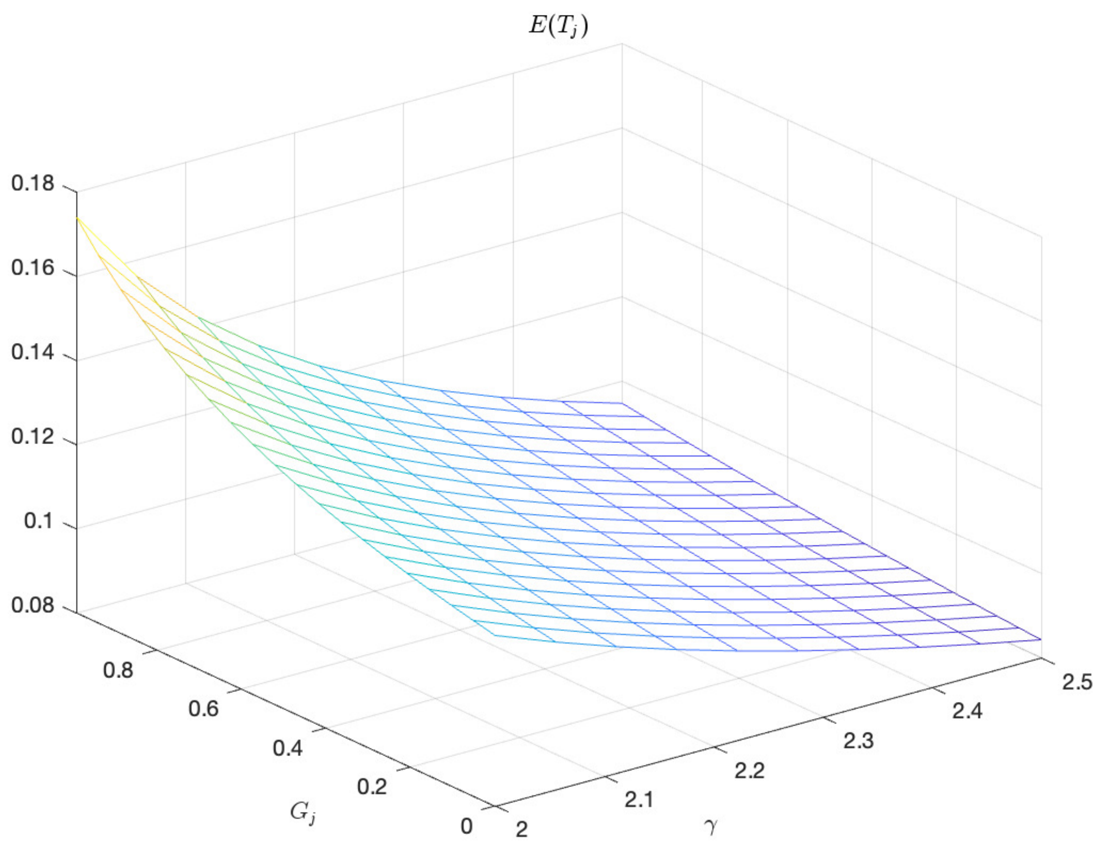

Figure 3: Expected turnover, gambling propensity, and the affect heuristic

This graph plots the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j and the affect parameter A_j. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. We assume that each type of investor's endowments equal per capita supplies (i.e., $\forall j\ Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 1$) in Panel A, and that all endowments equal zero (i.e., $\forall j\ Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 0$) in Panel B.

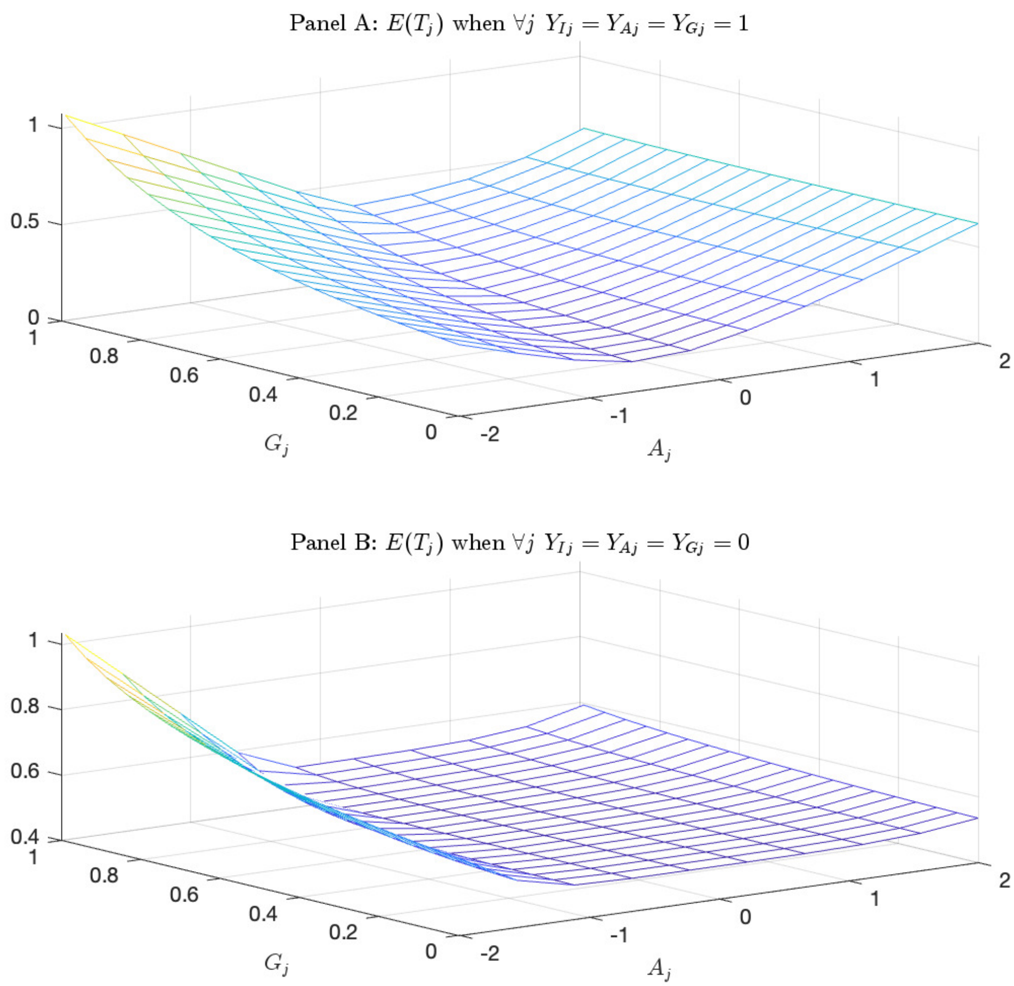

Figure 4: Relative beta as a function of gambling propensity, for different levels of overconfidence

This graph plots the relative beta β_{jM}/ℓ_j as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$.

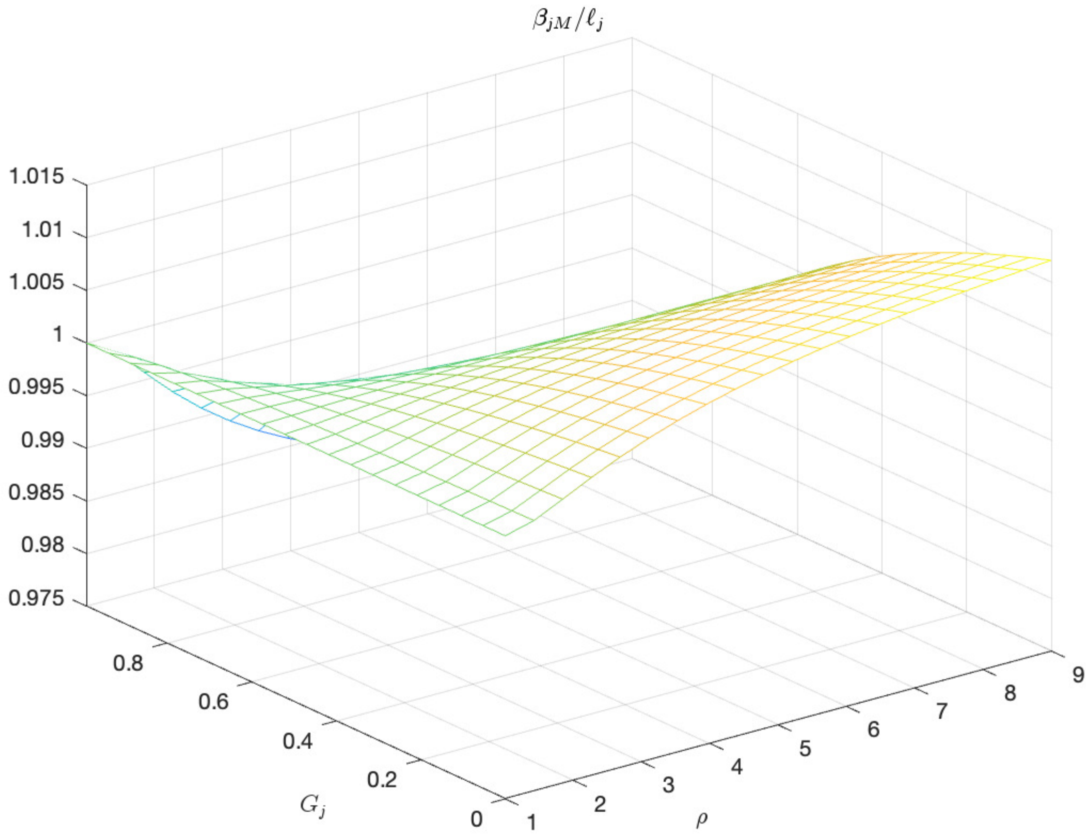

Figure 5: Relative beta as a function of gambling propensity, for different levels of investors' risk aversion

This graph plots the relative beta β_{jM}/ℓ_j as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$.

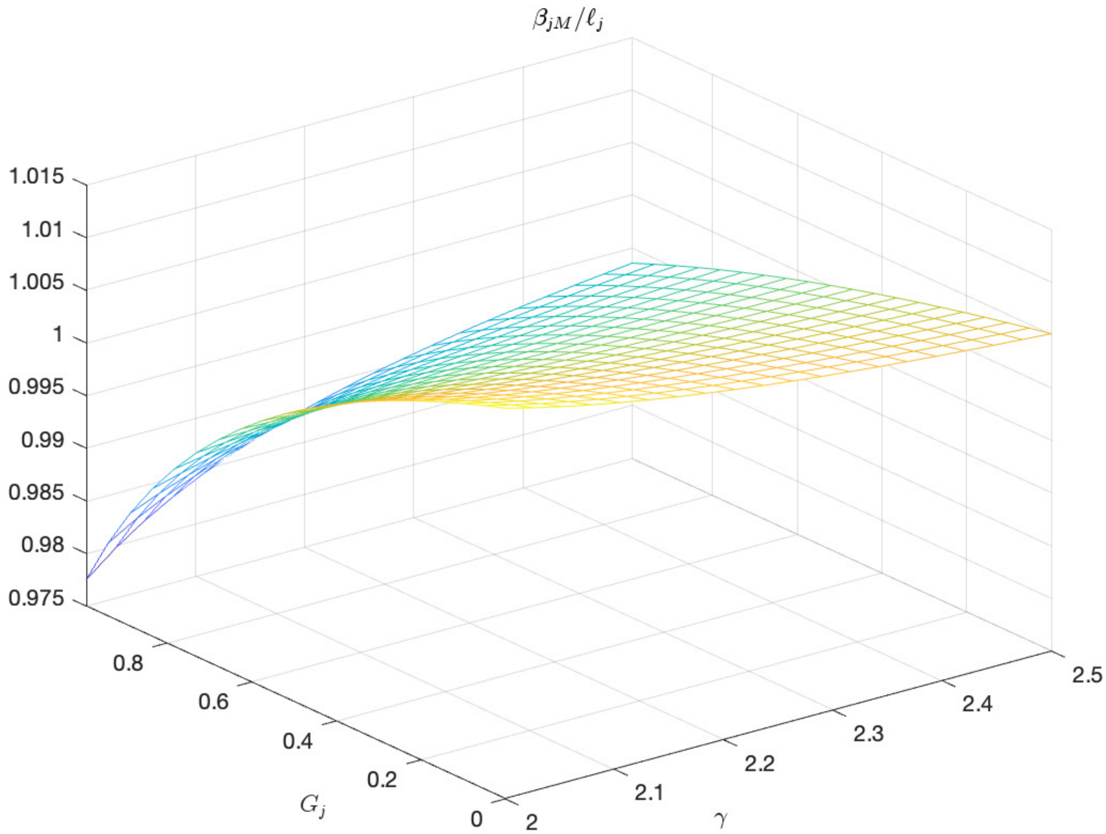

Figure 6: The degree of underreaction to public announcements as a function of gambling propensity, for different levels of overconfidence

This graph plots the level of underreaction to public announcements (as measured by $\mathrm{Cov}(r_j, S_j)$) as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\mu = 0.8$.

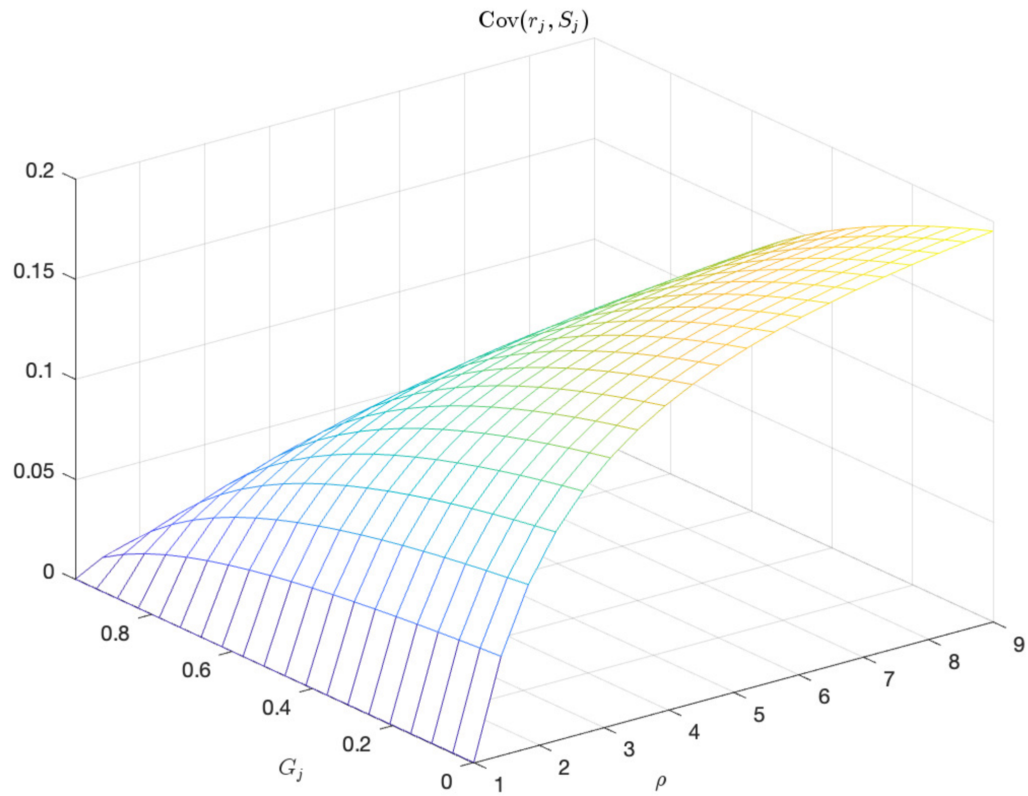

Figure 7: The degree of underreaction to public announcements as a function of gambling propensity and the affect parameter, for different levels of investors' risk aversion

This graph plots the level of underreaction to public announcements (as measured by $\text{Cov}(r_j, S_j)$) as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; $\rho = 9$; and $\mu = 0.8$.

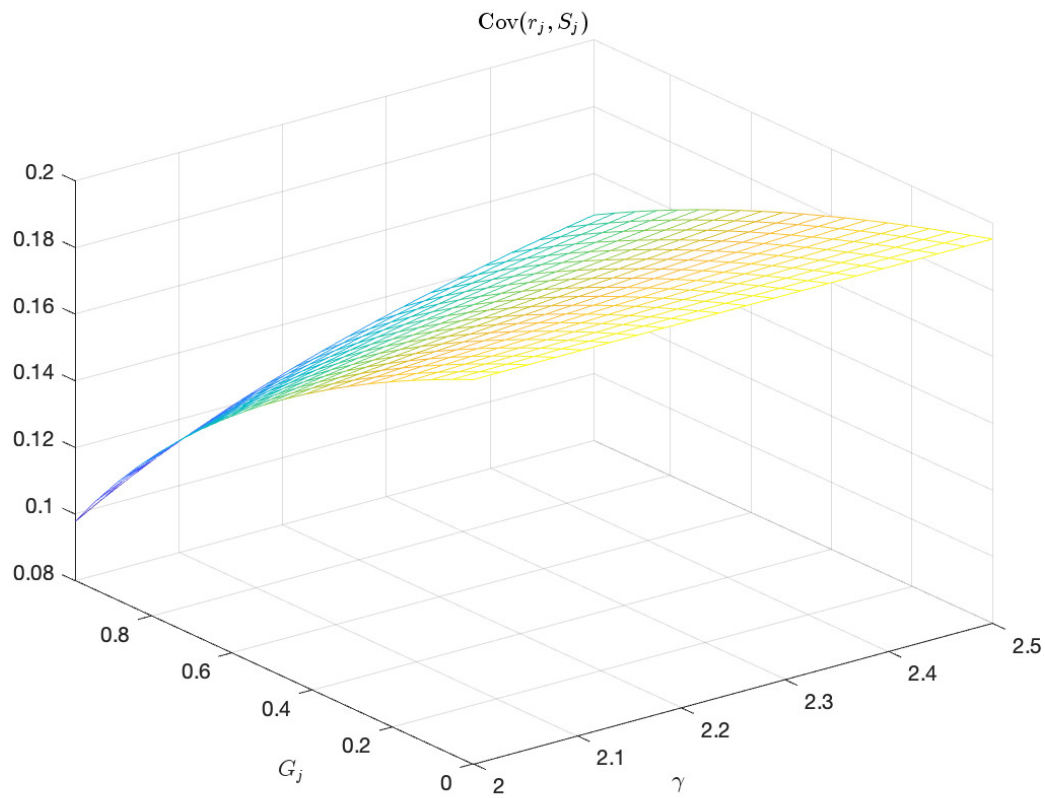

Figure 8: Risk-adjusted expected returns, gambling propensity, and the affect heuristic
This graph plots the risk-adjusted expected return α_j for each stock as a function of the gambling propensity G_j and the affect parameter A_j. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$.

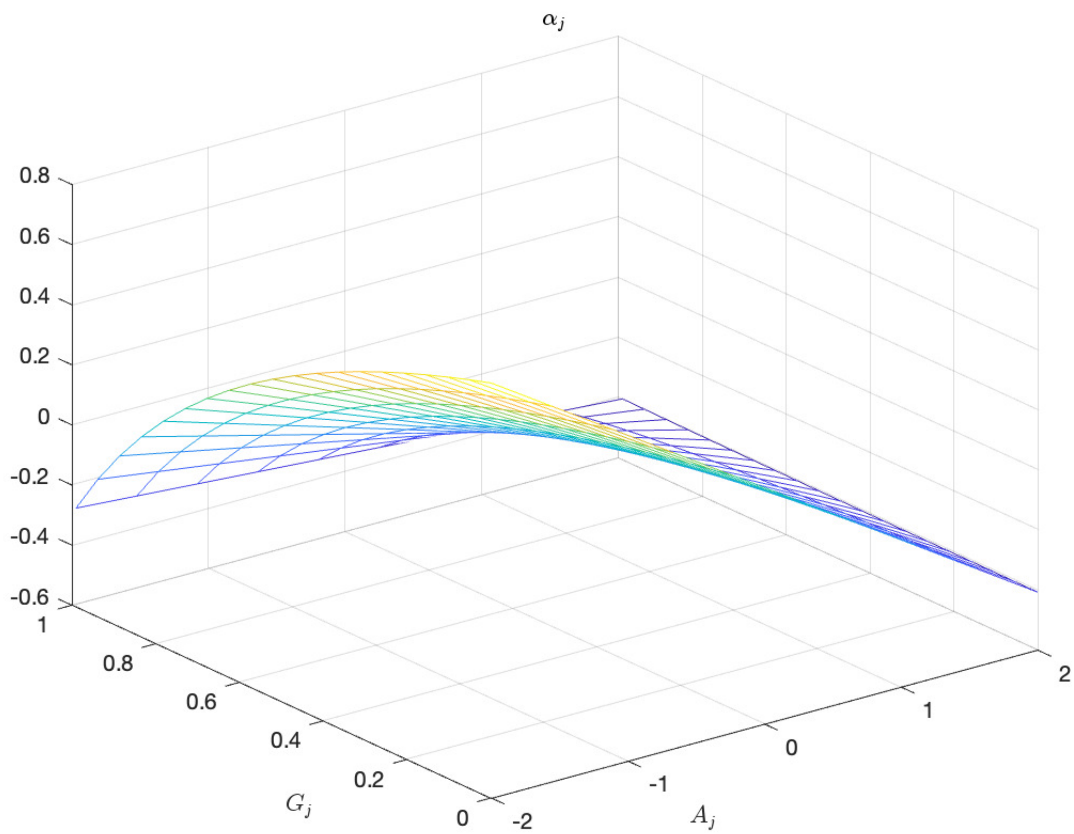

Figure 9: Risk-adjusted expected returns as functions of the gambling and affect heuristics, for different levels of investors' risk aversion

Panel A of this graph plots the risk-adjusted expected return α_j for each stock as a function of the gambling propensity G_j and the affect parameter A_j, when investors' risk-aversion parameter $\gamma = 2.5$. Panel B of this graph plots the difference between the α_j's for cases where γ equals 2.5 and 2. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 1]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$.

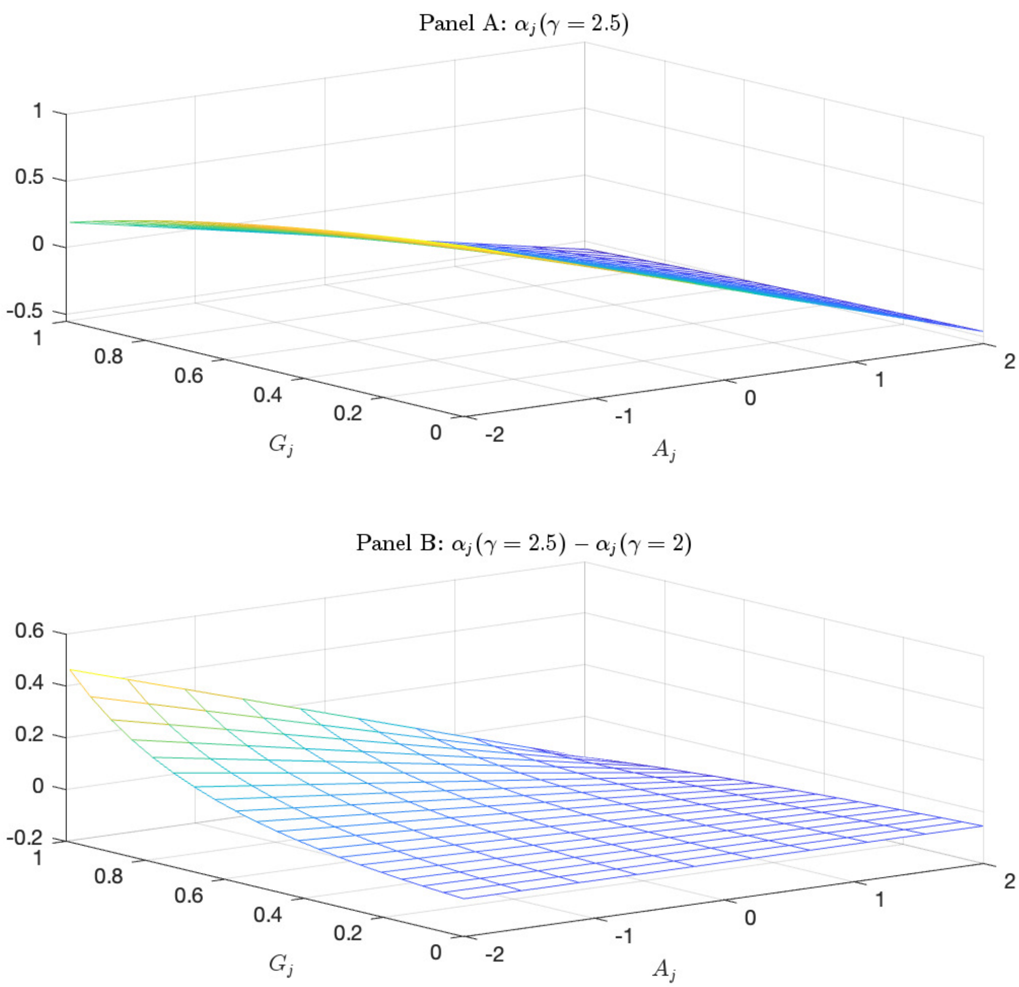

Figure 10: The cross-sectional dispersion of the industry-adjusted measure of the advertising expense XAD_3

For stock-month t in the period from July of year y to June of the next year $y+1$, we compute each firm's three-year moving average of advertising expense as of the end of the past fiscal year $y-1$. We then adjust this expense by subtracting the industry average (where the industry is defined as the first two digits of the SIC code in month $t-1$). To address extreme values, we further winsorize the industry-adjusted expense for each cross-section at the 1% and 99% levels, using its distribution every month. Panel A shows the cross-sectional standard deviation (SD) of the industry-adjusted expense (XAD_3), while Panel B shows the 10th, 25th, 75th, and 90th percentiles of the cross-section of XAD_3 over the sample period from July 1981 to December 2021.

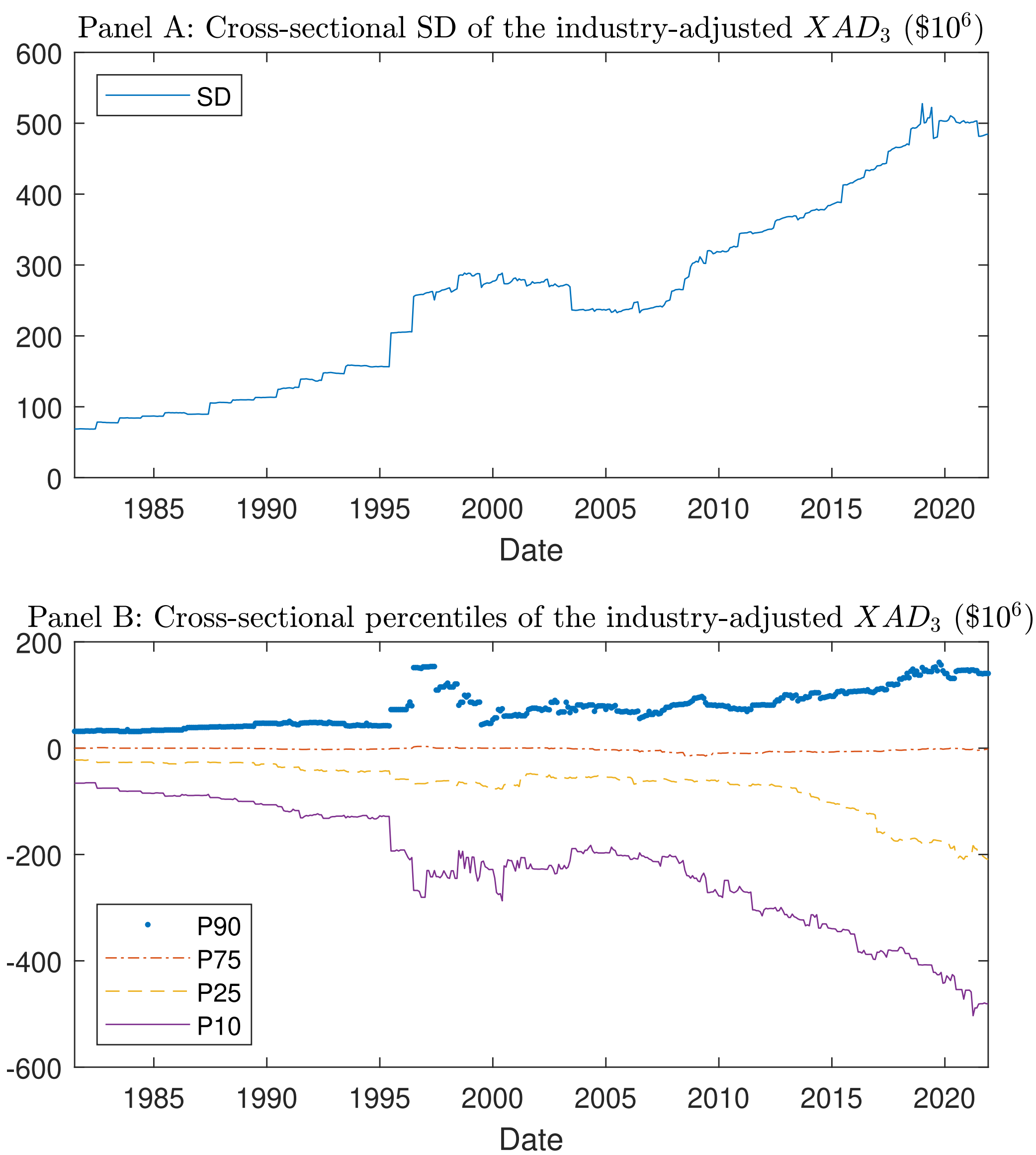

Table 1: Summary statistics

For stock-month t, $Turnover$ represents the monthly shares traded in month t, scaled by the number of outstanding shares at the end of the past month $t-1$. $Relative\ beta$ is the ratio of the return beta (estimated using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$)) to cash flow beta in the month. Given month t in quarter q, we estimate the cash flow beta as the coefficient of the market-level ROE in the regression of the firm's ROE on the value-weighted market-level ROE using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$). $r_{j,t}$ is the return of the stock in month t. If month t is in the period from July of year y to June of the next year $y+1$, we use the Catholic-to-Protestant (CP) ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We compute the CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for any missing year between 1980 and 2020 using linear interpolation. XAD_3 is the firm's three-year moving average of advertising expense as of the end of the past fiscal year $y-1$. We use the quarterly report that is announced in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter. ME is the stock's market value of equity as of the end of the past month $t-1$; $Firm\ age$ is the number of months since the stock's first CRSP appearance as of the end of the past month $t-1$; ANA is the number of analysts who follow the firm and report forecasts to the I/B/E/S database in the past month $t-1$; $FDISP$ is the stock's analyst forecast dispersion, which is computed as the standard deviation of EPS forecasts reported by analysts in the I/B/E/S database in the past month $t-1$. BM is the book-to-market ratio computed using the book equity as of the end of the past fiscal year $y-1$, and the market value of equity as of the end of the past calendar year $y-1$; D/A is the stock's book debt (i.e., the sum of short-term and long-term debt) divided by total assets as of the end of the past fiscal year $y-1$; OL is the operating leverage computed as the yearly sum of the stock's cost of goods sold and selling, general and administrative expenses divided by total assets as of the end of the past fiscal year $y-1$. For month t in quarter q, $EVOLA$ is the earnings volatility of the stock computed as the standard deviation of EPS using at least 12 quarters in the past 20 quarters (from quarters $q-20$ to $q-1$). To address extreme values, we winsorize the continuous variables (i.e., all variables except $Firm\ age$ and ANA) at the 1% and 99% levels, using each of their distributions every month. The exception is $Stock\ return$ ($r_{j,t}$), which is not winsorized. Table 1 presents the time-series averages of the cross-sectional statistics of the variables. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

Table 1 (continued)

	Mean	Median	SD	Percentile 10th	25th	75th	90th
$Turnover$	0.128	0.087	0.146	0.021	0.045	0.155	0.269
$Relative\ beta$	2.344	0.777	5.732	0.153	0.348	1.809	4.527
$Return\ beta$	1.239	1.14	0.666	0.474	0.769	1.598	2.132
$Cash\ flow\ beta$	3.801	1.546	8.173	0.254	0.643	3.491	7.578
$Stock\ return\ (r_{j,t})$	0.014	0.005	0.145	–0.131	–0.06	0.074	0.161
CP	1.705	1.392	1.428	0.232	0.558	2.374	3.787
XAD_3 (\$$10^6$)	89.687	6.708	273.772	0.278	1.142	38.59	186.882
$Profitability$	0.099	0.09	0.079	0.022	0.052	0.137	0.196
ME (\$$10^6$)	3158.772	450.281	9288.735	34.238	109.492	1740.906	6580.083
$Firm\ age$	178.415	157.348	122.51	29.048	77.591	270.697	377.035
ANA	4.97	2.145	6.875	0	0.086	7.106	14.545
$FDISP$	0.121	0.067	0.161	0.017	0.032	0.139	0.277
BM	0.663	0.527	0.526	0.173	0.307	0.858	1.291
D/A	0.219	0.182	0.203	0.003	0.04	0.334	0.491
OL	1.113	0.949	0.758	0.352	0.611	1.405	2.041
$EVOLA$	0.508	0.262	0.79	0.075	0.133	0.538	1.096

Table 2: Gambling propensity and turnover

The table presents the estimated coefficients and the corresponding t-statistics (in parentheses) for the following regression:

$$Turnover_{j,t} = b_0 + b_1 \times CP_{j,t} + b_2 \times Controls + \epsilon_{j,t}.$$

$Turnover_{j,t}$ is the turnover of the j'th stock in month t. Given month t in the period from July of year y to June of the next year $y+1$, $CP_{j,t}$ is the Catholic-to-Protestant ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We compute CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for each missing year between 1980 and 2020 using linear interpolation. We control for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME and firm age as of the end of the past month $t-1$, book-to-market ratio BM and leverage D/A as of the end of the past year $y-1$, earnings volatility $EVOLA$, and the number of analysts following the firm and reporting forecasts, ANA, and analyst forecast dispersion $FDISP$ in the past month $t-1$. To address extreme values, we winsorize the continuous variables (i.e., all variables except $Firm\ age$ and ANA) at the 1% and 99% levels, using each of their distributions every month. We estimate the regression using the Fama-MacBeth procedure and correct the standard errors using the Newey-West method with up to three lags. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

Dependent variable: $Turnover_{j,t} \times 100$	(1)	(2)
CP	0.097***	0.260***
	(3.48)	(8.98)
$r_{j,t-1}$	2.726***	2.779***
	(2.65)	(3.72)
$r_{j,t-2}$	1.788**	3.213***
	(2.04)	(5.13)
$r_{j,t-12\ to\ t-3}$	1.736***	4.190***
	(4.46)	(12.96)
$\ln(ME)$	1.216***	-1.660***
	(8.02)	(-11.51)
$\ln(Firm\ age)$	-1.829***	-0.659***
	(-18.03)	(-9.50)
BM	-0.684***	-1.244***
	(-3.37)	(-5.52)
D/A	0.362	-0.804***
	(1.48)	(-2.86)
$EVOLA$	1.183***	0.956***
	(10.52)	(9.85)
$\ln(1+ANA)$		6.895***
		(18.01)
$FDISP$		14.065***
		(13.50)
Observations	930,463	545,803
R-squared	0.163	0.213

Table 3: Gambling propensity and the relative beta

The table presents the estimated coefficients and the corresponding t-statistics (in parentheses) for the following regression:

$$Relative\ beta_{j,t} = b_0 + b_1 \times CP_{j,t} + b_2 \times Controls + \epsilon_{j,t}.$$

$Relative\ beta_{j,t}$ is the relative beta of the j'th stock (i.e., the ratio of the return beta, which is estimated using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$), to the cash flow beta, which is estimated as the coefficient of the market-level ROE in the regression of the firm's ROE on the the value-weighted market-level ROE, using at least 12 quarters of the past 20 quarters) in month t. Given month t in the period from July of year y to June of the next year $y+1$, $CP_{j,t}$ is the Catholic-to-Protestant ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We compute CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for each missing year between 1980 and 2020 using linear interpolation. We control for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM, leverage D/A, and operating leverage OL as of the end of the past year $y-1$. To address extreme values, we winsorize continuous variables at the 1% and 99% levels, using each of their distributions every month. We estimate the regression using the Fama-MacBeth procedure and correct the standard errors using the Newey-West method with up to three lags. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

Dependent variable: $Relative\ beta_{j,t}$		
	(1)	(2)
CP	-0.031***	-0.021*
	(-2.68)	(-1.81)
$r_{j,t-1}$	-0.179	-0.186
	(-1.58)	(-1.55)
$r_{j,t-2}$	-0.170	-0.175
	(-1.53)	(-1.47)
$r_{j,t-12\ to\ t-3}$	-0.037	-0.044
	(-0.54)	(-0.65)
$\ln(ME)$	0.181***	0.172***
	(14.58)	(14.08)
BM	0.685***	0.658***
	(6.03)	(5.84)
D/A	-0.680***	-0.717***
	(-8.49)	(-8.13)
OL		0.006
		(0.22)
Observations	470,033	436,865
R-squared	0.018	0.021

Table 4: Gambling propensity and the post-profitability-announcement drift

The table presents the estimated coefficients and the corresponding t-statistics (in parentheses) for the following regression:

$$\begin{aligned} \textit{Risk-adjusted return}_{j,t} \;=\; & \sum_{m=1}^{3} \big(b_{0,m} \times D_{-m} + b_{1,m} \times Profitability_{j,t} \times D_{-m} + b_{2,m} \times Profitability_{j,t} \times CP_{j,t} \times D_{-m} \\ & + b_{3,m} \times CP_{j,t} \times D_{-m}\big) + b_4 \times Controls + \epsilon_{j,t}. \end{aligned}$$

$\textit{Risk-adjusted return}_{j,t}$ is the j'th stock's excess return over the risk-free interest rate (Exret) or the difference between the stock return and the benchmark-implied return in month t. We estimate the benchmark models, including the CAPM, the Fama and French (1993) and Carhart (1997) 4-factor model (FFC4), and the Fama and French (2015) 5-factor model (FF5), using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$). If month t is in the period from July of year y to June of the next year $y+1$, we use the quarterly report that is announced in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter; D_{-m} (where $m = 1, 2,$ or 3) is a dummy variable indicating that the announcement is made in month $t-m$. CP is the Catholic-to-Protestant ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We compute CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for each missing year between 1980 and 2020 using linear interpolation. We consider controlling for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12 \text{ to } t-3}$), market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM as of the end of the past year $y-1$. To address extreme values, we winsorize all continuous variables at the 1% and 99% levels, using each of their distributions every month. The exception is the dependent variable $\textit{Risk-adjusted return}_{j,t}$, which is not winsorized. We estimate the regression using the Fama-MacBeth procedure and correct the standard errors using the Newey-West method with up to three lags. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

Table 4 (continued)

Dependent variable: $Risk\text{-}adjusted\ return_{j,t} \times 100$

	Exret	Exret	CAPM	CAPM	FFC4	FFC4	FF5	FF5
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Profitability \times D_{-1}$	5.581***	6.128***	5.627***	6.745***	5.301***	6.822***	4.506***	6.450***
	(5.55)	(6.64)	(5.71)	(7.36)	(4.81)	(6.61)	(4.47)	(6.55)
$Profitability \times D_{-2}$	3.695***	4.059***	3.585***	4.324***	2.968***	3.679***	3.319***	3.783***
	(3.73)	(4.37)	(3.64)	(4.61)	(2.84)	(3.60)	(3.01)	(3.95)
$Profitability \times D_{-3}$	4.662***	4.324***	4.733***	4.558***	3.927***	3.850***	3.846***	3.415***
	(3.78)	(3.65)	(3.62)	(3.80)	(3.19)	(3.16)	(2.90)	(2.72)
$Profitability \times CP \times D_{-1}$	-0.974**	-1.016***	-0.938**	-1.078***	-0.832*	-0.945**	-1.221***	-1.367***
	(-2.53)	(-2.69)	(-2.29)	(-2.81)	(-1.81)	(-2.26)	(-2.83)	(-3.27)
$Profitability \times CP \times D_{-2}$	-0.066	-0.146	0.113	-0.049	0.193	0.123	-0.297	-0.241
	(-0.17)	(-0.40)	(0.30)	(-0.13)	(0.48)	(0.31)	(-0.68)	(-0.57)
$Profitability \times CP \times D_{-3}$	0.080	0.368	0.179	0.400	0.619	0.786	0.494	0.834
	(0.15)	(0.68)	(0.32)	(0.74)	(1.08)	(1.41)	(0.82)	(1.42)
$CP \times D_{-1}$	0.092	0.082	0.092	0.084	0.087	0.073	0.172***	0.152***
	(1.53)	(1.46)	(1.54)	(1.55)	(1.51)	(1.35)	(3.19)	(2.73)
$CP \times D_{-2}$	-0.014	-0.016	-0.023	-0.025	-0.028	-0.033	0.082	0.050
	(-0.23)	(-0.31)	(-0.39)	(-0.47)	(-0.50)	(-0.62)	(1.40)	(0.86)
$CP \times D_{-3}$	0.008	-0.038	-0.016	-0.055	-0.069	-0.105	0.010	-0.051
	(0.10)	(-0.54)	(-0.21)	(-0.80)	(-0.95)	(-1.57)	(0.14)	(-0.73)
$r_{j,t-1}$		-3.597***		-3.920***		-5.170***		-5.981***
		(-8.63)		(-8.23)		(-8.28)		(-9.86)
$r_{j,t-2}$		-0.026		-0.158		-1.282**		-1.276**
		(-0.06)		(-0.39)		(-2.48)		(-2.37)
$r_{j,t-12\ to\ t-3}$		0.374**		0.337*		0.115		-0.060
		(1.98)		(1.88)		(0.62)		(-0.32)
$\ln(ME)$		-0.144***		-0.144***		-0.149***		-0.191***
		(-3.53)		(-3.57)		(-4.98)		(-6.24)
BM		0.338***		0.352***		0.229**		-0.026
		(2.91)		(3.10)		(2.28)		(-0.24)
Observations	966,313	888,011	894,557	867,572	894,557	867,572	894,557	867,572
R-squared	0.018	0.053	0.018	0.051	0.015	0.049	0.014	0.051

Table 5: Gambling propensity, advertising, and risk-adjusted returns

The table presents the estimated coefficients and the corresponding t-statistics (in parentheses) for the following regression:

$$Risk\text{-}adjusted\ return_{j,t} \quad = \quad b_0 + b_1 \times ADV_{j,t} + b_2 \times ADV_{j,t} \times CP_{j,t} + b_3 \times CP_{j,t} + b_4 \times Controls + \epsilon_{j,t}.$$

$Risk\text{-}adjusted\ return_{j,t}$ is the j'th stock's excess return over the risk-free interest rate (Exret) or the difference between the stock return and the benchmark-implied return in month t. We estimate the benchmark models, including the CAPM, the Fama and French (1993) and Carhart (1997) 4-factor model (FFC4), and the Fama and French (2015) 5-factor model (FF5), using at least 18 monthly returns in the past 36 months (from months $t-36$ to $t-1$). If month t is in the period from July of year y to June of the next year $y+1$, $ADV_{j,t}$ is the cross-sectional percentile rank of the industry-adjusted XAD_3 (i.e., the difference between the firm's three-year moving average of advertising expense, XAD_3, and the industry average) as of the end of the past year $y-1$. CP is the Catholic-to-Protestant ratio in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. We compute CP in a county for each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for each missing year between 1980 and 2020 using linear interpolation. We consider controlling for $Profitability$. We use the quarterly report that is announced in the past three months (from months $t-3$ to $t-1$) to compute $Profitability$ as the ratio of a firm's gross profits (revenues minus cost of goods sold) in the quarter to its assets at the beginning of the quarter; D_{-m} (where $m = 1, 2,$ or 3) is a dummy variable indicating that the announcement is made in month $t-m$. We also consider controlling for past stock returns in month $t-1$ ($r_{j,t-1}$), in month $t-2$ ($r_{j,t-2}$), and from months $t-12$ to $t-3$ ($r_{j,t-12\ to\ t-3}$), market equity ME as of the end of the past month $t-1$, and book-to-market ratio BM as of the end of the past year $y-1$. To address extreme values, we winsorize all continuous variables at the 1% and 99% levels, using each of their distributions every month. The exceptions are the dependent variable $Risk\text{-}adjusted\ return_{j,t}$ and the standardized advertising measure (ADV), which are not winsorized. We estimate the regression using the Fama-MacBeth procedure and correct the standard errors using the Newey-West method with up to three lags. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

Table 5 (continued)

Dependent variable: $Risk\text{-}adjusted\ return_{j,t} \times 100$.

	Exret	CAPM	CAPM	FFC4	FFC4	FF5	FF5
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ADV	-0.504***	-0.559***	-0.284**	-0.593***	-0.251*	-0.620***	-0.315**
	(-3.64)	(-3.98)	(-2.13)	(-4.31)	(-1.77)	(-4.24)	(-2.08)
$ADV \times CP$	0.215***	0.222***	0.221***	0.237***	0.231***	0.190***	0.247***
	(3.48)	(3.42)	(3.39)	(3.36)	(3.26)	(2.68)	(3.39)
CP	-0.087*	-0.088*		-0.096*		-0.054	
	(-1.96)	(-1.90)		(-1.94)		(-1.22)	
$Profitability \times D_{-1}$			6.608***		7.035***		5.894***
			(5.23)		(5.08)		(4.26)
$Profitability \times D_{-2}$			4.169***		3.552**		3.433**
			(3.06)		(2.56)		(2.41)
$Profitability \times D_{-3}$			5.245***		4.873***		2.921
			(3.19)		(2.70)		(1.64))
$Profitability \times CP \times D_{-1}$			-1.007*		-1.246**		-1.143*
			(-1.78)		(-2.00)		(-1.81)
$Profitability \times CP \times D_{-2}$			0.640		1.117		0.887
			(0.96)		(1.53)		(1.25)
$Profitability \times CP \times D_{-3}$			-0.149		-0.034		0.606
			(-0.17)		(-0.04)		(0.63)
$CP \times D_{-1}$			0.006		0.051		0.018
			(0.08)		(0.53)		(0.18)
$CP \times D_{-2}$			-0.204*		-0.257**		-0.201*
			(-1.68)		(-1.98)		(-1.68)
$CP \times D_{-3}$			-0.121		-0.156		-0.204
			(-0.94)		(-1.16)		(-1.54)
$r_{j,t-1}$			-3.934***		-5.253***		-5.998***
			(-7.73)		(-8.23)		(-8.68)
$r_{j,t-2}$			0.225		-0.739		-0.787
			(0.47)		(-1.23)		(-1.28)
$r_{j,t-12\ to\ t-3}$			0.080		-0.090		-0.229
			(0.39)		(-0.42)		(-1.04)
$\ln(ME)$			-0.096**		-0.111***		-0.120***
			(-2.28)		(-3.37)		(-3.58)
BM			0.483***		0.356***		0.233*
			(3.88)		(3.05)		(1.95)
Observations	335,341	326,623	286,647	326,623	286,647	326,623	286,647
R-squared	0.007	0.007	0.077	0.006	0.077	0.006	0.079

Table 6: Gambling propensity, advertising, and the risk-adjusted return: Portfolio analysis

For month t from July of year y to June of the next year $y+1$, we divide the sample stocks into two equal groups based on the standardized advertising expense ADV (i.e., the cross-sectional percentile rank of the industry-adjusted XAD_3, representing the difference between the firm's three-year moving average of advertising expense, XAD_3, and the industry average) from the past fiscal year $y-1$, and independently into two equal groups based on the Catholic-to-Protestant ratio (CP) in the county where the firm of the stock is headquartered as of the end of the past fiscal year $y-1$. This leads to 2×2 portfolios (where "Hi" indicates "High"; and "Lo" indicates "Low"). We compute CP in each of the years 1980, 1990, 2000, 2010, and 2020 using survey data, and for each missing year between 1980 and 2020 using linear interpolation. We compute the return of each portfolio by weighting stocks in the portfolio equally. This table presents the excess return over the risk-free interest rate, and benchmark-adjusted returns based on the CAPM, the Fama and French (1993) and Carhart (1997) 4-factor model (FFC4), and the Fama and French (2015) 5-factor model (FF5)) for each portfolio, as well as each high-ADV minus low-ADV (conditional on high/low CP ratio) portfolio, and the return difference between high-ADV minus low-ADV portfolios conditional on high and low CP ratios. We correct standard errors using the Newey-West method with up to three lags. t-statistics are reported in parentheses. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from July 1981 to December 2021. See Appendix B for variable definitions.

$Risk\text{-}adjusted\ return \times 100$

	Exret		CAPM		FFC4		FF5	
	Lo CP	Hi CP	Lo CP	Hi CP	Lo CP	Hi CP	Lo CP	Hi CP
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Lo ADV	1.333***	1.186***	0.534***	0.329**	0.488***	0.329***	0.414***	0.375***
	(5.11)	(4.35)	(3.24)	(2.18)	(4.6)	(3.64)	(3.53)	(4.06)
(2) Hi ADV	1.145***	1.335***	0.311**	0.413***	0.254***	0.418***	0.219**	0.498***
	(4.49)	(4.76)	(2.5)	(3.38)	(3.01)	(4.47)	(2.15)	(4.25)
(2)-(1)	-0.188**	0.149	-0.223**	0.085	-0.234***	0.089	-0.196**	0.123
	(-2.1)	(1.65)	(-2.43)	(0.93)	(-2.81)	(1.03)	(-2.27)	(1.3)
Difference		0.336***		0.308***		0.323***		0.319**
		(3.08)		(2.7)		(2.78)		(2.53)

Internet Appendix for
Asset Pricing with Affect Investing, Gambling, and Overconfidence:
Theory and Evidence

A Traditional Form of Overconfidence

In the main paper (see Section 2), we assume that both the signal about the common factor, τ, and the signal about the j'th ($j = 1, ..., J$) stock's firm-specific cash flow, s_j, are public information; and that overconfident investors are skeptical about the quality of the information since they do not produce the information themselves (Odean (1998) and Luo, Subrahmanyam, and Titman (2021)).

Here we modify the setting in Section 2 of the main paper by assuming the presence of private information. Specifically, we assume that for the j'th stock, overconfident investors also observe a private signal, $s_j^* = \theta_j + \xi_j$, where the noise term ξ_j is drawn from a normal distribution with zero mean and variance ν_{ξ_j}. Further, we add a traditional form of overconfidence as in Daniel, Hirshleifer, and Subrahmanyam (1998). Specifically, overconfident investors overestimate the precision of the private signal, $s_j^* = \theta_j + \xi_j$, in that they believe that ξ_j has a smaller variance ξ_j/ρ^* than the actual variance ν_{ξ_j}, where $\rho^* > 1$ is a constant parameter representing the scale of the traditional overconfident bias. We denote the unbiased (overconfident) belief about the variance of s_j^* as $\nu_{s_j^*} = \nu_{\theta_j} + \nu_{\xi_j}$ ($\omega_{s_j^*} = \nu_{\theta_j} + \nu_{\xi_j}/\rho^*$). Affect investors and gamblers hold unbiased beliefs about the variance ν_{ξ_j}.

Denote the vector of the private signals as

$$s^* = \begin{pmatrix} s_1^* \\ s_2^* \\ \vdots \\ s_J^* \end{pmatrix}; \text{ and let } \mathcal{S} = \begin{pmatrix} s \\ s^* \end{pmatrix}.$$

Note that the per capita supply of each stock is fixed at unity. Stock prices fully reveal the private signals s^*; therefore, there is no asymmetric information. However, investors have heterogeneous assessments of signal informativeness; that is, they agree to disagree. Specifically, overconfident investors believe that $\theta|\mathcal{S} \sim N\left(\hat{q}\mathcal{S}, \hat{\Gamma}\right)$, where $\hat{q}$ and $\hat{\Gamma}$ are constants depending on the variance-covariance matrix of θ and $\mathcal{S}$ based on overconfident investors' biased beliefs. Affect investors and gamblers believe that $\theta|\mathcal{S} \sim N\left(q\mathcal{S}, \Gamma\right)$, where q and Γ are constants depending on the variance-covariance matrix of θ and $\mathcal{S}$ based on their unbiased beliefs.

See Equation (3) of the main paper; we redefine the following parameters:

$$\begin{aligned}\Omega &\equiv \hat{q} - q, \quad \phi_\kappa \equiv \ell\ell' \kappa_{F|\tau} + \hat{\Gamma}, \quad \phi \equiv \ell\ell' \nu_{F|\tau} + \Gamma, \quad \phi_G \equiv \phi - G/\gamma^2, \text{ and} \\ \Phi &\equiv \eta_\kappa \phi_\kappa^{-1} + \eta_A \phi^{-1} + \eta_G \phi_G^{-1}.\end{aligned}$$

We obtain the following theorem (the proof of which is at the end of this Internet Appendix).

Theorem 1 *With overestimation of the precision of private information, in equilibrium:*

(i) Stock prices are given by:

$$P = \frac{1}{R_f}\left[\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} + \Phi^{-1}\left[\eta_\kappa\phi_\kappa^{-1}\left(\ell\Delta\tau + \Omega\mathcal{S}\right) + \eta_A\phi^{-1}A - \gamma\mathbf{1}\right]\right].$$

(ii) Each overconfident and affect investor's, and each gambler's, demands for a stock are respectively given by:

$$\begin{aligned}X_\kappa &= \frac{\phi_\kappa^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \hat{q}\mathcal{S} - PR_f\right), \\ X_A &= \frac{\phi^{-1}}{\gamma}\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} - PR_f\right), \\ X_G &= \frac{\phi_G^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} - PR_f\right).\end{aligned}$$

We consider a simple economy like in Section 3.1 of the main paper, where firm-specific payoff variances are identical in the cross-section of stocks (i.e., $\forall j\ \nu_{\theta_j} = \nu$), and signals about the firm-specific components θ_j, s_j, and s_j^* are uninformative (i.e., $\forall j$ $\nu_{\epsilon_j} \to \infty$). In this case, we obtain the same analytical results as in Propositions 1, 2, 3, and 4. In what follows, we examine the general case numerically as in Section 3.2 of the main paper.

IA.1 Turnover

We can use Theorem 1 to obtain the demand for the j'th stock by each overconfident and affect investor, and gambler, denoted by $X_{\kappa j}$, X_{Aj}, and X_{Gj}, respectively, which are the j'th elements of the vectors X_κ, X_A, and X_G. We can then use Equation (4) of the main paper to compute the expected turnover of the j'th stock.

We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$. The overconfidence parameter ρ takes values from 1 to 9. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).[1]

Figure IA.1 plots the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, for different values of overconfidence (ρ), ranging from 1 to 9. As in Figure 1, we assume here that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j\ Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$); in this case, the turnover is not affected by the affect parameter A_j. Figure IA.1 shows that when overconfident investors correctly assess the precision of the public signals (i.e., $\rho = 1$), trading still occurs. This is caused by heterogeneous assessments of private information precision, due to informed investors' traditional form of overconfidence (i.e., $\rho^* > 1$). Consistent with Figure 1 of the main paper, an increase in the gambling propensity G_j also leads to an increase in $E(T_j)$, and an increase in the overconfidence parameter ρ results in an increase in the expected turnover $E(T_j)$.

Figure IA.2 displays the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, when investors' risk aversion γ varies from 2 to 2.5 (fixing $\rho = 9$). Consistent with Figure 2 of the main paper, as γ increases, the expected turnover $E(T_j)$ decreases; also an increase in the gambling propensity G_j leads to an increase in $E(T_j)$ in the cross-section of stocks.

In Panel A of Figure IA.3, we assume that each investor's endowment equals the per capita supply (i.e., $\forall j, Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 1$); in Panel B, we assume that investor endowments are identically zero (i.e., $\forall j, Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 0$). We plot the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j and the affect parameter A_j. Like in Figure 3 of the main paper, the cross-sectional relationship between $E(T_j)$ and A_j is non-monotonic; $E(T_j)$ is higher for stocks with extreme positive/negative values

[1] We set $\nu_{\xi_j} = \nu_{\theta_j}/2$ to indicate a significant magnitude of noise in the signal s_j^*. The traditional overconfidence parameter $\rho^* = 2$ is consistent with the range used in Odean (1998) (pp. 1909–10). See Footnote 15 of the main paper for justification of other parameter values.

of A_j. Further, in the cross-section of stocks, the expected turnover $E(T_j)$ increases as gambling propensity G_j increases.

IA.2 Return co-movement with the market

Using Theorem 1, we can express the realized excess returns of the j'th stock and the market as

$$r_j = \mathbf{1}_j'(V - PR_f) \text{ and } r_M = \frac{1}{J}\mathbf{1}'(V - PR_f), \tag{IA.1}$$

where

$$V - PR_f = \ell\left(F - \frac{\nu_F}{\nu_\tau}\tau\right) + \theta - q\mathcal{S} - \Phi^{-1}\left[\eta_\kappa \phi_\kappa^{-1}\left(\ell\Delta\tau + \Omega\mathcal{S}\right) + \eta_A \phi^{-1} A - \gamma\mathbf{1}\right].$$

We can compute $\beta_{jM} = \dfrac{\text{Cov}(r_j, r_M)}{\text{Var}(r_M)}$.

Figure IA.4 depicts the relative beta β_{jM}/ℓ_j for each stock as a function of gambling propensity G_j, for different levels of overconfidence (ρ). Like in Figure 4 of the main paper, in the cross-section of stocks, β_{jM}/ℓ_j decreases as G_j increases, which accords with Proposition2. Also, as overconfident investors underestimate the precision of the signals to a greater extent (i.e., a higher ρ), a greater discrepancy arises between β_{jM} and ℓ_j.

Figure IA.5 depicts the relative beta β_{jM}/ℓ_j for each stock as a function of gambling propensity G_j, when investors' risk aversion (indicated by the parameter γ) varies. Like in Figure 5 of the main paper, as G_j increases, β_{jM}/ℓ_j decreases. As investors become more risk averse (i.e., as γ increases), the discrepancy between β_{jM} and ℓ_j diminishes.

IA.3 Underreaction to public information

Recall that the public signal for the j'th stock (see Equation (11) of the main paper) takes the following form:

$$S_j = \mu s_j + (1-\mu)\tau,$$

where $\mu \in (0,1]$ is a constant parameter. We can then use r_j from Equation (IA.1) to compute the underreaction measure $\text{Cov}(r_j, S_j)$.

Figure IA.6 plots $\text{Cov}(r_j, S_j)$, for each stock as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We set $\mu = 0.8$ (our results are robust to different values of μ); the other parameters are the same as those used in the earlier figures. Like in Figure 6 of the main paper, when overconfident investors assess the precision of the signals properly (i.e., $\rho = 1$), we observe $\text{Cov}(r_j, S_j) = 0$ indicating no underreaction. However, if overconfident investors underestimate the precision of the signals s and τ (i.e., $\rho > 1$), then $\text{Cov}(r_j, S_j) > 0$, and an increase in the gambling propensity G_j leads to a decrease in $\text{Cov}(r_j, S_j)$. Also, if overconfident investors underestimate the precision of the signals to a greater extent (i.e., a higher ρ), $\text{Cov}(r_j, S_j)$ is higher.

Figure IA.7 depicts $\text{Cov}(r_j, S_j)$, for each stock as a function of the gambling propensity G_j, when investors' risk aversion (indicated by the parameter γ) varies. Like in Figure 7 of the main paper, as G_j increases, $\text{Cov}(r_j, S_j)$ decreases. As investors become more risk averse (i.e., as γ increases), $\text{Cov}(r_j, S_j)$ increases.

IA.4 The (unconditional) risk-adjusted expected return

Using Theorem 1 and Equation (IA.1), we the express the expected excess returns of the j'th stock and the market, respectively, as

$$E(r_j) = \mathbf{1}_j' \Phi^{-1} \left(\gamma \mathbf{1} - \eta_A \phi^{-1} A \right), \quad \text{and} \quad E(r_M) = \frac{1}{J} \mathbf{1}' \Phi^{-1} \left(\gamma \mathbf{1} - \eta_A \phi^{-1} A \right);$$

note that these expected returns are unconditional on the public information τ and s, and the private information s^*. We can compute the (unconditional) risk-adjusted expected returns of the stocks as

$$\alpha_j = E(r_j) - \beta_{jM} E(r_M),$$

Figure IA.8 plots the risk-adjusted return α_j as a function of gambling propensity G_j and affect parameter A_j. Like in Figure 8 of the main paper, we observe that in the cross-section of stocks, α_j decreases as the affect parameter A_j increases. Also the negative relationship between α_j and A_j attenuates when gambling propensity G_j is high.

In Panel A of Figure IA.9, we set γ to a higher value (2.5), compared to Figure IA.8, where $\gamma = 2$. Like in the main paper (see Panel A of Figure 9 and the ensuing discussion), we observe that the negative relationship between α_j and the affect parameter A_j becomes more pronounced for higher γ. We corroborate this finding in Panel B of Figure IA.9, where we plot the difference between the α_j's when $\gamma = 2.5$ and when $\gamma = 2$. The difference decreases as the affect parameter A_j increases.

References

Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998), "Investor psychology and security market under-and overreactions," *Journal of Finance*, vol. 53, 1839–1885.

Luo, J., A. Subrahmanyam, and S. Titman (2021), "Momentum and reversals when overconfident investors underestimate their competition," *Review of Financial Studies*, vol. 34, 351–393.

Odean, T. (1998), "Volume, volatility, price, and profit when all traders are above average," *Journal of Finance*, vol. 53, 1887–1934.

Proof of Theorem 1:

Denote $\mathcal{S} = \begin{pmatrix} s \\ s^* \end{pmatrix}$.

(a) The i'th overconfident investor believes that

$$F|\tau \sim N\left(\frac{\nu_F}{\kappa_\tau}\tau, \kappa_{F|\tau}\right) \text{ and } \theta|\mathcal{S} \sim N\left(\hat{q}\mathcal{S}, \hat{\Gamma}\right),$$

where $\hat{q}$ and $\hat{\Gamma}$ are constants depending on the variance-covariance matrix of θ and $\mathcal{S}$ based on the investor's biased belief. Write the investor's wealth at Date 1 as

$$W_{i1} = W_{i0}R_f + X_i'(V - PR_f) = W_{i0}R_f + X_i'(\bar{V} + \ell F + \theta - PR_f).$$

The investor's demand X_i maximizes

$$\begin{aligned} & \hat{E}\left[U(W_{i1})|\tau, \mathcal{S}\right] \\ = \; & \hat{E}\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(\bar{V} + \ell F + \theta - PR_f)\right]|\tau, \mathcal{S}\right] \\ = \; & -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \hat{q}\mathcal{S} - PR_f\right) + 0.5\gamma^2 X_i'\phi_\kappa X_i\right], \end{aligned}$$

where $\phi_\kappa = \ell\ell'\kappa_{F|\tau} + \hat{\Gamma}$. The f.o.c. w.r.t. X_i implies that the optimal demand is:

$$X_\kappa = \frac{\phi_\kappa^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\kappa_\tau}\tau + \hat{q}\mathcal{S} - PR_f\right). \qquad \text{(IA.2)}$$

(b) The i'th affect investor perceives that in addition to the actual liquidation values $V = \bar{V} + \ell F + \theta$, there are extra non-monetary payoffs as indicated by A, and has unbiased beliefs that

$$F|\tau \sim N\left(\frac{\nu_F}{\nu_\tau}\tau, \nu_{F|\tau}\right), \text{ and } \theta|\mathcal{S} \sim N\left(q\mathcal{S}, \Gamma\right),$$

where q and Γ are constants depending on the variance-covariance matrix of θ and $\mathcal{S}$ based on the investor's unbiased belief. The investor's demand X_i maximizes:

$$\begin{aligned} & E\left[U(W_{i1}^A)|\tau, \mathcal{S}\right] \\ = \; & E\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(A + \bar{V} + \ell F + \theta - PR_f)\right]|\tau, \mathcal{S}\right] \\ = \; & -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} - PR_f\right) + 0.5\gamma^2 X_i'\phi X_i\right], \end{aligned}$$

where $\phi = \ell\ell'\nu_{F|\tau} + \Gamma$. The f.o.c. implies that the demand is expressed as:

$$X_A = \frac{\phi^{-1}}{\gamma}\left(A + \bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} - PR_f\right). \tag{IA.3}$$

(c) The i'th gambler also has unbiased beliefs. The investor's demand X_i maximizes:

$$\begin{aligned} & E\left[U_G(W_{i1})|\tau, \mathcal{S}\right] \\ = \;& E\left[-\exp\left[-\gamma W_{i0}R_f - \gamma X_i'(\bar{V} + \ell F + \theta - PR_f) - 0.5X_i'GX_i\right]|\tau, s\right] \\ = \;& -\exp\left[-\gamma W_{i0}R_f - \gamma X_i'\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + qs - PR_f\right) + 0.5\gamma^2 X_i'\left(\phi - \frac{G}{\gamma^2}\right)X_i\right]. \end{aligned}$$

The f.o.c. implies that the demand is expressed as:

$$X_G = \frac{\phi_G^{-1}}{\gamma}\left(\bar{V} + \ell\frac{\nu_F}{\nu_\tau}\tau + q\mathcal{S} - PR_f\right), \tag{IA.4}$$

where $\phi_G = \phi - G/\gamma^2 > 0$.

(d) From Equations (IA.2), (IA.3), and (IA.4), the market-clearing condition, $\eta_\kappa X_\kappa + \eta_A X_A + \eta_G X_G = \mathbf{1}$, implies that the equilibrium prices P take the form as given in this theorem. □

Figure IA.1: Expected turnover as a function of gambling propensity, for different levels of overconfidence, when there is private information

This graph plots the expected turnover $E(T_j)$ for each stock as a function of gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$). We assume that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j$ $Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$).

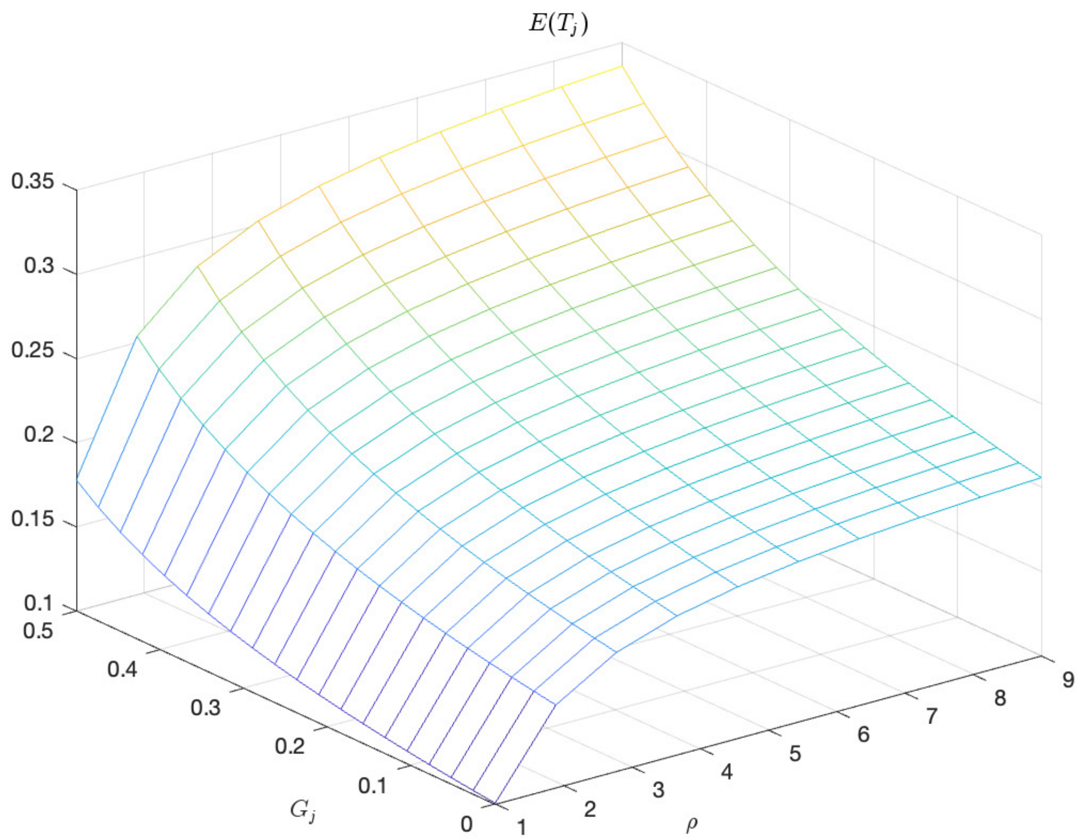

Figure IA.2: Expected turnover as a function of gambling propensity, for different levels of risk aversion, when there is private information

This graph plots the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$). We assume that each type of investors' endowments equal the ex ante means of their optimal demands (i.e., $\forall j\ Y_{\kappa j} = E(X_{\kappa j})$, $Y_{Aj} = E(X_{Aj})$, and $Y_{Gj} = E(X_{Gj})$).

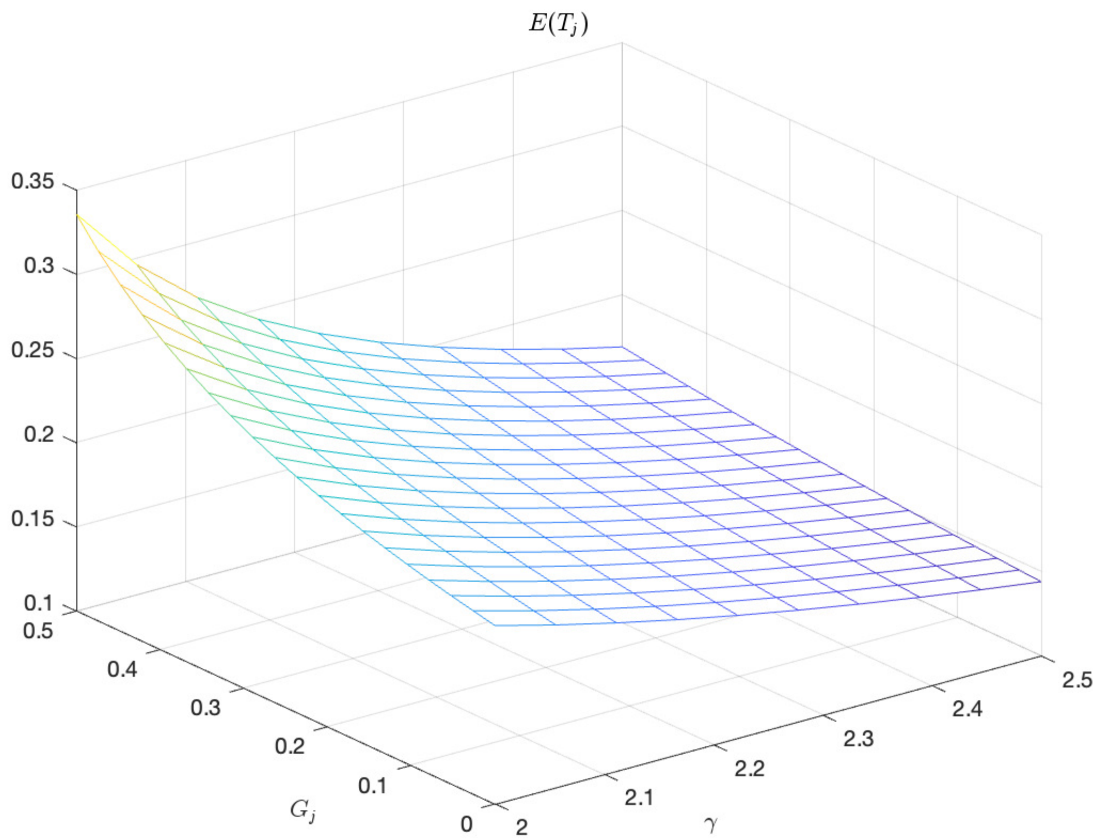

Figure IA.3: Expected turnover, gambling propensity, and the affect heuristic, when there is private information

This graph plots the expected turnover $E(T_j)$ for each stock as a function of the gambling propensity G_j and the affect parameter A_j. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$). We assume that each type of investor's endowments equal per capita supplies (i.e., $\forall j\ Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 1$) in Panel A, and that all endowments equal zero (i.e., $\forall j\ Y_{\kappa j} = Y_{Aj} = Y_{Gj} = 0$) in Panel B.

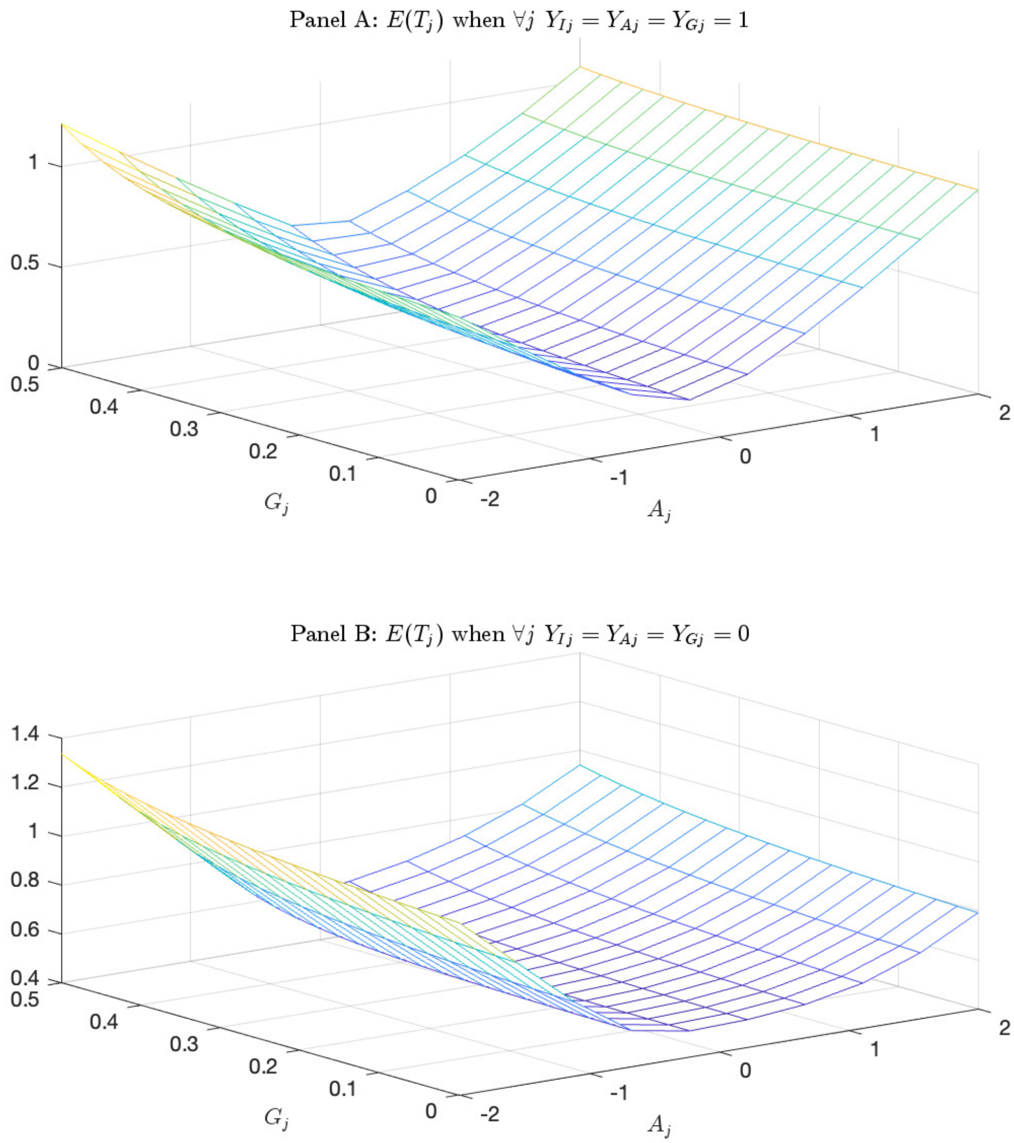

Figure IA.4: Relative beta as a function of gambling propensity, for different levels of overconfidence, when there is private information

This graph plots the relative beta β_{jM}/ℓ_j for each stock as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).

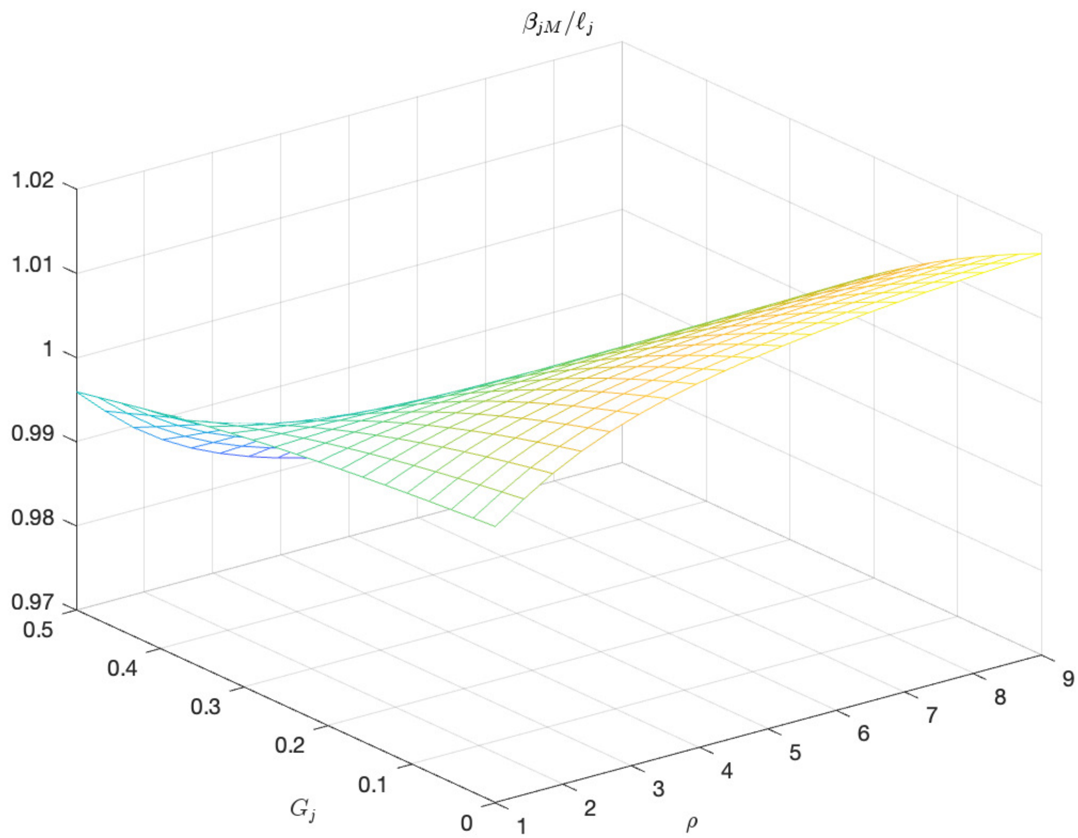

Figure IA.5: Relative beta as a function of gambling propensity, for different levels of risk aversion, when there is private information

This graph plots the relative beta β_{jM}/ℓ_j for each stock as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).

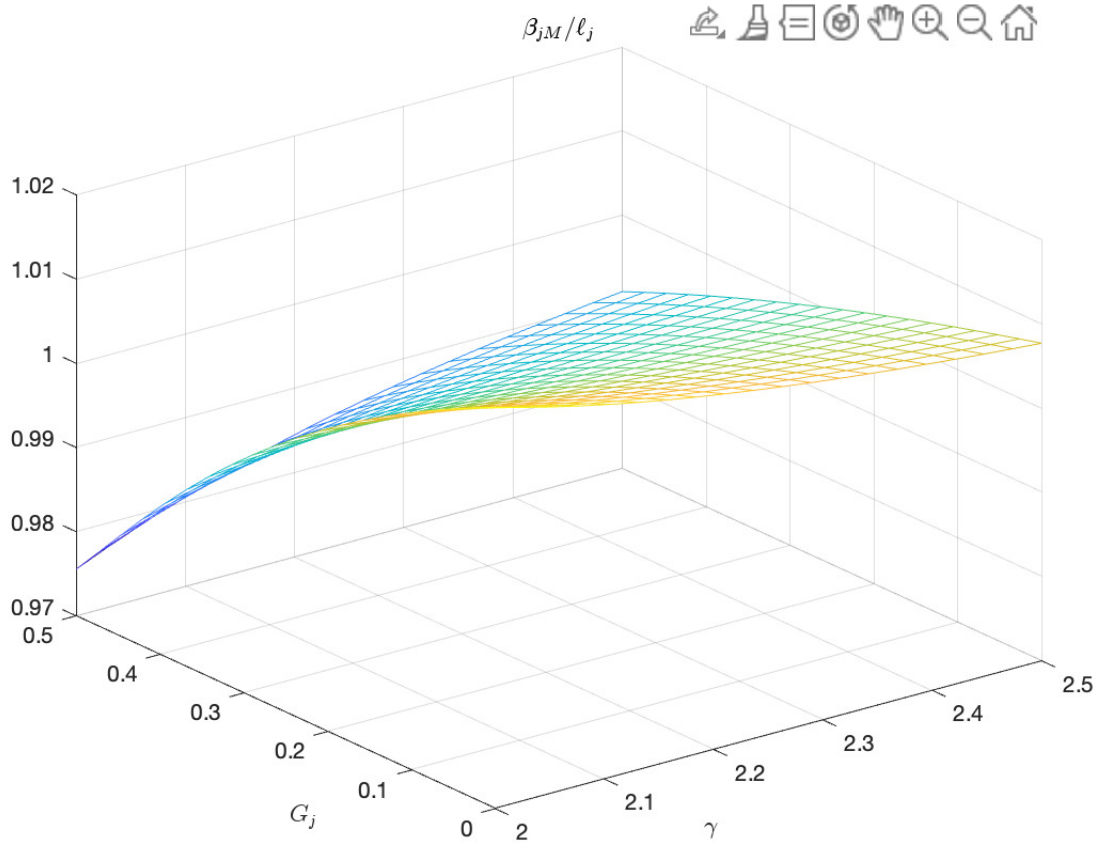

Figure IA.6: Level of underreaction to public information as a function of gambling propensity, for different levels of overconfidence, when there is private information

This graph plots the post-public-announcement drift (as measured by $\mathrm{Cov}(r_j, S_j)$) for each stock as a function of the gambling propensity G_j, for different levels of the overconfidence bias (ρ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\mu = 0.8$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).

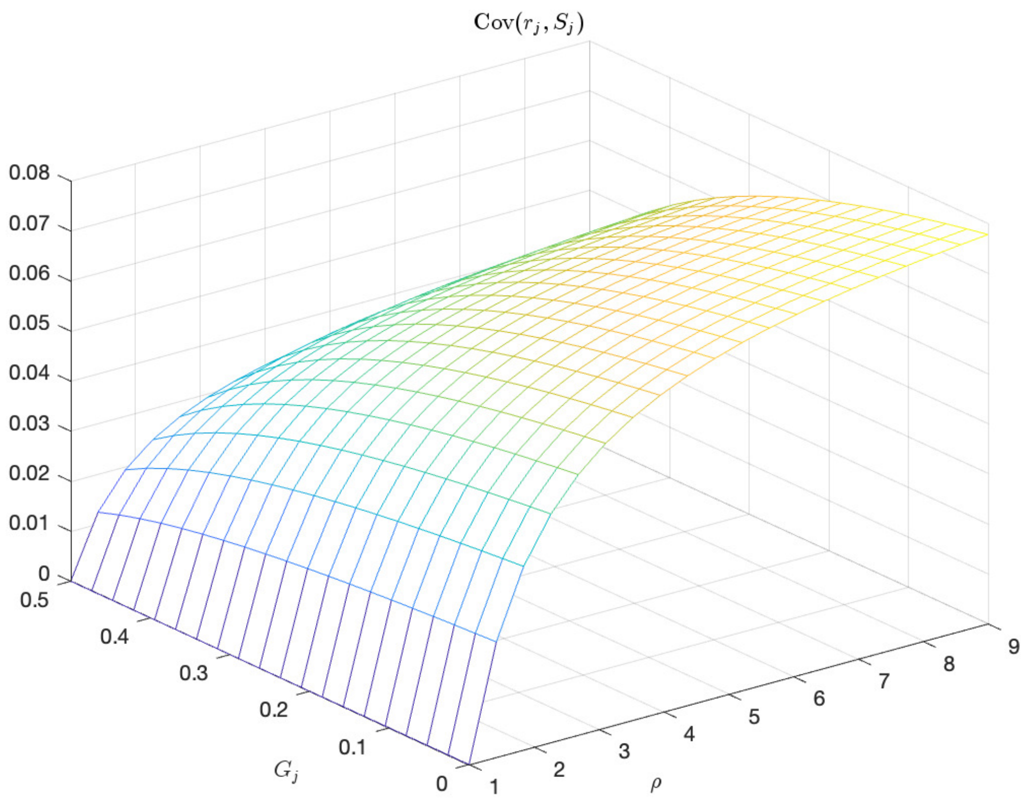

Figure IA.7: Level of underreaction to public information as a function of gambling propensity, for different levels of investors' risk aversion, when there is private information

This graph plots the post-public-announcement drift (as measured by $\mathrm{Cov}(r_j, S_j)$) for each stock as a function of the gambling propensity G_j, for different levels of investors' risk aversion (γ). We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; $\rho = 9$; and $\mu = 0.8$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).

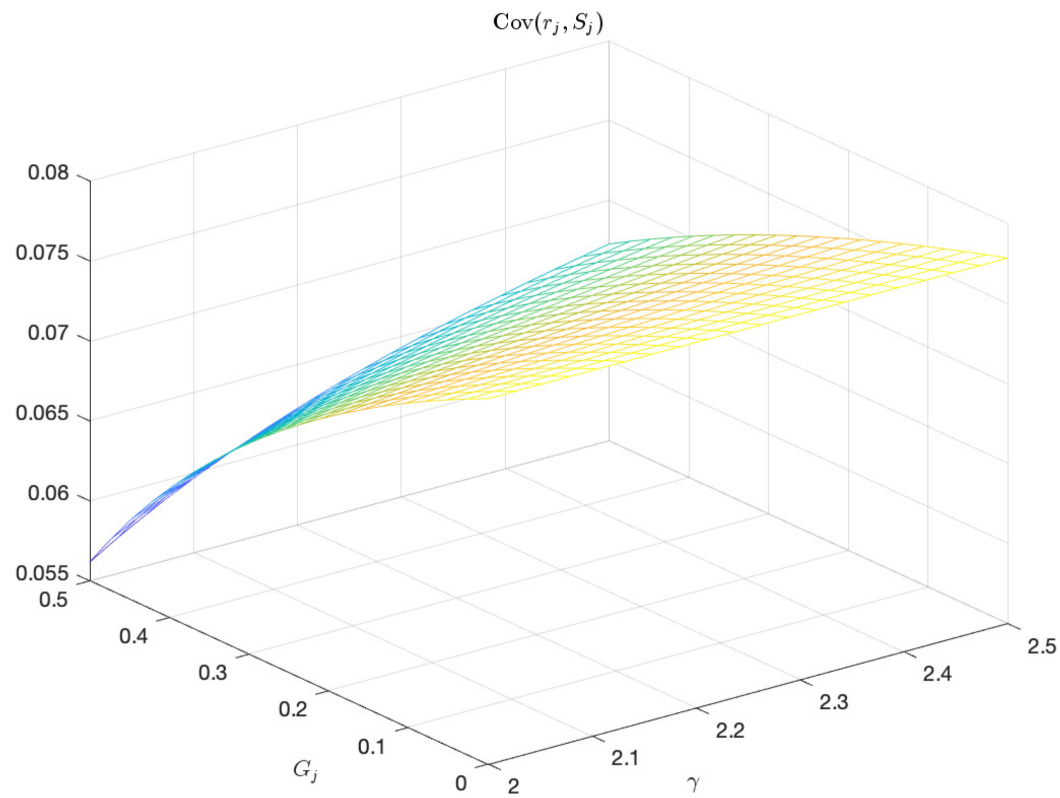

Figure IA.8: Risk-adjusted expected returns, gambling propensity, and the affect heuristic, when there is private information

This graph plots the risk-adjusted expected return α_j for each stock as a function of the gambling propensity G_j and the affect parameter A_j. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\gamma = 2$, $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j\ \ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).

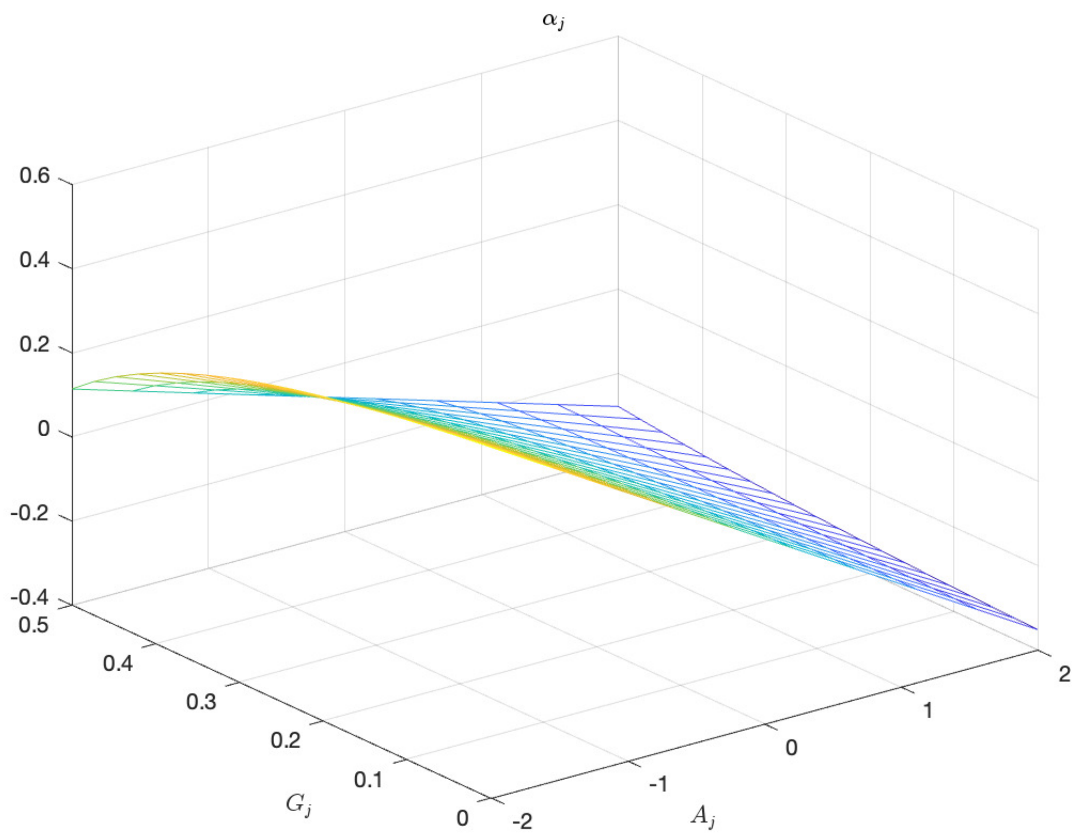

Figure IA.9: Risk-adjusted expected returns, gambling propensity, and the affect heuristic, for different levels of risk aversion, when there is private information

Panel A of this graph plots the risk-adjusted expected return α_j for each stock as a function of the gambling propensity G_j and the affect parameter A_j, when investors' risk-aversion parameter $\gamma = 2.5$. Panel B of this graph plots the difference between the α_j's for cases where γ equals 2.5 and 2. We consider an economy with $J = 200$ stocks, where the G_j values are evenly distributed on the support $[0, 0.5]$, and the A_j values are evenly distributed on the support $[-2, 2]$. The other parameter values are $\eta_\kappa = 0.5$, $\eta_A = 0.2$, $\eta_G = 0.3$; $\forall j$ $\ell_j = 1$, $\nu_{\theta_j} = 1$ and $\nu_{\epsilon_j} = \nu_{\xi_j} = \nu_{\theta_j}/2$; $\nu_F = 0.01$ and $\nu_\zeta = \nu_F/2$; and $\rho = 9$. There is also a traditional form of overconfidence (as indicated by $\rho^* = 2$).