A Jumping Index of Jumping Stocks? An MCMC Analysis of Continuous-Time Models for Individual Stocks

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Abstract This paper examines continuous-time models for the S&P 100 index and its constituents. First, we find that the stylized facts found in the index literature do not carry over to single stocks. Second, parameter estimates for the stochastic processes for single stocks imply pronounced heterogeneity in the cross-section. Third, we find that a jump in the index is not necessarily accompanied by a large number of contemporaneous jumps in its constituents stocks. Consequently, fourth, index jumps can be classified as induced by either a strongly increasing correlation between the returns on individual stocks or by macroeconomic events.

Key Words: Jump-diffusion models; individual stocks; Markov Chain Monte Carlo.

JEL Classifications: G11; G12

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1 Introduction

This paper analyses continuous time jump diffusion models for single stock returns. In particular we are interested in the question of how the dynamics for index constituents differ from the index, and whether a jump in the index is triggered by jumps in the constituents? By answering these questions we contribute to the understanding of return dynamics in stock markets.

The analysis of the statistical properties of stock returns has been one of the main questions in empirical finance research. There is a large body of literature on continuous-time models designed to capture essential features of observed stock price movements, including time-varying variance and jumps, i.e., sudden large movements in prices. Empirical testing of these continuous time equity price models has been at the center of many empirical studies. Starting with models with stochastic volatility (see, e.g. Jacquier, Polson, and Rossi (1994) and Jacquier, Polson, and Rossi (2004)) the literature has evolved to models with jump components in returns (see, e.g. Bakshi, Cao, and Chen (1997) and Pan (2002)) and to models containing jumps in returns and in volatility (see, among others, Eraker, Johannes, and Polson (2003a), Eraker (2004), Broadie, Chernov, and Johannes (2007)).[1]

Empirical investigations of equity asset pricing models have been carried out mainly with major equity market indices like the S&P 500 or the NASDAQ 100 as the main economic object of investigation. From the literature one can identify certain characteristics of stock

[1]Recently, it has been shown that to further improve the models' ability to consistently reproduce stylized facts in the data, it seems helpful to include non-affine terms in the variance process. Examples for papers from this strand of the literature are Christoffersen, Jacobs, and Mimouni (2010), Chourdakis and Dotsis (2011), Mijatovic and Schneider (2014), and Ignatieva, Rodrigues, and Seeger (2015). Nevertheless, for the analysis in this paper we opt to analyze only affine jump diffusion models. We do so for a number of reasons. First, we learn from the above papers that the jump components for non-affine specifications usually do not differ substantially from those for affine models, and since jumps are the key object of our investigation, we do not expect to gain much from using non-affine models. Second, building on the latter argument we do not want to introduce an additional layer of complexity by using non-affine models, but rather go with the cleanest possible setup in which we can still analyze our research question.

price models that can be termed as stylized facts. First, models with just a stochastic volatility component, but without jumps, appear to be significantly mis-specified. Second, when jumps are included in the model, they turn out to be rare, negative and large in absolute value. Third, there is a negative correlation between the innovations in the return and the variance process. Since all these findings were made exclusively for indices, the big question is if they also remain valid for individual stocks. This represents the major motivation for our paper.

In the first step of our analysis we compare estimated model parameters for the S&P 100 index versus with the parameters of the single stock constituents. We employ a pure stochastic volatility model (SV), an SV including jumps in prices (SVJ), and an SV model featuring correlated jumps in the stock price and the conditional variance (SVCJ). Estimation of the models analyzed is based on the Markov Chain Monte Carlo (MCMC) approach developed in Eraker et al. (2003a). At the center of our study lies the analysis of index versus single stock jumps. For this reason we run in a second step a simulation study on the jump models considered in our setup to gain insights on how to properly identify jump days. The simulation study provides us with a posterior jump probability that allows us to cleanly differentiate between jump days and non-jump days. In the main body of our study we analyze the relationship of index jumps and how those are related to the jumps in the single stock constituents of the index.

Our first set of results shows that the stochastic processes for the 'typical' (i.e., average) individual stock is significantly different from that for the market index, in our case represented by the S&P 100. The average size of the jumps in the stock price is smaller in absolute terms than for the index, and jumps in individual stock prices are even in many cases on average positive. The frequency of jumps in prices is more than twice as high for the representative stock as for the index, so, although jumps are still somewhat rare, they tend to occur much more frequently for the average stock than for the S&P 100. Furthermore, the correlation between stock returns and volatility changes is estimated at values which are much less negative than for the index. This last result is, of course, derived under the physical probability measure, but is nevertheless related to empirical findings concerning the pricing of options on indices and individual stocks. As shown by Bollen and Whaley (2004) the implied volatility curves for stock market indices tend to be negatively sloped and much steeper than those for the index component stocks. Also Bakshi, Kapadia, and Madan (2003), Dennis and Mayhew (2002) and Dennis, Mayhew, and Stivers (2006) provide evidence for structural differences in the pricing of index and individual stock options. In line with these results from empirical option pricing research, our findings support the notion that one cannot simply extend the results from the analyses of major equity indices to single stocks.

The main contribution of our empirical analysis concerns the behavior of the individual stocks on days when the index exhibits a jump. Surprisingly, we find index jump days where only relatively few stocks also exhibit jumps. For the SVJ model we find days like February 24, 1994, or January 4, 2000, where the number of jumping stocks is 1. On the other extreme, again for the SVJ model, the highest number of such contemporaneous stock jumps is 75 on October 13, 1989 and 63, observed on October 27, 1997. These findings immediately raise the question of whether the models estimated for the index and the individual stocks are compatible at all. The solution to this apparent 'puzzle' can be found via a detailed analysis of the mechanics of the models employed in our study. A jump is considered likely (in terms of posterior probability), if the price move of an asset on a given day is large relative to the conditional variance. The index as a diversified portfolio of stocks then naturally jumps on days when most of the component stocks exhibit large *returns* (but not necessarily jumps) in the same direction. Since the average local variance of the stocks in our sample is pretty stable around index jump days, at least some of these jumps must be caused by a substantial increase in the correlation between the index component stocks, generating the large and negative index return, which is then identified as a jump. We label these jumps as 'correlation jumps'. The remaining jumps, which we call 'macro-driven jumps', are more interesting from an economic point of view. They are characterized by a large number of stocks or sectors exhibiting high jump probabilities simultaneously, and we can clearly link these jumps to important macroeconomic events.

A further surprising result of our analysis is that jump models unexpectedly do not identify an unusually large number of jumps during the time of the financial crisis in 2008. This is surprising since 13 of the 20 largest daily return movements in our sample occurred during the financial crisis period. We show that this can be explained by the fact that during the financial crisis we observe a prolonged period of high volatility and the large return movement during that time were within two standard deviations of the diffusive volatility component. This implies that these large return movements can be generated by a pure stochastic volatility model without relying on jumps.

An important overall conclusion we draw from our analysis concerning the 'right' model for the index itself and its constituents is the following: even if a pure stochastic volatility model were sufficient to represent the dynamics of the index constituents (single stocks) properly, there would still be strong evidence for jumps in the index dynamics, so that here a jump model seems appropriate.

A study that is related to our paper in terms of analyzing the properties of individual stock returns is Maheu and McCurdy (2004), who estimate the parameters of a discrete-time GARCH model with jumps via maximum likelihood for a rather small set of selected stocks. They attribute jumps to corporate news events and show that this hypothesis is supported by the data. In contrast to their approach we consider a richer set of models by using a SV, a SVJ, and a SVCJ model, and investigate these models for the large cross-section of stocks constituting the S&P 100 index. Jiang and Yao (2008) employ a methodology developed in

Barndorff-Nielsen and Shephard (2004) and analyze a long time series of stock returns to decompose price changes into a continuous and a jump part. Their focus is on cross-sectional return predictability characteristics like size and book-to-market, and their results show that a large part of this predictability is due to differences in the jump part of returns. Şerban, Lehoczky, and Seppi (2008) propose a model where stock returns are driven by a market factor, a 'common idiosyncratic' component and a factor which is truly idiosyncratic to the respective stock. Their findings indicate that the common factor in idiosyncratic volatility is relevant for option pricing. A more recent strand of literature examines co-jumps between stock returns. Examples can be found in Caporin, Kolokolov, and Renò (2014) and Gilder, Shackleton, and Taylor (2014). Our research question differs to these papers in that we analyze the relation between the stock index and its constituents, whereas these paper investigate the relation between single stocks.

The remainder of the paper is structured as follows. In Section 2 we present the model and describe our estimation approach. The results are then discussed in Section 3. Section 4 concludes.

2 Model and Estimation Approach

2.1 Model

Our model specification follows Eraker et al. (2003a). Their approach provides a flexible model structure, which allows for jumps both in prices and the conditional volatility process. The logarithm of the stock price (Y) and the variance (V) are assumed to follow the continuous-time processes

$$dY_t = \mu \, dt + \sqrt{V}_t \, dW_t^y + d\left(\sum_{j=1}^{N_t} \xi_j^y\right) \tag{1}$$

$$dV_t = (\alpha_0 + \alpha_1 V_t) \; dt + \sigma_v \sqrt{V_t} \, dW_t^v + d\left(\sum_{j=1}^{N_t} \xi_j^v\right), \tag{2}$$

where dW_t^y and dW_t^v denote Brownian increments with correlation $E(dW_t^y dW_t^v) = \rho \, dt$. The fact that there is a (empirically mostly negative) correlation between returns and variance innovations is often called the 'leverage effect'. The term μ captures the expected return, and N_t denotes a Poisson process with constant intensity λ. The Poisson process enters both the return and variance equation, thus generating simultaneous jumps. ξ^y and ξ^v denote the jump sizes in returns and variance, respectively, and these jump sizes can be correlated. In more detail, the jump size in variance follows an exponential distribution with parameter μ_v^{-1}, i.e., $\xi_t^v \sim Exp(\mu_v^{-1})$ with expected value and standard deviation equal to μ_v, while, conditional on ξ_t^v, the jump size in the log price at time t follows a normal distribution with mean $\mu_y + \rho_j \xi_t^v$ and variance σ_y^2, i.e., $\xi_t^y | \xi_t^v \sim N\left(\mu_y + \rho_j \xi_t^v, \sigma_y^2\right)$. The variance process follows an affine structure. For the SV model the long-term mean is given by $-\alpha_0/\alpha_1$, whereas for a jump model we have a long-term mean of $(-\alpha_0 - \lambda \mu_v)/\alpha_1$. The speed of mean reversion given by $-\alpha_1$.

The general case represented by the dynamics in Equations (1) and (2) is called the stochastic volatility model with contemporaneous jumps in the return and the conditional variance (SVCJ model). This model turned out to be performing best in the analysis by Eraker et al. (2003a). Besides the general model we also consider two special cases. By setting $\lambda = 0$ the SVCJ model reduces to the well-known stochastic volatility (SV) model proposed by Heston (1993). If we include jumps, but only in returns, we obtain the stochastic volatility model with jumps (SVJ), first considered by Bates (1996). In this case we restrict the jump size in volatility to be identically equal to zero $\xi^v \equiv 0$, implying that jumps in returns are normally distributed with mean μ_y and variance σ_y^2.

To estimate the models, we use an Euler discretization scheme and set the interval at $\Delta = 1$ (day). Denoting the log return of the asset $Y_t - Y_{t-1}$ by R_t, we can write the discretized version of the system in (1) and (2) as

$$\begin{aligned} R_t &= \mu + \sqrt{V_{t-1}}\varepsilon_t^y + \xi_t^y J_t \\ V_t &= V_t + \alpha_0 + \alpha_1 V_{t-1} + \sigma_v \sqrt{V_{t-1}}\varepsilon_t^v + \xi_t^v J_t, \end{aligned} \tag{3}$$

where shocks to returns and volatility, $\varepsilon_t^y = W_t^y - W_{t-1}^y$ and $\varepsilon_t^v = W_t^v - W_{t-1}^v$, follow a bivariate normal distribution with zero expectation, unit variance, and correlation ρ. In the Euler discretization scheme, we assume at most one jump per day. This is not a problem, since, given the observation frequency, we cannot distinguish anyway between situations where multiple (smaller) jumps or only one larger jump have occurred during a day. J_t thus represents an indicator, which is set equal to one in the case of a jump and to zero otherwise. Due to the assumption of contemporaneous jumps in returns and volatility this indicator is of course the same in the dynmics for R and V. The jump sizes retain the distributional assumptions described above.

For technical details concerning the above discretization schemes the reader is referred to Jones (2003b), Eraker, Johannes, and Polson (2003b), Jones (2003a), Ait-Sahalia (1996), and Conley, Hansen, Luttmer, and Scheinkman (1997).

2.2 Estimation Approach

The underlying model setup includes latent variables such as volatility, jump times, and jump sizes. Each of these latent states are treated as a parameter to be estimated in a Bayesian context. This leads to a high dimensional posterior distribution, which is not equal to a known statistical distribution. We therefore rely on Markov Chain Monte Carlo (MCMC) techniques to compute the posterior moments. We report the posterior means and the posterior standard deviations in the estimation results.

In a nutshell, MCMC allows us to draw from a high dimensional distribution by breaking it down into draws from a series of lower dimensional conditional distributions.[2] We are thus able to construct a Markov Chain that converges to the desired posterior distribution. After convergence, we draw N times from that posterior to perform Monte Carlo integration.[3] In the following, we provide a brief overview of the algorithm by using the SVCJ model specification since this yields the most complex structure. For more details on the sampling algorithm we refer to Ignatieva et al. (2015).

According to Bayes Theorem, the posterior distribution of the parameters and the latent states is proportional to the likelihood times the prior distribution

$$p(\boldsymbol{\Theta}, \boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J} | \boldsymbol{R}) \propto p(\boldsymbol{R} | \boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta}) p(\boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta}) \tag{4}$$

where we denote by $\boldsymbol{\Theta} = (\mu, \alpha(V_t), \gamma(V_t), \rho, \mu_y, \sigma_y, \rho_j, \mu_v, \lambda)^\top$ the vector of model parameters, with the expression $\alpha(V_t)$ denoting the parameters entering the drift specifications, and $\gamma(V_t)$ denoting the parameters entering the diffusion specification of the variance. The time series of state variables are collected into $\{\boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J}\}$, and $\boldsymbol{R}$ denotes the time series of observed returns. Note that it is possible to give the prior a hierarchical structure, which is resulting

[2]For a detailed discussion of this algorithm in a financial econometrics context, see Johannes and Polson (2006).

[3]In our empirical analysis we use a burn-in period of 600,000 and then draw 1.4 million times from the posterior distribution. This large number of draws is necessary to ensure convergence in the estimation of the polynomial models. The remaining models will converge after a much smaller number of overall draws.

from our model specifications. Therefore,

$$p(\boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta}) = p(\boldsymbol{V}|\boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta})p(\boldsymbol{\xi}^y|\boldsymbol{\xi}^v, \boldsymbol{\Theta})p(\boldsymbol{\xi}^v|\boldsymbol{\Theta})p(\boldsymbol{J}|\boldsymbol{\Theta})p(\boldsymbol{\Theta}). \tag{5}$$

Given our model framework, the only component of this prior distribution not determined by the model is the component given by $p(\boldsymbol{\Theta})$. We use the same set of independent conjugate priors described in Ignatieva et al. (2015). Given the Markov property of the model we can rewrite the remaining components of the posterior distribution as follows

$$\begin{aligned}
p(\boldsymbol{R}|\boldsymbol{V}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta}) &= \prod_{t=1}^{T} p(R_t|V_t, V_{t-1}, \xi_t^y, \xi_t^v, \boldsymbol{\Theta}) \\
p(\boldsymbol{V}|\boldsymbol{\xi}^v, \boldsymbol{J}, \boldsymbol{\Theta}) &\propto \prod_{t=1}^{T} p(V_t|V_{t-1}, \xi_t^v J_t, \boldsymbol{\Theta}) \\
p(\boldsymbol{\xi}^y|\boldsymbol{\xi}^v, \boldsymbol{\Theta}) &= \prod_{t=1}^{T} p(\xi_t^y|\xi_t^v, \boldsymbol{\Theta}) \\
p(\boldsymbol{\xi}^v|\boldsymbol{\Theta}) &= \prod_{t=1}^{T} p(\xi_t^v|\boldsymbol{\Theta}) \\
p(\boldsymbol{J}|\boldsymbol{\Theta}) &= \prod_{t=1}^{T} p(J_t|\boldsymbol{\Theta})
\end{aligned}$$

The MCMC sampler now samples iteratively through the following complete conditional distributions

$$\begin{aligned}
\text{Parameters}: \quad & p(\boldsymbol{\Theta}_i|\boldsymbol{\Theta}_{-i}, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{V}, \boldsymbol{R}),\ i = 1, \ldots, K \\
\text{Jump times}: \quad & p(J_t|\boldsymbol{\Theta}, \boldsymbol{J}_{-t}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v, \boldsymbol{V}, \boldsymbol{R}),\ t = 1, \ldots, T \\
\text{Jump sizes}: \quad & p(\xi_t^y|\boldsymbol{\Theta}, \boldsymbol{J}, \boldsymbol{\xi}_{-t}^y, \boldsymbol{\xi}^v, \boldsymbol{V}, \boldsymbol{R}),\ t = 1, \ldots, T \\
& p(\xi_t^v|\boldsymbol{\Theta}, \boldsymbol{J}, \boldsymbol{\xi}_{-t}^v, \boldsymbol{\xi}^y, \boldsymbol{V}, \boldsymbol{R}),\ t = 1, \ldots, T \\
\text{Volatility}: \quad & p(V_t|\boldsymbol{\Theta}, V_{t+1}, V_{t-1}, \boldsymbol{J}, \boldsymbol{\xi}^v, \boldsymbol{\xi}^y, R_{t+1}, R_t),\ t = 1, \ldots, T
\end{aligned}$$

where we denote the i-th element of a vector by $\boldsymbol{\Theta}_i$. The remaining parameters in a vector excluding a particular parameter i is indicated by a minus sign in front of the index. That is, $\mathbf{\Theta}_{-i}$ indicates the elements of the vector without the i-th element.

By relying on conjugate priors for the model parameters, we are able to use a Gibbs steps for updating all parameters, jump times, and jump sizes. The only parameters not having a recognizable complete conditional distribution are the variances, denoted by V_t above. The complete conditional distribution for V_t is given by

$$p(V_t|\mathbf{\Theta}, V_{t+1}, V_{t-1}, \boldsymbol{J}, \boldsymbol{\xi}^v, \boldsymbol{\xi}^y, R_{t+1}, R_t) = p(R_{t+1}, V_{t+1}|\mathbf{\Theta}, V_t, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v)p(V_t|\mathbf{\Theta}, R_t, V_{t-1}, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v)$$

where the first component on the right-hand side of the equation denotes a bivariate normal distribution and the second component denotes a univariate normal distribution. The Metropolis-Hastings step proposes a new variance $V_t^{(g)}$ in iteration g by drawing from $p(V_t|\mathbf{\Theta}, R_t, V_{t-1}, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v)$ and accepting that draw with probability

$$\min\left\{\frac{p(R_{t+1}, V_{t+1}|\mathbf{\Theta}, V_{t+1}, V_t^{(g)}, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v)}{p(R_{t+1}, V_{t+1}|\mathbf{\Theta}, V_{t+1}, V_t^{(g-1)}, \boldsymbol{J}, \boldsymbol{\xi}^y, \boldsymbol{\xi}^v)}, 1\right\}$$

which shows that candidate draws are proposed consistent with V_{t-1} and R_t and accepted consistent with V_{t+1} and R_{t+1}.

3 Empirical Analysis

3.1 Data

We obtain stock return data for the period from 1980 to 2014 from the Center for Research in Security Prices (CRSP) database. We use the index constituents file from Compustat to identify the companies which were included in the S&P 100 index on any given day in the sample. The launch date for the S&P 100 index is June 15, 1983, but Compustat provides information on index constituents only from September 1989 onwards. Therefore, we confine the analysis of index jump days to the time period from 1989 on.

In total we find 205 companies that where at one point in time in the S&P 100 index, we matched these companies with the return information provided by CRSP using the cusip identifier. From the 205 companies we are able to unambiguously match 201 by cusip which end up to be the companies used in our analysis. On a daily basis we can match between 92 and 99 stocks with Compustat listing between 94 and 100 stocks per day within the index. This indicates that we are able to almost completely replicate the index constituents with our sample. Table A1 of the online appendix shows the list of companies included in our analysis together with descriptive statistics on the returns as well as information on the estimation period and the period the company was included in the index.

3.2 Parameter Estimates and Model Choice

We will first discuss the parameter estimates for the different models. Our estimation results for the S&P 100 index are comparable, as expected, to those shown in other papers for indices like the S&P 500. This result ensures that the differences between the index constituents and the index can be carried over to a more widely used index like the S&P 500 and are not due to our index selection.

We are interested in the differences of the parameter estimators for the single stocks and the stock index constituted by these stocks. In particular the jump parameters are at the center of our analysis as well as the posterior model odds comparing a pure stochastic volatility model to the jump diffusion specifications.

3.2.1 SV

In Column 1 of Table 1 we can see that, as in basically all empirical studies for the Heston SV model, the correlation between diffusive price changes and diffusive volatility changes is strongly negative for the index with a value of roughly -0.6. For the typical stock, however, ρ is much less negative with a cross-sectional average estimate of -0.27 with 95% percent of the estimates ranging between -0.51 and -0.06. Although we do not analyze options data in this paper, this result for ρ provides support (under the $\mathbb{P}$-measure) for the finding that implied volatility smiles for individual stocks tend to be much flatter than those for the major equity indices around the world (see, e.g. Bollen and Whaley (2004)).

Not surprisingly, the parameter estimators for the single stocks imply a widely differing dynamic structure for the single stocks. The variance parameter α_0 has a mean of 0.0264 for the index whereas the 95% quantile for the single stocks ranges from 0.0367 to 0.3759. The parameter α_1 ranges from -0.0941 to -0.0061, with the value for the index being -0.0216. These parameter values imply large differences in long run mean or speed of mean reversion for the index and the single stocks. For the long run mean of volatility we have an approximate average value of 35% annually for the single stocks versus 17.5% for the index. However, since the parameter α_1 is negative we have mean reverting variances for all return series.

3.2.2 SVJ

Compared to the SV model we now introduce jumps in the price process, which constitutes the SVJ model for which parameter estimates are reported in Table 2. The estimates for α_0, α_1, and ρ as well as the differences between the index and the single stocks are very similar to the SV case, so we will not discuss them in detail.

One of the key parameters in the SVJ model is the mean jump size. A stylized fact from empirical research is that this quantity is negative and large in absolute value for the major equity indices. This is confirmed in our analysis, since μ_y for the S&P 100 is estimated at -2.37 with a standard error of 0.87, i.e. significantly different from zero. In contrast to this, the typical stock exhibits a weakly positive expected jump size (0.79) with a very large cross-sectional variation with 2.5%- and 97.5%-quantiles equal to -1.11 and 3.37, respectively. Looking deeper into the results for the single stocks, we find examples for both significantly positive and significantly negative jump sizes, again evidence for the wide variation in the characteristics of the stochastic processes for the stocks in our sample. These findings clearly show that the estimation results for indices cannot be generalized to individual stocks and that there is no 'law' that jump sizes can only be chosen to be negative in applications of SVJ models. As one might expect the estimated standard deviation of the jump size σ_y, is much smaller for the index (2.76) than for the average stock (5.38). Again this is evidence for the large cross-sectional variation across stocks.

Another key parameter of a jump process is the intensity or, loosely speaking, the probability of a jump over the next time interval (here one day). Again, the differences between the stocks and the index are striking. For the S&P 100 λ is estimated at 0.0051, corresponding to an expected number of roughly 1.26 jumps per year. For the average stock λ is estimated to be about seven times larger (0.0360). Again there is pronounced cross-sectional variation in this estimate for individual stocks with a 2.5%-quantile of 1.43 jumps per year, while the stock representing 97.5%-quantile would on average exhibit 25.9 jumps annually.

3.2.3 SVCJ

Compared to the SVJ case the SVCJ model additionally includes jumps in the local variance process V. In Table 3 we see again that jumps are rare events for the index with an estimate for λ of 0.0057, implying again about 1.4 jumps per year. For the stocks we obtain a cross-sectional average for λ of 0.0267, resulting in an average number of 6.7 jumps annually. As in the cases of the models analyzed above there is considerable variation in these estimates across stocks with 95% of the estimates ranging between roughly 1 and 19 jumps per year.

Similar to the other models the correlation between diffusive price changes and diffusive volatility changes is strongly negative for index (roughly -0.66) and much less negative for the typical stock with a cross-sectional average estimate of -0.37. Nevertheless, this correlations tends to be negative also for the stocks, with a cross-sectional 97.5%-quantile of -0.09.

One has to be careful when interpreting the expected jump size for the price process. Conditional on the current jump size in volatility this is given by $\mu_y + \rho_j \xi_t^V$, which yields an unconditional expectation equal to $\mu_y + \rho_j \mu_v$. For the index this expected value is equal to -2.68, while for the average stock we obtain an estimate of 1.52. Clearly there is again no such thing as *the* representative stock, with both positive and negative mean jump sizes, depending on the stock under consideration. For the remaining parameters we obtain results that are qualitatively very similar to those shown for the SV and SVJ model.

Another way of visualizing structural differences in the parameter estimates for index and single stocks is to use box-plots as given in Figure 1 and 2. These figures give box-plots for the estimated SVJ and SVCJ jump parameters. We see in both graphs that all jump parameters exhibit a huge variation in values across stocks. In particular, however, we observe that the estimated parameters for the index (circled observation) is always located outside the inter-quartile range indicating that the index value is not representative for a typical stock.

3.2.4 Model Choice

Model comparison is undertaken by computing Bayes factors for the models under consideration. Eraker et al. (2003a) showed that the odds ratio for the SV relative to the SVJ can be estimated as

$$\frac{p(\mathrm{SV}|R)}{p(\mathrm{SVJ}|R)} = \frac{B(\alpha_0, \beta_0)}{B(\alpha_0, \beta_0 + T)} \frac{1}{G} \sum_{g=1}^{G} \frac{B(\alpha_0 + \sum_{t=1}^{T} J_t^g, \beta_0 + 2T - \sum_{t=0}^{T} J_t^g)}{B(\alpha_0 + \sum_{t=1}^{T} J_t^g, \beta_0 + T - \sum_{t=0}^{T} J_t^g)}, \tag{6}$$

where G is the number of non-discarded draws in the MCMC sample. We follow Kass and Raftery (1995) and interpret evidence against a model is positive if the log Odds Ratio is in between 2 and 6, strong if it is in between 6 and 10, and very strong if it is larger than 10.

Table 4 shows that for all of the individual stocks as well as for the index, the SV model is rejected against the more flexible alternatives, and that is why we do not consider this model any more in the subsequent analysis. This is a rare exception where the result from the analysis of index returns carries over to the results when analyzing single stocks. The SV model specification is outperformed by the jump diffusion setup.

Somewhat surprisingly the SVCJ model does not consistently outperform SVJ. One reason may be that the SVJ model is not exactly nested in the SVCJ approach, so that one cannot obtain the SVJ model by meaningfully restricting certain parameters in the SVCJ model. The latter model is somewhat more flexible, but at the same time imposes the strong requirement that every price jump has to be accompanied by a variance jump. This may make it difficult for the SVCJ model to outperform the SVJ approach, since it is not clear that indeed price and variance jumps occur in a strictly simultaneous fashion.

3.3 When (and why) does the Index Jump?

3.3.1 Simulation Results

We conduct a simulation study to a) demonstrate that our estimation method is able to correctly identify jump days in the empirical data and b) determine the appropriate threshold for the posterior jump probability used to separate jump from non-jump days.

For our simulation study we simulate 1000 paths based on the SVJ and SVCJ models. We generate two simulation results for each model. First, we use the estimated parameters for the index and second, we use the average parameter estimates from the single stock estimation. This ensures that we understand how sensitive the estimation procedure of posterior jump probabilities is relative to different model choices and different parameter values. We apply the Euler discretization scheme given in Equation (3) to simulate 4000 days per path using parameter estimates for the index and average single stocks as given in Tables 2 and 3.

Our simulation study confirms in particular two results that have been documented in prior papers. First, Eraker et al. (2003b) show that the estimation method is able to identify the model parameters. Second, Jones (2003a) shows that the Euler discretization at a daily frequency is a good approximation for the continuous time setup. A very interesting and novel result (to the best of our knowledge) is that we show how well the estimation setup is able to identify jump days.

The results of our simulation study can be found in Table 5. Panel A and B show the results for the SVJ and SVCJ model, respectively. On the left side we show posterior jump probabilities using the parameter values for the index and on the right side we show the results for the single stocks. We see a clear difference in the results for the posterior jump probabilities on jump and non-jump days in all four cases. The most important lesson we learn from the table is that the posterior jump probabilities for a jump day range from 0 to 100%, whereas on non-jump days for both models combined we observe an upper bound for the probability of around 10% for the index and 25% for the single stocks. These results clearly show that we are able to rule out false positives (identifying a jump day when there is none) when identifying jump days when we set the threshold for the posterior jump probabilities to some value higher than 10% for the index and 25% for the single stocks. However, we are unable to rule out false negatives (not identifying a jump day when there is one) when identifying jump days.

Based on these results we could use a threshold of 10% for the posterior jump probability for the index to statistically significantly differentiate between non-jump and jump days. To be more conservative and minimize the probability of identifying false positives we set this threshold to 15%.

3.3.2 Jump Day Analysis

The issue at the core of our analysis is to study index jumps in detail. In this section we will discuss three main results: a) jumps are identified relative to the level of return variance b) we identify to distinct types of index jumps that we coin 'correlation jumps' and 'macro jumps' c) we show that the two jump types are driven by different economical environments.

The first step in this exercise is to identify the days when the index jumps. Given our simulation results, we chose to select those days as jump days where the posterior probability for a jump was greater than 10% for the SVJ model and greater than 15% for the SVCJ model. We note that these thresholds are very conservative in the sense that the probability of identifying false jump days is virtually zero. This will become important later on when we identify the two different types of index jumps.

For the SVCJ model we identify 31 jump days in our sample period, whereas for the SVJ model we observe 28 index jump days (see Tables 6 and 7). Not surprisingly jump days exhibit mostly large negative index returns; we observe only two jump days with a positive return for the SVJ and one day in the case of the SVCJ model in our sample. In the following we focus the discussion on the results of the SVJ, since a) the SVJ model outperforms the SVCJ model more often than the other way around and b) the results we are interested in are virtually the same for SVJ and SVCJ model.

One first surprising result from Table 6 is that we do not observe an increase in jump events during the financial crisis. This conclusion can be drawn from the fact that Table 6 shows only two days in 2008 where the index jumps. A value that is comparable to the number of jumps found in other years. However, when inspecting a list of the 20 largest index movements in Table 8 we see that 13 of the largest index movements actually took place in the year 2008. We would expect such an unusual number of large daily movements to trigger a large number of jump days. The SVJ model identifies two jump days for the year 2008 at the 10% threshold used as seen in Table 6, whereas the SVCJ model identifies three jump days in 2008.

Table 8 provides intuition on why we do not see a large number of jump occurrences coinciding with the unusually large index returns. We limit the following discussion to the columns 3-5 of the table, which shows the results for the SVJ model, since we obtain similar results for the SVCJ model. As can be seen from the Table 8, most days with large movements and a low posterior jump probability also have a large value for the variance. For example, the movements of about -6% on October 7 and November 19 of 2008 are within the bounds induced by two standard deviations of the volatility value for these days. This means that a pure stochastic volatility model would be able to generate these movements. This explanation is not applicable to all our observations. For example in Table 8 we observe large positive movements on October 13 and 28 of 2008. On both days the index gained more that 10%, a movement outside of the bounds induced by the diffusive component, but no jump was identified on both days. The reason for this can be found in the fact that we are not able to rule out false negatives when identifying jump days. As discussed, our estimation strategy is able to rule out false positives, it is therefore the case that days where the return observation is outside the diffusive bounds and are not identified as jumps can be considered as false negatives.

The main takeaway from Table 8 is that given our model framework jump days are not identified only by large index movements. The index movement has to be seen relative to the underlying value of the variance process. If the variance for a particular day is large the diffusive component of the return process can generate a high index return. As shown in both tables, the diffusive component is even able to capture a number of the 20 highest returns in absolute value in our sample. However, if the variance is small a relatively modest index movement can be identified as a jump event since we would need a jump component on top of the diffusive part to generate such an observation.

The next question arising naturally is how individual stocks behave on these index jump days. A prior would certainly be that many individual stocks also exhibit jumps on the same days the index jumps. We identify jump events for single stocks by setting the posterior jump probability to 25%, as indicated by our simulation study with the parameter values set to the average value for the stocks. Unexpectedly, our results show that we find days on which the index jumps but only a very small number of stocks jump also. For example, in Table 6 we see that on January 4th 2000 only one stock jumps on the day where the index exhibits a posterior jump probability of over 50%, which constitutes a very high value as demonstrated by our simulation exercise. On the other hand we observe 75 stocks jumping on October 13th 1989 when we have a posterior jump probability of 100% for the index.

To facilitate understanding of such different jump patterns we list the 5 jump days with the largest and lowest number of stocks jumping for the SVJ model in Table 9. In Panel A, we observe days where only one to five stocks jump despite the fact that the posterior jump probability for the index is higher than 10% for all five days. What does this table imply in terms of how we interpret the occurrence of an index jump for these days? Obviously, given the small number of stocks jumping simultaneously with the index, there is no additivity in the strict sense that an index jump is the sum of jumps in individual stock prices. On the other hand, the index return *has to be* equal to the weighted sum of individual stock returns.

The hypothesis therefore is that index jumps are generated to a very large extent by diffusive price movements in the individual stocks, which happen to move mostly in the same direction.

This reasoning can be verified by the results in Table 10, which show that in all of the five days in Panel A the overwhelming majority of stocks exhibits a returns on index jump days that moves into the same direction. So why is it that the individual stocks do not also jump on these days? A look at the average conditional variance of stock returns around index jump days confirms the intuition that individual stock return volatility is much higher than index volatility, so that it is more likely for stocks to have large returns in absolute value generated by just the diffusive component of the stochastic process. On 'normal' days diversification would result in an index return which is small enough in absolute value not to be considered an index jump. However, when the correlation between the individual stocks goes up significantly, the large stock returns will basically all have the same sign (here: negative), destroying the diversification effect and resulting in a rather large and negative overall index return. Conditional on the index parameters such large movement constitutes a nearly impossible event and therefore cannot be explained by the diffusive component of the index alone. Hence this large negative return on the index then 'has to be' identified by the model as a jump.

However, correlation does not give the full picture yet. To find out if *all* the index jumps in our sample are likely to be the result of a correlation moving (or jumping) towards one, we analyze the jump probabilities of the different industries our sample firms belong to.[4]

[4]The companies are divided in industries according to the first two digits of the SIC codes that can be found at siccode.com. The sector that we label *Primary* groups all the companies belonging to the SIC codes corresponding to Agriculture, Forestry, Fishing and Mining. In the sector labeled *Manufacturing* we group companies that have SIC codes corresponding to Manufacturing and Construction. The thirds sector that we label *Transport* corresponds to the companies related to Transport and Public utilities. The sector that we label as *Trade* is comprised of the companies under the SIC codes of Wholesale Trade and Retail Trade. The sector we call *Finance* is constituted by the companies that relate to the siccode category called Finance, Insurance and Real Estate. We label the second last sector *Services* as on the SIC webpage. For sake of convenience we call the last sector *PA*, which is the short version of Public administration as on the SIC webpage.

Again, for the sake of brevity we concentrate our following analysis on the results found for the SVJ model and the threshold for the jumps in the single stocks is set to 25% posterior jump probability. Panel B of Table 9 shows the five days with the largest number of index constituents jumping when the index jumps. The table also contains information on the average posterior jump probabilities across industry sectors. Here we see a clear difference between the index jump days listed in both panels. For Panel B we observe an average jump probability across the sectors that are significantly higher than the ones observed for the days in Panel A. We see that the index jump is accompanied by jumps in constituents of all sectors on these days. This is a clear distinction to the occurrences of the days listed in Panel A, where we have significant average jump probabilities of at most one sector on these days. The important conclusion we draw from this difference is that the index jumps listed in Panel A are generated from diffusive movements in the stocks that go largely into the same direction. We term these jumps 'correlation jumps'. Whereas the jumps in Panel B are generated by jump movements in the constituents across all sectors. We term these jump events 'macro-driven jumps'.

At this point it is instructive to shortly come back to our discussion on the identification of jump days. Since our estimation method rules out false positives with a probability close to one, we are very confident that the 'correlation jumps' we identify are indeed jump events induced by the models. This is important, since one of the central contributions of our analysis is the identification of two economically distinct jump events, and we have to make sure that the identification of these jump days does not come merely from a false rejection of an hypothesis.

To analyze our distinction of the differences in the jump days in more detail we turn to Figure 3, which shows the sector jump probabilities for the five days with the largest number of stocks jumping in the top panel and the five days with the lowest number of stocks jumping in the bottom panel. We find big differences in the overall level of sector jump probabilities between both panels. It can easily be seen that the sector averages in the upper panel of Figure 3 are much higher than the averages in the lower panel. We can also see that for every day in the upper panel at least 6 of the seven sectors exhibit a posterior jump probability above 10% (see also Table 9). This implies that the jump in the index on the days depicted in the upper panel of Figure 3 is accompanied by jumps in single stocks of at least 6 out of seven industry sectors. This stands in clear contrast to the jump days depicted in the lower panel of the same figure. Here at most one sector has an average jump probability larger than 10%, implying that the index movement is not accompanied by unusually large movements in the sectors.

It is interesting to note that a quick Google search for the days depicted in the upper panel of Figure 3 gives back stories of stock market crashes as first or second hit. For example, the jumps on October 13, 1989 and October 27, 1997 can be linked to crucial market wide events. The first event was termed a "mini crash" relating to a drop in prices in the junk bond markets, whereas the second relates to an economic crisis in Asia (sometimes called the 'Asian flu'). These two days also represent the two largest percentage drops in the index for our sample. Also, stock market reports for February 27, 2007, frequently mention a large drop in the Chinese stock market as an important reason for the big loss in the S&P 100 on that day, so the reasons for this jump are similar to the ones described above. In summary, since many industries jump on these days, and since the reasons can be traced back to macroeconomic events, again this is the reason why we refer to these jumps as 'macro-driven jumps' jumps?

On the other hand, the jump days depicted in the lower panel of Figure 3 are characterized by very small sector jump probabilities, so the fact that we identify an index jump at all is more due to the statistical properties of the stochastic process for stock prices, which tends to label a price move as a jump mostly when it is too large to be generated with a sufficiently high probability by the diffusive component. Again, we therefore refer to these jumps as 'correlation jumps', driven mostly by the correlation effect described above.

4 Conclusion

This paper analyzed stochastic equity price models for a large cross section of individual stocks, i.e. 201 stocks that were included in the S&P 100 index at some time in the period we analyze. Furthermore, we compared the estimation results for the individual stocks to the results for the S&P 100 index. The models under consideration allowed for a jump component in the return as well as in the variance process. The estimation method employs a Bayesian econometric framework using a MCMC algorithm to compute the moments of the posterior distribution. A first novel result of our study is that we show via an simulation study that our estimation is able to cleanly identify jump days versus non-jump days.

Our results show that the parameters governing the stochastic processes for individual stocks are very heterogeneous. Unsurprisingly, the volatility process for individual stocks is much higher than the volatility process for the index. Furthermore, we find a less pronounced leverage effect in the individual stocks compared to the index. Considering the distribution of the jumps in returns we can show that the stylized fact of negative average jump size found in the analysis of indices does not carry over to individual stocks; some stock exhibit negative mean jump sizes where others have positive mean jump sizes.

Considering the behavior of the individual stocks from which the index is constructed on index jump days we find a rather surprising result. We identify two types of index jumps. The first jump type arises due to many stocks exhibiting a diffusive movement in the same direction (usually negative return movement) and we therefore call those jumps 'correlation jumps'. The second index jump type is generated by jumps in a large number of stocks across all sectors of the economy which is why we name those jumps 'macro-drive jumps'. Furthermore, we surprisingly find that the models under consideration do not identify an unusually large number of jumps during 2008, i.e. the time of the financial crisis. Intuitively this can be explain by the prolonged high levels of volatility during that time which implies that large index returns can be generated by a pure stochastic volatility model.

Table 1: Parameter Estimates (SV Model)

	Index		Individual Stocks			
	mean	std. err.	average	std. dev.	2.5%	97.5%
μ	0.0305	0.0087	0.0281	0.0382	−0.0365	0.0930
α_0	0.0264	0.0027	0.1376	0.1179	0.0367	0.3759
α_1	−0.0216	0.0026	−0.0299	0.0225	−0.0941	−0.0061
σ_v	0.1791	0.0082	0.3722	0.1318	0.2043	0.6507
ρ	−0.5899	0.0322	−0.2673	0.1095	−0.5127	−0.0574

NOTE: The table shows the estimated parameters for the index (posterior mean and standard error) and cross-sectional summary statistics for the posterior means of the parameter estimates for the individual stocks. An overview of the stocks included in our sample and the respective estimation period can be found in Table A1 of the online appendix.

Table 2: Parameter Estimates (SVJ Model)

	Index		Individual Stocks			
	mean	std. err.	average	std. dev.	2.5%	97.5%
μ	0.0325	0.0087	0.0175	0.0596	-0.0803	0.0829
α_0	0.0219	0.0025	0.0657	0.0936	0.0097	0.1699
α_1	-0.0181	0.0023	-0.0180	0.0281	-0.0500	-0.0016
σ_v	0.1625	0.0081	0.2322	0.0861	0.1322	0.3577
ρ	-0.6418	0.0297	-0.3476	0.1256	-0.6194	-0.0650
μ_Y	-2.3672	0.8727	0.7901	1.0811	-1.1148	3.3742
σ_Y	2.7602	0.6041	5.3834	3.6092	1.7476	14.3364
λ	0.0051	0.0021	0.0360	0.0431	0.0057	0.1026

NOTE: The table shows the estimated parameters for the index (posterior mean and standard error) and cross-sectional summary statistics for the posterior means of the parameter estimates for the individual stocks. An overview of the stocks included in our sample and the respective estimation period can be found in Table A1 of the online appendix.

Table 3: Parameter Estimates (SVCJ Model)

	Index		Stocks			
	Mean	S. E.	Mean	S. E.	2.5%	97.5%
μ	0.0365	0.0087	0.0238	0.0457	-0.0736	0.0837
α_0	0.0215	0.0025	0.0605	0.1032	0.0059	0.2540
α_1	-0.0266	0.0030	-0.0259	0.0344	-0.1321	-0.0043
σ_v	0.1450	0.0084	0.2113	0.0904	0.1228	0.3432
ρ	-0.6597	0.0333	-0.3763	0.1448	-0.6310	-0.0878
μ_Y	-2.4269	0.5475	0.6106	1.0363	-1.4616	2.9216
σ_Y	2.1621	0.4693	5.7855	3.3338	2.1613	12.7180
λ	0.0057	0.0015	0.0267	0.0183	0.0045	0.0753
μ_V	1.8524	0.4195	1.8190	3.7166	0.5047	9.0287
ρ_J	-0.0010	0.0164	-0.0005	0.0111	-0.0134	0.0239

NOTE: The table shows the estimated parameters for the index (posterior mean and standard error) and cross-sectional summary statistics for the posterior means of the parameter estimates for the individual stocks. An overview of the stocks included in our sample and the respective estimation period can be found in Table A1 of the online appendix.

Table 4: Model Comparison via Bayes Factors

Evidence is ...	**SVJ vs. SV**	**SVCJ vs. SV**	**SVCJ vs. SVJ**	**SVJ vs. SVCJ**
... positive	0	1	9	30
... strong	0	0	0	15
... very strong	198	191	0	18

NOTE: This table shows the Bayes factors in a comparison of nested model specifications. The first two columns show the results for comparing the SVJ and the SVCJ models respectively with the SV model specification. The third column shows the results for comparing the SVJ and the SVCJ models. A positive value for the Bayes factor shows a preference for the model mentioned first in the column heading. We follow Kass and Raftery (1995) and interpret evidence against a model is positive if the log Odds Ratio is in between 2 and 6, strong if it is in between 6 and 10, and very strong if it is larger than 10. An overview of the stocks included in our sample and the respective estimation period can be found in Table A1 of the online appendix.

Table 5: Posterior Jump Probability on Jump vs. Non-jump Days (SVCJ Model)

Panel A - SVJ Model

	Index		Single Stocks	
	Non-jump days	Jump days	Non-jump days	Jump days
Mean	0.0060	0.3596	0.0311	0.3468
Median	0.0030	0.0658	0.0194	0.1094
Q_{99}	0.0543	1.0000	0.2465	1.0000
Q_{95}	0.0164	1.0000	0.0847	0.9999
Q_{05}	0.0006	0.0014	0.0087	0.0127
Q_{01}	0.0003	0.0006	0.0061	0.0086

Panel B - SVCJ Model

	Index		Single Stocks	
	Non-jump days	Jump days	Non-jump days	Jump days
Mean	0.0078	0.3230	0.0266	0.3126
Median	0.0024	0.1183	0.0155	0.0931
Q_{99}	0.0912	1.0000	0.2208	1.0000
Q_{95}	0.0268	0.9993	0.0782	0.9999
Q_{05}	0.0002	0.0026	0.0036	0.0099
Q_{01}	0.0000	0.0008	0.0017	0.0051

NOTE: The table shows descriptive statistics for the posterior jump probability estimated by the MCMC algorithm for the SVJ and the SVCJ model for non-jump and jump days. We perform a simulation study where 1,000 price paths are simulated based on the estimated parameters for the S&P 100 index. In columen 2 and 3 we report results based on the parameters estimated for the Index whereas in column 4 and 5 we report results for the parameters based on the single stocks. A day is labeled a jump day in the simulation when a the draw from the uniform distribution over the interval $[0, 1]$ is less than or equal to $\lambda\Delta t$, where λ is the estimated intensity for the Poisson process driving jumps in the S&P 100 index (see Table 3), and Δt is equal to 1 day.

Table 6: Index Jump Days (SVJ Model)

Date	Return	Jump Prob.	# Jum.	Prim.	Man.	Transp.	Trade	Fin.	Serv.	PA
13/10/1989	-6.53	100.00	75	78.19	69.96	65.38	50.80	69.29	65.39	89.09
16/10/1989	3.20	24.12	26	60.04	18.32	28.81	32.10	17.96	2.83	44.13
22/01/1990	-3.00	21.97	6	10.24	7.71	9.62	5.27	7.37	3.08	23.00
15/11/1991	-4.32	99.78	29	7.97	24.37	24.87	29.69	22.42	29.07	33.45
16/02/1993	-2.29	28.47	10	6.27	8.94	6.03	16.48	7.13	8.60	13.41
04/02/1994	-2.36	93.75	12	6.80	9.09	10.90	3.31	7.85	23.88	30.96
24/02/1994	-1.38	14.29	1	9.07	2.58	3.13	3.57	3.77	1.29	13.12
08/03/1996	-3.17	65.52	11	6.71	9.79	7.43	4.35	27.98	3.60	27.10
27/10/1997	-7.09	94.03	63	71.46	49.35	45.94	51.36	37.56	47.30	14.24
28/10/1997	5.61	18.23	29	16.84	22.81	18.07	35.16	19.26	38.05	39.43
31/08/1998	-7.52	53.34	22	11.11	18.14	15.31	23.92	6.86	29.41	11.51
20/07/1999	-2.50	10.09	2	7.03	3.05	2.29	3.79	5.85	11.87	2.96
04/01/2000	-3.85	50.79	1	4.10	3.17	5.46	2.91	6.96	3.39	5.10
14/04/2000	-6.01	30.55	7	14.49	7.72	6.43	5.99	12.89	13.03	2.83
17/09/2001	-5.42	30.44	31	4.56	31.91	21.14	30.17	22.66	16.27	39.38
24/03/2003	-3.73	11.50	5	4.10	7.64	6.52	10.11	9.69	12.31	11.02
20/01/2006	-1.91	23.73	6	24.53	4.31	4.20	2.96	12.66	14.68	97.02
25/01/2007	-1.29	23.48	5	4.32	3.27	3.69	16.56	6.20	7.01	2.20
27/02/2007	-3.63	99.42	53	18.25	35.67	43.28	39.44	33.27	20.59	10.26
06/06/2008	-3.16	12.72	7	2.97	9.43	8.48	5.31	4.41	4.21	6.96
01/12/2008	-8.94	10.45	2	11.81	8.43	5.60	5.35	5.33	5.98	3.41
04/02/2010	-3.04	26.10	10	16.58	11.30	9.56	11.35	15.35	6.44	18.53
11/08/2010	-2.69	15.21	8	8.13	12.52	9.55	4.10	12.39	2.09	8.11
08/08/2011	-6.44	14.16	24	26.88	15.89	28.32	28.50	21.86	11.41	5.31
07/11/2012	-2.63	19.25	14	19.22	7.61	14.13	2.10	38.26	7.69	4.28
20/06/2013	-2.46	13.64	13	12.27	11.22	15.97	16.80	4.55	11.08	12.74
10/04/2014	-2.04	13.11	8	9.83	8.06	6.55	6.89	14.47	10.18	2.77
31/07/2014	-2.07	29.15	12	5.52	14.08	16.79	6.73	14.41	5.42	4.35

NOTE: The table reports the returns and the jump probability for the S&P 100 index on the days where the SVJ model identifies a jump. Furthermore, the table shows the number of stocks that exhibit a jump probability of more than 25% on the days the index jumps and the average posterior jump probabilities for the sectors under consideration.

Table 7: Index Jump Days (SVCJ Model)

Date	Return	Jump Prob.	# Jum.	Prim.	Man.	Transp.	Trade	Fin.	Serv.	PA
13/10/1989	-6.53	100.00	72	78.97	70.10	58.48	46.57	71.85	61.29	90.97
15/11/1991	-4.32	99.85	29	6.29	24.21	22.72	31.04	20.17	32.91	24.50
16/02/1993	-2.29	26.11	9	3.28	7.62	4.10	17.09	7.52	7.92	1.44
04/02/1994	-2.36	80.83	6	2.50	5.88	9.07	2.51	4.77	23.98	8.94
18/05/1995	-1.65	25.63	2	4.00	4.32	2.12	2.87	1.23	5.86	0.30
08/03/1996	-3.17	48.69	8	6.88	8.52	4.16	2.73	19.41	5.26	13.71
05/07/1996	-2.32	15.78	6	6.46	3.18	10.85	2.74	10.41	5.25	2.78
23/06/1997	-2.27	18.07	4	2.42	5.91	2.16	4.12	6.51	5.78	8.46
27/10/1997	-7.09	87.93	69	71.11	51.26	46.91	53.72	40.87	52.79	28.92
27/08/1998	-3.97	64.72	9	20.19	10.07	3.33	18.09	9.31	7.49	27.24
31/08/1998	-7.52	42.49	25	13.67	22.37	17.82	31.45	9.66	35.40	30.60
30/11/1998	-2.33	18.52	1	8.16	2.87	5.18	2.97	2.34	3.52	0.63
04/01/2000	-3.85	87.16	3	2.68	4.10	9.33	3.99	2.38	6.88	3.18
12/04/2000	-2.96	35.89	1	4.54	5.04	3.36	4.06	1.37	3.63	1.48
14/04/2000	-6.01	38.04	6	15.11	7.81	6.74	5.50	5.91	13.87	0.70
09/03/2001	-2.81	30.28	0	2.69	2.57	2.89	1.28	1.55	3.75	10.58
12/03/2001	-5.17	30.83	5	2.49	5.28	6.72	6.18	5.75	5.00	37.36
17/09/2001	-5.42	39.77	35	7.70	35.89	26.77	34.21	25.73	18.27	84.21
29/01/2002	-3.11	15.24	5	1.82	3.86	6.56	1.81	16.76	5.82	5.40
18/07/2002	-2.77	15.17	2	8.85	6.03	8.33	6.01	3.99	4.70	3.29
19/07/2002	-4.21	18.57	9	19.66	14.53	5.96	6.72	1.68	3.49	2.70
27/02/2007	-3.63	98.87	37	10.93	28.95	38.93	41.21	19.18	15.44	0.51
19/10/2007	-2.54	28.88	5	56.69	7.80	3.25	3.29	3.39	5.13	0.50
04/09/2008	-3.08	27.27	2	7.51	7.62	5.68	4.51	4.07	7.89	8.31
15/09/2008	-5.02	67.12	7	14.31	8.45	12.95	3.78	10.96	6.15	18.63
29/09/2008	-9.19	25.76	15	21.89	17.68	17.61	15.41	12.06	31.81	6.27
23/04/2010	0.60	20.85	2	15.30	4.91	3.91	4.98	2.88	1.69	2.78
27/04/2010	-2.21	32.87	10	6.29	9.98	12.72	9.23	6.76	2.23	28.83
02/08/2011	-2.38	24.83	5	6.90	7.73	8.16	6.25	7.76	4.27	21.40
04/08/2011	-4.64	59.98	27	43.86	22.16	17.99	15.18	7.45	20.77	14.61
08/08/2011	-6.44	15.62	33	29.56	18.57	24.01	37.01	32.72	18.98	9.33

NOTE: The table reports the returns and the jump probability for the S&P 100 index on the days where the SVCJ model identifies a jump. Furthermore, the table shows the number of stocks that exhibit a jump probability of more than 25% on the days the index jumps and the average posterior jump probabilities for the sectors under consideration.

Table 8: Posterior Jump Probabilities and Diffusive Bounds

		SVJ			SVCJ		
Date	Return	Prob.	UB	LB	Prob.	UB	LB
13/10/1989	-6.5273	1.0000	1.7596	-1.6947	1.0000	1.3308	-1.2577
27/10/1997	-7.0927	0.9403	2.9335	-2.8686	0.8793	2.7582	-2.6851
31/08/1998	-7.5165	0.5334	3.8614	-3.7965	0.4249	4.3498	-4.2767
14/04/2000	-6.0088	0.3055	3.5328	-3.4678	0.3804	3.7395	-3.6664
29/09/2008	-9.1862	0.0649	5.5909	-5.5259	0.2576	7.2023	-7.1292
07/10/2008	-5.9769	0.0087	6.2020	-6.1370	0.0856	7.9580	-7.8850
09/10/2008	-7.9154	0.0156	6.5012	-6.4362	0.1447	8.1528	-8.0797
13/10/2008	10.6551	0.0158	6.8262	-6.7613	0.0073	8.4501	-8.3771
15/10/2008	-8.7550	0.0291	6.5080	-6.4431	0.0428	8.0643	-7.9912
22/10/2008	-5.9697	0.0103	6.5767	-6.5117	0.0140	7.8162	-7.7432
28/10/2008	10.2961	0.0098	6.8499	-6.7850	0.0079	7.7964	-7.7233
13/11/2008	6.3306	0.0039	6.5990	-6.5340	0.0134	7.0829	-7.0098
19/11/2008	-6.0350	0.0126	6.4398	-6.3749	0.0257	6.8154	-6.7423
20/11/2008	-6.6214	0.0160	6.6009	-6.5360	0.0248	6.9304	-6.8574
21/11/2008	6.0205	0.0034	6.7677	-6.7027	0.0023	7.0510	-6.9779
24/11/2008	5.8915	0.0036	6.5296	-6.4646	0.0021	6.8025	-6.7294
01/12/2008	-8.9448	0.1045	5.9946	-5.9296	0.0637	6.2029	-6.1298
10/03/2009	6.0499	0.0056	5.3532	-5.2882	0.0079	5.4093	-5.3362
23/03/2009	6.7381	0.0156	4.6809	-4.6159	0.0052	4.7052	-4.6321
08/08/2011	-6.4430	0.1416	3.8892	-3.8242	0.1562	5.0169	-4.9438

NOTE: This table shows the 20 largest daily returns observed for the S&P 100 index. We show the return, the posterior jump probability of the Index, the upper and lower bound for the diffusive part of the model computed as $\mu \pm 2\sqrt{V_{t-1}}$ for both jump models.

Table 9: Jump Days with Largest and Smallest Number of Stocks Jumping (SVJ Model)

Date	Return	Jump Prob.	# Jum. Stocks	Prim.	Man.	Transp.	Trade	Fin.	Serv.	PA
Panel A: Jump days with lowest number of stocks jumping										
25/01/2007	-1.29	23.48	5	4.32	3.27	3.69	16.56	6.20	7.01	2.20
20/07/1999	-2.50	10.09	2	7.03	3.05	2.29	3.79	5.85	11.87	2.96
01/12/2008	-8.94	10.45	2	11.81	8.43	5.60	5.35	5.33	5.98	3.41
24/02/1994	-1.38	14.29	1	9.07	2.58	3.13	3.57	3.77	1.29	13.12
04/01/2000	-3.85	50.79	1	4.10	3.17	5.46	2.91	6.96	3.39	5.10
Panel B: Jump days with largest number of stocks jumping										
13/10/1989	-6.53	100.00	75	78.19	69.96	65.38	50.80	69.29	65.39	89.09
27/10/1997	-7.09	94.03	63	71.46	49.35	45.94	51.36	37.56	47.30	14.24
27/02/2007	-3.63	99.42	53	18.25	35.67	43.28	39.44	33.27	20.59	10.26
17/09/2001	-5.42	30.44	31	4.56	31.91	21.14	30.17	22.66	16.27	39.38
15/11/1991	-4.32	99.78	29	7.97	24.37	24.87	29.69	22.42	29.07	33.45

NOTE: The table reports the returns and the jump probability for the S&P 100 index for the 5 days with the largest and smallest number of single stocks jumping where the SVJ model identifies a jump for the index. Furthermore, the table shows the number of stocks that exhibit a jump probability of more than 25% on the days the index jumps and the average posterior jump probabilities for the sectors under consideration.

Table 10: Signs of Individual Stock Returns on and around Index Jump Days (SVJ Model)

Date	$\#R > 0$	$\%R > 0$	$\#R < 0$	$\%R < 0$
Panel A: Lowest number of stocks jumping				
25/01/2007	12.00	0.13	81.00	0.87
20/07/1999	19.00	0.20	75.00	0.80
01/12/2008	0.00	0.00	93.00	1.00
24/02/1994	13.00	0.14	77.00	0.86
04/01/2000	12.00	0.13	83.00	0.87
Panel B: Largest number of stocks jumping				
13/10/1989	0.00	0.00	92.00	1.00
27/10/1997	0.00	0.00	96.00	1.00
27/02/2007	0.00	0.00	93.00	1.00
17/09/2001	14.00	0.14	85.00	0.86
15/11/1991	2.00	0.02	90.00	0.98

NOTE: The table shows in Panel A the five days where an index jump is accompanied by the lowest number of single stock jumps and Panel B shows the five days where an index jump is accompanied by the highest number of single stock jumps. Column 2 and 4 show the number of positive and negative stock returns, respectively. Column 3 and 5 show the same information as percentages.

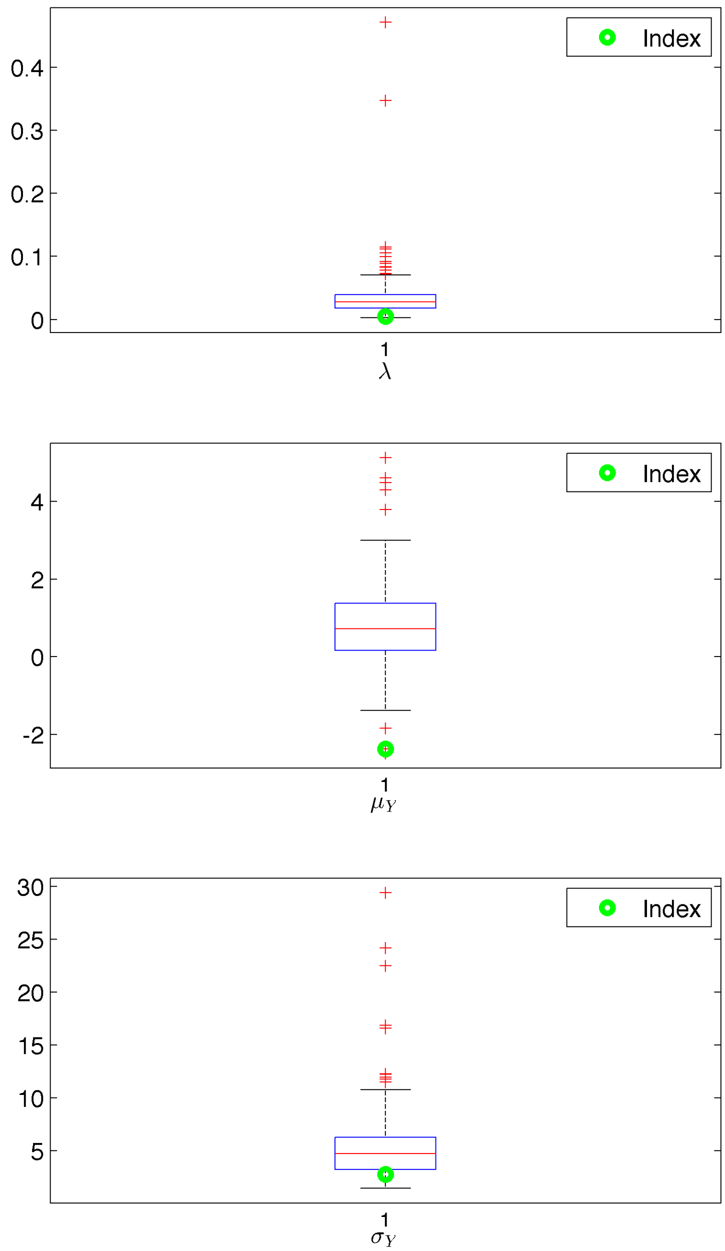


Figure 1: Box Plots for Parameters of SVJ Model

This figure shows the box plots for the parameters characterizing the SVJ model. The top box plot shows the estimates of jump intensity λ for the single stocks and the index in green. The middle box plot shows the estimates for the expected jump size in return μ_Y for the single stocks and the index. The bottom box plot shows the estimates for the standard deviation of the jump size in return σ_Y.

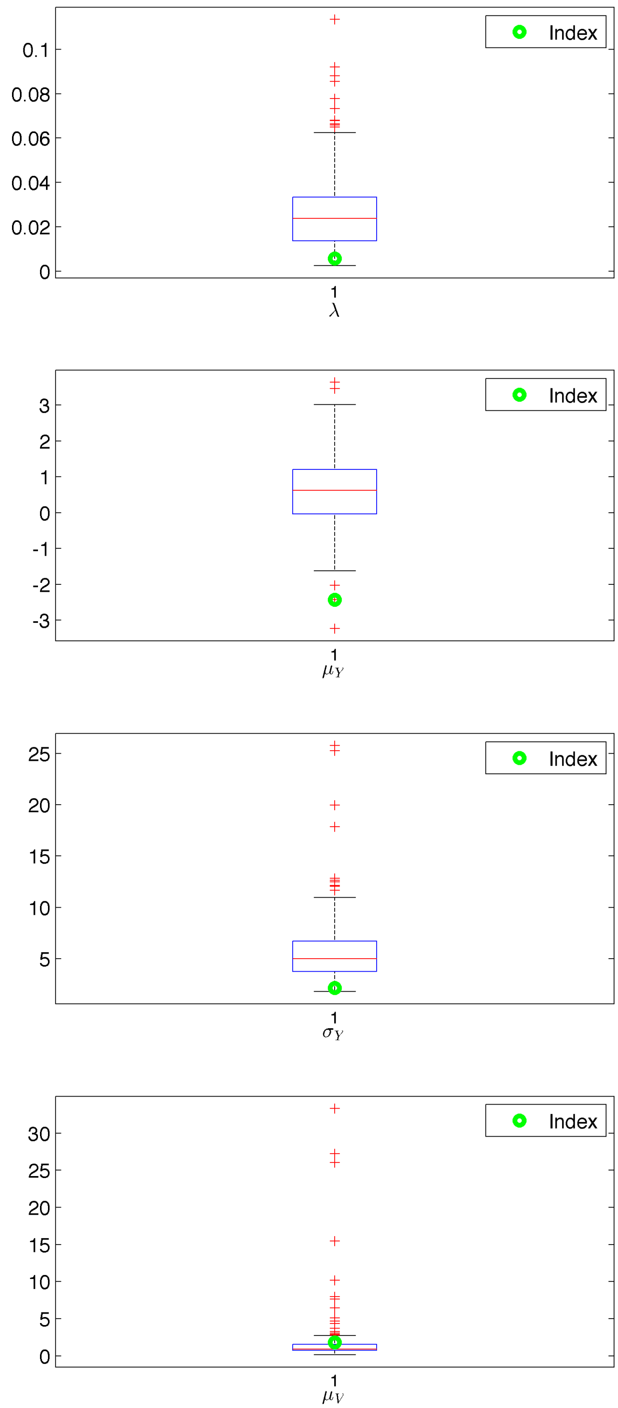
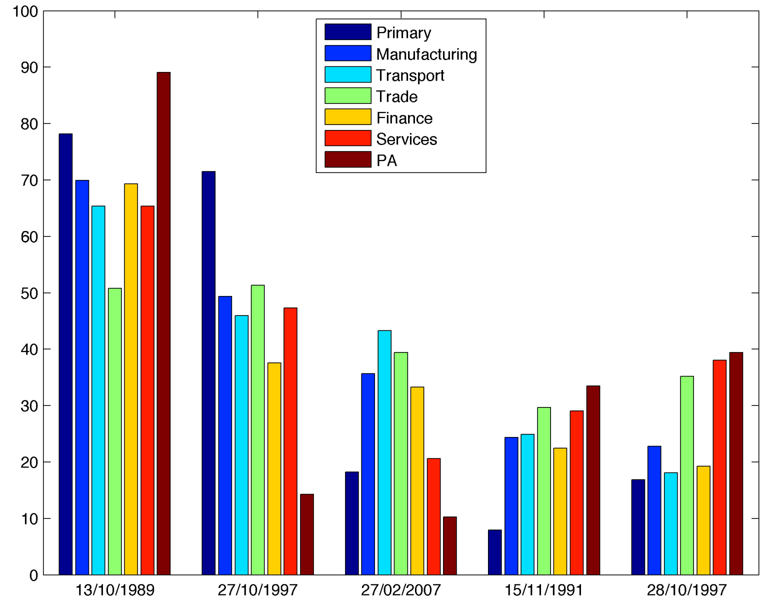


Figure 2: Box Plots for Parameters of SVCJ Model

This figure shows the box plots for the parameters characterizing the SVCJ model. The top box plot shows the estimates of jump intensity λ for the single stocks and the index in green. The second box plot shows the estimates for the expected jump size in return μ_Y for the single stocks and the index. The third box plot shows the estimates for the standard deviation of the jump size in return σ_Y. The bottom box plot shows the estimates for the expected jump sizes in variance μ_V.



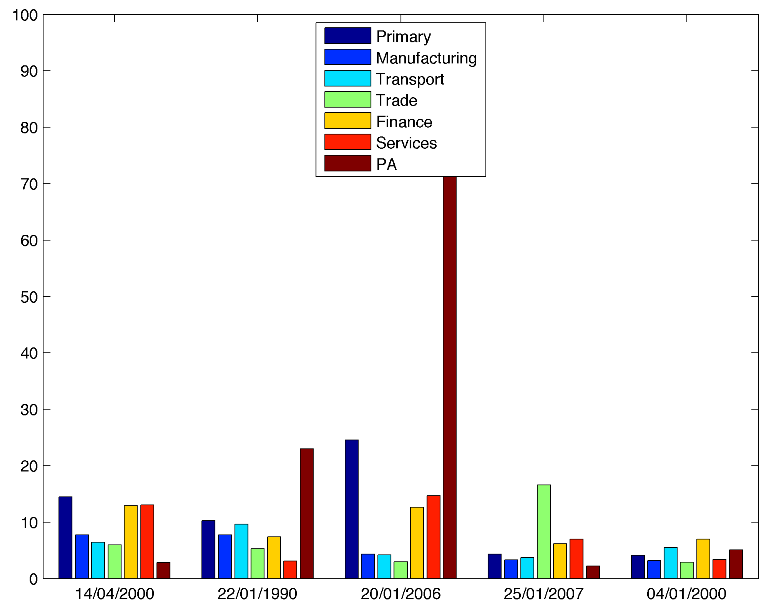


Figure 3: Industry Jump Probabilities on Index Jump Days (SVJ Model)

This figure shows the average posterior jump probabilities for the different sectors on the jump days identified by the SVJ model. The upper graph shows the five days where an index jump is accompanied by the highest number of single stock jumps and the lower graph shows the five days where an index jump is accompanied by the lowest number of single stock jumps.

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