VIX Decomposition, the Price of Fear and Stock Return Predictability

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ABSTRACT

The VIX is not just a volatility index but a fear gauge. We formalize this market perception with a linear decomposition of the VIX that consists with four fundamentally different elements: the realized variance (RV), the variance risk premium (VRP), the realized tail (RT), and the tail risk premium (TRP), respectively. The VRP compensates the anticipated (normal) market volatility, and the TRP prices the fear of potentially (unusual) large market movements. Empirically, approximate one-third of the VIX's formation is attributed to the TRP. In addition to VRP, RT and TRP are crucial components for predicting future returns on equity portfolios.

Keywords: Variance Risk Premium, Tail Risk Premium, Quadratic Variation, Polynomial Variation, Realized Variance, Realized Tail.

JEL classification: C22, C51, C52, G1, G12, G13

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I. Introduction

The conventional view of the (squared) VIX index sees it as a risk-neutral forward-looking measure of market return volatility. Statistically, it is a time-series conditional expectation of the future realized variance (i.e. quadratic variation) in a risk-neutral probability space. Thus, the conventional measure of equity variance risk premium (hereafter VRP^c), the difference between the physical measure of the realized variance (hereafter RV) and the squared VIX, serves as an important indicator of aggregate risk-aversion of market participants.[1] Mounting empirical evidence suggests that the variance risk premium is a superior predictor of future aggregate market returns compared to the traditional predictor variables such as the dividend-price and other valuation ratios, particularly for shorter time horizons.[2] Andreou and Ghysels (2013) and Bondarenko (2014) demonstrate that the variance risk premium is unique in that it cannot be explained by other traditional risk factors. Nevertheless, a puzzle regarding the variance risk premium described by Bollerslev, Tauchen, and Zhou (2009; BTZ hereafter) exists: neither the square VIX nor the realized variance predict stock market returns, so why does their difference (i.e. VRP^c) provide strong predictive power?

Many studies address this issue by empirically identifying attributions of the predictability. Empirical findings of Todorov and Tauchen (2011) suggest that the volatility risk either coincides or is highly correlated with the price jump risk, while Bollerslev and Todorov (2011) show that the risk premium for unusual tail events cannot be explained exclusively by the level of volatility

[1] See Campbell and Cochrane (1995), Bekaert and Engstrom (2010), Bollerslev, Gibson and Zhou (2011), Bekaert, Hoerova, and Lo Duca (2013), and Bekaert and Hoerova (2014).

[2] These studies include but are not limted to Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), Han and Zhou (2012), Du and Kapadia (2012), Eraker and Wang (2015), Almeida, Vicente, and Guillen (2013), Bekaert and Hoerova (2014), Bali and Zhou (2014), Camponovo, Scaillet, and Trojani (2014), Kelly and Jiang (2014), Li and Zinna (2014), Vilkov and Xiao (2013) and Bollerslev, Marrone, Xu, and Zhou (2014).

and argue that the jump-tail risk is still present even if the investment opportunity set does not change over time, remains and it a force even for short time-intervals where the investment opportunity set is approximately constant. Cremers, Halling and Weinbaum (2015) also show that aggregate jump and volatility risk collectively explain variation in expected returns, and aggregate stock market jump risk is priced in the cross-section. Thus, volatility and price jump-tail risk premia share compensations for similar risks and therefore should be modeled jointly. Most notably, Bollerslev, Todorov and Xu (2015; BTX hereafter) reveal that most of the predictability for the aggregate market portfolio previously attributed to the variance risk premium stems from not just the volatility jump but the tail risk component, and the compensation for tail risk drives out most of the predictability stemming from the part of the variance risk premium associated with "normal" sized price fluctuations.[3] Intuitively, the compensation demanded by investors for bearing tail risk (*jump fear*) contributes to the expectation as well as the predictability of future market returns.

Du and Kapadia (2012) and Chow, Jiang and Li (2014) observe that the VIX index rapidly deviates from the true volatility measure when a larger proportion of stock return variability is determined by fears of substantial jumps. This indicates that the VIX index may be an inappropriate measure for volatility per se—the tail process (i.e. large jumps) is embedded in the index.[4] Specifically, from a statistical viewpoint, the VIX is a measure above and beyond quadratic variation in that the embedded tail variation in the VIX is determined specifically by higher-order

[3] In addition, a number of papers have related jump-tail risk to asset risk premia. For example, Naik and Lee (1990), Longsta and Piazzesi (2004), Liu, Pan, and Wang (2005), Bollerslev and Todorov (2011, 2014), Kelly and Jinag (2014), and Andersen, Fusari and Todorov (2015) model jump-tail risk premia in equity returns, while Gabaix (2008) and Wachter (2013), extending initial work of Rietz (1988) and Barro (2006), relate equity risk premia to time-varying consumption disaster risk.

[4] In addition, Broadie and Jain (2008), Cont and Kokholm (2013), as well as Carr, Lee, and Wu (2012) also observe that jump-induced tail bias the VIX as a volatility measure.

moments of the jump distribution and is therefore distinguishable from the quadratic variation. Recently, Amaya et al. (2015) empirically examine time-series and cross-sectional properties of realized moments based on the standard diffusion process of returns with jumps and found that the realized third moment, skewness, has significant and reliable predictive power for future stock returns, but, unsurprisingly, there is little evidence of a reliable relation between realized volatility and future stock returns. Many other studies also suggest that higher-order moments of the return stochastic process contribute significantly to return predictability. For example, BTZ (2009) argues that the volatility-of-volatility (VOV) constitutes the dominant source of the variation in the equity premium. Park (2013) further develops a model-free and risk-neutral measure of the VOV implied by a cross section of the VIX options which is called the VVIX index. Park (2013) states that the VOV risk premium significantly contributes to the forecasting power of the VVIX index, and that the predictability of the VVIX largely results from the incorporated VOV rather than volatility jumps.

In summary, we note that empirical evidence has strongly shown that tail risk premium does not just afford some additional predictability for the market portfolio over and above that of the variance risk premium—it is the *main source* for the predictability of future returns. As elaborated below, the source of return predictability from the conventional measure of variance risk premium is ambiguous in that VIX is not simply a volatility measure. Thus, using VIX to measure variation premium (i.e., VIX^2 minus RV) makes it very difficult to distinguish between volatility or tail effects.

Unambiguously, the VIX index was originally designed to measure the *quadratic variation* (hereafter $\mathbb{QV}$) of a *jump-free* process.[5] Nevertheless, the deviation of VIX from $\mathbb{QV}$ estimation is

[5] See Carr and Madan (1999), Demeter, Derman, Kamal and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000).

proportional to the jump intensity. As discussed above, the accuracy of the VIX to $\mathbb{QV}$ deteriorates rapidly when a larger proportion of stock return variability is determined by fear of discordantly large jumps (i.e., the tail risk). In fact, it has been often overlooked that the Bakshi-Kapadia-Madan's (2003; BKM hereafter) measure of the variance (hereafter V^{BKM}) is insensitive to tail variation and can serve as an unbiased ex-ante estimate of quadratic variation.[6] A puzzle then arises: if the VIX is not simply a $\mathbb{QV}$ measure, then what does it really measure? We identify that the underlying variation process of the VIX measure actually follows a *polynomial variation* (hereafter $\mathbb{PV}$) which is much more general than the $\mathbb{QV}$ in form. The $\mathbb{PV}$ contains time variation in the jump intensity process in addition to the $\mathbb{QV}$, where this additional jump-induced tail of the distribution in the $\mathbb{PV}$ is determined by high-order moments of the jump distribution in a polynomial form. We further determine that the physical measure for the $\mathbb{PV}$, named the *Realized* VIX (hereafter RVIX) analogous to the realized variance, can be simply measured by twice the arithmetic-logarithmic forward return difference. Thereby, it is evident that the VIX index is indeed a risk-neutral forward-looking measure of the RVIX and not that of the RV.

Next, according to Bollerslev and Todorov's (2011) definition of risk premium, the difference between the physical measure of RVIX and squared VIX is then the appropriate risk premium for the polynomial variation of returns. For convenience, we call this the VIX *risk premium* (hereafter VIXRP). Analogously, the correct definition of *variance risk premium* (hereafter VRP) corresponding to quadratic variation should be the difference between RV and V^{BKM} (not squared VIX). Finally, since $\mathbb{QV}$ is the second order $\mathbb{PV}$, the RV (the physical measure of $\mathbb{QV}$) is a special case of the RVIX (the physical measure of $\mathbb{PV}$), when the stochastic process of

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[6] Du and Kapadia (2012) and Chow, Jiang and Li (2014) explicitly demonstrate that the BKM's measure of the variance of the holding period return is the most appropriate for measuring quadratic variation.

return is *tail-free*. The premium-differentiation between VIX risk and variance risk (i.e., VIXRP minus VRP) thus logically quantifies the tail risk premium (hereafter TRP) or the compensation for investors' fear risk.

The main goals of the present manuscript are twofold. First, by explicitly recognizing the underlying stochastic process of the VIX index that follows the polynomial (not quadratic) variation, we decompose the (squared) VIX into four fundamentally different components: the conditional realized variance, the variance risk premium, the conditional realized tail, and the tail risk premium. Second, relying on our decomposition of the VIX index, we seek to clarify where the inherent market return predictability of the conventional VRP^c (i.e., VIX^2 - RV) is coming from and how it plays out over different return horizons and for different portfolios with different risk exposures.

Based on the notion of BTZ, the risk premium of return variation is defined as the difference between the time series conditional expected future return variation in the (options based) risk-neutral ($\mathbb{Q}$) framework and that in the physical probability ($\mathbb{P}$) space. Conventionally, the physical measures employed are backward-looking (past) sample estimations, where the options based $\mathbb{Q}$ measures are forward-looking. This counterintuitive approach used for calculating variation risk premium could naturally produce biased results. Fortunately, to resolve this problem, Bekaert and Hoerova (2014; BH hereafter) developed a robust selection procedure for identifying stable volatility forecasting models that are used for estimating $\mathbb{P}$ spaced conditional realized variation. We apply both the BTZ unconditional ex-post and the BH conditional ex-ante methods in our empirical analysis. Using 5-minute high-frequency data for the S&P 500 index, we found that statistically the realized tail as well as both the unconditional and conditional tail risk premium are significantly positive. Our empirical results confirm that the return predictability for the aggregate market portfolio afforded by the conventional variance risk premium (VRP^c) is attributed to the return predictability of the decomposed components: the unbiased variance risk premium (VRP), the realized tail (RT), and the tail risk premium (TRP). Importantly, the tail variation and its risk premia do not just offer some additional predictability for the market portfolio over and above that of the variance risk premia but provides the main impetus for the total predictability. This is consistent with recent findings of BTX (2015) that most of the predictability for market return previously ascribed to the variance risk premium actually originates from the tail risk component. In addition, we found even greater increases in predictive performance of RT and TRP from decomposed market portfolios: Size, Value, Momentum as well as Industrial Sectors. In summary, the significant empirical evidence of the market return predictability stemming from our measure of the tail risk premium explains the predictability appearing in the variance risk premium as previously documented in the literature. Consequently, our measure of the tail risk premium as a component to the risk premium of total market return variation provides a proxy for market fears.

The rest of the paper is organized as follows: Section II begins with a simple derivation of the VIX formulation, wherein the realized VIX and polynomial variation are formally defined. A simple approach for determining the market tail risk premium (the price of *fear*) as well as our decomposition of the VIX index are also presented, and we show the sample estimating procedures of the statistics in Section III. Section III also presents the statistical estimations for both unconditional and conditional risk premiums of return variation. Section IV describes the data and illustrates our empirical analysis of the VIX decomposition. Section V reports our empirical findings of equity return predictability of the four VIX decomposed components, and Section VI contains brief concluding remarks.

II. VIX Decomposition

The Chicago Board Options Exchange's (CBOE) VIX index is the most widely used option-based (forward-looking) measure of stock return variability. Nevertheless, it is well known that the index contains compensation for risk in addition to that for the time-varying volatilities. Those include risk premium of jump intensities as well as that of fears for jump tail events. As such, this does lend acceptance to the common use of the term "investor's fear gauge" as an epithet for the VIX volatility index, although admittedly a rather imperfect proxy. This section presents an unambiguous approach to distinguish risk between volatility and fear embedded in the VIX index. We begin with a simple formulating process of the VIX index.

A. A Simple VIX Formulation

Without any specification of the return generating process, Chow, Jiang and Li (2014) show that the formulation of VIX can be derived mathematically and straightforwardly as follows: Let $R_{t+1}\left(=\frac{S_{t+1}-S_t}{S_t}\right)$ be the forward arithmetic return and $r_{t+1}\left(=\ln\left(\frac{S_{t+1}}{S_t}\right)\right)$ denote the logarithmic forward return over a period from t to $t+1$. Employing Taylor series expansion and the expansion with remainder, the difference between the arithmetic and logarithmic returns can be expressed as follows:

$$R_{t+1}-r_{t+1}=\left[\int_{S_t}^{\infty}\frac{1}{K^2}(S_{t+1}-K)^{+}dK+\int_{0}^{S_t}\frac{1}{K^2}(K-S_{t+1})^{+}dK\right]=\sum_{n=2}^{\infty}\frac{1}{n!}r_{t+1}^{n}. \quad (1)$$

Now, let $\mathbb{Q}$ denote the risk-neutral distribution associated with the time dynamic of forward returns. Under the no-arbitrage framework, the time-series conditional expected return-difference can be measured by current option prices, which is equivalent to the basic formulation of the (squared) VIX:[7]

$$E_t^{\mathbb{Q}}(R_{t+1} - r_{t+1}) = e^{r_f}\left\{\int_{S_t}^{\infty} \frac{1}{K^2} C_{t,t+1}(K)dK + \int_0^{S_t} \frac{1}{K^2} P_{t,t+1}(K)dK\right\} \tag{2}$$
$$= \frac{1}{2}\mathrm{VIX}_t^2 = \frac{1}{2}\left[E_t^{\mathbb{Q}}(r_{t+1}^2) + \sum_{n=3}^{\infty} \frac{2}{n!} E_t^{\mathbb{Q}}(r_{t+1}^n)\right]$$

where $E_t^{\mathbb{Q}}(\cdot)$ is the risk-neutral conditional expectation operator at time *t*, r_f is the annualized risk-free rate corresponding to expiration date $t+1$, and $C_{t,t+1}(K)$ and $P_{t,t+1}(K)$ are the *current* (at time *t*) premiums of call and put option contracts with a strike *K* and expiration $t+1$, respectively. That is, the arbitrage-free argument implies that the VIX index can be extracted from the market price of a portfolio composed of all possible out-of-the-money (OTM) call/put options of the underlying index with weight inversely proportional to the square value of the strike price. Equivalently, equation (2) shows that instead of employing a long list of OTM options, the VIX can also be simply replicated by a portfolio of only two assets: a long position of a forward contract with a settlement price, S_{t+1} and a short position of a *log contract* with a settlement price, $\ln(S_{t+1})$, where the *log contract* has been proposed by Neuberger (1994) for hedging volatility.[8]

B. The Polynomial Variation and the Realized Tail

The most notable result from equation (2) is that the VIX index, calculated from the fair market price of either an options portfolio or that of long-short forward contracts, provides not only a forward-looking estimate of the market volatility but information about the future return distribution in its entirety. The distributional information in addition to the volatility (the second

[7] Under a purely continuous process of the quadratic variation, equation (2) serves as a basis for the derivation of the VIX. See Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999a, 1999b), and Britten-Jones and Neuberger (2000) and others.

[8] Precisely, the replicated portfolio consists $\frac{1}{S_t}$ long position for every short position.

moment) is characterized by a polynomial combination of a series of all higher distributional moments (e.g. skewness, kurtosis, etc.). This aggregate of high moments implanted in the VIX formulation perhaps explains why the VIX index is often referred to as the investor fear gauge. To examine and analyze the VIX index as a market fear indicator, decomposing the index in terms of different risk characteristics is necessary. For convenience, we define RVIX_{t+1} as the future realized outcomes of the VIX such that

(3) $$\text{RVIX}_{t+1}^2 = 2(R_{t+1} - r_{t+1}).$$

Then, the (squared) VIX is a conditionally risk-neutral estimate of twice the future arithmetic and logarithmic return differences (as called RVIX_{t+1}):

(4) $$\text{VIX}_t^2 = E_t^{\mathbb{Q}}(\text{RVIX}_{t+1}^2).$$

Next, following the classical approach and without losing generality, we assume that asset returns follow Merton's (1976) diffusion-jump process:

(5) $$R_{t+1} = \int_t^{t+1} (\alpha_t - \lambda\mu_J) dt + \int_t^{t+1} \sigma_t dW_t + \int_t^{t+1} \int_{\mathbb{R}^0} (e^x - 1)\, \mu[dx, dt].$$

(6) $$r_{t+1} = \int_t^{t+1} \left(\alpha_t - \frac{1}{2}\sigma_t^2 - \lambda\mu_J\right) dt + \int_t^{t+1} \sigma_t dW_t + \int_t^{t+1} \int_{\mathbb{R}^0} x\, \mu[dx, dt],$$

where α_t is the instantaneous expected return of the asset, σ_t is the volatility, W_t is standard Brownian motion, $\mathbb{R}^0$ is the real line excluding zero, and $\mu[dx, dt]$ is the Poisson random measure for the compound Poisson process with compensator equal to $\lambda \frac{1}{\sqrt{2\pi}\sigma_J^2} e^{-\frac{1}{2}(x-\alpha)^2}$, with λ as the jump intensity. Now, by taking the square of (6) and based on the Brownian properties, the future quadratic return, r_{t+1}^2, can be expressed by a sum of two decomposed components: the integrated value of continuously instant variance ($\mathbb{CV}$ hereafter) and that of discontinuously (or jump) quadratic variability (hereafter $\mathbb{JQV}$). This decomposed process of return variability is the *quadratic variation* ($\mathbb{QV}$), and r_{t+1}^2 is the future realized outcome of the quadratic variation (denoted RV_{t+1}).[9] We summarize this as follows:

$$
\begin{aligned}
\text{(7)} \qquad \mathbb{QV}_{[t,t+1]} = r_{t+1}^2 &= \int_t^{t+1} \sigma_t^2 \, dt + \int_t^{t+1} \int_{\mathbb{R}^0} x^2 \mu(dx, dt) \\
&= \mathbb{CV}_{[t,t+1]} + \mathbb{JQV}_{[t,t+1]}.
\end{aligned}
$$

Carr and Wu (2009) have shown that the theoretical determination of the VIX is inconsistent with the quadratic variation in that

$$
\text{(8)} \qquad \text{VIX}_t^2 = E_t^{\mathbb{Q}}\left(\mathbb{QV}_{[t,t+1]}\right) + \frac{1}{2} E_t^{\mathbb{Q}} \left(\int_t^{t+1} \int_{\mathbb{R}^0} (e^x - 1 - x^2) \, \mu[dx, dt] \right).
$$

A question then arises: what the fundamental process of determining the VIX value should be? To answer this question, we consider (3), (5) and (6) and define a generalized stochastic process of return variations, *polynomial in form*, as follows:

Definition 1. *The infinite-order polynomial variation* ($\mathbb{PV}$) *of returns, based on the return generating process of* (5) and (6), *from time t to $t+1$ is defined as*

$$
\begin{aligned}
\text{(9)} \qquad \mathbb{PV}_{[t,t+1]} = 2(R_{t+1} - r_{t+1}) &= \int_t^{t+1} \sigma_t^2 dt + \sum_{n=2}^{\infty} \frac{2}{n!} \int_t^{t+1} \int_{\mathbb{R}^0} x^n \, \mu(dx, dt) \\
&= \mathbb{CV}_{[t,t+1]} + \mathbb{JPV}_{[t,t+1]}.
\end{aligned}
$$

where $\mathbb{JPV}$ denotes a weighted sum of the all predictable jumps of the $\mathbb{PV}$. The linear combination of all orders of return variability in (9) characterizes the entire probability distribution of R_{t+1}, and thus the $\mathbb{QV}$ is just a special case of the $\mathbb{PV}$, if $n = 2$. It is also important to note that since the continuous component of the polynomial variation converges to that of the quadratic variation under the Brownian motion, $\mathbb{PV}$ equals $\mathbb{QV}$ with the absence of jump.

[9] See Andersen et al. (2001), Cont and Tankov (2003) and others.

Theorem 1. *Based on Definition 1 as well as equations* (3) *and* (4), *the theoretical value of the VIX index at time* t *is the* (*squared-rooted*) *risk neutral estimate of the polynomial variation from time* t *to* $t+1$:

$$\text{VIX}_t = \sqrt{E_t^{\mathbb{Q}}\left(\mathbb{PV}_{[t,t+1]}\right)} \tag{10}$$

In short, the VIX index is a risk-neutral forward looking measure of the *polynomial variation* of log-returns: not that of the quadratic variation. Consequently, the realized variance (RV_{t+1}) is not generally the future realized outcome of the VIX. This highlights the potential bias of the conventional calculation of the variance risk premium by simply taking the difference between the squared VIX and the RV.

Structurally, although polynomial and quadratic variations are similar in form, $\mathbb{PV}$ provides additional information beyond the jump process of return variability. That is, statistically, the difference between $\mathbb{PV}$ and $\mathbb{QV}$ simultaneously captures the asymmetry, tail thickness, and other characteristics of the return distribution. We refer to this difference as the *tail variation* (hereafter $\mathbb{TV}$) or whose physical measure is the *realized tail* (hereafter RT) of returns:

Corollary 1. *From* (10) *in Theorem 1 and* (7), *the difference between the polynomial and the quadratic variations of returns characterizes the jump tail variation* (*denoted* $\mathbb{TV}$), *which can be measured by the realized tail* (*denoted* RT). *The realized tail is a polynomial combination of all possible higher orders of log-returns that is calculated by the spread between the squared realized VIX and the realized variance:*

$$\begin{aligned} \text{RT}_{t+1} \equiv \mathbb{TV}_{[t,t+1]} &= \mathbb{PV}_{[t,t+1]} - \mathbb{QV}_{[t,t+1]} \\ &= \left[2(R_{t+1} - r_{t+1}) - r_{t+1}^2\right] = \sum_{n=3}^{\infty} \frac{2}{n!} r_{t+1}^n . \end{aligned} \tag{11}$$

Note that, based on (6), the higher order of the jump process, x^n for $n > 2$, is equivalent to the same order of the log-returns, r^n for $n > 2$. Therefore, the expected RT is a polynomial sum of all higher order moments of an asset's log-return distribution. Corollary 1 highlights the important relationship between the $\mathbb{QV}$ and the VIX: Under the risk-neutral framework as well as from (4) and (11),

$$(12) \qquad E_t^{\mathbb{Q}}\left(\mathbb{QV}_{[t,t+1]}\right) = \mathrm{VIX}_t^2 - E_t^{\mathbb{Q}}\left(\mathbb{TV}_{[t,t+1]}\right).$$

Consistent with Proposition 1 of Carr and Wu (2009), we show that the (risk-neutral) conditional $\mathbb{QV}$ is just a *tail-free* VIX^2. Further, the option based conditional tail variation can then be measured by the spread between the squared VIX and the BKM's unbiased variance measure (V^{BKM}): [10]

$$(13) \qquad E_t^{\mathbb{Q}}(\mathrm{RT}_{t+1}) = \mathrm{VIX}_t^2 - \mathrm{V}_t^{BKM} = \sum_{n=3}^{\infty} \frac{2}{n!} E_t^{\mathbb{Q}}(r_{t+1}^n),$$

where

$$(14) \qquad \mathrm{V}_t^{BKM} \equiv E_t^{\mathbb{Q}}\left(\mathbb{QV}_{[t,t+1]}\right) = e^{r_f}\left[\int_{S_t}^{\infty} \frac{2\left[1-\ln\left(\frac{K}{S_t}\right)\right]}{K^2} C_{t,t+1}(K)dK + \int_0^{S_t} \frac{2\left[1+\ln\left(\frac{S_t}{K}\right)\right]}{K^2} P_{t,t+1}(K)dK\right],$$

that serves as an appropriate (risk-neutral) forward looking measure of the quadratic variation.

C. VIX Decomposition

Following BTZ's basic notion, we define formally three different risk premiums: the VIX risk premium (VIXRP), the unbiased variance risk premium (VRP), and the tail risk premium (TRP) as follows: First,

$$(15) \qquad \mathrm{VIXRP}_{[t,t+1]} = E_t^{\mathbb{Q}}\left(\mathbb{PV}_{[t,t+1]}\right) - E_t^{\mathbb{P}}\left(\mathbb{PV}_{[t,t+1]}\right) = \mathrm{VIX}_t^2 - E_t^{\mathbb{P}}(\mathrm{RVIX}_{t+1}^2),$$

[10] This spread is the negative value of Du and Kapadia (2012) jump and tail index.

where $E_t^{\mathbb{P}}(\text{RVIX}_{t+1})$ is the physical measure of the polynomial variation in the actual probability space $\mathbb{P}$, and VIX_t^2, as shown in (4), is the risk-neutral estimation of $\mathbb{PV}$. Since $\mathbb{PV}$ identifies the overall variation of returns, VIXRP contains both the risk premium of return volatility and that of potentially abnormal variability. Second,

$$\text{(16)} \qquad \text{VRP}_{[t,t+1]} = E_t^{\mathbb{Q}}\left(\mathbb{QV}_{[t,t+1]}\right) - E_t^{\mathbb{P}}\left(\mathbb{QV}_{[t,t+1]}\right) = \text{V}_t^{BKM} - E_t^{\mathbb{P}}(\text{RV}_{t+1}).$$

VRP serves as a risk premium proxy for ordinary price fluctuation with *normal* jumps. Third,

$$\text{(17)} \qquad \text{TRP}_{[t,t+1]} = (\text{VIX}_t^2 - \text{V}_t^{BKM}) - E_t^{\mathbb{P}}(\text{RT}_{t+1}),$$

where

$$\text{(18)} \qquad E_t^{\mathbb{P}}(\text{RT}_{t+1}) = E_t^{\mathbb{P}}(\text{RVIX}_{t+1}^2) - E_t^{\mathbb{P}}(\text{RV}_{t+1})$$

TRP is the difference between VIXRP and VRP, which characterizes the compensation for the prospectively *unusual* jumps of market return distribution (the *fear* risk). Finally, the VIX index can then be decomposed into four fundamentally different constituents such that:

$$\text{(19)} \qquad \text{VIX}_t^2 = \left[E_t^{\mathbb{P}}(\text{RV}_{t+1}) + \text{VRP}_{[t,t+1]}\right] + \left[E_t^{\mathbb{P}}(\text{RT}_{t+1}) + \text{TRP}_{[t,t+1]}\right].$$

Intuitively, the first two components of the (squared) VIX index reflect the conditional (physical) expectation of future volatility and the risk compensation of the future variability from normal economic uncertainty. The third and fourth elements characterize the conditional (physical) tail variation of returns and the corresponding (tail) risk premium for compensating the fear of a potential abnormal market disaster. The RT and TRP could be negative, if market returns are negatively skewed. This implies that the VIX index could understate the true return volatility due to negative RT and/or TRP, although VIX tends to be highly correlated with return volatility. Importantly, the conventional BTZ variance risk premium (VRP^c), is also biased toward the true variance risk premium in that from (19),

$$\text{(20)} \qquad \mathrm{VRP}^c_{[t,t+1]} = \mathrm{VIX}^2_t - E^{\mathbb{P}}_t(\mathrm{RV}_{t+1}) = \mathrm{VRP}_{[t,t+1]} + \left[E^{\mathbb{P}}_t(\mathrm{RT}_{t+1}) + \mathrm{TRP}_{[t,t+1]}\right].$$

It is clear that the widely used VRP^c is actually influenced by not only the volatility risk premium but the realized tail and its associated risk premium. Consequently, the impact of tail risk on future market price fluctuation could be the source of the predictability of the BTZ's VRP^c to US aggregate equity returns. This paper addresses this issue by empirically examining the return predictability of our four decomposed VIX measures.

Traditionally, the past realized variation is often used as the $\mathbb{P}$ estimate of the conditional variation of stock market returns, which is, in fact, an unconditional sample estimate of the historical return variability. Consequently, to insure the accuracy of risk estimation, developing robust statistical methods for measuring conditional (physical) return variation is necessary. We present our estimation procedures of conditional RVIX, RV and RT based on BH's forecasting models in the next section.

III. Unconditional and Conditional Estimates

To quantify the actual return variations, standard approaches employ high-frequency price observations, and the time interval $[t-1, t]$ is split into n equally spaced increments. (e.g. 78, 5-minute trading intervals in a day). Let p_t denote the logarithmic price of the asset. The j^{th} intraday return r_j on day t is defined as $r_j = p_{t-1+\frac{j}{n}} - p_{t-1+\frac{j-1}{n}(\Delta)}$. According to Andersen and Bollerslev (1998), the unconditional (ex-post) estimate of the realized variance can be defined:

$$\text{(21)} \qquad \widehat{\mathrm{RV}}_t = \sum_{j=1}^{n} r_j^2 \ \xrightarrow{p}\ \mathbb{QV}_{[t-1,t]}, \qquad \text{for } n \to \infty$$

where $\xrightarrow{p}$ standard for convergence in probability. Analogous to RV estimation, Jiang and Oomen (2008) show that the sum of the twice difference between arithmetic and logarithmic returns convergence in probability limit to quadratic variation pluses jumps in exponential form. Mathematically, that is, $\underset{n \to \infty}{plim} \sum_{j=1}^{n} 2(R_j - r_j) = \mathbb{QV}_{[t-1,t]} + 2\int_{t-1}^{t}[\exp(J_u) - J_u^2 - J_u - 1]dq_u = \mathbb{PV}_{[t-1,t]}$, with J being the jump process. Therefore, the sample estimate of our realized VIX can be calculated as:

$$\widehat{\mathrm{RVIX}}_t^2 = \sum_{j=1}^{n} 2(R_j - r_j) \overset{p}{\to} \mathbb{PV}_{[t-1,t]}, \qquad \text{for } n \to \infty, \tag{22}$$

and the asymptotically unbiased, unconditional measure of the realized tail can thus be computed by $\widehat{\mathrm{RT}}_t = \sum_{j=1}^{M}[2(R_j - r_j) - r_j^2] \overset{p}{\to} [\mathbb{PV}_{[t-1,t]} - \mathbb{QV}_{[t-1,t]}]$, for $n \to \infty$. Further, the estimation of VIX (denoted $\widehat{\mathrm{VIX}}_t$) based on finite options prices can be obtained from the Chicago Board of Option Exchanges (CBOE). We also apply the same procedure as the CBOE's VIX formulation to the unbiased variance measure of V_t^{BKM} (denoted $\hat{\mathrm{V}}_t^{BKM}$).[11] Then, the calculation of our risk premiums can be summarized as follows: [12]

Unconditional VIX Risk Premium:	$\widehat{\mathrm{VIXRP}}_t = \widehat{\mathrm{VIX}}_t^2 - \widehat{\mathrm{RVIX}}_t^2$,
Unconditional Unbiased Variance Risk Premium:	$\widehat{\mathrm{VRP}}_t = \hat{\mathrm{V}}_t^{BKM} - \widehat{\mathrm{RV}}_t$,
Unconditional Tail Risk Premium:	$\widehat{\mathrm{TRP}}_t = (\widehat{\mathrm{VIX}}_t^2 - \hat{\mathrm{V}}_t^{BKM}) - \widehat{\mathrm{RT}}_t$.

Economically, the return variation risk premium, as shown in (15), (16), and (17), is the difference between the conditional variation using a risk-neutral probability measure and that using the actual physical probability measure. Both the options based estimates of $\hat{\mathrm{V}}_t^{BKM}$ and $\widehat{\mathrm{VIX}}_t$ are risk-neutral conditional measures. Since the empirical projection of the physically realized return variations relies on variables in the information sets, the common approach is to employ

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[11] See the VIX white paper, URL: http://www.cboe.com/micro/vix/vixwhite.pdf.

[12] All variables are annualized whenever appropriate.

forecasting models. Recently, BH (2014) evaluate a plethora of state-of-the-art volatility forecasting models based on the decomposition of the squared VIX index to produce an accurate measure of the conditional variance. We adopt one of the BH's winning models (model 11) as our forecasting model for estimating conditional return variation. BH's Model 11 features continuous and jump variations at three frequencies: 1-day, 5-day, and 22-day, respectively, in that the presence of realized variability at all three frequencies is important in delivering lower error statistics. We present the application of BH's Model 11 to our variables as follows.

We begin with daily measures of RV, RVIX and RT, calculated from 5-minute intraday returns as well as an overnight close-to-open return (79 increments in total per day). They are $\widehat{\mathrm{RV}}_t^{(1)} = \frac{79}{\kappa}\sum_{i=1}^{\kappa} r_i^2$, $\widehat{\mathrm{RVIX}}_t^{2(1)} = \frac{79}{\kappa}\sum_{i=1}^{\kappa} 2(R_i - r_i)$, and $\widehat{\mathrm{RT}}_t^{(1)} = \frac{79}{\kappa}\sum_{i=1}^{\kappa} 2(R_i - r_i) - r_i^2$, respectively, where κ is the actual trading increment. Next, the *h*-day estimate of the continuous as well as the discontinuous components of the quadratic and polynomial variations in (2.7) and (2.10) are calculated: $\mathbb{CV}_t^{(h)} = \left(\frac{22}{h}\sum_{j=1}^{h} \widehat{\mathrm{RV}}_{t-j+1}^{(1)}\right) - \mathbb{JQV}_t^{(h)}$, $\mathbb{JQV}_t^{(h)} = \frac{22}{h}\sum_{j=1}^{h} max\left[\left(\widehat{\mathrm{RV}}_{t-j+1}^{(1)} - \mathrm{TBPV}_{t-j+1}^{(1)}\right), 0\right]$, and $\mathbb{JPV}_t^{(h)} = \frac{22}{h}\sum_{j=1}^{h}\left(\widehat{\mathrm{RVIX}}_{t-j+1}^{2(1)} - \mathbb{CV}_{t-j+1}^{(1)}\right)$, where $\mathrm{TBPV}_t^{(1)}$ stands for the daily threshold bipower variation defined in (Corsi et al. (2010), eq. 2.14). Note that we scale up all measures to the monthly (22-day) basis. Then, three rollover series of continuous and discontinuous sample estimates, daily (*h* = 1), weekly (*h* = 5), and monthly (*h* = 22), accordingly, are used as independent variables for the following forecasting models:

$$
\begin{aligned}
2\left[R_t^{(22)} - r_t^{(22)}\right] = \;& \alpha + \beta^m \mathbb{CV}_{t-22}^{(22)} + \beta^w \mathbb{CV}_{t-22}^{(5)} + \beta^d \mathbb{CV}_{t-22}^{(1)} \\
& + \gamma^m \mathbb{JPV}_{t-22}^{(22)} + \gamma^w \mathbb{JPV}_{t-22}^{(5)} + \gamma^d \mathbb{JPV}_{t-22}^{(1)} + \varepsilon_t,
\end{aligned}
\tag{23}
$$

$$\left[r_t^{(22)}\right]^2 = a + b^m \mathbb{CV}_{t-22}^{(22)} + b^w \mathbb{CV}_{t-22}^{(5)} + b^d \mathbb{CV}_{t-22}^{(1)} + c^m \mathbb{JQV}_{t-22}^{(22)} + c^m \mathbb{JQV}_{t-22}^{(5)} + c^d \mathbb{JQV}_{t-22}^{(1)} + e_t, \tag{24}$$

and

$$2\left[R_t^{(22)} - r_t^{(22)}\right] - \left[r_t^{(22)}\right]^2 = \mathcal{A} + \mathcal{C}^m\left[\mathbb{JPV}_{t-22}^{(22)} - \mathbb{JQV}_{t-22}^{(22)}\right] + \mathcal{C}^w\left[\mathbb{JPV}_{t-22}^{(5)} - \mathbb{JQV}_{t-22}^{(5)}\right] + \mathcal{C}^d\left[\mathbb{JPV}_{t-22}^{(1)} - \mathbb{JQV}_{t-22}^{(1)}\right] + \epsilon_t \tag{25}$$

where $R_t^{(22)}$ and $r_t^{(22)}$ are the monthly rollover arithmetic and logarithmic returns over the time interval $[t-22, t]$, respectively. Consequently, the conditional measures of return variations as well as their risk premiums can be computed using the estimated coefficients from regressions of (23), (24), and (25), accordingly. We summarize the calculation as follows: Let $\overline{\text{RVIX}}_t^2 = \hat{E}_t^{\mathbb{P}}(\text{RVIX}_{t+22}^2)$, $\overline{\text{RV}}_t = \hat{E}_t^{\mathbb{P}}(\text{RV}_{t+22})$, and $\overline{\text{RT}}_t = \hat{E}_t^{\mathbb{P}}(\text{RT}_{t+22})$ be the empirical conditional estimates of next month's return variations.

Conditional VIX Risk Premium:	$\overline{\text{VIXRP}}_t = \frac{1}{12}\widehat{\text{VIX}}_t^2 - \overline{\text{RVIX}}_t^2$, where $\overline{\text{RVIX}}_t^2 = \hat{\alpha} + \hat{\beta}^m \mathbb{CV}_t^{(22)} + \hat{\beta}^w \mathbb{CV}_t^{(5)} + \hat{\beta}^d \mathbb{CV}_t^{(1)} + \hat{\gamma}^m \mathbb{JPV}_t^{(22)} + \hat{\gamma}^w \mathbb{JPV}_t^{(5)} + \hat{\gamma}^d \mathbb{JPV}_t^{(1)}$,
Conditional Unbiased Variance Risk Premium:	$\overline{\text{VRP}}_t = \frac{1}{12}\hat{\mathbb{V}}_t^{BKM} - \overline{\text{RV}}_t$, where $\overline{\text{RV}}_t = \hat{a} + \hat{b}^m \mathbb{CV}_t^{(22)} + \hat{b}^w \mathbb{CV}_t^{(5)} + \hat{b}^d \mathbb{CV}_t^{(1)} + \hat{c}^m \mathbb{JQV}_t^{(22)} + \hat{c}^w \mathbb{JQV}_t^{(5)} + \hat{c}^d \mathbb{JQV}_t^{(1)}$,
Conditional Tail Risk Premium:	$\overline{\text{TRP}}_t = \frac{1}{12}\left(\widehat{\text{VIX}}_t^2 - \hat{\mathbb{V}}_t^{BKM}\right) - \overline{\text{RT}}_t$, where $\overline{\text{RT}}_t = \hat{\mathcal{A}} + \hat{\mathcal{C}}^m\left[\mathbb{JPV}_t^{(22)} - \mathbb{JQV}_t^{(22)}\right] + \hat{\mathcal{C}}^w\left[\mathbb{JPV}_t^{(5)} - \mathbb{JQV}_t^{(5)}\right] + \hat{\mathcal{C}}^d\left[\mathbb{JPV}_t^{(1)} - \mathbb{JQV}_t^{(1)}\right]$.

IV. Empirical Analysis of VIX Decomposition

This section describes data and empirical analysis of our VIX decomposition. Particularly, the focus is on examining the source of the intrinsic market return predictability with respect to different return horizons as well as different decomposed aggregate market portfolios with various types of risk exposures.

A. Data Description

We employ the aggregate S&P 500 composite index as a proxy for the aggregate market portfolio. Our high-frequency data for the S&P 500 index span the period January 2, 1990 to October 10, 2014. The prices are recorded at 5-minute intervals, with the first price for the day at 9:30 A.M. and the last price at 4:00 P.M.[13] Along with the close-to-open overnight return, this leaves us with a total of 79 intraday return observations for each of the 5,979 trading days in the sample. In addition, the daily VIX index is obtained directly from the website of the Chicago Board Options Exchange (CBOE). For calculating V^{BKM}, we use closing bid and ask quotes for all S&P 500 options traded on the CBOE.[14] Further, for analyzing predictive performance of variance and tail risk premium on various size, book-to-market, and momentum sorted portfolios, we downloaded return data from Kenneth R. French's data library.[15] Finally, the data of the control variables on our analytical models are from Compustat and the Federal Reserve Bank dataset and the Federal Reserve Bank of St. Louis website.

[13] The source of our high-frequency data is from Genesis Financial Technologies.

[14] We obtained options data from Ivolatility.com

[15] Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

B. Sample Estimates of the VIX Decomposed Components

Basic summary statistics for the daily, weekly and monthly measures of return variations and risk premiums are provided in Table 1. In addition to ex-post (unconditional) sample estimates, we calculate the daily conditional measures of return variations using the resulting coefficients from the forecasting regressions of (23), (24) and (25) over the full sample as follows:

(26)
$$\overline{\text{RVIX}}_t^2 = \underset{(2.184)}{12.747} - \underset{(0.132)}{0.164}\,\mathbb{CV}_t^{(22)} + \underset{(0.169)}{0.456}\,\mathbb{CV}_t^{(5)} - \underset{(0.089)}{0.071}\,\mathbb{CV}_t^{(1)} + \underset{(0.226)}{0.544}\,\mathbb{JPV}_t^{(22)} + \underset{(0.316)}{0.370}\,\mathbb{JPV}_t^{(5)} - \underset{(0.041)}{0.028}\,\mathbb{JPV}_t^{(1)},$$

(27)
$$\overline{\text{RV}}_t = \underset{(2.212)}{12.752} - \underset{(0.143)}{0.171}\,\mathbb{CV}_t^{(22)} + \underset{(0.180)}{0.487}\,\mathbb{CV}_t^{(5)} - \underset{(0.091)}{0.086}\,\mathbb{CV}_t^{(1)} + \underset{(0.229)}{0.530}\,\mathbb{JQV}_t^{(22)} + \underset{(0.334)}{0.392}\,\mathbb{JQV}_t^{(5)} + \underset{(0.042)}{0.029}\,\mathbb{JQV}_t^{(1)},$$

(28)
$$\overline{\text{RT}}_t = \underset{(0.195)}{-0.234} - \underset{(1.300)}{2.013}\left[\mathbb{JPV}_t^{(22)} - \mathbb{JQV}_t^{(22)}\right] - \underset{(0.890)}{0.608}\left[\mathbb{JPV}_t^{(5)} - \mathbb{JQV}_t^{(5)}\right] + \underset{(0.123)}{0.129}\left[\mathbb{JPV}_t^{(1)} - \mathbb{JQV}_t^{(1)}\right];$$

The heteroscedasticity-robust standard errors are reported in parentheses. Numerically, due to the similarity of scale between RVIX^2 and RV, the magnitude of the realized tail (RT) measures is quite small in that the means of daily, weekly and monthly RT are only -0.172, -0.312, and -0.206 percentage points, respectively.[16] Nevertheless, the significant *t*-statistics for all RT estimates indicate that RVIX^2 is statistically different from RV, and thus the higher order jump (tail) process of market returns cannot be ignored. Implicitly, it shows that market return variability results from two parts: volatility as well as tail variation. Therefore, the risk compensation of market variation can be decomposed by the risk premium of the variance (VRP) and that of the tail (TRP), accordingly. Empirically, both VRP and TRP are statistical non-zero. The mean of variance risk

[16] Bondarenko (2014) also shows the numerical similarity between $\sum_{j=1}^{n} 2(R_j - r_j)$ and $\sum_{j=1}^{n} r_j^2$.

premium is generally negative, but that of the tail risk premium appears to be positive. To illustrate, Figure 1 plots the daily time series of VRP and TRP based on conditional measures. Consistent with empirical evidence in previous studies, the spread between the unbiased implied (risk-neutral, V^{BKM}) and realized variance is generally positive. We show that the spread between the realized and implied tail variation, on the other hand, is mostly negative and seems to be highly and negatively correlated with the variance risk premium. Back in Table 1, conditional and unconditional risk premium measures are alike on average. However, conditional (unconditional) VRP tends to be positively (negatively) skewed. This highlights the potential difference between ex-ante and ex-post approaches in market return predictability analysis.

[Insert TABLE 1 here]

[Insert FIGURE 1 here]

C. Decomposable Goodness-of-Fit Test

To examine fundamental attributions of the four individual components (RV, VRP, RT, and TRP) to the variation of VIX, we employ Klein and Chow (2013, KC hereafter) decomposed *R-square* approach. Based on an optimal simultaneous orthogonal data transformation, KC methodology allows us to identify the underlying uncorrelated components of RV, VRP, RT, and TRP, respectively. Since squared VIX is a linear combination of the four decomposed factors as shown in (19), without losing generality and to avoid the problem of multicollinearity, a multiple factor regression model with orthogonally transformed variables can be set up as follows:

$$\text{(29)} \qquad \mathrm{VIX}_t^2 = \alpha_t + \beta_{RV}^{\perp}\mathrm{RV}_t^{\perp} + \beta_{VRP}^{\perp}\mathrm{VRP}_t^{\perp} + \beta_{RT}^{\perp}\mathrm{RT}_t^{\perp} + \beta_{VRT}^{\perp}\mathrm{TRP}_t^{\perp} + e_t,$$

where $\perp$ denotes variables or coefficients after orthogonal data transformation. Specifically, the inherent components of the four factors retain their original variances before orthogonalization, but their cross-sectional covariances are zero. In addition, the multi-factor regression (29) retains the same coefficient of determination (*R-square*, i.e. the ratio of systematic variation to the overall variability of the VIX) as that using the original, non-orthogonalized factors. Since *R-square* represents a goodness-of-fit of the VIX from data of the four components, disentangling the *R-square*, based on factors' volatility and their corresponding betas, is thus able to determine individually the contribution of to the VIX's variation from different components. Statistically, this decomposition of the *R-square* can be expressed as:

$$
\begin{aligned}
R^2_{\text{VIX}} &= \left(\beta^{\perp}_{\text{RV}}\frac{\sigma_{\text{RV}}}{\sigma_{\text{VIX}}}\right)^2 + \left(\beta^{\perp}_{\text{VRP}}\frac{\sigma_{\text{VRP}}}{\sigma_{\text{VIX}}}\right)^2 + \left(\beta^{\perp}_{\text{RT}}\frac{\sigma_{\text{RT}}}{\sigma_{\text{VIX}}}\right)^2 + \left(\beta^{\perp}_{\text{TRP}}\frac{\sigma_{\text{TRP}}}{\sigma_{\text{VIX}}}\right)^2 \\
&= DR^2_{\text{RV}} + DR^2_{\text{VRP}} + DR^2_{\text{RT}} + DR^2_{\text{TRP}}
\end{aligned}
\tag{30}
$$

where DR^2 denotes the *decomposed R-square*. Further, note that from (19), since the squared VIX is a sum of the four factors, R^2_{VIX} in (30) is one.

Table 2.1 and 2.2 report the empirical results of (29) and (30) with the unconditionally and conditionally daily estimations of RV, VRP, RT and TRP, respectively. To prevent bias results due to measurements at different scales, all variables are standardized for the analyses. Over the sample period January 1993 to September 2014, the (unconditional) realized volatility characterizes more than two-third (66.83 percent) of the VIX daily variation. Notably, the *decomposed R-square* of the TRP is 26.64 percent, which almost four time larger than that of the VRP. The impact of both unconditional and conditional RT on VIX's variability appears to be small. This demonstrates that the investors' required compensation of potential large market movements (tail risk) is a major factor in determining the variation of the VIX.

Nevertheless, as shown in Table 2.2, the influence of conditional RV to the VIX is much smaller than that of unconditional RV, where the *decomposed R-square* drops to 29.8 percent. Since the conditional RV is an ex-ante measure calculated from on the forecasting model (27), the results from both unconditional and conditional analyses (Tables 2.1 and Table 2.2) strongly indicate that although the formulation of the VIX is forward-looking (options) base, the major determinant of the VIX is actually the physically (ex-post) realized volatility.

To further analyze the impact of the decomposed components on the VIX variability under different market conditions, we divide the overall sample into sub-samples based on three different levels of the VIX: (1) Nervous Market Condition for VIX $\geq$ 23.32 (75 percentile), (2) Normal Market Condition for 14.17 $\leq$ VIX < 23.32, and (3) Calm Market Condition for VIX < 14.17 (25 percentile), correspondingly. It appears that the tail risk premium (the variance risk premium) has the largest influence on the VIX determination during Nervous Market Condition (Calm Market Condition). This clearly reveals that the VIX is not only a volatility index but also the market's fear gauge.

V. Stock Return Predictability

Mounting empirical evidence suggests that equity market future returns could be predicted by the long-term variance risk premium, defined as the difference between the risk-neutral and the actual expectations (i.e. VIX^2 – RV), especially over a 3- to 6-month time horizon. BTX (2015) argues that the variance risk premium can be naturally decomposed into two fundamentally different sources of market variance risk: normal size price fluctuations and jump tail risk. Specifically, by differentiating the *left* and *right* (risk-neutral) jump components from the quadratic variation based on a threshold of log-jump size, the part of the variance risk premium associated with compensation for *left* jump (tail) risk may be seen as a proxy for market *fears*. BTX show that the *left* jump (or tail variation) serves as a predictor variable for market future returns. Instead of discriminating the quadratic jump variation between left and right, we measure tail risk based on the spread between the polynomial and quadratic variations (i.e. $\mathbb{TV} = \mathbb{PV} - \mathbb{QV}$).

A. S&P 500 Index Return Predictability

Following the analytical procedures of BTZ and BH, we investigate the relationship between aggregate stock market (the S&P 500 Index) monthly excess returns and a set of lagged predictor variables with a focus on the realized tail and the tail risk premium. The main predictive variables include the four decomposed VIX risk factors: RV, VRP, RT and TRP, respectively. In addition, to ensure the robustness of our analysis, we also include a set of control variables employed by BH that consists of the real 3-month rate (the 3-month T-bill minus CPI inflation, denoted 3MTB), the logarithm of the dividend yield (denoted Log(DY)), the credit spread (the difference between Moody's BAA and AAA bond yield indices, denoted CS) and the term spread (the difference between the ten-year and the 3-month Treasury yields, denoted TS). Table 3 reports two correlation matrices of predictor variables with respect to the unconditional and conditional measures. The realized tail has relatively low cross-sectional correlations with other variables. It ranges from -0.18 (with VRP) to 0.24 (with TRP) for the unconditional RT, and from -0.28 (with TRP) to 0.50 (with RV) for the conditional RT.

[Insert TABLE 3 here]

Our main analytical results of stock market predictability appear in Table 4. We employ the standard approaches of BTZ and BH by regressing excess stock returns (the annualized monthly S&P500 return in excess of the annualized 3-month T-bill rate) against the risk factors described above. All variables, except RT, are expressed in annualized percentages; realized tail is expressed in basis points. The analysis is also based on three different horizons, monthly, quarterly and annual (denoted by 1, 3 and 12, respectively), averaging returns over a quarter/year. To correct for serial correlation, the Newey–West *t* statistics with a relatively large number of lags is adopted. For each Panel of the Table 4, we report the results from simple regressions with respect to each risk variable and their risk premium individually as well as with multiple regressions that consider jointly individual risk factors, its premium and control variables. Panel A reveals monthly return predictability.

There are fairly different outcomes between unconditional and conditional measurements. Based on a conventional ex-post approach of simple historical (unconditional) estimation, individual *t*-statistics for all risk factors (except the realized tail), extending from -2.311 to 3.204, are significant at the 5% level. At monthly prediction horizon, TRP is significant for both unconditional and conditional measures, this result is in line with previous Table 2.1 and 2.2 that TRP is of larger significance among the four decomposed VIX components.

[Insert TABLE 4 here]

Importantly, shown on Panel A of Table 4, almost an opposite result appears when we employ the BH conditional approach. From simple return predictability regressions, the realized tail (RT) and its risk premium (TRP) are the only significant predictors for future monthly market returns. The similar result holds from the multiple variable regression, except that VRP is significant, where Newey-West *t*-statistics of conditional VRP, RT, and TRP regressor coefficients are 3.172, 2.651, and 2.513, respectively. By extending the prediction period from a month to a quarter, Panel B of Table 4 shows that from the regression with multiple control variables, both conditional and unconditional RT still retain their predictive power of stock market returns. However, Panel C of Table 4 reports that both conditional RT and TRP fail to predict stock market returns. Therefore, the tail risk factor and its risk premium have predictive power for stock return over a relatively short period of time. On the other hand, the predictability VRP increases as the time horizon increases from a month to a quarter. In short, the empirical evidence from Table 4 concludes that from multiple regressions including control variables, RV does not predict S&P

500 index returns almost for all time horizons (except unconditional monthly prediction). Nevertheless, the time series conditional tail risk factor and its premium proxy, on the contrary, statistically predict the next month's (and quarter's) stock market returns.

Next, consider that the two decomposed components of the VIX risk premium derived from the polynomial variation (i.e., VRP and TRP) are separate potential predictors of stock market returns. To compare the predictability of VRP with that of TRP, we plot the corresponding Newey-West *t*-statistics and adjusted regression R^2s for all of the 1- through 12-month return regressions in Figure 2. The *t*-statistics from the simple regressions based on unconditional (conditional) VRP are all significant (insignificant), and the R^2s increase with the return horizons. However, the R^2s of the unconditional VRP regression decreases after they reach the maximum value of 10% at the four-month horizon. Consistent with the results in Table 4, the *t*-statistics from the simple regressions based on either unconditional or conditional TRP are significant in the short time horizon (shorter than two-month), and the R^2s decrease with the return horizon. In addition, the adjusted R^2s from the multiple regressions based on both unconditional (conditional) predictor variables are higher but close to those from the simple regressions based on unconditional VRP (conditional TRP) only. In summary, the risk premium of the market return variation contains two components: compensation for economic uncertainty, measured by the VRP, and that for market fear, measured by the TRP. To further examine the sources of the predictability, we follow BTX by analyzing a series of predictability regressions for various style portfolios.

[Insert FIGURE 2 here]

B. Return Predictability of Style Portfolios

Portfolios with different styles represent different risk characteristics and exposures. Therefore, their reaction to a change in aggregate risk and risk-aversion could vary. Table 5 reports the results from multiple regressions based on lagged RV, VRP, TRP, RT, and control variables similar to those in Table 5. The dependent variables are based on monthly excess returns of different style portfolios. The style portfolios are classified by three different risk factors of Fama-French-Carhart: *Size*, *Value*/*Growth* and *Momentum*, accordingly. The six equally weighted portfolios, obtained from the data library of Kenneth R. French, made up of the top and bottom quintiles for each of the three different stock sorts according to their market capitalization, book-to-market (B/M) value, and most recent annual return. The predictability analysis is again based on three different horizons: monthly, quarterly and annual.

The most notable result shown in Table 5 is that neither conditional nor unconditional realized variance (RV) predicts style portfolios for all time horizons. Now, we begin with the analysis relating to the size-sorted portfolios. From the monthly and quarterly results, both unconditional and conditional measures of VRP and those of TRP are significant predictors for the small-stock portfolio. The influence of the conditional realized tail to the small-stock portfolio is insignificant till the predictive time horizon increases to 1 quarter (1-year), where the *t*-statistics of conditional (unconditional) RT reaches 2.967 (2.536). The predictability of a big-stock portfolio mainly comes from the variance risk premium, although conditional TRP and RT show some influence on monthly and quarterly predictability. Further, the zero-cost long-short portfolio of SMB (small minus big) is a proxy portfolio that removes the market systematic risk but retains only the size effect. From Panels A and B of Table 5, in contrast to BTX, we find that the tail risk premium (TRP) contributes to the predictability of the SMB portfolio, where the variance risk premium (VRP) shows no impact on SMB prediction at all.

For the book-to-market sorted value and growth portfolios, both the conditional and unconditional variance risk premiums (tail risk premiums) seem to be significant predictors for the monthly and quarterly (annual) returns on the zero-cost High-Minus-Low (HML) portfolios. The *t*-statistics of conditional TRP and RT predictors for the next month returns on the growth (low Book-to-Market) portfolios are significant at the 5% percent level. But, this tail risk influence on the value portfolios declines as the predictive time horizon increases. Both the VRP and TRP appear to have an impact on the monthly and quarterly return prediction for the value portfolios. Our VIX decomposed measures seem to have relatively low predictability for the returns on the momentum (WML) portfolios. Particularly, none of the *t*-statistics of the quarterly predictive regression coefficients is significant. However, the unconditional VRP and RT as well as the conditional TRP retains some predictive power on the monthly return prediction of the WML portfolios. Both the winner and loser portfolios have some influence from the VRP, TRP and RT.

[Insert TABLE 5 here]

Figure 3 shows the predictability patterns (*t*-statistics and R^2) of VRP (solid lines) and TRP (dashed lines) over time for size, value/growth, and momentum portfolios. Generally, the patterns are similar between unconditional and conditional measures. The impact of TRP (VRP) on SMB appears to be relative short-term (long-term). For the HML portfolios, the predictive power of TRP seems to be much larger than that of VRP, where the R^2s of TRP for the HML portfolio appear to be maximized at the intermediate four-month horizon. Finally, the pattern of increasing (decreasing) predictability from TRP (VRP) on the WML portfolio indicates that the short-term (long-term) predictability of momentum portfolios is attributable to variance (tail) risk premium. In summary, the results of Table 5 and Figure 3 describes that variance and tail risk have various impacts on portfolios with different fundamental risk exposures. In addition to style portfolios, we further investigate the effects of our decomposed VIX premiums to disintegrative equity market portfolio based on different mutually exclusive industrial sectors.

[Insert FIGURE 3 here]

C. Return Predictability of Industrial Portfolios

Table 6 reports results from multiple predictability regressions that include the four conditional measures of the VIX decomposed components (VR, VRP, TRP and RT) as well as all control variables. Once again, the realized variance (RV) has no influence on return predictability for all sector portfolios. The conditional realized tail of the S&P index return distribution (RT), on the other hand, significantly attributes monthly return predictability to industrial sectors of Non-Durables, Chemicals, Equipment, Telecommunication, Utilities, and Wholesale. By extending the predictive time horizon from a month to a quarter, RT has significant impact on eleven of the twelve sectors. Although both VRP and TRP have predictive power for monthly and quarterly returns on some industrial stocks, it is less significant than the predictability of RT. This suggests that the realized jump tail could be an important risk factor in determining future returns on disintegrative market portfolios or even on individual assets. The insignificance of *t*-statistics of all our predictor variables in Panel C of Table 6 suggests that the influence of variance risk and tail risk premium to less diversified market portfolios (e.g., industrial equity funds) occurs only in the relative short run. Interestingly, from our empirical outcomes shown in all Panels of Table 6, returns on Energy stocks appear to be independent from both equity market volatility and jump-tail (fear) risk.

[Insert TABLE 6 here]

VI. Conclusion

We explicitly formulate the price of aggregate *fear* embedded in the VIX index. Instead of quadratic variation, the underlying stochastic process of the VIX index actually follows a polynomial form of market return variation. Based on our notion of the polynomial variation, the

VIX index is composed of four fundamentally different elements: the realized variance (RV), the underlying (unbiased) variance risk premium (VRP), the realized tail (RT), and the tail risk premium (TRP). RV measures the current (normal) volatility of returns, VRP quantifies the risk premium of anticipated (normal) market volatility, RT captures the present (abnormal) jumps of market returns, and TRP compensates the *fear* of potentially (unusual) large market price movements, respectively. In short, the VIX index consists of investors' required compensations to two separately expected market risks: the volatility risk (*normal* price fluctuations from economic uncertainty) and jump-tail risk (potentially and abnormally large price movement caused by investors' *fear*). Empirically, although the daily variation of the VIX index is largely attributed to the contemporarily realized volatility, premiums of both the volatility and tail risk play an important role in formatting the VIX.

Our VIX decomposition also highlights the bias of the conventional measure of variance risk premium (VRP^c; the squared VIX minuses RV) toward the true premium of its underlying variance risk (VRP) in that VRP^c is actually the sum of VRP, RT, and TRP. The question then arises: could the strong predictive power of the popular VRP^c as previously reported in the literature be actually from the predictability of conditional realized tail and that of tail risk premium? We look for the answer by investigating empirically the joint predictive ability of the decomposed VIX components for future returns on the S&P 500, style, and sector portfolios. To insure the accuracy of risk estimation, we employ both the BTZ *unconditional* and the BH *conditional* approaches for calculating RV, VRP, RT, and TRP, respectively. Statistically, our analysis, consistent with previous researchers' findings, also shows that the realized variance (RV) has <u>no</u> predictive power of future market returns. However, the realized tail (RT), on the other hand, has significant influence on market return prediction, particularly, for relatively short time horizons. In addition, both the unbiased premiums of variance risk and tail risk play an important role in predicting future returns on the market, style as well as different sector portfolios. Specifically, the predictability of the zero-cost small-minus-big (*size*) portfolios appears to be driven by the tail risk premium. The variance risk premium has a significant impact on the return prediction of the high-minus-low book-to-market (*growth/value*) portfolios. Nevertheless, the influence of the four VIX decomposed components on return prediction of the winners-minus-losers (*momentum*) portfolios is quite weak. Finally, although none of our VIX decomposed measures has long-term predictive power for forecasting (annual) returns on industrial portfolios, the conditional RT and TRP, particularly, appear to be strong return predictors for monthly and quarterly returns on almost all sector portfolios. Interestingly, the insignificance of all of our predictors for predicting returns on the Energy portfolio demonstrates the unique pricing behavior of Energy stocks from other sectors.

Perceptibly, in spite of the fact that the physical measure of realized tail (RT) is numerically unnoticeable, our empirical evidence reveals that its impact on future returns is statistically significant and should not be ignored. Particularly, the increase in statistical significance from the market indexes to less diversified industrial portfolios indicates that the influence of tail risk on individual stocks could be nontrivial. Therefore, mapping the cross-sectional dynamics of time-varying tail variations in individual asset prices so that the asset pricing model can generate sufficient compensations for investors' fear of potential disasters becomes a consequential line of further research.

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TABLE 1
Summary Statistics

This table reports descriptive statistics for our realized volatility (i.e., RV), realized tail (i.e., RT) as well as both the conditional and unconditional (annualized) risk premiums of the variance (i.e., VRP) and these of the jump-tail (i.e., TRP) with respect to no overlapping 1-day, 5-day (weekly) and 22-day (monthly) time horizon, respectively. The sample of 5-minute returns of S&P 500 index extends from January 31, 1990, to September 10, 2014. The conditional measures are based on the forecasting models shown in equations (26), (27) and (28), accordingly. All our measures are on daily overlapping basis with 5979 observations in total. RT is in annualized basis point, and all other variables are in annualized percentage. In addition, all numbers are scaled up by a factor of 100.

			Unconditional		*Conditional*	
	RV	RT	VRP*	TRP	VRP*	TRP
Panel A. Daily Measure						
Mean	3.803	-0.172	1.013	-0.179	2.051	-0.156
Std. Dev.	8.716	6.523	6.241	0.740	3.971	0.744
Skewness	11.912	-49.538	-15.959	-5.038	5.014	-5.051
Max	282.739	50.690	25.947	3.606	59.072	3.289
Min	0.099	-429.90	-226.595	-9.521	-5.869	-9.620
t-value	33.611	-2.035	12.505	-18.673	39.791	-16.160
Panel B. Weekly Measure						
Mean	3.943	-0.312	0.901	-0.183	2.054	-0.161
Std. Dev.	7.479	4.770	4.752	0.701	3.784	0.708
Skewness	6.910	-15.493	-8.144	-4.913	4.480	-4.846
Max	109.415	26.492	14.991	1.328	43.839	1.325
Min	0.234	-100.389	-69.897	-7.625	-3.886	-7.545
t-value	18.723	-2.320	6.736	-9.264	19.274	-8.100
Panel C. Monthly Measure						
Mean	3.879	-0.206	0.970	-0.177	2.044	-0.154
Std. Dev.	6.154	1.548	2.840	0.637	3.606	0.641
Skewness	6.616	-7.694	-5.208	-4.600	4.076	-4.515
Max	74.069	4.630	9.610	0.648	31.758	0.490
Min	0.389	-19.569	-30.766	-5.611	-2.166	-5.555
t-value	10.790	-2.276	5.848	-4.752	9.705	-4.122

FIGURE 1
Variance and Tail Risk Premiums

The monthly conditional estimates of VRP and TRP are based on 5-minute sample returns of S&P 500 index extends from January 31, 1990 to September 10, 2014 for a total of 5979 trading days. VRP = V^{BKM} – RV, TRP = (VIX^2– RVIX) – VRP, and the conditional measures are based on the forecasting models shown in equations (26), (27) and (28), accordingly. Both VRP and TRP are reported in annualized percentage and scaled up by a factor of 100.

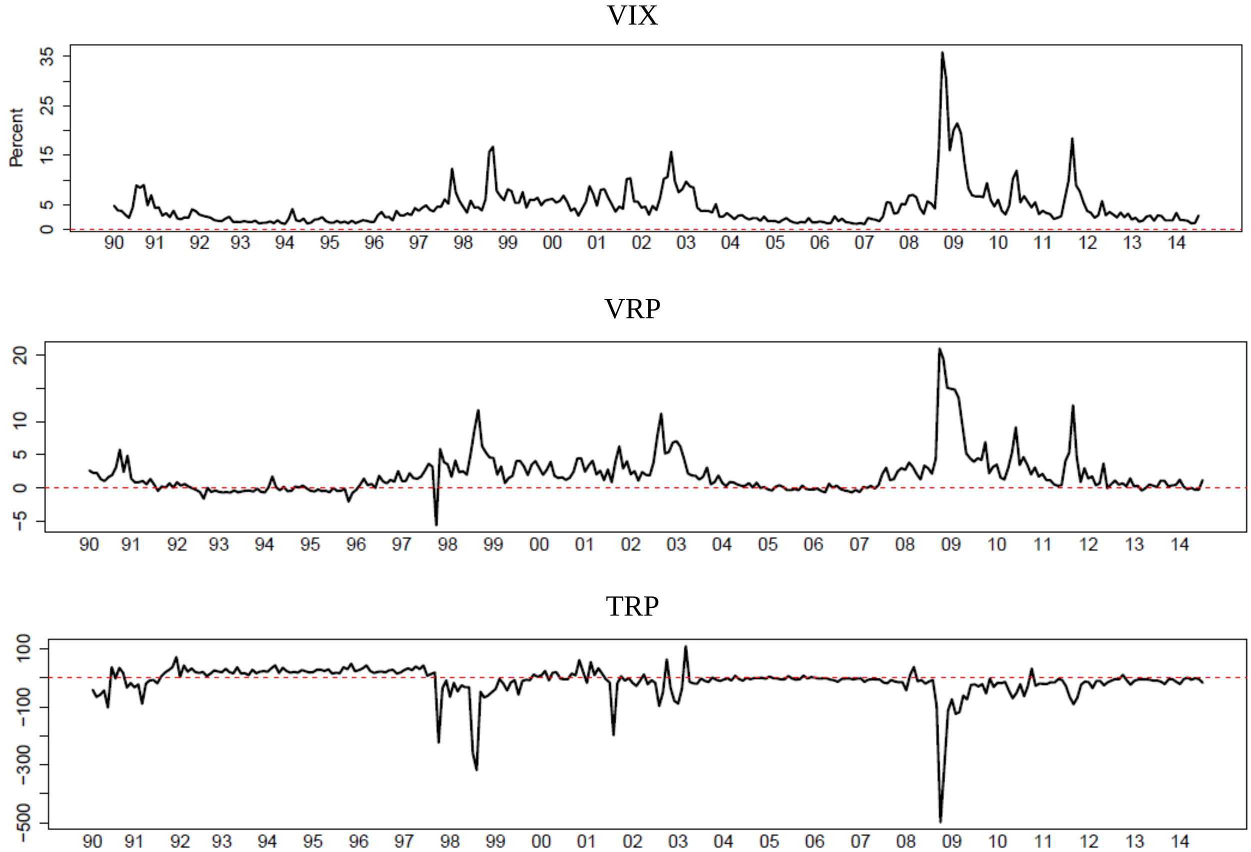

TABLE 2.1
Goodness-of-Fit of the VIX Decomposition (Unconditional Estimates)

This table reports the *decomposed* R^2 of the orthogonalized VIX components, $\text{RV}^{\perp}$, $\text{VRP}^{\perp}$, $\text{RT}^{\perp}$ and $\text{TRP}^{\perp}$, respectively. In addition to the overall sample analysis, we also examine the decomposed goodness of fit from three different sub-samples. The sub-samples are classified by three different levels of the VIX. These include (1) *Nervous Market Condition*: VIX ≥ 23.32 (75 percentile), (2) *Normal Market Condition*: 14.17 ≤ VIX < 23.32, and (3) *Calm Market Condition*: VIX < 14.17 (25 percentile), correspondingly. We employ Klein and Chow (2013) to orthognalize the VIX's decomposed variables and further calculate their *decomposed* R^2. In addition, to avoid bias results due to measurements at different scales, all variables are standardized for the analyses. The overall sample period of daily data ranges from January 1993 to September 2014.

A: Overall Sample	$\mathbf{RV^{\perp}}$	$\mathbf{VRP^{\perp}}$	$\mathbf{RT^{\perp}}$	$\mathbf{TRP^{\perp}}$
Coefficient	0.82	0.26	-0.01	-0.53
Decomposed R^2 (%)	66.83	6.52	0.01	26.64
B: Subsample for VIX ≥ 23.32 (*Nervous Market Condition*)				
Coefficient	0.68	0.16	-0.06	-0.49
Decomposed R^2 (%)	63.70	3.84	0.12	32.35
C: Subsample for 14.17 ≤ VIX < 23.32 (*Normal Market Condition*)				
Coefficient	0.34	0.16	-0.00	-0.05
Decomposed R^2 (%)	69.93	28.37	0.04	1.67
D: Subsample for VIX < 14.17 (*Calm Market Condition*)				
Coefficient	0.55	0.29	-0.02	0.02
Decomposed R^2 (%)	65.81	33.32	0.01	0.87

TABLE 2.2

Goodness-of-Fit of the VIX Decomposition (Conditional Estimates)

This table reports the *decomposed* R^2 of the four orthogonalized VIX components, $RV^{\perp}$, $VRP^{\perp}$, $RT^{\perp}$ and $TRP^{\perp}$, respectively. In addition to the overall sample analysis, we also examine the decomposed goodness of fit from three different sub-samples. The sub-samples are classified by three different levels of the VIX. These include (1) *Nervous Market Condition*: VIX ≥ 23.32 (75 percentile), (2) *Normal Market Condition*: 14.17 ≤ VIX < 23.32, and (3) *Calm Market Condition*: VIX < 14.17 (25 percentile), correspondingly. We employ Klein and Chow (2013) to orthognalize the VIX's decomposed variables and further calculate their *decomposed* R^2. In addition, to avoid bias results due to measurements at different scales, all variables are standardized for the analyses. The conditional measures are based on the forecasting models shown in equations (26), (27) and (28), accordingly. The overall sample period of daily data ranges from January 1993 to September 2014.

A: Overall Sample	$\mathbf{RV^{\perp}}$	$\mathbf{VRP^{\perp}}$	$\mathbf{RT^{\perp}}$	$\mathbf{TRP^{\perp}}$
Coefficient	0.55	0.76	0.03	-0.34
Decomposed R^2 (%)	29.80	58.37	0.08	11.74
B: Subsample for VIX ≥ 23.32 (*Nervous Market Condition*)				
Coefficient	0.46	0.71	-0.01	-0.34
Decomposed R^2 (%)	28.82	55.24	0.01	15.93
C: Subsample for 14.17 ≤ VIX < 23.32 (*Normal Market Condition*)				
Coefficient	0.34	0.68	-0.00	-0.02
Decomposed R^2 (%)	22.86	76.94	0.00	0.20
D: Subsample for VIX < 14.17 (*Calm Market Condition*)				
Coefficient	0.29	0.62	-0.01	0.04
Decomposed R^2 (%)	24.70	72.94	0.00	2.36

TABLE 3
Correlation Matrices

The table depicts pairwise correlations for monthly non-overlapping measures of variation (i.e., VIX, RV, V^{BKM}, and RT), these of risk premiums (i.e., VRP and, TRP) as well as these of our control variables including 3MTB (3-month T-bill minus CPI inflation), Log(DY), the log-dividend yield, CS (the spread between Moody's BAA and AAA bond yield), and TS (the spread between 10-year and 3-month Treasury yields), respectively. The sample period extends from January 31, 1990 to September 10, 2014.

		Unconditional							
		RV	VRP	RT	TRP	3MTB	log(DY)	CS	TS
Unconditional	VIX	0.86	0.35	-0.18	-0.69	-0.17	0.12	0.66	0.09
	RV	1.00	-0.17	-0.10	-0.71	-0.15	0.16	0.59	0.08
	VRP		1.00	-0.18	-0.13	-0.06	-0.04	0.20	0.02
	RT			1.00	0.24	-0.06	0.04	0.09	0.08
	TRP				1.00	0.18	-0.23	-0.49	-0.04
	3MTB					1.00	-0.66	-0.43	-0.72
	log(DY)						1.00	0.31	0.08
	CS							1.00	0.31

		Conditional							
		RV	VRP	RT	TRP	3MTB	log(DY)	CS	TS
Conditional	VIX	0.80	0.93	0.15	-0.69	-0.17	0.12	0.66	0.09
	RV	1.00	0.53	0.50	-0.71	-0.07	0.08	0.43	0.03
	VRP		1.00	-0.09	-0.62	-0.21	0.14	0.68	0.11
	RT			1.00	-0.28	0.08	-0.04	-0.10	-0.08
	TRP				1.00	0.17	-0.22	-0.46	-0.03
	3MTB					1.00	-0.66	-0.43	-0.72
	log(DY)						1.00	0.31	0.08
	CS							1.00	0.31

Table 4

S&P 500 Return Predictability Regressions

This table reports the estimated regression coefficients and adjusted R^2's from return predictability regressions for monthly, quarterly as well as annual excess returns on the S&P 500 market portfolio, respectively. RV is the realized variance, VRP is the unbiased variance risk premium, RT is the realized tail, and TRP is the tail risk premium. The term 3MTB is the 3-month T-bill minus CPI inflation, Log(DY) is the log-dividend yield, CS is the spread between Moody's BAA and AAA bond yield, and TS is the term spread between 10-year and 3-month Treasury yields. The sample extends from January 31, 1990 to September 10, 2014. Newey-West *t*-statistics are reported in parentheses. *Adj.* R^2 is the adjusted coefficient of determination. RT is in annualized basis point, and all other variables are measured by annualized percentage.

Panel A. Monthly Return Prediction

Unconditional												
VIX	-0.281									0.523		
	(-0.221)									(0.408)		
RV		-1.788									1.668	
		(-2.311)									(1.260)	
VRP			4.598								6.597	
			(3.204)								(4.756)	
RT				0.110							0.543	
				(0.036)							(0.132)	
TRP					16.164						26.064	
					(2.448)						(1.961)	
Conditional												
RV						-2.127						-2.207
						(-0.835)						(-0.660)
VRP							-0.052					5.168
							(-0.033)					(3.172)
RT								0.484				2.376
								(2.056)				(2.651)
TRP									15.155			29.260
									(2.470)			(2.513)
3MTB										-1.784	1.260	0.769
										(-0.358)	(0.328)	(0.195)
Log(DY)										-5.023	15.831	12.016
										(-0.224)	(0.843)	(0.656)
CS										-13.477	-12.808	-12.593
										(-0.911)	(-0.923)	(-0.806)
TS										-0.970	2.674	2.126
										(-0.140)	(0.503)	(0.388)
Constant	5.618	8.850	-5.819	4.341	6.600	10.243	4.430	5.489	6.113	36.084	-52.486	-19.072
	(1.126)	(3.390)	(-1.543)	(1.342)	(2.409)	(1.599)	(1.469)	(1.692)	(2.174)	(0.392)	(-0.680)	(-0.253)
Adj. R^2*(%)*	-0.290	2.022	4.034	-0.343	2.573	0.405	-0.343	0.183	2.341	-1.046	6.766	4.602

Panel B. Quarterly Return Prediction

Unconditional												
VIX	0.086									0.720		
	(0.106)									(0.732)		
RV		-1.149									0.480	
		(-2.588)									(0.630)	
VRP			4.096								4.765	
			(6.855)								(6.228)	
RT				-2.288							-0.910	
				(-1.940)							(-0.923)	
TRP					5.501						8.678	
					(0.898)						(1.707)	
Conditional												
RV						-0.937						-2.673
						(-0.457)						(-1.668)
VRP							0.401					3.553
							(0.288)					(3.313)
RT								0.562				1.779
								(1.631)				(3.699)
TRP									4.851			10.824
									(0.795)			(2.161)
3MTB										-3.582	-1.043	-1.612
										(-0.929)	(-0.346)	(-0.480)
Log(DY)										-12.740	1.878	-2.160
										(-0.637)	(0.109)	(-0.123)
CS										-11.194	-9.818	-10.985
										(-0.814)	(-0.799)	(-0.806)
TS										-3.787	-0.461	-1.139
										(-0.720)	(-0.107)	(-0.241)
Constant	3.878	7.198	-4.776	4.011	5.049	6.889	3.488	5.621	4.847	64.995	1.279	33.498
	(1.114)	(4.059)	(-1.516)	(1.489)	(1.759)	(1.330)	(1.220)	(1.841)	(1.671)	(0.824)	(0.020)	(0.500)
Adj. R^2(%)	-0.332	2.356	9.265	0.192	0.589	0.056	-0.177	0.253	0.416	0.084	10.200	4.820

Panel C. Annual Return Prediction

Unconditional												
VIX	0.460									0.353		
	(1.327)									(0.820)		
RV		0.057									-0.112	
		(0.193)									(-0.177)	
VRP			1.452								1.144	
			(3.099)								(2.079)	
RT				-0.125							0.764	
				(-0.144)							(0.735)	
TRP					-2.897						-3.594	
					(-1.369)						(-0.755)	
Conditional												
RV						0.416						-0.767
						(0.904)						(-0.994)
VRP							0.722					0.629
							(1.524)					(0.798)
RT								0.074				0.225
								(0.585)				(0.827)
TRP									-2.813			-2.519
									(-1.380)			(-0.591)
3MTB										-3.007	-2.281	-2.581
										(-0.985)	(-0.788)	(-0.813)
Log(DY)										-20.012	-16.750	-18.599
										(-0.872)	(-0.748)	(-0.793)
CS										2.325	1.466	1.718
										(0.385)	(0.267)	(0.297)
TS										-3.173	-2.039	-2.467
										(-0.682)	(-0.467)	(-0.521)
Constant	2.144	4.153	1.052	4.287	3.892	3.129	2.844	4.479	3.967	72.049	58.536	68.997
	(0.668)	(1.421)	(0.319)	(2.531)	(1.545)	(0.945)	(0.937)	(1.460)	(1.690)	(0.917)	(0.762)	(0.861)
Adj. R^2(%)	0.942	-0.335	3.614	-0.352	0.500	-0.097	1.445	0.3224	0.490	2.976	4.612	2.685

FIGURE 2
S&P 500 Return Predictability Regressions

The left panels show the Newey-West *t*-statistics from the simple return predictability regressions for the S&P 500 portfolio based on the unbiased variance risk premiums VRP (solid line) and the tail risk premium TRP (dashed line), respectively. The right panels depict the corresponding R^2s along with the R^2s from multiple regressions including both VRP and TRP (dotted line). The results shown on the top (bottom) panels are based on unconditional (conditional) measures of risk premiums.

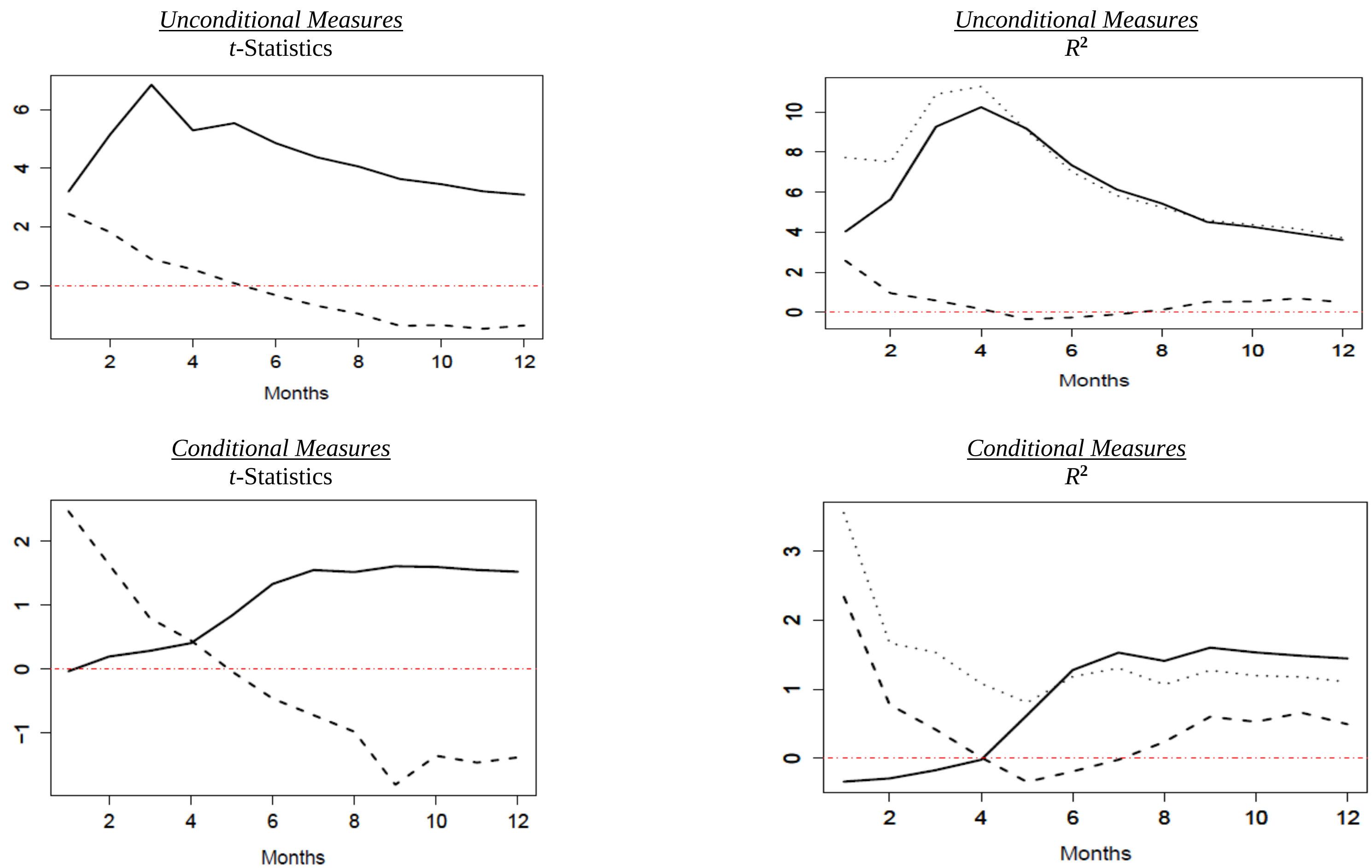

TABLE 5
Style Portfolio Return Predictability Regressions

This table reports the predictability regression results from excess returns on *Size* (20% smallest and biggest firms), *Book-to-Market* (20% highest and lowest B/M ratios), and *Momentum* (20% top and bottom performance), along with the corresponding zero-cost portfolios. All other variables are described in Table 3.

Panel A: Monthly Return Prediction

		Unconditional								
	Constant	RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj. R^2(%)*
Small	3.804	0.110	0.568	3.133	0.504	-0.326	-1.101	0.265	-0.059	6.238
	(0.429)	(0.805)	(3.236)	(2.207)	(1.049)	(-0.699)	(-0.491)	(0.201)	(-0.098)	
Big	-4.792	0.152	0.599	1.957	0.003	0.147	1.464	-1.187	0.273	6.797
	(-0.776)	(1.331)	(4.930)	(1.737)	(0.011)	(0.463)	(0.985)	(-1.029)	(0.604)	
SMB	8.596	-0.042	-0.032	1.176	0.501	-0.474	-2.565	1.453	-0.332	3.707
	(1.479)	(-0.390)	(-0.215)	(1.774)	(2.017)	(-1.369)	(-1.810)	(1.776)	(-0.669)	
High	0.098	0.085	0.447	2.300	0.195	-0.124	0.259	-0.586	0.028	4.213
	(0.014)	(0.763)	(3.029)	(1.986)	(0.476)	(-0.313)	(0.148)	(-0.391)	(0.057)	
Low	-8.822	0.140	0.701	1.821	0.020	0.376	2.289	-0.761	0.527	7.276
	(-1.290)	(0.995)	(4.935)	(1.479)	(0.074)	(0.990)	(1.378)	(-0.742)	(0.937)	
HML	8.920	-0.054	-0.255	0.479	0.175	-0.499	-2.030	0.174	-0.499	2.693
	(2.071)	(-0.658)	(-2.164)	(0.769)	(0.742)	(-1.894)	(-2.025)	(0.213)	(-1.094)	
Winners	-2.819	0.021	0.557	1.124	0.270	0.132	1.048	-1.600	0.472	4.932
	(-0.452)	(0.202)	(4.338)	(1.100)	(0.920)	(0.438)	(0.698)	(-1.684)	(0.909)	
Losers	-2.436	0.318	1.080	4.450	0.141	-0.265	0.114	0.779	-0.069	8.492
	(-0.191)	(1.577)	(4.622)	(2.419)	(0.317)	(-0.382)	(0.036)	(0.309)	(-0.079)	
WML	-0.384	-0.297	-0.523	-3.326	0.129	0.398	0.934	-2.378	0.540	5.392
	(-0.040)	(-1.836)	(-2.566)	(-2.507)	(0.460)	(0.773)	(0.385)	(-1.261)	(0.777)	
		Conditional								
		RV	VRP	TRP	RT					
Small	6.534	-0.205	0.406	3.561	0.144	-0.379	-1.465	0.428	-0.123	4.461
	(0.755)	(-0.662)	(2.436)	(2.920)	(1.587)	(-0.831)	(-0.660)	(0.286)	(-0.213)	
Big	-1.564	-0.161	0.455	2.251	0.203	0.087	1.053	-1.186	0.200	3.906
	(-0.257)	(-0.555)	(3.513)	(2.330)	(2.909)	(0.264)	(0.730)	(-0.916)	(0.431)	
SMB	8.098	-0.044	-0.049	1.309	-0.059	-0.465	-2.518	1.614	-0.324	3.142
	(1.413)	(-0.198)	(-0.338)	(1.978)	(-0.883)	(-1.298)	(-1.751)	(1.836)	(-0.622)	
High	2.736	-0.101	0.293	2.628	0.127	-0.196	-0.152	-0.493	-0.066	2.502
	(0.381)	(-0.249)	(1.850)	(2.781)	(1.105)	(-0.474)	(-0.089)	(-0.282)	(-0.127)	
Low	-5.075	-0.331	0.560	2.130	0.254	0.331	1.893	-0.828	0.481	4.073
	(-0.779)	(-0.895)	(3.529)	(1.754)	(3.001)	(0.913)	(1.244)	(-0.677)	(0.912)	
HML	7.810	0.231	-0.268	0.498	-0.128	-0.526	-2.046	0.335	-0.547	2.716
	(1.879)	(1.200)	(-2.964)	(0.830)	(-1.897)	(-1.915)	(-2.046)	(0.500)	(-1.202)	
Winners	1.053	-0.267	0.344	1.566	0.145	0.031	0.457	-1.558	0.342	1.266
	(0.153)	(-0.885)	(2.437)	(1.617)	(2.004)	(0.080)	(0.288)	(-1.266)	(0.587)	
Losers	2.275	-0.386	0.913	4.873	0.379	-0.298	-0.308	0.739	-0.090	6.781
	(0.197)	(-0.585)	(2.949)	(2.941)	(1.964)	(-0.452)	(-0.115)	(0.217)	(-0.101)	
WML	-1.222	0.118	-0.569	-3.307	-0.235	0.329	0.765	-2.297	0.431	5.860
	(-0.126)	(0.374)	(-2.967)	(-2.513)	(-3.070)	(0.664)	(0.293)	(-1.230)	(0.652)	

Panel B: Quarterly Return Prediction

	Constant	RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj. R^2(%)*
		Unconditional								
Small	7.424	0.111	0.458	2.142	0.041	-0.501	-2.061	0.527	-0.396	8.769
	(0.881)	(1.342)	(4.690)	(3.478)	(0.229)	(-1.063)	(-0.967)	(0.408)	(-0.654)	
Big	-0.507	0.041	0.412	0.471	-0.101	-0.026	0.350	-0.855	0.029	10.240
	(-0.096)	(0.611)	(5.910)	(1.084)	(-1.164)	(-0.104)	(0.252)	(-0.868)	(0.078)	
SMB	7.931	0.071	0.046	1.671	0.141	-0.475	-2.410	1.383	-0.425	9.358
	(1.569)	(0.901)	(0.509)	(3.049)	(1.168)	(-1.577)	(-1.946)	(2.117)	(-0.945)	
High	3.051	0.034	0.320	1.617	-0.200	-0.267	-0.610	0.122	-0.239	7.480
	(0.507)	(0.468)	(3.833)	(3.296)	(-1.546)	(-0.830)	(-0.388)	(0.086)	(-0.579)	
Low	-3.182	0.057	0.466	0.148	-0.054	0.142	0.900	-0.716	0.217	9.634
	(-0.564)	(0.776)	(5.800)	(0.303)	(-0.527)	(0.499)	(0.626)	(-0.757)	(0.509)	
HML	6.233	-0.023	-0.146	1.469	-0.146	-0.408	-1.511	0.839	-0.456	11.250
	(1.710)	(-0.326)	(-1.932)	(2.866)	(-1.505)	(-1.912)	(-1.721)	(1.286)	(-1.136)	
Winners	1.331	0.004	0.365	0.337	0.019	-0.049	-0.048	-1.325	0.236	6.655
	(0.246)	(0.059)	(4.030)	(0.646)	(0.186)	(-0.170)	(-0.035)	(-1.482)	(0.533)	
Losers	6.077	0.118	0.713	1.757	-0.178	-0.613	-2.145	1.750	-0.689	13.910
	(0.545)	(1.041)	(4.094)	(2.082)	(-0.677)	(-1.127)	(-0.733)	(0.823)	(-0.981)	
WML	-4.746	-0.114	-0.348	-1.420	0.197	0.564	2.097	-3.075	0.924	16.450
	(-0.591)	(-1.078)	(-1.746)	(-1.664)	(0.884)	(1.370)	(1.040)	(-2.206)	(1.622)	
		Conditional								
		RV	VRP	TRP	RT					
Small	9.976	-0.129	0.352	2.345	0.136	-0.551	-2.400	0.459	-0.457	6.659
	(1.193)	(-0.734)	(3.403)	(4.404)	(2.967)	(-1.204)	(-1.090)	(0.338)	(-0.784)	
Big	2.268	-0.242	0.313	0.643	0.155	-0.071	0.013	-0.976	-0.024	4.543
	(0.414)	(-1.759)	(3.304)	(1.398)	(3.454)	(-0.257)	(0.009)	(-0.890)	(-0.060)	
SMB	7.709	0.113	0.039	1.703	-0.018	-0.479	-2.413	1.435	-0.433	9.289
	(2.200)	(0.798)	(0.458)	(3.835)	(-0.479)	(-2.280)	(-2.757)	(2.710)	(-1.346)	
High	5.899	-0.051	0.180	1.809	0.123	-0.361	-1.106	0.071	-0.365	4.082
	(1.022)	(-0.262)	(1.800)	(3.084)	(2.604)	(-1.119)	(-0.746)	(0.049)	(-0.851)	
Low	-0.382	-0.300	0.378	0.320	0.167	0.112	0.612	-0.860	0.188	4.895
	(-0.066)	(-1.867)	(3.477)	(0.642)	(3.261)	(0.357)	(0.417)	(-0.817)	(0.409)	
HML	6.281	0.249	-0.198	1.489	-0.043	-0.472	-1.718	0.931	-0.553	13.020
	(1.844)	(2.369)	(-2.354)	(2.945)	(-1.477)	(-2.251)	(-2.046)	(1.588)	(-1.436)	
Winners	3.881	-0.271	0.266	0.533	0.132	-0.090	-0.359	-1.401	0.188	3.140
	(0.666)	(-1.925)	(2.565)	(1.196)	(3.226)	(-0.289)	(-0.247)	(-1.321)	(0.396)	
Losers	11.109	-0.200	0.498	2.078	0.208	-0.742	-2.907	1.533	-0.856	8.772
	(1.558)	(-0.578)	(3.083)	(2.121)	(1.983)	(-1.915)	(-1.656)	(0.988)	(-1.652)	
WML	-7.228	-0.072	-0.231	-1.545	-0.076	0.652	2.549	-2.934	1.043	14.560
	(-0.852)	(-0.354)	(-1.314)	(-1.629)	(-0.954)	(1.517)	(1.101)	(-1.868)	(1.738)	

Panel C: Annual Return Prediction

		Unconditional								
	Constant	RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj. R^2(%)*
Small	12.109	0.071	0.169	0.270	0.229	-0.521	-3.518	0.908	-0.544	21.380
	(1.698)	(1.519)	(2.548)	(0.819)	(2.536)	(-1.828)	(-1.697)	(1.962)	(-1.402)	
Big	4.090	-0.013	0.102	-0.393	0.050	-0.132	-1.127	0.096	-0.097	4.825
	(0.670)	(-0.236)	(2.202)	(-0.976)	(0.638)	(-0.567)	(-0.637)	(0.214)	(-0.266)	
SMB	8.019	0.084	0.068	0.663	0.178	-0.389	-2.390	0.812	-0.447	19.060
	(2.279)	(1.689)	(1.035)	(2.352)	(2.613)	(-2.348)	(-2.655)	(1.608)	(-1.488)	
High	5.118	0.009	0.025	0.201	0.030	-0.166	-1.545	0.942	-0.154	5.108
	(0.750)	(0.191)	(0.421)	(0.594)	(0.291)	(-0.558)	(-0.784)	(1.730)	(-0.378)	
Low	1.883	-0.009	0.149	-0.582	0.084	-0.014	-0.602	0.157	0.012	8.814
	(0.326)	(-0.154)	(3.234)	(-1.438)	(1.012)	(-0.061)	(-0.365)	(0.376)	(0.030)	
HML	3.236	0.018	-0.124	0.783	-0.054	-0.152	-0.943	0.785	-0.166	10.510
	(0.869)	(0.390)	(-2.266)	(2.594)	(-0.840)	(-0.722)	(-1.051)	(2.143)	(-0.378)	
Winners	4.437	-0.035	0.144	-0.503	0.065	-0.087	-1.224	-0.261	0.154	10.170
	(0.657)	(-0.625)	(3.014)	(-1.192)	(0.866)	(-0.339)	(-0.633)	(-0.583)	(0.365)	
Losers	11.586	0.050	0.112	-0.382	0.127	-0.578	-3.814	2.101	-0.597	26.440
	(1.200)	(0.622)	(1.347)	(-0.682)	(1.150)	(-1.570)	(-1.338)	(3.553)	(-1.188)	
WML	-7.149	-0.085	0.032	-0.121	-0.061	0.491	2.590	-2.363	0.751	37.160
	(-1.231)	(-1.510)	(0.443)	(-0.383)	(-0.922)	(1.982)	(1.574)	(-4.887)	(2.353)	
		Conditional								
		RV	VRP	TRP	RT					
Small	13.353	0.125	0.079	0.442	-0.041	-0.591	-3.870	0.998	-0.651	19.820
	(1.769)	(1.873)	(1.042)	(1.621)	(-1.816)	(-1.932)	(-1.770)	(2.085)	(-1.537)	
Big	5.058	-0.074	0.055	-0.300	0.023	-0.159	-1.296	0.113	-0.136	2.579
	(0.794)	(-1.088)	(0.821)	(-0.823)	(1.036)	(-0.621)	(-0.699)	(0.233)	(-0.344)	
SMB	8.295	0.199	0.023	0.741	-0.064	-0.432	-2.574	0.885	-0.515	20.370
	(2.360)	(2.419)	(0.317)	(3.019)	(-2.689)	(-2.580)	(-2.854)	(1.899)	(-1.760)	
High	5.590	0.060	-0.012	0.259	-0.011	-0.199	-1.698	0.994	-0.204	5.213
	(0.787)	(1.097)	(-0.171)	(0.840)	(-0.482)	(-0.632)	(-0.826)	(1.721)	(-0.471)	
Low	3.209	-0.091	0.085	-0.451	0.027	-0.052	-0.835	0.179	-0.042	4.902
	(0.529)	(-1.209)	(1.226)	(-1.219)	(1.210)	(-0.203)	(-0.481)	(0.381)	(-0.098)	
HML	2.381	0.151	-0.096	0.710	-0.039	-0.147	-0.863	0.814	-0.162	8.095
	(0.634)	(2.258)	(-1.773)	(2.451)	(-2.401)	(-0.676)	(-0.927)	(2.104)	(-0.360)	
Winners	6.120	-0.103	0.060	-0.346	0.031	-0.143	-1.545	-0.229	0.074	5.520
	(0.846)	(-1.310)	(0.849)	(-0.956)	(1.278)	(-0.493)	(-0.748)	(-0.458)	(0.158)	
Losers	12.340	0.074	0.057	-0.273	-0.021	-0.619	-4.021	2.173	-0.659	26.040
	(1.231)	(0.763)	(0.589)	(-0.524)	(-0.734)	(-1.573)	(-1.351)	(3.343)	(-1.218)	
WML	-6.220	-0.177	0.002	-0.073	0.052	0.476	2.475	-2.402	0.733	36.110
	(-1.053)	(-1.662)	(0.035)	(-0.260)	(1.950)	(1.835)	(1.473)	(-4.643)	(2.149)	

FIGURE 3
Sorted Zero-Cost Style Portfolio Return Predictability Regressions

This figure depicts the Newey-West *t*-statistics and the corresponding R^2s from simple return predictability regressions for the sorted zero-cost style portfolios based on the unbiased variance risk premiums VRP (solid lines) and the tail risk premium TRP (dashed lines), respectively. The dotted lines are the *t*-statistics and adjusted R^2s from multiple regressions including both VRP and TRP. SMB, HML and WML stands for Small Minus Big, High Minus Low, and Winners Minus Losers, accordingly.

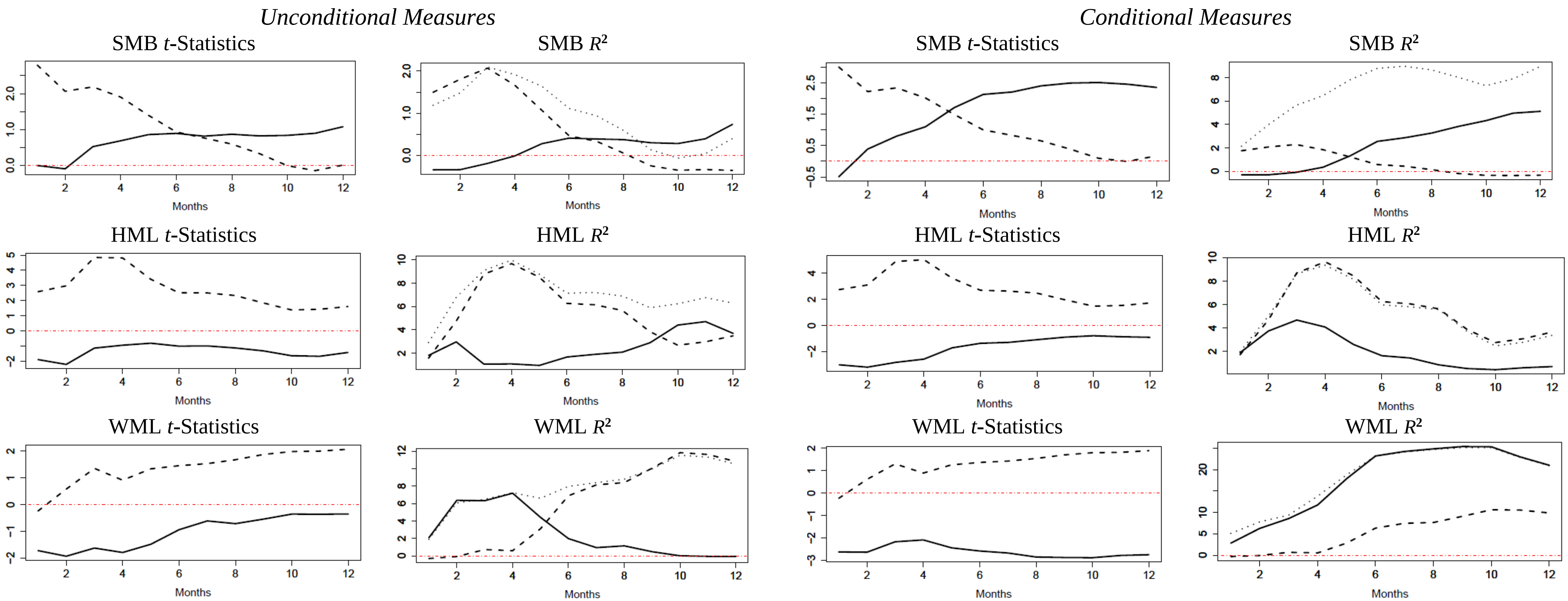

TABLE 6
Industry Portfolio Return Predictability Regressions

This table reports the predictability regression results from excess returns on twelve industry portfolios. All other variables are described in Table 3. We employ the data directly from the Fama-French Research Library.

Panel A: Monthly Return Prediction

		Conditional								
	Constant	RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj. R^2(%)*
Non-Durables	-2.939	-0.230	0.229	1.706	0.213	0.202	1.099	0.408	0.271	2.619
	(-0.576)	(-1.063)	(1.584)	(1.942)	(3.556)	(0.696)	(0.893)	(0.403)	(0.745)	
Durables	4.102	-0.560	0.751	3.806	0.269	-0.252	-0.655	0.033	0.182	6.054
	(0.395)	(-1.122)	(3.068)	(2.614)	(1.931)	(-0.438)	(-0.262)	(0.015)	(0.251)	
Manufacturing	-0.374	-0.265	0.525	3.126	0.244	0.002	0.704	-0.432	0.154	4.296
	(-0.047)	(-0.584)	(2.466)	(2.460)	(1.925)	(0.004)	(0.374)	(-0.233)	(0.270)	
Energy	4.095	-0.136	0.181	0.700	0.009	-0.286	-0.190	-1.306	-0.344	-1.131
	(0.605)	(-0.461)	(1.258)	(0.822)	(0.137)	(-0.754)	(-0.114)	(-1.533)	(-0.651)	
Chemicals	-2.197	-0.220	0.325	2.197	0.202	0.124	0.986	-0.251	0.417	3.268
	(-0.362)	(-0.601)	(1.791)	(2.466)	(1.986)	(0.346)	(0.697)	(-0.163)	(0.833)	
Equipment	-4.373	-0.364	0.968	3.899	0.350	0.281	2.006	-1.902	0.543	5.055
	(-0.416)	(-0.957)	(4.549)	(2.321)	(4.053)	(0.533)	(0.764)	(-1.246)	(0.615)	
Telecommunications	3.529	0.046	0.404	1.766	0.196	-0.329	-0.080	-1.789	-0.234	1.621
	(0.501)	(0.153)	(2.059)	(1.451)	(2.384)	(-0.895)	(-0.044)	(-1.281)	(-0.405)	
Utilities	-0.912	-0.090	0.026	-0.037	0.108	0.034	0.915	-0.925	0.068	-0.764
	(-0.168)	(-0.482)	(0.223)	(-0.046)	(2.544)	(0.127)	(0.657)	(-0.921)	(0.192)	
Wholesale	-2.329	-0.187	0.509	2.662	0.233	0.108	1.073	-0.439	0.272	4.310
	(-0.325)	(-0.566)	(3.236)	(2.647)	(2.785)	(0.241)	(0.641)	(-0.356)	(0.488)	
Healthcare	-4.089	-0.182	0.264	0.552	0.111	0.325	1.667	-0.831	0.176	-0.599
	(-0.617)	(-0.555)	(1.973)	(0.577)	(1.296)	(0.842)	(1.037)	(-0.742)	(0.316)	
Finance	3.581	-0.096	0.429	3.661	0.183	-0.112	-0.588	-0.463	-0.045	4.746
	(0.433)	(-0.178)	(2.075)	(3.145)	(1.173)	(-0.217)	(-0.309)	(-0.201)	(-0.071)	
Other	-0.372	-0.159	0.507	2.976	0.167	-0.101	0.798	-1.358	0.154	4.674
	(-0.053)	(-0.337)	(2.635)	(2.166)	(1.419)	(-0.259)	(0.491)	(-0.785)	(0.291)	

Panel B: Quarterly Return Prediction										
		Conditional								
	Constant	RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj.* R^2*(%)*
Non-Durables	-2.202	-0.202	0.204	0.752	0.133	0.184	0.889	-0.021	0.269	2.398
	(-0.502)	(-1.524)	(2.711)	(1.916)	(3.636)	(0.724)	(0.827)	(-0.025)	(0.845)	
Durables	10.763	-0.312	0.438	1.904	0.214	-0.637	-2.576	0.642	-0.442	7.689
	(1.221)	(-1.180)	(2.906)	(2.410)	(3.205)	(-1.341)	(-1.144)	(0.364)	(-0.692)	
Manufacturing	5.166	-0.155	0.312	1.435	0.152	-0.297	-0.798	-0.392	-0.268	3.469
	(0.765)	(-0.846)	(3.263)	(2.785)	(2.907)	(-0.848)	(-0.465)	(-0.280)	(-0.584)	
Energy	3.168	-0.116	0.147	0.489	-0.006	-0.223	-0.037	-1.080	-0.308	0.883
	(0.634)	(-0.643)	(1.574)	(0.998)	(-0.117)	(-0.860)	(-0.029)	(-1.225)	(-0.758)	
Chemicals	3.169	-0.176	0.135	0.906	0.119	-0.151	-0.537	0.226	-0.027	2.202
	(0.664)	(-0.855)	(1.309)	(1.886)	(2.140)	(-0.560)	(-0.465)	(0.229)	(-0.073)	
Equipment	3.566	-0.300	0.597	0.866	0.210	-0.097	-0.180	-1.670	-0.004	5.009
	(0.456)	(-1.297)	(3.607)	(1.035)	(2.691)	(-0.235)	(-0.089)	(-1.152)	(-0.006)	
Telecommunications	2.124	-0.140	0.358	0.612	0.178	-0.207	0.289	-1.690	-0.028	5.359
	(0.322)	(-0.836)	(2.344)	(1.006)	(3.905)	(-0.638)	(0.164)	(-1.361)	(-0.053)	
Utilities	-4.198	-0.029	0.132	0.516	0.069	0.192	1.726	-1.296	0.347	1.978
	(-0.896)	(-0.259)	(1.553)	(1.343)	(2.113)	(0.841)	(1.379)	(-1.349)	(1.269)	
Wholesale	3.803	-0.256	0.370	0.670	0.163	-0.171	-0.485	-0.611	-0.094	5.679
	(0.787)	(-1.499)	(3.272)	(1.261)	(3.107)	(-0.538)	(-0.418)	(-0.730)	(-0.231)	
Healthcare	-2.107	-0.183	0.210	-0.171	0.099	0.241	1.136	-0.913	0.096	2.957
	(-0.392)	(-1.439)	(2.306)	(-0.394)	(2.742)	(0.767)	(0.815)	(-0.988)	(0.202)	
Finance	9.651	-0.241	0.290	2.013	0.188	-0.370	-2.253	0.231	-0.457	7.465
	(1.605)	(-0.921)	(2.279)	(2.982)	(2.505)	(-1.025)	(-1.533)	(0.169)	(-0.964)	
Other	5.336	-0.254	0.329	1.324	0.172	-0.367	-0.746	-0.688	-0.265	5.683
	(0.912)	(-1.203)	(3.159)	(2.523)	(3.057)	(-1.116)	(-0.516)	(-0.571)	(-0.610)	

Panel C: Annual Return Prediction

	Constant	*Conditional* RV	VRP	TRP	RT	3MTB	Log(DY)	CS	TS	*Adj. R^2(%)*
Non-Durables	0.170	-0.010	0.043	0.186	0.004	0.072	-0.082	0.447	0.088	1.750
	(0.032)	(-0.213)	(0.834)	(0.916)	(0.237)	(0.305)	(-0.055)	(1.213)	(0.233)	
Durables	6.142	0.015	0.064	0.174	0.030	-0.244	-2.259	1.863	-0.067	17.170
	(0.617)	(0.213)	(0.597)	(0.404)	(1.060)	(-0.593)	(-0.763)	(2.435)	(-0.111)	
Manufacturing	7.961	-0.029	0.034	-0.155	0.000	-0.345	-2.121	0.671	-0.380	6.759
	(1.052)	(-0.579)	(0.508)	(-0.460)	(0.017)	(-1.210)	(-0.944)	(1.215)	(-0.917)	
Energy	5.663	-0.074	0.012	-0.359	-0.017	-0.300	-1.018	-0.484	-0.288	2.595
	(0.969)	(-1.366)	(0.200)	(-1.254)	(-0.863)	(-1.272)	(-0.604)	(-0.962)	(-0.883)	
Chemicals	4.607	-0.029	-0.018	-0.057	0.001	-0.154	-1.353	0.872	-0.143	6.068
	(0.905)	(-0.633)	(-0.326)	(-0.221)	(0.044)	(-0.751)	(-0.916)	(2.250)	(-0.455)	
Equipment	8.948	-0.124	0.129	-0.803	0.037	-0.285	-2.279	-0.144	-0.254	6.417
	(1.146)	(-0.923)	(1.292)	(-1.278)	(1.202)	(-0.925)	(-1.070)	(-0.199)	(-0.393)	
Telecommunications	3.302	-0.142	0.059	-0.609	0.052	-0.166	-0.700	-0.044	0.085	4.184
	(0.422)	(-1.151)	(0.501)	(-1.087)	(1.601)	(-0.473)	(-0.323)	(-0.059)	(0.153)	
Utilities	-2.808	-0.051	0.054	0.040	0.015	0.171	0.956	-0.317	0.358	-0.894
	(-0.466)	(-0.996)	(0.807)	(0.146)	(0.633)	(0.687)	(0.561)	(-0.777)	(1.124)	
Wholesale	2.535	-0.017	0.123	0.015	0.024	-0.064	-0.632	0.084	-0.003	8.430
	(0.478)	(-0.311)	(1.989)	(0.061)	(1.237)	(-0.246)	(-0.420)	(0.180)	(-0.008)	
Healthcare	2.354	-0.009	0.060	-0.002	0.008	-0.025	-0.452	0.124	-0.235	1.360
	(0.414)	(-0.134)	(0.905)	(-0.006)	(0.322)	(-0.083)	(-0.283)	(0.264)	(-0.524)	
Finance	9.994	0.057	0.041	0.371	-0.009	-0.252	-3.104	0.770	-0.292	10.720
	(1.059)	(0.823)	(0.502)	(0.961)	(-0.350)	(-0.622)	(-1.122)	(1.379)	(-0.540)	
Other	5.799	-0.048	0.028	-0.108	0.003	-0.282	-1.681	0.649	-0.152	7.538
	(0.746)	(-0.854)	(0.414)	(-0.319)	(0.112)	(-0.919)	(-0.741)	(1.368)	(-0.352)	